ON THE WELFARE EFFECTS OF FOREIGN INVESTMENT

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Abstract

Modern growth theory emphasizes endogenous technological change as the engine of growth. A policy implication for developing countries that has been drawn from this theory is that foreign direct investment increases growth. Foreign producers with more advanced technology increase the rate of innovation and the rate of growth of GDP. In this paper we show that, given the knowledge spillovers and creative destruction effects existent in these models, even for a closed economy, an increase in the rate of growth does not always imply an increase in welfare, there may be an immiserizing growth effect. In an open economy welfare assessments must, also, recognize that national investment may become unprofitable due to foreign competition. Taking into account all the relevant effects, we characterize the conditions that imply a positive or a negative welfare effect.

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1. Introduction

Recent literature in growth theory points to research and innovation as the engine of growth. A policy implication for developing economies that has been drawn from this theory is that foreign investment increases growth. Romer (1993) emphasizes the point and gives developing countries the counsel of opening their economies to foreign investment. The argument is that better technologies or lower costs of research increase the rate of innovation and the rate of growth of the economy.

This paper discusses the role of foreign direct investment on growth and welfare in this literature. The idea is that foreign investors already use better technologies, so they have lower costs to introduce them in the economy. Hence, a fundamental effect of foreign investment is a decrease in the cost of introducing new goods into the economy. This is the effect of foreign investment that this paper studies, it is present when foreign investment creates firms that produce new goods, or "old" goods with new technologies.¹ Two other effects are intrinsically linked to the first one, the loss of capital returns to foreigners and the change in the cost of capital.

Sources of suboptimality are pervasive in endogenous growth models. In the R&D sector private costs diverge from social costs and private benefits diverge from social benefits. If there were no distortions, the increase in R&D productivity brought on by foreign investment would certainly increase welfare. In the presence of distortions the sign of this welfare effect is not clear.

The main claim of this paper is that the new theory of endogenous growth gives new and more fundamental relevance to the possibility of immiserizing growth. When the economy's equilibrium is not efficient, an increase in the productivity of a sector

¹An exhaustive treatment of all the issues raised by the multitude of forms that foreign investment may take is outside the range of this paper.
expands the production possibility boundary, but may decrease welfare. This is the immiserizing growth effect originally exposed by Bhagwati (1958). Stokey (1995), considering a more general setting, emphasizes that the existence of market failures is constitutive of the R&D activity. Thus, there are always distortions in these endogenous growth models and this raises the possibility of immiserizing growth.

In this paper, the existence of knowledge spillovers, decreases the social cost of innovation relative to the private cost. The social benefit from innovation may be larger than the private benefit due to the difference between total consumer surplus and the part that is appropriated by the producer, and may be smaller than the private benefit due to the existence of creative destruction effects. Thus, the equilibrium rate of innovation may be larger or smaller than the optimum.\textsuperscript{2} When in equilibrium there is more research than in the social optimum, an expansion of the R&D sector caused by an increase in R&D productivity strengthens the existing distortion. The welfare cost of the increase in the distortion may dominate the welfare benefit derived from growth.

The effect of an increase in research productivity is first studied for a closed economy. We show that an immiserizing growth effect is an actual possibility if the equilibrium rate of growth is higher than the optimal rate of growth.

We then study the effect of opening the economy to foreign investors. This is the main contribute of this paper. We assume that foreign investors have lower costs of introducing new goods into the economy as they already produce them.\textsuperscript{3} This is why foreign investment is expected to increase the rate of innovation and the rate of

\textsuperscript{2} In a partial equilibrium setting, the results from the study of patent races as surveyed by J. Reinganum (1989) are: "the typical outcome is that aggregate expenditure in R&D is too high relative to the cooperative optimum" but "comparison with surplus-maximizing investment is more difficult since the innovator is typically unable to appropriate the full surplus [and] this will tend to depress investment in R&D, making the comparison ambiguous".

\textsuperscript{3} This holds as long as the home economy remains technologically behind the foreign economy.
growth. The increase in R&D productivity brought on by foreign investment, again may increase or decrease welfare.

The second effect of foreign investment is that national investment becomes unprofitable. This reduces national income and consumption and reinforces the possibility of a negative effect of foreign investment on welfare. The third effect of foreign investment is a change in the cost of capital. Foreign investors have access to the international capital market, so when there is foreign investment, the relevant cost of capital is the international interest rate.

Section 2 presents the model. Section 3 studies the welfare effect of an increase in research productivity for a closed economy. Section 4 studies an increase in productivity brought by foreign investment. The entire analysis is carried out from the perspective of the home economy, which is assumed to be a small economy. The assumption of no trade is maintained throughout the paper.

2. The basic model

This section describes the model used in the rest of the paper which builds on Grossman and Helpman (1991, Ch. 4).4 Technological change is endogenous and is formalized as rising product quality.5 Population is constant and normalized to one.

Consumption of final goods

The utility of the representative household is given by

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4 In Grossman and Helpman (1991) growth derives from innovation in consumption goods. If we used instead, the models presented on Romer (1990) or Barro and Sala-i-Martin (1993) where growth derives from innovation in intermediate goods, the qualitative results would be maintained but the analysis would be more involved.

5 We could use an increasing variety specification of technological change which has its roots in Romer (1990). The stronger creative destruction effect implied by the quality specification makes it more interesting for the study of foreign investment.
(1) \[ U = \int c \cdot e^{-\alpha U} \, dt, \]

where,

(2) \[ \log c_i = \frac{1}{n} \log \left( \sum_{j=1}^{n} q^j \cdot x_j \right) \quad q > 1 \]

The set of goods in the economy is fixed and given by the interval \([0, 1]\). Research allows an increase in the quality of each good. \( q^j \) is an index for quality, which increases from 1 to \( q \), \( q^2 \ldots \) when a new quality is developed. So \( q \) measures the step or "jump" in the quality ladder each time a better quality is developed and \( q^j \) is the best quality known of good \( j \) at each moment. The expression \( \left( \sum_{i=1}^{n} q^i \cdot x_i \right) \) represents the effective quantity of good \( j \) with each unity weighted by its quality. This specification of \( c \) implies an infinite elasticity of substitution between different qualities of the same good. Thus, for each good, only the brand with higher quality-price ratio is demanded. As shown below, this implies that only the higher quality of each good is produced.

From the maximization of the utility function (1) we obtain the rate of growth of consumption:

(3) \[ \frac{\dot{c}}{c} = r - \rho \]

where \( r \) is the real rate of return. Consumption is the numeraire, \( \rho = 1 \). From the formulation of \( c \) given in (2) we obtain the aggregate demand for each good which has positive demand. \( p_j \) is the price of the best quality of good \( j \).

