Trade Policy and Incentives for Cost Reduction in a Differentiated Industry: Price vs. Quantity Competition* 

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December 28, 1998

Abstract

The incentives for governments to impose subsidies and tariffs on R&D and output is analysed in a differentiated good industry where firms invest in a cost saving technology. When government commitment is credible, subsidies to R&D and output are positive both under Bertrand and Cournot competition. However, if the government cannot commit to a policy action, it chooses a tariff under Bertrand competition and a subsidy under Cournot competition.

JEL Classification: F12, F13, L13.

Keywords: Product Differentiation, Trade Policies, Commitment, Tariffs, Subsidies.

* Celia acknowledges grant support from NOVA-FORUM and Acção Integrada Luso-Espanhola E-96/98. Kujal acknowledges support from INOVA, Ações Integradas Hispano-Portuguesas 690 and TEEB-630. All errors are our own. Corresponding author: Kujal, Universidad Carlos III de Madrid, Calle Madrid 136, 28965 Getafe, Madrid SPAIN, tel: 34 91 624 4651, e-mail:kujal@econo.uc3m.es.
1. Introduction

Since the seminal Spencer and Braude (1983) paper, there has been a growing literature in strategic trade policy. Braude and Spencer (1993) use a third market model where two firms produce a homogeneous good and compete in quantities in the second stage of a game, after having invested in cost-reducing R&D. This is a previous stage. In this paper, they show that, under free trade, firms overinvest in the cost-saving technology. They also show that if governments have the possibility of unilaterally (or bilaterally) subsidizing/taxing R&D, they will choose to subsidize R&D. If, however, the governments have the possibility of subsidizing/taxing both output and R&D, they will tax R&D and subsidize output. An underlying assumption in their paper is that governments credibly commit to a policy action before firms choose their strategic variables.

Bester and Petrakis (1993) use a model with product differentiation where, again, firms invest in cost-reducing R&D before engaging in market competition. They show that investment in R&D not only depends on the degree of product substitutability between the two goods but also on the assumption made on the nature of market competition (i.e., whether engaging in Bertrand or Cournot competition). In their model, if the degree of product substitutability is high, there are stronger incentives to invest in R&D under Bertrand competition than under Cournot competition. For low good substitutability, the result is reversed. They further show that, unlike the homogeneous good, Spencer and Braude's (1992) results, underinvestment in R&D is obtained under both Cournot and Bertrand competition if the goods are poor substitutes. They do not, however, analyze incentives to invest in R&D under strategic trade policy instruments.

The question of whether the choice of the optimal policy instrument is sensitive to the nature of market competition was addressed by Eaton and Grossman (1986). These authors use a model of conjectures to show that, with Bertrand market competition, the optimal policy instrument is a tax on exports. However, under Cournot competition (as in Spencer and Braude (1993)) subsidies on exports are optimal. Eaton and Grossman (1986) do not explicitly model firm investment in R&D or product differentiation and assume that firms set output or prices taking subsidies and taxes as given.

Reites (1991) used a Cournot duopoly model to study firm incentives to invest in cost-reducing technology under quantity restrictions. Firms invest in R&D initially and then compete in quantities. The introduction of a VER (or a quota)
at the free trade production level results in both the domestic and the foreign firms choosing lower levels of cost reducing R&D than they would under free trade. In the presence of a quota, domestic investment in R&D declines because the strategic advantage of R&D vanishes for the domestic firm — with the quota, the domestic firm becomes a monopolist on the residual demand and chooses the cost-maximizing level of R&D expenditures. The foreign firm, constrained by the quota, also has less incentives to invest in R&D. Reitzes (1991) also shows that a quota and a tariff may often produce opposite effects on domestic R&D since while a tariff preserves the strategic link between R&D and foreign output, a quota does not.

Cabral et al. (1998) examine the effect of imposing a VER (or a quota) under Bertrand competition, following Reitzes (1991) and motivated by Eeles and Grossman's (1986) findings that the optimal policy is reversed from Cournot to Bertrand competition. They find that under Bertrand competition Reitzes's (1991) results are only partially reversed. Similarly to the Cournot competition case, the domestic firm invests less in R&D relatively to the free trade case. The foreign firm, however, invests more in R&D than under free trade. This result is explained by the fact that under Bertrand competition foreign investment in R&D has a negative (indirect) strategic effect on foreign firms' profits: higher foreign investment in R&D makes the domestic firm lower its price which in turn results in lower prices and profits for the foreign firm. This makes the foreign firm “underinvest” in R&D (under Cournot competition this indirect strategic effect is of the opposite sign and consequently the firm overinvests in R&D). With the imposition of the quota the negative strategic effect disappears and investment in R&D necessarily increases for the constrained case. Contrarily, the domestic firm invests less in R&D as it faces lesser competition from the foreign firm (this effect is similar to the one present in Reitzes' (1991) model).

Another related issue concerns the timing of players' moves. Recent work by Mann (1985) has shown that the timing of players' moves is relevant for the policy choice. Cacciatore (1987) and Hermann et al. (1997) show that if firms know that the subsidy level depends on their choice of price or quality, firms respond optimally.

1 As Brender (1995) notes, “An intriguing but underappreciated aspect of trade policy analysis is the crucial importance of the timing of decisions.” (p. 1418).
and choose the maximum price or maximum quality. In this scenario, domestic welfare is lower than if the government moves first and commits to a given subsidy level. In the Huguenet et al. (1997) framework, where the foreign firm sells in the domestic market, non-commital to a tariff results in the foreign firm exiting the industry only if it produces the high quality good. Domestic welfare is in this case higher than under free trade due to the possibility of leapfrogging. Leechy and Neary (1994, 1995) show that domestic welfare under commitment is always higher if the government can commit to an R&D subsidy. By committing to an R&D subsidy the government decreases the R&D expenditure of the foreign firm. In this way it decreases the loss in the domestic firm’s market share which results from the foreign firm’s overinvestment in R&D.

