Multimarket Competition in Banking, with Application to Portugal

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Working Paper nº 215
Fevereiro de 1994
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February 16, 1994

draft

Abstract

Bank activities usually span over several markets and branches. The multi-location nature of the bank may originate equilibrium price dispersion. The implication of it for applied work of market power measurement in the banking industry are investigated. The aim of the paper is to evaluate how a spatial model can explain price differences across banks in the deposits market. Also of interest is the identification of a general pattern of conduct in the market, that is, how collusive is market equilibrium. An application to Portuguese commercial banking is reported. The results show a satisfactory performance of the spatial framework. As to conduct, Nash behavior for all banks and collusion among firms of the same economic group receive support from the data, although the evidence in favor of the first is stronger. Market-wide collusion is rejected as description of oligopolistic interaction.

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*Financial support from EC Human Capital and Mobility Fellowship grant # ERBCHBGCT920147 is gratefully acknowledged.
1 Introduction

The banking industry presents characteristics that make it different from other sectors. One of them is the multi-location nature of its activities and localized competition. Typically, some banks have more branches and cover a more extensive territorial area while others have just a small number of branches. Thus, not all banks operate in every local market. The existence of local markets gives banks more market power than an unified national market and assessment of the exercise of market power should take into account this phenomenon.

Several studies of the banking industry have tested for the existence of market power. In one line of research banks are modelled as deciding on prices and product differentiation is assumed (Hannan, 1991). One of the main hypotheses tested is that the exercise of market power varies positively with market concentration. Product differentiation is usually controlled for through the inclusion, in the econometric analysis, of bank specific characteristics. This is a correct procedure if a bank operates only in one market. However, recent research has found localized competition effects and spatial competition variables to be of importance (Fuentelsaz and Salas, 1992; Barros and Leite, 1994; Hannan and Liang, 1993).

Banks operating over several markets are usually restricted to no price discrimination across their own branches (at least in small areas). This is due, for instance, to the organizational costs involved or by an implicit ethical code, specially in small countries. In large countries, one may see price discrimination across broad regions, but not in small areas within a region. In any case, one does not see extensive decentralization of price policies to the branch level, specially in deposits. Managers may have some discretionary power to adjust marginally interest rates on deposits, within tight bounds defined at the central office. Special treatment to some customers is usually required to be cleared by the central office.

This constraint on a market where a bank has several branches is also present across local markets. In such case, price discrimination across markets would yield higher profits to the bank. Its existence may be justified by reputational costs or by internal costs of implementation of price discrimination, or both. An alternative explanation is that local markets seemingly independent may constitute a single market from the point-of-view of the bank. Besides those factors, a rationale for such constraint to be profit maximizing for a bank in a single market can be presented. Multibranching considerations give
raise to different policies across banks. This is specially true in the deposits side, since with respect to
loans, manager discretion at the branch level may allow effective price decentralization.

The constraint of a common pricing policy over all local markets creates spillovers from one
market to the other. Such constraint has important implications for the optimal choice of rates by
banks. Pricing policies of banks can change significantly due to it. Important consequences for the
assessment of market power based on bank data aggregated over local markets follow. Namely, banks
operating in different local markets will have, probably, distinct prices, originating equilibrium price
dispersion. This is an important issue for applied work on the banking sector. Unlike the U.S., most
countries do not have data for banks on a local basis neither are banks restricted in branching over
several local markets. To characterize the precise way the analysis must be adjusted for this effect is
the purpose of the paper.

I shall set forth that only pricing decisions are analyzed. Branching decisions as well as the choice
of the local markets for a bank to operate are taken as given. The main question to be addressed is how
does the pricing policy of a bank changes when the same price must be charged in all of its branches
and whether this consideration can explain interest rate differentials across banks. To focus exclusively
on spatial competition, a version of the Salop (1979) circular city model is extended to multimarket
considerations.

One similar model, in the approach, can be found in Fuentelsaz and Salas (1992). They also present
a model of banking spatial competition inspired by Salop's (1979) model. Their aim is, however, quite
different. The authors look at space per se. That is, the circle is interpreted as geographic space. Both
pricing and branching decisions are considered. Despite its simplicity, the model works remarkably
well in the explanation of banks' pricing policies across Spanish regions and across European countries.

A feature of the present work is the introduction of explicit multibranch banks and the constraint
of an equal pricing rule across branches of the same bank. A theoretical justification for this constraint
is presented. The structural model of spatial competition yields a precise rule (first-order condition
for profit maximization) for the optimal choice of rates by banks. The estimation of the model with
Portuguese data is performed.

