**Hic's Optimal Trade Openness and the
Modelling of the Default Penalty**

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Abstract: When it is costly to impose sanctions on a defaulting country, there is a penalty structure alternative to Aizenman's (1990) which will be preferred by creditors. Aizenman's results on debtor's optimal openness and credit market policies may then be reversed.

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1. Introduction

Aizenman (1990) argued that highly indebted countries should follow credit market policies that warrant a higher degree of trade dependency than produced by the market, since this lowers the probability of default and therefore allows the country to face better conditions in international credit markets (i.e., lower interest rates). As will be shown, this result depends on the form used to model the default penalty. Aizenman (1990) assumed that, in the case of default on international loans, the indebted country faces a deterioration of the terms of trade which increases the price of a required imported input by a fixed amount. The resulting reduction in output constitutes the "collateral" on the loan.

The debtor defaults whenever debt repayment is larger than the default loss. Under uncertainty, which in Aizenman's model arises from a multiplicative productivity shock, default may become the debtor's choice for some states of the world. Denoting the value of the multiplicative output shock which makes the output loss equal to debt repayment by θ_0 , the probability of repayment will simply be the probability of this random variable assuming a value greater than θ_0 . Since θ_0 increases with the amount borrowed and decreases with the dependency on the imported input, Aizenman showed that the interest rate charged on loans turns out to be positively related to the amount borrowed and, moreover, increases in trade dependency should push down this positively sloped supply curve. In such a framework, it is optimal to subsidize borrowing for investment in activities that are highly trade dependent and to tax borrowing for investment in activities that reduce the degree of the economy's dependency.

However, the above analysis overlooks the existence of costs to the creditor of enforcing the default penalties. To accomplish the deterioration of the defaulter's terms of trade the creditor will incur in costs such as the costs of imposing a trade embargo, costs of monitoring an embargo, political costs due to lobbies interested in trade with

the defaulter, output losses in exporting sectors, costs of persuading other countries to join in, etc.

We show that the type of penalty used by Aizenman is not optimal from the creditor's point of view, when imposing trade sanctions involve costs to the creditor. There is one alternative penalty structure which, while not necessarily optimal, is better from the creditor's point of view. Under this alternative penalty regime, it may be optimal for debtors to reduce its degree of trade dependency on international trade, contrary to Aizenman's policy recommendations. This suggests that more work needs to be done on the choice of the default penalty used by creditor nations before any general policy recommendations can be made.

2. The Model

The model draws on Aizenman (1990)¹. There are two periods. In the first period the country borrows for first-period consumption (B_c) and for investment in each of the n productive sectors of the economy ($I_i, i=1, \dots, n$). In the second period, production is uncertain and debt repayment is due. After observing production levels, the debtor country decides to either repay its debt or to default, and consumes whatever production is left after repayment or default losses take place, respectively. Sector i 's production is given by

$$X_i = \theta (K_i)^\alpha (M_i)^{\beta_i}, \quad i=1, \dots, n. \quad (1)$$

¹ Aizenman (1990) uses a three period model because there is an additional aspect analyzed: the possibility that the central planner's discount rate differs from the social discount rate. For the purpose of studying credit market policies two periods only are needed, as was done in Aizenman (1989), the previous (unpublished) version of the model.

where X_i is sector i 's output, θ is a random productivity shock that affects production in all productive activities, K_i is the capital stock in sector i (initial capital stock plus investment borrowing I_i), and M_i is the amount of the imported input used in sector i . Since the imported input receives a portion β_i of sector i 's output, sector i 's contribution to national income (Y_i) equal $(1-\beta_i) p_i X_i$, where p_i is sector i 's output price. National income (Y) will therefore equal $\sum_{i=1}^n (1-\beta_i) p_i X_i$.

In case of default, the creditor raises the price of the imported input from p_m to p_m' . We consider the case where the creditor can change the degree of trade retaliation depending on the total volume of the imported input the debtor nation purchases:

$$p_m' = p_m (1 + z_0 + z \sum_{i=1}^n M_i) \quad (2)$$

In Aizenman (1990) it is assumed that $z=0$, $z_0=\exp(P_m)$ with $P_m>0$, and $p_m=1$.

