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Abstract

Nonrenewable resource scarcity has been a traditional concern when designing optimal growth models. Technological change has played an important role in those models, since its presence mitigates the depletion effect on extraction paths over time. Moreover, in empirical research, when studying the time trends of prices and other scarcity measures, the role played by technology is crucial. The effects of technological change have been studied in previous literature in several different ways. More emphasis has been placed on capital-resource growth models than on specific sectoral behaviour models. However, in order to predict actual market dynamics the latter are more appropriate.

In this paper, we examine the impact of different kinds of technological change on extraction path over time, in the context of a competitive resource industry. Two cases are considered. In the first case, prices are assumed to be exogenous, while in the second demand is introduced. Therefore, both price and extraction paths result from demand/supply dynamics. Implications for the evolution of scarcity measures (price, marginal extraction cost and user cost) are derived.

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1. Introduction

Simple models of nonrenewable resource extraction consider the case of a firm that has a fixed production process, implying that the firm's cost function does not change throughout the entire period of extraction activities. However, the assumption of no technical improvements in production is empirically inappropriate for most resources. Technology can be (and historically appears to have been) an important way to decrease the scarcity problems inherent of nonrenewable resources, since it allows the extraction of resource stocks that were previously too expensive. Thus, the analysis of extraction behavior in the presence of technical change is quite pertinent.

The impact of technological change on resource extraction can be modeled in several different ways. When using a basic profit maximization model to analyze resource extraction, technological progress is generally represented by a downward shift in the extraction cost function and/or in the exploration cost function. A number of authors have done that in a simple way by including time as an argument of cost[1]. However, this implies that progress is basically an exogenous process, whereas most of the literature suggests that technological change is at least partly endogenous[2]. Furthermore, some general results can be obtained even without this simplification.

Farzin [7] studies the impact of technological change on the behavior of scarcity measures (price, marginal cost of extraction and scarcity rent) for a competitive firm. The author begins by considering the case of no technological change, and presents the dynamics of scarcity measures (with special emphasis to the relationships between their movements). These results are then extended to the case of technological change, but his conclusions rely on using a *given extraction path* to compare the case of progress with the case of no progress. This seems quite illogical, for it implies that the competitive firm does not change the chosen path for its decision variable even though its cost function has been altered, and hence it is not behaving as a profit maximizer.

In this paper, a more adequate study of technological change is presented,

[1] See, for example, Dagupta and Heal [4], Slade [11], and Berck and Roberts [1]. An example of a specific cost function that is estimated including time is given in Lasserre and Ouellette [10]. The same strategy is often applied to production functions in capital-resource models with consumption, although these have limited interest for empirical studies (again, see Dasgupta and Heal [4]).

[2] The methods used to explain technical progress are similar to those used in investment theory. For a good introduction to the subject, see Kamien and Schwartz [9].

using comparative dynamic methods from Caputo [2] and Sweeney [12]. The basic behavior of scarcity measures is sketched, then the impact of technical progress on those measures (as well as on the extraction path) is derived. Finally, an attempt is made to take demand into consideration, thus gaining some insight into market dynamics. This first part considers only the case of a single change in technology. However, a more realistic approach requires the consideration of several stages of technological improvement. That is the purpose of the second part of this research study.

2. A Basic Model of Extraction

Consider the general profit maximization problem for an individual owner of a nonrenewable resource deposit when price is taken as given:

$$\begin{aligned} \underset{E_t}{Max} \quad \Pi &= \int_0^T \left[P_t E_t - C(E_t, S_t, Z_t)\right] e^{-rt} dt \\ s.t. \quad \dot{S} &= -E_t \\ S_0 &= \Sigma \\ S_T &\geq 0 \\ E_t &\geq 0 \end{aligned} \tag{2.1}$$

where S_t is the existing stock of resource in moment t, E_t is the extraction at t and Z_t indicates the level of technology attained at t (assumed to be exogenous). Σ is the known initial number of resource units available[3].

The cost function is assumed to have the following properties (all derivatives evaluated at t):

1. twice continuously differentiable;

[3]Farzin ([6] and [7]) criticizes the use of a finite, known stock of the resource, and replaces the state equation in 2.1 with $\dot{X} = E_t$, where $X_0 = 0$. However, the necessary assumption that $\lim_{t\to\infty} X_t = \overline{X}$ seems to mean exactly the same thing if one is trying to calculate the extraction path. Moreover, the author's justification for the existence of a finite maximum for cumulative extraction is that "*barring technological change*, increasingly large quantities of the resource can be exploited only at increasingly high incremental costs...it will never make economic sense to extract infinite amount of the resource" ([6], p.815-6). If it is true that the finite initial stock assumption is unrealistic, then a better way to represent the state equation would be to include discoveries, which could encompass the effect of new geological information as well as technological progress.

2. strictly convex, thus $\frac{\partial^2 C}{\partial E^2} > 0$, $\frac{\partial^2 C}{\partial S^2} > 0$, $\frac{\partial^2 C}{\partial Z^2} > 0$ and $\frac{\partial^2 C}{\partial E^2}\frac{\partial^2 C}{\partial S^2} - \left(\frac{\partial^2 C}{\partial E \partial S}\right)^2 > 0$;

3. according to intuition, in the absence of learning by doing one expects $\frac{\partial C}{\partial E} > 0$, $\frac{\partial C}{\partial S} < 0$, $\frac{\partial C}{\partial Z} < 0$, and $\frac{\partial^2 C}{\partial E \partial S} < 0$, $\frac{\partial^2 C}{\partial E \partial Z} < 0$, $\frac{\partial^2 C}{\partial S \partial Z} > 0$;

   Thus, marginal extraction cost is positive and increasing (reflecting diminishing returns to extraction); there are stock effects in both total and marginal cost (when the stock decreases, extraction becomes more costly); as for technology, it is assumed to lower total and marginal extraction cost, and to decrease the impact of stock effects on total cost (note that $\frac{\partial C}{\partial S}$ becomes smaller in absolute value when the level of technology improves)[4].

The Hamiltonean (in current value) for this problem is:

$$H_c = P_t E_t - C(E_t, S_t, Z_t) + \phi_t(-E_t)$$

with first order conditions[5]:

$$\begin{array}{lll} i) & \frac{\partial H_c}{\partial E_t} & = \left[P_t - \frac{\partial C_t}{\partial E_t}\right] - \phi_t \leq 0;\ E_t \geq 0;\ E_t\left(\left[P_t - \frac{\partial C_t}{\partial E_t}\right] - \phi_t\right) = 0 \\ ii) & \dot{\phi} - \phi_t r & = -\frac{\partial H_c}{\partial S_t} = \frac{\partial C_t}{\partial S_t} \\ iii) & \dot{S} & = -E_t \\ iv) & \lim_{t \to T} \phi_t e^{-rt} S_t & = 0;\ \lim_{t \to T} \phi_t e^{-rt} \geq 0;\ \lim_{t \to T} S_t \geq 0 \end{array} \tag{2.2}$$

Given the properties of the cost function, H_c is strictly concave in (S_t, E_t). Hence these conditions are both necessary and sufficient.

From condition ii), solving the differential equation:

$$\phi_t = e^{-r(T-t)}\phi_T - \int_t^T \frac{\partial C_\tau}{\partial S_\tau} e^{-r(\tau-t)} d\tau \tag{2.3}$$

Substituting 2.3 into condition ii) again, the behavior of ϕ can be summarized through [6]:

$$\dot{\phi} = e^{-r(T-t)}\left(\phi_T r + \frac{\partial C}{\partial S}\right) - \int_t^T \frac{\partial \dot{C}_\tau}{\partial S_\tau} e^{-r(\tau-t)} d\tau \tag{2.4}$$

[4]These assumptions are similar those found in the literature. See, for example, Farzin [6].

[5]If the problem has an infinite horizon, then the transversality condition $\lim_{t\to\infty} e^{-rt}H_c = 0$ must also hold, but the discount factor ensures that the condition is verified as long as H_c is finite.

[6]The expression $\frac{\partial \dot{C}}{\partial S}$ refers to $\frac{\partial\left(\frac{\partial C}{\partial S}\right)}{\partial t}$.