Earlier work on the Portuguese market has found spatial competition variables to be significant in
the explanation of dispersion of margins across banks (Barros and Leite, 1994). This paper attempts
to give theoretical support to that empirical finding.
The organizational of the paper is as follows. Section 2 presents the structure of the theoretical model. The next section, Section 3 explores the effects on pricing rules of multimarket/multibranch considerations. Section 4 presents some issues pertaining to the empirical implementation of the model. Section 5 discusses the data of the study and the econometric methods. Section 6 presents the empirical results for the Portuguese banking sector. Section 7 explores a simpler empirical model. Finally, Section 8 concludes.

2 The Model

The fact that banks can be arguably taken as price-taker in one of the markets in which they operate, namely a government securities or a money market, allows to dichotomize the analysis treating separately the deposits market and the loans market.

A simple intuitive explanation for this feature can be given. Suppose that banks are price-takers in the money market. The opportunity cost of lending one unit of money is its interest income on the money market, no matter what happens in the deposits market. This exogenous rate, together with real resources costs incurred, is the only element needed to investigate pricing strategies in the credit market. By the same reasoning, the deposits market can be treated on its own.\(^1\) Given this characterization, the analysis is cast in general terms. Applications of the model can be made to other products where local competition matters. The interpretation of the variables in each market is straightforward. It is made precise for the deposits market in the application developed below. The terms firm and bank are used with the same meaning.

A spatial competition model in the vein of Salop (1979) is considered. The adoption of a model of horizontal product differentiation, instead of vertical differentiation, is not obvious. Certain characteristics of products offered by banks are clearly of horizontal differentiation while others correspond more to vertical differentiation. In the latter category, an example is the (in)existence of lines in branches. In the former type of characteristics, geographic space and complexity and sophistication of services are good examples. In the case of banking, complexity of additional services may increase its user cost to consumers. Certain level of services may be more preferred by some consumers and less by others. We believe that horizontal differentiation is more important in consumers' decisions in retail banking than vertical differentiation.

\(^1\)For derivations and a more complete discussion see Hannan (1991).
Each local market is represented by a one-dimensional characteristics space (circle) of length one. Bank customers are located continuously, with an uniform distribution of density \( \delta \). The circle describing each local market should be interpreted as a summary description of consumers' preferences over characteristics of banks. Geographic location is one important element on the definition of the characteristics space but by no means the only one. It justifies, for example, that a deposit featuring the same conditions can be differently located in terms of characteristics across local markets or across branches of a bank in a specified local market. Other characteristics, as the existence of lines or a personalized service, interact in some unspecified way to generate the location of each branch in consumers' preferences.

The spatial model focus on location and localized competition. Multibrand competition issues, in the sense that consumers buy several brands, are of somewhat secondary importance in retail banking. Consumers (depositors and borrowers) operate mainly with one bank even if they have accounts in more than one bank.\(^2\)

The branching decision itself is not considered explicitly. It is taken as given from a previous stage. This decision is, in the context of the present model, more sophisticated that usually modelled. A bank must decide the local markets it enters, the number of branches in each of them and the geographical location and other characteristics that determine the location of the bank in consumer preferences.

Addition of the variable "length of circle" can be easily done. However, it increases the number of parameters to be identified, without adding a special insight to the theoretical model. In fact, it creates an interpretation problem: explaining changes from market to market in preferences of consumers, which in turn determine the set of possible characteristics. For this reason, the characteristics circle is assumed to be equal in all markets.

An inelastic demand structure is assumed. Each customer buys one unit of of the goods (borrows one unit of money or deposits one unit), which he values \( v \). This value is high enough for individuals to buy at present prices. Consumers have also to pay a unit transport cost. The transportation cost reflects the utility cost to consumers of not buying their most preferred variety. The assumption of inelastic demand is potentially more restrictive. However, in the application performed, retail banking for deposits, it is perhaps more reasonable than in other contexts. Empirical estimates for the supply

\(^2\)Models with differentiated products characterized by competition of each brand/product with every other brand/product are in the tradition of the Chamberlinian model.
elasticity of deposits are low and indicate inelastic demand, although not perfectly inelastic.\footnote{The extension to elastic demand schedules can, in principle, be made along well-known lines. See Greenhut, Norman and Harg (1987).}

Since the emphasis is on spillovers across markets, various local markets are considered simultaneously. Several degrees of freedom in the modelling procedure are then available: the number of branches of a bank in each market, the number of local markets a bank operates, different transport costs and consumers’ density across markets.