The above results indicate that assumptions both on market structure and on the timing of moves are crucial in determining policy outcomes. Further, the policy instrument reversal suggested by Eaton and Grossman’s (1986) one-stage game may not extend to multi-stage games (as seen in Cabral et al. (1998)).

In this paper, we study the choice of strategic trade policy instruments when firms invest in R&D and then compete in prices or in quantities. Given that (i) firm incentives to invest in R&D depend on the nature of product market competition and on the degree of substitutability between the goods, (ii) quotas lead only to a partial reversal of results between Cournot and Bertrand competition when one considers a differentiated goods industry with firm investment in R&D, and (iii) the timing of the government move (i.e. moving before or after the firm has chosen its R&D) may alter policy outcomes, we use the Bester and Petrakis (1993) differentiated goods model, where firms invest in R&D before the market competition stage to consider and compare both Bertrand and Cournot market competition equilibria under the following policy instruments:

(i) Credible unilateral and bilateral subsidies to R&D.
(ii) Credible unilateral and bilateral subsidies to output.
(iii) Credible unilateral and bilateral subsidies to both, R&D and output.
(iv) Non-credible unilateral and bilateral subsidies to output.

We show that, when governments commit to a policy before the firms choose R&D, unilateral and bilateral subsidies to R&D are positive both under Cournot

3Given that R&D costs are sunk in our model and that firms invest in cost reducing innovation prior to the market competition stage, allowing for the government to move after the choice of R&D does not change any of the results.
firm relative to free trade regardless of the nature of market competition. World output also increases both for unilateral and bilateral R&D subsidies. When both governments subsidize output we find that they choose a positive subsidy on output both under both Bertrand and Cournot competition, unlike Eaton and Grossman's (1986) result. However, under non-credible, our results change: governments subsidize under Cournot competition and tax exports under Bertrand competition. Surprisingly, the Eaton and Grossman's (1986) policy reversal result is obtained under non-credible policies. This highlights the role of the timing of moves in the choice of trade policy instruments.

The paper is structured as follows. In section 2 the model is laid out and the free-trade results are derived. In Section 3 and 4 R&D and production subsidies are studied, respectively (both the unilateral and the bilateral subsidy cases are analysed). In Section 5 the issue of non-credible policies are analysed. Section 6 concludes.

2. The model under Free Trade

We use a third-country model to consider the case of two firms, located in two different countries, that produce a differentiated good which they sell in a third country. There is a competitive numerical sector. The two firms operate under constant returns to scale and initially have the same marginal costs of production. Firms can invest in a cost-saving technology prior to engaging in market competition and are able to reduce its marginal cost by $\Delta$ by spending $\gamma^2$. Both firms face the following symmetric demand functions:

$$x_i = \frac{1}{1 - \gamma^2} [a (1 - \gamma) - p_i + \gamma x_j], \ i, j = 1, 2. \quad (2.1)$$

$\gamma$ measures the degree of product differentiation. As $\gamma$ approaches zero each firm becomes a local monopolist and as $\gamma$ approaches one, goods become almost perfect substitutes. To avoid corner solutions in the Bertrand game we limit our attention to the cases where $\gamma \leq 0.82769$.

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5 It should be noted that Eaton and Grossman (1986) assume government commitment.

6 These are the demand functions of a consumer with utility $u(x_1, x_2) = a(x_1 + x_2) - \frac{1}{2} (x_1^2 + x_2^2) + m$ with $m$ representing money, following Dixit (1979). Resulting inverse demand is $p_i = a - x_i - \gamma x_j$. 


Firms play a two-stage game. In stage one, firms simultaneously decide how much to invest in cost saving R&D ($\Delta_i$). In stage two, given the reduced unit cost, firms simultaneously decide on which price/output to set. In this context, investment in R&D has a commitment value, as firms can use R&D strategically to improve their position in the subsequent market competition stage. The problem is solved using sub-game perfect equilibria.

We analyze both the quantity competition and the price competition cases.

2.1. Cournot competition

2.1.1. The output choice stage

Firm $i$ chooses $x_i$ to maximize profits which, given inverse demand ($p_i = a - x_i - \gamma x_j$) and reduced unit cost ($c - \Delta_i$), are:

$$\max \left( (a - x_i - \gamma x_j) x_i(x_i, p_i) \right).$$

(2.2)

$p_i$ and $\Delta_i$ are taken as given. Each firm's reaction function is thus derived:

$$x_i(x_i) = \frac{1}{2} \left( a - c + \Delta_i - \gamma x_j \right)$$

(2.3)

The intersection of the two reaction functions gives us the equilibrium quantities ($x_1, \phi$), each chosen given the output of the other firm. The equilibrium outputs and profits are respectively:

$$x^*_i = \frac{(2\Delta_i + (a - c)(2 - \gamma) - \Delta_i \gamma)}{(4 - \gamma^2)}$$

(2.4)

and

$$\pi^*_i = \frac{[(2\Delta_i + (a - c)(2 - \gamma) - \Delta_i \gamma)] - \Delta_i^2}{2 (4 - \gamma^2)}.$$

(2.5)

2.1.2. The R&D stage

Firm $i$ given $\Delta_i$, chooses $\Delta_i$ to maximize its profits (defined above). First-order conditions and symmetry allow us to derive optimal R&D spending, output and price for each firm.
\[ \Delta^* = \frac{4(a-c)}{(4+4\gamma-2\gamma^2-\gamma^3)} \] \hfill (2.6)

\[ \sigma^* = \frac{(a-c)(2-\gamma^2)}{(4+4\gamma-2\gamma^2-\gamma^3)} \] \hfill (2.7)

\[ \nu^* = \frac{a\gamma^2 - c(4-\gamma^2)(1+\gamma)}{{(-4-4\gamma+2\gamma^2+\gamma^3)}}. \] \hfill (2.8)

Firms' profits are then given by

\[ \pi^* = (a-c)^2 \frac{(8-8\gamma^2+\gamma^4)}{(-4-4\gamma+2\gamma^2+\gamma^3)^2}. \] \hfill (2.9)

One should note that a firm has more incentive to invest in cost-reducing R&D under Cournot competition than under a pure cost-minimizing strategy, since there is a positive strategic effect of R&D on profits.