(4) \[ x_j = c_i / p_j \]

\(^6\text{Grossman and Helpman (1991) use expenditure as numeraire, so in their paper, expenditure is constant and there is growth when prices decrease. Using consumption as numeraire the rate of growth we obtain for consumption is the real rate of growth.}\)
Production of final goods

Each good is produced by a monopolist who owns the respective patent. Labor is the only factor of production in the economy and \( L_j \) is the quantity of labor used in the production of good \( j \). The production function is,

\[
(5) \quad x_j = L_j
\]

Each monopolist maximizes profits taking as given the demand (4) and the wage rate, \( w \). Given the characteristics of demand, profits always grow with the price of the good but only the good with higher quality for price is demanded. So, the producers of the best quality of each good use limit pricing. The price of each good is the highest price for the best quality that implies negative profits for the producer of the following quality and only the higher quality of each good is produced. All prices are equal and given by:

\[
(6) \quad p_j = qw, \quad \text{for all } x_j
\]

Substitute from (6) and (4) in (2), to get the equilibrium wage rate:

\[
(7) \quad w = Q / q
\]

where \( Q \) is an index of knowledge (or technology) in the economy and is defined as,

\[
Q = q^T \quad \text{where} \quad T = \int k, dj
\]

\( Q \) increases when a new product is developed in any sector \( j \) and the respective \( k_j \) increases. At each moment \( Q \) is given. The wage rate is exogenous at each moment and varies with \( Q \). When \( Q \) increases, the same quantity of labor is used to produce the same quantity of a better good. In terms of \( e \), the productivity of work in the production of final goods increases with \( Q \).

From (6), (5) and (4) we get the following expression for profits,
market.

\( \tilde{r} \) is a random variable whose behavior results from the R&D technology described above.

\[
(10) \quad v_i = \int_{0}^{\tilde{r}} e^{-[R(t)-R(\tau)]} \pi(\tau) d\tau, \quad \text{where} \quad R(t) = \int_{0}^{t} r(s) ds
\]

We consider a symmetric equilibrium. Expected profits are equal in all sectors and so \( i \) is equal in all sectors and equal to aggregate R&D. The equilibrium aggregate rate of innovation is given by the equilibrium probability of success in R&D, \( i \).

Private researchers do not take into account the consumer surplus which they do not appropriate, nor the loss of profits for current producers. Thus, private returns from innovation may be smaller or larger than social returns. This is important to evaluate the welfare effects of the change on innovation costs.

**Labor-Market Equilibrium**

\( L \) is the quantity of labor in the economy. The labor-market equilibrium condition is,

\[
(11) \quad \alpha i + \frac{c_i}{q \omega_i} = L,
\]

where the first term is labor used in R&D and the second term is labor used in the production of all varieties of goods \( x \) and is obtained from conditions (4) to (6).

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7 As we have constant returns to scale in research, the scale of the individual researcher is not defined.

8 All research is carried out by followers. Industry leaders undertake no research because they have the same costs as outsiders but their reward is strictly smaller as they care only for the increase in profits.

9 By the properties of the Poisson distribution, at each moment the expected number of improvements in the economy is \( i \). As the number of sectors is infinite, the law of large numbers implies that the variation of \( i \) on time is given by \( i \).

10 Dixit and Stiglitz (1977) note the relevance of the non-appropriated consumer surplus when consumers like variety. Romer (1994) notes the relevance of the non-appropriated consumer surplus for the decision to produce or not to produce a new good, when there are fixed costs.
Steady-State

We solve for the steady-state obtaining one relationship between the rate of innovation and the interest rate, from consumers' preferences and another from the production side of the economy. As we have seen, the rate of growth is proportional to the rate of innovation.

The steady-state is defined by constant rates of growth of consumption and innovation. By the labor market equilibrium condition this implies that the quantity of work used in the production of goods is constant and that consumption grows at the same rate as the wage rate. The rate of growth of $c$ and $w$ is by condition (7) given by the rate of growth of the index of knowledge $Q$, which is $i\log q$.

Condition (3), obtained from the maximization of consumers utility can be understood as the supply of credit, derived from the agents’ preferences:

\[ r = \frac{c}{c} + \rho = i \log q + \rho \quad \text{(PP)} \]

As consumption grows at a constant rate, the real rate of return is constant in the steady-state. This together with the fact that the rate of innovation is constant implies that the value of a patent given in condition (9) can be evaluated as: \[ v_t = \frac{(1-1/q)c_t}{r+i-\frac{c}{c}} = \frac{(1-1/q)c_t}{r+i(1-\log q)} \]

$(r+i)$ is the relevant discount factor for the investor as $i$ is the instant probability of losing the whole value of the firm because someone develops the next state-of-the-art.

\[ v_t = \frac{(1-1/q)c_t}{r+i-\frac{c}{c}} = \frac{(1-1/q)c_t}{r+i(1-\log q)} \]

With a constant real rate of return the rate of growth of consumption is constant and $R(t) = \nu$. For the quality developed at $t$, given the properties of the Poisson distribution, the probability that at each moment $t$ next quality was not yet discovered is $e^{-\nu t}$. So the value of a patent given in (9) can be written as:
quality. The rate of growth of consumption determines the rate of growth of demand for each good. Substituting condition (13) and the free-entry condition (9) in the labor-market equilibrium condition, we get a relationship between the rate of innovation and the rate of return that results from the technologies of production and research, and can be understood as the demand for credit from the investors in the research sector. For a higher interest rate the discounted value of the returns from innovation is smaller and investment decreases.

\[ i = \frac{q-1}{q - \log q} \frac{L}{a} - \frac{r}{q - \log q} \]  
(RD)

(14)

To obtain condition (14) we used the free-entry condition (9) assuming that \( i > 0 \). Thus, condition (14) is valid only for \( i > 0 \).

The rate of innovation and the rate of return are endogenously determined by conditions (12) and (14). This equilibrium, represented in Fig.1 by point 0, is given by the intersection of curve PP (condition 12) and curve RD (condition 14).

\[ i = \max \{ (1 - 1/q) L/a - \rho / q, 0 \} \]

(15)

![Fig.1](image)

\[ v_t = \int e^{-r(t-t^*)} \pi(t)e^{-\theta(t-t^*)} dt = \int e^{-r(t+1/q)c(t)\tau}d\tau = \int e^{-r(t+1/q)c(t)\tau} (1-1/q)c\tau d\tau \]
Once we have determined the steady-state values for the interest rate and the rate of innovation, we can obtain the steady-state solutions for all other variables. We are mainly interested in the values for the rates of growth. The rate of growth of $c$ and $w$ is determined by the rate of innovation and equal to the rate of growth of $Q$, if the rate of growth diminishes with the discount factor, $\rho$, and the innovation cost, $\sigma$, and increases with the quantity of labor, $L$, and with $q$, the size of the quality jump.\footnote{The rate of growth cannot be negative as there is no knowledge depreciation.} The next section uses these results to study the welfare effect of an increase in research productivity.