The unit of reference is the branch, indexed by $m$. The following notation will be used throughout: $t_{k(m)}$ denotes the transport cost in market $k$ where branch $m$ operates; $\delta_{k(m)}$ is the density of consumers (market size); and $n_{k(m)}$ is the total number of branches in market $k$. For tractability, branches within each market are assumed to locate symmetrically. A consumer located at $x$ is indifferent between going to branch $m$ of bank $i$ or to branch $m+1$ if

$$v - t_{k}x - p_{i} = v - t_{k} \left( \frac{1}{n_{k}} - x \right) - p_{i+1}$$

(1)

where $v$ denotes consumer valuation of the good.

Demand of branch $m$ of bank $i$ is given by the sum of $D^{m+1}$ and $D^{m-1}$, the local demands in each side of the branch, where

$$D^{m+1} = \frac{1}{2n_{k}} + \frac{p_{m+1} - p_{i}}{2t_{k}}$$

(2)

and

$$D^{m-1} = \frac{1}{2n_{k}} + \frac{p_{m-1} - p_{i}}{2t_{k}}$$

(3)

Total demand for branch $i$ in market $k$ is defined by

$$D^{ik} = \delta_{k} \left( \frac{p_{i+1} + p_{i-1}}{2t_{k}} - \frac{p_{i}}{t_{k}} + \frac{1}{n_{k}} \right)$$

(4)

The symmetric Nash equilibrium with price discrimination across markets will be our benchmark equilibrium. It also corresponds to the case of one bank per market only. In this case, the equilibrium price in market $k$ is just:\footnote{The index $m$ will be omitted whenever no confusion arises.}

$$p_{i} = c + \frac{t_{k}}{n_{k}}$$

(5)
where \( c \) is the marginal cost, assumed to be equal for all firms. All banks name the same price. The equilibrium price is independent of the size of the market, varies positively with transportation costs and negatively with the number of competitors present in the market.

The profit function of the bank \( i \) is

\[
\Pi^i = \sum_{m=1}^{n_i} \delta_{k(m)} \left[ \frac{p_{m+1} + p_{m-1}}{2\ell_{k(m)}} - \frac{p_i}{\ell_{k(m)}} + \frac{1}{n_{k(m)}} \right] (p_i - c)
\]  

(6)

The first-order condition for profit maximization in the choice of price is

\[
\frac{\partial \Pi^i}{\partial p_i} = \sum_{m=1}^{n_i} \delta_{k(m)} \left[ \frac{p_{m+1} + p_{m-1}}{2\ell_{k(m)}} - \frac{p_i}{\ell_{k(m)}} + \frac{1}{n_{k(m)}} \right] - (p_i - c) \sum_{m=1}^{n_i} \delta_{k(m)} \ell_{k(m)} = 0
\]  

(7)

The equilibrium price of each bank is indexed to the equilibrium interest rates of its neighbors in all branches, which need not to practice the same interest rate. The presence of multibranch banks under the constraint of no price discrimination across branches together with differences either in transportation costs or number of branches operating in each local market is sufficient to generate asymmetric equilibria. An asymmetric equilibrium means that in equilibrium each bank may name a different value for the interest rate. Locations are still symmetric within each local market for tractability of the model.

Unless additional assumptions are made, model possibilities are too rich to make possible a precise identification of the effects present. To identify the effects at play in this particular extension of the Salop (1979) model, several simplified cases are presented in the next section.

3 Individual effects across local markets

To look at one effect at a time, it is assumed that each bank has at most one branch in each local market. The case of multibranch banks is treated afterwards.

As to the remaining variables, we assume first an equal number of banks in each local market but different transport costs. Second, equal transport costs across markets are considered but the number of banks operating in each market differs. Third, consumers’ density differ across markets. Those three effects work together to make price-cost margins differ across banks.

3.1 Differences in transport costs

Suppose that there are two separate markets (indexed by \( A \) and \( B \)) and three banks (indexed by 1, 2 and 3). As a rule, uppercase letters will index markets and roman numericals index banks. Bank 1
and bank 3 operate in one market only, respectively market A and market B. Bank 2 operates in both markets.

