Non-manipulability in Walrasian Cost Games*

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Abstract. In this paper, we propose a perfect competition test which checks whether arbitrarily small coalitions of firms, which behave strategically on costs, are able to manipulate prices in their own benefit. We apply this test to economies with a continuum of differentiated producers. We show that, under thickness conditions on preferences, implying a high degree of substitutability among commodities, there is no monopolistic power neither in economies with constant marginal costs nor in economies with cubic costs. We also give an example of thin markets where monopolistic power prevails.

Key words: Perfect competition, monopolistic power, Walrasian equilibrium, thick markets.

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1 Introduction

Manipulability of the Walrasian mechanism is a subtle issue in economies where both the number of agents and the number of goods are quite large. In particular, when there is a large number of firms and each one produces a distinct commodity, manipulability may arise as a monopolistically competitive phenomenon. Chamberlin's (1933) theory of monopolistic competition captured the idea that, in a world of product differentiation, firms of relatively insignificant size can have some monopoly power. The source of this imperfection has been controversial in the literature, since the publication of Chamberlin's work.

Long ago, Kaldor (1935) and, more recently, Jones (1987), suggested that, in a product differentiation framework, any imperfection of competition, in a large numbers environment, is due to deviations from constant returns to scale. Precisely, Kaldor (1935) remarked that "the possibility of product differentiation will by itself never prevent the establishment of perfect competition if economies of scale are completely absent". However, according to Chamberlin (1937), what marks the contrast between monopolistic competition and perfect competition is the shape of demand curve and not the shape of the cost curve. The inadequate substitutability implied by consumers' preferences is responsible for the imperfect elasticity of the demand curve facing each firm and, therefore, is the ultimate source of monopolistic competition.

In modern attempts to model Chamberlinian competition the inadequate substitutability is achieved by breaking away with the idea that proliferation of products leads to increasingly more neighboring goods serving as close substitutes. Dixit-Stiglitz's (1977) large finite symmetric model, Hart's (1985) countable infinite non-neighboring goods model and Páscoa's (1993, 1997) continuum models illustrate this approach. In contrast, modern perfectly competitive models of commodity differentiation depend crucially on the assumption that nearby commodities are regarded as close substitutes, uniformly among consumers. The weak* continuity of preferences defined on measure-bundles, introduced by Mas-Colell (1975), and the uniform condition on rates of substitution among nearby commodities, proposed by Jones (1983, 1984) and Ostroy-Zame (1994), give a precise mathematical characterization of the substitutability required for perfect competition.

Representing the set of suppliers by the unit real interval $[0, 1]$, endowed with Lebesgue measure, guarantees nonatomicity and gives a precise content to the notion of negligible agents. However, this nonatomicity is compatible with either Cham-

berlin's monopolistic competition or with price-taking behaviour. As Ostroy-Zame (1994) already remarked in the pure exchange context, it is thickness of markets (a neighboring goods situation implying a high degree of substitutability) rather than nonatomicity which leads to perfect competition.

The main purpose of this paper is to explore the contrast between monopolistic power and perfect competition, in the light of a manipulability test. We will check whether asymptotically negligible coalitions of firms behaving strategically in costs are able to manipulate prices in their benefits. More precisely, firms' strategies consist in misrepresenting their real cost functions. We say that there is no monopolistic power provided that the possible gains (with respect to Walrasian profits) that coalitions of firms can achieve, by adopting a strategic behavior, goes to zero as the coalition size decreases.

In pure exchange economies with a continuum of agents and commodity differentiation, related manipulability tests were considered by Ostroy and Zame (1994), and Hervés, Moreno and Páscoa (1998). Ostroy-Zame's (1994) test checks the non manipulability of equilibrium prices by withholding endowments, that is, whether an equilibrium price can be approximated by a sequence of equilibrium prices associated with economies where endowments have been withheld by decreasing coalitions. Hervés, Moreno and Páscoa (1998) propose as an alternative test of perfect competition checking whether the coalitional incentives to deviate from a price-taking behavior, by misrepresenting not only endowments but also preferences, goes to zero as the coalition size decreases. Even when we restrict ourselves to price manipulation by changing endowments, Ostroy-Zame test and Hervés-Moreno-Páscoa test are not immediately comparable. On the one hand, in Ostroy-Zame (1994) own words, "the withholding test does not distinguish between (utility-) favorable and unfavorable price manipulations ...", whereas the latter does. On the other hand, the withholding test is related to the lower hemi-continuity of the Walrasian correspondence and the coalitional incentive compatibility test is related to the upper hemi-continuity instead. The test we propose in the current paper is in the spirit of the latter and is therefore closely related to the upper hemi-continuity of the equilibrium correspondence.

