A Work Project, presented as part of the requirements for the Award of a Masters Degree in Finance from the Faculdade de Economia da Universidade Nova de Lisboa.

THE HIDDEN VALUE BEHIND CAPITAL STRUCTURE DECISIONS

CARLOS AFONSO BI FRANÇA E SILVA

Nr. 175

A Project carried out on the financial area, with the supervision of:

Professor Pedro Santa-Clara

2009, June

Final Report
ABSTRACT

I estimate the optimal capital structure for a growth company, for which market value is dictated by its highly volatile nature. I study the welfare impact of corporate taxes, analyzing their economic effect of inducing higher bankruptcy levels. Assuming that management always seeks to optimize the market value of company’s assets, I find a significant loss of value that varies negatively with volatility. Flow and stock insolvency are important for the maximization of capital structure, and I compare both, modeling the value of the company as an option on its revenues. These are not only highly significant for R&D and startup companies but also have significant welfare consequences. I compare the options of liquidation and re-financing and find a clearly important role of early liquidation for R&D frameworks. I complement the study with a comparative statics analysis estimating the impact of risk to the value of the firm and the optimal capital structure decision, for a cross-section of firms. The results present quantitative evidence that reinforce the literature of trade-off and capital structure applied to growth companies.

Keywords: Capital Structure, Real Options, R&D valuation, Simulation.

INTRODUCTION

We know a lot about what conditions capital structure decisions, but not much about either how to estimate accurately the real value-maximizing level of debt for a specific firm, or how to estimate the impact of these decisions for the whole economy. While there is an extensive number of economically significant factors that affect capital structure decisions,
balancing tax advantages against bankruptcy costs is consensually accepted in the profession as a significant aspect to be taken into account. Decisions based upon this premise may nevertheless lead to a perverse social effect. Corporate taxes impose a distortion on economy by giving managers the incentive to choose higher leverage structures. I will show that as a direct consequence of this, the number of bankruptcies increase leading to a market inefficiency, as part of the firm’s value can be appropriated by neither government, nor debt/stock-holders and is therefore lost. This paper explores these two concepts. I propose a valuation framework\(^1\) for the estimation of optimal debt levels and use the results to deduce the impact of taxes on social welfare resulting from bankruptcy levels.

The capital structure decision, key to every company, assumes a particularly interesting role for R&D or startup companies exhibiting high operational volatility. There is empirical evidence of R&D managerial preference for conservative levels of debt (Long and Malitz (1985), Long and Malitz (1983), Myers (1977), etc.). A proposed explanation\(^2\) for this conservativism is the presence of costs\(^3\) other than bankruptcy, varying positively with debt ratios, impacting such firms to a wider extent than they do for value stocks. Amongst other contributions that extend the bankruptcy costs, under-investment is of most importance to R&D (for instance Myers\(^4\) (1977)). Growth companies that by definition have their value linked to cash-flows deferred to the future are heavily dependent on ensuring the

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1 I explore the extensive empirical research on capital structure in appendix I, and find that most tests focus on proving a relation between debt ratios and a multitude of factors (volatility, growth options, size, etc), but there are not as many contributions towards measuring these relationships.

2 Other explanations include the need to control agency costs of debt, that can only be done through limiting debt outstanding, since the effectiveness of bond covenants is reduced if a firm’s value is strongly based on intangible growth options (Long and Malitz (1983)); (O’Brien (2003)) points out both the necessity to keep a steady flow of cash for R&D windows of opportunity and product market introduction (financial slack).

3 The cost of under-investment is described for example in Myers (1977).

4 Myers proposes a theory under which firms limit borrowing even in the same perfect and complete capital markets proposed by Modigliani and Miller.
availability of funds to carry-out their growth and investment options. Myers builds on this notion to include the cost of under-investment, for which management will rationally follow a different decision path than they would for an unlevered investment (or riskless debt) and will tend to pass up positive opportunities, investing only when the expected gain is higher than the promised payment to bond-holders.

To fully capture the impact of these intrinsic R&D costs and the value of management decisions\(^5\) for highly volatile investments, different valuation techniques besides the traditional discounted cash-flow approach must be used. Traditional techniques value investments in one step accounting neither the importance of flexibility on investment decisions nor the possibility of risk changing over the course of the project. I believe that the study of R&D companies is pointless if such features are not captured\(^6\).

I propose the use of a framework that allows the liquidation of the company based on its market value (stock distress), at each period, long before the project takes a negative turn with debt-holders taking possession of company’s assets (flow distress). I do this by modeling the firm’s value as an American call option on the firm’s revenue (that in turn follow a stochastic process with time-varying volatility).

Research is vast in models for company valuation and capital structure (nevertheless, contributions to the latter, model mostly flow distress, choosing to invest until firm’s assets reach zero, regardless of its economic value). Previous studies in this area that have developed concepts relevant to this paper include Schwartz and Moon\((2001)\) and Schwartz and Gorostiza \((2000b)\) that take into account stock distress utilizing an option theory framework similar to the one I apply; Strebulaev \((2007)\) estimates the optimal capital structure maximizing the value of the company at time zero, allowing for costly re-

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\(^5\) The option value and undervaluation of the DCF approach is extensively explained in Athanassakos \((2007)\).

\(^6\) Option theory addresses these issues with the viewpoint of stepped investment decisions, acknowledging that risk varies as a project evolves.
financing in liquidity stress which is also included in my contribution; Goldstein, Ju and Leland (2001) model the cash flows as a geometric Brownian motion and solve for the optimal coupon and bankruptcy level allowing for debt levels to change through time; Miao (2005) models the level of technology idiosyncratic shocks (innovation level) and chooses to invest while above a certain threshold\(^7\), but does so using an equilibrium model not suitable for an individual firm calculation and therefore difficult to apply in this case.

I adapt the model derived by Schwartz and Moon (2000)\(^8\) in which risk takes the central role, adding capital structure features such as in Strebulaev (2007). Schwartz and Moon developed this model to value Internet companies and make sense of the irrational exuberance of stock prices\(^9\) prior to the dot.com crisis. They model stochastic revenues highlighting volatility in its several forms\(^10\) to a much larger extent than other authors, which is important if we expect the option value to hold a significant share of the firm’s value.