2.2. Bertrand competition

2.2.1. The price choice stage

Firm \( i \) chooses \( p_i \) so as to maximize profits:

\[ \max [p_i - (c - \Delta_i)] x_i(p_i, p_j). \] \hfill (2.10)

\( p_j \) and \( \Delta_i \) are taken as given. This defines each firm's reaction function:

\[ p_i = \delta_i(p_i; \Delta_i) \equiv \frac{a(1-\gamma) + c - \Delta_i + \gamma p_i}{2}. \] \hfill (2.11)

Once more, the intersection of the two reaction functions determines the equilibrium prices \( (p_1, p_2) \), each chosen given the price of the other firm. The equilibrium prices and profits are:

\[ p_i = \frac{1}{4-\gamma^2} \left[ \{a(1-\gamma) + c\} (2+\gamma) - 2\Delta_i - \gamma \Delta_j \right], \] \hfill (2.12)

and

\[ \pi_i = \frac{(p_i - c + \Delta_i)^2}{1-\gamma^2} - \frac{\Delta_i^2}{2}. \] \hfill (2.13)
2.2.2. the R&D stage

Firm i, given $\Delta_i$, chooses $\Delta$ to maximize its profits (defined above). From the first-order conditions and symmetry we obtain optimal R&D spending, output and price for each firm:

$$\Delta^* = \frac{2(2 - \gamma)}{D_i(\gamma)}(a - c), \quad (2.14)$$

$$x_i^* = \frac{(4 - \gamma)}{2(2 - \gamma^2)} \Delta^*, \quad (2.15)$$

$$p^* = \frac{1}{2 - \gamma} [a(1 - \gamma) + c - \Delta^*]. \quad (2.16)$$

Firms’ profits are then given by

$$\pi^* = \frac{8 - 16\gamma^2 + 7\gamma^4 - \gamma^6}{D_i(\gamma)^2} (a - c)^2. \quad (2.17)$$

Where, $D_i(\gamma) = (1 + \gamma)(2 - \gamma)(4 - \gamma^2) - 2(2 - \gamma^2)$.

It should be noted that a firm has less incentive to invest in cost-reducing R&D under price competition than under a pure cost-minimizing strategy, since there is a negative strategic effect of R&D on profits – as a response to firm i’s reduction of unit costs, its rival decreases its price, thus shifting i’s demand upwards. Firm i has then to reduce its price in order to sell the same output. By lowering its R&D expenditure beyond the cost minimizing level, a firm can commit to softer competition in the subsequent market game.

It now can be clearly seen that firms invest more in R&D under Cournot competition than under Bertrand competition. Further, due to the competitive nature of the Bertrand game, output under Bertrand competition is greater than under Cournot competition. Domestic welfare is higher under Bertrand competition for most values of $\gamma$. Only when $\gamma$ is high enough ($\gamma > 78445$), i.e. the goods are close substitutes, is domestic welfare under Cournot competition higher. This is related to the Bester and Petralia’ (1993) result that welfare does not only depend on the nature of competition but also of the degree of product differentiation.
3. Optimal R&D subsidies

In this section we analyse the effect of imposing optimal R&D subsidies upon the firms. We assume that the government can credibly commit to a policy action. The game is modelled as follows: Governments first choose and announce their policy actions. After this announcement, firms decide how much to invest in cost reducing R&D. They then proceed to compete in prices or in quantities in the final stage of the game.

We consider unilateral subsidies and bilateral subsidies towards R&D. We first analyze both the cases of Cournot competition and Bertrand competition.

3.1. Cournot competition

3.1.1. Unilateral subsidies

Market competition stage. We initially assume that only the government of country 1 subsidizes R&D. A fraction \( z_i \) of firm 1's expenditures on R&D is subsidized by the government. The problem is then no longer symmetric. Firm 1 and firm 2 again maximize profits:

\[
\pi_1 = (a - \gamma x_2 - c + \Delta_1)x_1 - \left( \frac{\Delta_1^2}{2} \right)(1 - z_1) \quad (3.1)
\]

\[
\pi_2 = (a - \gamma x_1 - c + \Delta_2)x_2 - \left( \frac{\Delta_2^2}{2} \right) \quad (3.2)
\]

From the first order condition we get the reaction functions for firms 1, 2:

\[
x_i = \frac{1}{2}(a - c + \Delta_1 - \gamma x_j), i, j = 1, 2
\]

that gives us the equilibrium outputs for firms 1, 2:

\[
x_i^* = \frac{2(a - c + \Delta_1) - (a - c - \gamma \Delta_1)}{4 - \gamma^2}, i, j = 1, 2
\]

The possibility of non-credibility is analysed in section 3.

