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António Nogueira Leite*, Vasco Santos*
and Maria do Carmo Seabra*

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António Nogueira Leite* Vasco Santos*† Maria do Carmo Seabra*

Universidade Nova de Lisboa, Faculdade de Economia
Travessa Estêvão Pinto (Campolide), P–1070 Lisboa, Portugal

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Abstract

Access prices to bottleneck facilities are usually set by regulatory authorities due to anti-trust reasons: owners of bottleneck facilities have an incentive to charge socially undesirable access prices. We show that welfare-maximizing access prices may not be implementable due to what we term instrument insufficiency, and relate the likelihood of instrument insufficiency to cost and demand parameters, industry structure and ownership, type of competitive behavior in the downstream market, and (other) economic policies followed.

We conduct the inquiry by considering a market structure and regulatory setup closely adhering to the reality of telecommunications markets. We consider a two-tier industry structure in which the monopolist producer of a bottleneck input also operates in the downstream market, facing competition from a fixed number of non-integrated firms. The regulator sets the upstream and access prices, but not the downstream price, and the number of downstream firms (and their ownership or control) is exogenously given, thus there not being free entry and profits not being nil.

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1 Introduction

Firms owning a bottleneck facility needed by others as an input have an incentive to abuse their position when setting the access charge to the bottleneck facility.[1] The realization of this has lead authorities to regulate access prices. We show that socially-optimal access prices may not be implementable, a situation that we term *instrument insufficiency*. The fact that instrument insufficiency may (or may not) arise for a given market and regulatory structure raises the question of its *likelihood* as a function of cost and demand parameters, industry structure and ownership, type of competitive behavior in the downstream market, and (other) economic policies followed by regulatory authorities. We determine how all these factors influence

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†Corresponding author: António Nogueira Leite, Universidade Nova de Lisboa, Faculdade de Economia, Travessa Estêvão Pinto (Campolide), P–1070 Lisboa, Portugal, telephone no. (+351 1) 388 73 81, facsimile no. (+351 1) 386 42 87, e-mail: anleite@feunix.fe.unl.pt.

[1] See Baumol and Sidak (1994, pp. 172–173, 179) on this matter.

instrument insufficiency and the likelihood of its occurrence. These questions, though of independent theoretical interest, also are presently of acute relevance to the regulation of the telecommunications industry. The presentation reflects this dual use of the results.

Telecommunications markets are characterized in many countries by the coexistence of a state-owned monopoly over fixed-link services and competition in mobile (cellular) services. Typically, this competition takes the form of a duopoly where one of the players is owned by the (monopolist) provider of fixed-link services and the other player is a privately owned firm. This is the case in countries such as Belgium, France, Germany, Italy, Portugal, Spain, and the US.[2]

Since most mobile calls require the use of the fixed-link network, access to the latter provides the opportunity for strategic and often times anti-competitive behavior on the part of the monopolist fixed-link operator.[3] This, in turn, gives rise to a regulatory problem, namely that in which price ought to be set for access to the fixed-link network, which thus constitutes the bottleneck facility. The question we ask is whether the access price (together with direct regulation of the monopolist's fixed-link output sold directly to consumers) is a sufficient regulatory instrument for achieving optimal resource allocation in markets such as telecommunications, bearing in mind that regulating the access price is tantamount to regulating an input price, an indirect and possibly imperfect regulatory mechanism compared to, say, price or quantity regulation. Furthermore, since the answer is that regulatory setting of the access charge may or may not be sufficient for allocational optimality, the likelihood of sufficiency as a function of numerous economic factors has to be assessed. We enumerate them now, but not before pointing out that the problem of instrument insufficiency cannot be reduced to merely counting regulatory instruments versus objectives. In fact, in most of the paper, regulatory authorities have two objectives, specifically ensuring that the socially optimal quantities are consumed in the fixed-link and mobile markets, and two regulatory instruments, namely the fixed-link and access prices. This constitutes *prima facie* evidence in favor of instrument sufficiency, an impression dispelled by the paper's analysis.

Most of the literature on these issues devotes no more than scant attention to imperfect competition in the downstream market, a realistic feature in most countries.[4] Instead of presuming free entry and lack of profits in the mobile market as, for the most part, has been heretofore postulated in the literature (of which a synopsis follows) we assume a fixed number of downstream competitors, in an effort to adhere more strongly to reality.[5] This is crucial for instrument insufficiency. Presuming free entry, and therefore implicitly assuming

[2] See Parker and Röller (forthcoming) for a description of this industry's market structure in the US. The regulatory authorities of some of these countries have already announced that a third operator will be licensed.

[3] The incentives for anti-competitive behavior on the part of the vertically integrated firm associated with this market structure have been repeatedly recognized by the European Commission in the last few years.

[4] See Parker and Röller (forthcoming) for empirical tests of the US telecommunications market indicating prices in significant excess of costs. This is attributed by the authors to the duopoly market structure imposed by the regulatory authority.

[5] In many European countries and in the US, the number of mobile operators is exogenously set according to spectrum restrictions or other considerations—see Parker and Röller (forthcoming) on the US telecommunications market—and as a result, entry is not free.

competitively-determined output in the mobile market, sweeps the problem of instrument insufficiency away: regulatory control of the access charge is then enough to ensure allocational optimality (with or without fixed entry costs).

The type of competitive behavior adopted by firms in the downstream market will also be seen to affect the likelihood of instrument insufficiency. We will begin by assuming Cournot-Nash behavior on the grounds of realism for the telecommunications industry (see footnote 13). Later, by focusing on other types of competitive behavior, we will draw attention to what ultimately influences the likelihood of instrument insufficiency, namely, the competitiveness of the outcome of the game played in the downstream market, defined as how close this outcome is to the socially optimal one.

Differences in the efficiency of mobile operators resulting from their diverse ownership are also a possibility in reality. This is to be expected when one mobile firm is controlled by the fixed-link provider, usually a firm with a long experience in the telecommunications industry and "deep pockets" capable of financing sustained inefficiency. As will be seen, this feature is of some consequence for regulatory purposes. This aspect has not figured explicitly as a regulatory factor in most discussions of these issues despite its potentially important impact when efficiency differences are pronounced. It too has an impact on the likelihood of instrument insufficiency.

Liberalizing policies have become fashionable. These, however, are hardly ever of the free-for-all kind that is usually depicted in a stylized manner as free entry. Instead, authorities typically give away or auction operating licenses and increase their numbers slowly.[6] We discuss how two types of liberalizing or competition-increasing policies—increases in the number of firms operating in the mobile market and divestiture by the fixed-link service provider of the mobile operator it controls—affect the likelihood of instrument insufficiency.

Regulatory authorities often refrain from directly regulating price in (perfectly or imperfectly) competitive markets—a stylized fact also pointed out by Vickers (1995) in a related paper—a feature that we capture by assuming that only the (monopolist) fixed-link provider has its output directly regulated. Performance in the mobile market is controlled by regulatory authorities only indirectly via the access charge, even when one of the firms operating in this market is controlled by the monopolist. Though realistic, this regulatory environment has received fleeting attention in the literature. By leaving the regulator with only an indirect instrument for regulating the mobile market, this feature will be shown to have a strong bearing on instrument insufficiency and its likelihood.

Rather, the number of firms is not only exogenously set by the regulatory authority but also made common knowledge, something that eliminates the incentive for preemptive behavior of further entry when players decide what quantities to produce or prices to set. This further vindicates our modeling strategy of taking the number of firms as exogenously set at an imperfectly competitive level and focusing on a Nash solution.

[6]As mentioned in a previous footnote, regulatory authorities in some countries have announced the upcoming licensing of a third operator.

Interdependence of demands for the fixed-link and mobile services is also a possibility. As the price of mobile phones becomes lower, the substitutability between a cellular hand-set and a fixed-link connection increases.[7] We deal with this issue, showing how such considerations may strongly alter the likelihood of instrument insufficiency by comparison with the case of independent demands.

Since profits in the mobile market are not zero due to imperfect competition, allocational optimality is an issue. The access price plays a crucial role in this matter through its effect on the output in the cellular market.[8] Socially undesirable low output in the mobile market calls for a low access charge—an input price from the mobile-market firms' viewpoint—to induce output to reach the socially optimal level. That such aim may not be achievable gives rise to what we describe as instrument insufficiency. The likelihood of such insufficiency thus needs to be related to the factors presented in the previous paragraphs.

The access charge however also affects the revenue of the fixed-link provider, being an output price from the fixed-link firm's standpoint. Avoiding fixed-link provider losses raises the question of the extent to which either market should be distorted, thus obviously leading to Ramsey-type pricing. We show how our regulatory structure leads to results contrasting sharply with those obtained elsewhere in the literature.[9] It should be clearly understood that we are not concerned with how fixed costs attributable to the setting up of the fixed-link network itself, universal-service obligations, contribution to common costs, or any other *exogenous* cost sources are financed through the distortion of both markets.[10] This problem, though obviously also leading to Ramsey-pricing considerations, is studied for instance in Laffont and Tirole (1994, 1996). Our results spring instead from an *endogenous* source: the possible losses that optimally setting the access price may entail for the fixed-link operator.

The problem of access pricing was first analyzed by Willig (1979) and Baumol (1983). In a contestable market's framework, assuming that the input provider is an integrated monopolist, and that prices of final goods are fixed, they show that the access price should reflect the incremental cost of providing access plus the opportunity cost to the incumbent of units sold by the entrant. This formula, known in the literature as the Efficient Component Pricing Rule (ECPR), was later popularized by Baumol and Sidak (1994) and has since been the subject of several analyses and extensions. These have shown that when the prices of the final goods and the access price are optimally set—the former thus becoming Ramsey-type—, the resulting access price is in general larger than would be implied by the ECPR, the reason being that increasing the access price allows for a decrease in the price of final goods, enhancing allocative efficiency and consumers'

[7] Paraphrasing Laffont and Tirole (1996, p. 233) makes the point clearly: "While few customers forego telephone service altogether for small increases in the current subscriber line charges, they may forego the use of multiple lines at home or in secondary dwellings. And, more and more, they may start bypassing the network (trough mobile phones)."

[8] See Spencer and Brander (1983) on this issue.

[9] Armstrong and Doyle (1994) model these two markets similarly but, by assuming that the regulator is capable of choosing price for the mobile market, end up concluding that pure Ramsey pricing should be set in both markets, a conclusion that will be shown not to hold in our setting.

[10] See Kahn and Taylor (1994, pp. 229, 236–237) on this matter.

surplus (Armstrong and Doyle (1995), Laffont and Tirole (1994)). It has also been shown that the relevant notion of opportunity cost changes whenever input-substitution possibilities are considered, the entrant has the possibility of bypass, or imperfect substitutability between competitive outputs is allowed for: each of these effects calls for a decrease in the optimal access price (Economides and White (1995), Cave and Doyle (1994), Armstrong and Doyle (1994, 1995), Armstrong and Vickers (1995), Vickers (1995), Armstrong, Doyle and Vickers (1996), Laffont and Tirole (1994, 1996)). If, due to the existence of barriers to entry, the regulator deliberately wishes to promote entry (even inefficient entry), again a price lower than the ECPR is required (Economides and White (1995), Cave and Doyle (1994)). Armstrong, Doyle and Vickers (1996) assume that the prices of final outputs are directly regulated, thus obtaining pure Ramsey pricing when dealing with the simplest case, and modified Ramsey pricing when some of the complications referred to above are present. Finally, Laffont and Tirole (1994) emphasize asymmetry of information between the regulator and the monopolist.[11] A salient feature of their approach is that lump-sum transfers from the regulator to the monopolist can be made, thus recovering marginal-cost pricing as an optimal solution.

Another strand of the literature has addressed the issue of the adequate structure of a regulated industry in which there is a monopolist provider of a bottleneck facility, e.g., Armstrong and Doyle (1995), Economides and Woroch (1994), Gilbert and Riordan (1995) and Vickers (1995). Armstrong and Doyle (1995) stress that not allowing integration increases social costs when firms try to fight or circumvent the integration prohibition, either through litigation and regulatory hearings or by inefficiently creating close substitutes that fall outside the scope of the integration prohibition. Vickers (1995) analyzes the welfare effects of vertical integration in an industry where there is Cournot competition and free entry in the downstream market, the monopoly price is regulated by a welfare-maximizing regulator, and the regulator has asymmetric information concerning the regulated incumbent's costs. It is shown that, in such a setting, the welfare effects of vertical integration versus divestiture are ambiguous. He also discusses whether the monopolist should be allowed to operate in the downstream industry through control of one of the downstream competitors. His results are basically driven by two considerations: (i) avoiding fixed-cost duplication inherent to free entry when such costs are present and (ii) the altered incentive of the monopolist operating upstream to raise competitors' costs when also operating in the downstream industry. Avoiding too much duplication of fixed costs due to excessive entry recommends access charges *in excess* of marginal costs. As we shall see, we find rationales for *low* access charges at reasonable parameter values, a feature that will be shown to raise the serious regulatory issue of instrument insufficiency. In a different vein, Economides and Woroch (1994) analyze the private and social incentives for network interconnection as well as the impact of different industry structures on consumers' welfare. Gilbert and Riordan (1995) also analyze the issue of the optimal degree of vertical integration in the presence of optimal regulation and asymmetric information. They conclude that vertical disintegration

[11] Laffont and Tirole (1994) consider interdependence of demands at a level different from ours: they assume that the services provided in the downstream market by the integrated firm are not a perfect substitute for the services provided by its competitors.

induces additional (informational) costs akin to the "double marginalization" phenomenon associated with monopoly pricing of complementary products, whereas vertical integration can increase cost through reduced competition among suppliers. Their results are predicated on the assumption of regulation of all firms in the case of vertical disintegration.