There is a multitude of factors influencing capital structure decisions, but the most decisive can be classified into two main families\(^11\): the Trade-Off between taxes and several costs (bankruptcy, agency, under/over-investment, etc.) or the Information Asymmetry theories (market timing, pecking order and signaling). The Trade-Off concept is the basis of this paper as it not only has an important economic reasoning when applied to R&D valuation but is also easy to model in simulation.

\(^7\) He chose this setup up to capture the under-investment problem in Myers (1977).

\(^8\) The original model is disregards capital structure; I introduce it and include the cost of re-financing under liquidity stress.

\(^9\) A reference to Shiller’s book that forecasted the dot.com crash. A lot of literature was developed in the late 90’s by researchers and practitioners making use of real options to explain the difference in valuation found by DCF approaches and observed market prices.

\(^10\) They model high (stochastic) time-varying expected rates of growth and unanticipated time-varying shocks to the expected rate of growth.

\(^11\) I propose my categorization in appendix I, where I modestly survey the capital structure literature, recognizing that a lot has been proposed since the last survey available (e.g. Market Timing theory, etc).
Based on the results of the simulation framework that models the company operations as an American real option on the revenues, with capital structure decisions, I will measure the perverse social welfare impact of taxes and perform a comparative statics analysis to capture the impact of risk to financing decisions for a cross-section of firms in order to demonstrate the impact of capital structure decisions both from the viewpoint of rational management and governmental taxation policy makers.

The rest of the paper is organized as follows, section 2 presents details of the model, data, simulation methods and assumptions. Section 3 analyses the findings and valuation, using amazon.com to calibrate the model. Here are included sub-sections for flow and economical distress, comparative statics and welfare analysis. Section 4 analyses the results and summarizes the conclusions. Lastly the appendix reviews the capital structure literature, classifying the vast contributions (from 1958 to 2008) to capital structure including empirical evidences into a survey that I very modestly attempt at.

THE MODEL

To value a company that holds significant growth options, some stylized facts are of crucial importance. First, volatility is what drives value. Therefore, revenues are modeled using a geometric brownian motion but accounting not only the volatility in revenues but also making the drift stochastic (and time-varying) and bringing unanticipated changes of the drift into the equation\textsuperscript{12}. Second, startup companies exhibit extremely high volatility and growth rates when they initiate operations, but converge after a period to average industry values. In the model, these processes are made mean-reverting to capture this feature.

\textsuperscript{12} In their original contribution, Schwartz and Moon find that the variable that most impacts firm value is in fact the time-varying volatility in the drift.
Lastly, the model is developed assuming a limited liability company and for simplicity, a time horizon after which the company terminates operations and distributes the remaining cash-flow to share-holders.

To solve for the optimal capital structure I maximize the company value at time zero by simulation, for several debt levels\(^1\). According to the trade-off theory, the total value will be given by the sum of the equity-holder value, debt-holder value and taxes.

\[
V_0 = V_{0,E} + V_{0,D} + V_{0,T}
\] (1)

While the total value will remain constant\(^2\), management will rationally choose to increase the share available to investors at the expense of taxes. Specifically, managers will choose to optimize the first two components on the right side of equation (1).

The value of the equity is the present value\(^3\) of the assets \(\hat{A}_T\) remaining at the end of the firm’s life (time \(T\)). The dynamics of the assets evolve, by accumulating net income, as all earnings are retained in the company until the horizon to avoid complicating the model with a dividend policy.

\[
V_{0,E} = E_Q \left( \hat{A}_T e^{-rfT} \right)
\] (2)

\[
d\hat{A}_t = NI dt
\] (3)

The value of debt in time zero is the sum of the present value of all debt payments (DP). As DP is constant, the value depends only on the liquidation or bankruptcy of the company.

\[
V_{0,D} = E_Q \left( \int_{t=1}^{T} DPe^{-rfT} dt \right)
\] (4)

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\(^1\) To find an accurate debt level I developed a maximization algorithm that iteratively decides for a direction of search and divides value-debt_level curve in half towards a neighbor known point.

\(^2\) The value remains constant for the same amount of bankruptcies. Increasing debt levels will increase the number of bankruptcies and therefore will reduce the total value.

\(^3\) The value is discounted under the risk neutral measure at a constant risk-free rate.
The present value of taxes (TX) is given by equation (5). T can be either the horizon or the period when bankruptcy (flow or stock) happen.

\[ V_{0,e} = E_0 \left( \int_{t=1}^{T} TX_t e^{-tf} dt \right) \]  

(5)

The net income (NI\textsubscript{t}) is calculated from equation (6)-(11), where DP is the debt payment and TX\textsubscript{t} the tax expense in period t. Taxes are only paid if the loss carry forward (L\textsubscript{t}) equals zero. Its dynamics are given by equation (8).

\[ NI_t = EBIT_t - DP - TX_t \]  

(6)

\[ TX_t = (EBIT_t - DP)r_c \]  

(7)

\[ dL_t = -NI_t dt, (if \quad L_t > 0) \quad \land \quad dL_t = max(-NI_t dt, 0), (if \quad L_t = 0) \]  

(8)

\( R_t \) is the revenue at time t, COGS the cost of goods sold and SGA the selling, general and administrative expenses. COGS is proportional\textsuperscript{16} to the revenues and SGA has both a proportional part of variable costs and a fixed cost FC constant in time. These parameters were estimated in Schwartz and Moon (2000) and are shown in table 3 in the appendix.

\[ EBIT_t = R_t - COGS_t - SGA_t \]  

(9)

\[ COGS_t = \alpha R_t \]  

(10)

\[ SGA_t = \beta R_t + FC \]  

(11)

There are two sources of uncertainty in this model both incorporated in the revenues which follow the geometric Brownian motion of equation (12).

\[ \frac{dR_t}{R_t} = \left[ (\mu - \lambda_1 \sigma) dt + \sigma dz_t \right] \]  

(12)

The stochastic equation captures the high uncertainty of an R&D or startup company. Both the drift and the volatility are time-varying. The drift has been adjusted downwards

\textsuperscript{16} The constants \( \alpha \) and \( \beta \) were estimated by regression in the original contribution.
with the market price of risk \((\lambda_1)\) so that valuation in equations (2), (4) and (5) can be done using the risk-free rate using the adjusted expectation operator.

