Industrial Policy and Firm Heterogeneity

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Abstract

Our concern is about a firm-specific industrial policy. When R&D subsidies or taxes are differentiated among firms, the question arises which firms in an industry should receive such support. We analyze a situation where firms differ in their R&D technologies in two distinct ways: They differ both in the costs of performing R&D activities and in the output obtained from such activities. The introduction of several domestic firms creates a corrective motive for government intervention with the firms' R&D activities in addition to Spencer and Brand's strategic motive.

We find that the optimal firm-specific industrial policy is affected differently by the two sources of firm heterogeneity. Another finding is that a change in a firm's R&D productivity has an ambiguous effect on the optimal policy towards the firm.

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1 Introduction

A country's trade policy has many facets, among them measures directed towards production, such as export subsidies, and others directed towards innovation, such as R&D subsidies or taxes. The former measures are increasingly difficult for governments to pursue, due to successful trade negotiations in GATT and elsewhere. This leads to a need for further analysis of trade-policy measures directed towards innovation. Our concern in the present paper is about one important aspect of R&D subsidies or taxes that distinguishes this policy instrument, in our view, from other trade-policy instruments: Whereas these other instruments tend to be industry-specific, aimed at industries in particular need of government support (or taxation, as the case may be), the support of R&D activities is, in its nature, firm-specific and even project-specific.

When R&D subsidies or taxes are differentiated among firms, the question arises which firms should receive such support. We attack this question on the industry level, asking which firms in an industry should receive the highest R&D subsidy, or pay the lowest R&D tax. In order to do this, we model an industry with several domestic firms that, by way of simplification, export all of their production to the world market. These firms, together with a number of foreign firms operating in the same industry, participate in a non-tournament R&D competition à la Spencer and Brander (1983): During an initial stage, each firm invests in process-innovation activities that bring down its production costs in the ensuing production stage, in which firms compete in quantities.1

Our focus is on a situation where domestic firms differ both in their costs of doing R&D and in their R&D productivities, i.e., the rates at which their R&D activities transform into reduced production costs. In order to model such a heterogeneity among domestic firms, it is, of course, essential that the model features more than one domestic firm. With several domestic firms and a government that is restricted to intervene at the R&D stage only, it is possible to distinguish three different motives for government intervention. First, there is the profit-shifting motive; when firms compete in quantities on the product market, this motive calls for an R&D subsidy (Spencer and Brander, 1983): Such a subsidy makes domestic firms produce more, entailing a contraction of foreign firms' R&D and production and therefore leaving more of the available profit to be earned by the domestic firms. Secondly, there is the need to correct for the incentives that each firm has to overinvest in R&D beyond what cost minimization prescribes in situations where lower marginal costs entail a higher market share.

1See Beath et al. (1995) for a survey of various models of R&D competition.
(Brander and Spencer, 1983); this calls for a tax on R&D. Finally, with several domestic firms exporting their production, there is a need to intervene in order to keep the domestic firms from competing too heavily with each other on the world market; while no discussion exists in the literature on industrial policy in this respect, see Dixit (1984) and Klette (1994) on production subsidies/taxes when there are several domestic firms.

In Spencer and Brander's (1983) seminal discussion, there is one domestic firm only, so that only the first two motives for government intervention are present. When the government is able to intervene at both the R&D and the production stages, the first motive entails a production subsidy and the second one an R&D tax, according to Spencer and Brander. When government intervention is restricted to the R&D stage, as in the present analysis, they find under reasonable assumptions, that the optimum policy is an R&D subsidy, i.e., the profit-shifting argument dominates the overinvestment one. Although we will be concerned mainly with the design of a firm-specific industrial policy in the following analysis, we obtain, as a by-product, an extension of the Spencer-Brander analysis to the case of multiple domestic and foreign firms.

Industrial policy in non-tournament models of international R&D competition is discussed in a few papers since Spencer and Brander (1983), such as Bagwell and Staiger (1994) and Leahy and Neary (1996, 1997). A firm-specific industrial policy does not seem to have been studied in the received literature, though. The studies closest to ours are those by Leahy and Montagna (1997) and Long and Soubeyran (1997a, 1997b) on firm-specific production subsidies (or taxes). However, as noted above, we believe that firm-specific policies more naturally occur at the R&D level than at the production level.

A building block in our analysis is a model of non-tournament R&D competition among firms that differ in their R&D technologies, and this model is in itself a novelty. The models of R&D competition that are closest to ours in the received literature are by Rosen (1991), Poyago-Theotoky (1996), and Yin and Zuscovitch (1998). However, in these models, firms have identical R&D technologies and differ only in their initial, or pre-R&D, production costs.

With our focus on firm-specific government policy, our analysis is closely related to studies of R&D cooperation, or research joint ventures, among heterogeneous firms (Veugeler and

\(^2\)In Collie's (1993) analysis of the optimum export policy when firms' production costs differ, the policy is restricted to be uniform rather than firm-specific.

\(^3\)The literature on targeting, such as Dixit and Grossman (1986) and Neary (1994), is mainly concerned with which industry to support, rather than which firms in an industry. A crude kind of firm-specific support is, however, discussed by Dixit (1988) in a tournament model of R&D competition with asymmetric R&D efficiency. Here, the government controls how many domestic firms participate in an international patent race. In a sense, then, the government performs an industrial policy that discriminates on the firm level.
Kesteloot 1996, Kesteloot and Veugelers 1997, Long and Soubeyran 1997c, Röller et al. 1997): With a firm-specific policy and all domestic production being exported, the government's optimum level of R&D activity in each domestic firm coincides with what the firm itself would choose if all domestic firms were cooperating at the R&D stage.

Long and Soubeyran (1997c, 1997d) and Salant and Shaffer (1998a, 1998b) find that, even if the firms participating in a research joint venture are ex-ante identical, the optimum R&D efforts may be asymmetric, because increased differences in production costs lead to an increase in industry profit; in our analysis, we invoke so much convexity in R&D costs that this phenomenon does not occur. Salant and Shaffer (1998b) are particularly interesting, since they, too, note the formal resemblance between R&D cooperation and firm-specific R&D subsidies, as described above; however, they discuss only cases with identical firms and are concerned with the possible optimality of treating firms differently even if they are identical.

In Section 2 below, we present our model of the following three-stage situation. In the first stage, the government decides on firm-specific taxes on the R&D activities of the domestic firms in a particular industry. In the second stage, firms, both domestic and foreign, decide on how much R&D to perform when each firm's R&D affects its production costs but the cost per unit of R&D effort as well as the rate at which R&D effort is transformed into production-cost reductions differ among the firms. In the third stage, the firms compete on the world market by simultaneously deciding on produced quantities, and all domestic production is exported. In this Section, we also discuss the relevance of having firms differ in their R&D efficiency along two dimensions, both R&D costs and R&D productivity. In the subsequent sections, the gain in insights obtained from allowing both dimensions is made clear. In Section 3, we focus on the R&D game between the firms, which is stages two and three above. In particular, we discuss the comparative statics of firms' R&D activities and production quantities with respect to changes in R&D efficiency.