Demand for fixed-link and mobile services and their elasticities are obviously of the essence when studying these issues: their forms and behavior are crucial in determining regulatory instruments and their sufficiency. They should thus be kept fairly general and any assumptions made on them be justified. We do both in the sequel.

Introducing asymmetry of information when studying these problems adds no further insight. *Ergo*, we refrain from further complicating our setting. Also, we do not explicitly address the issue of the difficulties faced by regulators when actually implementing Ramsey-type prices. These are associated with informational problems, and the concomitant possibility of capture when estimates (of demand elasticities, cost parameters, etc.) are needed.[12] However, we return to the issue in the concluding section.

As briefly mentioned before, the careful reader will probably be struck at some point by the fact that the telecommunications industry, besides providing a benchmark against which the realism of the assumptions can be judged, does not play any further relevant role in the analysis. This is in fact the case: our results are of use when studying other industries with similar structure. This stratagem of presentation is used because (i) the issue at hand is currently of particular importance in the economics of telecommunications literature; (ii) the reader is provided with a less abstract frame of presentation, with the corresponding added ease of interpretation of both arguments and results; (iii) it brings to the fore the rationale behind each assumption, thus rendering more obvious any adaptations needed for the application of the results to other markets with similar but not absolutely equal structures.

To summarize, this paper identifies a new source of regulatory failure (instrument insufficiency) and relates its likelihood to several structural and policy variables. It is organized as follows. Section 2 presents the model, followed by Section 3 were a simple version later used as benchmark is analyzed. Section 4 then proceeds to analyze the effects of differences of efficiency in the downstream market. Section 5 considers the consequences of imposing a break-even constraint upon the monopolist. Divestiture is analyzed in Section 6 and interdependent demands are considered in Section 7, whose subsections retrace the issues discussed before for the case of independent demands and draw comparisons. Finally, Section 8 discusses the introduction of fixed costs, non-constant-returns-to-scale and general versions of competitive behavior in the downstream market, while Section 9 concludes. All proofs are relegated to an Appendix, with one exception constituting integral part of the text that should thus be read as the reader progresses.

[12]See Laffont and Tirole (1996, p. 253) on these issues.

2 The Model

We study a model involving two markets and $n+1$ players. One of the players, the fixed-link provider, monopolistically operates in the fixed-link market and competes in the mobile-telephony market against n non-integrated firms. The latter need to accede the former's network in order to provide their service. Access cannot be rationed by the fixed-link provider.

Let I denote the fixed-link provider who also owns one of the mobile operators, E_i denote its competitors, with $i \in \{3, \ldots, n+2\}$, and subscripts 1 and 2 refer not only to the fixed-link and mobile markets' prices, respectively, but also to the quantity sold by firm I in each of these markets. The firms' profits can be written as

$$\pi^I = p_1 q_1 + p_2 q_2 + p_a \sum_{i=3}^{n+2} q_i - c_1 (q_1 + q_2 + \sum_{i=3}^{n+2} q_i) - c_2 q_2$$

$$\pi^{E_i} = p_2 q_i - p_a q_i - c q_i, \qquad i \in \{3, \ldots, n+2\},$$

where $c_1 > 0$ represents the constant marginal cost of carrying calls in the fixed-link network, and $c_2 > 0$ and $c > 0$ are the additional constant marginal costs of providing cellular calls by firms I and E_i, respectively.

By allowing c_2 and c to differ, we capture efficiency differences in the downstream market, a feature that will be seen to be of relevance. Finally, p_a denotes the access price. Quantities, denoted q_i, $i = 1, 2, \ldots, n+2$, are decision variables of the players.

We are assuming that cellular calls are always routed through the fixed-link network, a simplifying but fairly realistic assumption. Furthermore, we are setting fixed costs equal to zero. This entails no loss of generality when optimality of operation rather than entry is of importance. However, the existence of significant fixed cost when entering the mobile market is to be expected: introducing fixed costs at a later stage will prove straightforward, once the main results on instrument-insufficiency likelihood are apprehended. We do so in the penultimate substantial section. Suffice it to say at this juncture that considering fixed-cost issues increases the likelihood of instrument insufficiency. Linearity of all relationships seems to be fairly realistic (at least up to capacity) and has been an assumption maintained in this literature. We retain it throughout the paper. Later, departure from this constant-returns-to-scale hypothesis will also be dealt with. Finally, even though calls to a mobile phone originated in the fixed-link network induce an access charge too, we believe that the relative economic impact of this interconnection charge on the industry's structure and performance is (currently) empirically small enough to warrant no further cluttering of the model. The same applies to access charges resulting from calls involving two different mobile providers.

Demands are represented with generality: besides assuming that they are negatively sloped with respect to own price and differentiable, no other assumption is made; in particular, no particular functional form is assumed. We will begin by assuming that demands in both markets are independent of each other, and then relax this assumption in Section 7. Some mild regularity assumptions will also be made subsequently, as will assumptions ensuring that second-order conditions prevail.

The effects of increased competition in the mobile market on instrument-insufficiency likelihood will later prove crucial. Thus, using a solution concept that allows for a gradual increase in competitiveness as the number of players increases when solving the game played in the downstream market is illuminating. This is a feature of the Cournot solution (not shared, for instance, by the Bertrand solution) that recommends its assumption. We do so also on the grounds of realism.[13] Later, the general argument relating competitiveness of the game solution to instrument-insufficiency likelihood will became apparent and will thus be discussed. Firms set quantities in the downstream market in accordance with the following first-order conditions:

$$\begin{cases} \max\limits_{q_2} \pi^I \\ \\ \max\limits_{q_i} \pi^{E_i} \end{cases} \Leftrightarrow \begin{cases} p_2 - c_1 - c_2 + \frac{\partial p_2}{\partial q_2} q_2 = 0 \\ \\ p_2 - p_a - c + \frac{\partial p_2}{\partial q_i} q_i = 0 \end{cases} \Leftrightarrow \begin{cases} \frac{p_2-(c_1+c_2)}{p_2} = -\frac{\alpha}{\eta_2} \\ \\ \frac{p_2-(p_a+c)}{p_2} = -\frac{1-\alpha}{\eta_2 n}, \end{cases} \tag{1}$$

where $q \equiv q_2 + \sum_{i=3}^{n+2} q_i$, $\alpha \equiv \frac{q_2}{q}$, and $\eta_2 \equiv \frac{\partial q}{\partial p_2} \frac{p_2}{q}$.

We are not concerned with asymmetric behavior on the part of the n non-integrated mobile firms. Thus, we assume symmetry, i.e., $\sum_{i=3}^{n+2} q_i = nq_3$.

For what follows we need the following assumption, whose mildness is self-evident.

Assumption 1 *Demand is not too convex:*

$$\frac{\partial p_2}{\partial q} + q\frac{\partial^2 p_2}{\partial q^2} < 0, \forall q \geq 0.$$

Linear demand is clearly consistent with this assumption. The assumption is also sufficient for concavity of the integrated and non-integrated firms' profit equations.

The effect of changes in p_a on q_2 and q_3 will become crucial in the sequel. This is elucidated by

Lemma 1

$$\frac{dq_2}{dp_a} > 0, \quad \frac{dq_3}{dp_a} < 0, \quad \frac{dq}{dp_a} < 0.$$

Social welfare can be written as

$$\begin{aligned} W &= V(q_1, q_2 + nq_3) + p_1 q_1 + p_2 q_2 + p_a n q_3 - c_1(q_1 + q_2 + nq_3) - c_2 q_2 \\ &\quad + p_2 n q_3 - p_a n q_3 - cnq_3 \\ &= V(q_1, q) + p_1 q_1 - c_1 q_1 + p_2 q - q_2(c_1 + c_2) - nq_3(c + c_1), \end{aligned} \tag{2}$$

where

$$V(q_1, q) = \int_0^{q_1} p_1(q_1) dq_1 - p_1 q_1 + \int_0^{q} p_2(q) dq - p_2 q$$

represents the sum of the net consumer surplus in both markets.

[13] Again, Parker and Röller (forthcoming) uncover significant economic profits in duopoly markets in the US, suggesting that assuming, say, Bertand competition at this juncture would be lacking in realism.

3 Cost Symmetry in the Downstream Market

We begin by making the simplifying assumption that both firms are equally efficient in the production of the non-common input, i.e., $c_2 = c$. This simplest version of the problem will become the benchmark against which subsequent results will be evaluated.

Lemma 2 *Attaining the social-welfare maximizing allocation requires marginal-cost pricing:*

$$\begin{cases} p_1^* = c_1 \\ p_2^* = c + c_1. \end{cases}$$

The rationale behind this result is straightforward. Since in the downstream market all firms are equally efficient, how output is shared between them is of no consequence to welfare because productive efficiency is assured. Thus, the only requirement for allocational optimality is that prices faced by consumers are aligned with social costs, i.e. the regulator prefers marginal-cost pricing to prevail. First impressions would suggest that the regulator simply has to correctly set the access charge to achieve this *desideratum*. However, there are circumstances where the pair of regulatory instruments (q_1, p_a) is insufficient for the social optimum to be achieved.[14]

Proposition 1 *A necessary and sufficient condition for instrument insufficiency is*

$$c_1 < \frac{c + c_1}{-\eta_2 n}. \tag{3}$$

Proof Take the first-order conditions in (1) and add them to obtain $(n+1)p_2 - (n+1)c = np_a + c_1 - \frac{\partial p_2}{\partial q} q$. Comparing this expression with the first-order condition of the regulator's problem, $(n+1)p_2 - (n+1)c = (n+1)c_1$, yields that achieving the social welfare optimum through (q_1, p_a) implies setting p_a as follows:

$$p_a^* = c_1 + \frac{q^*}{n} \frac{\partial p_2}{\partial q}\bigg|_{q=q^*} \Leftrightarrow p_a^* = c_1 + \frac{p_2^*}{\eta_2 n} \Leftrightarrow p_a^* = c_1 + \frac{c + c_1}{\eta_2 n}. \tag{4}$$

Note that since decreases in p_a lead to an increase in q, an outcome desired by the regulator in view of the social insufficiency of market output under imperfect competition, the regulator uses p_a, setting it below the marginal cost of the associated input, c_1, to achieve the social optimum.

Instrument insufficiency arises if p_a^* is negative, an access price that, if implemented, would turn firm I into a "money pump," since non-integrated firms would then unboundedly increase their profits by buying and disposing of infinite quantities of the common input. This outcome obviously is not socially optimal, leading as it does to resource waste. Therefore, we conclude that there are cases in which p_a is not a sufficient

[14] With independent demands and constant marginal costs, the two markets are totally separate, and the question is really whether each instrument is sufficient for achieving optimality in the market it affects. This no longer will be the case in latter sections, when the optimal choice of the two regulatory instruments will have to be calculated jointly.

instrument for achieving social optimality, since the market structure is such that the desirable regulatory outcome cannot be attained. □

Simple computations make clear that, for reasonable values of the demand elasticity, instrument insufficiency ensues. In fact, consider $\eta_2 = -1$ and $n = 1$ and check that the inequality above is satisfied.

Thus, the higher is p_a^*, the higher is the likelihood of the pair of regulatory instruments (q_1, p_a) being sufficient for social optimality in resource allocation. We thus equate higher values of p_a^* with better regulatory scenarios from a sufficiency standpoint.

The effect of the degree of competition on the likelihood of instrument insufficiency deserves to be emphasized. In fact, as $n \to \infty$, the inequality in Proposition 1 becomes unfulfilled and instrument insufficiency is no longer a possibility. Why? As the number of competitors in the mobile market increases, competition also increases and output approaches the first-best quantity associated with marginal-cost pricing. Thus, p_a^* becomes higher since the inefficiency associated with a low output is less. This, in turn, has the desirable consequence of making it more likely that (q_1, p_a) are sufficient for attaining allocational optimality.

The demand elasticity affects the sufficiency of the regulatory instruments in a comparable fashion: the more inelastic demand is, the higher is the likelihood of instrument insufficiency, as (3) makes clear.[15]

How should the optimal access price be set as n, and therefore the competitiveness of the outcome of the game played in the downstream market, increases?

