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HOSPITAL COSTS AND RANDOM DEMAND

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Abstract

This research note's primary objective is to assert for the impact of random demand for emergencies in Portuguese hospital costs. In order to do so, three different estimation methods are applied: Pooled OLS, Fixed-effects and Stochastic Frontier Analysis. Some conclusions of this note point out that dispersion measures of demand for emergencies are not significant in explaining total costs for the preferred models. Moreover, following Battese and Corra (1977), 58% of the total variance of the disturbance is due to the inefficiency term. Finally, predicting Coelli's cost efficiency (1996), Portuguese hospitals have shown not to be far from the efficiency frontier.

I. Introduction

The Health Sector is one characterized by some peculiarities that differentiate it from other sectors. In that sense, when addressing any issue regarding health economics, one must have in mind that usual market and economic thinking may not verify.

One good example of this is the demand for health and health care, which is a well-known derived demand function, in other words, its amount depends directly on how much is supplied, being the supply determined by the health production function.

Furthermore, demand in the health sector may also be characterized as being partially stochastic. As a way to better to understand this let us analyze emergency hospital admissions, which usually accounts for a considerable portion of total demand for hospital health care. In fact, emergency admissions may suffer fluctuations over time, varying a lot from one period to another, a variation that is in part unpredictable (even in the presence of very good estimations), raising the following question: Does random demand for emergencies effects, or not, hospital costs?

This research question goes the following path. On Section II a literature review is presented, regarding notorious research papers on the topic of stochastic demand and forms of addressing it. Section III describes the data, briefly presenting the panel construction and the reasons behind the chosen variables. Afterwards, Section IV is concerned with the chosen methodology, mainly, with the stochastic frontier method, referring to annex the derivation of both Pooled OLS and Fixed Effects models. The empirical part of the paper is presented in Section V that starts by grasping each estimation method results individually and finishes by constructing the best model, according to Likelihood tests for every estimation process. Conclusion is made on Section VI, where a summary of procedures and results is shown, enabling a quick understanding of the paper.

II. Literature Review

Several research studies on the aforementioned matter were produced, specifically on how fluctuations in stochastic demand influence hospital costs. Friedman and Pauly (1981) were the first to establish such relationship, discussing the importance to take into account that for some firms the choice of inputs is made before knowing which demand will be faced in the future, in other words, in the presence of random demand. Moreover they were interested in correctly specifying the cost functions of firms facing such demand, always having in mind that a period with abnormally high demand could worsen the quality of the final output, by either reducing its values or even by lowering the utility of the firm's manager.

Their first step towards a consistent construction of a cost function for hospitals was to assume that each hospital sets its own level of quality. In the case of a demand burst in a given period of time, hospitals are ready to assume a temporary decrease of the level of quality, which may be translated as an implicit or unobserved penalty cost. Nonetheless, if the effect persists over time, the hospital will increase their inputs in order to attain their previous level of quality, since by then the cost of quality reduction becomes higher than the cost of adding more inputs.

Some empirical implications are to be taken from their models. Firstly, costs have shown to be highly sensitive to variations of the ratio expected output over the actual one; however they almost don’t change with respect to actual costs alone. Secondly they justify differences in their coefficients with differences in the size of hospitals, since this will consequently determine different levels of occupancy rates, average costs and case-mix composition; an important point to take into consideration in further studies. They also agree on the idea that the suppression of excess beds won’t translate in a huge increase of savings. Finally, a policy proposition is made, in the presence of prospective budgets implemented by government and other insurers (both unwilling to specify an admission level), proportional revenue allowances are to be implemented, as a way to increase gains or to respond to unexpected losses in short-run.

Later on, Gaynor and Anderson (1995) argue that uncertainty over demand enhances standby capacity of hospitals to reach a point that is considered to be excessive. The reason behind such behaviour is that hospitals look to avoid patients’ rejections when these are admitted into the hospital under emergency status.

Standard theory of cost and production implies that technical efficiency in the production process is achieved (production “frontier”), however in the presence of stochastic demand such assumption does not hold. Therefore, the authors tried to derive a cost function for hospitals that differs from the usual approach, from which they were able to conclude that uncertainty impacts hospital costs, and only by taking this into consideration, unbiased cost function parameters may be computed.

After estimating the cost of an empty bed, they found out that a one percent decrease in the number of beds has a small impact on decreasing costs (only one-third of one percent). Nonetheless, if the number of beds diminishes largely, occupancy rates increase, resulting on a huge costs decrease. In their example, going from a 65 percent occupancy rate to a 76 percent occupancy rate, will decrease costs for the average hospital by almost 9,5 percent of total costs.

Gaynor and Anderson conclude by stating one should merge these results with measures of social benefit of excess capacity in hospitals, so that optimal occupancy rates, and consequently the optimal number of excess beds, are to be correctly computed.

Deepening the previous relationship, Hughes and McGuire (2003) seek for responses from hospitals' production to demand uncertainty. They agree that an optimal level of reserve capacity is achieved given the existence of trade-off between costs of holding unused capacity available for unpredictable demand versus rejection of patients on emergency status due to operating at full capacity. Moreover, they also agree with Gaynor and Anderson (1995) when pointing out limitations to previous studies, such as, loss of information due to the use of aggregate measures.

As for fluctuations in demand, these were previously relying on annual or quarterly periods of time that smoothed the behaviour throughout the entire time-series. In response to this, a distinction between elective and emergency admissions is to be made, from which they established two important assumptions for their model: emergency services' demand is randomly distributed and follows a known probability density function, and, there is an observable excess demand for elective treatments.

From here they conclude not only that demand uncertainty impacts hospitals' costs, but also that by incurring in costs from holding standby capacity and being these much higher than costs with elective admissions, an increase in emergency cases will be of high concern in the budgetary perspective.

There is also another important point to take into consideration on the analysis of stochastic demand and hospital costs relationship. In one hand costs are expected to increase due to the existence of idle capacity that ensures emergency admissions, in the other hand by allocating more resources such as beds to emergency admissions, one is overcrowding elective admissions, originating the so called waiting-lists.

Siciliani, Stanciole and Jacobs (2009) address this particular issue with success. Waiting-lists are frequently seen as a way to effectively ration demand for health care (by reducing patients' benefits from asking treatment). Until a certain point, waiting times may reduce idle capacity in the presence of stochastic demand; however, higher waiting times may also increase costs, usually due to increasing costs in managing the waiting-list, since one may have repeated examinations and increase in treatment costs, length of stay and cancelation rates. This is why the effect of waiting time on hospital costs is a non-linear one, having a U-shaped relationship, meaning there is a level of waiting time which minimises total costs. Nevertheless, results show that waiting times do not impact significantly hospital costs, implying that patients loose but at the same time providers do not benefit from such situation.

III. Data

Data related with the Health Sector in Portugal is spread over several institutions. Not only this but data is gathered with different purposes, which leads to the creation of databases with different sets of time (annual, monthly, daily) in order to answer to different objectives.

Despite all their different characteristics, it may be interesting to make an evaluation that encloses information from different sources, since only by doing so one may induce a broader perspective on specific matters. In particular, when evaluating hospital costs and the impact hospitals' inputs and outputs have on its structure, one must gather different types of figures.

Therefore, three different data sets are used when addressing this research question: the BDEA (Base de Dados dos Elementos Analíticos), the Diagnosis Related Groups and a Mixed Panel-Data (built by resorting to several sources of information).

III.1 - BDEA (Base de Dados dos Elementos Analíticos)

ACSS (Administração Central do Sistema de Saúde) provides information regarding hospitals' accounting performances since 2002, from which we gather data regarding the dependent variable of this study, Total Costs.

We use only hospitals that face emergencies. Specialty hospitals, such as oncology, maternities or even psychiatry, are also excluded from the analysis due to their specificities[1]. Some hospitals fail in reporting their accounts in some of the years[2]. This leads to an unbalanced panel data, a feature aggravated once all data is gathered.

[1] Oncology Hospitals: IPO's Coimbra, Lisboa and Porto. Psychiatric Hospitals: Júlio de Matos, Lorvão, Magalhães Lemos, Miguel Bombarda, Sobral Cid. Maternitites: Alfredo da Costa, Júlio Dinis. Pediatric Hospitals: Maria Pia.

[2] 2003: Centro Hospitalar de Coimbra. 2004: Hospital de Águeda, Guarda e Viseu.

III.2 - Diagnosis Related Groups (DRG)

DRG's are a classification system for hospital inpatients grouped accordingly to similarities in their clinical profiles and consumption of resources.

Each group is accredited with a certain *relative weight*, where this reflects the expected cost of a particular procedure for the medium inpatient inserted in a given group, relative to the average cost for the medium national inpatient. From such weighting coefficient one is able to design the *case-mix index* that results from the ratio between the number of weighted equivalent inpatients (admission episodes classified in DRG are converted to *equivalent inpatients* by taking the length of stay of each particular case and the normalized interval defined for each DRG) and the total number of equivalent inpatients (Fetter 1980).

Mainly two reasons force the construction of an annual based dataset. Firstly, daily information presents a huge number of observations, which leads to heavy computation when applying statistical methods. Moreover one has that daily, weekly and monthly information may be gathered into annual data, while the reverse path is not possible.

Having these restrictions in mind one must compress this dataset, which is originally composed by a total of 4,292,378 million daily observations and 146 variables comprehended between the years of 2003 and 2006, into annual data.

Thus, two important questions arise: Will all this information be needed? How to deal with such huge amount of information?

While the answer to the first question is an unquestionable and straightforward "no", the last one requires a much more complex resolution. In order to address it one must answer another question: Which variables are to be used in our analysis? The response to such interrogation is that the relevant variables to be computed from the DGR dataset are the *annual average of the standard deviation of the number of emergencies* taking place in a given year and hospital (***sd***), and its *coefficient of variation* (***varco***).

After dropping irrelevant variables, one is left with 16 variables, each with a particular piece of information. These 16 variables enables the computation of our two final variables.

The creation of three different indicators will lead to annual observations. The first is an hospital identification variable across all three datasets (***id***).

The existence of hospital centers (two or more hospitals under the same management board) was taken into account, implying that several hospitals have seen their id correspond to the same of others.[3] These transformations lead, once again, to the creation of an unbalanced panel data.

The second indicator (***ind***) will assume the following format, YMMID, where Y stands for the year in question (Y = {3, 4, 5, 6} = {2003, 2004, 2005, 2006}), MM stands for the month in question (MM = {01, 02… 11, 12} = {Jan, Feb … Nov, Dec}) and ID stands for the indicator built in the first place – ***id***, implying that ***ind*** = {30101, 30102 … 61297, 61298}.