The main results of the analysis are in Sections 4 and 5. Central to the understanding of how firm heterogeneity affects industrial policy is, in our view, the distinction between a government's corrective and rent-shifting motives for intervention. In order to make this distinction clear, we start our analysis of the government's optimum firm-specific industrial policy with restricting foreign firms to do no R&D, since this restriction eliminates the rent-shifting motive for intervention (in the absence of any production subsidy). We characterize the optimum firm-specific policy in this case, which is always a tax on R&D efforts when the only motives for intervention are the corrective ones, and find, somewhat counter-intuitive, that a firm with a relatively high R&D productivity does not necessarily pay a relatively high
tax. The reason for the ambiguity is the interplay of two forces. On one hand is the direct
effect: A higher R&D productivity makes a firm do more R&D, ceteris paribus, i.e. the firm’s
incentive to overinvest in R&D increases; this calls quite naturally for a higher tax the higher
the R&D productivity is. On the other hand, there is a strategic effect: The more a firm’s
domestic rival produces, the larger is the negative externality that the firm imposes upon
them, and the higher is the tax necessary to impose on this firm. Since an increase in R&D
productivity lowers a firm’s costs and therefore lowers its rivals’ production, this effect calls
for a lower tax the higher the R&D productivity is.

In Section 5, we introduce R&D activities among the foreign firms, so that the optimum
policy now also has an element of rent-shifting, with the consequence that the optimum policy
towards a particular firm may be either a tax or a subsidy. The comparative-statics results
however, stand essentially unaltered from the previous analysis. In particular, the ambiguity
in the relation between the R&D productivity and the optimum policy persists, for the same
reason as outlined above. Section 6 provides some concluding comments.

2 The model

To set our analysis in the simplest framework possible, we consider a foreign market with
demand given by a linear inverse demand function,

\[ P = a - Q \]  \hspace{1cm} (1)

where \( P \) is price and \( Q \) is total quantity supplied in the market. On the supply side, there
are \( n \) firms, out of which \( m \) firms are domestic and the other \( n - m \) firms are foreign (that is,
firms located in other countries). Denote by \( M \) the set of domestic firms and by \( N \) the set
of all firms in the market. Domestic firms are labelled from 1 through \( m \) and, thus, foreign
firms from \( (m + 1) \) through \( n \). Let \( q_i \) be the quantity produced by firm \( i \).

The production technology is characterized by constant returns to scale. The level of a
firm’s unit production cost depends on the R&D activity performed by the firm. In particular,
the unit production cost of firm \( i \) is given by:

\[ c_i = \bar{c}_i - \theta_i x_i \]  \hspace{1cm} (2)

where \( \bar{c}_i \) is the initial level of unit production cost and \( \theta_i x_i \) is the reduction in cost obtained
by firm \( i \), depending on its R&D investment, \( x_i \), and its R&D productivity, measured by \( \theta_i \). A
higher \( \theta_i \) means a more efficient firm in doing R&D activities. As indicated by the subscripts
on \( c_i \) and \( \theta_i \), we do not restrict firms to be equal.\(^4\)

Denote R&D costs by \( \varphi(x_i; \gamma_i) \), where \( \gamma_i \) is a firm-specific parameter, i.e. R&D costs are not restricted to be equal across firms. A higher \( \gamma_i \) means a less efficient firm, with \( \varphi(0; \gamma_i) = 0, \partial \varphi / \partial \gamma_i > 0 \) and \( \partial^2 \varphi / \partial x_i \partial \gamma_i > 0 \), that is, both total costs and marginal costs increase with \( \gamma_i \). Thus, firms' R&D technologies may vary in two different ways: differences in R&D productivity and differences in R&D costs.\(^5\)

Government intervention is assumed to occur at the R&D stage. No production subsidies exist. We allow for a tax on each domestic firm per unit of its R&D investment. Denote by \( \sigma_i, i \in M \), the tax rate. We put \( \sigma_i = 0 \) for each foreign firm, assuming that foreign governments are passive. This is made for convenience as our interest lies in how asymmetries across domestic firms affect domestic industrial policy.

Firm \( i \) has a profit function given by

\[
\Pi_i = \Pi_i(\eta, x, \sigma) = (P - c_i) \eta_i - \varphi(x_i; \gamma_i) - \sigma_i x_i
\]

(3)

where \( \eta, x \) and \( \sigma \) are vectors of firms' production quantities, research activities, and taxes, respectively.

One idiosyncratic feature of our model is the two-dimensional heterogeneity in R&D efficiency. A simple reparametrization of parameter can serve to clarify the meaning of this double dimensionality. Let \( y_i = \theta_i x_i \) be the decision variable of each firm, i.e., let firms decide on R&D output rather than R&D input. Then, firm \( i \)'s profit function can be written as:\(^6\)

\[
\Pi_i = \left( P(Q) - c_i + y_i - \sum_{j \in M \setminus \{i\}} y_j \right) \eta_i - \varphi\left( y_i; \gamma_i \right)
\]

(4)

where we made use of the definition of \( c_i \). Thus, we can interpret our model in a different way, stating that it reflects two different sources of heterogeneity in R&D costs: one multiplicative (given by \( 1/\theta_i \)) and another having any form. In particular, for some specifications, the effect of \( \gamma_i \) will be undistinguishable from that of \( 1/\theta_i \) (for example, \( \varphi = \gamma_i x_i^2 = (\gamma_i/\theta_i)^2 y_i^2 \)). In this sense, our model comprises more standard R&D technology formulations as special cases. However, in a cost function of the type

\[
\varphi = \gamma_i x_i + x_i^2 = \gamma_i y_i + \frac{1}{\theta_i} y_i^2
\]

(5)

\(^4\)Note the absence of technological spillovers in the proposed formulation. The results obtained below cannot, thus, be attributed to spillover effects.

\(^5\)Another interpretation can also be given. See below.

\(^6\)Omitting the tax component. We will come to it later on.
the two sources of heterogeneity have different implications. Having said this, let us stress the point that our preferred interpretation is in terms of heterogeneity in both R&D productivity and R&D costs, with R&D effort as the choice variable, rather than R&D output.

Like Spencer and Brander (1983), we assume the market is abroad, so that all production of domestic firms is exported and none of the foreign firms' production is imported. Thus, the government, maximising national welfare, takes no notice of consumers surplus and simply maximises the sum of domestic firms' profits, net of the R&D tax; i.e., the government maximises:

\[
W = \sum_{i \in M} [\Pi_i + \sigma_i x_i]
\]  

Our interest is with the following three-stage situation: In stage 1, the government decides on a vector \((\sigma_1, \ldots, \sigma_m)\) of R&D taxes, one for each domestic firm. In stage 2, firms, domestic and foreign, choose their level of R&D activities, thus determining their costs in the subsequent production. In stage 3, finally, firms choose production quantities. We will be looking for the subgame-perfect equilibrium of this game.

3 R&D competition with firm heterogeneity

Before dealing with the general case, in an international competition setting, it is useful to characterize and interpret the R&D stage competition. To this purpose, assume that only two (domestic) firms are present in the market \((m = n = 2)\). We consider the following two-stage game: first, firms decide on R&D investments; and, second, after R&D investments have been made and become common knowledge, firms decide their production levels.