Proposition 2 *The socially-optimal access charge approaches marginal-cost pricing as the number of downstream firms increases:*

$$p_a^* \to c_1 \text{ as } n \to \infty.$$

As the number of firms in the downstream market increases and output approaches the socially optimal level, the regulatory necessity for keeping the access charge below the marginal cost of the corresponding input becomes also smaller. Note that this result (and similar ones that follow) is *not* predicated on the assumption of Cournot play. Rather, it depends solely on q increasing and approaching the socially optimal level with n, thus making it less necessary for the access charge to be used to force output in the downstream market to increase. This feature is shared by many other solution concepts, Bertrand being perhaps the illustrative example springing to mind. In this case, full competitiveness is achieved with $n \geq 2$ and thus marginal-cost pricing in both the two output markets and the input (bottleneck) market becomes optimal with as few as two firms. Therefore, except for the element of realism concerning the telecommunications market it introduced in the analysis, the assumption of Cournot competition was immaterial, being rather a proxy for capturing the crucial feature: increased competitiveness as a result of increases in the number of downstream firms.

[15] Parker and Röller (forthcoming) estimate the demand elasticity for mobile calls as being equal to -2.5.

4 Efficiency Differences in the Downstream Market

We will now consider the possibility of the two firms differing in their cost of the non-common input, i.e., that their efficiencies differ. This feature is of interest because efficiency differences between the mobile firm owned by the fixed-link provider and its competitors can arise for several reasons: (i) the fixed-link provider's "deep pockets" can support a lower efficiency of its integrated mobile provider; (ii) technological or marketing knowledge possessed by the fixed-link provider can be made available to its mobile firm, thus making it more efficient. These effects influence cost differences in opposite ways. We thus accommodate all possible cost differences and show how they affect the optimal regulatory policy.[16]

Lemma 3 *Attaining the social-welfare maximizing allocation requires a price in excess of marginal cost in the mobile market when the integrated firm is more efficient ($c_2 < c$), and marginal-cost pricing when the non-integrated firms are equally or more efficient ($c_2 \geq c$):*

$$\begin{cases} p_1^* = c_1 \\ p_2^* = c + c_1 + (c_2 - c)\frac{\frac{dq_2}{dp_a}}{\frac{dq}{dp_a}} \end{cases} \text{if } c_2 < c, \qquad \begin{cases} p_1^* = c_1 \\ p_2^* = c + c_1 \end{cases} \text{if } c_2 \geq c.$$

This result is again easy to interpret. When firm I is more efficient than its competitors ($c_2 < c$), decreasing p_a in order to increase q, though increasing total output towards the social optimum, has the drawback of inducing an increase in the output of the less-efficient firms (through the lowering of an input cost) and decreasing that of the more-efficient firm (due to increasingly aggressive competition on the part of the non-integrated firms). Thus, the social optimum in the mobile market involves a higher price and a lower output than when all firms are equally efficient.[17][18] The size of the divergence is proportional to the difference between c_2 and c, and how sensitive q and q_2 are to changes in p_a, since the latter determine the efficiency of p_a^* in affecting output shares in the mobile market.

However, setting $p_2 = c + c_1$ is still optimal when $c_2 > c$ since it yields marginal-cost pricing *and* leads the less-efficient integrated firm to leave the mobile market (price falling short of its marginal cost, $c_1 + c_2$). Marginal-cost pricing thus prevails in this case.

How do efficiency differences affect the likelihood of the regulatory instruments being sufficient for social optimality?

[16]Colombo and Valetti (1996) argue that "... integration between cellular and fixed network operators permits a more efficient use of both tangible and intangible assets... ", this being another reason why differences in efficiency are to be expected.

[17]Another way of seeing this is by noting that were $p_2 = c + c_1$ and an infinitesimal increase in p_a would lead to (i) a decrease in nq_3 with no impact on welfare since willingness to pay, p_2, equals the marginal cost of non-integrated firms, $c + c_1$, and (ii) an increase in q_2 increasing welfare be $c - c_2$ per unit of change in q_2.

[18]Laffont and Tirole (1996, p. 242) obtain a contrasting result when dealing with the case where the services of the integrated and a single non-integrated mobile operator are not perfect substitutes, and demands are linear. Their result is a direct consequence of linear demand inducing cost absorption, i.e., cost differences are less than reflected in price differences. The access charge is then set such that the cost advantage in favor of the integrated firm is optimally "shared" between markets.

Proposition 3 *A sufficient condition for instrument insufficiency is*

$$c_1 < \frac{c+c_1}{-\eta_2 n} + \frac{c-c_2}{n}\left[\left(\frac{1}{\eta_2}+n+1\right)\frac{\frac{dq_1}{dp_a}}{\frac{dq}{dp_a}} - 1\right] \text{ if } c_2 < c, \qquad c_1 < \frac{c+c_1}{-\eta_2 n} \text{ if } c_2 \geq c.$$

When I is more efficient than its opponent ($c > c_2$) instrument insufficiency is less likely since cumbersome algebra reveals the expression in square brackets to be negative, thus rendering the inequality less likely to be fulfilled. Thus, since one of the cases of cost asymmetry is identical to that treated in Proposition 1, and the other yields a case more favorable to sufficiency, we can unequivocally conclude that instrument sufficiency is more likely when cost asymmetry is a possibility.

We can now characterize the optimal setting of the access charge as the number of firms in the downstream market increases.

Proposition 4 *The socially-optimal access charge exceeds marginal-cost pricing if the integrated firm is more efficient ($c_2 < c$), and approaches marginal-cost pricing if the non-integrated firms are equally or more efficient ($c_2 \geq c$), as the number of downstream firms increases:*

$$p_a^* \to c_1 - (c-c_2)\frac{\frac{dq_1}{dp_a}}{\frac{dq}{dp_a}} > c_1 \text{ if } c_2 < c; \qquad p_a^* \to c_1 \text{ as } n \to \infty \text{ if } c_2 \geq c.$$

The rationale for an access charge in *excess* of marginal cost when $c < c_2$ is of interest. Increased competition in the downstream market insures that output approaches its first-best quantity. Thus, insufficient output in the mobile market becomes less of a concern as $n \to \infty$. The access charge is now set *above* marginal cost, c_1, to steer production away from the less-efficient non-integrated producers and towards the more-efficient integrated firm.

5 Monopolist Losses: Break-even Constraint

The problem analyzed in Section 3 shows that the regulator would like to price the fixed-link calls sold directly to the public at marginal cost and set p_a below c_1, features that would induce a loss to firm I. However, a stylized fact of regulatory activity is that losses are typically avoided by regulatory authorities. Thus, we now analyze the problem including firm I's break-even constraint in the regulator's problem. For simplicity and comparability of results, we still make use of the simplifying assumption that both firms are equally efficient in the production of the non-common input.[19]

Lemma 4 *Attaining the social-welfare maximizing allocation requires Ramsey pricing:*

[19]The effect of efficiency differences in this model is well understood and can be added if needed by analysis of an all-encompassing version of the model. We do not do so for expositional reasons.

$$\begin{cases} \frac{p_1-c_1}{p_1} = -\frac{\lambda}{1+\lambda}\frac{1}{\eta_1}, \\ \frac{p_2-(c+c_1)}{p_2} = -\frac{\lambda}{1+\lambda}\frac{1}{\eta_2} + \frac{\lambda}{1+\lambda}\frac{q_1}{p_2}\left[-2\frac{\partial p_2}{\partial q} - q_2\frac{\partial^2 p_2}{\partial q^2} - 2\frac{\partial p_2}{\partial q} - q\frac{\partial^2 p_2}{\partial q^2}\right]. \end{cases}$$

Since the term in square brackets is positive (see proof of Lemma 1), this result allows us to conclude that marginal-cost pricing is abandoned in both markets in favor of Ramsey-type prices. The price in the first market is pure-Ramsey, i.e., is given be the inverse-elasticity rule, whereas the second market is further distorted beyond pure Ramsey pricing, a result contrasting sharply with those obtained elsewhere in the literature. The rationale for this apparently extreme bias is important. Were the regulatory instruments to affect each market directly (say, by having the regulatory authority set quantities or prices), pure Ramsey pricing would be obtained, as is usually the case in the literature. However, p_a affects the output in the mobile market only indirectly (being the cost of an input) and, crucially, does so by imposing further distortion on the fixed-link market (increased losses for firm I due to a lower access charge require an increased price in the fixed-link market). Therefore, the indirect nature of the regulatory instrument, p_a, together with its effect on fixed-link market profits, makes a further distortion in the mobile market socially optimal.

How does the imposition of the non-negative profit constraint affect the likelihood of instrument insufficiency?

Proposition 5 *The imposition of the non-negative profit constraint reduces the likelihood of instrument insufficiency.*

This result should be read with care. It implies that instrument insufficiency is less likely to be a problem when regulatory authorities take into consideration the possible losses that setting the access charge in an optimal manner may generate. However, it would be hasty to conclude that welfare is higher because of this. In fact, the opposite is true. Note that the regulatory authority is now operating under a new restriction, and that restriction is active, as pointed out above. Welfare is therefore strictly lower in this case. The proposition above should thus be interpreted as pointing a desirable side effect, in terms of likelihood of instrument insufficiency, of imposing an additional (active and welfare-reducing) constraint on the activities of the regulatory authority.[20]

Again, increased competition in the mobile market has several important implications.

Lemma 5 *Pure Ramsey pricing in the downstream market is approached as the number of firms increases.*

This result is not hard to understand. The further distortion of the downstream market associated with the second term on the right-hand side of its Lerner index in Lemma 4 results from the use of an indirect and

[20] This again makes clear that our problem cannot be reduced to simply counting instruments versus objectives since introducing a new binding regulatory objective *decreases* the likelihood of instrument insufficiency.

imperfect policy instrument, the access charge. As the number of firms increases, heightened competition permits a lesser use of this instrument and the corresponding reduction in the extra distortionary term associated with it.

Importantly, marginal-cost pricing is approached in both output markets as competition in the mobile market increases.

Proposition 6 *Marginal-cost pricing in the output markets is approached as the number of firms increases:*

$$\begin{cases} p_1^* \to c_1 \\ p_2^* \to c + c_1 \end{cases} \quad \text{as } n \to \infty.$$

As n increases, output in the mobile market converges to the first-best optimum $q(c + c_1)$. The regulator can thus increase p_a, the regulatory instrument used to partially correct for the mobile-market distortion. This, in turn, alleviates the profit constraint, thus allowing λ to converge to 0. *Ergo*, prices in *both* output markets converge to marginal cost due to increased competition in the mobile market (under optimal setting of the regulatory instrument pair), a first-best outcome from a welfare viewpoint.

It should be clear by now that marginal-cost pricing of the bottleneck input is also approached as the number of firms in the downstream market increases, since the need for setting it below marginal cost correspondingly decreases.

Proposition 7 *The socially-optimal access charge approaches marginal-cost pricing as the number of downstream firms increases:*

$$p_a^* \to c_1 \text{ as } n \to \infty.$$

Thus, we conclude that marginal-cost pricing in all three markets and a first-best allocational outcome is approached when the number of firms increases.

6 Divestiture

One form of liberalization policy often discussed has the fixed-link provider divest itself of the mobile firm it controls. We model this policy by studying a model involving two markets and $n + 2$ players. The fixed-link provider monopolistically operates in its market and provides access to the fixed link to $n + 1$ non-integrated mobile operators. In the previous section we had n non-integrated mobile providers plus another (integrated) mobile firm, for a total of $n + 1$ competitors. We will continue to assume the number of firms in the mobile market to equal $n + 1$ in order to ensure comparability of results, since we thus keep the degree of competition in the downstream market equal in both the original model and this one involving divestiture.

The downstream firms' profits can be written as

$$\pi^{I} = p_1 q_1 + p_a \sum_{i=2}^{n+2} q_i - c_1 (q_1 + \sum_{i=2}^{n+2} q_i)$$

$$\pi^{E_i} = p_2 q_i - p_a q_i - c q_i,\ i \in \{2, \ldots, n+2\}.$$

The game played in the mobile market yields the following first-order conditions:

$$\max_{q_i} \pi^{E_i} \Leftrightarrow p_2 - p_a - c + \frac{\partial p_2}{\partial q_i} q_i = 0 \Leftrightarrow \frac{p_2 - (p_a + c)}{p_2} = -\frac{1}{\eta_2 (n+1)}, \tag{5}$$

where $q \equiv \sum_{i=2}^{n+2} q_i$. For the reasons given in Section 2, we will again assume symmetry, thus writing $q = (n+1) q_3$.

Adding up the first-order conditions we obtain $(n+1)p_2 - (n+1)p_a - (n+1)c = -\frac{\partial p_2}{\partial q} q$. Differentiating this expression yields

$$\frac{dq}{dp_a} = \frac{n+1}{(2+n)\frac{\partial p_2}{\partial q} + q \frac{\partial^2 p_2}{\partial q^2}}. \tag{6}$$

Assumption 1 is enough to sign this derivative, yielding $\frac{dq}{dp_a} < 0$.

As before, we begin by characterizing the pricing to be followed for achieving a socially-optimal resource allocation.

Lemma 6 *Attaining the social-welfare maximizing allocation requires marginal-cost pricing:*

$$\begin{cases} p_1^* = c_1 \\ p_2^* = c + c_1. \end{cases}$$

This result is not surprising since the objectives to be achieved by the regulatory authority are the same as in Section 3. However, the market structure is now different and, crucially, this changes the effectiveness of the regulatory instrument. Thus, we must again ask whether the pair of regulatory instruments (q_1, p_a) is sufficient for achieving the social optimum, and compare this case to that without divestiture.