When the time horizon is far enough into the future ($T \to \infty$), 2.4 becomes:

$$\dot{\phi} = -\int_t^{\infty} \frac{\partial \dot{C}_\tau}{\partial S_\tau} e^{-r(\tau - t)} d\tau \tag{2.5}$$

Equation 2.5 shows the close relationship between the behavior of stock effects throughout time and that of opportunity cost (namely, if $\frac{\partial \dot{C}_\tau}{\partial S_\tau} > 0$ for $\tau > t$ then $\dot{\phi} < 0$ at t and vice-versa for $\frac{\partial \dot{C}_\tau}{\partial S_\tau} < 0$).

Note, from condition i), that whenever extraction is positive:

$$P_t = \frac{\partial C_t}{\partial E_t} + \phi_t \tag{2.6}$$

The dynamic version of 2.6 is:

$$\dot{P} = \frac{\partial \dot{C}}{\partial E} + \dot{\phi} \tag{2.7}$$

Note also that:

$$\frac{\partial \dot{C}}{\partial S} = \frac{\partial^2 C}{\partial E \partial S}\dot{E} + \frac{\partial^2 C}{\partial S^2}\dot{S} + \frac{\partial^2 C}{\partial S \partial Z}\dot{Z} \tag{2.8}$$

$$\frac{\partial \dot{C}}{\partial E} = \frac{\partial^2 C}{\partial E^2}\dot{E} + \frac{\partial^2 C}{\partial E \partial S}\dot{S} + \frac{\partial^2 C}{\partial E \partial Z}\dot{Z} \tag{2.9}$$

Equations 2.7, 2.8, 2.9, and 2.5 or 2.4 can be used to study the relationship between opportunity cost, price, and extraction cost along the optimal extraction trajectory.

2.1. Extraction with Parameter Invariance

In this section it will be assumed that the parameters of problem 2.1 (price, interest rate, and the level of technology) remain constant and that the time horizon is arbitrarily large. Under those conditions, equations 2.7, 2.9, and 2.8 can be combined to yield:

$$0 = \frac{\partial \dot{C}}{\partial E} + \dot{\phi} \tag{2.10}$$

$$\frac{\partial \dot{C}}{\partial S} = \frac{\frac{\partial^2 C}{\partial E \partial S}}{\frac{\partial^2 C}{\partial E^2}}\left(\frac{\partial \dot{C}}{\partial E}\right) + \frac{F}{\frac{\partial^2 C}{\partial E^2}}\left[\left(\frac{\partial^2 C}{\partial E \partial S}\right)^2 - \frac{\partial^2 C}{\partial E^2}\frac{\partial^2 C}{\partial S^2}\right] \tag{2.11}$$

From 2.10, marginal cost and opportunity cost will necessarily be moving in opposite directions along the optimal path. However, with cost function convexity 2.11 shows that if $\frac{\partial \dot{C}}{\partial E} > 0$ then $\frac{\partial \dot{C}}{\partial S} < 0$; if that happens for a long enough period of time, $\dot{\phi} > 0$. Hence, for the case of invariant price it will generally be true that $\frac{\partial \dot{C}}{\partial E} < 0$ and $\dot{\phi} > 0$, since $\dot{\phi} < 0$ would mean $\frac{\partial \dot{C}}{\partial S} > 0$ for at least some periods, implying $\frac{\partial \dot{C}}{\partial E} < 0$ which contradicts 2.10[7]. Under these conditions, equation 2.9 indicates that extraction will be decreasing along the optimal trajectory.

3. Technological Progress with exogenous prices

3.1. Single dZ

In this section the role of technology will be examined in a comparative dynamics perspective, although to begin with the comparative static effects on the steady state shall be ascertained. When the parameters of the model are invariant (price P, interest rate r, and technological progress Z), the system described by the first order conditions will eventually reach a steady state, where $\dot{S} = 0$ and $\dot{\phi} = 0$. This steady state may be characterized either by the exhaustion of the initial resource stock or by the abandonment of remaining units (if price falls below marginal extraction cost plus opportunity cost, extraction becomes unprofitable and $E = 0$, as can be seen from first order condition i)). Note that if the terminal time is sufficiently far into the future it is possible to have both $\overline{S} > 0$ and $\overline{\phi} > 0$ (steady state values) without breaking the transversality condition. This is the case that will be analyzed throughout the rest of the paper[8]. It can be shown that under strict concavity of the Hamiltonean and with $\frac{\partial^2 C}{\partial E \partial S} < 0$ the steady state is a saddle point.

The first order conditions imply that at the steady state:

$$\overline{\phi} = -\frac{\left.\frac{\partial C}{\partial S}\right|_{E=0,S=\overline{S}}}{r} \tag{3.1}$$

[7] It is possible to have $\frac{\partial \dot{C}}{\partial E} > 0$ and $\dot{\phi} < 0$ for a period of time if $\frac{\partial \dot{C}}{\partial S}$ behaves non-monotonically, namely if it starts out negative (which it will have to remain as long as marginal cost is growing) and then turns positive. Thus it would be possible, but not necessary, for extraction to grow for some time and then start decreasing.

[8] As Sweeney [12] remarks, the situation where $\overline{\phi} > 0$ is usually associated with a negative, stock-dependent externality that would linger even after extraction had ceased.

$$P = \left.\frac{\partial C}{\partial E}\right|_{E=0,S=\overline{S}} + \overline{\phi} \tag{3.2}$$

Now, assuming that the level of technology was higher than in the previous case ($Z' > Z$, hence $dZ > 0$) the effects on the optimal extraction path are examined, beginning with the steady state. A phase diagrammatic approach similar to the one in Sweeney [12] is presented in the Appendix.

Total differentiation of 3.1 and 3.2 yields:

$$d\overline{\phi} = -\frac{1}{r}\left(\frac{\partial^2 C}{\partial S^2}d\overline{S} + \frac{\partial^2 C}{\partial S \partial Z}dZ\right) \tag{3.3}$$

$$dP = \frac{\partial^2 C}{\partial E \partial Z}dZ + \frac{\partial^2 C}{\partial E \partial S}d\overline{S} + d\overline{\phi} \tag{3.4}$$

Maintaining the assumption of constant prices, $dP = 0$, equation 3.4 becomes:

$$d\overline{\phi} = -\frac{\partial^2 C}{\partial E \partial Z}dZ - \frac{\partial^2 C}{\partial E \partial S}d\overline{S} \tag{3.5}$$

Solving 3.3 and 3.5, and simplifying yields:

$$d\overline{S} = \frac{\frac{\partial^2 C}{\partial S \partial Z} - r\frac{\partial^2 C}{\partial E \partial Z}}{r\frac{\partial^2 C}{\partial E \partial S} - \frac{\partial^2 C}{\partial S^2}}dZ \tag{3.6}$$

$$d\overline{\phi} = -\frac{\left(\frac{\partial^2 C}{\partial S \partial Z}\frac{\partial^2 C}{\partial E \partial S} - \frac{\partial^2 C}{\partial E \partial Z}\frac{\partial^2 C}{\partial S^2}\right)}{r\frac{\partial^2 C}{\partial E \partial S} - \frac{\partial^2 C}{\partial S^2}}dZ \tag{3.7}$$

Using the properties assumed for the cost function, it is shown that the steady state stock will decrease with a single technological improvement, which means that cumulative extraction is higher. This result can be explained intuitively if one remembers that extraction costs are lower, which combined with a constant price means that some units that were previously abandoned now become sufficiently attractive to be extracted. In general, nothing can be said about the sign of $d\overline{\phi}$ when $dZ > 0$. The explanation for this indeterminacy lies with the double effect of technology on costs: on one hand, marginal extraction costs are decreased, making the value of each remaining stock unit higher; but on the other hand, the negative impact of depletion on costs becomes smaller, meaning that each unit of remaining stock has less cost-reducing value. A clear illustration of these effects is obtained if one looks at each of them separately, as follows.