Debtor's default losses (Ω) will equal

$$\Omega = - \sum_{i=1}^n \frac{dY_i}{dX_i} \frac{dX_i}{dp_m} dp_m = \Delta(\theta) \sum_{i=1}^n \gamma_i Y_i \quad (3)$$

with $\gamma_i = \frac{\beta_i}{1-\beta_i} p_i$ and $\Delta(\theta) = z_0 + z \sum_i M_i(\theta)$. Default (repayment) occurs when default losses are smaller (larger) than repayment of total borrowed funds $\bar{B}$ ($\bar{B}=B_c + \sum_{i=1}^n I_i$).

θ_0 , defined above, is given by

$$\sum_{i=1}^n \gamma_i \Delta(\theta_0) Y_i(\theta_0) = (1+r^*(\theta_0)) \bar{B} \quad (4)$$

and the probability of repayment (π) will be defined as the probability of θ falling above the critical value θ_0 :

$$\pi = \int_{\theta_0}^1 f(\theta) d\theta, \quad (5)$$

where f is the density function of the θ on $[0,1]$.

The external interest rate charged by creditors (r^*) is a function of the probability of repayment π since, as suggested by Aizenman, the creditor must in expected value receive a return on the loan equal to the risk free interest rate (r_f)²:

$$(1+r^*) \pi = (1+r_f) \quad (6)$$

3. Results

How is the probability of default affected by a change in z ? Is it optimal for the creditor to set $z=0$, which would result in a default penalty such as the one modelled by Aizenman, or should instead the creditor make the penalty dependent on the level of trade dependency of the debtor country?

Proposition 1: If there is a cost to the creditor of imposing sanctions on a defaulting debtor and this cost is fixed, then (a) the creditor chooses to make the default penalty an increasing function of the imported input level, setting $z>0$, (b) z is set such that the probability of repayment π is maximized.

² The cost of imposing trade penalties are assumed to be borne by the lending nations but not by the direct lenders so that the rule determining the external interest rate is not affected.

The intuition behind this result is that, if imposing sanctions involves a fixed cost, the creditor will also want to reduce the probability of having to impose such sanctions to a minimum, i.e., will choose to maximize the probability of repayment (π).

Proof:

Step 1:

To compute $\frac{d\theta_0}{dz}$ we will start by differentiating condition (4):

$$\begin{aligned} \frac{d(1+r^*)}{d\theta_0} \frac{d\theta_0}{dz} \bar{B} - \sum_{i=1}^n \gamma_i Y_i(\theta_0) \frac{d\Delta(\theta_0)}{dz} - \sum_{i=1}^n \gamma_i Y_i(\theta_0) \frac{d\Delta(\theta_0)}{d\theta_0} \frac{d\theta_0}{dz} - \\ \sum_{i=1}^n \gamma_i \Delta(\theta_0) \frac{dY_i(\theta_0)}{d\theta_0} \frac{d\theta_0}{dz} = 0 \end{aligned} \quad (7)$$

From differentiation of (6) we obtain:

$$\frac{d(1+r^*)}{d\theta_0} = \frac{(1+r^*) f(\theta_0)}{\pi} \quad (8)$$

Substituting in (7) and evaluating $\frac{d\Delta(\theta_0)}{d\theta_0}$, $\frac{dY_i(\theta_0)}{d\theta_0}$ and $\frac{d\Delta(\theta_0)}{dz}$ we can rewrite (7)

as:

$$\begin{aligned} \left\{ (1+r^*(\theta_0)) \bar{B} \frac{f(\theta_0)}{\pi} - \frac{z}{p_m \theta_0} \left(\sum_{i=1}^n \gamma_i Y_i(\theta_0) \right)^2 - \Delta(\theta_0) \frac{\sum_{i=1}^n \gamma_i Y_i}{\theta_0} \right\} \frac{d\theta_0}{dz} - \\ z \sum_{i=1}^n \frac{dM_i}{dz} \sum_{i=1}^n \gamma_i Y_i(\theta_0) - \sum_{i=1}^n M_i(\theta_0) \sum_{i=1}^n \gamma_i Y_i(\theta_0) = 0. \end{aligned}$$

Finally, using (4) again and rearranging terms:

$$\frac{d\theta_0}{dz} = \frac{\sum_{i=1}^n M_i(\theta_0) + z \sum_{i=1}^n \frac{dM_i(\theta_0)}{dz}}{\Delta(\theta_0) \left[\frac{f(\theta_0)}{\pi} - \frac{1}{\theta_0} \right] - \frac{z}{p_m \theta_0} \sum_{i=1}^n \gamma_i Y_i(\theta_0)} \quad (9)$$

To evaluate the sign of the above derivative we need to determine the sign of $t = \left[\frac{f(\theta_0)}{\pi} - \frac{1}{\theta_0} \right]$. Notice that the sign of t depends on the elasticity of π with respect to θ_0 being greater or less than one.