The drift is given by equation (13). It is a mean reverting process, also risk adjusted by \((\lambda_2)\), modeling an initial high expected rate of return that with time falls to a more conservative level. The unanticipated change in the growth rate of revenues is given deterministically by the zero-reverting equation (14).

\[
d\mu_t = [k(\bar{\mu} - \mu_t) - \lambda_2 \eta_t]dt + \eta_t dz_t^2
\]

\[
d\eta_t = -k_2 \eta_t dt
\]

The volatility in the revenues is given by the deterministic mean reverting equation (15).

\[
d\sigma_t = k_1(\bar{\sigma} - \sigma_t)dt
\]

Both volatility parameters start high but volatility in revenues converges to a lower value while shocks to the rate of growth of revenues disappear as the company matures.

Cash varies with each period’s net income\(^{17}\), until either it is exhausted and reaches zero or a decision of liquidation is made, based on the market value of the company. When assets are exhausted bankruptcy happens and is accounted to the calculation of default probability. At each period the value of future cash-flows is forecasted using the Longstaff and Schwartz\(^{18}\) (2001) method. Forecasting the market value models the management decision to liquidate or continue in operation. The case when the company continues to operate and cash is below zero, models a creditors protection in which the company can negotiate and eventually leave the troubled situation, evolving towards positive grounds. To avoid that the model allows the company to stay in operation despite very negative results,

\(^{17}\) To make share-holders indifferent to when they receive the company cash-flows and therefore avoid defining a dividend policy, assets compound each period at the risk-free rate.

\(^{18}\) The decision of continuing or liquidating the company is made each period by comparing the current value of the company with the fitted value from a cross-sectional OLS, regressing next period’s value on the state variables of the model (current growth rate in revenues and the current revenues in levels and raised to the second and third power).
I introduce a parameter to emulate the increased difficulty in raising new financing (therefore forcing stock distress sooner than would otherwise occur).

Some simplifications are obvious from the equations. First, the model has no investment decisions and no CAPEX. An investment policy would be interesting to add to the framework, especially accounting the role of innovation. Several studies that use Poisson draws to model the discovery of ideas would be easily adapted to bring more realism into the setup of real options (Schwartz and Gorostiza (2000a), Schwartz and Gorostiza (2000b), Schwartz (2004), etc.).

Second, depreciation tax shield was not included in the model, even though it would also be easy to do. Depreciation would nevertheless complicate the analysis of the tax effects to debt level (contributions that could be used are Schwartz and Moon (2001)).

Third, costs: variable costs are a constant percentage of revenues, imposing a deterministic effect in EBIT in each period. This feature does not account for a learning curve that is part of every company. More importantly, fixed costs stay constant throughout time, eventually leading to an underestimation of bankruptcies in the model. Catastrophic events are also important for the value of these companies (e.g. competition and technology shocks). There is a significant amount of contributions that make use of Poisson variables to include negative value-destroying jumps ((Pyndick (1993), etc.).

Fourth, interactions with other companies would be interesting to model in order to capture a company much dependant on a single product, and thus largely impacted by competitor’s actions. A recent area, that mixes real options with game theory to value R&D companies has given interesting contributions for a case such as this one (e.g. Miltersen and Schwartz (2004)).

Lastly, debt payments are kept constant throughout the lifetime of the firm. The most simple approach, while there are recent empirical contributions that point to time-varying
target debt ratios (Leary and Roberts (2005) and Hovakimian and Opler and Titman (2001)).

The reason not to model all these features was the trade-off between realism and simplicity. While the inclusion of some of those would not be difficult to achieve, it would complicate the analysis and deviate the attention from the debt-equity choice focus.

Data used to calibrate the model is reported in appendix III. All values, including some that are used to compare simulation results are based on published financial information from Bloomberg and amazon.com’s financial statements reported until the 3rd quarter of 1999 (the same data utilized in the Schwartz and Moon (2000)). Amazon.com is an iconic example of the .com era. Heavily biased towards growth, amazon exhibits a p/e ratio of 78 and is heavily dependent on its innovation capabilities (it invests 9.9% of its total sales in R&D expenditures). What may appear at first a simple business model is in fact a multi-level e-commerce platform based on state-of-art patented algorithms for recommendation and customer tracking.

RESULTS

I. Basic Results

The optimal estimated capital structure for amazon.com is reported in table 1 along with the corresponding debt-to-value and equity-to-value ratios. The model estimates significantly conservative levels of debt, in line with the empirical evidence for growth companies. (Myers (2001)) reports debt levels for different industries, examplifying the case of pharmaceutical companies typically operating at negative debt ratios19, besides the

19 They hold cash and marketable securities on excess of their outstanding debt, being therefore net lenders.
overly quoted case of Microsoft. Notice that amazon exhibits a large default probability of 35.7% (stock distress is 36.6%) after optimizing the debt level, from an already high starting all-equity probability of 30.5% (33.4% stock distress). It is not surprising that amazon was already near its debt capacity before the optimization.

<table>
<thead>
<tr>
<th>Capital Structure</th>
<th>Value (Bio US$)</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>7.659</td>
<td>97.3%</td>
</tr>
<tr>
<td>Debt</td>
<td>0.216</td>
<td>2.7%</td>
</tr>
<tr>
<td>Debt plus Equity</td>
<td>7.875</td>
<td></td>
</tr>
<tr>
<td>Total value (with taxes)</td>
<td>11.426</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Optimal capital structure at time zero.

The value of amazon for each level of fixed debt payment is reported in figure 1. The optimal debt has an expected value of 0.216 billion US$ and implies fixed responsibilities of 6 million US$/quarter.

Figure 1: Optimal Debt Values. The debt payment levels that maximize debt, from the perspective of Debt plus Equity.