Technological progress might not affect both marginal extraction cost and marginal depletion cost. Two special cases will be analyzed[9]:

- Extraction-biased technological change: $\frac{\partial^2 C}{\partial E \partial Z} < 0$ and $\frac{\partial^2 C}{\partial S \partial Z} = 0$

  In this case equations 3.6 and 3.7 become:

$$d\overline{S} = \frac{-r\frac{\partial^2 C}{\partial E \partial Z}}{r\frac{\partial^2 C}{\partial E \partial S} - \frac{\partial^2 C}{\partial S^2}} dZ \tag{3.8}$$

$$d\overline{\phi} = \frac{\frac{\partial^2 C}{\partial E \partial Z}\frac{\partial^2 C}{\partial S^2}}{r\frac{\partial^2 C}{\partial E \partial S} - \frac{\partial^2 C}{\partial S^2}} dZ \tag{3.9}$$

  Note that the impact on steady state scarcity rent is no longer ambiguous. In the presence of this type of technological change, $\overline{\phi}$ increases with $dZ > 0$.

- Depletion-biased technological change: $\frac{\partial^2 C}{\partial E \partial Z} = 0$ and $\frac{\partial^2 C}{\partial S \partial Z} > 0$

  Now the relevant equations are:

$$d\overline{S} = \frac{\frac{\partial^2 C}{\partial S \partial Z}}{r\frac{\partial^2 C}{\partial E \partial S} - \frac{\partial^2 C}{\partial S^2}} dZ \tag{3.10}$$

$$d\overline{\phi} = -\frac{\frac{\partial^2 C}{\partial S \partial Z}\frac{\partial^2 C}{\partial E \partial S}}{r\frac{\partial^2 C}{\partial E \partial S} - \frac{\partial^2 C}{\partial S^2}} dZ \tag{3.11}$$

  Thus, this type of technological change causes a decrease of steady state scarcity rent.

Unlike what has been done so far, a true comparative dynamic analysis requires taking into account impacts on the entire optimal path. For this purpose a variational differential equation approach similar to that in Caputo [2] is appropriate[10]. It was shown above that first order conditions for the single competitive producer problem, with parameter invariance, are given by:

$$P = \frac{\partial C}{\partial E}(E, S; Z) + \phi \tag{3.12}$$

$$\dot{\phi} = r\phi + \frac{\partial C}{\partial S}(E, S; Z) \tag{3.13}$$

$$\dot{S} = -E \tag{3.14}$$

---

[9] Farzin [7] also presents the case of neutral technological change, where $\frac{\frac{\partial^2 C}{\partial E \partial Z}}{\frac{\partial^2 C}{\partial S \partial Z}}$ does not change with the level of technology. This case brings no additional information to our model, hence it will not be considered here.

[10] See also Epstein [5] for the infinite horizon case.

Condition 3.12 defines optimal extraction:

$$E^* = E(S, \phi; P, Z) \tag{3.15}$$

The partial derivatives of 3.15 can be calculated implicitly:

$$\begin{aligned}
\frac{\partial E}{\partial S} &= -\frac{\frac{\partial^2 C}{\partial E \partial S}}{\frac{\partial^2 C}{\partial E^2}} > 0 \\
\frac{\partial E}{\partial \phi} &= -\frac{1}{\frac{\partial^2 C}{\partial E^2}} < 0 \\
\frac{\partial E}{\partial Z} &= -\frac{\frac{\partial^2 C}{\partial E \partial Z}}{\frac{\partial^2 C}{\partial E^2}} > 0 \\
\frac{\partial E}{\partial P} &= \frac{1}{\frac{\partial^2 C}{\partial E^2}} > 0
\end{aligned} \tag{3.16}$$

Upon substitution of 3.15 into 3.13 and 3.14, one obtains the following system:

$$\begin{aligned}
\dot{\phi} &= r\phi + \frac{\partial C}{\partial S}\left(S, E(S, \phi; P, Z); Z\right) \\
\dot{S} &= -E(S, \phi; P, Z)
\end{aligned} \tag{3.17}$$

To complete the problem, boundary conditions are necessary. The terminal conditions are replaced with the steady state equations, $\dot{S} = 0$ and $\dot{\phi} = 0$. The optimal state and costate arcs thus obtained from 3.17, $(S(t;\beta), \phi(t;\beta))$, where β denotes the set of parameter values (P, r, Z and S_0), can be plugged back into 3.17, producing:

$$\begin{aligned}
\dot{\phi}(t;\beta) &= r\phi(t;\beta) + \frac{\partial C}{\partial S}\left(S(t;\beta), E(S(t;\beta), \phi(t;\beta); P, Z); Z\right) \\
\dot{S}(t;\beta) &= -E(S(t;\beta), \phi(t;\beta); P, Z)
\end{aligned} \tag{3.18}$$

The variational differential equations are derived from 3.18 and from the boundary conditions, by differentiating with respect to Z. The result, using 3.16, is[11]:

$$\begin{aligned}
\frac{\partial \dot{\phi}}{\partial Z} &= r\frac{\partial \phi}{\partial Z} + \left(\frac{\partial^2 C}{\partial S^2} - \frac{\left(\frac{\partial^2 C}{\partial E \partial S}\right)^2}{\frac{\partial^2 C}{\partial E^2}}\right)\frac{\partial S}{\partial Z} - \frac{\frac{\partial^2 C}{\partial E \partial S}}{\frac{\partial^2 C}{\partial E^2}}\left(\frac{\partial \phi}{\partial Z} + \frac{\partial^2 C}{\partial E \partial Z}\right) + \frac{\partial^2 C}{\partial S \partial Z} \\
\frac{\partial \dot{S}}{\partial Z} &= \frac{\frac{\partial^2 C}{\partial E \partial S}}{\frac{\partial^2 C}{\partial S^2}}\frac{\partial S}{\partial Z} + \frac{1}{\frac{\partial^2 C}{\partial E^2}}\left(\frac{\partial \phi}{\partial Z} + \frac{\partial^2 C}{\partial E \partial Z}\right) \\
\frac{\partial S_0}{\partial Z} &= 0
\end{aligned} \tag{3.19}$$

as well as equations 3.6 and 3.7.

[11] Notation has been simplified for clarity, but it should be remembered that from now on the values presented are optimal ones.

All derivatives are evaluated at $\beta = \beta_0$, where β_0 denotes the initial set of parameter values. The solution to 3.19, $\left(\frac{\partial \phi}{\partial Z}, \frac{\partial S}{\partial Z}\right)$, provides the impact of changes in Z on the entire optimal paths of scarcity rent and stock. Instead of studying this general case, a distinction will once again be made between extraction and depletion-biased technological change.

- In the extraction-biased case, 3.19 becomes:

$$\begin{aligned}
\frac{\partial \dot{\phi}}{\partial Z} &= r\frac{\partial \phi}{\partial Z} + \left(\frac{\partial^2 C}{\partial S^2} - \frac{\left(\frac{\partial^2 C}{\partial E \partial S}\right)^2}{\frac{\partial^2 C}{\partial E^2}}\right)\frac{\partial S}{\partial Z} - \frac{\frac{\partial^2 C}{\partial E \partial S}}{\frac{\partial^2 C}{\partial E^2}}\left(\frac{\partial \phi}{\partial Z} + \frac{\partial^2 C}{\partial E \partial Z}\right) \\
\frac{\partial \dot{S}}{\partial Z} &= \frac{\frac{\partial^2 C}{\partial E \partial S}}{\frac{\partial^2 C}{\partial S^2}}\frac{\partial S}{\partial Z} + \frac{1}{\frac{\partial^2 C}{\partial E^2}}\left(\frac{\partial \phi}{\partial Z} + \frac{\partial^2 C}{\partial E \partial Z}\right) \\
\frac{\partial S_0}{\partial Z} &= 0
\end{aligned} \tag{3.20}$$

as well as equations 3.8 and 3.9

Using cost function properties, a study of variable movements through the construction of a diagram in $\left(\frac{\partial \phi}{\partial Z}, \frac{\partial S}{\partial Z}\right)$can be undertaken. Figure 3.1 represents such a diagram, using simplified notation[12]. The arrows indicate motion and are drawn using the following procedure: a sign for $\left(\frac{\partial \phi}{\partial Z}, \frac{\partial S}{\partial Z}\right)$ is assumed, then used to calculate the corresponding sign for $\left(\frac{\partial \dot{\phi}}{\partial Z}, \frac{\partial \dot{S}}{\partial Z}\right)$, which is then used to draw the appropriate motion arrows. The results obtained are summarized in Proposition 1.