Some authors justify this modeling by arguing that governments are not playing strategies actively in the markets. They tend to commit to policy actions before the market participants.
R&D stage. Given the equilibrium outputs the firms maximize their profits with respect to $\Delta$. From the first order conditions we get,

$$\Delta_1 = \frac{4([c-a/(2-\gamma) - \Delta_1 \gamma])}{(-4 + \gamma^2)(1 - \frac{8}{(-4 + \gamma^2)}) - z_l)}$$

$$\Delta_2 = \frac{4([c-a/(2-\gamma) - \Delta_2 \gamma])}{(-8 + \gamma^2 + \gamma^4)}$$

This gives us the equilibrium R&D investment for each firm,

$$\Delta_1^* = \frac{4([c-a/(4-4\gamma - 2\gamma^2 + \gamma^4)])}{(16 + 12\gamma^4(1 - z_l) - \gamma^2(1 - z_l) - 32z_l + 8\gamma^2(2z_l - 4))}$$

(2.4)

$$\Delta_2^* = \frac{4([c-a/(4-4\gamma(1 - z_l) - 2\gamma^2(1 - z_l) + \gamma^2(1 - z_l - 8z_l)])}{(16 + 12\gamma^4(1 - z_l) - \gamma^2(1 - z_l) - 22z_l + 8\gamma^2(5z_l - 4))}$$

(3.5)

The two equations above can be solved for the optimal subsidy, $z^*_l$, and total welfare for both countries can be derived:

$$z^*_l = \frac{2\gamma^2}{(8 - 6\gamma^2 + \gamma^4)}$$

(3.6)

$$TW_1^* = \frac{4(-4\gamma - 2\gamma^2 + \gamma^4)^2}{32 - 96\gamma^2 + 72\gamma^4 - 16\gamma^6 + \gamma^8}$$

(2.7)

$$TW_2^* = \frac{(8 - 8\gamma^2 + \gamma^4)(-8 + 8\gamma + 12\gamma^2 - 8\gamma^3 - 2\gamma^4 + \gamma^6)^2}{32 - 96\gamma^2 + 72\gamma^4 - 16\gamma^6 + \gamma^8}.$$ 

(3.8)

It can be seen that the optimal subsidy is an increasing function of $\gamma$ and reaches its maximum when $\gamma = 1$. The subsidy is zero when firm 1 has a monopoly on the production of the good $(\gamma = 0)$ and reaches its maximum as the market becomes increasingly competitive. Welfare for country 1 is positive if $\gamma$ is low, i.e., when goods are (high) imperfect substitutes. It becomes negative when goods are close substitutes. This is because the optimal subsidy is an increasing function of $\gamma$. Country 1 thus has no incentive to unilaterally subsidize R&D when market competition is high. For a high degree of product substitution, total welfare is greater than under free trade for country 1 while it is lower for country 2.
Innovation expenditures depend upon the degree of product differentiation and on whom receives the subsidy. Firm 1 (the subsidized firm) invests more in R&D relative to free trade when the degree of product differentiation is greater. The opposite applies for the other firm.

3.1.2. Bilateral Subsidies

Market competition stage. Suppose now that each country chooses to subsidize R&D, paying a fraction \( \sigma \) of firm \( f \)'s R&D expenditures. Profit functions change to include an extra term. \((1 - z_i)\) multiplies firm \( f \)'s R&D expenditures. Once more, profit maximization yields the reaction functions:

\[
x_i = \frac{1}{2}(a - c + \Delta_i - \gamma z_j), \quad i \neq j.
\]

Equilibrium output levels are then derived:

\[
z_i = \frac{(2\Delta_i + a(2 - \gamma) - c(2 - \gamma) - \Delta_i \gamma)}{(4 - \gamma^2)}
\]

(3.9)

The R&D stage. Substituting the equilibrium outputs into profits, maximizing with respect to \( \Delta_i \) and solving the first-order condition, we obtain:

\[
\Delta_i = \frac{8(a(2 - \gamma) - c(2 - \gamma) - \Delta_i \gamma)}{(4 + \gamma^2)(1 - z_i)}
\]

The equilibrium level of innovation expenditures, \( \Delta_i \), for both the firms is then derived:

\[
\Delta_i^* = \frac{(4(a - c)(4 - 4\gamma(1 - z_j) - 2\gamma^2(1 - z_j) + \gamma^2(1 - z_j) - 8z_j))}{D_\gamma(\gamma)}
\]

(3.10)

where, \( D_\gamma(\gamma) = (12\gamma^4(1 - z_i)(1 - z_j) - \gamma^2(1 - z_i)(1 - z_j) + 16(1 - 2z_i)(1 - 2z_j) - 8\gamma^2(z_i + z_j(3 - 6z_j))). \)

Substituting the equilibrium cost reduction expenditures in profits and maximizing with respect to \( z_i \) we can next solve for the subsidies:

\[
z_i = \frac{2\gamma^2}{(4 - \gamma^2)(4z_i - 2 + \gamma^2(1 - z_j))}
\]
The solution to the equilibrium subsidies is non-unique. Two values are obtained for both firm $i$ and firm $j$. We choose the relevant subsidy as the one which results in a greater domestic welfare level.

Plotting the subsidy as a function of the degree of product differentiation it can be seen that both the countries subsidize the good only for low values of $\gamma$ ($\leq 0.5878$). Looking at total welfare, it can be seen that both the countries choose the following optimal subsidies if they are to subsidize:

$$z_1 = \frac{\gamma^2}{(2 - \gamma^2)^2(2 - \gamma^2 - \sqrt{4 - 12\gamma^4 + \gamma^4})}$$

Plotting $z_1$ and $z_2$ as a function of $\gamma$ it is seen that the subsidies are positive only for gamma $\gamma \leq 0.5878$.

Discussion: As in Spencer and Brander (1983), we find that optimal R&D subsidies are positive when both the countries choose subsidies. Total welfare under bilateral R&D subsidies is lower than under free trade. However, given that under unilateral subsidies welfare is greater for the subsidizing country and lower for the other country, both the countries choose to subsidize R&D. This results in lower domestic welfare for both the countries. Both firms invest more in R&D under bilateral subsidies and world output increases relative to free trade.