Proposition 8 *A necessary and sufficient condition for instrument insufficiency is*

$$c_1 < \frac{c + c_1}{-\eta_2 (n+1)}. \tag{7}$$

Direct comparison with Proposition 1 shows that the inequality in Proposition 8 is now less likely to be fulfilled, allowing us to conclude that the social optimum is more likely to be achieved through optimal setting of (q_1, p_a) in the model with divestiture than in the original model. The difference between the right-hand sides of the expressions (3) and (7) can easily be seen to equal $\frac{c+c_1}{-\eta_2 (n^2+n)}$. Thus, it is inversely related to the number of competitors in the downstream market. This is easy to understand: the regulatory authorities affect the output in the mobile market by influencing the price of an input to all firms with divestiture, and all but one in the absence of divestiture; this difference becomes less important as n increases. Moreover, the difference is governed by the term in n^2, showing that the increase in regulatory instruments' sufficiency brought about by divestiture becomes quite small for relatively low values of n. This argument can be interpreted as suggesting that liberalization by divestiture is less useful from the viewpoint of instrument sufficiency than increasing n since, for a relatively small number of competitors in the mobile market, the optimally-set access charges in the models with and without divestiture are very similar.

When efficiency differences in the original model are taken into consideration, the difference between the two variants of the model becomes smaller when c exceeds c_2, i.e., the non-integrated firms are less efficient than the integrated one, as comparing Propositions 3 and 8 makes clear. Again, this result is easy to understand: in this case, p_a^* in the model without divestiture becomes higher, thus making it more likely that it (jointly with q_1) will be sufficient for allocational optimality to be achieved. This, in turn, renders the regulatory advantage of divestiture even less pronounced.

As before, marginal-cost pricing implies setting p_a below c_1, thus inducing a loss for firm I. Thus, we now analyze the problem including firm I's break-even constraint on the regulator's problem, and compare the results with those obtained in the previous section.

Lemma 7 *Attaining the social-welfare maximizing allocation requires Ramsey pricing:*

$$\begin{cases} \frac{p_1-c_1}{p_1} = -\frac{\lambda}{1+\lambda}\frac{1}{\eta_1}, \\ \\ \frac{p_2-(c+c_1)}{p_2} = -\frac{\lambda}{1+\lambda}\frac{1}{\eta_2} + \frac{\lambda}{1+\lambda}\frac{q_2}{p_2}\left[-2\frac{\partial p_2}{\partial q} - q\frac{\partial^2 p_2}{\partial q^2}\right]. \end{cases}$$

The term in square brackets is positive (see proof of Lemma 1). Once more, we conclude that marginal-cost pricing is abandoned in both markets in favor of Ramsey-type prices, the first market being priced in accordance with the inverse-elasticity rule, and the second being further distorted. The rationale for this conclusion is the same as before.

How do the Lerner indices for the downstream market in the original and divestiture models compare?

Proposition 9 *With divestiture, the downstream market is relatively less distorted vis-à-vis the upstream market.*

This result needs to be understood. Though still an indirect and imperfect regulatory instrument, access-pricing regulation is now more effective since all firms are affected by the regulatory instrument whereas without divestiture one was not. Insofar as the regulatory influence over the downstream market is more efficient with divestiture, the relative distortion of the two markets is now closer to pure Ramsey pricing.

How does the model with divestiture compare with the original model from a welfare viewpoint? Given the nature of the problems, the question comes down to which of the two market structures yields a less restrictive break-even constraint, and thus has less-distorted markets. Divestiture has three effects: (i) it eliminates firm I's profit from its mobile provider, (ii) it creates another firm buying access below marginal cost and thus inducing an additional loss for firm I, and (iii) increases output in the mobile market for a given p_a since all firms obtain access below marginal cost in the model with divestiture, whereas in the model without divestiture the vertically integrated firm obtains access at marginal cost. This allows for an increase in p_a^*, since the output distortion is now smaller, which reduces firm I's losses of providing access.

The net effect on firm I's profit constraint is ambiguous, since effects (i) and (ii) make the profit constraint more binding under divestiture, whereas (iii) has the opposite effect. To see why intuitively, consider the difference between equations (A3) and (A6) at a common (q_1, q). After some manipulation, we obtain

$$(p_a^* - \tilde{p}_a^*)q + (p_2 - p_a^* - c)q_2,$$

where $\tilde{p}_a$ (p_a, respectively) is the access charge in the model with (without, resp.) divestiture, and the asterisks denote the optimal values of the access charge at the common quantities. When demand and cost conditions are such that q_2 is small, effects (i) and (ii) are also small as the second term of the last equation reveals. To put it another way, removing the integrated firm has a negligible impact on the mobile market's equilibrium. It is then effect (iii), associated with $(p_a^* - \tilde{p}_a^*)q$, that predominates, thus indicating that the profit constraint is less binding with divestiture. In this case, social welfare increases with divestiture.

Finally, note that the results obtained in Sections 3 and 5 concerning the effects of increased competition in the downstream market apply to the case of divestiture, *mutatis mutandis*. In an attempt to conserve space, we will thus not restate such results in this section.

7 Interdependent Demands

Up to now, we have assumed that demands are independent, i.e., that consumers' willingness to pay for one service is not affected by the quantity of the other service consumed. This assumption is clearly unrealistic in telecommunications markets. For instance, consumers with a town residence and a weekend house can substitute away from having two fixed-link connections in favor of a single fixed-link one plus a mobile phone. The latter ensures coverage of the secondary location plus the added convenience of portability and coverage while on the move.[21] This behavior induces substitutability between fixed and mobile services. Network externalities can also induce interdependent demands. This feature, on the other hand, gives birth to complementarity between services. Interdependence in demand is thus of the essence in this inquiry.[22]

We are, at this stage, adding exclusively the relationship between the two markets through demand. We have already accounted for another type of relationship arising through technology: calls originated in the

[21] This is recognized by Laffont and Tirole (1996, p. 241) when they write: "Suppose further that mobile communications do not compete with local calls (this might be a rough approximation of the current situation, although this assumption will become quite unrealistic when the price of mobile calls plummets)."

[22] See Baumol and Sidak (1994, p. 197) on this matter.

mobile network have to be routed through the fixed one. The structure of the model took this complementarity into account from the outset.

Formally, demand is now expressed as $p_i(q_1, q), i = 1, 2$. Thus, willingness to pay for each unit of either service is also a function of the quantity consumed of the other. In the following, we restrict attention to the original version of the model. Comparison with the model with divestiture is straightforward, even if at times computationally cumbersome.

The firms' profits can be written as

$$\pi^I = p_1(q_1, q)q_1 + p_2(q_1, q)q_2 + p_a \sum_{i=3}^{n+2} q_i - c_1(q_1 + q_2 + \sum_{i=3}^{n+2} q_i) - c_2 q_2$$

$$\pi^{E_i} = p_2(q_1, q)q_i - p_a q_i - c q_i, \qquad i \in \{3, \dots, n+2\}.$$

First-order conditions in the two markets now simultaneously determine their equilibrium,

$$\begin{cases} \max\limits_{q_1} \pi^I \\ \max\limits_{q_2} \pi^I \\ \max\limits_{q_i} \pi^{E_i} \end{cases} \Leftrightarrow \begin{cases} p_1 - c_1 + \frac{\partial p_2}{\partial q_1} q_2 + \frac{\partial p_1}{\partial q_1} q_1 = 0 \\ p_2 - c_1 - c_2 + \frac{\partial p_1}{\partial q} q_1 + \frac{\partial p_2}{\partial q} q_2 = 0 \\ p_2 - p_a - c + \frac{\partial p_2}{\partial q} q_i = 0. \end{cases} \tag{8}$$

Changes in p_a now affect not only q_2 and q_3, but also q_1.

We will now make two simplifying assumptions that will prove useful as we continue. Their rationale will be carefully discussed. Without them the algebra would be too cumbersome, while their assumption will be seen not to preclude the discussion of effects stemming from interrelated demands.

Assumption 2 *Demand $p_i(q_1, q), i = 1, 2$ is such that:*

$$\frac{\partial^2 p_i}{\partial q_i \partial q_j} = 0, \qquad \frac{\partial^2 p_i}{\partial q_j^2} = 0, \qquad i \neq j.$$

We assume that an increase in the quantity consumed of one service induces a "parallel" shift in the other service's demand. More precisely, the change in willingness to pay for one service brought about by a given variation in the quantity consumed of the other is assumed to be uniform in the quantity of the former. This assumption is tantamount to requiring parallel shifts in the case of linear demand, indeed being a generalization of that idea to more general demand curves. Figure 1 graphically illustrates the assumption, where segments A and B have the same length. Formally, it implies $\frac{\partial^2 p_i}{\partial q_i \partial q_j} = 0$.

We also assume that the change in the willingness to pay for one service brought about by a given change in the quantity consumed of the other is independent of the quantity initially consumed of the latter. Graphically, this is illustrated in figure 1 where segments A and C have the same length. This assumption implies $\frac{\partial^2 p_i}{\partial q_j^2} = 0$.

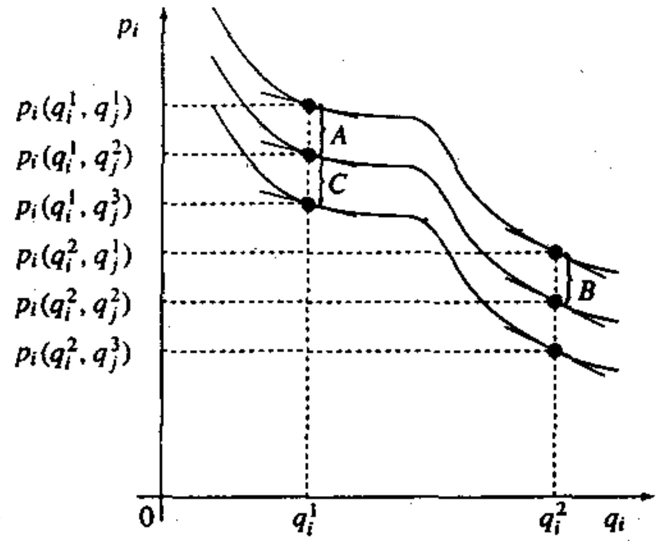

FIGURE 1. *Assumption 2: "parallel" shift of demand* ($i \neq j, q_j^1 - q_j^2 = q_j^2 - q_j^3$).

The motivation for these assumptions is the following: since we are only interested in how substitutability or complementarity between markets affects firms' behavior, optimal regulatory policies, and actual outcomes, we considerably simplify the analysis by assuming that cross-demand effects are uniform across quantities. The aforementioned simplification immediately becomes apparent when we totally differentiate (8) to obtain

$$\begin{bmatrix} 2\frac{\partial p_1}{\partial q_1} + q_1\frac{\partial^2 p_1}{\partial q_1^2} & \frac{\partial p_1}{\partial q} + \frac{\partial p_2}{\partial q_1} & \frac{\partial p_1}{\partial q} \\ \frac{\partial p_1}{\partial q} + \frac{\partial p_2}{\partial q_1} & 2\frac{\partial p_2}{\partial q} + q_2\frac{\partial^2 p_2}{\partial q^2} & \frac{\partial p_2}{\partial q} + q_2\frac{\partial^2 p_2}{\partial q^2} \\ n\frac{\partial p_2}{\partial q_1} & n\frac{\partial p_2}{\partial q} + nq_3\frac{\partial^2 p_2}{\partial q^2} & (1+n)\frac{\partial p_2}{\partial q} + nq_3\frac{\partial^2 p_2}{\partial q^2} \end{bmatrix} \begin{bmatrix} \mathrm{d}q_1 \\ \mathrm{d}q_2 \\ n\mathrm{d}q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ n \end{bmatrix} \mathrm{d}p_a,$$

from which we get

$$\frac{\mathrm{d}q_1}{\mathrm{d}p_a} = \frac{n\frac{\partial p_2}{\partial q_1}\left(\frac{\partial p_2}{\partial q} + q_2\frac{\partial^2 p_2}{\partial q^2}\right) - n\frac{\partial p_1}{\partial q}\frac{\partial p_2}{\partial q}}{\Delta}$$

$$\frac{\mathrm{d}q_2}{\mathrm{d}p_a} = \frac{n\frac{\partial p_1}{\partial q}\left(\frac{\partial p_1}{\partial q} + \frac{\partial p_2}{\partial q_1}\right) - n\left(2\frac{\partial p_1}{\partial q_1} + q_1\frac{\partial^2 p_1}{\partial q_1^2}\right)\left(\frac{\partial p_2}{\partial q} + q_2\frac{\partial^2 p_2}{\partial q^2}\right)}{\Delta}$$

$$\frac{\mathrm{d}q_3}{\mathrm{d}p_a} = \frac{\left(2\frac{\partial p_1}{\partial q_1} + q_1\frac{\partial^2 p_1}{\partial q_1^2}\right)\left(2\frac{\partial p_2}{\partial q} + q_2\frac{\partial^2 p_2}{\partial q^2}\right) - \left(\frac{\partial p_1}{\partial q} + \frac{\partial p_2}{\partial q_1}\right)^2}{\Delta}$$

$$\frac{\mathrm{d}q}{\mathrm{d}p_a} = \frac{n\frac{\partial p_2}{\partial q}\left(2\frac{\partial p_1}{\partial q_1}+q_1\frac{\partial^2 p_1}{\partial q_1^2}\right)-n\frac{\partial p_2}{\partial q_1}\left(\frac{\partial p_1}{\partial q}+\frac{\partial p_2}{\partial q_1}\right)}{\Delta},$$

where

$$\begin{aligned}\Delta = & \left(2\frac{\partial p_1}{\partial q_1}+q_1\frac{\partial^2 p_1}{\partial q_1^2}\right)\left(\left[(2+n)\frac{\partial p_2}{\partial q}+q\frac{\partial^2 p_2}{\partial q^2}\right]\frac{\partial p_2}{\partial q}\right)-\left[\frac{\partial p_1}{\partial q}\right]^2\frac{\partial p_2}{\partial q}- \\ & -2(1+n)\frac{\partial p_1}{\partial q}\frac{\partial p_2}{\partial q_1}\frac{\partial p_2}{\partial q}-\left[\frac{\partial p_2}{\partial q_1}\right]^2\frac{\partial^2 p_2}{\partial q^2}n(q_3-q_2)-\left[\frac{\partial p_2}{\partial q_1}\right]^2\frac{\partial p_2}{\partial q}- \\ & -\frac{\partial p_2}{\partial q_1}\frac{\partial p_1}{\partial q}\frac{\partial^2 p_2}{\partial q^2}nq_3.\end{aligned}$$