Proposition 1. *In the case of an extraction-biased technological change, that is, $\frac{\partial^2 C}{\partial E \partial Z} < 0$ and $\frac{\partial^2 C}{\partial S \partial Z} = 0$, where $dZ > 0$,*

1. $\frac{\partial \bar{S}}{\partial Z} < 0$ and $0 < \frac{\partial \bar{\phi}}{\partial Z} < -\frac{\partial^2 C}{\partial E \partial Z}$
2. $\frac{\partial S}{\partial Z} < 0$ *and* $\frac{\partial \phi}{\partial Z} > 0\ \forall t \in]0, +\infty[$
3. $0 < \frac{\partial \phi}{\partial Z} < -\frac{\partial^2 C}{\partial E \partial Z}$ *at* $t = 0$, *hence* $\frac{\partial \dot{S}}{\partial Z} < 0$, *which implies* $\frac{\partial E}{\partial Z} > 0$, *i.e.. initial extraction increases.*

Proof: *Equations 3.8 and 3.9 are used directly to obtain the steady state results. This defines an area in Figure 3.1. Looking at the motion arrows and recalling that $\frac{\partial S}{\partial Z} = 0$ at the initial moment, if the steady*

[12] For the diagram it is assumed that $\frac{\partial^2 C}{\partial E \partial Z}$ does not change with stock or extraction, allowing one to plot the line $\frac{\partial \phi}{\partial Z} = -\frac{\partial^2 C}{\partial E \partial Z}$.

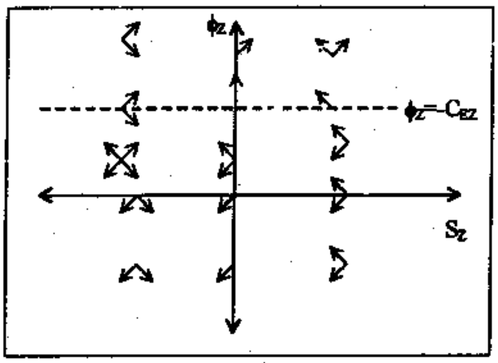


Figure 3.1: Extraction-biased technological change

state is to be reached then necessarily $\frac{\partial \phi}{\partial Z} > 0\ \forall t \in [0, +\infty[$ *and* $\frac{\partial S}{\partial Z} < 0$ $\forall t\ \in]0, +\infty[$. *For* $t = 0$, *plugging* $\frac{\partial S}{\partial Z} = 0$ *and* $\frac{\partial \phi}{\partial Z} > 0$ *into the* $\frac{\partial \dot{S}}{\partial Z}$ *equation in 3.20 provides the result.*

All subsequent Propositions derive from using the same approach.

- For the depletion-biased case, 3.19 turns into:

$$
\begin{aligned}
\frac{\partial \dot{\phi}}{\partial Z} &= \left(r - \frac{\frac{\partial^2 C}{\partial E \partial S}}{\frac{\partial^2 C}{\partial E^2}}\right)\frac{\partial \phi}{\partial Z} + \left(\frac{\partial^2 C}{\partial S^2} - \frac{\left(\frac{\partial^2 C}{\partial E \partial S}\right)^2}{\frac{\partial^2 C}{\partial E^2}}\right)\frac{\partial S}{\partial Z} + \frac{\partial^2 C}{\partial S \partial Z} \\
\frac{\partial \dot{S}}{\partial Z} &= \frac{\frac{\partial^2 C}{\partial E \partial S}}{\frac{\partial^2 C}{\partial S^2}}\frac{\partial S}{\partial Z} + \frac{1}{\frac{\partial^2 C}{\partial E^2}}\frac{\partial \phi}{\partial Z} \\
\frac{\partial S_0}{\partial Z} &= 0
\end{aligned}
\tag{3.21}
$$

as well as equations 3.10 and 3.11

Using a procedure analogous to the one described above, this case is pictured in Figure 3.2 and the associated results are in Proposition 2.

Proposition 2. *In the case of a depletion-biased technological change, that is,* $\frac{\partial^2 C}{\partial S \partial Z} > 0$ and $\frac{\partial^2 C}{\partial E \partial Z} = 0$, *where* $dZ > 0$,

1. $\frac{\partial \overline{S}}{\partial Z} < 0$ and $\frac{\partial \overline{\phi}}{\partial Z} < 0$
2. $\frac{\partial S}{\partial Z} < 0\ \forall t\ \in]0, +\infty[$
3. $\frac{\partial \phi}{\partial Z} < 0$ *at* $t = 0$, *hence* $\frac{\partial \dot{S}}{\partial Z} < 0$ *and once more initial extraction increases.*

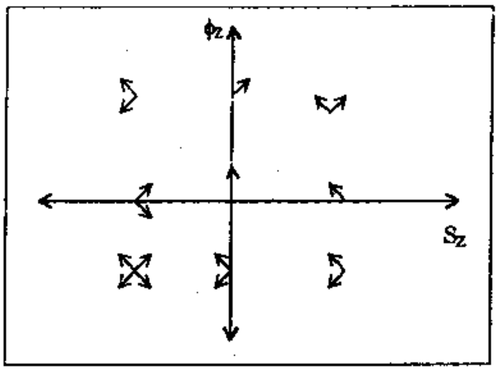


Figure 3.2: Depletion-biased technological change

3.2. Single dZ and dP

In this section, the combined effect of a single change in technology (dZ) and in the price of the extracted resource (dP) is examined in a comparative dynamics perspective. The case of a single change in the price of the good (dP) was already examined in Caputo [2]. Except for the transversality conditions, and, consequently, for what occurs at the boundary, the motion diagram is the same as in Caputo [2]. Using these results, and those obtained in the previous section, the two special cases for technological change are analyzed. The comparative statics results are as follows:

- Extraction-biased technological change: $\frac{\partial^2 C}{\partial E \partial Z} < 0$ and $\frac{\partial^2 C}{\partial S \partial Z} = 0$

  In this case equations 3.6 and 3.7 become:

$$d\overline{S} = \frac{-r\frac{\partial^2 C}{\partial E \partial Z}}{r\frac{\partial^2 C}{\partial E \partial S} - \frac{\partial^2 C}{\partial S^2}} dZ + \frac{r}{r\frac{\partial^2 C}{\partial E \partial S} - \frac{\partial^2 C}{\partial S^2}} dP \tag{3.22}$$

$$d\overline{\phi} = \frac{\frac{\partial^2 C}{\partial E \partial Z}\frac{\partial^2 C}{\partial S^2}}{r\frac{\partial^2 C}{\partial E \partial S} - \frac{\partial^2 C}{\partial S^2}} dZ - \frac{\frac{\partial^2 C}{\partial S^2}}{r\frac{\partial^2 C}{\partial E \partial S} - \frac{\partial^2 C}{\partial S^2}} dP \tag{3.23}$$

  Note that when $dZ > 0$ and $dP > 0$, the impact both on steady state scarcity rent and stock is amplified, as it is obtained by adding up the separate effects of the same sign. Technological improvement and price increase affect the problem in a similar manner, since both contribute to make extraction more attractive and stock more valuable. Therefore, the steady state stock decreases by more, and the steady state scarcity rent also increases by more. However, when $dZ > 0$ and $dP < 0$ the signs both on $d\overline{S}$ and on $d\overline{\phi}$ are undetermined.

- Depletion-biased technological change: $\frac{\partial^2 C}{\partial E \partial Z} = 0$ and $\frac{\partial^2 C}{\partial S \partial Z} > 0$

  Now the relevant equations are:

$$d\overline{S} = \frac{\frac{\partial^2 C}{\partial S \partial Z}}{r\frac{\partial^2 C}{\partial E \partial S} - \frac{\partial^2 C}{\partial S^2}} dZ + \frac{r}{r\frac{\partial^2 C}{\partial E \partial S} - \frac{\partial^2 C}{\partial S^2}} dP \tag{3.24}$$

$$d\overline{\phi} = -\frac{\frac{\partial^2 C}{\partial S \partial Z}\frac{\partial^2 C}{\partial E \partial S}}{r\frac{\partial^2 C}{\partial E \partial S} - \frac{\partial^2 C}{\partial S^2}} dZ - \frac{\frac{\partial^2 C}{\partial S^2}}{r\frac{\partial^2 C}{\partial E \partial S} - \frac{\partial^2 C}{\partial S^2}} dP \tag{3.25}$$

  This type of technological change lowers costs but also lowers scarcity rent, whereas a price increase lowers costs but raises stock value. Thus, if $dZ > 0$ and $dP > 0$ there is an additional decrease of steady state stock, while steady state scarcity rent may increase or decrease. On the contrary, when $dZ > 0$ and $dP < 0$, the impact on the steady state stock is not clear, while the steady state scarcity rent decreases by more.

