Meaningful Cheap Talk Must Improve Equilibrium Payoffs

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Abstract

We generalize Farrell's (1987) idea of coordination by means of cheap talk. We show that if cheap talk is meaningful (in a specific sense) and if there is room for cooperation (namely if there exists at least one pure-strategy equilibrium Pareto superior to the default equilibrium), then cheap talk must increase the equilibrium expected payoff relative to the play of the game without pre-play communication.

1 Introduction

In a seminal paper, Farrell (1987) has shown that non-binding, costless pre-play communication (cheap talk) may improve the equilibrium payoff in a game with the structure of the “battle of the sexes.”

The “battle of the sexes” is a game that entails both elements of competition and of coordination. It possesses two pure-strategy equilibria which are not Pareto ranked, and a third equilibrium in mixed strategies which is Pareto inferior to the pure-strategy equilibria. The first fact induces the competitive element of the game, while the second one adds the cooperative element. Farrell’s (1987) idea is that cheap talk may help players coordinate in playing one of the pure-strategy equilibria. The idea is that, while players’ preferences are opposite regarding the pure-strategy equilibria, they both prefer any of these equilibria to the mixed-strategy equilibrium, which, by assumption, is the default equilibrium to be played in case no “agreement” is reached at the communication stage.

We generalize Farrell’s (1987) idea in the following sense. We show that if there is room for cooperation, namely if there exists at least one pure-strategy equilibrium that is Pareto superior to the default equilibrium, then cheap talk “must” increase the equilibrium expected payoff. The qualification implicit in the quotes around “must” is that the result is only true if messages are paid attention to, that is, if cheap talk is meaningful. In fact, for any complete information game, there will always exist equilibria in which communication is simply ignored.

The intuition for the result is quite simple. At the communication stage, player i has no incentive to announce a strategy associated with an equilibrium that yields player j a bad payoff. This is so because, in equilibrium, player j will not announce the action corresponding to that equilibrium. In fact, players will only make announcements associated with superior equilibria (equilibria mutually beneficial for both players), which in turn implies that cheap talk improves equilibrium payoffs (relative to the case when communication is not allowed).

2 Basic results

Consider the following two-player game in normal form, denoted by G : S_i is the space of player i 's pure strategies and s_i is a generic element of S_i , $i = 1, 2$; $S \equiv S_1 \times S_2$ and $s \equiv (s_1, s_2)$; finally, $u_i(s)$ denotes player i 's utility.

Based on game G , we now define an extended game, denoted by Γ , which comprises a communication stage prior to playing G . At the communication stage, players simultaneously announce actions from their own pure-strategy sets S_i (the terms "action" and "pure strategy" mean the same thing). After all messages are sent and received, game G is played.

As noted before, games of this sort always admit equilibria in which cheap talk has no influence on the outcome of G ("babbling equilibria").¹ Following Farrell (1987), we concentrate on *no-babbling equilibria*, that is, equilibria with the following properties:

- (i) If announcements correspond to a Nash equilibrium of G , then that equilibrium is played.
- (ii) If announcements do not correspond to a Nash equilibrium of G , then a "default" equilibrium is played independently of which announcements are made. Payoffs in this default equilibrium are given by $\bar{u}_i$. By assumption, this is also the equilibrium when no communication is possible.

Notice that properties (i)–(ii) induce a well-defined reduced game at the communication stage, so the set of no-babbling equilibria (Nash, perfect Nash, or proper Nash) is non-empty.

Finally, we define s to be a superior equilibrium of G if and only if s is a pure-strategy equilibrium and $u_i(s) > \bar{u}_i$, $\forall i$. Our main result will be based on the hypothesis that there exists at least one superior equilibrium. This hypothesis implies that G is to some extent a game of coordination, specifically, a game with at least two Nash equilibria which are Pareto ranked.

¹See, for example, Farrell and Gibbons (1989). Seidmann (1992) shows that the same is not necessarily true in games of incomplete information.