Note that the total effect of a change in p_a on quantities is now the result of a direct effect, represented by the terms without cross derivatives and associated with the direct impact that changes on p_a have on the quantities transacted by the downstream firms, and a cross effect, represented by terms where cross derivatives are present and associated with further changes in quantities transacted resulting from demand interdependencies. Obviously, the total derivatives above collapse to those in the proof of Lemma 1 when cross effects are nil. Direct observation reveals that the signs of the derivatives above can differ from those in Lemma 1 if cross effects are strong enough when compared to direct effects.

Without loss of generality, let units of measurement be chosen such that $\left|\frac{\partial p_1}{\partial q}\right| = \frac{\partial p_2}{\partial q_1}$. Even though we will not explicitly use this simplification in the following, the experimenting reader will note significant simplifications when manipulating expressions. Two cases (exhausting all possibilities) are of interest: (i) $\mathrm{sign}\frac{\partial p_1}{\partial q} = \mathrm{sign}\frac{\partial p_2}{\partial q_1}$, in which case the two services can unequivocally be classified as *substitutes* if $\frac{\partial p_i}{\partial q_j} < 0, i \neq j$ or *complements* if $\frac{\partial p_i}{\partial q_j} > 0, i \neq j$, and (ii) $\mathrm{sign}\frac{\partial p_1}{\partial q} \neq \mathrm{sign}\frac{\partial p_2}{\partial q_1}$, in which case such classification is no longer clear-cut.

Optimality in resource allocation requires output pricing to be as follows:

Lemma 8 *Attaining the social-welfare maximizing allocation requires the following output pricing:*

$$\begin{cases} p_1^* = c_1 - \int_0^q \frac{\partial p_2}{\partial q_1}\mathrm{d}q \\ p_2^* = c + c_1 - \int_0^{q_1} \frac{\partial p_1}{\partial q}\mathrm{d}q_1. \end{cases}$$

We encounter a new reason for departing from marginal-cost pricing. If demands are interrelated, i.e., $\frac{\partial p_i}{\partial q_j} \neq 0, i \neq j$, a unitary increase in the consumption of one service changes the welfare derived from the consumption of the other by $\int_0^{q_i} \frac{\partial p_i}{\partial q_j}\mathrm{d}q_i$. Internalizing such externalities thus recommends prices that differ from marginal costs by exactly those amounts.

How does sufficiency of regulatory instruments compares with the case of independent demands?

Proposition 10 *The likelihood of instrument insufficiency of the access charge increases (decreases, resp.) vis-à-vis the case of independent demands iff*

$$\frac{\partial p_1}{\partial q} > (<) 0.$$

Instrument insufficiency of the fixed-link price occurs iff

$$\frac{\partial p_2}{\partial q_1} q > c_1.$$

The rationale behind this result is simple and important. If the use of mobile phones increases the usefulness of fixed-link network services (through increased demand, but not as a result of routing requirements), then a lower price for mobile calls is socially optimal *vis-à-vis* the case where demands are independent. This implies a higher quantity consumed that can only be effected through a lower optimally-set access charge to be paid by the non-integrated mobile service providers. This adds another reason for setting the access charge below marginal cost, thus making it even less likely that (q_1, p_a) are sufficient regulatory instruments for achieving allocational efficiency. Instrument insufficiency concerning the *fixed-link* market is now also a possibility. If there is a positive externality caused by the fixed-link market on the mobile one, and such externality is strong enough, than a negative price for the fixed-link may become optimal. Consumers could then unboundedly collect money by demanding infinite quantities of the service. This would obviously lead to wasteful overuse of the fixed-link, a socially inferior result.

Expression (A8) is obtained in the proof of Proposition 10:

$$p_a^* = c_1 + \frac{p_2^*}{\eta_2 n} + \frac{\frac{\partial p_1}{\partial q} q_1 - (n+1) \int_0^{q_1} \frac{\partial p_1}{\partial q} \mathrm{d}q_1}{n}.$$

The numerator of the last term is essentially composed of two terms: (i) the first takes into account that the integrated mobile provider does consider the cross-demand effect between markets insofar as the latter affects its own revenue through the willingness to pay of the *marginal* consumer, whereas (ii) the second term considers the (total) cross effect on *all* consumers (both marginal and inframarginal). Therefore, (ii) net of (i) determines the optimal correction to the access charge resulting from consideration of interdependent demands. This phenomenon in the context of optimal access charges also underlies the work of Spence (1975, 1976) in the context of models with quality choice.

The presence of interdependent demands also strongly affects the optimal regulatory setting of the access charge when the number of firms increases.

Proposition 11 *The socially-optimal access charge approaches the following limit as the number of downstream firms increases:*

$$p_a^* \to c_1 - \frac{\partial p_1}{\partial q} q_1.$$

Note that increased competition in the downstream market is no longer enough to ensure both convergence of the optimal access charge to marginal-cost pricing and the concomitant guarantee of instrument sufficiency. This is so because increased competitiveness in the downstream market does not ensure the internalization of externalities across markets. Instead, this task has still to be performed by the regulatory instruments. Insofar as an externality is positive and pronounced *vis-à-vis* the marginal cost of carrying call through the fixed link, the likelihood of instrument insufficiency becomes again positive, even when the downstream market is extremely competitive.

7.1 Efficiency Differences in the Downstream Market

Lemma 9 *Attaining the social-welfare maximizing allocation requires the following output pricing:*

$$\begin{cases} p_1^* = c_1 - \int_0^q \frac{\partial p_2}{\partial q_1} \mathrm{d}q \\ p_2^* = c + c_1 - \int_0^{q_1} \frac{\partial p_1}{\partial q} \mathrm{d}q_1 + (c_2 - c)\frac{\frac{\mathrm{d}q_2}{\mathrm{d}p_a}}{\frac{\mathrm{d}q}{\mathrm{d}p_a}} \end{cases} \text{if } c_2 < c, \qquad \begin{cases} p_1^* = c_1 - \int_0^q \frac{\partial p_2}{\partial q_1} \mathrm{d}q \\ p_2^* = c + c_1 - \int_0^{q_1} \frac{\partial p_1}{\partial q} \mathrm{d}q_1 \end{cases} \text{if } c_2 \geq c.$$

The interpretation of and intuitions behind these equations are the same as in Section 4, except that

$$\frac{\frac{\mathrm{d}q_2}{\mathrm{d}p_a}}{\frac{\mathrm{d}q}{\mathrm{d}p_a}} = \frac{n\frac{\partial p_1}{\partial q}\left(\frac{\partial p_1}{\partial q} + \frac{\partial p_2}{\partial q_1}\right) - n\left(2\frac{\partial p_1}{\partial q_1} + q_1\frac{\partial^2 p_1}{\partial q_1^2}\right)\left(\frac{\partial p_2}{\partial q} + q_2\frac{\partial^2 p_2}{\partial q^2}\right)}{n\frac{\partial p_2}{\partial q}\left(2\frac{\partial p_1}{\partial q_1} + q_1\frac{\partial^2 p_1}{\partial q_1^2}\right) - n\frac{\partial p_2}{\partial q_1}\left(\frac{\partial p_1}{\partial q} + \frac{\partial p_2}{\partial q_1}\right)}$$

may no longer be negative (as it was in Section 4), its sign now depending on the relative strength of the direct and cross effects. This obviously can affect the optimal departure from marginal-cost pricing: a lower efficiency of the non-integrated firms no longer implies that p_2^* should exceed $c + c_1 - \int_0^{q_1} \frac{\partial p_1}{\partial q} \mathrm{d}q_1$ when $c_2 < c$, as was the case in Section 4. The relative strength of the direct versus cross effects determines the optimal policy. This, in turn, affects the likelihood of regulatory instrument insufficiency. If $\frac{\frac{\mathrm{d}q_2}{\mathrm{d}p_a}}{\frac{\mathrm{d}q}{\mathrm{d}p_a}} > 0$, since lower optimal values of p_2 are effected through lower access charges, the likelihood of insufficiency is now increased by the consideration of efficiency differences in the downstream market. Thus, the conclusion that efficiency differences decreased the likelihood of instrument insufficiency reached when assuming independent demands, is no longer necessarily true with interdependent demands.[23]

7.2 Monopolist's Losses: Break-even Constraint

Suppose that the optimal regulatory policy involves losses for the monopolist. Then,

[23] Another way of seeing this is by noting that, for the case $c_2 < c$, were $p_2 = c + c_1 - \int_0^{q_1} \frac{\partial p_1}{\partial q} \mathrm{d}q_1$ and an infinitesimal variation in p_a would lead to (i) a variation in nq_3 with no impact on welfare since willingness to pay plus the external effect, $p_2 + \int_0^{q_1} \frac{\partial p_1}{\partial q} \mathrm{d}q_1$, equals the marginal cost of non-integrated firms, $c + c_1$, and (ii) a variation in q_2 increasing welfare be $c - c_2$ per unit of *increase* in q_2. If $\frac{\frac{\mathrm{d}q_2}{\mathrm{d}p_a}}{\frac{\mathrm{d}q}{\mathrm{d}p_a}} > 0$ then q and q_2 move in tandem in response to changes in the access price. Therefore, increasing q_2 requires an increase in q brought about by a lower access charge. This increases the likelihood of instrument insufficiency.

Lemma 10 *Attaining the social-welfare maximizing allocation requires the following output pricing:*

$$\left\{\begin{array}{rcl}\frac{p_1-c_1}{p_1} & = & -\frac{\lambda}{1+\lambda}\frac{1}{\eta_1}-\frac{\lambda}{1+\lambda}\frac{\partial p_2}{\partial q_1}\frac{1}{p_1}[q_2+\frac{q}{\lambda}]=-\frac{\lambda}{1+\lambda}\frac{1}{\eta_1}-\frac{\partial p_2}{\partial q_1}\frac{q_2}{p_1}-\frac{\lambda}{1+\lambda}\frac{\partial p_2}{\partial q_1}\frac{nq_3}{p_1} \\ \frac{p_2-(c+c_1)}{p_2} & = & -\frac{\lambda}{1+\lambda}\frac{1}{\eta_2}-\frac{\partial p_1}{\partial q}\frac{q_1}{p_2}-\frac{\lambda}{1+\lambda}\frac{q_3}{p_2}\frac{1}{\frac{\partial p_2}{\partial q}\left(2\frac{\partial p_1}{\partial q_1}+q_1\frac{\partial^2 p_1}{\partial q_1^2}\right)-\frac{\partial p_2}{\partial q_1}\left(\frac{\partial p_1}{\partial q}+\frac{\partial p_2}{\partial q_1}\right)} \\ & & \left\{\left(2\frac{\partial p_1}{\partial q_1}+q_1\frac{\partial^2 p_1}{\partial q_1^2}\right)\left(4\frac{\partial p_2}{\partial q}+q\frac{\partial^2 p_2}{\partial q^2}+q_2\frac{\partial^2 p_2}{\partial q^2}\right)\frac{\partial p_2}{\partial q}-2\left[\frac{\partial p_1}{\partial q}\right]^2\frac{\partial p_2}{\partial q}\right. \\ & & -(4+n)\frac{\partial p_1}{\partial q}\frac{\partial p_2}{\partial q_1}\frac{\partial p_2}{\partial q}-\left[\frac{\partial p_2}{\partial q_1}\right]^2\frac{\partial^2 p_2}{\partial q^2}n(q_3-q_2)-(n-2)\left[\frac{\partial p_2}{\partial q_1}\right]^2\frac{\partial p_2}{\partial q} \\ & & \left.-\frac{\partial p_2}{\partial q_1}\frac{\partial p_1}{\partial q}\frac{\partial^2 p_2}{\partial q^2}nq_3\right\}.\end{array}\right.$$

It is easy to see that the first market is now distorted also in accordance with the cross effect: if an increase in the consumption of the fixed-link service increases (decreases, resp.) the demand for the mobile service, then the price distortion allowed in the fixed-link market falls short of (exceeds, resp.) what pure Ramsey pricing would yield. The same applies in the case of the mobile market, with the added proviso that now the extra term already discussed in section 2 can have a strong impact on the relative distortion in this market, depending as it does on both the direct and cross effects. Thus, the conclusion that the mobile market should be distorted over and above pure Ramsey pricing is now not necessarily true.