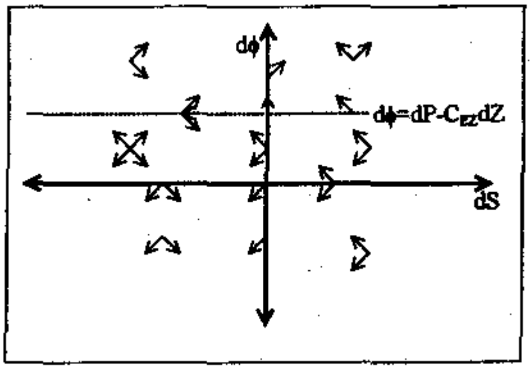
Proceeding similarly as in section 3.1, comparative dynamics results are the following:

- In the extraction-biased case, the variational differential equations become:

$$\begin{aligned} d\dot{\phi} &= rd\phi - \frac{\frac{\partial^2 C}{\partial E \partial S}}{\frac{\partial^2 C}{\partial E^2}}\left(d\phi + \frac{\partial^2 C}{\partial E \partial Z}dZ - dP\right) + \left(\frac{\partial^2 C}{\partial S^2} - \frac{\frac{\partial^2 C}{\partial E \partial S}}{\frac{\partial^2 C}{\partial E^2}}\right) dS \\ d\dot{S} &= \frac{\frac{\partial^2 C}{\partial E \partial S}}{\frac{\partial^2 C}{\partial S^2}} dS + \frac{1}{\frac{\partial^2 C}{\partial E^2}}\left(d\phi - dP + \frac{\partial^2 C}{\partial E \partial Z}dZ\right) \\ dS_0 &= 0 \end{aligned} \tag{3.26}$$

  as well as equations 3.22 and 3.23

Using cost function properties, a study of variable movements through the construction of a diagram in $d\phi$ and dS[13] can now be undertaken. Again, the arrows indicate motion and are drawn using the following procedure: a sign for $d\phi$ and dS is assumed, then used to calculate the corresponding sign for $d\dot{\phi}$ and $d\dot{S}$, which is then used to draw the appropriate motion arrows. Figure 3.3 depicts the case of a rise in price and Figure 3.4 represents the



Figurc 3.3: Extraction-biased change with a price increase

first possibility discussed in Proposition 4 for a fall in price. As usual, all results are summarized in the following Propositions.

Proposition 3. *In the case of an extraction-biased technological change, that is, $\frac{\partial^2 C}{\partial E \partial Z} < 0$ and $\frac{\partial^2 C}{\partial S \partial Z} = 0$, where $dZ > 0$ and $dP > 0$, then,*

1. $d\overline{S} < 0$ and $0 < d\overline{\phi} < dP - \frac{\partial^2 C}{\partial E \partial Z} dZ$
2. $dS < 0$ and $d\phi > 0$ $\forall t \in]0, +\infty[$
3. $0 < d\phi < dP - \frac{\partial^2 C}{\partial E \partial Z} dZ$ *at* $t = 0$, *hence* $d\dot{S} < 0$ *and* $dE > 0$, *i.e., initial extraction increases.*

Note that Figures 3.1 and 3.3 look the same except for the ridge line. This is to be expected given the similar effects of extraction-biased technological improvement and price increase.

Proposition 4. *In the case of an extraction-biased technological change, that is, $\frac{\partial^2 C}{\partial E \partial Z} < 0$ and $\frac{\partial^2 C}{\partial S \partial Z} = 0$, where $dZ > 0$ and $dP < 0$, several possibilities can occur:*

– *(i) if* $dP - \frac{\partial^2 C}{\partial E \partial Z} dZ < 0$, *then:*

1. $d\overline{S} > 0$ and $dP - \frac{\partial^2 C}{\partial E \partial Z} dZ < d\overline{\phi} < 0$

[13] From now on, $d\phi = \frac{\partial \phi}{\partial Z} dZ + \frac{\partial \phi}{\partial P} dP$, and $dS = \frac{\partial S}{\partial Z} dZ + \frac{\partial S}{\partial P} dP$. Likewise for $d\dot{\phi}$ and $d\dot{S}$.

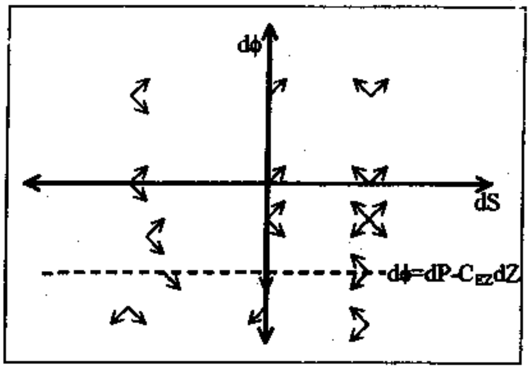


Figure 3.4: Extraction-biased change with a price decrease (case (i))

2. $dS > 0$, $d\phi < 0$, $\forall t \in]0, +\infty[$

3. $dP - \frac{\partial^2 C}{\partial E \partial Z} dZ < d\phi < 0$ at $t = 0$, *thus* $d\dot{S} > 0$ and *initial extraction decreases.*

– *(ii) if* $dP - \frac{\partial^2 C}{\partial E \partial Z} dZ > 0$, *then:*

1. $d\overline{S} < 0$ and $0 < d\overline{\phi} < dP - \frac{\partial^2 C}{\partial E \partial Z} dZ$

2. $dS < 0$, $d\phi > 0$, $\forall t \in]0, +\infty[$

3. $0 < d\phi < dP - \frac{\partial^2 C}{\partial E \partial Z} dZ$ at $t = 0$, *thus* $d\dot{S} < 0$ and *initial extraction increases.*

The diagram for this case also looks exactly the same as that in Figure 3.1, except for the ridge line, hence it is not presented here. Intuitively, this means a small enough price decrease only dampens the effects of technological change. The qualitative results are exactly the same as in Proposition 1.

– *(iii) if* $dP - \frac{\partial^2 C}{\partial E \partial Z} dZ = 0$, *then the two effects cancel out, and in diagrammatic terms we would never leave the origin.*

- For the depletion-biased case, the variational differential equations are:

$$
\begin{aligned}
d\dot{\phi} &= r d\phi + \left(\frac{\partial^2 C}{\partial S^2} - \frac{\left(\frac{\partial^2 C}{\partial E \partial S}\right)^2}{\frac{\partial^2 C}{\partial E^2}} \right) dS - \frac{\frac{\partial^2 C}{\partial E \partial S}}{\frac{\partial^2 C}{\partial E^2}} (d\phi - dP) + \frac{\partial^2 C}{\partial S \partial Z} dZ \\
d\dot{S} &= \frac{1}{\frac{\partial^2 C}{\partial E^2}} (d\phi - dP) + \frac{\frac{\partial^2 C}{\partial E \partial S}}{\frac{\partial^2 C}{\partial S^2}} dS \\
dS_0 &= 0
\end{aligned}
\tag{3.27}
$$

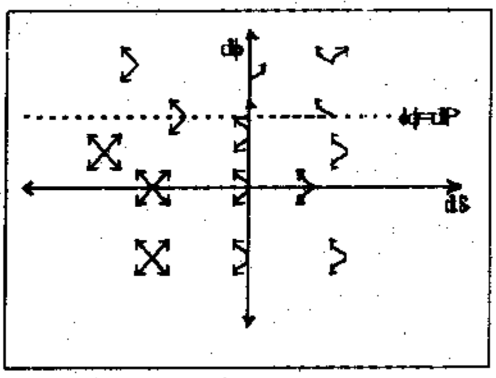


Figure 3.5: Depletion-biased change with a price increase

as well as equations 3.24 and 3.25

Using Figures 3.5 and 3.6 respectively, Propositions 5 and 6 are obtained.