The third and final indicator (***ind2***) assumes a similar form when compared with the previous one, that is YID, where once again Y is the year and ID is the hospital ***id*** code, therefore ***ind2* =** {301, 302 … 697, 698}.

As it is easily observable the indicator ***ind*** will enable the data aggregation in monthly averages, from which one will take its annual standard deviation.

[3] Gathered hospitals: Hospital de Setúbal with Hospital Ort. Outão (for every year); Hospital de Beja with Hospital de Serpa (for 2005 and 2006); Hospital Bragança with Hospital Mirandela and with Hospital Macedo de Cavaleiros (for 2005 and 2006); Hospital Vila Real with Hospital Peso da Régua (for 2003); Hospital Portimão with Hospital Lagos (for 2005); Hospital Egas Moniz with Hospital Sª Cruz and with Hospital S. Francisco Xavier (for 2005 and 2006)

Taking a look to the variable “admission type”, different values arise, each of them corresponding to either programmed, urgent or other different types of admission in a given hospital. Thus, taking only the information related with urgent admissions one is apt to generate a dummy variable for emergencies where $\boldsymbol{emg_{adm\_type}(urgent) = 1}$ if $\boldsymbol{j = i}$ and $\boldsymbol{emg_{adm\_type}(\sim urgent) = 0}$ if $\boldsymbol{j \neq i}$.

Consequently, using the information captured by this dummy variable one is capable of counting the number of emergencies occurred in a given hospital in a given month of a given year, by resorting to our ***ind*** variable. Once this computation is completed it becomes easy to get the annual standard deviation of emergencies in a given hospital in a given year, since the variable ***ind2*** allows monthly data to be aggregated annually. These transformations result on a solid construction of the *annual average of the standard deviation of emergencies* taking place in a certain hospital – ***sd***.

A similar procedure to the one used to compute ***sd*** may be applied in order to calculate the *average annual number of emergencies* in a given hospital (***md***), which is of high importance in the construction of our final variable, the *coefficient of variation* $\left(\boldsymbol{varco = \frac{\sigma}{\mu} = \frac{sd}{md}}\right)$. The usefulness of such variable is the fact of being a dimensionless measure of the spread of the distribution of a random variable (Encyclopaedia of Mathematics, 1988), meaning it has the ability to compare data sets that are in the same unit of measurement but present widely different mean values for each individual (Sharma, 2007), which in case of hospitals is sure to take place due to differences in size, and therefore in admissions capacity.

III.3 – Mixed Panel Data

This Mixed Panel Data comes from Fortuna (2009), which has different sources for each set of variables. Built for the years between 2003 and 2006, it comprises outputs (production) and inputs (technology) recovered from DGS[4] and dummy variables constructed throw the access to different sources (e.g. hospitals reports, hospital websites, among others).

Despite the existence of important variables in this dataset in order to address our research question, one should still make reference to three important issues. Preferred to the *number of deaths*, the variable *rate of deaths* is adjusted to initial risk, as higher technological differentiation is translated into worst patients who will forcibly increase the *number of deaths*. However, in our dataset, its application means loss of information, in terms of observations. One must resort to the variable *number of deaths* as a proxy.

The computation of a gross death rate (dividing the *number of deaths* by the *number of discharges*) would not solve the problem, as by taking logarithms in the empirical implementation, it would only change the interpretation of the variable *number of discharges*.

The second issue is concerns the *average annual wage (**w**)*. It is a proxy for expenses with personnel, even though it is not possible to distinguish the average annual wage of a physician from that of a nurse, as this is an aggregated variable. Its formula is the quotient between *total costs with personnel* and the *total number of staff members working*, in a given hospital in a given year.

[4] Direcção Geral de Saúde – the general health directorate from the Ministry of Health.

A huge *average annual wage* dispersion is observed, ranging from 3,304.09€ to 41,233.54€, something that may be the result of payments to staff through service companies, in which case the costs are registered in a different account.

Following Jacobs, Smith and Street (2006) the functional form is expressed in the double log form.

Greene (1990) alerts for the impact that a single errant observation may have in the estimation of our last models, whose effect may dominate an entire set of observations, even if large. Thus one must take into account all observations dropped due to unrealistic values or even not accommodated outliers, meaning once all three datasets are gathered one is left with a sum of 67 different hospitals. Not every hospital is observed yearly between the period of 2003 and 2006, meaning that estimations are produced using an unbalanced dataset[5].

The complete dataset accounts for a total of 239 observations across 4 years, and a total of 26 key explanatory variables[6] and 1 dependent variable[7] (Annex.DataDescription). When doing so, one is avoiding three main issues in the computation of a stochastic frontier model (the latest to be constructed): non-convergence, no-concavity and no decreasing loglikelihood function.

Summarizing, the unrestricted model that will follow is built upon 26 independent variables, divided into eight different groups following the line of thought usually linked to cost functions. Since hospitals are expected to produce multiple outputs a first set of variables is chosen, being its quantities represented by the *number of deaths* and the *number of discharges*, both likely to impact *total costs* in an *a priori* analysis.

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[5] 45 hospitals with 4 observations; 11 hospitals with 3 observations; 10 hospitals with 2 observations and 1 hospital with 1 observation.

[6] In logarithms if not dummy variables

[7] ln(total costs)

Input prices are usually preferred to input quantities when computing a costs function, for a question of efficiency, nevertheless, the latter are much easier to gather since hospitals fail to report prices related with their intakes. Input prices are represented by *average annual wage*, while inputs are *number of emergencies*, *number of outpatients*, and *number of beds*.

Other four variables, *annual average of the standard deviation of emergencies* and *its coefficient of variation*, *occupancy rate* and *average length of stay*, are added to the models in order to assert their impact in *total costs*, being the first two the key variables in addressing our question. Furthermore, the *average length of stay* may be interpreted as an intermediary output, in the sense it works simultaneously as an input, the time a patient stays hospitalized directly affects his or her treatment, and as an output, in the sense it results from a choice of inputs and their allocation.

Finally a set of dummy variables are added depending on the estimation method, to allow for time-invariant effects that a group or all hospitals may face. Being time dummies (from 2003 to 2006), regional dummies (North, Center, Lisboa e Vale do Tejo, Alentejo and Algarve), district dummies (central hospital, district hospital and small district hospital) and type dummies (university hospital, EPE and separate buildings) the observable effects for each hospital.

IV. Methodology

In order to better estimate results from our panel data, characterized by repeated measures for each hospital, three methods are to be applied. The first one will be a simple **Pooled-OLS**, due to the existence of serial correlation. Secondly, **Fixed-Effects** estimation as a way to capture the role of hospital characteristics that are not likely to change with time, such as managerial skills.

Both models are characterized by computing average values for the coefficients, which seems not to be the best suitable option when analysing hospitals' total costs. Consequently, a third and last method will be put into place, the **Stochastic Frontier Analysis**. Enabling the estimation of a Cost Efficiency, the analysis to be performed will be a result from an input-oriented approach.

IV.1 – Stochastic Frontier Analysis

Despite its usefulness in addressing a potential problem of endogeneity once Pooled OLS is performed, coming from the fact that bigger hospitals will probably present higher total costs and higher emergency inpatients, the Fixed-effects Model does not enable an error term decomposition in random statistical noise (exogenous shocks that affect each hospital) and time-invariant cost inefficiency. A solution for this problem is the application of the Stochastic Frontier Analysis, where the error term will present these two components.

A more complete description of the method is reported in **Annex.IV.1.2**.

IV.1.1 – Single-Equation Cost Frontier Models – The Model

The panel data constructed encloses several observations for H hospitals through T time periods. In the construction of a single-equation model Kumbhakar and Knox Lovell (2000) point out that panel data is not required to be balanced. For notation, one must assume a well-balanced panel, and assume that the deterministic kernel of the stochastic cost frontier follows a Cobb-Douglas form. They also state that one must assume time invariant cost efficiency, allowing the formulation of the following cost frontier model:

1) $\ln E_{ht} \geq \beta_0 + \beta_y lny_{ht} + \sum_n \beta_n lni_{nht} + \varepsilon_{ht} \Leftrightarrow$

2) $\Leftrightarrow \mathbf{ln}\, \boldsymbol{E_{ht}} = \boldsymbol{\beta_0} + \boldsymbol{\beta_y} \boldsymbol{lny_{ht}} + \sum_{\boldsymbol{n}} \boldsymbol{\beta_n} \boldsymbol{lni_{nht}} + \boldsymbol{v_{ht}} + \boldsymbol{u_h}$

$\boldsymbol{h}$: hospital (=1,...,H) $\boldsymbol{t}$: year (=1,...,T = 2003,…, 2006) $\boldsymbol{\beta_0}$: constant

$\boldsymbol{y_{ht}} = (\boldsymbol{y_{1ht}}, \dots, \boldsymbol{y_{Mht}}) \geq \mathbf{0}$: vector of outputs produced by hospital h in period t

$\boldsymbol{i_{ht}} = (\boldsymbol{i_{1ht}}, \dots, \boldsymbol{i_{Nht}}) > \mathbf{0}$: vector of prices and/or quantities of the inputs employed by hospital h in period t

$\boldsymbol{E_{ht}} = \boldsymbol{p_{ht}^T q_{ht}} = \sum_{\boldsymbol{n}} \boldsymbol{p_{nht} q_{nht}}$: total expenditure incurred by hospital h in period t

$\boldsymbol{v_{ht}}$: random statistical noise $\boldsymbol{u_h} \geq \mathbf{0}$: time invariant cost inefficiency

$\boldsymbol{\varepsilon_{ht}} = \boldsymbol{v_{ht}} + \boldsymbol{u_h}$: this composed error term is asymmetric but positively skewed since $u_h \geq 0$

Assuming that $\boldsymbol{c}(\boldsymbol{y_{ht}}, \boldsymbol{i_{ht}}, \boldsymbol{\beta})$ is the cost frontier common to all hospitals, where β is a vector of technology parameters to be estimated, it implies that:

$\boldsymbol{E_{ht}} \geq \boldsymbol{c}(\boldsymbol{y_{ht}}, \boldsymbol{i_{ht}}, \boldsymbol{\beta}). \boldsymbol{exp}(\boldsymbol{v_{ht}})$, being $c(y_{ht}, i_{ht}, \beta)$ the deterministic part common to all hospitals, and $exp(v_{ht})$ a hospital-specific random part (random shocks faced by each hospital).