A starting question is whether initial low-cost firms perform more R&D, or not. The way to obtain an answer to this question is to assume that firm 1 has higher initial costs \((\bar{c}_1 > \bar{c}_2)\). Otherwise, firms are identical.

The solution to the second-stage problem is the following pair of quantities:

\[
q_1 = \frac{\alpha_1 + \theta(2x_1 - x_2)}{3}, \quad \alpha_1 = a - 2\bar{c}_1 + \bar{c}_2
\]
\[
q_2 = \frac{\alpha_2 + \theta(2x_2 - x_1)}{3}, \quad \alpha_2 = a - 2\bar{c}_2 + \bar{c}_1
\]

First-stage equilibrium profits are

\[
\Pi_i = q_i^2 - \varphi(x_i)
\]  

\(^7\)Note the implicit assumption of absence of a distortionary cost of public funds.
The first-stage optimal R&D choices satisfy

\[ 2q_i \frac{\partial q_i}{\partial x_i} - \frac{\partial \rho}{\partial x_i} = 0, \quad i = 1, 2 \]  

(8)

or, substituting by the relevant expressions,

\[ \frac{4\theta}{9} (\alpha_i + \theta(2x_i - x_j)) - \frac{\partial \rho}{\partial x_i} = 0, \quad i = 1, 2 \]  

(9)

Even if we cannot explicitly solve for \( x_i, i = 1, 2 \), without specification of a particular functional form for R&D costs, it is nonetheless possible to show the next result.

**Remark 1** Suppose \( \phi_{ii} > 3\theta i^2 / 2 \), \( i = 1, 2 \), where \( \phi_{ii} = \frac{\partial^2 \phi}{\partial x_i^2} \). When firms are equal, except with respect to initial costs, the initial low-cost firm does more R&D than the initial high-cost firm.

Thus, the initial cost gap among firms is broadened by the activities at the R&D stage. The simplest way to show our claim is to consider first the case of identical firms (\( \bar{c}_1 = \bar{c}_2 \)), and then take an increase in \( \alpha_1 \) and a simultaneous decrease in \( \alpha_2 \) (both resulting from an increase in \( \bar{c}_1 \)). If the induced changes in \( x_1 \) and \( x_2 \) keep the sign constant, a difference \( \bar{c}_1 - \bar{c}_2 \) can be seen as a series of infinitesimal changes starting from an initial identical position.

Total differentiation of the first-order conditions (9) yields:

\[
\left( \frac{8}{9} \theta^2 - \phi_{11} \right) dx_1 - \frac{4}{9} \theta^2 dx_2 = \frac{8\theta}{9} d\bar{c}_1 \\
\left( \frac{8}{9} \theta^2 - \phi_{22} \right) dx_2 - \frac{4}{9} \theta^2 dx_1 = -\frac{4\theta}{9} d\bar{c}_1
\]

where \( \theta_1 = \theta_2 = \theta \). Solving, we obtain

\[
\frac{dx_1}{d\bar{c}_1} = \frac{1}{\tilde{H}} \left[ \frac{8\theta}{9} \left( \frac{8\theta^2}{9} - \phi_{22} \right) - \frac{16\theta^3}{9} \right] < 0 \\
\frac{dx_2}{d\bar{c}_1} = \frac{4\theta \phi_{11}}{9 \tilde{H}} > 0
\]

where \( \tilde{H} = |\frac{\partial^2 \Pi}{\partial x_i \partial x_j}| > 0 \), from which results

\[
\frac{d\Delta}{d\bar{c}_1} < 0
\]

where \( \Delta = x_1 - x_2 \). We use the stability condition of Dixit (1986), \( \tilde{H} > 0 \), to get this inequality sign. The stability condition is satisfied for \( \phi_{ii} > 3\theta i^2 / 2 \).\(^8\) This condition also

\(^8\)In the two-firms case, this is more restrictive than necessary. Later on, however, this same condition will be used.
means fulfillment of second-order conditions \((\varphi_{ij} > \beta_i^2/9)\). This remark shows that ex-ante asymmetries in production costs result in a higher incentive to invest in R&D by the lower cost firm. The result is not novel, but the intuition behind it will prove useful below. Suppose that no R&D effort is present. The low-cost firm produces more in equilibrium. Since R&D reduces the constant marginal cost of producing the final good, an equal marginal R&D outcome is applied to a greater mass of production by the low cost firm. Therefore, this firm has a higher marginal benefit from R&D, which leads to a higher equilibrium R&D investment and to an increase in production cost asymmetries.

To focus on the role played by R&D productivity heterogeneity, we assume next no initial asymmetry on the basic cost parameter: \(\bar{\theta}_i = \bar{\theta}, \forall i\). On the other hand, firms are allowed to differ on their ability to put R&D effort to use. The assumption of a common \(\bar{\theta}\) allows us to write equilibrium quantities produced by each firm as:

\[
q_i = \frac{\alpha + 2\bar{\theta}_1 x_1 - \bar{\theta}_1 x_2}{3}, \quad i = 1, 2; i \neq j
\]  

where \(\alpha := \alpha - \bar{\theta}\). For future use, it is convenient to establish the way equilibrium quantities vary with the changes in R&D choices and parameters.

Remark 2 The following comparative statics results hold in the quantity sub-game:

\[
\frac{\partial q_i}{\partial x_i} > 0; \quad \frac{\partial q_i}{\partial \theta_i} > 0
\]

\[
\frac{\partial q_i}{\partial x_j} < 0; \quad \frac{\partial q_i}{\partial \theta_j} < 0
\]

These results are intuitive ones, as they say that increases in R&D investment or in R&D productivity increase own production and reduce the other firm's production (in equilibrium).

Note that these are direct effects and should not be taken as changes in equilibrium values of the full game in the case of the R&D productivity parameter \(\theta_i\) (or \(\theta_j\)), as it is necessary to include the strategic effect through equilibrium choices of R&D efforts.

We are now ready to characterize the R&D competition stage. Equilibrium choices of R&D efforts solve the following set of first-order conditions:

\[
\frac{4}{9} \theta_i (\alpha + 2\theta_1 x_1 - \theta_2 x_2) - \frac{\partial \varphi}{\partial x_1} = 0
\]

\(\text{Poyago-Theotoky (1996) obtained the same result, under a similar demand structure, a linear cost reduction function (set as a convex function of R&D effort) and quadratic R&D costs. We have a simpler R&D productivity function but a more general R&D cost function.}\)

\(\text{The implications of differences in the costs of performing a given level of R&D activity can be derived in the same way. We will refer to them along the way.}\)

\(\text{Remember that } n = m = 2. \text{ The following sections generalise this assumption.}\)

\(\text{The proof of remark 2 requires only differentiation of equation (10).}\)
\[ \frac{4}{3} \theta_2 (\alpha + 2 \theta_2 x_2 - \theta_1 x_1) - \frac{\partial \varphi}{\partial x_2} = 0 \] (12)

Once again, without specifying a functional form for \( \varphi \), it is not possible to solve explicitly for \( x_i, i = 1, 2 \). Nonetheless, the following comparative statics can be obtained. \(^{13}\)

**Remark 3** The effects of changes in R&D parameters are:

\[ \frac{\partial x_i}{\partial \theta_i} > 0; \quad \frac{\partial x_i}{\partial \theta_j} < 0; \quad \text{for } i = 1, 2; j = 1, 2 \]

From this remark, it is also easy to obtain that \( \Delta = x_1 - x_2 \) is increasing in \( \theta_1 \) and decreasing in \( \theta_2 \). It is also straightforward to show that \( \Delta \) is increasing in \( \gamma_2 \) and decreasing in \( \gamma_1 \).