Proposition 5. *In the case of a depletion-biased technological change, that is,* $\frac{\partial^2 C}{\partial S \partial Z} > 0$ and $\frac{\partial^2 C}{\partial E \partial Z} = 0$, *where* $dZ > 0$ *and* $dP > 0$, *then*

1. *(i)* $d\overline{S} < 0$ and $d\overline{\phi} < 0$, *or (ii)* $d\overline{S} < 0$ and $0 < d\overline{\phi} < dP$
2. $dS < 0 \ \forall t \in]0, +\infty[$
3. $d\phi < dP$ *at* $t = 0$, *hence* $d\dot{S} < 0$, *and once more initial extraction increases.*

Proposition 6. *In the case of a depletion-biased technological change, that is,* $\frac{\partial^2 C}{\partial S \partial Z} > 0$ and $\frac{\partial^2 C}{\partial E \partial Z} = 0$, *where* $dZ > 0$ *and* $dP < 0$, *then*

1. *(i)* $d\overline{S} < 0$ and $d\overline{\phi} < dP < 0$, *or (ii)* $d\overline{S} > 0$ and $dP < d\overline{\phi} < 0$
2. dS *and* $d\phi$ *anywhere except in the first quadrant,* $\forall t \in]0, +\infty[$
3. *the sign for* $d\dot{S}$ *at* $t = 0$ *is uncertain, so initial extraction may increase or decrease.*

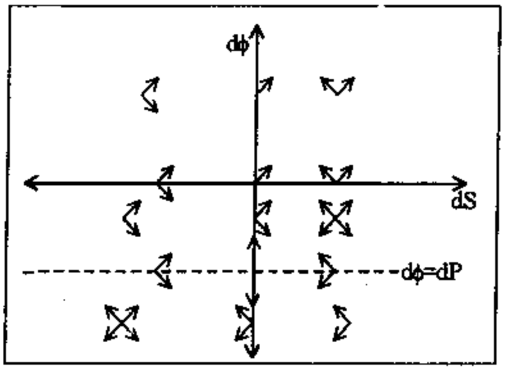


Figure 3.6: Depletion-biased change with a price decrease

3.3. The Impact on Scarcity Measures

3.3.1. Single dZ

The purpose of this section is to study which combinations of scarcity measures' movements are compatible with optimal behavior, when there is a single technological improvement, and assuming that price is exogenous and invariant. This assumption will be relaxed below.

If there is a single technological improvement, the comparative dynamics results obtained in the previous section can be combined with equations 2.10 and 2.11 to describe possible optimal paths for scarcity measures, namely scarcity rent and marginal extraction cost (price is still assumed constant at this point). To begin with, note that the conclusions of section 2.1 can be used in the analysis of the two separate optimal solutions (one for Z and another for Z'); the aforementioned equations hold for both of them. Hence, in either case scarcity rent is generally growing, and marginal extraction cost is decreasing (even though the exact amplitude of these movements cannot be compared). The next step is to consider what happens to the optimal path if technology is initially at Z and then increases to Z'.

In the extraction-biased case, it was shown in Proposition 1 that scarcity rent increases with technology. Considering that the price is constant, this implies that marginal extraction cost becomes lower, $\frac{d\left(\frac{\partial C}{\partial E}\right)}{dZ} < 0\ \forall t$. These effects have the same sign as when technology is invariant, therefore it can be concluded that the optimal path when there is a technological change is also characterized by $\dot{\phi} > 0$

and $\frac{\partial \dot{C}}{\partial E} < 0$. Extraction-biased technological change does not alter the direction of scarcity measure behavior.

For depletion-biased technological change, on the contrary, the nature of the optimal path is changed. Proposition 2 shows that both initial[14] and steady state scarcity rent decrease with technological change. Hence, scarcity rent cannot be growing monotonically, and likewise marginal extraction cost cannot always be decreasing.

It must be emphasized that, unlike what is done in Farzin [7], for a competitive producer the price path should be the starting point and not the result of other scarcity measure movements. That is the perspective of this paper; exogenous changes in price are analyzed in the following sections.

3.3.2. Single dZ and dP

In the presence of a single technological improvement and a single change in price, the consequences for scarcity measures' movements are now examined. Emphasis is placed on ruling out certain behaviors, namely monotonic ones, by proving them to be incompatible with optimal decisions.

In the extraction-biased case, for $dP > 0$ it was shown in Proposition 3 that both initial and steady state scarcity rent increase. Therefore, scarcity rent cannot decrease monotonically with marginal extraction cost always increasing. That is, $\dot{\phi} < 0$ and $\frac{\partial \dot{C}}{\partial E} > 0$ cannot characterize the optimal path[15]. When $dP < 0$, by Proposition 4, in case (i), both initial and steady state scarcity rent decrease. Therefore, scarcity rent cannot increase monotonically with marginal extraction cost always decreasing. In case (ii), initial and steady state scarcity rent increase. Therefore, scarcity rent cannot decrease monotonically, whether marginal extraction cost is always increasing, decreasing or constant. For case (iii), scarcity rent can either increase or decrease.

In the case of a depletion-biased technological change, for $dP > 0$, by Proposition 5, all paths are potential optimal paths. For $dP < 0$, Proposition 6 shows that initial and steady state scarcity rents decrease. Thus, scarcity rent cannot increase monotonically with marginal extraction cost also decreasing monotonically. These results can be summarized as follows:

- In the extraction-biased case, the following paths cannot be optimal:

[14] Here, initial means at the moment when technology changes.

[15] The cases where $\dot{\phi} < 0$ and $\frac{\partial \dot{C}}{\partial E} < 0$ or $\dot{\phi} < 0$ and $\frac{\partial \dot{C}}{\partial E} = 0$ are immediately ruled out by equation 2.6.

- $dP > 0$:
  - $\dot{\phi} < 0$ and $\frac{\partial \dot{C}}{\partial E} > 0$;
- $dP < 0$:
  - (i) $\dot{\phi} > 0$ and $\frac{\partial \dot{C}}{\partial E} < 0$;
  - (ii) $\dot{\phi} < 0$ and $\frac{\partial \dot{C}}{\partial E} < 0$; $\dot{\phi} < 0$ and $\frac{\partial \dot{C}}{\partial E} > 0$; $\dot{\phi} < 0$ and $\frac{\partial \dot{C}}{\partial E} = 0$;

  In the depletion-biased case, the following paths cannot be optimal:
- $dP < 0$:
  - (i) $\dot{\phi} > 0$, and $\frac{\partial \dot{C}}{\partial E} < 0$;
  - (ii) $\dot{\phi} < 0$, and $\frac{\partial \dot{C}}{\partial E} > 0$; $\dot{\phi} < 0$, and $\frac{\partial \dot{C}}{\partial E} < 0$; $\dot{\phi} < 0$, and $\frac{\partial \dot{C}}{\partial E} = 0$.

Thus, for $dZ > 0$ and $dP > 0$, or $dP < 0$, monotonic optimal paths for the scarcity rent, and, hence, for thc marginal extraction cost are only excluded in the case of a depletion-biased technological change, with $dP < 0$. In the case of an extraction-biased technological change, it can be optimal to have the scarcity rent increasing monotonically.

3.4. More comparative dynamics

Following the dynamic primal-dual methodology of Caputo [3], additional comparative dynamic results can be obtained. Specifically, the use of the dynamic envelope theorem and of second-order conditions for the primal-dual formulation yields a number of conclusions that are compatible with those in previous sections but contain more qualitative information. It can be to shown that (where Π^* denotes the optimal value of Π):

- $\frac{\partial \Pi^*}{\partial P} = \int_0^\infty E_t e^{-rt} dt > 0 \Rightarrow$ the impact of a price increase on the optimal profit is positive, as expected, and equal to discounted cumulative supply.
- $\frac{\partial \Pi^*}{\partial Z} = -\int_0^\infty \frac{\partial C}{\partial Z} e^{-rt} dt > 0 \Rightarrow$ the impact of a technological improvement on the optimal profit is also positive, as expected, and equal to discounted cost savings.
- $\int_0^\infty e^{-rt} \frac{\partial E}{\partial P} dt \geq 0 \Rightarrow$ discounted cumulativc supply has a nonnegative slope. Although this is compatible with the usual static result of upward-sloping supply, it shows that in a dynamic problcm such a result need not hold at every point in time.