3. Increase in productivity in the research sector

This section begins by studying the growth effect of an increase in productivity in R&D, for a closed economy. Then the welfare effect of this increase in productivity is determinate. It is shown that there may exist an innovating growth effect.

The Growth Effect

Using the results from the last section, it is straightforward to conclude that the rates of growth of GDP and consumption increase when the innovation cost, $\sigma$, decreases. This is represented in Fig.1. The curve RD moves to the right as the decrease in the cost of innovation makes it more profitable to invest in R&D. In the new equilibrium both the real interest rate and the rate of innovation increase.

The Welfare Effect

Let us suppose that before and after the increase in R&D productivity there is positive growth. So $\sigma < L(q-1)/\rho$. To determine the welfare effect of the decrease in the coefficient $\sigma$ we begin with the following statement.
**Lemma 1:** There is a value $a^*$ in the interior of the interval $(0, L(q-1)/\rho)$, such that $dU/da < 0$ for $a < a^*$ and $dU/da > 0$ for $a > a^*$.

*Proof:* From conditions (15) and (11) we get expression (16) for consumption. People consume labor income plus a constant part of their non-human wealth.

$$(16) \quad c_t = w_t L + w_t a \rho$$

Given expression (1) for utility, expression (16) for consumption, expression (7) for wages and the rate of growth of $Q$ which is $i \log q$, with $i$ given by (15), we get:

$$(17) \quad U = \frac{1}{\rho} \log(Q_t/q) + \frac{1}{\rho} \log(L + a \rho) + \frac{1}{\rho^2} (\log q)(1 - \frac{1}{q^2}) \frac{L}{a}$$

Differentiate this condition with respect to $a$ to obtain the welfare effect of an increase in research productivity:

$$(18) \quad \frac{dU}{da} = \frac{1}{L + a \rho} \frac{\log q}{\rho^2 (1 - \frac{1}{q})} \frac{L}{a^2} = \frac{a \rho^2 q^2 - (\log q)(q-1)L \rho a - (\log q)(q-1)^2 L^2}{(L + a \rho) q \rho^2 a^2}$$

We now want to determine the sign of this derivative for $a > 0$ and $a < L(q-1)/\rho$.

i) The denominator in expression (18) is always positive for $a > 0$, so it is the sign of the numerator that determines the sign of the derivative. The solutions to the quadratic equation in $a$ obtained by equating the numerator to zero are given by:

$$a = \frac{L}{2q \rho} \left[ (\log q)(q-1) \pm \sqrt{(\log q)^2 (q-1)^2 + 4q(\log q)(q-1)} \right]$$
ii) For  \( a = \frac{L(q - 1)}{\rho} \), \( \frac{dU}{da} = \frac{1}{Lq} \left( 1 - \frac{\log q}{q - 1} \right) \). \( q > 1 \), so \( \frac{\log q}{q - 1} < 1 \) and \( \frac{dU}{da} > 0 \) for this value of \( a \). Thus, the positive solution \( a^* \) must verify \( a^* < L(q - 1)/\rho \).

From i) and ii) we obtain Lemma 1.

Thus, \( U \) decreases with a fall in the innovation cost, near the zero-growth boundary \((a = L(q - 1)/\rho)\). As the cost of innovation decreases, \( dU/da \) becomes negative and \( U \) increases with a further decrease in the innovation cost. This leads to Proposition 1.

**Proposition 1:** The welfare effect of a decrease in the innovation cost depends on the initial position of the economy and on the change of the innovation cost. Notably, it is negative for small changes of the innovation cost near the zero-growth boundary.

Fig. 2 shows \( U \) as a function of the innovation cost for sensible values of \( q, \rho, L \) and the initial \( Q \). As the figure shows, welfare always increases with a decrease in the innovation cost if the change is large enough. But a small decrease in the innovation cost of a slowly growing economy decreases welfare.

![Fig. 2](image)

This figure shows utility, \( U \), as a function of the innovation cost \( a \), for \( \rho=0.1 \), \( Q=50 \) and \( q=2 \) in one case and \( q=1.25 \) in the other case.
An increase in the productivity in any sector implies an expansion of the production possibility boundary of the economy, but when the equilibrium is not efficient, welfare may decrease as originally described by Bhagwati (1958).\footnote{In Bhagwati (1958) growth (an increase in the stock of capital) increases production but deteriorates the terms of trade of a large open economy. The deterioration in the terms of trade more than offsets the positive welfare effect from growth. This happens due to the absence of an optimal tariff.} "What underlies the phenomena of immiserizing growth is the fact that the country experiences growth subject to some distortion."\footnote{Bhagwati and Srinivasan (1983, Chapter 25). In that chapter the problem of immiserizing growth is studied in the context of the classical "2*2*2" general equilibrium model for open economies.} In our economy the private value of innovation differs from its social value due to the difference between created surplus and the part that is appropriated by producers and due to creative destruction effects.\footnote{This is related to the business-stealing effect defined by G. Mankiw and M. Whinston (1986). In a partial equilibrium setting they show that in markets where there is free-entry, fixed set-up costs upon entry and product diversity, entry can be excessive, insufficient or optimal.} Also, the social cost of research is smaller then the private cost due to knowledge spillovers. This raises the possibility of immiserizing growth.\footnote{Final goods are produced in monopolies, but relative prices are not distorted because monopoly power is the same for all goods. The distortion in the relative price of labor is irrelevant as labor is the only factor of production. The existence of monopoly power is nevertheless important as the "creative destruction" effect is relevant only when equilibrium profits are not zero.}

If we solve the problem for the social optimum of the economy, we find that the static solution coincides with the market equilibrium. Taking as given the distribution of work between final production and R&D (which is investment in this economy), the production of each good is the same in the social optimum and in the equilibrium solution. However, the dynamic solution in the social optimum is different from the market equilibrium obtained in the last section. In the social optimum the rate of innovation is:\footnote{Grossman and Helpman (1991) showed that the equilibrium for this economy is not efficient (pages 101-106). To obtain the social optimum maximize utility subject to the equation of motion for $Q$ and the resource constraint: $\dot{Q} = i \log Q$ \quad $\int_{0}^{T} \sum_{x_{i}} d_{j} = L - ai$}
\[ g^* = i^* = \frac{L}{a - \rho \log q} + \rho \left( \frac{L}{q \rho} + 1 - \frac{q}{\log q} \right) \]

Thus, the equilibrium rate of innovation is higher than its optimal value if \( a > \frac{(L \log q)}{\rho(q - \log q)} \).

**Proposition 2:** Welfare decreases with a decrease in the innovation cost \( a \), only if the economy's rate of growth is larger than the optimum, \( g > g^* \).