Other manipulability tests for pure exchange economies, where a finite number of commodities are traded, were proposed by Roberts and Postlewaite (1976), Otani and Sicilian (1982, 1990), Makowsky, Ostroy and Segal (1995), and Jackson and Manelli (1997). Roberts-Postlewaite check whether the individual incentives to manipulate equilibrium prices vanish as the number of agents in the economy increases, whereas Otani-Sicilian analyze whether true Walrasian allocations in a

large economy can be claimed to be sufficiently robust against strategic manipulations of agents' demand maps. Makowsky-Ostroy-Segal identify conditions on the richness of the domain of preferences that allow to characterize efficient and incentive compatible mechanisms as perfectly competitive, whereas Jackson-Manelli identify conditions on the beliefs of agents under which all market-clearing prices and allocations of the reported economy approximate the competitive equilibria of the true economy.

The model contemplated in this paper has a representative consumer interested in a continuum of differentiated commodities, each one being produced by a different firm. Our paper starts by defining our test of manipulability and presenting an example of thin markets (that is, economies where physical differences among commodities are not superseded by economic substitutability) where asymptotically negligible coalitions can manipulate prices. In this example, the persistence of monopolistic power is achieved even with constant average costs. This example is presented in section 3 and shows that the sequence of profits resulting from manipulation, by decreasing coalitions, has a positive lower bound whereas Walrasian profits are necessarily zero. In section 4, we present a positive result asserting asymptotically non-manipulability under constant marginal costs and thickness conditions on the consumption sector. Sections 5 and 6 address more general costs. For more general cost structures equilibrium profits may be non-zero even though we have a continuum of firms. In fact, in our model of product differentiation we do not have free entry since entering firms do not replicate existing ones. Section 5 provides an example of an economy with non-zero Walrasian profits. Section 6 presents the non-manipulability result, under cubic costs and thickness conditions.

2 The Model

Let us consider a production economy $\mathcal{E}$ with a representative consumer and a continuum of firms represented by the unit real interval $T = [0, 1]$.

Each firm $t \in T$ produces only the commodity t , and is characterized by a cost function $\bar{c}(t) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$.

Let $\mathcal{M}(T)$ denote the space of Borel measures on T and $\mathcal{M}_+(T)$ the cone of positive measures.

The commodity space is $\mathcal{M}(T) \times \mathbb{R}$. The space of measures $\mathcal{M}(T)$ is endowed with the weak star topology σ^* . We recall that the σ^* -topology in $\mathcal{M}(T)$ is the weakest

topology for which the mapping $(\varphi, \alpha) \rightarrow \langle \varphi, \alpha \rangle \equiv \int \varphi(t) d\alpha(t)$ is continuous for every φ in the space $\mathcal{C}(T)$ of all continuous real-valued functions on T .

A commodity bundle is a pair (α, l) , where α is a measure which is absolutely continuous with respect to the Lebesgue measure μ on T and l is interpreted as leisure. So, there exists a consumption density $x : T \rightarrow \mathbb{R}_+$, such that $\alpha(B) = \int_B x(t) d\mu(t)$, for every Borel set $B \subset T$, and we write $\alpha = x\mu$.

The representative consumer is characterized by an initial endowment $\hat{l} \in \mathbb{R}_+$ of leisure and a preference relation $\succeq$ on her consumption set $\mathcal{M}_+(T) \times \mathbb{R}_+$.

We assume that $\succeq$ is σ^* -continuous. Therefore, since $(\mathcal{M}_+(T), \sigma^*)$ is a separable space, there exists an utility function $U : \mathcal{M}_+(T) \times \mathbb{R}_+ \rightarrow \mathbb{R}$, representing the preference relation $\succeq$ (see Eilenberg (1941), Debreu (1954) or Debreu (1983) p. 105-110).

A price system is a pair (p, ρ) , where $p \in \mathcal{C}_+(T)$, being $p(t)$ the price of commodity t , and $\rho \in \mathbb{R}_+$ is the price of leisure. Given $p \in \mathcal{C}_+(T)$, and $\alpha = x\mu \in \mathcal{M}_+(T)$, the price of α is given by $p \cdot \alpha = \int_T p(t) d\alpha(t) = \int_T p(t) x(t) d\mu(t)$.

A competitive firm $t \in T$ solves:

$$\begin{aligned} \max \quad & \pi_t(p(t), x(t)) = p(t) \cdot x(t) - \bar{c}(t)(x(t)) \\ \text{s.a.} \quad & x(t) \geq 0 \end{aligned}$$

where $x(t)$ denotes the quantity of commodity t produced by firm t .