All these effects are, again, in line with what economic intuition predicts. Increases in productivity (or cost savings) stimulate own R&D investment and reduce investment by competitors (in equilibrium).

## 4 Industrial Policy Implications: The corrective tax

The discussion of firm-specific industrial policy in open economies must take into account two different concerns: first, the rivalry between domestic firms; and second, the competition with foreign firms. In order to highlight the role of rivalry among heterogeneous domestic firms, we first consider the case where no foreign firm conducts R&D activities. This assumption excludes from the model the rent-shifting motive for intervention in the absence of any production subsidy (Spencer and Brander 1983). Later on, in Section 5, this assumption will be relaxed, so that both corrective and strategic motives are present.

Consider \( m \) domestic firms and \( n - m \) foreign firms, with \( x_i = 0, i \in N \setminus M, i.e. foreign firms do no R&D. We will solve for the subgame-perfect equilibrium of our full three-stage game by backwards induction. In stage 3, firm i's choice of production quantity \( q_i \) is given by the following first-order condition:\(^{14}\)

\[ \frac{\partial \Pi_i}{\partial q_i} = P - c_i + \frac{\partial P}{\partial q_i} q_i = a - c_i - q_i - Q = 0 \] (13)

Summing over all \( n \) first-order conditions and solving for the aggregate quantity, we obtain:

\[ Q = \frac{na - \sum_{i \in N} c_i}{n + 1} \] (14)

\(^{13}\)The results of Remark 3 can be seen as a particular case of the proof provided in the Appendix.

\(^{14}\)Second-order conditions are trivially satisfied.
Inserting this in each firm's first-order condition, we obtain each firm i's stage-3 production decision, given all firms' marginal costs:

$$q_i = a - c_i - Q = \frac{a - n\bar{c}_i + \sum_{j \in M(i)} \bar{c}_j}{n + 1}$$

(15)

Or, after inserting for marginal costs and rearranging,

$$q_i = \frac{\alpha_i + n\bar{\theta}_i \bar{x}_i - \bar{\theta}_j \bar{x}_j}{n + 1}, \quad i, j \in M, i \neq j$$

$$q_i = \frac{\alpha_i - \sum_{j \in M} \bar{\theta}_j \bar{x}_j}{n + 1}, \quad i \in N \setminus M,$$

where

$$\alpha_i := a - n\bar{c}_i + \sum_{j \in M(i)} \bar{c}_j, \quad i \in N$$

(16)

is assumed to be high enough such that all firms produce positive quantities in equilibrium. Differences in the basic marginal cost parameter, \( \bar{\theta}_i \), translate into differences in \( \alpha_i \). We can proceed to find second-stage equilibrium levels of R&D investment, given firms' R&D costs.

The reduced-form profit is given by:

$$\Pi_i = q_i^2 - \Phi_i(x_i; \gamma_i) - \sigma_i x_i$$

(17)

Each firm i's first-order condition with respect to the level \( x_i \) of its R&D investment is:

$$\frac{\partial \Pi_i}{\partial x_i} = 2q_i \frac{\partial q_i}{\partial x_i} - \frac{k_x}{x_i} - \sigma_i = 0$$

(18)

This provides us with a set of conditions, from which we have.\(^{15}\)

**Remark 4** An increase in the R&D tax of firm i decreases its own R&D effort and increases that of rival j:

$$\frac{dx_i}{ds_i} < 0; \quad \frac{dx_j}{ds_i} > 0$$

This result can be easily seen as a particular case of the comparative statics, above. A change in \( \sigma_i \) is essentially similar to a change in the cost parameter \( \gamma_i \). Basic economic intuition holds in this setting. An increase in the tax of firm i induces a lower R&D effort of this firm and a higher effort of rivals.

\(^{15}\) We assume that, whenever necessary, \( \Phi(x_i; \gamma_i) \) has the required concavity properties with respect to \( x_i \) to ensure that second-order conditions of firm i's maximization problem and the Dixit (1986) stability condition are satisfied.
Proceeding to the first stage, where the government chooses an R&D tax rate for each domestic firm, disregarding distortion costs in public costs of funds, we have the following objective function for the government:

\[ W = \sum_{i \in M} (\Pi_i + \sigma_i x_i) \]  

We obtain the optimal tax on domestic firm \( i \) in the following way. Suppose the government is able to choose R&D activities of each firm, \( x_i \), directly. The problem is:

\[ \max_{\{\sigma_i\}_{i \in M}} W = \sum_{j \in M} \left( q_j^r - \varphi(x_j; \gamma_j) \right) \]  

The optimal government choices solve:

\[ 2 \left[ \sum_{j \in M} q_j \frac{\partial \Pi_j}{\partial x_i} \right] = \frac{\partial \varphi}{\partial x_i} = 0, \quad i \in M \]  

To have the government's preferred outcome implemented through a set of subsidies \( \{\sigma_i\}_{i \in M} \), the optimal subsidy structure is given by the difference between each domestic firm's first-order condition (18) and the government's first-order condition for this firm (21):

\[ \sigma_i = -2 \left( \sum_{j \in M \setminus \{i\}} \frac{\partial \Pi_j}{\partial x_i} \right) \]  

Given our linear quantity stage,

\[ \frac{\partial q_j}{\partial x_i} = -\theta_j (n + 1) \]  

Thus,

\[ \sigma_i = \frac{2\theta_i}{n + 1} \sum_{j \in M \setminus \{i\}} q_j = \frac{2\theta_i}{n + 1} |Q_m - q_i| > 0 \]  

where \( Q_m \) is total production by domestic firms. For \( n = 2 \), it simplifies to

\[ \sigma_i = 2\theta_i \theta_i/(n + 1). \]

Inspection of the structure of the optimal choice of \( \sigma_i \) leads to the following proposition.

**Proposition 1** Suppose that, in equilibrium, each domestic firm does at least some R&D. The optimal policy taxes all domestic firms. The equilibrium tax is:

\[ \sigma_i = \frac{2\theta_i}{n + 1} \sum_{j \in M \setminus \{i\}} q_j > 0 \]  

It is immediate to see that
Corollary 1 If firms have equal R&D productivity \( \theta_i = \theta, \forall i \), then those that produce more are taxed less.

To see the implications of this proposition, take the case of all firms being equal except for efficiency in the costs of doing R&D. More efficient firms will conduct more R&D, have lower costs and face a lower tax.

This proposition implies that for small productivity asymmetries across firms, more efficient firms are less penalized by the government because they are better positioned from the start (lower \( \alpha_i \)) and/or more cost-effective in conducting R&D activities (lower \( \gamma_i \)). They are more quantity efficient in the sense that they produce more. They are, therefore, in a better position to extract rents in the foreign market. The government uses the tax to divert production to the more efficient firm.