Finally, increasing the number of downstream firms does not lead to marginal-cost pricing being approached, as is made clear by the second term on the left-hand side of the Lerner-index equalities of both markets. This is so for the reason already pointed out before. Since cross-market externalities are not corrected by increased competition in the downstream market, it is left for both regulatory instruments to internalize then, with the possibility of instrument insufficiency thus resurfacing.

8 Extensions

8.1 Fixed Costs

Fixed costs are likely to be incurred when entering the downstream market. For instance, operating in the cellular segment of the telecommunications market demands the installation of expensive antennas. Thus, obtaining increased competition in the downstream market entails duplicate use of resources. This introduces a tradeoff between costly duplication of fixed costs and undesirable low output stemming from imperfect competition in the downstream market. From a regulatory standpoint, increasing the number of firms, formerly akin to a "free good" from the regulator's viewpoint, is now a socially costly policy. The crucial consequence of this observation is that the optimal number of downstream firms is now not larger and possibly smaller than in the absence of fixed costs. The analysis carried out in the previous sections than shows that the optimal access charge now becomes smaller in order to contravene the now more clearly suboptimal output resulting from a less competitive outcome. This, as we have seen, increases the likelihood of instrument insufficiency.

8.2 Non-linear costs

We have assumed that marginal costs remain constant across quantities. This linearity, amounting to a constant-returns-to-scale assumption, seems to be quite realistic as far as the telecommunications industry is concerned. Insofar as increased quantities imply decreased costs, we again have a tradeoff analogous to the previous one, now between socially costly high marginal costs and undesirable low output stemming from imperfect competition in the downstream market. Again, consideration of this aspect calls for a smaller number of firms than in the previous sections, and a correspondingly lower optimal access charge. Therefore, the likelihood of instrument insufficiency increases. By the same token, the presence of increasing marginal costs has the opposite effect.

8.3 Competitiveness of game solution in downstream market

As the paper progressed it become obvious that the crucial element associated with the limit results obtained was not the number of firms *per se*, but rather the increased competitiveness in the downstream market—defined as how close the quantity traded was to the socially-optimal one—and the consequent increase in efficiency brought about by entry. Assuming Cournot competition was simply a way of obtaining a monotonic relationship relating competitiveness to the number of downstream firms. Other game solutions can be of interest, depending on the particular market under study. For instance, with Bertrand the optimal quantity is obtained in the downstream market with as few as two firms. Formally, our limit results are attained with $n \geq 2$. The implication of this observation is that, in the presence of fixed costs, regulators have to carefully ponder the type of competitive behavior to be expected in an imperfectly competitive downstream market and its evolution with the number of firms operating therein versus costs of duplication associated with entry: insofar as the latter are small and the former relationship is steep, liberalizing the number of mobile operators becomes a particularly attractive policy.

9 Conclusion

We have considered a market structure involving two markets where operating in one requires the use of the other as an input, and discussed the regulation of the access price associated with this transaction. The main conclusion is that there are many instances where regulatory instruments are insufficient for optimal resource allocation. The likelihood of this insufficiency has been related to several parameters characterizing the structure of these markets, some of which had not hitherto been discussed in the literature.

From a policy standpoint, we regard our paper as lending strong support to liberalizing policies that increase the number of firms operating in the downstream market and showing that divestiture policies are significantly less successful in promoting social welfare—insofar as only one downstream firm is owned by the upstream monopolist. In fact, as the number of firms operating in the mobile market increases, it becomes more likely that a first-best allocation of resources emerges. This is the more so as competitiveness increases quickly with entry. By the same token, the analysis also lends further support to policies other than entry that foster competition in the downstream market. Pursuing these, besides the usual effects on social welfare, also contributes to reducing the likelihood of instrument insufficiency.

Liberalizing policies that increase the number of firms, besides ensuring an increase in economic efficiency, also make the regulatory task much easier from an informational point of view. In fact, since marginal-cost pricing then becomes the optimal benchmark in many instances (interdependent demands being the important *caveat* here), regulators have only to collect information on marginal costs, and possibly fixed entry costs, whilst ignoring demand configuration altogether. This information seems quite easy to collect, thus reducing the effects of asymmetric information and attempted capture small. On the other hand, in order to be able to judge the effects of liberalizing the number of downstream firms, regulatory authorities need to have an in-depth knowledge of the competitive behavior to be expected of both incumbents and entrants.

On the other hand, our analysis suggests that in situations where the number of firms operating in the mobile market is small and regulation takes the indirect form described here, it is quite possible that regulatory instrument insufficiency may entail welfare losses.

There are many other policy conclusions to be obtained from our analysis. For instance, increased efficiency on the part of non-integrated firms can be non-optimally reflected in higher output in the cellular market if such change in costs induces the regulatory instrument(s) to become insufficient. However, these are not difficult to draw, and as a result, we will not extend this already long analysis by exhausting them.

Appendix

Proof of Lemma 1 Differentiating (1), we get

$$\begin{cases} \frac{\partial p_2}{\partial q}[\mathrm{d}q_2 + n\mathrm{d}q_3] + \frac{\partial p_2}{\partial q}\mathrm{d}q_2 + q_2\frac{\partial^2 p_2}{\partial q^2}[\mathrm{d}q_2 + n\mathrm{d}q_3] = 0 \\ n\frac{\partial p_2}{\partial q}[\mathrm{d}q_2 + n\mathrm{d}q_3] + \frac{\partial p_2}{\partial q}n\mathrm{d}q_3 + nq_3\frac{\partial^2 p_2}{\partial q^2}[\mathrm{d}q_2 + n\mathrm{d}q_3] = n\mathrm{d}p_a, \end{cases}$$

or

$$\begin{bmatrix} 2\frac{\partial p_2}{\partial q} + q_2\frac{\partial^2 p_2}{\partial q^2} & \frac{\partial p_2}{\partial q} + q_2\frac{\partial^2 p_2}{\partial q^2} \\ n\frac{\partial p_2}{\partial q} + nq_3\frac{\partial^2 p_2}{\partial q^2} & (1+n)\frac{\partial p_2}{\partial q} + nq_3\frac{\partial^2 p_2}{\partial q^2} \end{bmatrix} \begin{bmatrix} \mathrm{d}q_2 \\ n\mathrm{d}q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ n \end{bmatrix} \mathrm{d}p_a,$$

which yields

$$\frac{\mathrm{d}q_2}{\mathrm{d}p_a} = \frac{-n\left(\frac{\partial p_2}{\partial q} + q_2\frac{\partial^2 p_2}{\partial q^2}\right)}{\left[(2+n)\frac{\partial p_2}{\partial q} + q\frac{\partial^2 p_2}{\partial q^2}\right]\frac{\partial p_2}{\partial q}};\ \frac{\mathrm{d}q_3}{\mathrm{d}p_a} = \frac{2\frac{\partial p_2}{\partial q} + q_2\frac{\partial^2 p_2}{\partial q^2}}{\left[(2+n)\frac{\partial p_2}{\partial q} + q\frac{\partial^2 p_2}{\partial q^2}\right]\frac{\partial p_2}{\partial q}};\ \frac{\mathrm{d}q}{\mathrm{d}p_a} = \frac{n}{(2+n)\frac{\partial p_2}{\partial q} + q\frac{\partial^2 p_2}{\partial q^2}}. \tag{A1}$$

Assumption 1 implies $(2+n)\frac{\partial p_2}{\partial q} + q\frac{\partial^2 p_2}{\partial q^2} < 0, \forall n \in \{1, 2, \ldots\}$ and $\frac{\partial p_2}{\partial q} + q_2\frac{\partial^2 p_2}{\partial q^2} < 0$ since $q_2 < q$. It also implies concavity of the integrated and non-integrated firms' profit equations, $2\frac{\partial p_2}{\partial q} + q_2\frac{\partial^2 p_2}{\partial q^2} < 0$ and $2\frac{\partial p_2}{\partial q} + q_3\frac{\partial^2 p_2}{\partial q^2} < 0$. This, in turn, implies $\frac{\mathrm{d}q_2}{\mathrm{d}p_a} > 0$, $\frac{\mathrm{d}q_3}{\mathrm{d}p_a} < 0$, $\frac{\mathrm{d}q}{\mathrm{d}p_a} < 0$. □

Proof of Lemma 2 Social welfare in this case equals

$$\tilde{W} = V(q_1, q) + p_1 q_1 - c_1 q_1 + p_2 q - q(c + c_1).$$

The regulator wishes to

$$\max_{q_1, p_a} \tilde{W}. \tag{A2}$$

For computational convenience, we take q_1 rather than p_1 as the regulatory variable. This entails no loss of generality since demand is well-behaved and non-stochastic.

The first-order conditions equal

$$\begin{cases} \frac{\partial V}{\partial q_1} + p_1 + \frac{\partial p_1}{\partial q_1}q_1 - c_1 = 0 \\ \left[\frac{\partial V}{\partial q} + p_2 + \frac{\partial p_2}{\partial q}q - (c + c_1)\right]\frac{\mathrm{d}q}{\mathrm{d}p_a} = 0. \end{cases}$$

Using the fact that $\frac{\partial V}{\partial q_i} = -\frac{\partial p_i}{\partial q_i}q_i$ since, from $V(q_1, q) = \int_0^{q_1} p_1(q_1)\mathrm{d}q_1 - p_1 q_1 + \int_0^{q} p_2(q)\mathrm{d}q - p_2 q$ we obtain $\frac{\partial V}{\partial q_1} = p_1 - p_1 - \frac{\partial p_1}{\partial q_1}q_1 = -\frac{\partial p_1}{\partial q_1}q_1$ and $\frac{\partial V}{\partial q} = -\frac{\partial p_2}{\partial q}q$, and noting that $\frac{\mathrm{d}q}{\mathrm{d}p_a} \neq 0$, we finally get

$$\begin{cases} p_1^* = c_1 \\ p_2^* = c + c_1. \end{cases}$$

$\square$

Proof of Proposition 2 From the proof the Lemma 2 we have $p_a^* = c_1 + \frac{c+c_1}{\eta_2 n}$. Letting $n \to \infty$ yields $p_a^* \to c_1$. $\square$

Proof of Lemma 3 From (2), social welfare can now be written as

$$\begin{aligned} W &= V(q_1, q) + p_1 q_1 - c_1 q_1 + p_2 q - q_2(c_1 + c_2) - n q_3(c + c_1) \\ &\quad + q_2 c_2 - q_2 c_2 + q_2 c - q_2 c \\ &= V(q_1, q) + p_1 q_1 - c_1 q_1 + p_2 q - q(c + c_1) + q_2(c - c_2) \\ &= \tilde{W} + q_2(c - c_2). \end{aligned}$$

The regulator wishes to

$$\max_{q_1, p_a} \tilde{W} + q_2(c - c_2).$$

The first-order conditions equal

$$\begin{cases} \frac{\partial V}{\partial q_1} + p_1 + \frac{\partial p_1}{\partial q_1} q_1 - c_1 = 0 \\ \left[\frac{\partial V}{\partial q} + p_2 + \frac{\partial p_2}{\partial q} q - (c + c_1)\right] \frac{dq}{dp_a} - (c_2 - c)\frac{dq_2}{dp_a} = 0, \end{cases}$$

which yields the first case. The second one is obtained by noting that the second first-order condition and Lemma 1 imply $p_2 < c + c_1$. The hypothesis then yields $p_2 < c_2 + c_1$, thus implying that the non-integrated firm leaves the market. Therefore, $\frac{dq_2}{dp_a} = 0$, which eliminates the term determining the departure from marginal-cost pricing. $\square$

Proof of Proposition 3 Computations similar to those performed in the proof of Proposition 1 yield the following optimal p_a for the case $c_2 < c$:

$$\begin{aligned} p_a^* &= c_1 + \frac{c - c_2}{n}\left[1 - (n+1)\frac{\frac{dq_2}{dp_a}}{\frac{dq}{dp_a}}\right] + \frac{p_2^*}{\eta_2 n} \\ &= c_1 + \frac{c - c_2}{n}\left[1 - (n+1)\frac{\frac{dq_2}{dp_a}}{\frac{dq}{dp_a}}\right] + \frac{c + c_1 + (c_2 - c)\frac{\frac{dq_2}{dp_a}}{\frac{dq}{dp_a}}}{\eta_2 n}. \end{aligned}$$