The consumer behaves competitively as a price-taker and solves:

$$\begin{aligned} \max_{\alpha, l} \quad & U(\alpha, l) \\ \text{s.a.} \quad & p \cdot \alpha \leq \rho(\hat{l} - l) + \int_T \pi_t d\mu(t) \\ & \alpha \in \mathcal{M}_+(T) \\ & l \geq 0 \end{aligned}$$

A Walrasian equilibrium for the economy $\mathcal{E} \equiv (\mathcal{M}(T) \times \mathbb{R}, U, \hat{l}, \bar{c}(t), t \in T = [0, 1])$ is a price system (p^*, ρ^*) , an allocation (l^*, α^*) for the consumer and a supply function x^* , such that:

- (1) $x^*(t) \in \arg \max \pi_t(p^*(t), x^*(t))$ for almost all $t \in T$.
- (2) (l^*, α^*) solves consumer's problem.

$$(3) \quad (\text{Feasibility}) \quad \int_B x^*(t) d\mu(t) = \alpha^*(B) \text{ for all } B \subseteq T$$

$$\int_T \bar{c}(t)(x^*(t)) d\mu(t) = \rho^*(\hat{l} - l^*)$$

Let $\Pi(\mathcal{E})$ denote the set of Walrasian profit functions for the economy $\mathcal{E}$.

We consider that firms behave strategically on costs. So, a firm t , may declare a cost function $c(t)$ which differs from the real one $\bar{c}(t)$. In this way, a coalition of firms can manipulate prices, by pretending another economy $\mathcal{E}'$.

Precisely, each firm $t \in T$ has a strategy set Θ_t given by:

$$\Theta_t \subseteq \{ \theta : \mathbb{R} \rightarrow \mathbb{R}, \text{ where } \theta(x) \text{ is interpreted as the cost of producing } x \text{ units of commodity } t \text{ by firm } t \}$$

Given a coalition of firms $S \subset T$, a strategy profile for S is a measurable mapping $\theta : S \rightarrow \bigcup_{t \in S} \Theta_t$, such that $\theta(t) \in \Theta_t$ for almost all $t \in S$.

If a coalition S declares a strategy profile θ , then S creates an economy $\mathcal{E}_\theta$, which differs from the initial economy $\mathcal{E}$ only in the cost function of the firms in the coalition S . If (p, ρ) is a Walrasian equilibrium price for $\mathcal{E}_\theta$, we say that (p, ρ) is an attainable price system for the coalition S .

A coalition S has market power if there exists a strategy profile θ for S and there exists a Walrasian equilibrium (p, x, l, α) for $\mathcal{E}_\theta$, such that $\pi_t = p(t) \cdot x(t) - \bar{c}_t(x(t)) > \pi_t^*$, for any $\pi^* \in \Pi(\mathcal{E})$, for almost all $t \in S$. Then, we write $(\pi_t)_{t \in S} \in \Pi(S, \mathcal{E})$. That is, $\Pi(S, \mathcal{E})$ is the set of attainable profits by the coalition S in the economy $\mathcal{E}$. We are interested in checking whether the market power of a coalition vanishes as the coalition size tends to zero.

Definition 2.1 *There exists no monopolistic power in the economy $\mathcal{E}$ provided that: Given a sequence $(S_n)_n$ of coalitions of firms, with $\mu(S_n) \rightarrow 0$ when $n \rightarrow \infty$, it is verified that for any $\varepsilon > 0$ and for each $(\pi_t^n)_{t \in S_n} \in \Pi(S_n, \mathcal{E})$, there exists $\bar{n}$ and there exists $\pi \in \Pi(\mathcal{E})$, such that $|\pi_t^n - \pi_t| < \varepsilon$ for all $t \in S_n$, for all $n \geq \bar{n}$.*

We interpret the above definition as a perfect competition test which checks whether asymptotically negligible coalitions can manipulate prices in their own benefit. Notice that, in the case of null Walrasian profits, the test checks whether the manipulators' profits goes to zero when the size of the coalition vanishes.

3 An Economy with Monopolistic Power

Next we show that if $\mathcal{E}$ is an economy whose markets are not economically thick, then firms may have monopolistic power, i.e., the economy $\mathcal{E}$ does not pass our perfect competition test. For this, we state the next example, where the preference relation of the representative consumer is not σ^* -continuous and where there exists a sequence of firms $(S_n)_n$, such that $\mu(S_n) \rightarrow 0$ and $\pi_t^n \rightarrow a > 0$ for all $t \in S_n$ with $(\pi_t^n)_{t \in S_n} \in \Pi(S_n, \mathcal{E})$.

Let $\mathcal{E}$ be a production economy with a continuum of firms, represented by $T = [0, 1]$, and a representative consumer.

The representative consumer is endowed with $\hat{l}$ units of the numeraire and her preference relation is represented by the following utility function:

$$U(l, \alpha) = \ln l + \int_T \ln(x(t) + 1) d\mu(t)$$

where x is the derivative of the μ -continuous part of the bundle α .