Asymmetry of firms does not change the policy prescription of taxing firms to curtail the strategic incentive to over-invest in R&D from the point of view of the domestic government. However, it adds the insight that more quantity-efficient firms should be taxed at a lower rate.

A related question is whether all firms do R&D under the optimal tax structure (an assumption underlying the above characterization).

Proposition 2 If firms are not too quantity asymmetric then all domestic firms perform R&D at strictly positive levels.

Proof: For \( q_i = q_j, \forall i, j \in M \), it is the case that \( x_i > 0, \forall i \in M \). To see this, substitute the equilibrium value of the subsidy in a domestic firm's first-order condition:

\[
2q_i \frac{\partial q_i}{\partial x_i} - \frac{\partial \phi_i}{\partial x_i} + \frac{2\theta_i}{n + 1} \sum_{j \in M \setminus \{i\}} q_j = 0
\]  
(26)

Substituting for \( \frac{\partial q_i}{\partial x_i} = n\theta_i/(n + 1) \), we can re-write this expression as:

\[
\frac{2\theta_i}{n + 1} \left( nq_i - \sum_{j \in M \setminus \{i\}} q_j \right) - \frac{\partial \phi_i}{\partial x_i} = 0
\]  
(27)

and the term in parenthesis is positive for \( q_i = q_j \), yielding an optimal choice of \( x_i > 0 \). By continuity, there exists \( \epsilon > 0 \) such that \( |q_i - q_j| \leq \epsilon, \forall i, j \in M \) implies

\[
\sum_{j \in M \setminus \{i\}} q_j > 0, \forall i
\]  
(28)

which ensures a positive investment in R&D. ■
From the proof of the above result, it is also clear that the greater the number of foreign firms, the easier it is to fulfill the condition for positive R&D investment levels for domestic firms. Of course, if firms are sufficiently asymmetric, then some of them may not invest in R&D. For equal research productivity ($\theta_i = \theta$) the first firms to stop doing R&D are the less cost-efficient ones; and for the same R&D cost structure ($\gamma_i = \gamma$), the less productive ones in R&D will be the first ones to quit R&D activities.

A central question is, of course, how a domestic firm's R&D technology affects the R&D tax put upon it. The next proposition addresses the issue.\textsuperscript{16}

**Proposition 3** If a firm gets more efficient in the sense of a decrease on R&D costs, then its R&D tax decreases; while if it gets more efficient in the alternative sense of an increase in R&D productivity, then the R&D tax may increase or decrease:

\[
\frac{\partial \sigma_i}{\partial \gamma_i} > 0; \quad \frac{\partial \sigma_i}{\partial \theta_i} < 0
\]

\[
\frac{\partial \sigma_i}{\partial \theta_i} > 0; \quad \frac{\partial \sigma_i}{\partial \theta_j} > 0
\]

The surprising result in the proposition is the ambiguous effect of an increase in R&D productivity upon the firm-specific tax. The tax may increase or decrease, and this is so because two conflicting effects are present. First, the corrective tax is higher the larger the quantity produced by rival firms. An increase in the R&D productivity of one firm leads to a reduction in domestic rivals' production. Thus a lower tax on this firm is required. Second, a higher R&D productivity also means a stronger incentive for the firm to (over)invest in R&D. Thus, a heavier tax should be implemented. Taking together the two effects, an ambiguous qualitative implication results from the model.

5 Optimal taxes with foreign R&D

The assumption that foreign firms do not invest in R&D is unreasonable in many cases and it is unwarranted but for illustrative purposes. We now relax the assumption. On the other hand, we reduce the heterogeneity among firms to a minimum by dividing firms in three groups, two domestic and one foreign, with homogeneity within each group. Thus, there are three types of firms partitioned in two sets of domestic firms, denoted by $M_1$ and $M_2$, where $M_i$ denotes the set of domestic firms of type $i$, and $M = M_1 \cup M_2$, and a set of foreign firms,\textsuperscript{16}

\textsuperscript{16}The proof is a particular case of the one presented in the Appendix.
denoted by $M_3 = N \setminus M$. Firms within each group are equal with respect to R&D costs and R&D productivity technology. Foreign firms also do R&D, and they are endowed with parameters $(\theta_3, \gamma_3)$. Domestic firms of type $i$ are characterised by $(\theta_i, \gamma_i), i = 1, 2$. All three types of firms have identical initial production costs, $c_i$. Let $m_i$ be the number of firms in set $M_i, i = 1, 2, 3$.

In the first stage, the government chooses an R&D tax to apply to each domestic firm. The government’s objective function is the sum of profits and tax revenue. The existence of foreign firms doing R&D means that there is a strategic incentive for the government to do rent-shifting at the expense of foreign firms (Spencer and Brander 1983). In its choice of $x_i$, the government acts as a Stackelberg leader with respect to the foreign firm. The profit-shifting effect is tempered, in our model, by the government’s desire to exploit the heterogeneity among the domestic firms.

The stage of production of the final good is essentially the same, rendering second-stage profits given by:

$$
\Pi_i = q_i^2 - \varphi(x_i; \gamma_i)
$$

where

$$
q_i = \frac{1}{n + 1} (a - \bar{c} + n\theta_i x_i - m_j \theta_j x_j - m_k \theta_k x_k), \quad i, j, k = 1, 2, 3; i \neq j \neq k
$$

Profit maximisation yields the following set of first-order conditions:

$$
2q_i \frac{\partial q_i}{\partial x_i} - \frac{\partial \varphi}{\partial x_i} = 0, \quad i = 1, 2, 3.
$$

From these conditions, we get the analog of Remark 3. Moreover, it is straightforward to show that cost reductions are positively related to own efficiency and negatively associated with rivals' efficiency:

**Remark 5** Assume the following condition holds:

$$
\varphi_{ii} > \frac{2n(n + m_i)}{(n + 1)^2} \theta_i^2, \quad i = 1, 2.
$$

The effects of changes in R&D parameters are:

$$
\frac{\partial x_i}{\partial \theta_i} > 0, \quad \frac{\partial x_i}{\partial \theta_j} < 0, \quad \frac{\partial \theta_i x_i}{\partial \theta_i} > 0, \quad \frac{\partial \theta_i x_i}{\partial \theta_j} < 0
$$

$$
\frac{\partial x_i}{\partial \gamma_i} > 0, \quad \frac{\partial x_i}{\partial \gamma_j} < 0, \quad \frac{\partial \theta_i x_i}{\partial \gamma_i} < 0, \quad \frac{\partial \theta_i x_i}{\partial \gamma_j} > 0
$$

---

17The proof of remark 5 can be found in the Appendix.
The optimal government choices, if it could each domestic firm's R&D activity directly, solve:

\[
2 \sum_{j \in M} q_j \frac{\partial q_j}{\partial x_i} - \frac{\partial \varphi}{\partial x_i} = 2 \sum_{j \in M} \sum_{g \in \mathcal{N}_g} q_j \frac{\partial q_j}{\partial x_g} \frac{\partial x_g}{\partial x_i} = 0 \tag{35}
\]

To have this outcome implemented, the optimal tax must be:

\[
\sigma_i^* = -2 \sum_{k \in M \setminus \{i\}} \frac{\partial q_k}{\partial x_i} q_k - 2m_3 \frac{\partial x_g}{\partial x_i} \sum_{j \in M} q_j \frac{\partial q_j}{\partial x_g}, \quad i \in M \tag{36}
\]

It remains to show that it is not optimal for the government to have only one domestic firm active in equilibrium (with taxes inducing exit of all others). This issue is similar, under our structure, to the problem of merger profitability. From this literature, it is well-known that, in the linear oligopoly with constant marginal costs and quantity-setting firms, a merger is not profitable unless it encompasses a very significant share of existing firms.\(^{18}\) Thus, assuming that firms are not too heterogeneous and making the total number \(n\) of firms high enough, the above tax does characterise the optimal tax structure.