3.2. Bertrand competition

3.2.1. Unilateral subsidies

As before, we assume that it is firm $i$ that receives the subsidy. In this case, equilibrium prices for firm $i = 1, 2$ are:

$$p_i = \frac{2\Delta_i + \Delta_i \gamma - c(2 + \gamma) - a(2 - \gamma - \gamma^2)}{-4 + \gamma^2}$$  \hspace{1cm} (3.11)

Substituting equilibrium prices into profits and maximizing in order to $\Delta_i$, we obtain equilibrium investment levels in cost-saving innovation for both countries:

$$\Delta_i = \frac{2(a - c)(-2 + \gamma + \gamma^2)(-8 - 8\gamma + 12\gamma^2 + 6\gamma^3 - 6\gamma^4 - \gamma^6)}{D_2(\gamma)}$$  \hspace{1cm} (3.12)

*For the other root, total welfare is negative for both countries.*
\[ \Delta_2 = \frac{2(a - c)(-8 + 14y^2 - 7y^4 + y^6)(-4 + y^2(4 - 6z_1) - (4 - y^2 - y^4)(-1 + z_1) + 8z_1)}{D_b(y)} \]

(3.13)

Where, \( D_b(y) = y^2(240 - 448z_1) + y^6(236 - 336z_1) + y^{10}(14 - 16z_1) + y^{12}(-1 + z_1) + 64(-1 + 2z_1) + 8y^2(-44 + 7z_1) + y^4(-81 + 103z_1) \). Finally, by substituting the equilibrium values of \( \Delta_2 \), we can solve for the equilibrium R&D subsidy, R&D spending, output, and welfare.

\[ x_1^* = \frac{2y^2}{-8 + 18y^2 - 7y^4 + y^6} \]

\[ \Delta_1^* = \frac{2(-2 + \gamma + \gamma^2)(-8 + 18y^2 - 7y^4 + y^6)(-8 - 8y + 12y^2 + 6y^3 - 6y^4 - y^5 + y^6)}{D_b(y)} \]

\[ \Delta_2^* = \frac{2(8 - 14y^2 + 7y^4 - y^6)N_1(y)}{D_b(y)} \]

\[ x_2^* = \frac{\gamma^2N_2(y)}{D_b(y)} \]

\[ TW_1 = \frac{(-2 + \gamma + \gamma^2)(4 + 4y - 4y^2 - y^3 + y^4)^2}{D_b(y)} \]

\[ TW_2 = \frac{(128 - 448y^2 + 552y^4 - 280y^6 + 33y^{10} + 23y^{12} - 9y^{14} + 14y^{16})N_2(y)}{D_b(y)} \]

Where, \( D_b(y) = 512 - 2816y^2 + 6688y^4 - 8832y^6 + 6928y^8 - 3368y^{10} + 1031y^{12} - 195y^{14} + 21y^{16} - y^{18} \), \( N_1(y) = (-32 + 32y + 88y^2 - 72y^3 - 96y^4 + 44y^5 + 48y^6 - \)

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\[ 11\gamma^7 - 11\gamma^8 + \gamma^9 + \gamma^{10} \text{ and } N_5(\gamma) = (-256 + 256\gamma + 864\gamma^2 - 736\gamma^3 - 1240\gamma^4 + 744\gamma^5 + 952\gamma^6 - 380\gamma^7 - 424\gamma^8 + 107\gamma^9 - 111\gamma^{10} - 16\gamma^{11} + \gamma^{12} + \gamma^{13} + \gamma^{14}). \]

Note that all the above expressions are a function of the degree of product differentiation \( \gamma \). Comparing these results to the case of free trade it can be seen that:

(i) output of the firm that receives the subsidy is less (more) than the output under free trade when the degree of product differentiation is low (high). As markets get competitive a R&D subsidy in fact results in a lower level of output.

(ii) the converse is true for firm 2 (the non-subsidy-recipient).

(iii) innovation expenditure for the subsidized firm is greater than under free trade when the goods are not close substitutes.

(iv) firm-2 decreases its R&D expenditures relatively to free trade.

(v) Unilateral subsidies are positive for all \( \gamma \).

(vi) Domestic welfare is higher than free trade when goods are imperfect substitutes (near monopolies at home). Welfare of the non-subsidizing country is always less than under free trade.

### 3.2.2. Bilateral Subsidies

**Price competition stage** The problem of the firm is the same as in section 2.2 for the price selection stage. When both firms receive a subsidy of \( z_i^B \), profit maximization with relation to R&D expenditures yields:

\[
\Delta_i = \frac{2(-2 + \gamma^2)(-4 + \gamma^2(4 - 6z_i) - (4\gamma - \gamma^4)(-1 + z_i) + 8z_i)}{D_i(\gamma)}
\]

(3.14)

Where, \( D_i(\gamma) = (\gamma^8(-1 + z_i)(-1 + z_i) + 16(-1 + 2z_i)(-1 + 2z_i) + \gamma^9(-9 + z_i(11 - 13z_i) + 11z_i) - 8\gamma^7(6 - 9z_i + z_i(-9 + 14z_i)) + \gamma^6(32 - 44z_i + z_i(-44 + 60z_i))) \).

Substituting \( \Delta_i \) in total welfare we then solve for the equilibrium subsidy for each firm. This gives us the two roots (note we remove the subscripts as the roots are symmetric):

\[
z_{ik}^* = \frac{2 - 3\gamma^2 + \gamma^4 \pm \sqrt{S}}{2(4 - 5\gamma^2 + \gamma^4)}, i, k = 1, 2.
\]

(3.15)

where,

\[
S = \sqrt{(4 - 2\gamma^2 + 2\gamma^4 + 6\gamma^6 + \gamma^8)}
\]