*Proof:* \( i > i^* \) for \( a = \frac{(L \log q)}{\rho(q - \log q)} \). For \( a = \frac{(L \log q)}{\rho(q - \log q)} \) expression (18) implies that \( dU/da < 0 \). So \( \frac{(L \log q)}{\rho(q - \log q)} < a^* \). 

4. Foreign Investment

This section studies the welfare effect of foreign investment. Foreign investors introduce better qualities in the economy having, as domestic investors, an initial fixed cost which is a cost of adoption or adaptation. We assume that this cost is inferior to the domestic cost of discovering a new quality because foreign investors have already produced these better qualities. So one effect of foreign investment is an increase in research productivity. To study this first effect we build on the results of the previous section. Another effect depends on how this initial cost is paid. Who finances the capital is fundamental in the determination of the total welfare effect of foreign investment, as it will determine whether the domestic economy looses the opportunity to invest and delay consumption. We therefore consider two different cases: foreign investors borrow in the domestic market, they use only domestic capital, or foreign investors borrow in the international credit market.
We assume that nationals do not have access to the international capital market because we want to isolate the effect of direct foreign investment on national welfare. We do not want to mix it with effects of the access to the international capital market.

The idea that foreign investment is good for economies with a slow rate of growth is based upon the positive effect of foreign investment on the rate of innovation. However, as shown in the last section, an increase in the rate of innovation caused by a decrease in the research cost, as happens with foreign investment, may decrease welfare. We now adjust the effects described in the previous section, for a closed economy, to an increase in productivity brought on by foreign investors.

4.1 Foreign investors borrow in the domestic credit market

If foreign investors use domestic capital, they create new firms, borrowing from nationals and offering higher returns than national researchers because they have lower costs of introducing new goods in the economy. This has no opportunity costs in terms of what the foreign investor can do in other economies - he can use the same new technology at the same time everywhere. Free-entry in the research sector implies that the initial cost of research and adaptation is equal to the present value of all profits obtained with that research. So, when foreign investors use only domestic capital, they receive all profits but give them back as returns on domestic capital. In this case the only effect of foreign investment is a decrease in the cost of introducing new goods in the economy, or a decrease in the innovation cost. This is exactly the same effect studied in the last section for a closed economy.\(^{18}\)

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\(^{18}\) We assume that discoveries are sequential but if foreign investors already know various better qualities the introduction of new qualities does not have to go through the whole ladder. When we consider that the increase in the productivity in the introduction of new goods in the economy is brought by foreign investment we may have to determine which quality the foreign investor will introduce in the economy. We expect that both the cost of adapting a new good and the returns from
Proposition 3: When foreign investors use domestic capital the welfare effect of foreign investment is the welfare effect of a decrease in the innovation cost. It depends on the initial position of the economy and on the change in the innovation cost. The welfare effect is negative only if the rate of growth is larger than the optimum.

4.2 Foreign investors borrow in the international capital market

When foreign investors borrow in the international capital market, or use their own capital, the total effect of foreign investment must take into account that national investment ceases to exist. The cost of capital for foreign investors is given by the international interest rate. Foreign investors may prefer to use their own capital if the international interest rate is lower than the domestic interest rate or they may be forced to do that due, for instance, to difficulty in acting in local capital markets.

We now assume that foreign investors use only their own capital and evaluate the total effect of foreign investment. We must begin by studying the changes that occur when the economy opens to foreign investment. When we consider an increase in productivity in the R&D sector for a closed economy, there are no transitional dynamics and the economy goes immediately to the new steady-state. When the increase in productivity in R&D occurs because of foreign investors who use their own capital, there will be transfer of returns on capital and this will imply a form of transitional dynamics on national income and consumption for the home economy. In section 5 we do some calculations considering the whole transition path. This section

the introduction of that good in the economy increase with the "jump" in the quality ladder. We assume in the text that it is always optimal for foreign investors to introduce next quality, $k_j + 1$. 

16
ignores transitional dynamics and compare two steady-states, one with and one without foreign investment, as if this were the case from "the beginning of history". In the steady-state without foreign investment we consider that the economy has always been and will always be closed to foreign investment. In the steady-state with foreign investment we consider that the economy has always been open to foreign investment. This and the advantage of foreign investors on research imply that all creation of knowledge in the economy has been and will be due to foreign investors. Foreign investors use only their own capital, thus nationals lose the opportunity to invest in R&D. In the steady-state with foreign investment all profits belong to foreigners.

The steady-state without foreign investment corresponds to the closed economy studied above and condition (16) gives consumption for this steady-state. We must now determine the steady-state for the economy with foreign investment.

When we assume that foreign investors use their own capital, the cost of credit for investors in the economy is given by the world rate of return, $r^*$. The supply of credit, that was represented by curve PP and determined by the preferences of domestic consumers, is in this case just $r=r^*$. The effect of foreign investment in the demand for credit, determined by technology and resources in production and research, is the effect of the decrease in the innovation cost. So, in the steady-state with foreign investment, and where foreign investors do not have access to the domestic capital market, the rate of innovation is determined by the intersection of curve $RD'$ with $r=r^*$. The steady-state rate of innovation is given in condition (19) and shown in Fig.3.

\[
(19) \quad i = \frac{q-1}{q-\log q} \frac{L}{u_i} - \frac{r^*}{q-\log q}
\]
Foreigners have better productivity in R&D. We assume that their cost of innovation is given by \( a_i \) with \( a_i < a \) due to the cost advantage. The free-entry condition that applies to foreign investors implies that the cost of introducing a new good by foreign investment is equal to the revenues from being the only producer of that good in the home economy and is given by:

\[
\begin{align*}
  w^* &= a_i w_i \quad \text{if } i > 0 \\
  w^* &< a_i w_i \quad \text{if } i = 0
\end{align*}
\]

where \( v^* \) is the value of a patent for foreigners taking into account that for them the relevant rate of return is \( r^* \). The free-entry condition for national agents is the same as in the closed economy, given by condition (9). So, there will be no national research, and all profits belong to foreigners, as long as the absolute cost advantage of foreign investors is not dominated by a higher cost of capital.\(^9\) This is the interesting case, so we assume that it is verified.

\(^9\) The absolute cost advantage of foreign investors is not dominated by a higher cost of capital as long as \( v^* / a < v^* / a_i \), where agents take the rate of innovation as given. Total demand, \( D^* \), for the composite good \( \hat{c} \) includes part of the foreign profits. We assume that total demand behaves as before and grows at the same rate as \( c \). The domestic rate of return is determined by the equilibrium of the domestic market at zero credit demand. This is obtained from condition (12), given the rate of growth of consumption. Given the domestic and international rates of return and expression (13) for the value
With foreign investment the steady-state rate of innovation is given in (19), all investment is done by foreigners, as domestic agents will not find it profitable to invest in R&D. As nationals do no investment, they are not able to delay consumption and they consume all their income, which is only labor income. In the steady-state we have:

\[ (21) \quad \text{National Consumption} = w, L. \]

The Welfare Effect

Let \( r^* \) be the interest rate that would prevail in the economy after the increase in productivity in R&D if investors used domestic capital. This is the same interest rate that would prevail if the same increase in productivity in R&D occurred for a closed economy, as in section 3.