Furthermore, $\sum_{\boldsymbol{n}} \boldsymbol{\beta_n} = \mathbf{1}$ ensures homogeneity of degree +1 of the cost frontier in input prices, meaning that: $c(y_{ht}, \boldsymbol{\alpha} i_{ht}, \beta) = \boldsymbol{\alpha} c(y_{ht}, i_{ht}, \beta)$, where $\boldsymbol{\alpha} > \mathbf{0}$.

The error components of the stochastic cost frontier follow three assumptions:

1. $\boldsymbol{v_{ht}} \sim \boldsymbol{iid}\ \boldsymbol{N}(\mathbf{0}, \boldsymbol{\sigma_v^2})$
2. $\boldsymbol{u_h} \sim \boldsymbol{iid}\ \boldsymbol{N}^+(\mathbf{0}, \boldsymbol{\sigma_u^2})$
3. $\boldsymbol{u_h}$ and $\boldsymbol{v_{ht}}$ are independently distributed from each other and from the regressors

Given the marginal density function of $\boldsymbol{f(\varepsilon_{ht}) = f(v_{ht} + u_h)}$ one is able to retrieve the log likelihood function for a sample of H hospitals, each observed for T periods of time (in here it relies the importance of assuming a balanced panel data):

$$\boldsymbol{ln(L) = constant - \frac{H(T-1)}{2} ln(\sigma_v^2) - \frac{H}{2} ln(\sigma_v^2 + T\sigma_u^2)}$$

$$\boldsymbol{+ \sum_h ln\left[1 - \Phi\left(-\frac{\mu_{*h}}{\sigma_*}\right)\right] - \left(\frac{\varepsilon'\varepsilon}{2\sigma_v^2}\right) + \frac{1}{2}\sum_h \left(\frac{\mu_{*h}}{\sigma_*}\right)^2}$$

Where $\boldsymbol{\mu_{*h} = \frac{T\sigma_u^2\bar{\varepsilon}_h}{\sigma_v^2 + T\sigma_u^2}}$ and $\boldsymbol{\sigma_* = \frac{\sigma_v^2\sigma_u^2}{(\sigma_v^2 + T\sigma_u^2)}}$

Maximizing the log likelihood function with respect to the parameters one will be able to compute maximum likelihood estimates of $\boldsymbol{\beta, \sigma_v^2}$ *and* $\boldsymbol{\sigma_u^2}$.

Long panels usually imply that cost efficiency being time invariant will become a weaker assumption. Nevertheless all three methods (MLE; GLS; LSDV) lodge the possibility for time-varying cost efficiency.

Taking from the conditional distribution of $(\boldsymbol{u}|\boldsymbol{\varepsilon})$, corresponding to the density function of a variable distributed as $\boldsymbol{N^+(\mu_*, \sigma_*^2)}$, its mean or mode, or computing a minimum squared error predictor, one is able to correctly estimate cost efficiency. Hence, in order to appropriately measure cost efficiency given a stochastic cost frontier that follows all these assumptions, one must take the ratio of minimum cost attainable:

3) $\boldsymbol{CE_{ht} = \frac{c(y_{ht}, i_{ht}, \beta).exp(v_{ht})}{E_{ht}} = exp(-u_h)}$.

V. Results

It will become clear that both coefficients and p-values will change accordingly to the specified estimation method. Results for every estimated model, either using Pooled OLS or Fixed-effects are available in the annexes, with similar interpretations to the ones presented below, however it is of high importance to compare the three estimations methods under the same model specification (same variables), in order to follow a structured line of thought. The chosen specification was made with basis on likelihood-ratio (LR) tests, which consistently presented the same combination of variables as the best model, independently of the statistical method applied (Annex.Likelihood.Tests).

The inclusion of dummies, accounting for hospitals' location or hospitals' characteristics, is preferable in terms of LR test while estimating Pooled OLS and Fixed-effects, confirming Jacobs, Smith and Street (2006) prediction in improving estimations efficiency. Howbeit, the Stochastic Frontier Analysis does not allow the inclusion of such variables, leading to their exclusion and ultimately impacting the significance of important variables. Therefore, these models should still be taken into consideration, as the previously described dummies are used in the estimation of Pooled OLS and Fixed-effects, respectively in Annex.Table.PooledOLS and Annex.Table.Fixed-effects**.**

V.1 Pooled OLS and Fixed-effects – Within estimator

The main characteristic of the Pooled OLS model is that it treats each observation as being independent from all others; in that sense it produces average results for every observation when regressing the model.

The output produced by Fixed-effects estimation will consider that yearly observations are not independent from each other in case they were observed in a given hospital. The usage of this model is validated by the Hausman Test, that for every specification prefers the fixed effects over the random effects.

Best Model - 3 Estimations

	Pooled OLS	Fixed Effects	Stochastic Frontier
	coef/se	coef/se	coef/se
ln(outpatients)	0.537***	0.668***	0.531***
	(0.087)	*(0.064)*	*(0.052)*
ln(emergencies)	0.053	0.065	0.056
	(0.068)	*(0.112)*	*(0.049)*
ln(annual average of the standard deviation of emergencies)	0.158	0.014	0.134
	(0.148)	*(0.074)*	*(0.084)*
ln(coefficient of variation)	-0.130	0.036	-0.103
	(0.143)	*(0.079)*	*(0.081)*
ln(average annual wage)	-0.096*	-0.005	-0.102
	(0.055)	*(0.023)*	*(0.064)*
ln(discharges)	-0.463***	-0.342**	-0.434***
	(0.141)	*(0.154)*	*(0.098)*
ln(occupancy rate)	0.419**	0.001	0.349**
	(0.207)	*(0.124)*	*(0.136)*
ln(deaths)	0.274***	0.200***	0.278***
	(0.068)	*(0.045)*	*(0.044)*
ln(beds)	0.611***	0.321*	0.615***
	(0.110)	*(0.179)*	*(0.075)*
_cons	8.246***	9.447***	8.357***
	(0.948)	*(1.127)*	*(0.918)*
/lnsig2v	-	-	-3.754***
			(0.359)
/lnsig2u	-	-	-3.423***
			(0.734)
Number of observations	239	239	239
R2	0.971	0.709	-
Adjusted R2	0.969	0.697	-
sigma_v	-	-	0.153
sigma_u	-	0.294	0.181
sigma2	-	-	1.180
lambda	-	-	0.582
sigma_e	-	0.085	-
rho	-	0.923	-
corr	-	0.717	-

note: *** p<0.01, ** p<0.05, * p<0.1

The joint significance test has a p-value of zero for both Best Pooled OLS and Best Fixed-effects models, confirming that no variable is to be excluded from either regression. Furthermore, interpreting the Adjusted R^2, the independent variables explain 96.9% of the variation of total costs, in the case of Pooled OLS and 69.7% when applying the Fixed-effects (FE).

Analysing more deeply and focusing on our research question, one must notice the high individual statistical significance that all *number of outpatients* (***out***)**,** *number of* *deaths* (***dth***), *number of discharges* (***dis***) and *number of beds* (***beds***) persistently show for every type of estimation.

Starting by a *ceteris paribus* analysis of the *number of outpatients*, when ***out*** increases by one percentage point the ***total costs*** increase on average by 0.537 percentage points (0.668 for FE), a positive effect that may be explained due to higher costs of treating patients outside the hospital.

The variables *number of emergencies* (***emg***), *annual average of the standard deviation of emergencies* (***sd***) and *coefficient of variation* (***varco***) show no statistical significance for the usual levels of confidence (1%, 5% and 10%) in the Pooled OLS context (however last two become significant once dummies are added to the model). Thus one must conclude that given such model specification when demand for emergency services tends to fluctuate more across a given year than previously, its impact in ***tot*** will not be a significant one.

Regarding the volume of ***emg*** a direct interpretation would just mean that the number of emergencies would be indifferent on the capacity hospitals have in costs adjustment, nonetheless, a more plausible explanation is linked with the fact that this variable shows only small variations across our panel data, making it difficult to attribute explanation power to it.

In a *ceteris paribus* analysis of ***dth***, one has that for a one percentage point increase in this variable ***tot*** will on average increase by 0.274 percentage points (0.2 for FE). Once again, this effect could result from the composition of the patients each hospital receives and by their degree of technology, which has two main implications for hospitals. First more advanced techniques are usually linked to higher costs of treatments, since better technology is more expensive to use and to buy. Secondly this will attract more complicated cases in terms of resolution, meaning patients attracted to these hospitals will face, *a priori*, a higher probability of dying.

A negative effect is observed on a *ceteris paribus* evaluation of ***dis***, since for a one percentage point increase of this variable ***tot*** will decrease on average by 0.463 percentage points (0.342 for FE). For a higher number of treated patients in a certain hospital, cheaper resources will be spent, as patients at the end of their lives tend to account for higher treatment costs, declining therefore the costs of health production.

Recalling previous literature on the stochastic demand faced by hospitals there is no surprise in the result presented by ***beds***, which, *ceteris paribus*, impact positively ***tot***, leading to an increase of 0.611 percentage points (0.321 for FE) for a one percentage point increase in this variable. This is not only due to the directly associated cost of an extra bed, but more a question of correctly allocating it to either urgent or planned admissions and its associated costs.

The Fixed-effects model provides some extra information. The first to be noticed is a correlation of 71,7% between the errors within groups and the regressors invoked by the output. As one may also behold the standard deviation of the residuals within groups (***sigma_u***) is roughly three times that of the overall residuals (***sigma_e***). Finally it is important to interpret the intraclass correlation (***rho***), a value that transpires that 92,3% of the variance is due to differences across panels, attesting huge differences from one hospital to another in terms of data variation, but small variations within each hospital, something explained by one's short-term analysis.

V.3 Stochastic Frontier Analysis

The fixed-effects estimation method has two handicaps. In one hand there is the suppression of a large amount of variation in the data, which produces estimates that tend towards zero, meaning these are ones of almost no effect in the dependent variable. On the other hand this estimator finds it impossible to distinguish between time-invariant heterogeneity and inefficiency, thus the application of a stochastic frontier analysis.