Step 2:

We claim that the probability of repayment π is inelastic with respect to θ_0 . This claim can be established from the fact that the supply of credit is positively sloped. In fact, for a given level of trade dependency, in order to have $\frac{d(1+r^*(\theta_0))}{d\bar{B}} > 0$, we need $\frac{d\theta_0}{d\bar{B}} > 0$ (since $\frac{d(1+r^*)}{d\theta_0} > 0$).

Differentiating condition (4) with respect to consumption borrowing (the only type of borrowing which does not affect trade dependency, so that we are considering a move along a same supply curve), we obtain:

$$(1+r^*) + \bar{B} \frac{d(1+r^*(\theta_0))}{d\theta_0} \frac{d\theta_0}{dB_c} - \sum_{i=1}^n \gamma_i \frac{dY_i(\theta_0)}{d\theta_0} \frac{d\theta_0}{dB_c} = 0.$$

Using (8) and simplifying:

$$\frac{d\theta_0}{dB_c} = \frac{(1+r^*)}{(1+r^*) \bar{B} \left[\frac{1}{\theta_0} - \frac{f(\theta_0)}{\pi} \right]}$$

The expression in square brackets above must therefore be positive.

Step 3:

We can now evaluate the sign of $\frac{d\theta_0}{dz}$. From step 2 we conclude that the denominator of (9) is always negative. The numerator may be positive or negative depending on the value of z . Evaluated at $z=0$, $\frac{d\theta_0}{dz} < 0$. But as z increases, and since $\frac{dM_i}{dz} < 0$, $\frac{d\theta_0}{dz}$ increases also and will at some point become positive for a large enough z . We conclude that there is a positive value for z which will minimize θ_0 (maximize the probability of repayment π). ■

Proposition 2: Suppose that the creditor's cost of imposing sanctions on the defaulter ($C(\Omega)$) has a fixed component and a variable one which depends on the level of trade retaliation. Let $C(\Omega) = F + c(z \sum_{i=1}^n M_i)$, with c' increasing and $c'(0)=0$. Then (a) the creditor chooses to make the default penalty an increasing function of the imported input level, setting $z > 0$, (b) z is set below the value which maximizes the probability of repayment π .

Proof:

$$\frac{dC(\Omega)}{dz} = \int_0^{\theta_0} c' \left(z \sum_{i=1}^n M_i \right) \left(\sum_{i=1}^n M_i + z \frac{d \sum_{i=1}^n M_i}{dz} \right) f(\theta) d\theta + f(\theta_0) \left[F + c \left(z \sum_{i=1}^n M_i \right) \right] \frac{d\theta_0}{dz}$$

$$\text{Now, } \frac{\sum_{i=1}^n M_i}{dz} = \sum_{i=1}^n \left[\frac{-M_i(\theta) \sum_{i=1}^n M_i(\theta)}{1 + \Delta(\theta)} \right] \text{ and therefore}$$

$$\sum_{i=1}^n M_i + z \frac{d \sum_{i=1}^n M_i}{dz} = \sum_{i=1}^n \frac{M_i(\theta) [1 + \Delta(\theta) - z \sum_{i=1}^n M_i(\theta)]}{1 + \Delta(\theta)} = \sum_{i=1}^n \frac{M_i(\theta) [1 + z_0]}{1 + \Delta(\theta)} > 0$$

That implies that

$$\frac{d(z \sum_{i=1}^n M_i)}{dz} > 0 \quad (10)$$

Evaluated at $z=0$ ($z \sum M_i = 0$), it turns out that $\frac{dC(\Omega)}{dz} > 0$. Evaluated at z^* such that the probability of repayment is maximized ($\frac{d\theta_0}{dz} = 0$), it turns out that $\frac{dC(\Omega)}{dz} > 0$, given (10) and since $c' > 0$. One can thus conclude that $\frac{dC(\Omega)}{dz} = 0$ occurs for some $z: 0 < z < z^*$. ■

For other types of cost functions, namely when c' is constant, the outcome will depend on the specific parameters of the model.