Total demand of bank \( i \) in market \( k \) is given, in the case of two banks, by

\[
D_{ik}^k = \frac{1}{2} + \frac{p_i - p_j}{t_k}, \quad j \neq i;
\]

(8)

The profit of a bank located in one market only (\( i = 1, 3 \)) is

\[
\Pi_{ik}^k = (p_i - c) \left( \frac{p_j - p_i}{t_k} + \frac{1}{2} \right)
\]

(9)

The corresponding first-order condition for profit maximization is

\[
\frac{\partial \Pi_{ik}^k}{\partial p_i} = \frac{p_j - p_i}{t_k} + \frac{1}{2} - \frac{p_i - c}{t_k}
\]

(10)

For the multimarket bank, \( i = 2 \), profit under the constraint of no price discrimination across markets is given by

\[
\Pi^2 = (p_2 - c) \left[ \left( \frac{p_2 - p_3}{t_A} + \frac{1}{2} \right) + \left( \frac{p_2 - p_3}{t_B} + \frac{1}{2} \right) \right]
\]

The associated first-order condition for profit maximization is

\[
\frac{\partial \Pi^2}{\partial p_2} = \frac{p_1 - p_2}{t_A} + \frac{p_3 - p_2}{t_B} - \frac{p_2 - c}{t_A} - \frac{p_2 - c}{t_B} = 0
\]

(11)

Simultaneous resolution of all first-order conditions gives the equilibrium interest rates charged by each bank:

\[
p_1^* = c + \frac{t_A(t_A + 3t_B)}{4(t_A + t_B)}
\]

\[
p_2^* = c + \frac{t_A t_B}{t_A + t_B}
\]

\[
p_3^* = c + \frac{t_B(t_B + 3t_A)}{4(t_A + t_B)}
\]

In the case of equal transport costs for all markets, \( t_A = t_B \), the constraint of no price discrimination is not binding and the benchmark equilibrium prices result (see equation 5). Assume, without loss of generality, that \( t_A > t_B \). The change in equilibrium prices are as follows:

\[
p_1^* - p_1^0 = \frac{-t_A(t_A - t_B)}{4(t_A + t_B)} < 0
\]

\[
p_3^* - p_3^0 = \frac{t_B(t_A - t_B)}{4(t_A + t_B)} > 0
\]
\[
\begin{align*}
    p_{2,A}^* - p_{2,A}^0 &= \frac{t_A(t_A - t_B)}{2(t_A + t_B)} < 0 \\
    p_{2,B}^* - p_{2,B}^0 &= \frac{t_A(t_A - t_B)}{2(t_A + t_B)} > 0
\end{align*}
\]

Since local market power is greater in the market with higher transport, in the absence of any constraint upon its pricing policies, bank 2 would have charged a higher price in that market. When price discrimination is not possible, the bank will adjust to an intermediate value. The price in the market with higher transport cost decreases and the other bank also lowers its price in response. The opposite effects occur in the other local market. Thus, the presence of multimarket banks reduce the exercise of monopoly power in local markets, with high transportation costs, but increase it in markets with lower product differentiation, when no price discrimination is allowed. More importantly, the following ranking of equilibrium prices of each bank can be established: \( p_i^* > p_j^* > p_k^* \). That is, banks which operate in several markets will have higher prices than banks which work only in the local markets with lower product differentiation/transport costs. Similarly, they will set lower prices than banks working only in markets with substantial product differentiation/transport costs. The basic intuition of the example is formalized in the following proposition.

Proposition 1  Assume only one multimarket bank. The constraint of no price discrimination across markets generates, in the presence of different transport costs across markets, equilibrium price dispersion within local markets.

Proof: The profit of the multibranch bank is given by

\[
\Pi^i = \sum_{m=1}^{n_i} \left( \frac{1}{n} + \frac{p_{m+1}^k + p_{m-1}^k}{2t_k} - \frac{p_i}{t_k} \right) \left( p_i - c \right) 
\]

(12)

and profits for one-branch banks are

\[
\Pi^j = \left( \frac{1}{n} + \frac{p_{j+1}^k + p_{j-1}^k}{2t_k} - \frac{p_j}{t_k} \right) \left( p_j - c \right) 
\]

(13)

In the case of price discrimination across markets, the equilibrium price is equal for all banks in the same market and different across markets: \( p_0^k = c + t_k/n \). Imposing the constraint of no price discrimination across markets, the first-order condition for profit maximization of the multibranch bank is

\[
\frac{3\Pi^i}{\partial p_i} = \sum_{m=1}^{n_i} \left( \frac{1}{n} + \frac{p_{m+1}^k + p_{m-1}^k}{2t_k} - \frac{p_i}{t_k} \right) - (p_i - c) \sum_{k=1}^{n_i} \frac{1}{t_k} = 0
\]