Each firm $t \in T$ is characterized by the same cost function $\bar{c} : \mathbb{R} \rightarrow \mathbb{R}$, given by:

$$\bar{c}_t(x) = \bar{c} \cdot x \quad \text{with } \bar{c} < \hat{l}$$

The unique Walrasian equilibrium (p^*, x^*, l) for the above economy $\mathcal{E}$ is given by:

$$p^*(t) = \bar{c}, \quad x^*(t) = x = \frac{\hat{l} - \bar{c}}{2c}, \quad l = \frac{\hat{l} + \bar{c}}{2}$$

In equilibrium firms make zero profits.

Let (S_n) be a coalition of firms with $\mu(S_n) \rightarrow 0$. Consider that each $t \in S_n$ declares an identical cost function with constant marginal costs c , satisfying that

$$\bar{c} < c < \frac{\hat{l} + \bar{c}}{2}$$

Let θ_n be the strategy profile for S_n defined by the above cost functions.

Then, the price system given by:

$$p_n(t) = \begin{cases} \bar{c} & \text{if } t \in T \setminus S_n \\ c & \text{if } t \in S_n \end{cases}$$

is an equilibrium price for the apparent economy $\mathcal{E}_{c_n}$.

On the other hand, the equilibrium demand function of the representative consumer in $\mathcal{E}_{\theta_n}$, for each $t \in S_n$, is given by:

$$x_n(t) = \frac{\hat{l}}{2c} + \frac{\bar{c}}{2c} \mu(T \setminus S_n) + \frac{\mu(S_n)}{2} - 1$$

So, $x_n(t)$ converges to the strictly positive real number $\frac{\hat{l}}{2c} + \frac{\bar{c}}{2c} - 1$, when n goes to ∞ . Thus, there exists $a > 0$, such that $p^n(t) \cdot x_n(t) - \bar{c} \cdot x_n(t) > a$ for all n and for all $t \in S_n$.

Therefore, the economy $\mathcal{E}$ is a thin market economy where market power holds even when the measure of the coalition vanishes.

4 Nonmanipulability under constant marginal costs

In this section, we apply the perfect competition test introduced in section 2 to thick market economies with constant marginal costs and production sets uniformly bounded. For this, we consider that leisure l is numeraire, i.e., $\rho = 1$.

Given a measure $\gamma \in \mathcal{M}_+(T)$ and a Borel set B of T , let $\gamma|_B$ denote the restriction of γ to B . That is, $\gamma|_B(A) = \gamma(B \cap A)$, for every Borel set A .

Now we state the following assumptions:

- (A.1) The preference relation $\succeq$ of the representative consumer is σ^* -continuous and satisfies the next uniform substitutability condition (US), that we adapt from Ostroy-Zame (1994)

(US) Given $r > 1$ there exists $\delta > 0$ such that, if $\text{diameter}(Y) < \delta$ and β is any commodity bundle with $\beta(Y) \geq \gamma(Y)$, then $(\gamma, l) \prec (r\beta|_Y + \gamma|_{(X \setminus Y)}, l)$, whatever $l \in \mathbb{R}_+$ may be.

We recall that an economy is economically thick if (A.1) is satisfied.

- (A.2) There exists a real number $M > 0$ such that $x(t) < M$ for almost all $t \in T$. That is, the production sets of firms are uniformly bounded.

- (A.3) For each firm t , $\bar{c}(t)(x) = \bar{c}_t \cdot x$, and the function, which assigns to each firm t the marginal cost $\bar{c}_t$, is continuous on T .
- (A.4) We restrict strategies to those with constant marginal costs and we say that a strategy profile θ is admissible for the coalition S if the associated function f , given by $f(t) = c_t$, with $c_t \cdot x = \theta(t)(x)$, is continuous.

We remark that the example stated in previous section satisfies all the above assumptions, except (A.1).

Next result shows that, in this setting, thickness conditions (i.e. (A.1)) lead to perfect competition.

Theorem 4.1 *Under assumptions (A.1)-(A.4) there is no monopolistic power in the economy $\mathcal{E}$.*

Proof. Assume that there is monopolistic power in $\mathcal{E}$. Then, there exists a sequence of coalitions $(S_n)_n$, with $\mu(S_n) \rightarrow 0$ and there exist $\varepsilon > 0$ and $(\pi_t^n)_{t \in S_n} \in \Pi(S_n, \mathcal{E})$ such that $\pi_t^n > \varepsilon$ for almost all $t \in S'_n \subset S_n$, with $\mu(S'_n) > 0$, for all n .