Proposition 3: *For the default penalties considered in Propositions 1 and 2, trade dependency may be lower in the social optimum than in the market outcome, contrary to Aizenman's results. This case occurs for sufficiently high probabilities of default and, in this event, borrowing for investment in highly trade dependent activities should be taxed.*

The intuition behind this result is the following: more investment in a highly trade dependent activity increases demand for the imported input and therefore the size of the default penalty in the default states. This effect may dominate the openness effect described by Aizenman (the improvement in credit market conditions, relevant for the non-default states), for sufficiently high probabilities of default.

Proof: For the social optimum to be attained, the private cost of borrowing for investment must equal the corresponding social cost. To achieve this equality, taxes and subsidies on domestic credit are used by a planner.

The marginal cost of borrowing for investment is given by

$$\int_{\theta_0}^1 \frac{d(1+r^*)\bar{B}}{dl_i} f(\theta) d\theta + \int_0^{\theta_0} \sum_{i=1}^n \gamma_i Y_i \frac{d\Delta}{dl_i} f(\theta) d\theta.$$

The first term represents the expected marginal repayment and the second term the expected marginal penalty. The latter is absent under Aizenman's penalty regime (since for this regime $\Delta=z_0$, independent of investment)

The private cost of borrowing is given by $\pi(1+r_i)$ where r_i is the internal interest rate on borrowing for investment in activity i , taken as given by private agents. Borrowing for investment in activity i will be taxed at a rate τ_i such that

$$(1+r_i) = (1+r^*)(1+\tau_i)$$

and

$$(1+r_i) = \pi \frac{d(1+r^*)\bar{B}}{dl_i} + \frac{\int_0^{\theta_0} \sum_{i=1}^n \gamma_i Y_i(\theta) \frac{d\Delta(\theta)}{dl_i} f(\theta) d\theta}{\pi(1+r^*)}$$

implying that

$$\tau_i = \frac{d(1+r^*)}{dl_i} \frac{\bar{B}}{1+r^*} + \frac{\int_0^{\theta_0} \sum_{i=1}^n \gamma_i Y_i(\theta) \frac{d\Delta(\theta)}{dl_i} f(\theta) d\theta}{\pi(1+r^*)} \quad (15)$$

The first term is the same as in Aizenman's results. Additionally we have a second term which reflects the increase in the default penalty in the defaulting states. Rearranging the terms in the condition above, the following is obtained:

$$\tau_i = \frac{\pi \left(\frac{d(1+r^*)}{dl_i} \bar{B} \right) + \int_0^{\theta_0} \sum_{i=1}^n \gamma_i Y_i(\theta) \frac{d\Delta(\theta)}{dl_i} f(\theta) d\theta}{\pi(1+r^*)}, \quad (15')$$

where the numerator is an average of the interest-reducing openness effect (which is negative for highly trade dependent activities) and the penalty-increasing effect (which is always positive and stronger the more dependent the productive activity is on trade), weighed respectively by the probability of repayment and the probability of default.

An activity that is highly trade dependent (which reduces interest repayment in case of no default but substantially increases the cost of default in the default states) can be taxed if the probability of default is high enough or can be subsidized if this probability is low enough. ■

References:

Aizenmann, J., 1987, Investment, Openness and Country Risk, NBER Working Paper No. 2410.

Aizenman, J., 1989, Country Risk, Incomplete Information and Taxes on International Borrowing, *The Economic Journal* 99, 147-161.

Aizenmann, J., 1990, External Debt, Planning Horizon and Distorted Credit Markets, *Journal of International Money and Finance* 9 (2), 138-158.

Aizenmann, J., 1991, Trade Dependency, Bargaining and External Debt, *Journal of International Economics* 31, 101-120.

Cabral, C., 1996, Evaluating Debt Repurchases: What Are the Alternatives to Investment?, *Journal of International Economics*, forthcoming.

Diwan, I., 1990, Linking Trade and External Debt Strategies, *Journal of International Economics* 29, 293-310.

Eaton, J., M. Gersowitz, and J. E. Stiglitz, 1986, The Pure Theory of Country Risk, *European Economic Review*.

Rosenthal, R., 1991, On the Incentives Associated with Foreign Debt, *Journal of International Economics* 30, 167-176.