$p_2^* < 0$ yields the first inequality. By Lemma 3, the case $c_2 \geq c$ is identical to that in Proposition 1. $\square$

Proof of Proposition 4 From the proof of Proposition 3, $p_a^* = c_1 + \frac{c-c_2}{n}\left[1 - (n+1)\frac{\frac{dq_2}{dp_a}}{\frac{dq}{dp_a}}\right] + \frac{c+c_1+(c_2-c)\frac{\frac{dq_2}{dp_a}}{\frac{dq}{dp_a}}}{\eta_2 n}$. Letting $n \to \infty$ yields the first limit. The proof of Proposition 2 yields the second. $\square$

Proof of Lemma 4 Begin by writing π^I as follows:

$$\begin{aligned}\pi^I &= p_1 q_1 + p_2 q_2 + p_a n q_3 - c_1(q_1 + q_2 + n q_3) - c_2 q_2 \\ &= p_1 q_1 - c_1 q_1 + p_2 q - (c_1 + c) q - (p_2 - p_a - c) n q_3. \end{aligned} \tag{A3}$$

The regulator's problem is thus the following:

$$\max_{q_1, p_a} \quad \tilde{W} \qquad \text{s.t.} \quad \pi^I \geq 0,$$

or

$$\begin{aligned}\max_{q_1, p_a} \quad & \tilde{W} = V(q_1, q) + p_1 q_1 - c_1 q_1 + p_2 q - (c + c_1) q \\ \text{s.t.} \quad & p_1 q_1 - c_1 q_1 + p_2 q - (c_1 + c) q - (p_2 - p_a - c) n q_3 \geq 0.\end{aligned}$$

The first-order conditions to this problem are thus

$$\begin{cases} \frac{\partial \tilde{W}}{\partial q_1} = \frac{\partial V}{\partial q_1} + p_1 + \frac{\partial p_1}{\partial q_1} q_1 - c_1 + \lambda \left(p_1 + \frac{\partial p_1}{\partial q_1} q_1 - c_1 \right) = 0 \\ \\ \frac{\partial \tilde{W}}{\partial p_a} = \left[\frac{\partial V}{\partial q} + p_2 + \frac{\partial p_2}{\partial q} q - (c + c_1) \right] \frac{dq}{dp_a} \\ \qquad + \lambda \left\{ \left[p_2 + \frac{\partial p_2}{\partial q} q - (c + c_1) \right] \frac{dq}{dp_a} \right\} \\ \qquad - \lambda \left[n q_3 \frac{\partial p_2}{\partial q} \frac{dq}{dp_a} + (p_2 - p_a - c) n \frac{dq_3}{dp_a} - n q_3 \right] = 0 \\ \\ \frac{\partial \tilde{W}}{\partial \lambda} = p_1 q_1 - c_1 q_1 + p_2 q - (c_1 + c) q - (p_2 - p_a - c) n q_3 \geq 0 \qquad \lambda \frac{\partial \tilde{W}}{\partial \lambda} = 0. \end{cases}$$

The solution of (A2) ($p_1^* = c_1$, $p_2^* = c + c_1$, $p_a^* < c_1$) implies $\pi^I < 0$ and hence that the constraint is binding. This, in turn, implies $\lambda \neq 0$, which, considering the nature of the constraint, finally leads to $\lambda > 0$.

Simplifying the first-order conditions, one obtains

$$\frac{p_1 - c_1}{p_1} = -\frac{\lambda}{1 + \lambda} \frac{\partial p_1}{\partial q_1} \frac{q_1}{p_1} \Leftrightarrow \frac{p_1 - c_1}{p_1} = -\frac{\lambda}{1 + \lambda} \frac{1}{\eta_1},$$

and, noting from (1) that $n p_2 - n p_a - n c = -\frac{\partial p_2}{\partial q} n q_3$, using the total derivatives in (A1), and recalling that $\frac{dq}{dp_a} = \frac{dq_2}{dp_a} + \frac{dnq_3}{dp_a}$, we obtain after some manipulation

$$\frac{p_2 - (c + c_1)}{p_2} = -\frac{\lambda}{1 + \lambda} \frac{1}{\eta_2} + \frac{\lambda}{1 + \lambda} \frac{q_3}{p_2} \left[-2 \frac{\partial p_2}{\partial q} - q_2 \frac{\partial^2 p_2}{\partial q^2} - 2 \frac{\partial p_2}{\partial q} - q \frac{\partial^2 p_2}{\partial q^2} \right]. \tag{A4}$$

□

Proof of Proposition 5 Adding up and manipulating the fist-order conditions in (1) yields

$$\frac{p_2 - c - c_1}{p_2} = \frac{n}{n+1}\frac{p_a - c_1}{p_2} - \frac{1}{\eta_2\,(n+1)}.$$

Equating this expression to the second equation in Lemma 4 yields

$$p_a^* = c_1 + \frac{p_2^*}{\eta_2\, n} - \frac{n+1}{n}\frac{\lambda}{1+\lambda}\frac{p_2^*}{\eta_2} + \frac{n+1}{n}\frac{\lambda}{1+\lambda}q_3\left[-2\frac{\partial p_2}{\partial q} - q_2\frac{\partial^2 p_2}{\partial q^2} - 2\frac{\partial p_2}{\partial q} - q\frac{\partial^2 p_2}{\partial q^2}\right].$$

Comparing this expression with

$$p_a^* = c_1 + \frac{p_2^*}{\eta_2\, n}$$

from the proof of Proposition 1 and noting that the last two terms on the right-hand side are positive yields the conclusion that p_a^* is now higher. □

Proof of Lemma 5 Rewrite the first-order condition for the mobile market as

$$\frac{p_2 - (c + c_1)}{p_2} = -\frac{\lambda}{1+\lambda}\left\{\frac{1}{\eta_2} - \frac{nq_3}{np_2}\left[-2\frac{\partial p_2}{\partial q} - q_2\frac{\partial^2 p_2}{\partial q^2} - 2\frac{\partial p_2}{\partial q} - q\frac{\partial^2 p_2}{\partial q^2}\right]\right\},$$

note that $q(c + c_1) > q = q_2 + nq_3 > nq_3$ constitutes an upper bound on nq_3, and that $c + c_1 < p_2$ is a lower bound on p_2, to conclude that the term that yields the departure from pure Ramsey pricing in the mobile market approaches zero as $n \to \infty$, as long as the term in square brackets does not explode with n, a mild regularity assumption. □

Proof of Proposition 6 Write the downstream market first-order condition as

$$\frac{\lambda}{1+\lambda} = \frac{\frac{p_2-(c+c_1)}{p_2}}{-\frac{1}{\eta_2} + \frac{q_3}{p_2}\left[-2\frac{\partial p_2}{\partial q} - q_2\frac{\partial^2 p_2}{\partial q^2} - 2\frac{\partial p_2}{\partial q} - q\frac{\partial^2 p_2}{\partial q^2}\right]},$$

and note that the second term in the denominator approaches 0 when $n \to \infty$ (see proof of Lemma 5). Assuming that the first term remains bounded away from zero for $q \le q(c + c_1)$, a very mild assumption, we conclude that the behavior of $\frac{\lambda}{1+\lambda}$ as $n \to \infty$ is determined by the numerator. From (1), note that $\frac{p_2-(p_a+c)}{p_2} = -\frac{1-\alpha}{\eta_2 n} < -\frac{1}{\eta_2 n}$ and the latter approaches 0 when $n \to \infty$. Thus, $p_2 \to p_a + c$ as $n \to \infty$. Thus, the numerator approaches $\frac{p_a-c_1}{p_a+c}$ as $n \to \infty$. Finally, note that the optimal access charge in this problem exceeds p_a^* in the proof of Proposition 1, since the break-even constraint is binding, and falls short of c_1 since otherwise the break-even constraint would not bind, and, from Proposition 2, that p_a^* in approaches c_1 as $n \to \infty$. Therefore, so does p_a^* in the current problem. Thus, $\frac{\lambda}{1+\lambda} \to 0$ as $n \to \infty$. Therefore, marginal-cost pricing is recovered in both markets. □

Proof of Proposition 7 Take

$$p_a^* = c_1 + \frac{p_2^*}{\eta_2\, n} - \frac{n+1}{n}\frac{\lambda}{1+\lambda}\frac{p_2^*}{\eta_2} + \frac{n+1}{n}\frac{\lambda}{1+\lambda}q_3\left[-2\frac{\partial p_2}{\partial q} - q_2\frac{\partial^2 p_2}{\partial q^2} - 2\frac{\partial p_2}{\partial q} - q\frac{\partial^2 p_2}{\partial q^2}\right]$$

from the proof of Proposition 5, and use the result $\frac{\lambda}{1+\lambda} \to 0$ as $n \to \infty$ from the proof of Proposition 6 to obtain $p_a^* \to c_1$ as $n \to \infty$. □

Proof of Lemma 6 The social welfare function can be written as

$$\begin{aligned}\hat{W} &= V(q_1,(n+1)q_3) + p_1q_1 + p_a(n+1)q_3 - c_1(q_1+(n+1)q_3) - \\ &\quad +p_2(n+1)q_3 - p_a(n+1)q_3 - c(n+1)q_3 \\ &= V(q_1,q) + p_1q_1 - c_1q_1 + p_2q - (c+c_1)q,\end{aligned}$$

where $V(q_1,q)$ still represents the sum of the net consumer surplus in both markets. Noting that $\hat{W}$ above equals $\tilde{W}$, allows us to conclude that the regulator's problem,

$$\max_{q_1,p_a} \hat{W}, \tag{A5}$$

has exactly the same solution as the problem in the proof of Lemma 2. □

Proof of Proposition 8 Simple computations analogous to those performed in the proof of Proposition 1 yield

$$p_a^* = c_1 + \frac{q^*}{n+1}\frac{\partial p_2}{\partial q}\bigg|_{q=q^*} \Leftrightarrow p_a^* = c_1 + \frac{p_2^*}{\eta_2(n+1)} \Leftrightarrow p_a^* = c_1 + \frac{c+c_1}{\eta_2(n+1)}.$$

□

Proof of Lemma 7 Begin by writing π^I as follows:

$$\begin{aligned}\pi^I &= p_1q_1 + p_a(n+1)q_3 - c_1(q_1+(n+1)q_3) \\ &= p_1q_1 - c_1q_1 + p_2q - (c_1+c)q - (p_2-p_a-c)q.\end{aligned} \tag{A6}$$

The regulator's problem is thus the following:

$$\begin{aligned}\max_{q_1,p_a} &\quad \hat{W} \\ \text{s.t.} &\quad \pi^I \geq 0,\end{aligned}$$

or

$$\begin{aligned}\max_{q_1,p_a} &\quad \hat{W} = V(q_1,q) + p_1q_1 - c_1q_1 + p_2q - (c+c_1)q \\ \text{s.t.} &\quad p_1q_1 - c_1q_1 + p_2q - (c_1+c)q - (p_2-p_a-c)q \geq 0.\end{aligned}$$

The first-order conditions to this problem are thus

$$
\begin{cases}
\frac{\partial \hat{W}}{\partial q_1} = \frac{\partial V}{\partial q_1} + p_1 + \frac{\partial p_1}{\partial q_1} q_1 - c_1 + \lambda\left(p_1 + \frac{\partial p_1}{\partial q_1} q_1 - c_1\right) = 0 \\
\\
\frac{\partial \hat{W}}{\partial p_a} = \left[\frac{\partial V}{\partial q} + p_2 + \frac{\partial p_2}{\partial q} q - (c + c_1)\right] \frac{dq}{dp_a} \\
\qquad +\lambda\left\{\left[p_2 + \frac{\partial p_2}{\partial q} q - (c + c_1)\right] \frac{dq}{dp_a}\right\} \\
\qquad -\lambda\left[q \frac{\partial p_2}{\partial q} \frac{dq}{dp_a} + (p_2 - p_a - c) \frac{dq}{dp_a} - q\right] = 0 \\
\\
\frac{\partial \hat{W}}{\partial \lambda} = p_1 q_1 - c_1 q_1 + p_2 q - (c_1 + c)q - (p_2 - p_a - c)q \geq 0 \qquad \lambda \frac{\partial \hat{W}}{\partial \lambda} = 0.
\end{cases}
$$

Again, the solution of (A5) ($p_1^* = c_1$, $p_2^* = c + c_1$, $p_a^* < c_1$) implies $\pi^I < 0$ and hence that the constraint is binding. This, in turn, implies $\lambda \neq 0$, which, considering the nature of the constraint, finally leads to $\lambda > 0$.