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20 Amazon has at time zero revenues of 906 million US$ per quarter and variable costs that amount to 94% of sales, leaving a profit margin of 6%.
II. Flow Distress, Probability of bankruptcy

Valuation under different capital structures has to account the increasing probability of default (either flow or stock) that offsets the increase in value by means of tax shield. As the probability of default increases, the expected cost of bankruptcy\(^2\) also increases thus reducing the overall expected value of the firm. This might prove unbearable to most investors holding claims on the company cashflows. The bankruptcy distribution is reported in figure 2 for the unlevered and optimal structures of capital.

For all simulated scenarios, default occurs in the first years of operation when the initial cash balances are exausted. Increasing debt levels shifts default towards the beginning of the operations at the same time as it increases overall probability. In the not reported cases of extremelly high debt, bankruptcy happens 100% of the times in the first period. If the company survives the first years of operation, bankruptcy probability falls to insignificant values (after year 10 is less than 3%).

<table>
<thead>
<tr>
<th>Probability</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>1</td>
</tr>
<tr>
<td>0%</td>
<td>2</td>
</tr>
<tr>
<td>0%</td>
<td>3</td>
</tr>
<tr>
<td>0%</td>
<td>4</td>
</tr>
<tr>
<td>2%</td>
<td>5</td>
</tr>
<tr>
<td>10%</td>
<td>6</td>
</tr>
<tr>
<td>7%</td>
<td>7</td>
</tr>
<tr>
<td>4%</td>
<td>8</td>
</tr>
<tr>
<td>2%</td>
<td>9</td>
</tr>
<tr>
<td>1%</td>
<td>10</td>
</tr>
<tr>
<td>0%</td>
<td>Rest</td>
</tr>
</tbody>
</table>

Unlevered: 0.0% 0.0% 0.0% 0.0% 2.8% 10.0% 7.2% 4.2% 2.1% 1.5% 2.8%
Optimal D/V: 0.0% 0.0% 0.0% 0.0% 10.0% 12.5% 9.0% 3.0% 1.8% 0.7% 1.6%

Figure 2: Bankruptcy Probability Distribution.

This feature models the extremely high uncertainty of a venture such as amazon in its initial years, as it goes through an affirmation period. During this period its revenues are highly

\(^2\) The expected cost of bankruptcy is the direct and indirect costs weighted by the probability of default.
volatile and its fixed costs play a crucial role for its survival. After the first initial years, if cash balances are not exhausted by strongly negative results, the fixed costs that push bankruptcy scenarios in the first years will become less significant\textsuperscript{22}. In the long run the rate of growth of sales will diminish, its volatility will tend to zero, as the company reaches maturity and solidifies its position in the market.

Figure 3 depicts another perspective of the bankruptcy probability, reporting accumulated figures, where total results are easier to see. For the optimal scenario, default occurs 35.74\% of the times.

Allowing the liquidation of the company before it falls in distress, even though augmenting the number of bankruptcies, significantly increases the value for equity-holders as they do not have to wait until cash is exhausted and debt-holders take possession of the company\textsuperscript{23}. This model always forces optimal bankruptcies, therefore solving the Management Entrenchment problem\textsuperscript{24} that is found on numerous times in real cases. Management in this scenario always acts in the best interest of share-holders.

If we allow in the model for the company to recover after failing to match payments (entering a flow distress situation), introducing increased costs of re-financing, we reach exactly the same conclusions. The value of this (waiting-for-a-recovery) option proved insignificant in simulation. R&D companies are strongly affected by timing: failing the window of opportunity to introduce a product in the market leads to severe loss of competitiveness\textsuperscript{25} and devalue of the growth opportunities. While forecasting earlier the

\textsuperscript{22} This is modeled by the drift in the geometric brownian motion, making sales grow in the long run.

\textsuperscript{23} The value of the company is strongly dependant on the option value, which is to say that the value of ownership decision plays a significant role in the model.

\textsuperscript{24} Debt is issued in its optimal level (maximizing value), disregarding other considerations.

\textsuperscript{25} Not to mention the capacity to attract qualified labor force. The importance of human resources to Internet and software-based companies as a means to foster innovation and explore growth options is crucial.
lack of economic viability increases significantly the value of the firm, letting the venture continue, waiting for a recovery along the path is of much less interest for an R&D company. Having small cash balances and lacking sellable assets, it typically enters a stage in which death is inevitable.

![Bankruptcy Probability Distribution (Accumulated)](image)

**Figure 3: Bankruptcy Probability Distribution Accumulated.**

**II. Welfare Analysis**

Debt tax shield introduces an incentive for management to rationally increase debt levels. This has the perverse effect of inducing a larger number of bankruptcies with consequent loss of value. This inefficiency is reported in figure 1, where the total value\(^{26}\) of Amazon for its optimal estimated debt level is compared to its estimated value under a no-tax setup (thus with no incentive to have debt). The size of the charts captures the loss of value in the scenario of a 0.35 tax rate (the rightmost plot).

The estimated optimal value of the company would be 11.485 billion US$ for the no-tax scenario and 11.426 billion (less 59 million) induced by a higher level of bankruptcies, from 33.4% (default probability) to 36.6% after debt-equity optimization. The introduction of taxes leads to a destruction of 0.51% of total company value.

\(^{26}\) Total value is the expected value of equity, debt and taxes. While debt plus equity increases by means of tax shield, the total value diminished due to the increase in bankruptcy scenarios.
Figure 4: Optimal Capture Structure from the point of view of management, destroys value by inducing a higher number of bankruptcies.

Even for this conservative estimate of capital structure there is a significant effect of the tax shield when looking from the perspective of the overall value for society. There is a non negligible social loss to be considered by taxation policy makers. This value would even be more extreme for companies exhibiting state variables less extreme as those of amazon. For a company that started operations with less significant probability of default, the effect of optimizing its capital structure by means of tax shield, would lead to a higher change in default probability and thus to a more extreme destruction of value.

Figure 5: Effect of taxes to the probability of bankruptcy.
Figure 6 reports the comparative statics effect of tax shocks to the total value of Amazon and for its bankruptcy probability.

The right chart reports the value under each tax rate as a percentage of the value under the no-tax scenario. There is a loss of 0.51% for the normal US tax rate and 0.67% when taxes increase by 5%. In extreme (unreported) scenarios the drop becomes steeper and eventually taxes alone become unbearable for a company to operate.