So, there exists a strategy profile θ^n admissible for S_n , and there exists a Walrasian equilibrium (p^n, x^n, l^n) , such that $\pi_t^n = p^n(t) \cdot x^n(t) - \bar{c}_t \cdot x^n(t)$.

By assumption (A.2), this implies that we can take a fixed real number $a > 0$, such that:

$$p^n(t) > \bar{c}_t + a \text{ for almost all } t \in S'_n, \text{ for all } n.$$

Then by (A.3), given any $t_n \in S'_n$,

$$p^n(t') > \bar{c}_{t'} + \frac{a}{2}, \text{ for all } t' \in B_n = \{t' : |t_n - t'| < h\}$$

On the other hand, since p^n is a continuous function on S_n , given $t_n \in S'_n$, there exists a neighborhood A_n of t_n such that

$$p^n(t_n) > \bar{c}_{t'} + \frac{a}{2} \text{ for all } t \in A_n \cap S'_n \text{ and for all } t' \in B_n.$$

Since $\mu(S'_n) > 0$, without loss of generality, we can choose A_n such that $\mu(A_n \cap S'_n) > 0$, for all n large enough. Let $A'_n = A_n \cap S'_n$.

Let α^n be the corresponding Walrasian allocation in $\mathcal{E}_{\theta^n}$, for the representative consumer. That is, $\alpha^n(B) = \int_B x^n(t) d\mu(t)$, for each Borel set $B \subseteq T$.

Finally, we claim that, for each n large enough, there is $\tilde{\alpha}^n$, such that $p^n \cdot \tilde{\alpha}^n \leq p^n \cdot \alpha^n$ and $(\tilde{\alpha}^n, l^n) \succ (\alpha^n, l^n)$, which is in contradiction with the fact that (α^n, l^n) is a Walrasian allocation for $\mathcal{E}_{\theta^n}$.

To prove our point, let $\varepsilon^n(t) > 0$, such that $x^n(t) + \varepsilon^n(t) < M$ for each $t \in B'_n = B_n \cap S_n^c$. Define a measure γ^n restricted to B'_n such that

$$\gamma^n(B'_n) = \frac{1}{1 + \eta_n} \int_{B'_n} \varepsilon^n(t) d\mu(t), \text{ with } 0 < \eta_n \leq \frac{a}{\max_{t \in A'_n} \bar{c}_t}.$$

Note that, without loss of generality, we can consider B'_n such that $\alpha^n(A'_n) > \gamma^n(B'_n)$ by taking $\varepsilon^n(t)$ small enough.

Now, since $\alpha^n|_{A'_n}$ is an atomless measure, applying Liapunov's theorem, we can choose $A''_n \subseteq A'_n$, satisfying $\alpha^n(A''_n) = \gamma^n(B'_n)$. Let us consider the allocation $\tilde{\alpha}^n$ defined as

$$\tilde{\alpha}^n = \alpha^n - \alpha^n|_{A''_n} + (1 + \eta_n) \gamma^n|_{B'_n}$$

By construction $p^n \cdot \tilde{\alpha}^n \leq p^n \cdot \alpha^n$, and by (US) $(\tilde{\alpha}^n, l^n) \succ (\alpha^n, l^n)$ for all n , satisfying that $\text{diameter}(Y_n) \leq \delta$, being $Y_n = A''_n \cup B'_n$.

Q.E.D.

The next two sections will address the extension of the above theorem to economies with more general cost structures, actually to cubic cost functions. Clearly, for cubic costs, equilibrium profits may be non zero for operating firms, in spite of the fact that there is a continuum of firms. In a situation of product differentiation we can not say that there is free-entry. In general and, in particular, in the model considered in this paper, entering firms come with a new product, do not replicate existing firms. An example of non zero equilibrium profits will be given now.

5 An Example with non-zero Walrasian profits

In this section, we state an example of an economy with cubic costs, whose symmetric equilibrium output leads to a strictly positive Walrasian profit for every firm.

Let us consider as before a production economy $\mathcal{E}$ with a continuum of firms, represented by $T = [0, 1]$, and a representative consumer.

The representative consumer is endowed with $\hat{l} \geq 2$ units of the numeraire and her preference relation is represented by the following utility function:

$$U(l, \gamma) = l + \int_0^1 \log \frac{1}{2r} \gamma(t-r, t+r) d\mu(t), \quad 0 < r < 1$$

Each firm $t \in T$ is characterized by the cost function $c(t) : \mathbb{R} \rightarrow \mathbb{R}$ given by

$$c(t)(x) = a(t)x^3 + b(t)x$$

Given $\bar{t} \in T$, the Gateaux derivative of U at (l, γ) in the direction of $\delta_{\bar{t}}$ is

$$\begin{aligned} MU(l, \gamma; \delta_{\bar{t}}) &= \lim_{s \rightarrow 0^+} \frac{1}{s} [U(l, \gamma + s\delta_{\bar{t}}) - U(l, \gamma)] \\ &= \frac{1}{2r} \int_{\min(1, \bar{t}+r)}^{\max(0, \bar{t}-r)} \left[\frac{1}{2r} \gamma(t-r, t+r) \right]^{-1} d\mu(t) \end{aligned}$$

where the last equality is obtained by monotone convergence theorem applied to the second order remainder. Thus, the preference relation is thick. In fact, this preference relation is U -smooth as in Jones (1984) (i.e. MU is σ^* -continuous on γ and continuous on t).