**Proposition 4:**

i) When there is no change in the cost of capital \( r^* = r_i \) foreign investment adds a negative effect (due to the loss of profits) to the welfare effect of the increase in R&D productivity. So the total welfare effect is negative when it was negative in Proposition 2 and may be positive or negative when it was positive.

of a patent, the absolute cost advantage of foreign investors is not dominated by a higher cost of capital if:

\[ r^* < \left( \frac{a - l + \log q}{a_j + \frac{a}{a_j} p} \right). \]

For values of \( r^* \) and \( r_i \) that do not satisfy this condition the cost advantage of foreign investors is more than offset by their higher cost of capital and all investment is done by nationals. In this case there is no foreign investment in the steady-state. We may obtain a rate of innovation, \( \tilde{r} \), such that for rates of innovation inferior to this value, or world rates of interest larger than the corresponding value \( \tilde{r} \), the advantage foreigners have in the cost of introduction of new goods in the economy is more than compensated for by a disadvantage in the cost of capital. If \( r^* > \tilde{r} \) there will be no foreign investment and the steady-state will be the one for a closed economy determined in section 2.

\[ \tilde{r} = \frac{(1 - \bar{L} \bar{q}) a - \bar{q} \bar{p}}{\bar{q} + (1 - \bar{L} \bar{q}) a}, \quad \Rightarrow \tilde{r} < r_i \Rightarrow \tilde{r} > r_i. \]

In the text we assume \( r^* < \tilde{r} \), which is the interesting case.
ii) When the cost of capital increases ($r^* > r_i$) it implies an additional negative welfare effect. When the cost of capital decreases ($r^* < r_i$) there is a positive effect and the final conclusion depends on the relative strength of the various effects.

Proof: The welfare effect of the opening of the economy to foreign investors is,

$$
\Delta U = \int e^{-rt} \log c_i \, dt - \int e^{-rt} \log c_i \, dt
$$

where $c_i$ is national consumption when all research is done by foreign investors, given by (21), and $c$ is national consumption when there is no foreign investment and is given by (16). Substitute these conditions for consumption with and without foreign investment and the wage rate from condition (7) in expression (22) and obtain:

$$
\Delta U = \int_0^\tau e^{-rt} \log \left( \frac{LQ_t / q}{(L + ap)Q / q} \right) \, dt
$$

The level of knowledge $Q$ is given at each moment (each $k_f$ is given), so the initial $Q$ is the same for the two steady-states, only the growth rate is different. The growth rate of $Q$ is $I_0 q$. When there is no foreign investment $i$ is given in condition (15). When there is foreign investment $i$ is given in condition (19).

$$
\Delta U = \frac{1}{\rho} \log \left( \frac{L}{L + ap} \right) + \frac{1}{\rho^2} \log (q) \left( q - 1 \right) \left( \frac{L}{a_i(q - \log q) - \frac{L}{aq}} \right) + \frac{p q^{-r^*}}{q - q - \log q}
$$

The first term is a negative level effect on consumption due to the loss of profits and the increase in the interest rate, the second term is the effect of the increase in the rate of growth. This expression can be rewritten to discriminate the three effects present:

$$
\Delta U = \frac{1}{\rho} \log \left( \frac{L}{L + ap} \right) + \left[ \frac{1}{\rho} \log \left( \frac{L}{L + a_i \rho} \right) + \frac{\log q}{\rho^2} \left( 1 - \frac{1}{q} \right) \left( \frac{L}{a_i(q - \log q)} - \frac{L}{aq} \right) \right] + \frac{(\log q)(r_i - r^*)}{\rho^2(q - \log q)}
$$
The first term is the negative level effect in consumption because there is no national investment and so, returns on capital now belong to foreigners. The second term is the effect on consumption caused by the decrease in the coefficient \( a \). The last term is the effect of the change in the cost of capital.

Here we cannot evaluate the welfare effect by differentiating with respect to \( a \). Even if \( r^* = r_f \), and the last term in expression (25) is zero, the derivative of \( U \) with respect to the innovation cost does not include the level effect of the transfer of returns on capital. However, we use the results from section 3 about the sign of \( dU/da \) to obtain the sign of the effect of the change in the innovation cost, the second term in expression (25).

We proved that welfare may decrease with an increase in research productivity. If we differentiate the second term in (25) with respect to \( a \), we obtain exactly the expression of \( dU/da \) given in condition (18), so \( dU/da > 0 \) for \( a_f \in ]a^+, L(q-1)/\rho[ \) and \( dU/da < 0 \) for \( a_f \in ]0, a^+ [ \). We know that this term is zero for \( a_f = a \), so if the initial \( a \) is larger than \( a^* \), this second term is negative for \( a_f \in ]a^+, L(q-1)/\rho[ \) and \( a_f a \), but it is positive for small values of \( a_f \). If the initial \( a \) is already smaller than \( a^* \), this second term is always positive for \( a_f < a \). The effect on welfare of the decrease in the innovation cost may be negative or positive depending on the initial innovation cost and on the importance of the decrease. It is
For $r^* = r$, the last term in expression (25) is zero, so the total effect is the sum of the first two effects. Note that this may be negative even when the second term is positive. This completes the proof of part 1) of Proposition 4.

For $r^* \neq r$, the effect on growth of the change in the cost of capital is positive or negative depending on $r^*$ being larger or smaller than $r$. The total effect of foreign investment is the sum of these three effects, the increase in productivity in research, the transfer of returns on capital and the change in the cost of capital, and may be positive or negative. This proves Proposition 4.

To understand the effect of the loss of profits when national investment ceases to exist, we look at the effect of foreign investment when there is no change in the cost of capital, $r^* = r$. Free-entry in R&D implies that the initial cost of research and adaptation is equal to the present value of all profits obtained with that research. So, it implies a zero return for the investor. When investment is done by foreigners with foreign capital all profits belong to foreigners and compensate for the initial cost of research. Nationals loose the opportunity to invest in R&D and so, loose the returns from this activity. Nationals loose the returns on capital once and for all, this implies the level effect on consumption showed in Fig. 4.

Fig. 4 illustrates the effect of an increase in research productivity with and without the loss of the capital returns. GDP is the same in both cases. Consumption is less when the returns on capital are transferred to foreigners. Without this loss the solution is the same as for the closed economy, consumption is equal to income less investment. When foreign investors use their own capital, there is the loss of capital returns and consumption is equal to income less profits. We can show that in the
closed economy, for a given value of \( a \), investment in R&D is less than profits, so although nationals consume all their income, the loss of profits dominates the change of consumption: the decrease is equal to \( w_t u_p \).