Stochastic Frontier Analysis

	SF Full Model	SF Full Model - 1	SF Full Model - 2	SF Full Model - 3	SF Full Model - 4	SF Full Model - 5	SF Reduced Model
	coef/se	coef/se	coef/se	coef/se	coef/se	coef/se	coef/se
ln(outpatients)	0.532***	0.531***	0.611***	0.538***	0.533***	0.574***	0.557***
	(0.052)	*(0.052)*	*(0.060)*	*(0.065)*	*(0.065)*	*(0.067)*	*(0.067)*
ln(emergencies)	0.056	0.056	0.022	0.034	0.038	0.065	0.084
	(0.049)	*(0.049)*	*(0.057)*	*(0.069)*	*(0.068)*	*(0.071)*	*(0.069)*
ln(annual average of the standard deviation of emergencies)	0.133	0.134	0.079	0.221**	0.228**	0.494***	0.481***
	(0.084)	*(0.084)*	*(0.091)*	*(0.101)*	*(0.100)*	*(0.076)*	*(0.076)*
ln(coefficient of variation)	-0.103	-0.103	-0.035	-0.136	-0.146	-0.404***	-0.399***
	(0.081)	*(0.081)*	*(0.085)*	*(0.094)*	*(0.092)*	*(0.066)*	*(0.067)*
ln(average annual wage)	-0.102	-0.102	-0.159**	-0.180**	-0.183**	-0.195**	
	(0.064)	*(0.064)*	*(0.068)*	*(0.082)*	*(0.081)*	*(0.084)*	
ln(discharges)	-0.266	-0.434***	0.023	0.328***	0.329***		
	(0.451)	*(0.098)*	*(0.082)*	*(0.083)*	*(0.084)*		
ln(occupancy rate)	0.186	0.349**	-0.093	0.074			
	(0.448)	*(0.136)*	*(0.120)*	*(0.141)*			
ln(deaths)	0.277***	0.278***	0.428***				
	(0.044)	*(0.044)*	*(0.047)*				
ln(beds)	0.448	0.615***					
	(0.445)	*(0.075)*					
ln(averagelengthofstay)	0.169						
	(0.444)						
_cons	8.121***	8.357***	8.974***	8.235***	8.501***	9.066***	7.130***
	(1.109)	*(0.918)*	*(0.986)*	*(1.199)*	*(1.040)*	*(1.069)*	*(0.665)*
/lnsig2v	-3.752***	-3.754***	-3.828***	-2.880***	-2.940***	-2.846***	-2.823***
	(0.360)	*(0.359)*	*(0.364)*	*(0.498)*	*(0.367)*	*(0.423)*	*(0.403)*
/lnsig2u	-3.430***	-3.423***	-2.724***	-4.516	-3.903	-4.098	-4.087
	(0.744)	*(0.734)*	*(0.384)*	*(6.939)*	*(2.591)*	*(4.004)*	*(3.854)*
sigma_v	0.153	0.153	0.148	0.237	0.230	0.241	0.244
sigma_u	0.180	0.181	0.256	0.105	0.142	0.129	0.130
sigma2	0.056	0.056	0.087	0.067	0.073	0.075	0.076
lambda	1.174	1.180	1.736	0.441	0.618	0.535	0.532
gamma	0.580	0.582	0.751	0.163	0.276	0.222	0.220
Number of observations	239	239	239	239	239	239	239

note: *** p<0.01, ** p<0.05, * p<0.1

Looking to the key variables of this research question, ***sd*** and ***varco***, they are only statistically significant for more parsimonious models. For the reduced model, when ***sd*** increases by one percentage point, ***tot*** increases on average 0,481 percentage points, but as soon as one adds two variables to this model (*average annual wage* and *number of discharges*) this value immediately drops to 0,228 percentage points.

When statistically significant (two more parsimonious models), the ***varco*** works as an inverse force to the impact of ***sd***, forcing ***tot*** to decrease near 0,399 percentage points. Still, one must notice that ***varco*** may increase as a result of two distinct behaviours: an increase in the ***sd*** or a decrease in ***md*** (capacity indicator for each hospital), implying that for the same level of ***sd*** hospitals with higher annual flows in emergency admissions will face higher costs than hospitals with lower flows. Moreover, since the negative effect evidenced by ***varco*** never overcomes that of ***sd***, higher dispersion will always increase total costs, independently from the hospitals' size.

A strange result and one that makes no economic sense is that observed for the *average annual wage* (***w***), which has a negative impact. This cannot be fully explained. Nevertheless, after more detailed analysis one is able to conclude that in a simple relation with ***tot*** this variable has a positive impact, and only upon the addiction of volume variables, such as ***out*** and ***beds***, the sign becomes negative.

Under this estimation process the error term is decomposed in idiosyncratic error $(\boldsymbol{v_{ht}})$ and inefficiency error $(\boldsymbol{u_h})$, being the later a possible aggregation of technical with allocative inefficiency (something not distinguishable using this approach). Jacob, Smith and Street (2006) advert that mean level and variation efficiency computations are susceptible to models' specifications, in other words, they are sensitive to the chosen independent variables and to the functional form.

Following Battese and Corra (1977) and computing the *total variance of disturbance*, $\boldsymbol{\sigma_T^2 = \sigma_v^2 + \sigma_u^2}$, one is able to observe that the "full model – 1", or "best model", is the one presenting the lower overall disturbance, with a variance of 0,056. Afterwards it is important to build gamma, $= \frac{\boldsymbol{\sigma_u^2}}{\boldsymbol{\sigma_T^2}}$, a measure that tells us that this model allocates 58% of the total variance of the disturbance to the inefficiency term. In other words, 58% of the distance comprised between the Portuguese hospitals' operation point and the costs frontier is explain by inefficiencies within each of them, and only 42% is due to exogenous shocks common to all hospitals.

According to Coelli (1996), the frontier will represent costs minimization where the $\boldsymbol{u_h}$ estimates define how far each hospital runs above the cost frontier, empowering one to estimate cost efficiency for each model. Starting by the calculation of *lambda* proposed by Greene (1990), $=\frac{\sigma_u}{\sigma_v}$, one has a first indicator of efficiency, where whenever this ratio equals **zero** one will be in the presence of **no inefficiency**. The overall inefficiency for Portuguese hospitals is 1,180 for the best model. These efficiency results are still far from zero and state that inefficiency standard deviation is higher than the one coming from random factors, however they are not that high if one takes into consideration that it is comprehended between 0 and infinity.

Finally, predicting the cost efficiency for each hospital advanced by Coelli (1996), $CE_{ht} = exp(\boldsymbol{u_h})$, the inverse of what Kumbhakar and Knox Lovell (2000) considered. As a result, Coelli's estimation (in Annex.Table.CostEfficiency) will vary between 1 and infinity, in opposition to the production efficiency that is set between 0 and 1.

For the best model, one has a minimum of 1.033501 (most efficient hospital – Hospital de Santo António 2006) and a maximum of 1.603784 (least efficient hospital – Centro Hospitalar de Caldas da Rainha 2006), while the national average is that of 1.157327; meaning the Portuguese hospitals are on average not far from efficiency.

Even though evaluating production efficiency, Afonso and Fernandes (2008) and Gonçalves (2008) point out improvements in the efficiency frontier until 2004, with a slight decrease in 2005, mainly due to the performance of hospitals in the public sector. Furthermore Afonso and Fernandes observed significant yearly fluctuations regarding individual efficiency, showing that only 10% of the hospitals stayed on the production frontier across the studied time period.

It is also important to estimate functions in their usual form, this is, without emergencies' dispersion variables. These results are presented in Annex.BestModels.NoDispersionVariables and Annex.SF.NoDispersionVariables.

VI. Conclusion

This research paper tried to answer a research question regarding the impact that stochastic demand for emergency services in hospitals may have or not in hospital total costs. Using a Stochastic Frontier Analysis method, one may conclude, neither the *number of emergencies* nor its *dispersion* across a given year have a significant impact on *total costs* of hospitals. Regarding efficiency, Portuguese hospitals have shown not to be far from it, with about 58% of the distance towards full efficiency being explained by hospitals' within inefficiency and 42% being tied with exogenous shocks.

This study also enables the uncover of variables which are in fact important to explain total costs differences across hospitals, such as, *number of outpatients*, *number of deaths*, *number of discharges* and the *number of beds*.

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Annexes

Annex.DataDescription

Variable	Obs	Mean	Std. Dev.	Min	Max
Total Costs	239	61,600,000	69,000,000	4,818,909	338,000,000
Nº Outpatients	239	108,648.3	108,520.1	4,072	520,029
Nº Emergencies	239	94315.91	49705.43	24,987	249,420
ln(sd)	239	4.296703	0.8384274	2.36429	6.291573
ln(varco)	239	-2.371209	0.4893733	-3.624846	-0.9075351
Average Annual Wage	239	26,893.12	4,460.06	3,304.09	41,233.54
Nº Discharges	239	12,038.61	9,867.942	1,032	50,315
Occupancy Rate	239	74.02702	9.815144	42.6	98
Nº Deaths	239	572.795	448.326	30	2067

Variable	Description
Total Costs	sum of all accounts in Portuguese cost statements in euros, excluding amortizations and the noisiest accounts
Nº Outpatients	annual number of ambulatory patient visits, in a given hospital
Nº Emergencies	annual number of urgent admissions, in a given hospital
ln(annual average of the standard deviation of emergencies)	annual average of the standard deviation of emergencies taking place in a given hospital
ln(coefficient of variation)	quotient between annual average of the standard deviation of emergencies and the average annual number of emergencies, in a given hospital in a given year
Average Annual Wage	quotient between total costs with personnel and the total number of staff members working, in a given hospital in a given year
Nº Discharges	absolute annual number of inpatients discharges, in a given hospital
Occupancy Rate	(average length of stay X inpatients) / (365 X beds)
Nº Deaths	annual number of deaths, in a given hospital
Nº Beds	annual number of beds, in a given hospital
Average Length of Stay	average number of days a patient stays in a given hospital once admitted

A Work Project, presented as part of the requirements for the Award of a Masters Degree in Economics from the Faculdade de Economia da Universidade Nova de Lisboa.

HOSPITAL COSTS AND RANDOM DEMAND

ANNEXES

Francisco Cluny Parreira Rodrigues

Nº 522

A Project carried out on the Economics of Health and Health Care course, with the supervision of:

Professor Pedro Pita Barros

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Derivation of the Pooled Ordinary Least Squares and Fixed-effects Model

Annex.IV.PooledOLS

Enabling a general approach to panel data, enclosing a weighted (depending on the variability of the explanatory factors) average of both within and between estimators of hospitals, the Pooled OLS exploit the maximum information they possibly can.