(14)
No previous equilibrium price (in each of the markets where the multimarket bank operates) constitutes a Nash equilibrium. Take a price \( p_0 = e + t_k / n \). Marginal profit at that price is

\[
\frac{\partial \Pi_i}{\partial p_i} \bigg|_{p_0 = e + t_k / n} = 2 \frac{n_i}{n} - 2 \frac{t_k}{n} \sum_{k=1}^{n_i} \frac{1}{t_k}
\]  

(15)

To be an equilibrium,

\[
n_i - t_k \sum_{k=1}^{n_i} \frac{1}{t_k} = 0 \quad \text{or} \quad t_k = \frac{n_i}{\sum_{k=1}^{n_i} 1/t_k}
\]  

(16)

but for this \( t_k \), the first-order condition for profit maximization one-market banks adjacent to bank \( i \) is:

\[
\frac{\partial \Pi_j}{\partial p_j} \bigg|_{p_0 = e + t_k / n} = \frac{1}{2n} \left( \frac{t_k}{t_k} - 1 \right)
\]  

(17)

and for \( t_k \neq t_k \) the condition is not satisfied. So, none of the previous prices is an equilibrium.

The multimarket bank will adjust the price in a natural way, raising it in the markets where it is low and lowering in markets where high prices were prevalent. The other banks will also adjust its equilibrium prices. In each market, a chain effect on prices is created. Each bank will charge a different price. The structure of first-order conditions of one-market banks is similar to a second-order difference equation. Its resolution implies a different price named by each bank.\(^6\)

The overall level of prices increases or decreases according to whether the multimarket bank decreases or increases its price in a particular market. Neighbor banks follow the direction of change (by less than a one-to-one basis), and prices go down in markets with high transport costs (the ones where the multimarket bank lowers its price) and go up in the low transport costs markets. Price dispersion within each market is thus the result. \( \blacksquare \)

Proposition 1 shows the importance of looking not only at the concentration level or the degree of product differentiation in local markets as predictors of market power, but also at the existence of multimarket banks and whether they operate primarily in other markets, which may be more or less differentiated. Equilibrium prices are thus influenced by the degree of product differentiation in other markets when multimarket banks operate. Of course, with more than one multimarket bank, it is possible to design situations where symmetric price equilibria arise (for example, two markets and

\(^6\)For a full derivation of the equilibrium prices see proofs of Proposition 5 and Proposition 6.
two multimarket banks). The possible cases offer a too rich set of configurations. No general rule can be stated.

3.2 Differences in the number of operating banks

The second version of the model to be investigated characterizes the effect on equilibrium interest rates of multimarket banks when the number of banks differ across markets. Suppose that there are now identical transport costs, denoted by \( t \), and four banks. Banks 1 to 3 operate in market \( A \) and banks 2 and 4 in market \( B \). Bank 2 is again the only multimarket bank.

Under price discrimination across markets, equilibrium interest rates are \( p^0_{i,A} = c + t/3 \) in market \( A \) and \( p^0_{i,B} = c + t/2 \) in market \( B \). Demands in market \( A \) and market \( B \) can be readily computed and the ordering of equilibrium prices is given by \( p^*_1 = p^*_2 < p^*_3 < p^*_4 \).

Multimarket banks tend to lessen competition in local markets with many competitors. In markets with just a few competitors that type of banks increases the degree of local competition. In a sense, multimarket banks "shift" competitive pressure from markets where it is high to markets with low competition.

Different number of banks across local markets and existence of multimarket banks can generate dispersion in equilibrium prices. Again, it is possible to design situations where a symmetric outcome emerges. The next proposition states the result.

**Proposition 2** Assume only one multimarket bank. The constraint of no price discrimination generates, in the presence of a different number of competitors across markets, equilibrium price dispersion.