![Graph showing changes in GDP, consumption, and investment before and after the increase in productivity in R&D with and without loss of profits.]

From condition (16) consumption \( = n(L + u_p) \). From condition (7) \( w = Q/q \) and from the definition of \( Q \) the level of technology and the wage rate are given at any moment. So when \( a \) decreases, the levels of \( Q \) and \( w \) are given and consumption decreases.

For \( r^* \neq r_l \) we must take account of the effect of the change in the cost of capital. If \( r^* > r_l \), the cost of capital increases and this has a negative effect over the rate of innovation and the rate of growth and over welfare. If \( r^* < r_l \), the cost of capital decreases and this has a positive effect over the rate of growth and over welfare.

It is not clear if \( r^* \) should be larger or smaller than \( r_l \). \( r^* \) is the equilibrium real interest rate in the rest of the world. If the two economies are identical except in that the foreign economy is ahead in the innovation process then, in autarca, both economies grow at the same rate and have equal interest rates. In this case the domestic interest rate after the increase in R&D productivity, \( r_l \), is larger than \( r^* \).
Also, even if the two economies are identical in everything else, the home economy may take advantage of the delay in the innovation process and imitate, at lower costs, the goods already known abroad. This strengthens the case for \( r^* \) being larger than \( r \). On the other hand, if we want to consider that the home economy is a less developed economy we may not want to think about two identical economies.

In general, taking account of all three effects, the increase in productivity in research, the loss of returns on capital and the change in the cost of capital, foreign investment may increase or decrease welfare. This is true even when welfare increases with the decrease in the innovation cost.

5. The welfare effect of foreign investment considering the whole transition

Until now we studied the welfare effect of foreign investment considering no transitional dynamics. We assumed that the economy goes immediately to the new steady-state. In this section we perform some calculations to evaluate the change in welfare for an economy that opens to foreign investment, taking account of the whole transition dynamics for national consumption. We only consider the case where there is no change in the cost of capital, we assume \( r^* = r \).

The absolute cost advantage of foreign investors in the introduction of new goods implies that when the economy opens to foreign investment all national investment in R&D ceases immediately, thus GDP behaves as in section 4 from the moment the economy opens. However, during a transition period nationals own part of the profits, consumption is equal to wages plus that part of profits.

A national firm ceases production when a foreign investor introduces a better quality of the good it produced, during a transition period some sectors and the

---

20 Barro and Sala-i-Martin (1993, Chapter 8), discuss this point.
respective profits belong to nationals, as time goes on the percentage of sectors belonging to nationals goes to zero.\footnote{The economy converges to this steady-state for any $q$ such that $\log q < 1$.} At each moment a percentage $i$ of national producers ceases production and the respective sectors become property of foreigners. Let $\theta$ be the percentage of sectors that belongs to nationals, then $\theta = \exp(-it)$ and,

\begin{equation}
(26) \quad \text{National Consumption} = w_i L + \int \pi_t d\pi = w_i L + \theta \left( 1 - \frac{1}{q} \right) D_t
\end{equation}

$D$ is total demand for good $c$, it includes national consumption and part of foreign profits. The rate of growth of national consumption, when there is foreign investment, is obtained from (26) with the expression for demand given in (16). We assume that demand behaves as before and grows at the same rate as $c$. National profits increase at a slower rate than labor income, so the rate of growth of consumption is increasing through time. National consumption converges to $wL$ for all $q$ such that $\log q < 1$.

\begin{equation}
(27) \quad \frac{\dot{c}}{c} = \frac{\dot{GDP}_t}{GDP_t} \frac{(1 - 1/q)(1 + \rho i/L)}{1 + (1 - 1/q)(1 + \rho i/L)} e^{-\rho i}
\end{equation}

The welfare change when the economy opens to foreign investors at $t=0$ is given by (22) with $c_t$, national consumption when the economy is open to foreign investors, now given by (26), and $c$, national consumption when there is no foreign investment given by (16). We use the following expressions for consumption:

\begin{equation}
(28) \quad c_t = c_i e^{\epsilon_t} \quad \text{and} \quad c_t = c_i e^{\epsilon_t}
\end{equation}

$G$ is the growth factor of consumption when there is foreign investment. From (27):

\begin{equation}
(29) \quad G_t = \frac{1}{c_t} \int c_t (\tau) d\tau = \frac{GDP_t}{GDP_t} + \log \left( e^{\epsilon_t} \frac{(1 - 1/q)(1 + \rho i/L) + 1}{(1 + \rho i/L) + 1} \right)
\end{equation}

Substituting from (29) and (28) in (22), the welfare effect of foreign investment is,
\[
\Delta U = \frac{1}{\rho} \log \left( 1 - \frac{1}{q} + \frac{1}{1 + \rho t / L} \right) + \frac{1}{\rho^2} \left( \frac{\dot{GDP}_t}{GDP_t} - \frac{\dot{GDP}}{GDP} \right) + \int_0^\infty e^{-\rho t} \log \frac{e^{-\rho t} (1 - \frac{1}{q})(1 + \rho t / L) + 1}{(1 - \frac{1}{q})(1 + \rho t / L) + 1} \, dt
\]

The first term is the positive level effect in consumption at \( t=0 \), the second and third terms are the growth effect on consumption. At \( t=0 \) consumption increases as national investment goes to zero and there was still not any transfer of profits to foreigners. The growth effect may be positive or negative as it includes the increase of the growth rate of GDP and the decrease of the growth rate of consumption caused by the loss of profits. To compare this with the welfare effect of foreign investment when we assume that the economy goes immediately to the new steady-state, we obtain the difference between last expression and expression (25), with \( \nu^*=\eta_0 \).

\[
\text{Difference} = \frac{1}{\rho} \log \frac{1 + (1 - 1/q)(1 + \rho a / L)}{1 + (1 - 1/q)(1 + \rho a_1 / L)} + \int_0^\infty e^{-\rho t} \log \left[ 1 + e^{\rho t} \left( 1 - \frac{1}{q} \right) \left( 1 + \frac{\rho a_1}{L} \right) \right] \, dt
\]

This difference is positive as \( a_1 < a \). Thus, when we take into account the transition dynamics for national consumption, the welfare effect of foreign investment increases.

We can not obtain a definite sign for the general expression of the welfare effect. Thus, we carried out some calculations for some sensible values of the parameters that show that even when we take account of the whole transition, welfare may decrease, although this happens only for small changes in the rate of growth of GDP. The values used for the discount factor and for the R&D technology parameter were based on the work by Caballero and Jaffe (1993). For \( q \) it is more difficult to base
For given values of \( q, p \) and \( L/a_i \) we obtained numerically the value of \( L/a \)
that implies a zero welfare change, \((L/a)^*\). For any \( L/a \) larger than this threshold value
the welfare effect of the increase in R&D productivity is negative.