Now let us consider the map $d : T \rightarrow \mathbb{R}$ defined as

$$d(t) = \begin{cases} \frac{t+r}{2r} & \text{if } t \leq r \\ 1 & \text{if } r \leq t \leq 1-r \\ \frac{1-t+r}{2r} & \text{if } t \geq 1-r \end{cases}$$

Notice that $MU(l, \gamma; \delta_{\bar{t}}) = \frac{d(\bar{t})}{x}$ when $\gamma = x\mu$, $x \in \mathbb{R}_{++}$.

Suppose $a(t) = \frac{d(t)}{6}$ and $b(t) = \frac{9d(t)}{6}$.

Solving for an equilibrium with symmetric output per firm, $MU = MC$ if and only if $x^2 + 3x = 2$ and $l = \hat{l} - \int_T c(t) d\mu(t)$.

Then, this symmetric equilibrium is given by $x^* = 0.59607$, $p^*(t) = d(t)1.67765$ and $l = \hat{l} - 0.9294 \int_0^1 d(t) d\mu(t) = \hat{l} - 0.9294 \frac{2-r}{2}$.

Notice that $c(t)(x^*) = d(t)0.9294$ and $\hat{l} - l^* = \int_T c(t)(x^*) d\mu(t)$.

Therefore, the corresponding Walrasian equilibrium profit function π^* is given by $\pi^*(t) = d(t)0.0706$, which is strictly positive valued.

6 Non-manipulability under cubic costs

The aim of this section is to apply our test to economies where firms are characterized by cubic costs functions. We remark that, in this setting, demand functions are homogeneous of degree one. So, in order to obtain equilibrium prices uniformly bounded by 1, we consider the normalization $\sup(\sup_{t \in T} p(t), \rho) = 1$, being ρ the price of leisure. We assume that (A.1), as stated in section 4, holds. That is, the representative consumer's preference relation $\succeq$ is thick on $\mathcal{M}_+(T) \times \mathbb{R}_+$.

Now we state the following assumptions:

- (B.1) Each firm $t \in T$ is characterized by the cost function $\bar{c}(t)(x) = \bar{a}(t)x^3 + \bar{b}x$, where $\bar{b} > 0$ and $\bar{a} \in \mathcal{C}_+(T)$.
- (B.2) The function $\bar{a}$ verifies the next condition:

$$\int_T \bar{a}^{-1}(t) d\mu(t) < K, \text{ for some } K > 0$$

Note that if $\bar{a} \geq \varepsilon$, for some $\varepsilon > 0$, then assumption (B.2) is satisfied.

- (B.3) Strategic costs are of the form $c(t)(x) = a(t)x^3 + \bar{b}x$, with $a \in \mathcal{C}_+(T)$.
- (B.4) Strategic costs satisfy also the corresponding above condition (B.2). More precisely, if a defines a strategic profile, then $\int_T a^{-1}(t) d\mu(t) < K$.

Notice that if x is an equilibrium allocation for a strategic profile a , then $x(t) = \sqrt{\frac{p(t) - \bar{b}}{3a(t)}}$, for almost all t and that (B.2) and (B.4) imply $\int_T c^2(t) d\mu(t) < K \frac{(b+1)^2}{3}$

and $\int_T \pi^2(t) d\mu(t) < \frac{4K}{27}$. These facts will be used below. Notice also that prices can be restricted to $[b, 1]$.

Now, to ensure positive consumption of leisure, we assume the next lower bound for $\hat{l}$:

$$(B.5) \quad \hat{l} > \left(\min_{t \in T} a(t) \right)^{-\frac{1}{2}}$$

Notice that, under (B.5), $\hat{l} > \int_T \bar{c}(t) d\mu(t)$ and $l > 0$, in the absence of manipulation.

Theorem 6.1 *Under assumptions (A.1) and (B.1)-(B.5) there is no monopolistic power in the economy $\mathcal{E}$.*

Proof. Let (S_n) be a sequence of coalitions of firms, such that $\mu(S_n)$ converges to zero. Let (p_n, ρ_n) be attainable by S_n via the strategy profile $\theta^n(t)(x) = c^n(t)(x) = a^n(t)x^3 + bx$ and let (α^n, l^n) be a Walrasian allocation for the economy $\mathcal{E}_{\theta^n}$ at prices (p_n, ρ_n) .