Following Cameron & Trivedi (2005), the pair $\boldsymbol{\bar{E}} = \sum_{h=1}^{H} \sum_{t=1}^{T} \boldsymbol{E_{ht}}$ and $\boldsymbol{\bar{x}} = \sum_{h=1}^{H} \sum_{t=1}^{T} \boldsymbol{x_{ht}}$ define the overall means of, respectively, total expenditure incurred by hospitals and explanatory variables (outputs produced and prices and/or quantities of the inputs employed by hospitals). From here one may easily compute the moment matrices of the overall sum of squares and cross products:

$$\boldsymbol{S_{xx}^{O}} = \sum_{h=1}^{H} \sum_{t=1}^{T} (\boldsymbol{x_{ht}} - \boldsymbol{\bar{x}})(\boldsymbol{x_{ht}} - \boldsymbol{\bar{x}})'$$

$$\boldsymbol{S_{xE}^{O}} = \sum_{h=1}^{H} \sum_{t=1}^{T} (\boldsymbol{x_{ht}} - \boldsymbol{\bar{x}})(\boldsymbol{E_{ht}} - \boldsymbol{\bar{E}})'$$

Dividing it into matrices for both within ($\boldsymbol{w}$) and between (b) sum of squares and cross products yields:

$$\boldsymbol{S_{xx}^{w}} = \sum_{h=1}^{H} \sum_{t=1}^{T} (\boldsymbol{x_{ht}} - \boldsymbol{\bar{x}_h})(\boldsymbol{x_{ht}} - \boldsymbol{\bar{x}_h})'$$

$$\boldsymbol{S_{xE}^{w}} = \sum_{h=1}^{H} \sum_{t=1}^{T} (\boldsymbol{x_{ht}} - \boldsymbol{\bar{x}_h})(\boldsymbol{E_{ht}} - \boldsymbol{\bar{E}_h})'$$

$$\boldsymbol{S_{xx}^{b}} = \sum_{h=1}^{H} \boldsymbol{T}(\boldsymbol{\bar{x}_h} - \boldsymbol{\bar{x}})(\boldsymbol{\bar{x}_h} - \boldsymbol{\bar{x}})'$$

$$\boldsymbol{S}_{xE}^{b} = \sum_{h=1}^{H} \boldsymbol{T}(\overline{\boldsymbol{x}}_{\boldsymbol{h}} - \overline{\boldsymbol{x}})(\overline{\boldsymbol{E}}_{\boldsymbol{h}} - \overline{\boldsymbol{E}})'$$

As it may be shown $\boldsymbol{S}_{xx}^{O} = \boldsymbol{S}_{xx}^{w} + \boldsymbol{S}_{xx}^{b}$ and $\boldsymbol{S}_{xE}^{O} = \boldsymbol{S}_{xE}^{w} + \boldsymbol{S}_{xE}^{b}$. Consequently the OLS estimator may be constructed as it follows:

$$\boldsymbol{b}^{\boldsymbol{O}} = [\boldsymbol{S}_{xx}^{O}]^{-1}\boldsymbol{S}_{xE}^{O} = \left[\boldsymbol{S}_{xx}^{w} + \boldsymbol{S}_{xx}^{b}\right]^{-1}\left[\boldsymbol{S}_{xE}^{w} + \boldsymbol{S}_{xE}^{b}\right]$$

Taking this model formulation into account, one may ensure that Pooled OLS may in fact not be the most efficient to exploit jointly within and between variability. In case the individual specific effects are correlated with the explanatory variables, $\boldsymbol{\beta}_{\boldsymbol{x}}$ estimates will be inconsistent and biased.

Additionally, Pooled OLS estimation presents a constant intercept, denoting an inconsistent behavior of this model in case the correct model to apply is the Fixed-effects model, as one will study more carefully in the following section, due to the fact of the intercept being an individual-specific estimation, therefore impossible to consistently derive it for in a general approach.

Annex.IV.Fixed-effects

A possible problem of endogeneity may arise with the usage of Pooled OLS, since bigger hospitals will probably present higher total costs and higher emergency inpatients. Known as the simplest model applied in the treatment of panel data, the fixed-effects estimator surge as a possible solution for this issue, presenting the following general specification according to Cameron & Trivedi (2005):

$$\boldsymbol{E}_{\boldsymbol{ht}} = \boldsymbol{\gamma}_{\boldsymbol{h}} + \boldsymbol{\beta}_{x}\boldsymbol{x}_{\boldsymbol{ht}} + \boldsymbol{\varepsilon}_{\boldsymbol{ht}}$$

Its main characteristic relies on the variable $\boldsymbol{\gamma_h = \gamma_1, \dots, \gamma_H}$. Despite measuring time-invariant individual-specific effects, its unobserved heterogeneity may be correlated with the regressors $\boldsymbol{x_{ht}}$ and with their coefficients $\boldsymbol{\beta_x}$, assuming that the error term $\boldsymbol{\varepsilon_{ht}}$ is $\boldsymbol{iid}\ (\boldsymbol{0}, \boldsymbol{\sigma_v^2})$.

The main objective of such model is to estimate the coefficients $\boldsymbol{\beta_x}$, because they represent the marginal effect of change in regressors, as one may easily understand from $\frac{\partial E(\boldsymbol{E_{ht}})}{\partial \boldsymbol{x_{ht}}} = \boldsymbol{\beta_x}$. The variables $\boldsymbol{\gamma_h}$ are not of interest *per se*, however they are useful in the sense that only in their presence one may compute better estimations of $\boldsymbol{\beta_x}$.

Several consistent estimations may be constructed by resorting to a wide variety of models, more precisely four models: Least Squares Dummy Variables (LSDV), Within Estimator, First-differences and Differences-in-differences (DD). One must note that the first two methods produce identic $\boldsymbol{\beta_x}$ estimations.

Annex.IV.Fixed-effects.LSDV

Briefly describing the Least Squares Dummy Variables it is simply the inclusion of *N* dummy variables for each individual and constant over time, in a simple OLS regression. Moreover, one must include *N-1* dummy variables in a model with constant, or exclude the constant in order to put all the *N* dummies in the model. Constructing a model based on the last hypothesis one has the following form:

$$\boldsymbol{E_{ht} = d_1\gamma_1 + d_2\gamma_2 + \cdots + d_N\gamma_N + \beta_x x_{ht} + \varepsilon_{ht}}$$

Where $\boldsymbol{d_j(i) = 1}$ if $\boldsymbol{j = i}$ and $\boldsymbol{d_j(i) = 0}$ if $\boldsymbol{j \neq i}$. The error term is assumed to be well-behaved, enabling consistent and unbiased estimates, since the omitted variable bias effect is capture by the dummy variable for each individual. It is also important to point out that for huge datasets this model generates huge matrixes, that grow exponentially, which may lead to heavy computational requirements, making the estimation of parameters not feasible. Therefore, another approach may be required, such as the Within Estimator.

Annex.IV.Fixed-effects.Within Estimator

This method is derived by subtracting the time-averaged model from the original one, the within model comes as following:

$$[\boldsymbol{E_{ht}} - \overline{\boldsymbol{E_h}}] = \boldsymbol{\beta}_x[\boldsymbol{x_{ht}} - \overline{\boldsymbol{x_h}}] + [\boldsymbol{\varepsilon_{ht}} - \overline{\boldsymbol{\varepsilon_h}}]$$

Where $\overline{\boldsymbol{E_h}}$ is the mean of the $\boldsymbol{T}$ observations on the outcome for hospital $\boldsymbol{h}$, $\overline{\boldsymbol{x_h}}$ is the $\boldsymbol{K}$ row vector of the means of the $\boldsymbol{T}$ observations on explanatory factors $\boldsymbol{X}$ for hospital $\boldsymbol{h}$. Once the estimates of the parameter $\boldsymbol{\beta_x}$ are computed ($\boldsymbol{b_{FE}}$), it becomes possible to estimate the individual fixed effects for each hospital:

$$\boldsymbol{a_h} = \overline{\boldsymbol{E_h}} - \overline{\boldsymbol{x_h}}\boldsymbol{b_{FE}}$$

However for a small panel data this individual fixed effect estimate will be inconsistent, since $\boldsymbol{T}$ does not converge to infinity. Nonetheless knowing that the sufficient condition for consistency in the within model is given by $E[\boldsymbol{\varepsilon_{ht}} - \overline{\boldsymbol{\varepsilon_h}}|\boldsymbol{x_{ht}} - \overline{\boldsymbol{x_h}}] = \boldsymbol{0}$, in other words one assumes strict exogeneity, it may be ensured that the estimate for $\boldsymbol{\beta_x}$ is consistent. Such condition implies that strict orthogonality across time must be verified so that consistent OLS for the within estimator are computed.

The major flaw presented by the within estimator is related with its incapability of allowing estimates of the coefficients of time-invariant regressors, by virtue of the cancellation of such variables when the subtraction of one model with the other takes place.

Finally it is important to stress out that robust statistical inference may be designed for the within estimator if heteroskedasticity or serial correlation take place. Usually, the main drawback is indeed the existence of serial correlation, which in its turn leads to an underestimation of the standard errors, consequently inflating t-statistics and p-values, both of high importance in setting conclusions.

The degree of serial correlation diminishes in the presence of fixed or random effects; howbeit these procedures are not likely to fully eliminate this problem. The so-called panel-robust sandwich standard errors, which correspond to an extension of the white standard errors to panel data, solve this issue, considering that every time observations for each hospital from our fixed effects model are stacked:

$$\ddot{\boldsymbol{E}}_{\boldsymbol{h}} = \boldsymbol{\beta}_{\boldsymbol{x}}\ddot{\boldsymbol{x}}_{\boldsymbol{h}} + \ddot{\boldsymbol{\varepsilon}}_{\boldsymbol{h}}$$

Where both $\ddot{\boldsymbol{E}}_{\boldsymbol{h}}$ and $\ddot{\boldsymbol{\varepsilon}}_{\boldsymbol{h}}$ are $\boldsymbol{T} \times \boldsymbol{1}$ column vectors and $\ddot{\boldsymbol{x}}_{\boldsymbol{h}}$ is a $\boldsymbol{T} \times \boldsymbol{K}$ matrix of regressors. Stacking the time stacks over the $\boldsymbol{H}$ hospitals, one has the following matrix form:

$$\ddot{\boldsymbol{E}} = \boldsymbol{\beta}_{\boldsymbol{x}}\ddot{\boldsymbol{x}} + \ddot{\boldsymbol{\varepsilon}}$$