Making the relevant substitutions we can write the optimal tax as, for type 1 and type 2 firms respectively:

\[
\sigma_1^* = \frac{\theta_1((m_1 - 1)q_1 + m_2q_2)}{2} + \frac{m_3 \theta_3}{2} \frac{\partial x_g}{\partial x_1} (m_1q_1 + m_2q_2) \tag{37}
\]

\[
\sigma_2^* = \frac{\theta_2((m_2 - 1)q_1 + m_1q_1)}{2} + \frac{m_3 \theta_3}{2} \frac{\partial x_g}{\partial x_2} (m_1q_1 + m_2q_2) \tag{38}
\]

This tax is composed of two main parts. The first part of the tax is (for type 1 firms, a similar expression holds for type 2 firms):

\[
-2 \sum_{k \in M \setminus \{i\}} \frac{\partial q_k}{\partial x_i} q_k = \frac{\theta_1((m_1 - 1)q_1 + m_2q_2)}{2} > 0 \tag{39}
\]

and it is designed to internalise the effect of one domestic firm doing R&D on the profits of the other domestic firms. Since more R&D means lower costs in the final production stage, the firm is a tougher competitor. The cross-effect among domestic firms is negative, which justifies why a government that maximises industry profits has an interest in taxing R&D activities.

The second term in the tax structure is:

\[
-2m_3 \frac{\partial x_g}{\partial x_i} \sum_{j \in M} q_j \frac{\partial q_j}{\partial x_g} = \frac{m_3 \theta_3}{2} \frac{\partial x_g}{\partial x_1} (m_1q_1 + m_2q_2) < 0 \tag{40}
\]

\(^{18}\)See Salant et al. (1983).
and it constitutes the usual profit-shifting incentive of governments to subsidise domestic R&D efforts. Increasing R&D by a domestic firm induces a lower spending by the foreign firm, which benefits all domestic firms.¹⁹

This decomposition into corrective and profit-shifting components shows that the optimal tax can be positive or negative, depending on the relative strength of these two opposite forces, a result in line with the analysis of Dixit (1984) on multi-firm production subsidies.

We look now at the possibly different ways domestic firms are treated by the tailor-made industrial policy set by the government. The difference in tax rates among the two types of domestic firms is:

\[
\sigma^*_2 - \sigma^*_1 = -2 \left( q_2 \frac{\partial q_2}{\partial x_1} - q_1 \frac{\partial q_1}{\partial x_1} \right) - 2 \left( q_1 \frac{\partial q_1}{\partial x_3} + q_2 \frac{\partial q_2}{\partial x_3} \right) \left( \frac{\partial x_3}{\partial x_1} - \frac{\partial x_3}{\partial x_2} \right)
\]

(41)

Again, it is useful to look separately at the corrective and profit-shifting components of the optimal tax. Take first the difference due to the use of the tax structure for corrective purposes: In order to compare different firms' tax rates, we define \( \Omega \) as the difference between two firms' corrective tax components. Inserting the results of Section 4 in (41), we find:

\[
\Omega := \frac{2Q}{n+1} (\theta_1 - \theta_2) + \frac{2}{n+1} (\theta_2 q_2 - \theta_1 q_1)
\]

(42)

For \( \theta_1 = \theta_2, \Omega \leq 0 \) if \( q_1 < q_2 \), which is the result of lower R&D costs or lower initial costs of firm 2. If, on the other hand, the only difference across firms is on R&D productivity, there is an ambiguity: By doing less R&D the more R&D productive firm may still have a lower marginal cost of production, and consequently a greater production volume. The lower R&D productivity runs in favour of a smaller tax, but a lower quantity is associated with a lower tax for other firms (the externality imposed by other firms is smaller). Therefore, the evolution of the relative tax is ambiguous. Heterogeneity in R&D costs and heterogeneity in R&D productivity may have different implications for the optimal corrective tax imposed by the government.

A decrease in R&D costs always implies a decrease in the respective corrective tax. The effect works through the optimal choice of R&D. As costs of doing R&D decrease, a firm will increase its investment in R&D activities. This makes the firm relatively more efficient, which in turn makes it more attractive for the government to decrease its tax and divert production to this more efficient firm.

The same effect operates when a firm becomes more R&D productive. If the firm is more R&D productive, it will increase R&D efforts, giving rise to the effects previously described.

¹⁹Note that in the case of only one domestic and one foreign firm, we are back to the result of Spencer and Brander (1983), as in the case of a single domestic firm there is no corrective component in the tax.
The effect is, however, combined with the direct impact of R&D on the level of marginal cost of production. Even if R&D effort remains constant, an increase in R&D productivity entails a production cost reduction, which gives an incentive to the government to decrease the tax on this firm, as it becomes a more efficient producer. Nonetheless, the same intuition is also present: taxing more heavily less efficient firms is a way to redistribute production towards the more efficient firms.

Consider now the profit-shifting part. By construction, the linearity of the model in the quantity stage results in

\[
\frac{\partial x_y}{\partial (\theta_1 x_1)} = \frac{\partial x_y}{\partial (\theta_2 x_2)} \quad y \in M_3
\]

Thus, we can write the profit-shifting part of (41) as:

\[
-2 \left( m_1 q_1 \frac{\partial q_1}{\partial x_y} + m_2 q_2 \frac{\partial q_2}{\partial x_y} \right) (\theta_1 - \theta_2) \left( \frac{\partial x_y}{\partial (\theta_2 x_2)} \right) m_4
\]

(44)

Therefore, the firm with the higher R&D productivity receives a higher subsidy according to this effect. The reason is that a higher \( \theta_i \) also implies a relative efficiency of this firm in rent shifting. Consequently, the firm endowed with a higher \( \theta_i \) is used in a more intense way by the government for that purpose. On the other hand, differences in R&D costs alone are not sufficient to induce differential treatment of domestic firms. Again there is a difference. **Heterogeneity in R&D costs does not give rise to differences in the rent-shifting R&D tax, while heterogeneity in R&D productivity does.**

Taking the two effects together, if a firm is more efficient in both productivity and costs than the other, it has a higher subsidy. However, there is an indeterminacy if we cannot order firms in terms of the two efficiency characteristics.