**Proof:** In this case, transport costs are equal across markets. To economize notation, normalization of transport costs to unity is made. The profit of the multimarket bank is given by

\[
\Pi' = \sum_{m=1}^{n} \left( \frac{1}{n_k} + \frac{p^k_{n+1} + p^k_{n-1}}{2} \right) (p_i - c)
\]  

(18)

The first-order condition for profit maximization, under the constraint of an equal price across markets, is:

\[
\frac{\partial \Pi'}{\partial p_i} = \sum_{m=1}^{n} \left( \frac{1}{n_k} + \frac{p^k_{n+1} + p^k_{n-1}}{2} \right) - n_i p_i - n_i (p_i - c) = 0
\]  

(19)

With respect to one-market banks, profits and the first-order condition for profit maximization are
given, respectively, by:

\[
\Pi^i(k) = \left( \frac{1}{n_k} + \frac{p^k_{j+1} + p^k_{j-1}}{2} - p_j \right) (p_j - c)
\]

\[
\frac{\partial \Pi^i(k)}{\partial p_j} = \frac{1}{n_k} + \frac{p^k_{j+1} + p^k_{j-1}}{2} - 2p_j + c = 0
\]

The previous equilibrium prices, without the constraint of no price discrimination across markets, are no longer an equilibrium. Suppose that one-market banks set the previous equilibrium price: 

\[p^*_j = p^0_j = c + 1/n_k \]

Then the equilibrium price of bank \(i\) is found solving the first-order condition:

\[p^*_j = c + \frac{1}{n_i} \sum_{k=1}^{n_i} \frac{1}{n_k} \]

which is different from \(p^0_j\). Then one-market banks will also deviate from the proposed equilibrium price. A chain of price effects results. The price \(p^*_j\) implies that bank \(i\) wants to raise the price in markets where it faces many competitors and lower it in markets with a small number of competitors.

Again, price dispersion within the market characterizes the equilibrium outcome (except for the case of \(1/n_k = p^*_j - c\)). ■

3.3 Different demand densities

In the standard circular city model the equilibrium price is neutral to demand density. This result can be extended to the presence of multimarket considerations only if the all markets have the same number of firms and the same level of transport costs. The neutrality results from the fact that equilibrium prices in each market depend only on the number of firms and transport costs. If they are held constant across markets, equilibrium prices are identical in all markets. Densities of demand play no role in such particular framework. The profit of bank \(i\) is given by

\[
\Pi^i = \sum_{m=1}^{n_i} \delta_{k(m)} \left\{ \frac{1}{n_{k(m)}} + \frac{p^k_{m+1} + p^k_{m-1}}{2n_{k(m)}} - \frac{p}{t_{k(m)}} \right\} (p_i - c_i)
\]

The first-order condition is, holding the number of banks, \(n\), and transport costs (normalized to one) equal for all local markets,

\[
\frac{\partial \Pi^i}{\partial p_i} = \sum_{m=1}^{n_i} \delta_{k(m)} \left\{ \frac{1}{n} + \frac{p^k_{m+1} + p^k_{m-1}}{2} - p_i \right\} - p_i + c_i = 0
\]
Explicit resolution of the set of first-order conditions is not easy since it implies the knowledge of the neighbors of each bank in each market. It is however possible to characterize the equilibrium price without having to specify the exact location of a bank in each market and in which markets a particular bank operates.

**Proposition 3** Assume only one multimarket bank. The constraint of no price discrimination, in the presence of different consumer densities across markets, does not change the Nash equilibrium.

The Nash equilibrium has an equal equilibrium price for all banks and it is given by the unconstrained equilibrium price

$$p_i = \frac{1}{n} + c, \forall i$$  \hspace{2cm} (23)

**Proof:** The equilibrium is defined by the set of first-order conditions (22). Suppose that all banks charge the same price, $p_i = \tau, \forall i$. Then the first-order condition can be written as

$$\sum_{m=1}^{n_i} \frac{\delta_{k(m)}}{n} - \sum_{m=1}^{n_i} \delta_{k(m)}(\tau - c) = 0$$ \hspace{2cm} (24)

Solving this equation for $\tau$,

$$\tau = \frac{1}{n} + c$$ \hspace{2cm} (25)

This price, equal across banks, does satisfy all first-order conditions. To see that it constitutes a Nash equilibrium, if the other banks follow the rule, the optimal price of the bank is to set precisely the same price. Since the set of first-order conditions is a linear system with full rank, it has only one solution. $\blacksquare$

Different densities across markets do not, per se, change the symmetric equilibrium even in the presence of multimarket banks. Only when the number of competitors and/or the transport costs differ across markets the equilibrium price is determined also by local demand densities.