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Given the two solutions, we assume that the government selects the dominating one, i.e. the one that yields a greater domestic welfare level. This then gives us the equilibrium expenditure on R&D, output and total welfare for firm i, i \neq j:

$$\Delta_i^* = (a-c)\frac{(2 - \gamma^2)(2 - 3\gamma^2 + \gamma^4 + S)[-10\gamma^2 - 3\gamma^3 + 2\gamma^4 + \gamma^5(\gamma(-2 + S) + 2(2 + S))]}{D_2(\gamma)}$$  \hspace{1cm} (3.16)$$

$$z_i^* = (a-c)\frac{N_2(\gamma)}{D_2(\gamma)}$$

$$W_i^* = (a-c)\frac{(1 - \gamma^2)(-2 + 2\gamma + 3\gamma^2 + 3\gamma^3)}{D_2(\gamma)}$$  \hspace{1cm} (3.17)$$

where $N_2(\gamma) = 22\gamma^2 + 10\gamma^3 - 2\gamma^{10} - \gamma^{11} + 16(2 + S) - 43\gamma^2(4 + S) - \gamma(41 + S) - 2\gamma^2(56 + S) + 8\gamma(27 + 2S) - 2\gamma^2(18 + 75) + \gamma^2(76 + 75)$ and $D_2(\gamma) = (1 - \gamma)(-12\gamma^3 + \gamma^{12} + 16(2 + S) + \gamma^2(67 + S) - 3\gamma^2(68 + 3S) - 8\gamma(24 + 7S) + \gamma^2(304 + 34S))$.

It can be seen that:

(i) both governments choose to subsidize R&D.

(ii) investment in cost-reducing innovation is greater than under free trade.

(iii) total output sold in the market increases relatively to the free trade case.

(iv) when goods are imperfect substitutes, total welfare is lower relatively to free trade.

(v) with bilateral subsidies, profits are higher for both firms.

3.2.3. Discussion:

Under Bertrand competition we also get the classic prisoners' dilemma. The unilaterally subsidizing country has a higher level of welfare than under free trade. Since the non-subsidizing country’s welfare decreases, both the countries end up deciding to subsidize R&D, in spite of the fact that total welfare for both the countries is lower than under free trade. Comparing these results with the Cournot competition case, we see that in the Cournot case $\Delta$ is larger (smaller) for $\gamma < (>) 0.426$, output is lower and total welfare is higher (but still lower that in the free trade situation, as mentioned before). Further note that under Bertrand
competition total output with bilateral subsidies is higher than under unilateral subsidies. Compared to Cournot competition total world output is higher under Bertrand competition.

Note, however, that the qualitative policy results do not change from Bertrand to Cournot competition. That is, the optimal subsidy to R&D is always positive. We do not obtain the reversal of policy instruments as do Eaton and Grossman (1986). This suggests that policy instruments are sensitive not only to the nature of market competition but also to the degree of differentiation and to whether firms invest in a cost saving technology or not.

In our model, investment in R&D has a commitment value. Firms can strategically commit to a cost saving investment, thereby altering the best response of the other firm in the market competition stage.

4. Optimal output subsidies

If the output subsidy is unilaterally chosen, the subsidizing country makes higher profits than if it does not subsidize output. Given the prisoners dilemma nature of the policy instrument choice, both the countries end up subsidizing output. The equilibrium outcome of the subsidy choice game is both countries choosing to subsidize. Thus, we only present the results for the bilateral subsidy case for both, Cournot and Bertrand competition.

4.1. Bilateral output subsidy

4.1.1. Cournot competition

Adding the subsidy term to the profit function from the first order conditions we can easily solve for output, \( x_i \).

\[
x_i^* = \frac{(2 - \gamma)(a - c) + 2(\Delta_i + s_i) - \gamma(\Delta_i + s_i)}{4 - \gamma^2}.
\] (4.1)

Substituting \( x_i^* \) into the profits the expenditure on R&D, \( \Delta_i \), is obtained.

\[
\Delta_i = \frac{4[(a - c)(4 - 4\gamma - 2\gamma^2 + \gamma^3) + (4 - 2\gamma^2)s_i - \gamma(4 - \gamma^2)s_i]}{16 - 32\gamma^2 + 12\gamma^4 - \gamma^6}.
\] (4.2)
Substituting $\Delta_4$ into the profits and maximizing total welfare the equilibrium subsidy, $s^*$ is obtained,

$$s^* = (a - c)^2 \frac{48 - 40\gamma^2 + 2\gamma^4 - \gamma^6}{D_0(\gamma)}.$$  \hspace{1cm} (4.3)

where $D_0(\gamma) = 64 + 64\gamma - 128\gamma^2 - 64\gamma^3 + 72\gamma^4 + 20\gamma^5 - 16\gamma^6 - 2\gamma^7 - \gamma^8$.

Equilibrium production, innovation and welfare levels are,

$$x^*_1 = (a - c)^2 \frac{2(4 - \gamma^2)(2 - \gamma^2)}{D_0(\gamma)}$$  \hspace{1cm} (4.4)

$$\Delta^*_1 = (a - c)^2 \frac{8(8 - 6\gamma^2 + \gamma^4)}{D_0(\gamma)}$$  \hspace{1cm} (4.5)

$$W^*_1 = (a - c)^2 \frac{(4 - \gamma^2)^2(64 - 224\gamma^2 + 216\gamma^4 - 88\gamma^6 + 16\gamma^8 - \gamma^{10})}{D_0(\gamma)^2}$$ \hspace{1cm} (4.6)

We set the equilibrium profits ($s^*$) equal to zero in order to obtain the value of $\gamma$ below which the analysis of the profits and welfare is relevant. We restrict our attention to these $\gamma$ values only. In this relevant range, we find that the equilibrium subsidy is always positive. Total welfare is, however, maximized when $\gamma = 0$. Firm innovation under bilateral output subsidies is greater than under free trade. Total world output increases due to the bilateral subsidy.