Assuming that $E(\ddot{\boldsymbol{x}}'\ddot{\boldsymbol{\varepsilon}}) = \boldsymbol{0}$, the essential condition for consistency (weak exogeneity is not sufficient as it only implies contemporaneous exogeneity), and the model is correctly specified, the OLS estimator may be derived as it follows:

$$\widehat{\boldsymbol{\beta}}_{\boldsymbol{OLS}} = \boldsymbol{\beta} + [\ddot{\boldsymbol{x}}'\ddot{\boldsymbol{x}}]^{-\boldsymbol{1}}\ddot{\boldsymbol{x}}'\ddot{\boldsymbol{\varepsilon}}$$

Under strict exogeneity the asymptotic variance of the OLS estimators becomes:

$$\boldsymbol{Var}\left(\widehat{\boldsymbol{\beta}}_{\boldsymbol{OLS}}\right) = \boldsymbol{E}\left({\widehat{\boldsymbol{\beta}}_{\boldsymbol{OLS}}}^{\boldsymbol{2}}\right) - \boldsymbol{E}\left(\widehat{\boldsymbol{\beta}}_{\boldsymbol{OLS}}\right)^{\boldsymbol{2}} = \underbrace{[\ddot{\boldsymbol{x}}'\ddot{\boldsymbol{x}}]^{-\boldsymbol{1}}\ddot{\boldsymbol{x}}'}_{\boldsymbol{bread}}\underbrace{\boldsymbol{E}(\ddot{\boldsymbol{\varepsilon}}\ddot{\boldsymbol{\varepsilon}}')}_{\boldsymbol{cheese}}\underbrace{\ddot{\boldsymbol{x}}[\ddot{\boldsymbol{x}}'\ddot{\boldsymbol{x}}]^{-\boldsymbol{1}}}_{\boldsymbol{bread}}$$

From which one only needs to take the residuals $\hat{\boldsymbol{\varepsilon}}$, estimated with the within or fixed effects estimator, and plug them in the expression, in order to consistently estimate this variance.

Theoretical background on Stochastic Frontier Analysis

Annex.IV.1.2 – Cost vs Production

Further developing the Stochastic Frontier Analysis, one needs to differentiate the input-oriented cost efficiency (cost frontier) from the output-oriented technical efficiency (production frontier). Five major differences between these two processes are described by Kumbhakar and Knox Lovell (2000), which are imperative to better understand all computations and obtained results.

The first difference is related with data, as for cost efficiency estimation one needs to gather information regarding input prices or quantities (depending on the model), output quantities and total expenditure; for a production frontier one will only need employed quantities of input and the output provided by each producer.

Secondly, the number of outputs is also of high importance. It is possible to perform cost efficiency analysis for a firm that produces either one or multiple outputs, however for a production frontier estimation the firm will have to produce a single output. In case of multiple outputs a production frontier may be design recurring to an output distance function, dual to a revenue frontier, being one implication that joint production occurs, in other words, the total cost of producing both outputs jointly will be lower than producing them separately - *"Baumol Gama Economies"*.

A third difference arises due to the fact that inputs are treated equally in a stochastic production frontier, even if it is known in advance that different classifications are to be given to each input. No distinction is made between variable and quasi-fixed inputs, indicating that information will be lost, since there is no inference related with the variability of inputs. When looking to a stochastic cost frontier, inputs may be treated differently, a natural possibility as one is now working with an input-oriented model. Once such distinction is made and one knows exactly which inputs are and which are not quasi-fixed, one will construct a variable cost frontier.

In the fourth place, we have that no behavioural objectives must be set in advance to producers in an output oriented model, contrarily to what happens regarding the input-oriented model. This may sometimes become an unrealistic assumption; however, if for instance the producer faces fixed outputs, maybe due to short-run fixity or contract arrangement, one must only model a variable cost frontier in order to solve this problem. One must also take into account that in some sectors, as in the Health one, output is not storable and therefore the output maximization objective, indissociable from the output-oriented approach, will be inappropriate.

Finally, the last difference is related with the information given by each frontier's estimation. In one hand technical efficiency cannot be decomposed; in the other hand cost efficiency may be decomposed. The latest may have in fact two different sources, an input-oriented technical efficiency or an input allocative inefficiency, and understanding which of these is the main source of inefficiency may be an interesting exercise. It is important to take into consideration that in order to estimate cost efficiency, input-oriented technical efficiency is a necessary condition, however it is not sufficient per se, meaning that it will always have a lower magnitude than that of cost efficiency, being the difference the so-called input allocative inefficiency. Moreover, one must be careful when comparing input-oriented technical efficiency results with that of output-oriented technical efficiency, as they may not be the same; such will only take place if production is technically efficient or when inefficient production of technology still satisfies constant returns to scale. In case neither of this assumption does not hold one must be aware that the input-oriented technical efficiency will be lower than output-oriented for decreasing returns to scale, and greater for increasing returns.

Annex.IV.1.3 – Cross-section vs Panel Data in a Cost Frontier Analysis

Accessing cross-sectional data to perform efficiency estimates may raise several issues, mainly due to the fact that, in this case, each hospital would only be observed once, which would reduce one's confidence in the results. These data limitations may be solved by access panel data, otherwise as Schmidt and Sickles point out three problems arise when computing a stochastic frontier analysis.

The first issue addressed by them is related with Maximum Likelihood estimation of the stochastic cost frontier model, which will consequently allow for the decomposition of the residuals into cost efficiency and statistical noise. The limitation arises as all error terms follow strong assumptions regarding their statistical distribution. Panel data enables weaker distributional assumptions, because repeated observations for a given hospital are observed.

A second difficulty surges when assuming that in one's Maximum Likelihood Estimation the error related with cost efficiency must be independent from the regressors of the model (input prices, quasi-fixed quantities and output quantities). Nonetheless not every panel data estimation technique requires this independence assumption to hold, once again due to the existence of repeated observations.

Finally they point out that despite being possible to perform the JSLM technique, applying it to the estimation of cost efficiency, the estimator is not consistent since the variance of the conditional mean does not tend to zero when the size of the cross-section tends to infinity. By adding observations for each hospital the inconsistency problem is solved, meaning technical efficiency of hospitals is now consistently estimated.

Further Results

Annex.V.1.PooledOLS (Annex.Table.PooledOLS**)**

The main characteristic of this model is that it treats each observation as being independent from all others; in that sense it produces average results for every observation when regressing the model.

The joint significance test has a p-value of zero for all the Pooled OLS models, confirming that no variable is to be excluded from either regression. Furthermore, when looking for the Adjusted R^2, it is observable that as more variables are included in the model, the higher the explanation power gets. For a regression with all variables included (Full Model), the independent variables explain 98,1% of the variation of total costs. Nevertheless, for the Reduced Model, the value is only a bit lower, with a power of explanation of 94,4% , confirming the huge importance that three of the last four variables have on *total costs* (***tot***).

Analysing deeply these three variables and focusing more in our research question, one must notice the high individual statistical significance that all *number of outpatients* (***out*****),** *annual average of the standard deviation of emergencies* (***sd***) and *coefficient of variation* (***varco***) persistently show in every regression.

Starting by a *ceteris paribus* analysis of the *number of outpatients*, when ***out*** increases by one percentage point the ***total costs*** increase on average by 0,419 to 0,580 percentage points, a positive effect that may be explained due to higher costs of treating patients outside the hospital.

Depending on the model, *ceteris paribus*, for a one percentage point increase in ***sd***, ***total costs*** increase on average between 0,236 and 0,529 percentage points, meaning that when demand for emergency services tends to fluctuate more across a given year than previously, ***tot*** will also be higher than before.

One notorious fact is that the volume of the number of annual emergencies presents no statistical significance for the usual levels of confidence (1%, 5% and 10%). A direct interpretation would just mean that the number of emergencies would be indifferent on the capacity hospitals have in costs adjustment. A more plausible explanation is linked with the fact that the variable number of annual emergencies shows only small variations across our panel data, making it impossible to attribute explanation power to this variable.

From here comes the usefulness of the variable ***varco*** that enables the evaluation of the dispersion over the average number of emergencies in a given year, in a certain hospital. Once the computation of the average annual number of emergencies is computed one may better attain for the impact of dispersion; for a one percentage point *ceteris paribus* increase in ***varco***, ***total costs*** decrease, on average, between 0,223 and 0,463 percentage points.

These results show that higher dispersion on the number of emergencies for a given year increases hospitals' total costs, since the negative effect evidenced by ***varco*** never overcomes that of ***sd***, meaning that higher dispersion always increases total costs independently from the hospitals' size. Moreover, an increase in ***md*** also leads to an increase of ***tot***, in which case ***varco*** decreases, given a negative coefficient, ***tot*** must increase.

Regarding the dummy variables presented in the models, having the year of 2003 as base group one reaches the conclusion that in case one is in any of the other years, *ceteris paribus*, a statistically significant positive impact is observed. Regarding the regional location of each hospital, one may state that hospitals located in any place but the Northern region will see their total costs increase in a *ceteris paribus* analysis, except for hospitals located in the Centre region, which present no statistical significance. Hospital Centres will also present higher costs than district and small district hospitals, something that is easily understood due to their size. In the same line of thought, hospitals that are at the same time universities are also likely to present higher costs, as well as hospitals that perform their services in separate buildings, due to maintenance costs.

Both number of discharges and number of deaths usually present statistical significance, although never for a confidence level of 1%. The number of deaths always shows a positive coefficient, however discharges see its coefficient vary depending on the model that is used. Finally, neither the average annual wages, nor the occupancy rate, nor the average length stay, are statistically significant in explaining total costs.