The definition of the policy instrument as a value per unit of R&D effort is not the only one possible. A different tax structure could be defined in relation to R&D output, defined as the achieved cost reduction. Under a tax/subsidy definition based on R&D output a firm producing a higher quantity faces a lower tax. Define a tax \( \tau_i \) as proportional to \( \theta_i x_i \), thus \( \sigma_i = \tau_i \theta_i \). And

\[
\tau_i = \frac{\sigma_i}{\theta_i} = \frac{2}{n+1} \sum_{k \in M \setminus \{i\}} q_k + \frac{2 \theta_3}{n+1} m_3 \frac{\partial x_3}{\partial (\theta_2 x_2)} \sum_{j \in M} q_j, \quad i \in M
\]

(45)

In this case, the difference between the two groups of domestic firms in the corrective tax is:

\[
\tau_1 - \tau_2 = \frac{2}{n+1} (q_2 - q_1)
\]

(46)
With equal initial costs of production of the final good, quantities are directly related to R&D output, so that \( q_1 < q_2 \) if and only if \( \theta_1 x_1 < \theta_2 x_2 \). Thus, from expression (46):

\[
\tau_1 > \tau_2 \text{ if } \theta_1 x_1 < \theta_2 x_2 \tag{47}
\]

The profit-shifting part of \( \tau_1 \) is equal for both firms, as the focus on the output of R&D activities already corrects for the relative efficiency of domestic firms in rent shifting. Thus, in this special specification of the tax, the ex-post more efficient firm receives the lower tax. It may, or may not, correspond to a higher level of R&D effort. The ranking of firms according to R&D effort \( (x_i > x_j) \) may not be identical to that based on R&D output \( (\theta_i x_i > \theta_j x_j) \). A firm can do less R&D than the other and still obtain a higher R&D output due to differences in R&D productivity, as long as no clear ranking in efficiency parameters across domestic firms exists.

6 Final remarks

One message of this paper is that whenever industrial policy is targeted at the firm level in the presence of international rivalry, no clear policy prescription is available.

The optimal government intervention faces two conflicting forces. First, there is the (well-understood) strategic incentive to profit shifting of Spencer and Brander (1983), which prescribes a subsidy to R&D activities of domestic firms. Second, there is an incentive to correct the negative impact of one domestic firm’s R&D on the other through imposition of a tax. The Spencer-Brander subsidy is larger for the more R&D productive firm, while the corrective tax is lower for such a firm. Therefore, efficient firms receive a more favorable tax treatment. This is so because (a) they are relatively more successful in shifting profits from foreign firms; and (b) they impose a smaller external effect upon other domestic firms.

There is, however, an indeterminacy if firms cannot be ranked in terms of efficiency characteristics. The indeterminacy is resolved if the tax is defined in terms of R&D output, rather than effort. In this case, the tax automatically compensates for differences in R&D productivity.

The detection of a motive to tax domestic firms, instead of an incentive to subsidize, runs against what has, generally, been found in earlier literature. Leahy and Neary (1997), in a context of international as well as domestic interindustry R&D spillovers find a motive to tax domestic firms’ R&D activities: such a tax reduces the beneficial technological spillovers to foreign firms. As it is clear, our motive is quite different. Moreover, the Leahy-Neary incentive to tax R&D is absent as we assumed away spillovers.
The policy implications of the present analysis is more of a warning than it is a clear prescription. It shows that firm heterogeneity has a complex way of affecting the optimum industrial policy when firm-specific instruments are available. It is not clear how differences in R&D technology transform into differences in the government's firm-specific R&D policy. The formulae that we produce in the ensuing analysis may give the impression that there is a way out of this complexity. It is necessary to stress, however, that we do our analysis throughout with the assumption that the government has complete information. When firms are heterogeneous and the policy is firm-specific, then the government has a formidable task to pick winners. The informational requirements needed to fine-tune such a policy seem to be quite strong, as no robust prediction emerges. Therefore, we think it still may be wise to adhere to the advice of Nelson (1982), Stoneman (1987), and others, that picking winners is rarely successful. Our modest contribution is that, even if the information problem can be overcome, firm heterogeneity makes the link between firm characteristics and the optimal R&D firm-specific policy a complex one.

Appendix

Here we set up and solve our most general model, the one analysed in Section 5: three types of firms, characterised by equal ε. There are two types of domestic firms (sets $M_1$ and $M_2$) and one set of foreign firms ($M_3$).

The set of first-order conditions at the R&D stage is

$$n \theta_1 (\alpha + n \theta_1 x_1 - m_2 \theta_2 x_2 - m_3 \theta_3 x_3) - \frac{(n+1)^2}{2} \varphi_1 = 0$$

(A.1)

$$n \theta_2 (\alpha + n \theta_2 x_2 - m_1 \theta_1 x_1 - m_3 \theta_3 x_3) - \frac{(n+1)^2}{2} \varphi_2 = 0$$

(A.2)

$$n \theta_3 (\alpha + n \theta_3 x_3 - m_2 \theta_2 x_2 - m_1 \theta_1 x_1) - \frac{(n+1)^2}{2} \varphi_3 = 0$$

(A.3)

where $\varphi_i = \partial \varphi / \partial x_i$. To derive comparative statics, consider an infinitesimal change in some exogenous parameter $\mu$. Total differentiation of first-order conditions, after rearranging, gives, in matrix form:

$$
\begin{bmatrix}
  n^2 \theta_1^2 - \frac{(n+1)^2}{2} \varphi_{11} & -m_2 n \theta_1 \theta_2 & -m_3 n \theta_1 \theta_3 \\
  -m_1 n \theta_1 \theta_2 & n^2 \theta_2^2 - \frac{(n+1)^2}{2} \varphi_{22} & -m_3 n \theta_2 \theta_3 \\
  -m_1 n \theta_1 \theta_3 & -m_2 n \theta_2 \theta_3 & n^2 \theta_3^2 - \frac{(n+1)^2}{2} \varphi_{33}
\end{bmatrix}
\begin{bmatrix}
  dx_1 \\
  dx_2 \\
  dx_3
\end{bmatrix}
= 
\begin{bmatrix}
  \mu_1 d\mu \\
  \mu_2 d\mu \\
  \mu_3 d\mu
\end{bmatrix}
$$

(A.4)

where the $\mu_i$ are proportional to $-\partial^2 \Pi_i / \partial x_i \partial \mu$. In order to sign comparative statics, we impose necessary and sufficient conditions for stability, as described in Dixit (1986). The
necessary conditions amount to the requirement

\[
\frac{\partial^2 \Pi_i}{\partial x_i^2} < 0, \quad i = 1, 2, 3 \tag{A.5}
\]

which are just the second-order conditions of each firm’s profit maximization problem.