- $\int_0^\infty \left(\frac{\partial^2 C}{\partial S \partial Z}\frac{\partial S}{\partial Z} + \frac{\partial^2 C}{\partial E \partial Z}\frac{\partial E}{\partial Z} + \frac{\partial^2 C}{\partial Z^2}\right) e^{-rt}dt \leq 0 \Rightarrow$ the cumulative impact of technological change on marginal costs is nonpositive. Note that the term containing $\frac{\partial S}{\partial Z}$ would not appear in a static problem since the dynamics of the resource stock would not be considered.

- $\int_0^\infty \left(\frac{\partial^2 C}{\partial S \partial Z}\frac{\partial S}{\partial P} + \frac{\partial^2 C}{\partial E \partial Z}\frac{\partial E}{\partial P}\right) e^{-rt}dt = -\int_0^\infty \frac{\partial E}{\partial Z} e^{-rt}dt \Rightarrow$ the discounted marginal impact of technological change on extraction is equal to the discounted indirect effect of a price change on $\frac{\partial C}{\partial Z}$ over the entire time horizon, rather than at each point in time. Again, the inclusion of $\frac{\partial S}{\partial P}$ is due to the dynamic nature of the problem.

4. Endogenous price

Allowing price to be determined by market equilibrium requires the introduction of a demand function. Here it will be assumed that producers are competitive, so that the model described in the previous sections is an accurate description of resource supply behavior. However, although price is taken as given by each producer, price behavior will be determined by demand conditions.

For each period, the total quantity demanded of the resource will be equal to the sum of the quantities offered by each individual producer, $Q_t = \int_1^N E_t^i di$ (where i is the producer). Demand for the resource is assumed to be a continuous function of price at t, $Q^D(P_t, t)$, which can inverted to obtain $P(Q_t, t)$. There is a finite choke price in every period, i.e.. there is a price for which $Q^D = 0$. Market equilibrium is given by:

$$\left\{ \begin{array}{ll} Q_t = \int_1^N E_t^i di & = Q^D(P_t, t) \quad \forall t \\ \int_0^T Q^D(P_t, t) & = \int_0^T \int_1^N E_t^i di \leq \int_1^N S_0^i di = S \end{array} \right. \tag{4.1}$$

The first equation is a static equilibrium condition, whereas the second one is a dynamic equilibrium condition. The existence of an equilibrium can be demonstrated using cost function convexity (see Sweeney [12, pp.819-23]).

4.1. Equilibrium without technological improvements

Assuming an infinite horizon, each producer i will behave according to the first order conditions of problem 2.1, namely:

$$\begin{array}{llll} P_t & = \frac{\partial C_t^i}{\partial E_t^i} + \phi^i & if & E_t^i > 0 \\ P_t & < \frac{\partial C_t^i}{\partial E_t^i} + \phi^i & if & E_t^i = 0 \\ \dot{\phi}^i & = \phi_t^i r + \frac{\partial C_t^i}{\partial S_t^i} & & \\ \dot{S}^i & = -E_t^i & & \\ \lim_{t\to\infty} \phi_t^i e^{-rt} S_t^i & = 0 & \lim_{t\to\infty} \phi_t^i e^{-rt} \geq 0 & \lim_{t\to\infty} S_t^i \geq 0 \end{array} \tag{4.2}$$

If demand is constant or growing, prices must eventually grow as long as the resource is being extracted. This result can be obtained recalling equations 2.7, 2.9 and 2.11. Since there is no technological change:

$$\dot{P} = \frac{\partial^2 C^i}{\partial {E^i}^2}\dot{E}^i + \frac{\partial^2 C^i}{\partial E^i \partial S^i}\dot{S}^i + \dot{\phi}^i \tag{4.3}$$

If $\dot{P} < 0$ for all $\tau > t$ and demand is not decreasing, $\dot{E} > 0$ (for the resource market as a whole, and, since all producers are alike, for each individual deposit as well). Equation 4.3 can only be verified if $\dot{\phi}^i < 0$, which is incompatible with $\frac{\partial \dot{C}^i}{\partial E} > 0$ (see section 2.1). A constant price path, $\dot{P} = 0$ for all $\tau > t$, can also be ruled out using the same procedure[16].

If demand is actually constant, growing prices imply decreasing extraction. If demand is increasing, extraction may increase or decrease with growing prices.

4.2. Equilibrium with technological improvements

In this section, previous results are used to study the case in which a single, positive technological improvement occurs. In order to examine how this single change relates to market dynamics, we use the information provided by the time paths of the stock effect and of the marginal cost of extraction, as follows:

$$\frac{\partial \dot{C}}{\partial E} = \frac{\partial^2 C}{\partial E^2}\dot{E} + \frac{\partial^2 C}{\partial E \partial S}\dot{S} \tag{4.4}$$

[16] Note that a U-shaped price path is not ruled out, since $\dot{\phi}_t^i < 0$ can be compatible with an initially decreasing but then increasing stock effect, ie. $\frac{\partial \dot{C}}{\partial S} < 0$ for some time followed by $\frac{\partial \dot{C}}{\partial S} > 0$.

$$\frac{\partial \dot{C}}{\partial S} = \frac{\partial^2 C}{\partial E \partial S}\dot{E} + \frac{\partial^2 C}{\partial S^2}\dot{S} \tag{4.5}$$

These equations have to hold along the optimal path, after the single change in technology. Therefore, all the derivatives are evaluated at the new technology level as well as at the price at t, for all $t \in]0, +\infty[$, along the optimal path.

Recalling the results summarized in previous propositions, and following a similar procedure as the one adopted in section 3.3.2, an attempt will be made to eliminate the monotonic paths that are not compatible with optimality, taking market demand into consideration. However, only those results that concern the moment of the technological change (denoted by $t = 0$, as before) are applied since future price behavior is not constant, implying that a steady state will not necessarily be reached.

In the extraction-biased case, when $dP > 0$, at $t = 0$, extraction increases as well, from Proposition 3. For price and extraction to increase, demand has to increase, since neither a constant nor a decreasing demand are compatible with such a behavior. When $dP < 0$, from Proposition 4, in case (i), both price and extraction decrease at $t = 0$. Therefore, only a decreasing demand is compatible, while in case (ii), as extraction increases, demand can be constant, increasing or decreasing. Finally, as in case (iii) nothing happens to extraction at $t = 0$, it is again true that only a decreasing demand is compatible.

In the depletion-biased case, from Proposition 5, when $dP > 0$, extraction increases at $t = 0$. Therefore, only an increasing demand is possible. When $dP < 0$, from Proposition 6, as extraction can either increase or decrease at $t = 0$, demand can be constant, increasing or decreasing.

The possible monotonic paths once demand is considered become quite few. In fact, the above results imply that if demand is increasing, then prices may grow monotonically, but extraction cannot always be decreasing. If the improvement is extraction-biased, then scarcity rent may always be increasing. If demand is constant or decreasing, there are no possible monotonic paths for price. While it might be expected that technological change could allow price to decrease monotonically, it should be remembered that technology in this analysis changes only once; in a model with continuous progress, or with a number of discrete improvements, these results could be different. This is an area for further research.

5. Conclusions

This paper considers a model of nonrenewable resource extraction by a firm in a competitive framework. The main focus is the impact of technological change on relevant variables in that model, which is analyzed through a comparative dynamics perspective. The main results are summarized in this section.

When price is considered exogenous, steady state analysis indicates that with technological change leftover stock will decrease and scarcity rent may increase (if extraction-biased) or decrease (if depletion-biased). If, along with the technological improvement, price increases as well, then leftover stock will decrease even further, as expected. Furthermore, extraction will increase, at least initially. Scarcity rent will always be higher in the extraction-biased case. On the other hand, if price decreases, then leftover stock may increase or decrease, depending on which effect is dominant. The same can be said for initial extraction and scarcity rent.

Considering an exogenous demand function, the model predicts that with increasing demand prices may grow monotonically, but extraction cannot always be decreasing. If the technological improvement is extraction-biased, then scarcity rent may always be increasing. If demand is constant or decreasing, there are no possible monotonic paths for price. Here the emphasis was placed on the existence of optimal monotonic paths, given the multitude of possible paths for scarcity measures' optimal behavior. The results are compatible with the empirical findings that scarcity measures' paths have rarely shown monotonicity.