First, we will show that the sequence (p_n) is an equicontinuous set. If it is not so, then there exist sequences s_k, t_k converging to $t^* \in T$, such that $\lim_{k \rightarrow \infty} \frac{p_{n_k}(t_k)}{p_{n_k}(s_k)} > 1$, for some subsequence p_{n_k} of p_n . Since p_{n_k} is a continuous function, then for each k large enough there exists a neighborhood C_k of t_k , such that $\frac{p_{n_k}(t)}{p_{n_k}(s_k)} > 1$, for all $t \in C_k$. We choose C_k closed. Thus, for each k , there exists $y_k \in C_k$, such that $p_{n_k}(y_k) = \inf_{t \in C_k} p_{n_k}(t)$. Let β^k given by $\beta^k(B) = \alpha^{n_k}(B \cap C_k)$. Consider the bundle $g^k = \alpha^{n_k} - \beta^k + \frac{p_{n_k}(y_k)}{p_{n_k}(s_k)} \alpha^{n_k}(C_k) \delta_{s_k}$. Note that $p_{n_k} \cdot g^k \leq p_{n_k} \cdot \alpha^{n_k}$. Thus, by the uniform substitutability assumption, for all k large enough, one obtains that $(g^k, l^{n_k}) \succ (\alpha^{n_k}, l^{n_k})$, for all k , such that $\alpha^{n_k}(C_k) > 0$. But, for all k it is verified that $\alpha^{n_k}(C_k) > 0$; otherwise $p_{n_k}(t) \leq b$ for almost all $t \in C_k$, implying $p_{n_k}(t^*) < b$, which is impossible.

Secondly, since $\sup p_n < 1$ for all n , it follows, by Arzela-Ascoli, that some subsequence p_{n_k} will converge to some $p \in \mathcal{C}(T)$ uniformly. It will be shown that p is a true equilibrium price.

Let $x^{n_k} = \frac{d\alpha^{n_k}}{d\mu}$ given by $\sqrt{\frac{p^{n_k}(t) - b}{3a^{n_k}(t)}}$ for $t \in \text{supp } \alpha^{n_k}$. Notice that, since $\mu(S_n) \rightarrow 0$, $x^{n_k}(t)$ converges to $x(t)$, for almost all $t \in T$, where $x(t) = \sqrt{\frac{p(t) - b}{3a(t)}}$ and, therefore, α^{n_k} converges (in the σ^* topology) to α which is given by $\alpha(B) = \int_B x(t) d\mu(t)$, for every Borel set B (see Hildenbrand (1974) p. 48 example b).

Notice that ρ^{n_k}, l^{n_k} have cluster points ρ, l .

Now, we check the following conditions, which guarantee that (p, ρ, x, l) is a Walrasian equilibrium for the economy $\mathcal{E}$.

(1) $\hat{l} - l = \int_T \bar{c}(t)(x(t)) d\mu(t)$ or (1') $\int_T \tilde{p}(t) d\alpha = \hat{l} - l + \int_T \pi(t) d\mu(t)$, where $\pi(t) = \tilde{p}(t)x(t) - \bar{c}(t)(x(t))$ and $\tilde{p}(t) = \frac{p(t)}{\rho}$.

(2) If $(\gamma, L) \succ (\alpha, l)$, then $(p, \rho) \cdot [(\gamma, L) - (\alpha, l)] > 0$

Notice that $\rho > 0$, otherwise the leisure market would not clear for k sufficiently large. To see that (1) holds we use assumptions (B.2) and (B.4).

Notice that, by (B.2) and (B.4), $\int_T (c^{n_k}(t)(x^{n_k}(t)))^2 d\mu(t) < \frac{K(b+1)^2}{3}$ and, therefore, the sequence $(c^{n_k}(t)(x^{n_k}(t)))$ is uniformly integrable (see Appendix, Proposition 2). Now $c^{n_k}(t)$ converges to $\bar{c}(t)$ uniformly on compacta, for every t . Then, $c^{n_k}(t)(x^{n_k}(t))$ converges to $\bar{c}(t)(x(t))$, for a.e. $t \in T$ (see Royden (1968), problem 9.40 (b)). Thus, by Proposition 1 in Appendix,

$$\int_T c^{n_k}(t)(x^{n_k}(t)) d\mu(t) \rightarrow \int_T \bar{c}(t)(x(t)) d\mu(t)$$