In the leftmost chart, bankruptcy levels increase positively and significantly with taxes, explaining the loss of total value in the first chart.

### III Comparative Statics

I analyze the effect of shocks in the parameters of the model, reporting the ones that proved to be more significant in the simulation (variable costs and volatility). Their effect on value (debt and equity), probability of bankruptcy and change in the optimal debt-to-value ratio is discussed below.

*Variable cost parameters*
The first and most obvious parameters that significantly influence the company are variable costs. As the OLS-estimated variable costs ($\alpha$ and $\beta$) are assumed to remain constant throughout the lifetime of the company, they have a stronger importance than their fixed counterpart, as the latter while having the power of bankrupting the company in the first years, soon becomes less relevant\textsuperscript{27}. As would be expected, even a conservative change of 1% upwards or downwards, exerts a significant effect on the three analyzed aspects.

Optimizing the capital structure of the company for the new cost figures, the overall value moves from 7.875 billion US$ to [6.364; 9.177]. Overall this represents a variation of -19% in value when costs increase and 16% when costs drop. Debt-to-value varies between [2%; 4%], increasing the debt capacity of the company when costs drop. Bankruptcy also varies positively with costs from 37% to [33%, 44%].

\textsuperscript{27} Since fixed costs do not increase in time, its effect after the initial years becomes much less important.
Valuing any company is an exercise in which costs have to be carefully estimated. It becomes apparent the role of management in driving costs down, through a learning curve as it may be the difference between success and bankruptcy (notice the high variation in distress probability) as a 1% increase in costs lead to 19.5% increase in the probability of default.

**Volatility parameters**

I test the effect of change in both the volatility of revenues (from 0.1 to 0.05 and 0.15) and rate of growth of revenues (from 0.03 to 0.01 and 0.05). Their behavior is identical, but the change in the rate of growth is far more extreme. Value changes from 7.875 billion US$ to [7.813; 7.948] for the $\sigma$ parameter and [4.040; 24.146] for $\eta$. The volatility in the rate of growth has the capacity of increasing value in 200% or decreasing it to 50% of its value.

Debt ratios vary from 2.7% to [1%; 6%] for $\eta$ and [2.6%; 3.3%] for $\sigma$. Bankruptcy varies from [35%; 40%] for $\sigma$ and [21%; 43%] for $\eta$.

There is an important difference between the behavior of the two parameters. Volatility in the revenue drives the value slightly down as it increases default in 3%, while the shock in the rate of growth has the opposite effect (it doubles value). This is a finding in line with the observation performed in Schwart and Moon (2000). While volatility in revenues drives bankruptcy and therefore predicts lower debt-ratios, less volatility lead to stable revenues allowing for a higher debt capacity.

Volatility in the rate of growth has the opposite effect. It allows for extremely high (or low) rates of growth and therefore it captures the value of the option (as positive paths lead to large cash-flows and negative paths are liquidated early in the simulation). These are the

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scenarios where the option exerts its value, and what would be expected from the intuition of real options: the more volatility Amazon has, the more valuable it will be. Volatility parameters evolve according to: $\sigma$ increases, so does distress, but value and $D/V$ decrease; $\eta$ increases, so does distress and value, but $D/V$ decreases.

<table>
<thead>
<tr>
<th></th>
<th>D</th>
<th>U</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>3.3%</td>
<td>2.6%</td>
<td>2.7%</td>
</tr>
<tr>
<td>$\eta$</td>
<td>5.5%</td>
<td>0.8%</td>
<td></td>
</tr>
<tr>
<td>Cost</td>
<td>4.9%</td>
<td>2.0%</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Optimal Debt ratios for the perturbed parameters (U: upward change; D: downward change).

Welfare values can be directly inferred from the estimated bankruptcy values. For the perturbed parameters, the value lost varies significantly solely for the shock on the volatility in the rate of growth of revenues. In this case the loss varies between $[0.18\%; 1\%]$. Being that the highest loss (1%) happens when $\eta$ drops in value. A conclusion can be drawn, that the less volatile a company is (and the less valuable is the option) the higher will the social loss be. This is explained by the bankruptcy values. For the increase in volatility there is a probability of default of 43% for the optimized case compared to 40% for the no-tax. In negative shock, the values are 21% compared to 17%. As the value for the no-tax incorporates a very extreme probability driven by the high volatility values (not the debt level), the change introduced by taxation is less extreme. When volatility is high, the effect of taxes therefore is less extreme, as bankruptcies are already at a high level. Therefore the loss of welfare is higher for more stable industries, for which debt capacity is higher and plays a more important role in explaining bankruptcy levels.
CONCLUSIONS

The analysis of a growth company under a target static debt ratio, modeling the trade-off between taxes and bankruptcy leads to important conclusions, for investors doing R&D valuation, managers and governmental policy makers.

Social welfare is significantly affected by the way taxes are constructed. There is a significant loss of 0.5% of company value, which is predicted to be more extreme for less volatile cases (comparative statics proves that conclusion, estimating a 1% loss). Not always what is best for managers (and share-holders) is optimal for society.

There are also important conclusions to be drawn for R&D valuation, managers and investors. Estimating correctly (and managing) variable costs, together with the volatility in the expected rate of growth are the single most important factors for valuation. They exert a significant effect on bankruptcy levels and therefore have a strong relevance for the optimal structures of capital. Being able to forecast earlier the lack of economic viability increases significantly the value of the firm. Letting the venture continue, waiting for a recovery along the path is of much less interest for an R&D company. These findings pinpoint the role of management for the value of R&D ventures.