Throughout the paper it was considered that technology improved only once. Thus, comparative dynamics allows a thorough analysis of the effects of this disturbance in the model. The natural question at this point is how these results could be extended to the more realistic case where technology improves more often, either continuously or by spurts. That is an area where further research is important, and it is the purpose of the second part of this study.

A. Appendix

The representation of $\dot{S} = 0$ and $\dot{\phi} = 0$ in a phase diagram can be useful for comparative dynamics. The slope of $\dot{S} = 0$ is positive; it can be calculated implicitly using conditions 3.12 and 3.14:

$$\frac{\partial \phi}{\partial S} = -\frac{\partial^2 C}{\partial E \partial S} > 0 \tag{A.1}$$

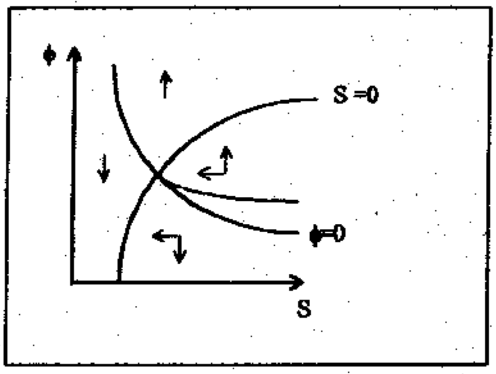


Figure A.1: Possible Phase Diagram

Using equation 3.13, the slope of $\dot{\phi} = 0$ is:

$$\frac{\partial \phi}{\partial S} = -\frac{\left(\frac{\partial^2 C}{\partial S^2} + \frac{\partial^2 C}{\partial E \partial S}\frac{\partial E}{\partial S}\right)}{r} \quad \text{(A.2)}$$

Substituting for $\frac{\partial E}{\partial S}$:

$$\frac{\partial \phi}{\partial S} = -\frac{\left(\frac{\partial^2 C}{\partial S^2}\frac{\partial^2 C}{\partial E^2} - \left(\frac{\partial^2 C}{\partial E \partial S}\right)^2\right)}{\frac{\partial^2 C}{\partial E^2} r} \quad \text{(A.3)}$$

With cost function convexity, this expression is negative.

The results attained so far are represented in Figure A.1.The optimal path has been drawn assuming monotonicity of $\frac{\partial \dot{C}}{\partial S}$, hence the negative slope (it was shown in section 2.1 that ϕ should be growing over time).

If $dZ > 0$, then the S-constant locus will be shifted upward, by equation 3.12:

$$d\phi = -\frac{\partial^2 C}{\partial E \partial Z} dZ > 0 \quad \text{(A.4)}$$

As for the ϕ-constant locus, it may be shifted upward or downward. Differentiation of equations 3.12 and 3.13 yields:

$$\begin{aligned} dE &= -\frac{\frac{\partial^2 C}{\partial E \partial Z} dZ + d\phi}{\frac{\partial^2 C}{\partial E^2}} && \text{(A.5)} \\ r d\phi &= -\frac{\partial^2 C}{\partial E \partial S} dE - \frac{\partial^2 C}{\partial S \partial Z} dZ && \text{(A.6)} \end{aligned}$$

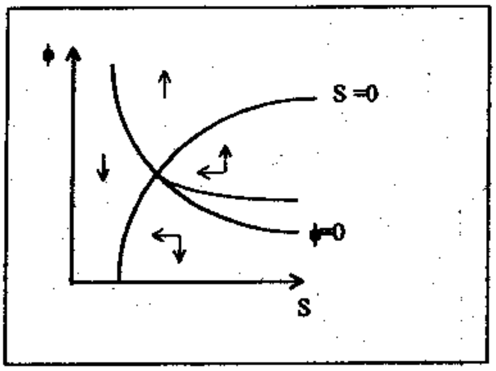


Figure A.1: Possible Phase Diagram

Using equation 3.13, the slope of $\dot{\phi} = 0$ is:

$$\frac{\partial \phi}{\partial S} = -\frac{\left(\frac{\partial^2 C}{\partial S^2} + \frac{\partial^2 C}{\partial E \partial S}\frac{\partial E}{\partial S}\right)}{r} \tag{A.2}$$

Substituting for $\frac{\partial E}{\partial S}$:

$$\frac{\partial \phi}{\partial S} = -\frac{\left(\frac{\partial^2 C}{\partial S^2}\frac{\partial^2 C}{\partial E^2} - \left(\frac{\partial^2 C}{\partial E \partial S}\right)^2\right)}{\frac{\partial^2 C}{\partial E^2} r} \tag{A.3}$$

With cost function convexity, this expression is negative.

The results attained so far are represented in Figure A.1.The optimal path has been drawn assuming monotonicity of $\frac{\partial C}{\partial S}$, hence the negative slope (it was shown in section 2.1 that ϕ should be growing over time).

If $dZ > 0$, then the S-constant locus will be shifted upward, by equation 3.12:

$$d\phi = -\frac{\partial^2 C}{\partial E \partial Z} dZ > 0 \tag{A.4}$$

As for the ϕ-constant locus, it may be shifted upward or downward. Differentiation of equations 3.12 and 3.13 yields:

$$dE = -\frac{\frac{\partial^2 C}{\partial E \partial Z} dZ + d\phi}{\frac{\partial^2 C}{\partial E^2}} \tag{A.5}$$

$$r d\phi = -\frac{\partial^2 C}{\partial E \partial S} dE - \frac{\partial^2 C}{\partial S \partial Z} dZ \tag{A.6}$$

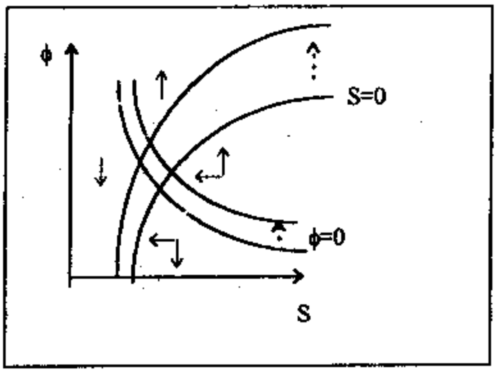


Figure A.2: Extraction-biased Technological Change

Combining these two equations and simplifying:

$$d\phi = \frac{\frac{\partial^2 C}{\partial E \partial S}\frac{\partial^2 C}{\partial E \partial Z} - \frac{\partial^2 C}{\partial S \partial Z}\frac{\partial^2 C}{\partial E^2}}{r\frac{\partial^2 C}{\partial E^2} - \frac{\partial^2 C}{\partial E \partial S}} dZ \tag{A.7}$$

The sign of this expression is indeterminate. Following what was done in the paper, two cases can be distinguished: extraction-biased and depletion-biased technological change. Clearly, if $\frac{\partial^2 C}{\partial E \partial Z} < 0$ and $\frac{\partial^2 C}{\partial S \partial Z} = 0$, the $\dot{\phi} = 0$ locus will be shifted upward, and if $\frac{\partial^2 C}{\partial E \partial Z} = 0$ and $\frac{\partial^2 C}{\partial S \partial Z} > 0$ it will be shifted downward, providing the steady state results that were previously obtained. The phase diagrams are presented in Figures A.2 and A.3 respectively (note that the $\dot{S} = 0$ locus does not shift for the depletion-biased case, since the marginal cost of extraction is not directly affected; as for the extraction-biased case, it can be shown that the upward shift of the $\dot{S} = 0$ locus is larger than that of the $\dot{\phi} = 0$ locus).

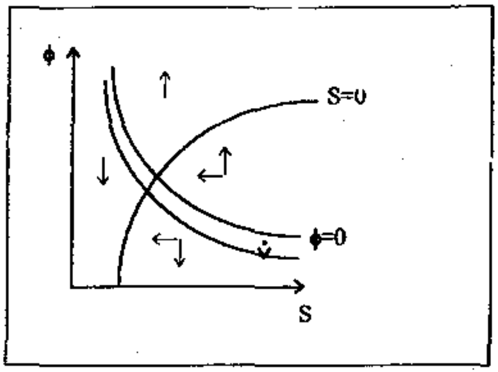


Figure A.3: Depletion-biased Technological Change

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