4.1.2. Bertrand competition

Adding the subsidy term to the profit function from the first order conditions we can easily solve for the price, $p_i$.

$$p_i = \frac{a^2 - 4 - \gamma^2 + \gamma^2}{2(2 + \gamma) - 2(s_1 + \Delta_1) - \gamma(s_2 + \Delta_2)}.$$  \hspace{1cm} (4.7)

Substituting $p^*_i$ into the profits the expenditure on R&D, $\Delta_i$, is obtained,

$$\Delta^*_i = \frac{2(2 - \gamma^2)(2 - \gamma^2 - \gamma(4 - \gamma^2))(a - c) + s_1(2 - \gamma^2)^2 - s_2\gamma(4 - \gamma^2)}{16 - 48\gamma^2 + 32\gamma^4 - 9\gamma^6 + \gamma^8}.$$  \hspace{1cm} (4.8)

It should be noted that the government only chooses a tax instead of a subsidy for very high values of $\gamma$. However, since for these values firm profits are negative, firms would not enter the market in this case.
Substituting $\Delta^*_t$ into the profits and writing the total welfare the subsidy, $s^*_t$, that maximizes total welfare is obtained,

$$s^*_t = (a - c) \frac{\gamma^2(16 + 24\gamma^2 - 28\gamma^6 + 9\gamma^8 - \gamma^8)}{D_t(\gamma)}$$

where $D_t(\gamma) = 64 + 64\gamma - 160\gamma^2 - 96\gamma^3 + 104\gamma^4 + 52\gamma^5 - 28\gamma^6 - 12\gamma^7 + 3\gamma^8 + \gamma^9$.

Equilibrium output, innovation and welfare are respectively given by the following expressions:

$$z^*_t = (a - c) \frac{(8 - 6\gamma^2 + \gamma^4)^2}{D_t(\gamma)}$$

$$\Delta^*_t = (a - c) \frac{2(1 - \gamma^2)(2 - \gamma^2)^2}{D_t(\gamma)}$$

$$W^*_t = (a - c)^2 \frac{2(8 - 6\gamma^2 + \gamma^4)^2(16 - 56\gamma^2 + 38\gamma^4 - 10\gamma^6 + \gamma^8)}{D_t(\gamma)^2}$$

Once more, setting equilibrium profits, $\pi^*_t$, equal to zero (and plotting) we obtain the relevant $\gamma$. This $\gamma$ then gives us the maximum value over which the analysis of the profits and welfare is relevant. Profits, $\pi_t$, equal zero for $\gamma = 0.327891$. We restrict our attention to these values of $\gamma$ only. It is easily seen that the equilibrium subsidy is always positive for all $\gamma < 0.827891$. However, note that total welfare becomes negative for $\gamma > 0.66639$. A welfare minimizing government would never choose to subsidize if the welfare is negative.\(^{10}\) The equilibrium subsidy is always positive in this range also. Firm innovation under bilateral output subsidies is greater/less than under free trade. Total world output increases due to the bilateral subsidy. The increase in output is greater than observed in the Cournot case.

5. Non-credible subsidies

In this section we assume that any policy announcement by the government is not credible. Hence, the firms will choose their policy actions as if the government chooses the policy variable after the firms move. First, the firm invests in

\(^{10}\)Note, the values for which the government chooses a tax instead of a subsidy is for very high values of $\gamma$. However, as firm profits are negative in this range a firm would not enter the industry.
R&D. After the firm has chosen its investment in cost reducing innovation the government chooses the optimal subsidy/tax announcement. Firms then play the market competition game. This only requires us to change the sequence of the moves in the first two stages of the game. As we use the sub-game perfect equilibrium concept this implies that we first solve for the market competition stage, the government(s) then chooses its policy action and finally the firms choose their R&D expenditure.

5.1. Bilateral output subsidy

5.1.1. Cournot competition

Total welfare is a decreasing function of $\gamma$ and is zero for $\gamma = 0.630804$. Total welfare is maximum for $\gamma = 0$. We focus on $\gamma$ in the range where total welfare is non-negative. Firm incentive to invest in cost reduction is maximum under a monopoly and only slightly changes with the degree of product differentiation. Under non-credible policies innovation is higher/less than under credible subsidies and higher/less than under free trade.

Adding the subsidy term to the profit function, $\max \{p_i - (c - \Delta_i - s_i)\} x_i(p_i, p_j)$, from the first order conditions we solve for output, $x_i$:

$$x_i = \frac{(2 - \gamma)(a - c) + 2(\Delta_i + s_i) - \gamma(\Delta_j + s_j)}{4 - \gamma^2}.$$  \hspace{1cm} (5.1)

Substituting $x_i$ into the profits we get, $\pi_i(x_i, \Delta_i, s_i) = (x_i(\Delta_i, s_i))^2 - \frac{a^2}{4}$, the policy announcement, $s_i$, by the government is obtained\(^1\):

$$s_i^* = \frac{\gamma^2(16 - 12\gamma^2 + \gamma^4)}{32 + 32\gamma^2 - 56\gamma^2 - 24\gamma^4 + 16\gamma^4 + 2\gamma^4 - 2\gamma^4 - \gamma^4}.$$  

Further substituting $s_i^*$ into the first order conditions the expenditure on R&D, $\Delta_i^*$, is obtained:

$$\Delta_i^* = \frac{8(4 - \gamma^4)}{32 + 32\gamma^2 - 56\gamma^2 - 24\gamma^4 + 16\gamma^4 + 2\gamma^4 - \gamma^4}. \hspace{1cm} (5.2)$$

Given $\Delta_i^*$ and $s_i^*$, equilibrium output and welfare are then derived:

\(^1\)The government decision is independent of firm investment in R&D.
\[ x^*_\gamma = \frac{2(16 - 12\gamma + \gamma^4)}{32 + 32\gamma - 56\gamma^2 - 24\gamma^3 + 16\gamma^4 + 2\gamma^5 - \gamma^6}. \] (5.3)

\[ W^*_\gamma = \frac{2(8(6\gamma)^2 - 256 - 720\gamma^4 + 224\gamma^6 - 26\gamma^8 + \gamma^{16})}{(32 + 32\gamma - 56\gamma^2 - 24\gamma^3 + 16\gamma^4 + 2\gamma^5 - \gamma^6)^2}. \] (5.4)

These results allow us to write the following propositions:

**Proposition 5.1.** Government subsidy increases with the degree of product differentiation. The government does not subsidize a domestic monopolist.