Lastly, I find clear evidence of the trade-off in the framework under analysis. This theory has been criticized as bankruptcy costs do not seem to explain why debt levels are observed at levels lower than would be expected. One has to take into account not only the cost of bankruptcy, but also its probability. Considering a high probability of default, the trade-off is observable, and it in fact predicts a very small debt capacity. These results reinforce the literature on capital structure for growth companies. Debt capacity is strongly dependant on bankruptcy levels, and the first years of operation call for low leveraged structures.
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Agency Costs


Asymmetric Information


Market Timing


Management entrenchment


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APPENDIX I

SURVEY OF THE CAPITAL STRUCTURE LITERATURE

Myers (1984) inspired by a prior research note\(^{29}\) sums up the capital structure problem as “How do firms choose their capital structures? We don’t know”. Later, Myers (2001) summarizes the state of current knowledge “There is no universal theory of the debt-equity choice, and no reason to expect one”. In fact, while apparently disappointing, this reveals the level to which this field has matured. We have obtained sufficient knowledge to understand and thrive in the absence of a single all-encompassing structure of strategic decision-making that fits every case, market, Industry and reality. We also managed to intuit the economic factors that influence capital structure decisions. We may have not reached a conclusion for a definitive financing strategy (e.g. the optimality of a target debt level) but we do have a sizable knowledge about financing tactics (e.g. how to decrease agency costs or increase tax shields). Moreover the most relevant theories coexist well, they do not contradict each other but rather focus on different non-exclusive factors, that can be applied in different scenarios. Thus all of these theories, in their own way, can be used to shed light on particular aspects of financing decisions and serve a financial manager in diverse situations.

I propose below one categorization of capital structure theories, surveying the most relevant literature. There are many surveys of capital structure but they all begin with Modigliani and Miller (1958) as I do.

Modern theories of capital structure begin with Modigliani and Miller (1958) defining under which conditions the choice of financing is irrelevant to the value of the firm. Their first two propositions of capital structure irrelevance were later enriched via a communication (Modigliani and Miller (1963)), correcting their conclusions in the presence of taxes, which they recognized as being wrong in their first work. Their overall contribution was extended with the introduction of offsetting costs, to avoid financing scenarios solely based on debt. This static trade-off theory anticipates the optimality of an equilibrium debt level between tax shields and costs arising from holding debt, which is to this day one of the most important capital structure models. Their fundamental and main conclusion that total company value does not vary regardless of the division of capital, is today undeniable.

There are, however, different theories that emphasize aspects other than taxes as the key to optimizing capital structure and to some extent researchers have been adjusting Modigliani and Miller’s assumptions to match their hypothesis with empirical observations. It has been realized that besides bankruptcy, additional costs have to be factored in. An example of a factor that has been proposed, is agency costs, first introduced by Jensen and Meckling.

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30 Perfect capital markets (with perfect competition and full access to all players), similar conditions of debt for firms and individuals and full symmetric information.

31 The aim of their paper was to lay the foundations of a theory of valuation of firms under uncertainty, to which the cost of capital is key.

32 Modigliani and Miller (without taxes) Proposition I argues that any advantage of leverage can be achieved through homemade leverage and thus there is no added value by the firm. The resulting conclusion is that the weighted average cost of capital should remain constant. They prove empirically this result by regressing cost of capital on debt/equity for oil and electric companies and finding no economically significant coefficients. Proposition II explains the first by saying that the cost equity increases with debt making \( \text{wacc} \) constant. There is no possibility of decreasing the cost of capital by accepting less-expensive debt. A third proposition reminded the separation from the financing instrument and the decision if a project is worthwhile.

33 Total value remains the same, but most importantly the value of equity on which managers focus is increased by capital structure decisions.

34 They first define the concept of Agency Costs, building on existing ideas traced back to Adam Smith (1776) “The director of such [joint-stock] companies, […] being the managers of other people’s money […] like the stewards of a rich man, they are apt to consider attention to small matters as not for their master’s honour”.


Jensen and Meckling developed a theory of the ownership structure of firms proving that agency costs result from the differing agendas of corporate agents (managers, shareholders and debt-holders), and lead to loss of efficiency as the firm bears all operational costs but is unable to capture its total gains.

A significant effort has been devoted to explaining the source of these costs and consequent mitigation strategies. One can differentiate between two major lines, the conflicts between share-holders/managers i.e. agency cost of equity (Jensen (86), Williamson (88), Stulz (88), etc.) and share-holders/debt-holders i.e. agency costs of debt (Diamond (1989), Ross, Westerfield and Jaffe (2003), etc).

Examples of the effect of agency costs of equity include the positive relationship between the amount of equity owned by management and either the amount of effort they are likely to put in or their propensity towards self-indulgent expenses (Jensen and Meckling (1976); over-investment, the role of free cash-flow and the use of debt to reduce cash-flows (Jensen (1986)); over-investment by managers driven by selfish aspirations towards business growth to the possible detriment of profitability (Ross, Westerfield and Jaffe (2003)); voting rights of equity and anti-takeover measures (Stulz (1988)); unwillingness of incumbent managers to liquidate companies and the role of debt in imposing scenarios in which liquidation is optimal (Harris and Raviv (1990)); (Decamps et al (2006)) follow a free cash-flow explanation using a stylized model to explain how market imperfections lead to...

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35 And consequent mitigation strategies based on capital structure choices.
36 For example through a leveraged stock repurchase, in the same way as dividends are used to reduce free cash-flow.
37 As a familiar over-investment example, one can observe management's tendency to diversify towards unfamiliar areas - acquisition or endogenously - when facing few growth opportunities and significant cash.
38 Stulz finds that managers are likely to use lower debt levels as a protection against takeovers. Convertible bonds as a mean to dilute equity and protect against takeovers are an example of poison pills.
not an optimal target debt level but rather an optimal target cash level that a firm is compelled to follow. One can draw two main conclusions from these theories. First, all these aforementioned costs are mitigated by increasing debt and thus reducing free cash to an optimal (Decamps et al (2006) even focus on the cash itself as the target level followed). Second, there is a distinction between value and growth companies, the latter having a wider amount of growth options and thus more free cash necessity typically having less debt while the former has limited growth potential.

Share-holder/debt-holder conflicts arise from the natural imbalance of equity-debt payoffs due to seniority characteristics of these types of securities, and examples include investment in value-decreasing high-risk projects (Diamond (1989)); less than optimal equity due to under-investment and reduction of equity through dividends (Ross, Westerfield and Jaffe (2003)). Observation estimates the direct costs (legal, administrative, etc.) in the magnitude of 1%-5% of market value (Warner (1977), Weiss(1990)), but indirect costs reach higher values of 10%-23% (Andrade and Kaplan (1998)). If one believes that highly leveraged companies are more likely to forego valuable investments and eventually have to cut-back R&D costs, training, etc. then it is obvious that growth companies are less likely to have large debt levels due to their under-investment costs being higher (Myers (1977)). Myers points out how growth companies in distress are more likely to see difficulties in raising capital to call debt.