Simplifying the first-order conditions, one obtains

$$
\frac{p_1 - c_1}{p_1} = -\frac{\lambda}{1+\lambda} \frac{\partial p_1}{\partial q_1} \frac{q_1}{p_1} \Leftrightarrow \frac{p_1 - c_1}{p_1} = -\frac{\lambda}{1+\lambda} \frac{1}{\eta_1},
$$

and, noting from (5) that $p_2 - p_a - c = -\frac{\partial p_2}{\partial q} \frac{q}{n+1}$,

$$
\frac{p_2 - (c + c_1)}{p_2} = -\frac{\lambda}{1+\lambda} \frac{1}{\eta_2} + \frac{\lambda}{1+\lambda} \frac{q}{p_2} \left[\frac{\partial p_2}{\partial q} \frac{n}{n+1} - \left(\frac{dq}{dp_a}\right)^{-1}\right].
$$

Plugging in the expression for $\frac{dq}{dp_a}$ in (6), one obtains

$$
\frac{p_2 - (c + c_1)}{p_2} = -\frac{\lambda}{1+\lambda} \frac{1}{\eta_2} + \frac{\lambda}{1+\lambda} \frac{q}{p_2} \frac{1}{n+1} \left[-2\frac{\partial p_2}{\partial q} - q \frac{\partial^2 p_2}{\partial q^2}\right]. \tag{A7}
$$

□

Proof of Proposition 9 Begin by multiplying and dividing the second part of the right-hand side of (A4) by $(n+1)q$, obtaining

$$
\frac{p_2 - (c + c_1)}{p_2} =
$$
$$
-\frac{\lambda}{1+\lambda} \frac{1}{\eta_2} + \frac{\lambda}{1+\lambda} \frac{q_3(n+1)}{q} \frac{q}{p_2} \frac{1}{n+1} \left[-2\frac{\partial p_2}{\partial q} - q_2 \frac{\partial^2 p_2}{\partial q^2} - 2\frac{\partial p_2}{\partial q} - q \frac{\partial^2 p_2}{\partial q^2}\right].
$$

Noting that $\frac{q_3(n+1)}{q} > 1$ because $q_2 < q_3$ (since the non-integrated firms obtain the input at a price below marginal cost, c_1, while the integrated firm acquires it at marginal cost) implies $(n+1)q_3 > q = nq_3 + q_2$ and, comparing the expressions in square brackets in (A4) and (A7), we conclude that the mobile market's Lerner index is smaller with divestiture. Thus, with divestiture the mobile market is relatively less distorted *vis-à-vis* the fixed-link market. □

Proof of Lemma 8 Social welfare equals

$$\tilde{W} = V(q_1, q) + p_1(q_1, q)q_1 - c_1q_1 + p_2(q_1, q)q - q(c + c_1).$$

The regulator wishes to

$$\max_{q_1, p_a} \tilde{W}.$$

The first-order conditions equal

$$\begin{cases} \frac{\partial V}{\partial q_1} + p_1 + \frac{\partial p_1}{\partial q_1}q_1 + \frac{\partial p_2}{\partial q_1}q - c_1 = 0 \\ \left[\frac{\partial V}{\partial q_2} + p_2 + \frac{\partial p_2}{\partial q}q + \frac{\partial p_1}{\partial q}q_1 - (c + c_1)\right]\frac{dq}{dp_a} = 0. \end{cases}$$

From $V(q_1, q) = \int_0^{q_1} p_1(q_1, q)dq_1 - p_1(q_1, q)q_1 + \int_0^q p_2(q_1, q)dq - p_2(q_1, q)q$ we obtain $\frac{\partial V}{\partial q_1} = p_1 - p_1 - \frac{\partial p_1}{\partial q_1}q_1 + \int_0^q \frac{\partial p_2}{\partial q_1}dq - \frac{\partial p_2}{\partial q_1}q = -\frac{\partial p_1}{\partial q_1}q_1 + \int_0^q \frac{\partial p_2}{\partial q_1}dq - \frac{\partial p_2}{\partial q_1}q$ and $\frac{\partial V}{\partial q} = -\frac{\partial p_2}{\partial q}q + \int_0^{q_1} \frac{\partial p_1}{\partial q}dq_1 - \frac{\partial p_1}{\partial q}q_1$. Assuming that cross effects are not such that $\frac{dq}{dp_a} = 0$, we obtain

$$\begin{cases} p_1^* = c_1 - \int_0^q \frac{\partial p_2}{\partial q_1}dq \\ p_2^* = c + c_1 - \int_0^{q_1} \frac{\partial p_1}{\partial q}dq_1 \end{cases}$$

□

Proof of Proposition 10 Equating the sum of the first-order conditions in (1) with the second first-order condition in Lemma 8, we obtain

$$p_a^* = c_1 + \frac{p_2^*}{\eta_2 n} + \frac{\frac{\partial p_1}{\partial q}q_1 - (n+1)\int_0^{q_1} \frac{\partial p_1}{\partial q}dq_1}{n}. \tag{A8}$$

Making use of Assumption 2, we obtain

$$p_a^* = c_1 + \frac{p_2^*}{\eta_2 n} - \frac{\partial p_1}{\partial q}q_1.$$

Comparing this expression to (4) in the proof of Proposition 1,

$$p_a^* = c_1 + \frac{p_2^*}{\eta_2 n},$$

we conclude that the optimal access price is lower with interdependent demands iff $\frac{\partial p_1}{\partial q} > 0$.

From the proof of Lemma 8 we have

$$p_1^* = c_1 - \int_0^q \frac{\partial p_2}{\partial q_1}dq$$

which is obviously also true when $n \to \infty$. Instrument insufficiency ensues iff $\int_0^q \frac{\partial p_2}{\partial q_1}dq > c_1$ which, by Assumption 2, yields $\frac{\partial p_2}{\partial q_1}q > c_1$. □

Proof of Proposition 11 From the proof of Proposition 10 we have

$$p_a^* = c_1 + \frac{p_2^*}{\eta_2 n} + \frac{\frac{\partial p_1}{\partial q} q_1 - (n+1)\int_0^{q_1} \frac{\partial p_1}{\partial q} \mathrm{d}q_1}{n},$$

whose limit, by Assumption 2, is $c_1 - \frac{\partial p_1}{\partial q} q_1$ when $n \to \infty$. □

Proof of Lemma 9 The regulator wishes to

$$\max_{q_1, p_a} \tilde{W} + q_2(c - c_2).$$

The first-order conditions are thus

$$\begin{cases} \frac{\partial V}{\partial q_1} + p_1 + \frac{\partial p_1}{\partial q_1} q_1 + \frac{\partial p_2}{\partial q_1} q - c_1 = 0 \\ \left[\frac{\partial V}{\partial q_2} + p_2 + \frac{\partial p_2}{\partial q} q + \frac{\partial p_1}{\partial q} q_1 - (c + c_1)\right] \frac{\mathrm{d}q}{\mathrm{d}p_a} - (c_2 - c)\frac{\mathrm{d}q_2}{\mathrm{d}p_a} = 0, \end{cases}$$

which yield the first case,

$$\begin{cases} p_1^* = c_1 - \int_0^q \frac{\partial p_2}{\partial q_1} \mathrm{d}q \\ p_2^* = c + c_1 - \int_0^{q_1} \frac{\partial p_1}{\partial q} \mathrm{d}q_1 + (c_2 - c)\frac{\frac{\mathrm{d}q_2}{\mathrm{d}p_a}}{\frac{\mathrm{d}q}{\mathrm{d}p_a}}. \end{cases}$$

The second is obtained by noting that, when $c_2 \geq c$, setting $p_2^* = c + c_1 - \int_0^{q_1} \frac{\partial p_1}{\partial q} \mathrm{d}q_1$ not only assures that the socially-optimal output level (inclusive of the externality arising from interdependent demands) is achieved, but that only the more-efficient integrated firms operate, since the previous expression implies $p_2^* - c_1 - c_2 + \int_0^{q_1} \frac{\partial p_1}{\partial q} \mathrm{d}q_1 < 0$ which, compared to the first-order condition for the integrated downstream firm in (8), garantees that this firm does not operate. □

Proof of Lemma 10 The monopolist's profit can be written as

$$\begin{aligned} \pi^I &= p_1(q_1, q)q_1 + p_2(q_1, q)q_2 + p_a n q_3 - c_1(q_1 + q_2 + nq_3) - c_2 q_2 \\ &= p_1(q_1, q)q_1 - c_1 q_1 + p_2(q_1, q)q - (c_1 + c)q - (p_2(q_1, q) - p_a - c)nq_3. \end{aligned}$$

The regulator's problem can be written as follows:

$$\begin{aligned} \max_{q_1, p_a} \quad & \tilde{W} \\ \text{s.t.} \quad & \pi^I \geq 0, \end{aligned}$$

or

$$\begin{aligned} \max_{q_1, p_a} \quad & \tilde{W} = V(q_1, q) + p_1(q_1, q)q_1 - c_1 q_1 + p_2(q_1, q)q - (c + c_1)q \\ \text{s.t.} \quad & p_1(q_1, q)q_1 - c_1 q_1 + p_2(q_1, q)q - (c_1 + c)q - (p_2(q_1, q) - p_a - c)nq_3 \geq 0. \end{aligned}$$

The first-order conditions to this problem are thus

$$
\begin{cases}
\frac{\partial \tilde{W}}{\partial q_1} = \frac{\partial V}{\partial q_1} + p_1 + \frac{\partial p_1}{\partial q_1} q_1 - c_1 + \frac{\partial p_2}{\partial q_1} q + \lambda \left(p_1 + \frac{\partial p_1}{\partial q_1} q_1 - c_1 + \frac{\partial p_2}{\partial q_1} q - \frac{\partial p_2}{\partial q_1} n q_3 \right) = 0 \\
\\
\frac{\partial \tilde{W}}{\partial p_a} = \left[\frac{\partial V}{\partial q} + \frac{\partial p_1}{\partial q} q_1 + p_2 + \frac{\partial p_2}{\partial q} q - (c + c_1) \right] \frac{dq}{dp_a} \\
\qquad + \lambda \left\{ \left[\frac{\partial p_1}{\partial q} q_1 + p_2 + \frac{\partial p_2}{\partial q} q - (c + c_1) \right] \frac{dq}{dp_a} \right\} - \\
\qquad - \lambda \left[n q_3 \frac{\partial p_2}{\partial q} \frac{dq}{dp_a} + (p_2 - p_a - c) n \frac{dq_3}{dp_a} - n q_3 \right] = 0 \\
\\
\frac{\partial \tilde{W}}{\partial \lambda} = p_1 q_1 - c_1 q_1 + p_2 q - (c_1 + c) q - (p_2 - p_a - c) n q_3 \geq 0 \qquad \lambda \frac{\partial \tilde{W}}{\partial \lambda} = 0.
\end{cases}
$$

where use was made of the fact that $\frac{\partial \tilde{W}}{\partial p_a} = \frac{\partial \tilde{W}}{\partial q_1} \frac{dq_1}{dp_a} + \frac{\partial \tilde{W}}{\partial q} \frac{dq}{dp_a}$ and $\frac{\partial \tilde{W}}{\partial q_1} = 0$ in the initial first-order condition.

Simplifying the first-order conditions, one obtains

$$
\frac{p_1 - c_1}{p_1} = -\frac{\lambda}{1+\lambda} \frac{1}{\eta_1} - \frac{\lambda}{1+\lambda} \frac{\partial p_2}{\partial q_1} \frac{1}{p_1} [q_2 + \frac{q}{\lambda}],
$$

and,

$$
\begin{aligned}
\frac{p_2 - (c + c_1)}{p_2} = {} & -\frac{\lambda}{1+\lambda} \frac{1}{\eta_2} - \frac{\partial p_1}{\partial q} \frac{q_1}{p_2} - \frac{\lambda}{1+\lambda} \frac{q_3}{p_2} \frac{1}{\frac{\partial p_2}{\partial q} \left(2 \frac{\partial p_1}{\partial q_1} + q_1 \frac{\partial^2 p_1}{\partial q_1^2} \right) - \frac{\partial p_2}{\partial q_1} \left(\frac{\partial p_1}{\partial q} + \frac{\partial p_2}{\partial q_1} \right)} \\
& \left\{ \left(2 \frac{\partial p_1}{\partial q_1} + q_1 \frac{\partial^2 p_1}{\partial q_1^2} \right) \left(4 \frac{\partial p_2}{\partial q} + q \frac{\partial^2 p_2}{\partial q^2} + q_2 \frac{\partial^2 p_2}{\partial q^2} \right) \frac{\partial p_2}{\partial q} - 2 \left[\frac{\partial p_1}{\partial q} \right]^2 \frac{\partial p_2}{\partial q} \right. \\
& - (4 + n) \frac{\partial p_1}{\partial q} \frac{\partial p_2}{\partial q_1} \frac{\partial p_2}{\partial q} - \left[\frac{\partial p_2}{\partial q_1} \right]^2 \frac{\partial^2 p_2}{\partial q^2} n (q_3 - q_2) - (n - 2) \left[\frac{\partial p_2}{\partial q_1} \right]^2 \frac{\partial p_2}{\partial q} \\
& \left. - \frac{\partial p_2}{\partial q_1} \frac{\partial p_1}{\partial q} \frac{\partial^2 p_2}{\partial q^2} n q_3 \right\}.
\end{aligned}
$$

$\square$

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