As seen from previous sections, multimarket firms will average, in a certain sense, the equilibrium prices in individual markets. Changes in prices in markets with higher demand densities have a higher impact on firms profits. A decrease in price is able to attract more consumers in high density markets.
3.4 Multibranch Banks

A bank can, in reality, have more than one branch in each local market. To investigate the implications of multibranch banks (defined as banks with more than one branch in each market) a single market is considered. This isolated the effect of multiple branches of the same bank on the market equilibrium. The crucial feature is the existence, or not, of adjacent branches of the same bank. Consider first the situation characterized by each branch having neighbors only from other institutions. Then the following result can be stated.

**Proposition 4** Multibranching in the same market does not change the symmetric Nash equilibrium corresponding to the case of one-branch – one-bank as long as branches of the same bank are not adjacent.

**Proof:** There are three types of banks: those with several branches, those with neighbors to a multibranch bank, and those which have only one branch banks as neighbors.

Let's start with second and third type of banks. If the adjacent banks set the equilibrium interest rate, then they have no gain in a deviation from it. The same reasoning applies to the second type of banks.

It now suffices to show that for a multibranch bank, under Nash behavior and restricting it to have no adjacent branches, since competition is localized, if all other banks charge the equilibrium interest rate of the symmetric Nash equilibrium, there is no incentive to deviate from the equilibrium. The constraint of no price discrimination across branches is not binding.

In a more rigorous way, a multibranch bank with no adjacent branches has the following profit function,

$$\Pi^i = \frac{n_i}{n} \left( \frac{1}{n} + \frac{p_{m+1}^k + p_{m-1}^k}{2t} - \frac{p_i}{t} \right) (p_i - c)$$  \hspace{1cm} (26)

which simplifies to

$$\Pi^i = \left( \frac{n_i}{n} + \frac{\sum_{m=1}^{m=n}(p_{m+1}^k + p_{m-1}^k) - n_i}{2t} \right) (p_i - c)$$  \hspace{1cm} (27)

The marginal profit when the bank takes price as its decision variable is

$$\frac{\partial \Pi^i}{\partial p_i} = \left( \frac{n_i}{n} + \frac{\sum_{j=1}^{j=n_i}(p_{j+1} + p_{j-1})}{2t} - \frac{n_j}{t} p_i \right) - (p_i - c) \frac{n_i}{t} = 0$$  \hspace{1cm} (28)
Evaluation in the symmetric equilibrium gives

$$\frac{\partial \Pi^f}{\partial p_i} \bigg|_{p_i = p_j} = \frac{n_i}{n} - (p_i - c) \frac{n_i}{t}$$  \hspace{1cm} (29)

For the equilibrium price of the case of one-branch banks,

$$\frac{\partial \Pi^f}{\partial p_i} \bigg|_{p_i = p_j = \infty} = 0$$  \hspace{1cm} (30)

So, if all other banks set this interest rate, the bank has no incentive to charge a different one. The symmetric Nash equilibrium interest rate is also an equilibrium here. ■

From this Proposition it follows that decisions at the bank level or at the branch level give the same outcome. There is no incentives to centralize or decentralize price decisions, as profits are equal in both circumstances. In the present framework, decentralization means that a bank allows its branches to set independently and non-cooperatively the interest rates charged. Thus, the following corollary results directly.

**Corollary 1** A decentralized or a centralized process of price decisions are equivalent if the bank does not hold adjacent locations.

The existence of adjacent branches of the same bank constitute another force to have differences in prices across banks. Inspection of first-order conditions of both types of banks (with and without adjacent banks) shows that we cannot have in equilibrium equal prices for all banks since they will exercise monopoly power on the range of consumers located between the two branches. Banks with adjacent branches set, in equilibrium, higher prices than other banks. This highlights the importance of allowing for such possibility in the empirical implementation of the model, although a complete treatment of the question would imply a precise characterization of location of banks around the circle.

Consider next the situation where a bank has two branches in adjacent locations. All consumers in the reach between the two branches are “captured” by the bank. To characterize equilibrium prices and profits is necessary to know whether the bank prefers to let its branches set prices independently or to centralize the decision. The exploitation of market power on “captured” consumers suggests intuitively that a centralized decision process is superior, from the point of view of the bank. The following proposition establishes the result.

15
Proposition 5 Assume that only one bank has two adjacent branches. A centralized process for setting prices equal for both branches yields higher profits than a decentralized process, with independent price-setting behavior by each branch.