Theorem 1 *Assume that all pure-strategy Nash equilibria of G are strict. If there exists some superior equilibrium of the original game, then, in a proper no-babbling equilibrium of the extended game, only actions corresponding to superior equilibria are announced, and expected payoff is strictly greater than in the game with no communication.*

Proof: Consider a “perturbed” game in the communication stage. In this game, every message is announced with positive probability. Therefore, it is a strictly dominated strategy for player i to announce an action corresponding to a pure-strategy Nash equilibrium, say s^* , yielding him or her a payoff less than $\bar{u}_i$. In fact, the expected payoff of announcing such an action would be lower than $\bar{u}_i$, by the assumption that all pure-strategy Nash equilibria are strict; whereas announcing an action corresponding to a superior equilibrium, say s^{**} , would yield player i an expected payoff greater than $\bar{u}_i$. By the requirement of properness (Myerson, 1978), the weight assigned by player i to the action corresponding to s^* must be at least one order of magnitude lower than the weight assigned to the action corresponding to s^{**} .

Now, given that, it is also a dominated strategy for player j to announce the action corresponding to s^* , assuming that payoffs are bounded and that the “perturbed” game is sufficiently close to the “unperturbed game.” To see why, notice that player j ’s expected payoff from announcing the action associated to s^* is

$$A = P(s_i^*)u_j(s^*) + (1 - P(s_i^*))\bar{u}_j,$$

where $P(s_i^*)$ is the probability that action s_i^* is announced by player i . On the other hand, announcing the action corresponding to s^{**} yields player j an expected payoff of

$$B = P(s_i^{**})u_j(s^{**}) + (1 - P(s_i^{**}))\bar{u}_j.$$

Since $u_j(s^{**}) > \bar{u}_j$ and $P(s_i^*) \leq \epsilon P(s_i^{**})$ (the latter by properness), we have $A < B$.

Finally, a similar argument, if somewhat simpler, applies to actions corresponding to no pure-strategy Nash equilibrium. Likewise, it is straightforward to show, by

Figure 1: Game G in first example.

	L	C	R
T	2 1	0 0	0 1
M	0 1	2 0	0 0
B	1 0	1 2	0 1

Figure 2: Game Γ in first example.

	L	C	R
T	2 1	2/3 2/3	0 1
M	2/3 2/3	2/3 2/3	2/3 2/3
B	2/3 2/3	2/3 2/3	2/3 2/3

contradiction, that at least some superior equilibrium must be played with positive probability. ■

3 Counterexamples

In this section, we present a series of counterexamples which elucidate on the importance of some of the assumptions and hypotheses implicit in the main result of the previous section.

The first example shows how the assumption that all pure-strategy Nash equilibria are strict is necessary for the result. In this example, the original game, shown in Figure 1, has two pure-strategy equilibria, (T, L) and (T, R) , none of which is a strict Nash equilibrium. There also exists a mixed-strategy equilibrium in which both players choose each action with equal probability. The expected payoff in this equilibrium, which we assume is the default equilibrium, is $2/3$ for each player. Since we restrict to no-babbling equilibria, expected payoffs at the communication stage are summarized by the payoff matrix in Figure 2. It can be seen that it is a proper equilibrium for the row player to choose M and the column player to choose L with

Figure 3: Game G in second example.

	L		C		R	
T	0	0	3	-3	9	1
M	-3	3	6	6	0	0
B	1	9	0	0	0	0

Figure 4: Game Γ in second example.

	L		C		R	
T	3/2	3/2	3/2	3/2	9	1
M	3/2	3/2	6	6	3/2	3/2
B	1	9	3/2	3/2	3/2	3/2

probability α and R with probability $1 - \alpha$, with $\alpha < 1/3$. Although there exists a superior equilibrium, namely (T, L) , the row player never announces the strategy corresponding to this equilibrium.

The second example shows that perfection is not a sufficient refinement to produce the main result. The original game in this example is described in Figure 3. The game has three pure-strategy equilibria, (T, R) , (M, C) and (B, L) . There also exists a mixed-strategy equilibrium in which players choose the first two actions with equal probability. Expected value in this equilibrium, which we assume is the default equilibrium, is $3/2$ for each player. The payoff structure at the communication stage is given by the matrix in Figure 4.

We will now show that it is a perfect equilibrium, although not a proper equilibrium, for the row player to announce T and the column player to announce L . For this purpose, consider the following strategies in an ϵ -perturbed game: each player chooses the first strategy with probability $1 - 2\epsilon$ and the two remaining strategies with probability ϵ each. Simple inspection reveals that this constitutes a Nash equilibrium given the constraint that all actions be chosen with probability greater than ϵ . Thence, the designated equilibrium is indeed perfect. However, the strategies in the perturbed game do not satisfy the requirement for properness, namely that the

Figure 5: Game G in third example.