Annex.Table.PooledOLS

	Pooled OLS											
	Robust Full model	Robust Full-1	Robust Full-2	Robust Full-3	Robust Full-4	Robust Full-5	Robust Full-6	Robust Reduced	Robust time	Robust region	Robust district	Robust type
	coef/se	coef/se	coef/se	coef/se	coef/se	coef/se	coef/se	coef/se	coef/se	coef/se	coef/se	coef/se
ln(outpatients)	0.444***	0.444***	0.479***	0.419***	0.422***	0.435***	0.436***	0.559***	0.535***	0.580***	0.454***	0.520***
	(0.071)	*(0.071)*	*(0.077)*	*(0.083)*	*(0.083)*	*(0.086)*	*(0.085)*	*(0.122)*	*(0.125)*	*(0.092)*	*(0.121)*	*(0.119)*
ln(emergencies)	-0.014	-0.014	-0.005	0.000	0.009	0.017	0.016	0.080	0.083	0.116	0.031	0.014
	(0.070)	*(0.069)*	*(0.097)*	*(0.090)*	*(0.090)*	*(0.094)*	*(0.095)*	*(0.122)*	*(0.124)*	*(0.122)*	*(0.111)*	*(0.112)*
ln(annual average of the standard deviation of emergencies)	0.264**	0.264**	0.236**	0.309**	0.288**	0.463***	0.464***	0.481***	0.502***	0.410***	0.499***	0.529***
	(0.122)	*(0.122)*	*(0.114)*	*(0.120)*	*(0.116)*	*(0.099)*	*(0.099)*	*(0.128)*	*(0.131)*	*(0.104)*	*(0.129)*	*(0.120)*
ln(coefficient of variation)	-0.240*	-0.239*	-0.191	-0.248**	-0.223*	-0.390***	-0.390***	-0.399***	-0.427***	-0.316***	-0.404***	-0.463***
	(0.127)	*(0.126)*	*(0.118)*	*(0.125)*	*(0.115)*	*(0.101)*	*(0.101)*	*(0.107)*	*(0.112)*	*(0.094)*	*(0.123)*	*(0.107)*
2004	0.037***	0.037***	0.042***	0.040**	0.041***	0.040**	0.039**		0.044*			
	(0.012)	*(0.012)*	*(0.015)*	*(0.016)*	*(0.016)*	*(0.016)*	*(0.017)*		*(0.023)*			
2005	0.042**	0.042**	0.042*	0.063***	0.062***	0.064***	0.065***		0.063***			
	(0.020)	*(0.020)*	*(0.022)*	*(0.022)*	*(0.022)*	*(0.023)*	*(0.022)*		*(0.024)*			
2006	0.080***	0.079***	0.064**	0.082**	0.082**	0.072**	0.073**		0.069*			
	(0.029)	*(0.029)*	*(0.031)*	*(0.034)*	*(0.034)*	*(0.034)*	*(0.033)*		*(0.036)*			
Lisboa e Vale do Tejo	0.211***	0.210***	0.213***	0.288***	0.279***	0.281***	0.280***			0.324***		
	(0.051)	*(0.050)*	*(0.055)*	*(0.048)*	*(0.049)*	*(0.050)*	*(0.050)*			*(0.067)*		
Centro	0.003	0.003	0.004	0.029	0.027	-0.002	-0.003			0.017		
	(0.035)	*(0.035)*	*(0.048)*	*(0.048)*	*(0.048)*	*(0.052)*	*(0.052)*			*(0.069)*		
Alentejo	0.190***	0.190***	0.293***	0.370***	0.366***	0.343***	0.341***			0.384***		
	(0.069)	*(0.068)*	*(0.074)*	*(0.066)*	*(0.068)*	*(0.072)*	*(0.075)*			*(0.095)*		
Algarve	0.372***	0.371***	0.458***	0.478***	0.463***	0.431***	0.429***			0.410***		
	(0.055)	*(0.055)*	*(0.056)*	*(0.055)*	*(0.056)*	*(0.049)*	*(0.051)*			*(0.102)*		
Districtal Hospital	-0.192***	-0.192***	-0.231***	-0.256***	-0.253***	-0.265***	-0.264***				-0.333***	
	(0.060)	*(0.060)*	*(0.073)*	*(0.072)*	*(0.074)*	*(0.071)*	*(0.071)*				*(0.087)*	
Small Districtal Hospital	-0.288***	-0.289***	-0.291***	-0.320***	-0.337***	-0.385***	-0.386***				-0.502***	
	(0.081)	*(0.080)*	*(0.107)*	*(0.115)*	*(0.112)*	*(0.110)*	*(0.110)*				*(0.146)*	
University Hospital	0.176**	0.177**	0.247***	0.268***	0.280***	0.298***	0.298***					0.323***
	(0.072)	*(0.071)*	*(0.077)*	*(0.081)*	*(0.079)*	*(0.078)*	*(0.078)*					*(0.106)*
Entidade Pública Empresarial	0.031	0.031	0.017	0.024	0.022	0.028	0.028					-0.096
	(0.041)	*(0.041)*	*(0.047)*	*(0.046)*	*(0.047)*	*(0.049)*	*(0.049)*					*(0.070)*
Seperate Buildings	0.138***	0.138***	0.117**	0.124***	0.117**	0.128***	0.128***					0.147**
	(0.052)	*(0.051)*	*(0.053)*	*(0.047)*	*(0.047)*	*(0.045)*	*(0.045)*					*(0.062)*
ln(average annual wage)	-0.006	-0.006	0.011	0.018	0.020	0.011						
	(0.051)	*(0.051)*	*(0.055)*	*(0.060)*	*(0.061)*	*(0.064)*						
ln(discharges)	-0.363*	-0.277**	0.089	0.224**	0.222**							
	(0.213)	*(0.117)*	*(0.097)*	*(0.093)*	*(0.091)*							
ln(occupancy rate)	0.251	0.167	-0.148	-0.144								
	(0.190)	*(0.154)*	*(0.146)*	*(0.152)*								
ln(deaths)	0.107	0.108*	0.181**									
	(0.066)	*(0.066)*	*(0.087)*									
ln(beds)	0.538***	0.450***										
	(0.205)	*(0.089)*										
ln(average length of stay)	-0.088											
	(0.184)											
_cons	9.816***	9.695***	9.334***	9.248***	8.660***	9.380***	9.498***	7.260***	7.290***	6.996***	9.224***	8.067***
	(1.077)	*(1.046)*	*(1.153)*	*(1.163)*	*(0.901)*	*(0.966)*	*(0.835)*	*(1.047)*	*(1.066)*	*(1.061)*	*(0.959)*	*(1.044)*
Number of observations	239	239	239	239	239	239	239	239	239	239	239	239
R2	0.983	0.983	0.979	0.978	0.978	0.976	0.976	0.945	0.946	0.966	0.954	0.953
Adjusted R2	0.981	0.981	0.977	0.976	0.976	0.974	0.975	0.944	0.944	0.964	0.953	0.951

note: *** p<0.01, ** p<0.05, * p<0.1

Annex.V.2.Fixed-effects.Within-estimator (Annex.Table.Fixed-effects**)**

Avoiding the aforementioned endogeneity issue that will probably arise with the estimation of the Pooled OLS, coming from the fact that bigger hospitals are expected to present higher total costs and higher emergency inpatients, one should estimate a Fixed-Effects model. The output produced by such estimation will consider that yearly observations are not independent from each other in case they were observed in a given hospital. The usage of this model is validated by the Hausman Test, that for every model prefers the fixed effects over the random effects.

Several differences arise when estimating our models through the method of fixed effects. The first concerns the drop of some of the dummy variables, mainly the ones regarding the regional location and district location of hospitals, in order to avoid collinearity problems. Howbeit, the inclusion of dummy variables, for instance hospitals' location or the hospitals' characteristics, is something that must be done, given its power in improving estimations efficiency (Jacobs, Smith and Street 2006).

Despite the joint significance test for every models also show a p-value of zero, making it possible to reject the null hypothesis of all variables being equal to zero, the Adjusted R^2's are much lower than the ones observed in the Pooled OLS models, with values around 73,3% and 66,5%, which is still a high value for this statistic.. Such Result is justified by the attempt of the Pooled OLS in artificially balancing out existing heterogeneity.

Once time-invariant individual-specific effects are taken into account, one comes across with models where the majority of variables display no statistical significance, which constitutes a major drawback in one's analysis.

The *number of outpatients* however, it is still significant across every constructed model, and in a *ceteris paribus* analysis, when ***out*** increases by one percentage point the ***total costs*** increase on average by 0,5 to 0,8 percentage points, reaffirming the importance of this variable in explain total costs increase.

In line with the already computed Pooled OLS models, another strong variable across models is the *number of deaths* (***dth***), which in a *ceteris paribus* analysis for a one percentage point increase leads ***tot*** to increase by at least 0,154 percentage points, implying the higher the number of deaths taking place in a given hospital in a given year, the higher will be total costs.

Previously of high importance in every model, *annual average of the standard deviation of emergencies* (***sd***) and *coefficient of variation* (***varco***) have only shown to be statistically significant in the reduced model where dummy variables for each type of hospital are included. Even more puzzling are the coefficient signs both exhibit, negative and positive signs respectively, going in the opposite direction of the previous explanations for their impact on total costs.

Notwithstanding such model specifications, some interesting results emerge from this estimation method. The first to be noticed is the correlation the strong correlation between the errors within groups and the regressors invoked by the output, reaching a maximum of 85,8%. As one may also behold the standard deviation of the residuals within groups is roughly three to four times that of the overall residuals for every model. Finally it is important to interpret the intraclass correlation, given by the estimation of *rho*, a calculation that makes clear that 94,5% to 97% of the variance is due to differences across panels, attesting a huge differences from one hospital to another in terms of data variation, but small variations within each hospital, an expect result as the dataset only comprises four years.