While the static trade-off theory elevates taxes and these theories emphasize agency costs and free-cash, still other theories focus on inside information. Given the characteristics of insiders (wielding better estimates of future cash flows), both asymmetric information and

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39 Positive results (negative) lead to high gains (losses) for equity-holders and fixed gains for debt-holders.
the signaling effect of capital structure decisions stand out as important explanations for financing decisions. Asymmetric information has been used in several forms to explain capital structure. (Ross (1977)) is one of the first to propose an explanation based on asymmetric information, in which investors take large amounts of debt as a signal of higher firm quality\textsuperscript{40}; (Leland and Pyle (1977)) propose an explanation based on management being risk-averse, in which increasing debt allows managers to retain a larger part of equity and thus of risk, reducing their welfare (since managers of good firms suffer a smaller decrease of their welfare, again debt will signal quality); (Myers and Majluf (1984)) and (Myers (1984)) propose an additional explanation based on asymmetry of information for the largely studied theory that firms choose their financing sources in a specific order of risk. They justify the ordering of financing sources (internal sources first\textsuperscript{41}, debt second, and finally equity) with the fact that investors see issuance of securities as a signal of a company’s value, thus forcing managers to always issue first the securities that change value the least upon information disclosure. Managers are forced by the market to follow a pecking order and will only issue equity when the level of debt is unbearable.

The Pecking Order, still today a very influential theory, predicts that growth companies will exhibit large amounts of debt (as their cash balances are typical low and they only raise capital after exhausting debt capacity), a clear contradiction to the aforementioned theories, based on agency costs or taxes (and in contradiction with observed data). Value companies

\textsuperscript{40} The reasoning is that if managers have their welfare linked to the success of the company, being penalized in the event of a bankruptcy, since lower quality firms have larger bankruptcy costs, higher quality companies tend to exhibit higher debt, with consequent signaling effect for investors.

\textsuperscript{41} The idea of prioritization can be tracked to several older contributions. Prior explanations (Donaldson (1961), etc.) saw the prioritization of internal funds as a way to avoid issuing costs (not information asymmetry as Majluf and Myers propose).
on their turn, having large amount of cash would tend to have no debt, as they would use the cash to buy back debt.

All of these theories stem from the same cause: information. They nevertheless differ in focus. The Pecking Order fully explains capital structure decisions with the maximization of company value by always seeking the least expensive instrument to finance growth options. Ross's Signaling theory, though similar, does not center around low-cost financing but rather as a way to signal quality and management confidence in the firm’s projects. Lastly, the quite recent and influential Market Timing theory (Baker and Wurgler (2002) and Alti (2006)\footnote{Alti’s focus is not on defining the theory but instead an attempt to empirically estimate market timing impacts on capital structure around IPOs and economical conditions around IPO times.}) also focus on information asymmetry but explains capital structure as the cumulative result of management attempts to time the market. The result is different from the Pecking Order (that predicts a clear order of financing) as market timing predicts that low-leveraged firms are the ones that raised capital when their market valuation was high while highly-leveraged did the same when their valuation was low (there is no rigid order). Welch (Welch (2004)) in an important contribution, reinforces this idea showing empirically that stock returns have a large explanatory power over debt ratios.

While theories of capital structure based on taxes, costs or information asymmetry are the most influential, other theories have arisen. The Management Entrenchment hypothesis (Berger, Ofek, Yermack (1997), Garvey, Hanka (1999), Zwiebel (1996), Lundstrum (2008), etc.) proposes that capital structure is influenced by management willingness to prevent takeovers (for some a way to perpetuate ineffective management, reducing company value); characteristic of products and competition particular to a firm (Maksimovic (1986) and Brander and Lewis (1986)) are also pointed out as key to the debt-equity choice, also
(Maksimovic and Titman (1989)) explain the different debt structures over a cross-section of firms of different industries with the characteristic of a firm’s products, suggesting that if firms can change the quality of their products in a way unperceivable by customers (prior to acquisition) then this leads to higher leveraged structures of capital.

Summing up, the capital structure research can be a confusing area, if one considers the amount of coexisting theories. Nevertheless, an important fact is that in most cases these theories are not mutually exclusive or contradictory: e.g., while trade-off proposes that too little debt does not optimize taxes and too much debt destroys value, cost-based theories (e.g., Free Cash-Flow) suggest that too little debt leads to under-investment and excessive indulging costs, which reinforces the virtue of debt. The same balance between debt and equity would be predicted by following any of these two theories.

One of the appointed difficulties in electing one theory is directly linked to the empirical research that has been published. While there is good support for most of these hypotheses, it has proven to be difficult to test one model against the other (thus deciding on the validity of a specific one), due to the fact that being non-mutually exclusive there is a strong correlation between variables that can proxy for each case and statistically significant coefficients are difficult to interpret as in favor of one single theory. Overall, empirical tests aimed at pointing out the definitive capital structure explanation have so far proven to be powerless. Some examples help clarify the link between theories: Trade-Off

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43 Building on the notion that a firm has an advantage when producing high-quality goods due to the reputation they create (market differentiation), they show that as debt increases the probability of bankruptcy, and thus threatens the firm’s reputation, then debt is negatively related to the incentive to increase quality.

44 With the exception of some interpretations of the Pecking Order, that leads to rather different conclusions, as already mentioned.

45 Besides the fact that all of them, most likely, capture some share of the overall truth about capital structure decisions.
theory despite its obvious appeal seems to fail in practice most of the times. Graham (2000) finds that in a large domestic sample the typical firm was so far from its optimality that by issuing more debt it could double their tax benefits. He points to Management Entrenchment as possible explanation. Some theories are even harder to test (e.g. testing the signaling effect of issuing debt is certainly not an easy task).