		<i>L</i>			<i>R</i>					<i>L</i>			<i>R</i>				
<i>T</i>		0	0	1	0	0	0			4	4	0	0	0	2		
<i>B</i>		1	3	3	1	0	0			0	0	2	4	4	4		
		<i>LM</i>								<i>RM</i>							

Figure 6: Game Γ in third example.

		<i>L</i>			<i>R</i>					<i>L</i>			<i>R</i>				
<i>T</i>		2	2	2	2	2	2			2	2	2	2	2	2		
<i>B</i>		1	3	3	2	2	2			2	2	2	4	4	4		
		<i>LM</i>								<i>RM</i>							

third strategy be chosen by each player with a probability one order of magnitude lower than the second one. In fact, our main result states that the only equilibrium at the communication stage consists of players announcing the middle strategy with probability one.

Finally, the third example shows that the result does not extend to games with more than two players. Figure 5 depicts the original game in this example. As before, players 1 and 2 choose rows and columns, respectively. Now we add a third player who picks one of the two matrices, LM or RM . The game has two pure-strategy equilibria, (B, L, LM) and (B, R, RM) , both of which are strict. In addition, there exists a mixed-strategy equilibrium in which the first two players mix with equal probability each of their actions and the third player chooses RM . Expected payoff under this equilibrium, which we assume is the default equilibrium, is given by 2 for all players. The payoff matrix at the communication stage is thus the one described in Figure 6.

We will now argue that (T, L, LM) constitutes a proper equilibrium of Γ , even though one of the players, player 1, chooses an action not associated with any Nash equilibrium. Consider the following equilibrium of the perturbed game. Player 1

chooses T with probability $1 - \epsilon$ and B with probability ϵ . Player 2 chooses L with probability $1 - \epsilon$ and R with probability ϵ . Finally, player 3 chooses LM with probability $1 - \epsilon$ and LM with probability ϵ . Clearly this equilibrium converges to the designated Nash equilibrium. We will now show that it constitutes an equilibrium of the properly perturbed game.² If player 1 deviates, it gets $1(1 - \epsilon)^2 + 4\epsilon^2 + 2 \cdot 2\epsilon(1 - \epsilon)$, which is lower than 2, its equilibrium payoff. If player 2 deviates, it gets $4\epsilon^2 + 2(1 - \epsilon^2)$, which is lower than $3\epsilon(1 - \epsilon) + 2(1 - \epsilon + \epsilon^2)$, its equilibrium payoff. Finally, if player 3 deviates, it gets $4\epsilon^2 + 2(1 - \epsilon^2)$, which is lower than $3\epsilon(1 - \epsilon) + 2(1 - \epsilon + \epsilon^2)$, its equilibrium payoff.

4 Final remarks

As Farrell (1988) has noted, “cheap talk is notoriously hard to model: there are no obviously ‘right’ rules about who speaks when, what he may say, and when discussion ends.” Not surprisingly, different structures of pre-play communication have been attempted in the literature, some of which present results related to ours.

Farrell (1988) and Watson (1991) consider the case in which one of the players unilaterally suggests the other which set of strategies to play. Watson (1991) shows that if the original game has a single Pareto-efficient outcome, then this is the only sensible outcome of the extended game.

Matsui’s (1991) assumes a structure of pre-play communication similar to ours, with players simultaneously sending messages. However, the equilibrium concept he considers is quite different from ours. He considers a large population matched to play a game of common interest with cheap talk. He shows that a unique cyclically stable set exists and this contains only Pareto optimal outcomes.

The paper which is closest to ours is Rabin (1994). As in our paper, he considers a two-player game and simultaneous message exchange. However, he considers a wider set of communication possibilities than our paper (and Farrell’s 1987). Rabin shows that, with enough rounds of cheap talk, each player’s expected payoff is at

²Since each player has only two pure strategies, properness corresponds to trembling-hand perfectness.

least as great as the payoff from his worst Pareto-efficient Nash equilibrium.

Our paper partly confirms the communication-yields-efficiency hypothesis, to borrow Rabin's (1994) expression, although we show some important qualifications need to be made.

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