<table>
<thead>
<tr>
<th>( 1/q=0.5 )</th>
<th>( p=0.1 )</th>
<th>( 1/q=0.48 )</th>
<th>( p=0.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L/a_i = 0.17 )</td>
<td>( L/a_i = 0.12 )</td>
<td>( L/a_i = 0.15 )</td>
<td>( L/a_i = 0.12 )</td>
</tr>
<tr>
<td>( \frac{\partial L}{\partial \Delta L} = 0.013862 )</td>
<td>( \frac{\partial L}{\partial \Delta L} = 0.006931 )</td>
<td>( \frac{\partial L}{\partial \Delta L} = 0.017328 )</td>
<td>( \frac{\partial L}{\partial \Delta L} = 0.001105 )</td>
</tr>
<tr>
<td>( (L/a)^* = 0.0691 )</td>
<td>( (L/a)^* = 0.1194 )</td>
<td>( (L/a)^* = 0.1483 )</td>
<td>( (L/a)^* = 0.1191 )</td>
</tr>
<tr>
<td>( \frac{\partial L}{\partial \Delta L} = 0.013551 )</td>
<td>( \frac{\partial L}{\partial \Delta L} = 0.006723 )</td>
<td>( \frac{\partial L}{\partial \Delta L} = 0.016739 )</td>
<td>( \frac{\partial L}{\partial \Delta L} = 0.009656 )</td>
</tr>
</tbody>
</table>

The first two rows indicate the values for \( q \) and \( p \). The third row gives the value considered for
\( L/a \) after the increase in productivity in R&D and the implied rate of growth of GDP. The last
row shows the threshold value obtained for \( L/a \) before the increase in productivity. For \( L/a \)
larger than this value the welfare effect of the increase in research productivity is negative.

6. Conclusion

The distortions that are always present when economic growth is based on
investment in R&D raise the possibility of immiserizing growth. For a closed economy,
a decrease in the innovation cost increases the rate of growth but may decrease
welfare. When the decrease in the innovation cost is brought on by foreign investment,
there are other effects that must be taken into account. Foreign investors introduce
new goods in the economy at lower costs, so foreign investors take away from
domestic producers the possibility to invest in innovation because they can pay better
returns. This has a negative effect on national income which is translated in the loss of
capital returns. We showed that, even when the increase in R&D productivity increases
welfare, foreign investment increases welfare only if the increase in productivity is
large enough to compensate for the loss of capital returns.
This paper studied the welfare effect of foreign investment for an unanticipated opening of the economy to foreign investment. In case this opening was previously announced, domestic investors would expect a rise in the rate of innovation. Taking these expectations into account decreases the expected return from research. Thus, the announcement of the opening to foreign investment implies an additional effect that should be added to our study.

For an unanticipated opening of the economy to foreign investment our results imply that for an economy with high costs of innovation and a low rate of growth the welfare effect of a small increase in R&D productivity is probably negative. We still have to add the negative welfare effect of the loss of profits as national investment vanishes. For a less-developed economy we would also expect a decrease in the cost of capital when investors have access to the international capital market, implying a positive welfare effect. The total welfare effect from foreign investment is positive only if this positive effect is large enough to compensate for the other negative effects.

A few numerical results suggest that there are small negative changes on welfare for a restricted range of values for the parameters. The choice of a formulation with a linear technology in R&D may justify these results. We chose this formulation because it leads to closed form solutions for welfare and welfare changes. However it excludes the “crowding-effect” in research costs that Stokey (1995) proposes as the most powerful reason for excessive R&D. As we only expect welfare to decrease with a decrease in the innovation cost if there is excessive R&D, a formulation including the “crowding-effect” would probably strengthen the case against foreign investment. Furthermore, even if we do not obtain strong results against foreign investment, these should be kept in mind when deciding how to attract foreign investors.
Appendix

This appendix formalizes technological change as increasing product variety as in Grossman and Helpman (1991, Ch. 3). We show how the results in the paper are modified for this formulation.

Utility is still given by (1), but $c$ is modified to include the taste for variety:

$$
U = \int e^{-\kappa u} \ln c_i \, dt,
$$

where

$$
(2a) \quad c_i = \left( \int x_i^n \, dj \right)^{-\alpha}, \quad 0 < \alpha < 1
$$

The set of goods in the economy is given by the interval $[0, \beta]$. Research allows an increase in the variety of goods. This specification of $c$ implies a constant and equal elasticity of substitution between any pair of goods, $\varepsilon = \frac{1}{(1-\alpha)}$. Utility maximization implies condition (3). From (2a) we obtain the aggregate demand for each good.

$$
(3) \quad \frac{c_i}{c} = \tau - \rho
$$

$$
(4a) \quad x_i = c_i p_i^{-\tau}
$$

The production function is given by condition (5). Profit maximization implies conditions (6a) and (7a) for prices and the equilibrium wage rate.

$$
(5) \quad x_i = L_i
$$

$$
(6a) \quad p_i = w_i / \alpha, \quad \text{for all } x_i
$$

$$
(7a) \quad w_i = \alpha Q_i
$$

where the index of knowledge, $Q_i$, is now defined as $Q_i = n_i^n$, implying that the rate of growth of the wage rate is $\frac{1}{n_i^n}$. Expression (8) for profits is in this case:

$$
(8a) \quad \pi_i = (1 - \alpha)c_i \, n_i
$$
The R&D technology is such that to discover a new good a researcher needs to use \( a/n \) of labor, the cost of developing new goods diminishes with the number of existing goods. Free-entry in R&D implies that the cost of research for a new good is equal to the expected return of that research. Thus,

\[
(9a) \quad v_r = a \frac{w_r}{n_t} \quad \text{for} \quad \frac{n_t}{n_r} > 1
\]

where \( v_r \) is the value of a patent. In this case a producer never looses its monopoly so,

\[
(11a) \quad v_r = \int e^{-\int k_{n_t} - R(t)} \pi(t) dt \quad \text{where} \quad R(t) = \int r(s) ds
\]

Let \( g = \frac{c}{n} \) be the economy’s rate of innovation. The labor-market equilibrium is,

\[
(11a) \quad a g + \frac{c}{w_r} = L
\]

**Steady-State**

At the steady-state the innovation rate and the rate of growth of consumption are constant. The labor market condition implies that consumption grows at the same rate as the wage rate, \( \frac{1 - \alpha}{\alpha} n \). The credit supply is obtained from the maximization of consumers’ utility:

\[
(12a) \quad r = \frac{c}{c + \rho} = \frac{1 - \alpha}{\alpha} g + \rho
\]

Given a constant interest rate and a constant innovation rate, we evaluate a patent as:

\[
(13a) \quad v_r = \frac{(1 - \alpha)c_t}{n_t} \left[ r - \left( \frac{c - n}{c - n} \right) \right]
\]
From (13a), the free-entry condition (9a) and the labor-market condition we obtain a relationship between the rate of innovation and the interest rate that can be understood as a demand for credit from the investors in R&D:

\[(14a) \quad g = \frac{1 - \alpha}{\alpha} \frac{L}{a} - r\]

Conditions (12a) and (14a) determine the rate of innovation in the steady-state.

\[(15a) \quad g = \max\left\{\frac{(1 - \alpha)L}{a} - \alpha \rho, 0\right\}\]

**Increase in productivity in the research sector**

We only consider positive rates of growth, so we assume that \( a < (1 - \alpha)L/\alpha \rho \).