It is easy to see that \( x^*_\gamma \) is zero for \( \gamma = 0 \). Further, plotting the optimal subsidy as a function of \( \gamma \) it is seen that it increases with \( \gamma \).

**Proposition 5.2.** Firm expenditure in R&D under Cournot competition is greater than under free trade. Total output increases and domestic welfare is lower.

### 5.1.2. Bertrand competition

Total welfare is a decreasing function of \( \gamma \) and is zero for \( \gamma = 0.974307 \). However, profits are non-negative only for \( \gamma \leq 0.938 \). Innovation is a decreasing function of product differentiation and is maximum under a domestic monopoly. Innovation expenditures under Bertrand competition are less than under Cournot competition under a duopoly. The government taxes exports for all degrees of product differentiation in the relevant range. This is the result similar to Eaton-Crosman. This suggests that the policy reversal result is more likely to be observed when governments cannot commit to a policy.

Adding the subsidy term to the profit function from the first order conditions and after some manipulations we obtain the equilibrium price, subsidy, and total welfare.

\[ p^*_\gamma = \frac{a(5\gamma^4 - 4\gamma^6) - c(32 + 32\gamma + 40\gamma^2 + 40\gamma^3 - 14\gamma^4 - 14\gamma^5 + \gamma^6 - \gamma^7)}{-32 - 32\gamma - 40\gamma^2 - 40\gamma^3 + 14\gamma^4 + 14\gamma^5 - \gamma^6 + \gamma^7}. \]

\[ \Delta^*_\gamma = \frac{2(2 - \gamma)^2(4 - 3\gamma^2)}{-32 - 32\gamma - 40\gamma^2 - 40\gamma^3 + 14\gamma^4 + 14\gamma^5 - \gamma^6 + \gamma^7}. \]

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\[ s_i^* = \frac{\gamma^2(-1 + \gamma^2)(16 - 12\gamma^2 + \gamma^4)}{-32 - 32\gamma - 40\gamma^2 - 144\gamma^3 + 14\gamma^2 - \gamma^6 + \gamma^7}. \]

\[ W_i^* = \frac{640\gamma^2 - 256 + 640\gamma^2 - 464\gamma^4 - 32\gamma^6 + 174\gamma^6 - 69\gamma^{10} + 8\gamma^{12}}{-32 - 32\gamma - 40\gamma^2 - 144\gamma^3 + 14\gamma^2 - \gamma^6 + \gamma^7}. \]


If governments can subsidize both R&D and production, we find that they will choose to subsidize both R&D and output under Bertrand competition. The optimal subsidy on R&D is given by \( z_i^b \), and the optimal subsidy on output is given by \( s_i^b \):}

\[ z_i^b = \frac{\gamma^2}{4 - \gamma^2} \]

\[ s_i^b = \frac{\gamma^2(a - c)}{1 + \gamma - 2\gamma^2 - \gamma^3} \]

However, under Cournot competition there is a partial reversal of results with both governments choosing to tax R&D while subsidizing output. This extends Spencer and Brander's (1983) result to the case of product differentiation. Subsidies to R&D and output are given by \( z_i^c \) and \( s_i^c \) respectively:

\[ z_i^c = \frac{\gamma^2}{4 - \gamma^2} \]

\[ s_i^c = \frac{\gamma^2(a - c)}{1 + \gamma - \gamma^2} \]

The reason why this happens is that under Cournot competition overinvestment in R&D is observed, unlike the Bertrand case. As a result, both governments find it in their incentive to tax R&D under Cournot competition.
7. Conclusion

When firms in a differentiated industry invest in cost reducing R&D, we show that the well known Eaton-Grossman's (1986) policy reversal result is reversed for all the relevant degrees of product differentiation, under credible government policies. That is, it is optimal for the governments to subsidize firms, if we restrict our attention to the cases where both firm profits and country welfare are positive. If there is no investment in cost reducing R&D the Eaton-Grossman policy reversal is obtained for all degree of product differentiation. In a simple way, investment in cost reducing R&D captures firm investment in sunk costs (entry barriers) before it enters an industry. Incorporating this feature in the model we see that the policy instrument reversal is no longer observed.

Our results also highlight the fact that market models that abstract from good differentiation may not be appropriate to the analysis of optimal trade policies when firms invest in cost reduction. As Bester and Petrats (1993) also found, we show that the degree of product differentiation is an important determinant of the policy instrument choice. This is true for both Cournot and Bertrand competition. Allowing for investment in cost reducing R&D we see that this has an important commitment value for both the firm. By committing to a level of cost reducing R&D firms are able to affect the price/output choice in the market selection stage.

Results on innovation expenditures under Cournot and Bertrand competition are similar to Bester and Petrats (1993), with firms investing more in cost-reducing innovation under Cournot than under Bertrand competition. Investment in R&D, however, depends on the degree of product differentiation. As the goods become closer substitutes, investment in R&D declines.

The importance of government commitment is further highlighted in our framework. We show that policy reversals occur when government policy announcements are not credible. However, a credible government will never choose to tax the domestic firm regardless of the degree of product differentiation. In all the cases domestic welfare is always greater under free trade.

Our results further indicate that when both the countries subsidize R&D and output a partial policy reversal is obtained moving from Bertrand to Cournot competition. Unlike Eaton and Grossman, where countries tax output under

\[12\] Allowing for no investment in cost reducing innovation, under collateral output subsidies the optimal subsidy, or tax, is positive only if the domestic firm is not a monopoly.
Bertrand competition and subsidize output under Cournot, we show that under a system of dual instruments both governments subsidize output and R&D under Bertrand competition. However, under Cournot competition R&D is taxed and output is subsidized.

We plan to further extend our work to several scenarios such as allowing for asymmetries in firm efficiency and country sizes.
References


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