The sufficient conditions are set to guarantee that the determinant of the matrix of second-order derivatives has the sign of \((-1)^p\), where \(p\) is the number of equations in the system (equal here to the number of sets of firms). Denote this determinant by \(H\):

\[
H = \left|\begin{array}{c}
\frac{\partial^2 \Pi_i}{\partial x_i \partial x_j}
\end{array}\right| \tag{A.6}
\]

In our model, the symmetry features of the equilibrium effectively reduce the number of relevant equations to \(p = 3\), and so we want \(H < 0\). We have:

\[
\begin{align*}
H &= \left( n^2 \theta_1^2 - \frac{(n+1)^2}{2} \varphi_{11} \right) \left( n^2 \theta_2^2 - \frac{(n+1)^2}{2} \varphi_{22} \right) \left( n^2 \theta_3^2 - \frac{(n+1)^2}{2} \varphi_{33} \right) - \\
&- m_1 m_3 (n \theta_1 \theta_3)^2 \left( n^2 \theta_2^2 - \frac{(n+1)^2}{2} \varphi_{22} \right) - m_1 m_2 (n \theta_1 \theta_2)^2 \left( n^2 \theta_3^2 - \frac{(n+1)^2}{2} \varphi_{33} \right) - \\
&- m_2 m_3 (n \theta_2 \theta_3)^2 \left( n^2 \theta_1^2 - \frac{(n+1)^2}{2} \varphi_{11} \right) - 2 m_1 m_2 m_3 n \theta_1^2 \theta_2^2 \theta_3^2
\end{align*}
\]

Several sets of conditions can be imposed that lead to \(H < 0\). The more convenient representation for our purposes is:

\[
n (\theta_i + n) \theta_i^2 - \frac{(n+1)^2}{2} \varphi_{ii} < 0, \quad i = 1, 2, 3. \tag{A.7}
\]

To see that \(H < 0\) follows, we can write \(H\) as

\[
\begin{align*}
H &= \left( n^2 \theta_2^2 - \frac{(n+1)^2}{2} \varphi_{22} \right) \left[ n^2 \theta_1^2 - \frac{(n+1)^2}{2} \varphi_{11} (n^2 \theta_2^2 - \frac{(n+1)^2}{2} \varphi_{33}) - n^2 \theta_2^2 \theta_3^2 \right] - \\
&- n^2 \theta_1^2 \theta_2^2 \left( 12 \theta_3^2 - \frac{(n+1)^2}{2} \varphi_{33} \right) - n^2 \theta_2^2 \theta_3^2 \left( 12 \theta_1^2 - \frac{(n+1)^2}{2} \varphi_{11} \right)
\end{align*}
\]

Assumption (A.7) implies:

\[
\left( n^2 \theta_1^2 - \frac{(n+1)^2}{2} \varphi_{11} \right) \left( n^2 \theta_3^2 - \frac{(n+1)^2}{2} \varphi_{33} \right) > n^2 m_1 m_3 \theta_1^2 \theta_3^2 \tag{A.8}
\]

and

\[
\left( n^2 \theta_2^2 - \frac{(n+1)^2}{2} \varphi_{22} \right) \left[ n^2 \theta_1^2 - \frac{(n+1)^2}{2} \varphi_{11} (n^2 \theta_2^2 - \frac{(n+1)^2}{2} \varphi_{33}) - n^2 m_1 m_3 \theta_1^2 \theta_3^2 \right] < \\
< -3 \theta_2^2 \left[ n^2 \theta_1^2 - \frac{(n+1)^2}{2} \varphi_{11} (n^2 \theta_2^2 - \frac{(n+1)^2}{2} \varphi_{33}) - n^2 m_1 m_3 \theta_1^2 \theta_3^2 \right]
\]

21
Note that the term in square brackets is positive. Thus,

\[ H < H' = -\theta_2^2 m_3 n \left( n^2 \theta_1^2 - \frac{(n+1)^2}{2} \varphi_{31} \right) \left( n^2 \theta_2^2 - \frac{(n+1)^2}{2} \varphi_{32} \right) < 0 \]  

by (A.7). Thus, \( H < H' < 0 \).

The comparative statics effects are

\[
\begin{align*}
\frac{dx_1}{d\mu} &= \frac{1}{H} \left[ \mu_1 \left( n^2 \theta_2^2 \varphi_{33} \right) \left( n^2 \theta_3^2 - \frac{(n+1)^2}{2} \varphi_{22} \right) - n^2 m_3 \theta_3^2 \theta_1^2 \right] + \\
&\quad + \mu_2 \left[ n m_3 \theta_2 \theta_1 \left( n m_2 \theta_3^2 + n^2 \theta_1^2 - \frac{(n+1)^2}{2} \varphi_{33} \right) \right] + \\
&\quad + \mu_3 [n m_3 \theta_1 \theta_3 (n^2 \theta_2^2 \varphi_{33} + n m_2 \theta_3^2)] \\
\frac{dx_2}{d\mu} &= \frac{1}{H} \left[ \mu_2 \left( n^2 \theta_3^2 - \frac{(n+1)^2}{2} \varphi_{33} \right) \left( n^2 \theta_3^2 - \frac{(n+1)^2}{2} \varphi_{11} \right) - n^2 m_1 \theta_1^2 \theta_3^2 \right] + \\
&\quad + \mu_1 \left[ n m_1 \theta_2 \theta_1 \left( n^2 \theta_3^2 + m n \theta_1^2 - \frac{(n+1)^2}{2} \varphi_{33} \right) \right] + \\
&\quad + \mu_3 [n m_1 \theta_2 \theta_3 (n^2 \theta_1^2 \varphi_{11} + n m_1 \theta_1^2)] \\
\frac{dx_3}{d\mu} &= \frac{1}{H} \left[ \mu_3 \left( n^2 \theta_1^2 - \frac{(n+1)^2}{2} \varphi_{11} \right) \left( n^2 \theta_2^2 - \frac{(n+1)^2}{2} \varphi_{22} \right) - n^2 m_1 \theta_1^2 \theta_2^2 \right] + \\
&\quad + \mu_1 \left[ n m_1 \theta_3 \theta_1 \left( m n \theta_2^2 + m^2 \theta_1^2 - \frac{(n+1)^2}{2} \varphi_{22} \right) \right] + \\
&\quad + \mu_2 [n m_2 \theta_2 \theta_3 (n^2 \theta_1^2 \varphi_{11} + m n \theta_1^2)]
\end{align*}
\]  

(A.10)  

(A.11)  

(A.12)

A shorter way of describing the effects is:

\[
\begin{bmatrix}
\frac{dx_1}{d\mu} \\
\frac{dx_2}{d\mu} \\
\frac{dx_3}{d\mu}
\end{bmatrix}
= 
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
\mu_1 \\
\mu_2 \\
\mu_3
\end{bmatrix}
\]  

(A.13)

where \( a_{ii} < 0, a_{ij} > 0, i = 1, 2, 3, j = 1, 2, 3, i \neq j \). These signs result from the sufficient and necessary conditions described above. With this structure it is a simple matter to show the results of Remark 5. For example, take an infinitesimal change in \( \gamma_1 \). In this case, \( \mu_1 = (1/2)(n+1)^2 \theta \partial^2 \varphi / \partial x_1 \partial \gamma > 0, \mu_2 = 0, \mu_3 = 0 \). It follows immediately that

\[
\frac{dx_1}{d\gamma_1} < 0, \quad \frac{dx_2}{d\gamma_1} > 0, \quad \frac{dx_3}{d\gamma_1} > 0
\]  

(A.14)
References


Leahy, D. and C. Montagna (1997), Strategic trade policy when firms have different efficiency levels, Discussion Paper 1549, CEPR, London.