Suppose (2) does not hold and let $(\hat{\gamma}, \hat{L}) = \lambda(\gamma, L) + (1 - \lambda)(\alpha, l)$, $0 < \lambda < 1$. Since $0 < l < \hat{l}$ (the first inequality by (B.5) and the second because $l = \hat{l}$ implies $x \equiv 0$) it follows that $0 < \hat{L} < \hat{l}$. Now, $p \cdot \hat{\gamma} = \rho(\hat{l} - \hat{L}) + \bar{\Pi}$, where $\bar{\Pi} = \int_T \pi(t) d\mu(t)$, and apply the implicit function theorem to obtain $\hat{L}^{n_k} = f(p^{n_k}, \rho^{n_k}, \bar{\Pi}^{n_k})$ such that $p^{n_k} \cdot \hat{\gamma} = \rho^{n_k}(\hat{l} - \hat{L}^{n_k}) + \bar{\Pi}^{n_k}$ (the function f exists since $\rho > 0$ and $\hat{L} \in (0, \hat{l})$, see Mas-Colell (1985)). Now, let $\pi^{n_k}(t) = \tilde{p}^{n_k}(t)x^{n_k}(t) - c^{n_k}(t)(x^{n_k}(t))$. Notice that $\pi^{n_k}(t)$ converges to $\pi(t)$ for almost all $t \in T$, and (π^{n_k}) is uniformly integrable, since $\int_T (\pi^{n_k}(t))^2 d\mu(t) < \frac{4K}{27}$, for all n_k . (See Appendix, Proposition 2)

Then, by Proposition 1 in Appendix, $\int_T \pi^{n_k}(t) d\mu(t) \rightarrow \int_T \pi(t) d\mu(t)$ and, therefore, $(\hat{\gamma}, \hat{L}^{n_k}) \preceq (\alpha^{n_k}, l^{n_k})$. But, by closed graph of $\preceq$ we have $(\hat{\gamma}, \hat{L}) \preceq (\alpha, l)$, contradicting the fact that $(\hat{\gamma}, \hat{L}) \succ (\alpha, l)$.

Finally, we check the non-manipulability. For this, let $\tilde{\pi}^{n_k}(t) \equiv p^{n_k}(t) x^{n_k}(t) - \bar{c}(t)(x^{n_k}(t))$. Notice that

$$\tilde{\pi}^{n_k}(t) \leq p^{n_k}(t) \sqrt{\frac{p^{n_k}(t) - b}{3a(t)}} - \bar{c}(t) \left(\sqrt{\frac{p^{n_k}(t) - b}{3a(t)}} \right) \equiv \hat{\pi}^{n_k}(t),$$

where $\hat{\pi}^{n_k}(t)$ is the maximal profit that firm t can make at prices $p^{n_k}(t)$ (given the true cost structure).

Recall $x^{n_k}(t) = \sqrt{\frac{p^{n_k}(t) - b}{3a^{n_k}(t)}}$ for $t \in \text{supp } \alpha^{n_k}$.

Now, since $p^{n_k} \rightarrow p$ uniformly, we obtain that $\tilde{\pi}^{n_k} \rightarrow \pi$ uniformly. Therefore, for all $\xi > 0$, there exists $\bar{n}$, such that $\tilde{\pi}^{n_k}(t) \leq \hat{\pi}^{n_k}(t) < \pi(t) + \xi$ for almost all $t \in S_{n_k}$, with $n_k \geq \bar{n}$.

Q.E.D.

Appendix

We collect some results which are particular adapted to the needs of this paper. For this, let (Ω, S, μ) be a finite measure space, let F be a Banach space and let $\mathcal{F}$ a set of functions from Ω to F , which are p -integrable (i.e. $\mathcal{F} \subset L^p$)

We recall the uniform p -integrability notion.

Definition. *The set $\mathcal{F}$ is uniformly p -integrable, $0 < p < \infty$, if*

- (i) *For every $\varepsilon > 0$ there exists $K \subset \Omega$, such that $\int_{\Omega \setminus K} \|f\|^p d\mu \leq \varepsilon$, for all $f \in \mathcal{F}$.*
- (ii) *For every $\varepsilon > 0$ there exists $\delta > 0$, such that if $A \in S$ and $\mu(A) < \delta$, then $\int_A \|f\|^p d\mu \leq \varepsilon$, for all $f \in \mathcal{F}$.*

The next result is Vitali convergence theorem, which is a generalization of Lebesgue dominated convergence theorem (see Schwartz (1993), p. 284)

Proposition 1. *Let (f_n) be a sequence of mappings from Ω to F , which are p -integrable. Suppose that (f_n) converge in measure (or almost everywhere) to a function f . Then $\int \|f_n - f\| d\mu$ converges to zero iff (f_n) is uniformly p -integrable.*

Proposition 2. *The unit ball of L^p is uniformly q -integrable for every $q < p$.*

(See Schwartz (1993), p. 289)

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