Empirical research aimed at quantifying the capital structure problem is also not abundant (one exception is the quantification of bankruptcy costs that has seen a number of contributions (Opler and Titman (1984), Andrade and Kaplan (1998), etc)). We do know for instance that companies should increase their tax shields, but by how much we cannot say. Still today the quantitative nature of the puzzle remains to be solved.

Empirical research proving the relation between leverage and specific factors has seen extensive and important contributions.

Cross-sectional studies prove the relationship between leverage and Market-to-Book ratio, that stands out as one of the most significant factor and one of the best predictors for leverage (with a strong negative relation). The main explanation is that growth firms are likely to keep a financial slack to ensure future capacity to invest and not miss important windows of opportunity (Smith and Watts (1992), Long and Malitz (1985)). Long and Malitz find the most highly leverage companies to be the most mature and capital-intensive firms while the less leveraged are the ones that exhibit high R&D levels or other proxies for growth opportunities (spending on advertising, book-to-market, etc). O’Brien (2003) goes one step further showing clear links between capital structure and being an industry innovator (measured as intensity of investment in R&D). He argues that low leverage helps to maintain competitiveness by ensuring availability of funds to continuous investments in R&D, launch new products and expansions through acquisitions He regresses leverage on
a set of independent variables such as relative R&D intensity, industry profitability, capital intensity, etc. and concludes that capital structure choice has clear impacts on strategy (thus not irrelevant).

Other factors that serve as cross-sectional interpretations include the volatility of cash flows, as more volatile ventures have higher bankruptcy probability, and thus higher expected bankruptcy costs (Long and Malitz (1985), Titman and Wessels (1988), etc); the Size factor, as a larger company under distress will have less fixed costs of refinancing and more assets that it can sell (Titman and Wessels (1988)); Asset Tangibility, is positively related to debt, since tangible assets are more likely to preserve their value in face of default than non-tangible ones (Titman and Wessels (1988)).

Time-series research focus on studying shocks that explain deviations from target structures. Important contributions have served as basis for the Market Timing theory (Baker and Wurgler (2002)) and Management Entrenchment (Friend and Lang (1988)).

Empirical work has therefore pointed important links between factors, while still a lot has to be done to quantify them. Market Timing seems to be, from an empirical point of view, one of the most promising theories to this day (the future will tell its robustness).

The overall puzzle is endowed with a stock of explanations, but still far from being solved. There may not be a single theory of capital structure as Myers points out, but rather practitioners must adapt the multitude of theories to every specific firm to find the optimally fitting solution. It might be that as the Trade-Off theory brings all existing costs together into a sliced pie, also as an analogy it may be that all existing theories contribute to the rational decision of financial managers and make sense only when combined.

Or it may well be that research will go on fulfilling Myer’s predictions in the beginning of this section.
The equations used in the simulation are in their discrete forms, presented below. The discretization was done by direct integration of their continuous counterparts or by applying the Ito’s lemma.

The revenues are given by

\[ R_{t+\Delta t} = R_t e^{\left[ \mu_t - \lambda \sigma_t - \left( \frac{\sigma_t^2}{2} \right) \Delta t + \sigma_t \sqrt{\Delta t} z_t \right]} \]  

(16)

The growth rate of revenues is given by

\[ \mu_{t+\Delta t} = e^{-k\Delta t} \mu_t + \left( 1 - e^{-k\Delta t} \right) \left( \mu - \frac{\lambda \mu e^{-kz}}{k} \right) + \frac{1 - e^{-2k\Delta t}}{2k} \eta_t \sqrt{\Delta t} z_t \]  

(17)

Where

\[ \sigma_t = \sigma_0 e^{-kz_t} + \bar{\sigma} \left( 1 - e^{-kz_t} \right) \]  

(18)

And

\[ \eta_t = \eta_0 e^{-kz_t} \]  

(19)
APPENDIX III

The parameter used to calibrate the model were estimated in the Schwartz and Moon (2000) model for amazon.com.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Estimated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial assets</td>
<td>( A_0 )</td>
<td>0.906</td>
</tr>
<tr>
<td>Initial revenue</td>
<td>( R_0 )</td>
<td>0.356 bio/quarter</td>
</tr>
<tr>
<td>Initial loss carry forward</td>
<td>( L_0 )</td>
<td>0.559 bio/quarter</td>
</tr>
<tr>
<td>Initial expected rate of growth in revenues</td>
<td>( \mu_0 )</td>
<td>0.11/quarter</td>
</tr>
<tr>
<td>Initial volatility of revenues</td>
<td>( \sigma_0 )</td>
<td>0.10/quarter</td>
</tr>
<tr>
<td>Initial volatility in the expected rate of growth in revenues</td>
<td>( \eta_0 )</td>
<td>0.03/quarter</td>
</tr>
<tr>
<td>Industry rate of growth in revenues</td>
<td>( \bar{\mu} )</td>
<td>0.015/quarter</td>
</tr>
<tr>
<td>Industry volatility</td>
<td>( \bar{\sigma} )</td>
<td>0.05/quarter</td>
</tr>
<tr>
<td>Corporate tax rate</td>
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</tr>
<tr>
<td>Risk-free constant rate</td>
<td>( \tau_f )</td>
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</tr>
<tr>
<td>Cost of financing in liquidity stress</td>
<td>( f_s )</td>
<td>0.05/quarter</td>
</tr>
<tr>
<td>Speed of adjustment for the rate of growth</td>
<td>( k )</td>
<td>0.07/quarter</td>
</tr>
<tr>
<td>Speed of adjustment for the volatility of revenues</td>
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<td>0.07/quarter</td>
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<tr>
<td>Speed of adjustment for the volatility of the rate of growth</td>
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<tr>
<td>COGS variable cost</td>
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<td>SGA variable cost</td>
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<tr>
<td>Fixed costs</td>
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<tr>
<td>Market price of risk for the revenues</td>
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<tr>
<td>Market price of risk for the expected rate of growth in revenues</td>
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<td>0.0/quarter</td>
</tr>
<tr>
<td>Horizon</td>
<td>( T )</td>
<td>25 years</td>
</tr>
</tbody>
</table>
Table 3: Parameters used in the simulation. These values were estimated from public data available at late 1999, from Amazon published results and analyst reports. Detail on estimation is available in Schartz and Moon (2000).