Annex.Table.Fixed-effects

	Fixed Effects Model									
	FE Robust Full model	FE Robust Full-1	FE Robust Full-2	FE Robust Full-3	FE Robust Full-4	FE Robust Full-5	FE Robust Full-6	FE Robust Reduced	FE Robust time	FE Robust type
	coef/se	coef/se	coef/se	coef/se	coef/se	coef/se	coef/se	coef/se	coef/se	coef/se
ln(outpatients)	0.500***	0.504***	0.606***	0.628***	0.629***	0.626***	0.626***	0.800***	0.691***	0.725***
	(0.072)	*(0.072)*	*(0.099)*	*(0.117)*	*(0.119)*	*(0.118)*	*(0.118)*	*(0.045)*	*(0.109)*	*(0.060)*
ln(emergencies)	0.072	0.061	0.036	0.065	0.069	0.075	0.076	0.121	0.108	0.085
	(0.118)	*(0.113)*	*(0.111)*	*(0.119)*	*(0.119)*	*(0.123)*	*(0.122)*	*(0.128)*	*(0.126)*	*(0.101)*
ln(annual average of the standard deviation of emergencies)	0.080	0.074	-0.013	-0.043	-0.056	-0.069	-0.068	-0.078	0.021	-0.164*
	(0.099)	*(0.103)*	*(0.125)*	*(0.133)*	*(0.127)*	*(0.135)*	*(0.135)*	*(0.103)*	*(0.138)*	*(0.088)*
ln(coefficient of variation)	-0.048	-0.043	0.059	0.089	0.104	0.117	0.117	0.136	0.026	0.224**
	(0.107)	*(0.111)*	*(0.140)*	*(0.149)*	*(0.141)*	*(0.150)*	*(0.150)*	*(0.109)*	*(0.151)*	*(0.094)*
2004	0.033**	0.033**	0.031**	0.024	0.024	0.025	0.025		0.023	
	(0.014)	*(0.014)*	*(0.015)*	*(0.016)*	*(0.016)*	*(0.016)*	*(0.016)*		*(0.016)*	
2005	0.044*	0.043*	0.035	0.041	0.040	0.041	0.040		0.045*	
	(0.023)	*(0.024)*	*(0.027)*	*(0.027)*	*(0.027)*	*(0.026)*	*(0.026)*		*(0.026)*	
2006	0.067**	0.065**	0.044	0.046	0.045	0.046	0.046		0.047	
	(0.029)	*(0.031)*	*(0.039)*	*(0.040)*	*(0.040)*	*(0.040)*	*(0.039)*		*(0.038)*	
University Hospital	0.057	0.061	0.107	0.160	0.159	0.160	0.161			0.178**
	(0.077)	*(0.078)*	*(0.097)*	*(0.110)*	*(0.112)*	*(0.112)*	*(0.111)*			*(0.089)*
Entidade Pública Empresarial	-0.003	-0.002	-0.006	-0.023	-0.022	-0.022	-0.021			-0.006
	(0.027)	*(0.027)*	*(0.026)*	*(0.025)*	*(0.024)*	*(0.025)*	*(0.024)*			*(0.019)*
Seperate Buildings	0.074	0.078	0.104	0.133*	0.137*	0.135*	0.134*			0.135**
	(0.058)	*(0.059)*	*(0.066)*	*(0.073)*	*(0.073)*	*(0.073)*	*(0.071)*			*(0.067)*
ln(average annual wage)	-0.021	-0.019	-0.019	-0.009	-0.008	-0.008				
	(0.013)	*(0.014)*	*(0.015)*	*(0.016)*	*(0.016)*	*(0.016)*				
ln(discharges)	-0.551	-0.348**	-0.075*	-0.020	-0.020					
	(0.407)	*(0.136)*	*(0.039)*	*(0.023)*	*(0.023)*					
ln(occupancy rate)	0.244	0.058	-0.139**	-0.058						
	(0.373)	*(0.105)*	*(0.070)*	*(0.080)*						
ln(deaths)	0.158***	0.154***	0.170***							
	(0.053)	*(0.051)*	*(0.059)*							
ln(beds)	0.562	0.362**								
	(0.408)	*(0.143)*								
ln(averagelengthofstay)	-0.209									
	(0.336)									
_cons	11.062***	10.831***	10.826***	10.537***	10.313***	10.180***	10.087***	7.787***	8.438***	9.570***
	(1.269)	*(1.293)*	*(1.440)*	*(1.604)*	*(1.604)*	*(1.661)*	*(1.565)*	*(1.199)*	*(1.429)*	*(1.228)*
Number of observations	239	239	239	239	239	239	239	239	239	239
R2	0.733	0.731	0.714	0.703	0.702	0.702	0.702	0.665	0.678	0.691
Adjusted R2	0.713	0.713	0.696	0.685	0.686	0.687	0.689	0.660	0.668	0.682
sigma_u	0.362	0.367	0.420	0.493	0.497	0.491	0.490	0.371	0.380	0.480
sigma_e	0.083	0.083	0.085	0.087	0.086	0.086	0.086	0.089	0.089	0.087
rho	0.950	0.951	0.960	0.970	0.971	0.970	0.970	0.945	0.948	0.969
corr	0.853	0.858	0.833	0.815	0.816	0.815	0.814	0.641	0.703	0.772

note: *** p<0.01, ** p<0.05, * p<0.1

Annex.BestModels.NoDispersionVariables

Best Models - No emergencies dispersion

	Pooled OLS	FE	SF
	coef/se	coef/se	coef/se
ln(outpatients)	0.592***	0.676***	0.569***
	(0.074)	*(0.068)*	*(0.049)*
ln(emergencies)	0.063	0.081	0.067
	(0.065)	*(0.107)*	*(0.049)*
ln(average annual wage)	-0.084	0.007	-0.093
	(0.054)	*(0.025)*	*(0.064)*
ln(discharges)	-0.382***	-0.363**	-0.362***
	(0.107)	*(0.163)*	*(0.080)*
ln(occupancy rate)	0.440**	-0.034	0.335**
	(0.207)	*(0.125)*	*(0.131)*
ln(deaths)	0.288***	0.200***	0.290***
	(0.065)	*(0.050)*	*(0.043)*
ln(beds)	0.601***	0.332*	0.613***
	(0.109)	*(0.185)*	*(0.076)*
_cons	7.543***	9.325***	7.876***
	(0.868)	*(1.167)*	*(0.872)*
/lnsig2v			-3.873***
			(0.311)
/lnsig2u			-3.182***
			(0.463)
Number of observations	239	239	239
R2	0.970	0.698	-
Adjusted R2	0.969	0.689	-
sigma_v	-	-	0.144
sigma_u	-	0.293	0.204
sigma2	-	-	0.062
lambda	-	-	1.412
gamma	-	-	0.666
sigma_e	-	0.086	-
rho	-	0.921	-
corr	-	0.717	-

note: *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Annex.SF.NoDispersionVariables

Stochastic Frontier Analysis - No emergencies dispersion

	SF No sd	SF No sd - 1	SF No sd - 2	SF No sd - 3	SF No sd - 4	SF No sd - 5	SF No sd - 6
	coef/se	coef/se	coef/se	coef/se	coef/se	coef/se	coef/se
ln(outpatients)	0.569***	0.569***	0.642***	0.614***	0.611***	0.946***	0.925***
	(0.049)	*(0.049)*	*(0.053)*	*(0.057)*	*(0.058)*	*(0.037)*	*(0.035)*
ln(emergencies)	0.067	0.067	0.033	0.066	0.069	0.170**	0.181**
	(0.049)	*(0.049)*	*(0.056)*	*(0.066)*	*(0.067)*	*(0.071)*	*(0.071)*
ln(average annual wage)	-0.093	-0.093	-0.150**	-0.157*	-0.161**	-0.161*	
	(0.064)	*(0.064)*	*(0.068)*	*(0.082)*	*(0.082)*	*(0.091)*	
ln(discharges)	-0.165	-0.362***	0.044	0.427***	0.434***		
	(0.448)	*(0.080)*	*(0.068)*	*(0.061)*	*(0.059)*		
ln(occupancy rate)	0.144	0.335**	-0.101	0.079			
	(0.447)	*(0.131)*	*(0.119)*	*(0.138)*			
ln(deaths)	0.289***	0.290***	0.437***				
	(0.043)	*(0.043)*	*(0.046)*				
ln(beds)	0.417	0.613***					
	(0.446)	*(0.076)*					
ln(averagelengthofstay)	0.198						
	(0.444)						
_cons	7.605***	7.876***	8.619***	7.111***	7.404***	6.446***	4.928***
	(1.063)	*(0.872)*	*(0.914)*	*(1.065)*	*(0.930)*	*(1.013)*	*(0.545)*
/lnsig2v	-3.871***	-3.873***	-3.821***	-2.934***	-2.976***	-2.748***	-2.714***
	(0.312)	*(0.311)*	*(0.363)*	*(0.309)*	*(0.287)*	*(0.266)*	*(0.262)*
/lnsig2u	-3.187***	-3.182***	-2.715***	-3.744**	-3.506***	-3.421**	-3.539**
	(0.468)	*(0.463)*	*(0.382)*	*(1.857)*	*(1.309)*	*(1.383)*	*(1.576)*
Number of observations	239	239	239	239	239	239	239
sigma_v	0.144	0.144	0.148	0.231	0.226	0.253	0.257
sigma_u	0.203	0.204	0.257	0.154	0.173	0.181	0.170
sigma2	0.062	0.062	0.088	0.077	0.081	0.097	0.095
lambda	1.408	1.412	1.739	0.667	0.767	0.714	0.662
gamma	0.665	0.666	0.751	0.308	0.371	0.338	0.305

note: *** p<0.01, ** p<0.05, * p<0.1

Annex.Likelihood.Tests

SF Likelihood-Ratio Tests	LR chi2(r)	Prob > chi2	Null hypothesis: r=0
Full VS Full -1	(1) = 0.14	0.7065	Not reject
Full -1 VS Full -2	(1) = 57.70	0.000	Reject
Full -1 VS Full -3	(2) = 128.57	0.000	Reject
Full -1 VS Full -4	(3) = 128.85	0.000	Reject
Full -1 VS Full -5	(4) = 143.97	0.000	Reject
Full -1 VS Full -6	(5) = 149.25	0.000	Reject
Best model	Stochastic Frontier Full -1		

Pooled OLS Likelihood-Ratio Tests	LR chi2(r)	Prob > chi2	Null hypothesis: r=0
Full VS Full -1	(1) = 0.16	0.6846	Not reject
Full -1 VS Full -2	(1) = 59.78	0.000	Reject
Full -1 VS Full -3	(2) = 127.81	0.000	Reject
Full -1 VS Full -4	(3) = 128.14	0.000	Reject
Full -1 VS Full -5	(4) = 143.23	0.000	Reject
Full -1 VS Full -6	(5) = 148.51	0.000	Reject
Best model	Pooled OLS Full -1		

Fixed-effects Likelihood-Ratio Tests	LR chi2(r)	Prob > chi2	Null hypothesis: r=0
Full VS Full -1	(1) = 0.82	0.3642	Not reject
Full -1 VS Full -2	(1) = 12.62	0.0004	Reject
Full -1 VS Full -3	(2) = 31.94	0.000	Reject
Full -1 VS Full -4	(3) = 32.81	0.000	Reject
Full -1 VS Full -5	(4) = 32.97	0.000	Reject
Full -1 VS Full -6	(5) = 33.09	0.000	Reject
Best model	Fixed-Effects Full -1		

Annex.Table.CostEfficiency

exp(u_h)	Obs	Mean	Std. Dev.	Min	Max	Most Efficient	Least Efficient
Full Model	239	1.15668	0.0798912	1.03351	1.599697	HOSPITAL SANTO ANTÓNIO 2006	CENTRO HOSPITALAR DE CALDAS DA RAINHA 2006
Full Model-1	239	1.15733	0.0805233	1.0335	1.603784	HOSPITAL SANTO ANTÓNIO 2006	CENTRO HOSPITALAR DE CALDAS DA RAINHA 2006
Full Model-2	239	1.23547	0.1516201	1.03404	1.880148	HOSPITAL SANTO ANTÓNIO 2006	CENTRO HOSPITALAR DO BARLAVENTO ALGARVIO -LAGOS 2004