Lemma 1A determines the welfare effect of an increase in R&D productivity, for a closed economy.

**Lemma 1A**: \( dU/da < 0 \) for any economy with a positive rate of growth.

**Proof**: From conditions (15a) and (11a) we get expression (16) for consumption.

\[(16) \quad c_t = w_tL + w_t\alpha \rho \]

Substitute on expression (1) for utility, expression (16) for consumption, expression (7a) for wages and expression (15a) for the rate of innovation:

\[(17a) \quad U = \frac{1}{\rho} \log(L + \alpha \rho) + \frac{1}{\rho} \log(\alpha Q_t) + \frac{1}{\rho^2} \frac{(1 - \alpha)}{\alpha} \left[ \frac{(1 - \alpha)L}{a} - \alpha \rho \right]\]

Differentiate this condition with respect to \( a \) to obtain the welfare effect of a decrease in the innovation cost:

\[(18a) \quad \frac{dU}{da} = \frac{1}{L + \alpha \rho} \left[ \frac{(1 - \alpha)^2}{\rho^2} \frac{L}{\alpha} \frac{\alpha \rho \tau}{a} - (1 - \alpha)^2 L \rho \alpha - (1 - \alpha)^2 \frac{L}{a} \right] = \frac{(L + \alpha \rho) \alpha \rho \tau}{a^2} \]
The denominator in this expression is always positive for \( a > 0 \), so it is the sign of the numerator that determines the sign of the derivative. The solutions to the quadratic equation in \( a \) obtained by equating the numerator to zero are \( a^* = \frac{(1 - \alpha)}{\alpha \rho} \) and \( a^- = \frac{-1 - \alpha}{\rho} \). Thus, \( \frac{dU}{da} < 0 \) for \( a < \frac{(1 - \alpha)L}{\alpha \rho} \).

**Proposition 1A:** For the economy with increasing product variety, a decrease in the innovation cost always increases welfare.

**Foreign Investment**

**Foreign investors borrow in the domestic capital market**

When foreign investors use domestic capital the total effect of foreign investment is the effect of an increase in research productivity as, in this case, there is no transfers of capital returns. This is exactly the effect studied for a closed economy so, in this case, foreign investment always increases welfare.

**Foreign investors borrow in the international capital market**

When foreign investors borrow in the international capital market we have to take account of the transfer of capital returns and also of the change in the relevant capital cost. We look at the two steady-states, with and without foreign investment, considered in section 4.2.

The cost of capital for foreign investors is in this case given by \( r^* \) so, in the steady-state with foreign investment the rate of innovation is determined by the intersection of the curve RD, given by condition (12a), with \( r = r^* \).
\[ g = \frac{1 - \alpha}{\alpha} \frac{L}{a_i} - r^* \]

The free-entry condition for foreign investors says that the cost of introducing a new good in the economy is equal to the returns from being the only producer of the good:

\[ v_i = a_i \frac{w_i}{n_i} \quad \text{for} \quad g > 0 \]

where \( v_i \) is the value of a patent for foreigners taking into account that for them the relevant interest rate is \( r^* \). The free-entry condition for national agents is the same as in the closed economy, given by condition (9a). Thus, there is no national research and all profits belong to foreigners. As nationals do no investment, they consume all their income, which is only labor income. In the steady-state with foreign investment:

\[ \text{National Consumption} = w_i L. \]

Let \( r_i \) be the rate of return that would prevail in the economy after the increase in R&D productivity if investors used domestic capital.

**Proposition 2A:**

i) When there is no change in the cost of capital \((r^* = r_i)\) foreign investment adds a negative effect (from the loss of profits) to the welfare effect of the increase in R&D productivity. The total welfare effect is positive or negative depending on the relative strength of the two effects.

ii) When the cost of capital increases \((r^* > r_i)\) it implies another negative effect. When the cost of capital decreases \((r^* < r_i)\) there is an additional positive effect. The total welfare effect depends on the relative strength of the three effects.
Proof: The change in welfare caused by the opening of the economy to foreign investors is given by:

\[ (22a) \quad \Delta U = \int_{0}^{\infty} e^{-\omega t} \log c_i \, dt - \int_{0}^{\infty} e^{-\omega t} \log c_i \, dt \]

where \( c_i \) is national consumption when all research is done by foreign investors and \( c \) is national consumption when there is no foreign investment. Substitute on (22a) the expressions for consumption with and without foreign investment given respectively by conditions (16) and (21a) and for the wage rate from condition (7a), and obtain:

\[ (23a) \quad \Delta U = \int_{0}^{\infty} e^{-\omega t} \log \frac{\alpha Q_i}{\alpha Q_i(L + a_{i} \rho)} \, dt \]

As we have seen the level of knowledge depends on \( n \), which is given at each moment so \( Q_n \) is the same for the two steady-states, only the growth rate is different. The growth rate of \( Q \) is \( \frac{1-\alpha}{\alpha} g \). When there is no foreign investment \( g \) is given in condition (15a). When there is foreign investment \( g \) is given in condition (19a). So, given the expressions for the growth rate, the total change in welfare is:

\[ (24a) \quad \Delta U = \frac{1}{\rho} \log \frac{L}{L + a_{i} \rho} + \left[ \frac{1}{\rho} \log \frac{L + a_{i} \rho}{L + a \rho} + \frac{1}{\rho^2} \log \frac{(1 - \alpha)^2}{\alpha} \left( \frac{L}{a_i} - \frac{L}{a} \right) + \frac{(1 - \alpha)(\mu_i - \mu)}{\rho^2 \alpha} \right] \]

The first term is the negative level effect in consumption because there is no national investment and so, returns on capital now belong to foreigners. The second term is the welfare effect of the decrease in the coefficient \( a \). The last term is the effect of the change in the cost of capital.

Proposition 1A showed that welfare increases with an increase in R&D productivity. We use this result to show that the second term in expression (24a) is always positive. If we differentiate this term with respect to \( a_i \) we obtain exactly the