Proof: The existence of adjacent branches implies that centralization of decisions sets a chain effect of price changes relative to the decentralized equilibrium. To establish the equilibrium prices we replicate here, for linear transport costs, the analysis of Eaton and Wooders (1985) for quadratic transport costs and a real line for characteristics space. The approach is taken as a reasonable approximation to the price changes in a circular city model.  

Denote by \( 1 \) and \( n \) the two adjacent branches of the same bank. The price of the multibranch bank is denoted by \( p_1 \). Locations are assumed to be symmetric. The first-order conditions for profit maximization are

\[
\frac{p_j + p_{j-1}}{2} - 2p_j + \frac{t}{n} = 0, \quad j = 2, \ldots, n-1 \\
\frac{p_{j+1} + p_j}{2} - 2p_j + \frac{t}{n} = 0, \quad j = 2, \ldots, n-1
\]

for the bank with adjacent branches and for all other banks, respectively.

Ignoring the finite number of firms, the first-order conditions (31) have the structure of a second-order difference equation. Solving appropriately, for one side (\( j=1,2,\ldots \)),

\[
p_j = \bar{p} + A_1 \lambda_1^{j-2} + A_2 \lambda_2^{j-2}, \quad j \geq 2
\]

where \( \bar{p} \) is a particular solution, \( \lambda_1 \) and \( \lambda_2 \) are the roots of the characteristic equation and \( A_1 \) and \( A_2 \) are constants. The values of the roots are \( \lambda_1 = 2 - \sqrt{3} < 1 \) and \( \lambda_2 = 2 + \sqrt{3} > 1 \). Requiring stability of the solution implies \( A_2 = 0 \). The chain of effects on the other side runs symmetrically, with parameters \( B_1 \) and \( B_2 \) associated with the same roots for the characteristic equation. We now concentrate on calculation of profits of bank 1. Prices of bank 1 and its direct neighbors are determined by the following conditions:

\[
4p_1 = p_2 + p_{n-1} + 2k_0, \quad k_0 = 2t/n \\
p_2 = k + A_1, \quad k = t/n
\]

For example, the percentual increase in the price of the fifth firm located away from the adjacent branches is of the order of the eighth decimal place.

Or the extreme points of the adjacent branches if more than two.
\[ p_{n-1} = k + B_1 \]
\[ 4p_2 = p_1 + p_3 + 2k = 0 \]
\[ p_3 = k_0 + \lambda_1 A_1 \]
\[ 4p_{n-1} = p_1 + p_{n-2} + 2k = 0 \]
\[ p_{n-2} = k_0 + \lambda_1 B_1 \]

It is straightforward to show that \( A_1 = B_1 \), so that prices are symmetric for neighbors on the right and on the left of the adjacent branches. The system to be solved then collapses to

\[ 2p_2 - 4p_1 + 2k_0 = 0 \]
\[ p_2 - k - A_1 = 0 \]
\[ p_1 + p_3 - 4p_2 + 2k = 0 \]
\[ p_3 - k - \lambda_1 A_1 = 0 \]

The solution to the system is given by

\[ A_1 = \frac{k_0 - k}{7 - 2\lambda_1} \]
\[ p_1 = \frac{k_0(4 - \lambda_1) + (3 - \lambda_1)k}{7 - 2\lambda_1} \]
\[ p_2 = \frac{k(6 - 2\lambda_1) + k_0}{7 - 2\lambda_1} \]
\[ p_3 = \frac{k(7 - 3\lambda_1) + \lambda_1 k_0}{7 - 2\lambda_1} \]

The equilibrium profit for firm 1 with centralization of decisions is given by

\[ \Pi^1(C) = \left[ \frac{k_0 - k}{7 - 2\lambda_1} + k \right] \frac{1}{k} \]

(34)

In the decentralized equilibrium, the profits are \( \Pi^1(D) = k^2 / l \). Since \( k_0 > k \) and \( 7 > 2\lambda_1 \) it follows \( \Pi^1(C) > \Pi^1(D) \) and centralization is preferred by the bank with adjacent branches.

Since \( \lambda_1 = 2 - \sqrt{3} \) and \( k_0 = 2k \) we can compute the relative increase in profits \( (\Pi^1(C) / \Pi^1(D)) \). It amounts to an almost 50% increase. This suggests that organization costs must be high if the bank is to decentralize, as an optimal decision, price setting to the branch level.
The intuition for the result is straightforward. The presence of adjacent branches gives to the bank monopoly power in the range of demand between branches. The exploitation of this monopoly power through a central decision on prices leads to an increase in equilibrium profits. "Captured" consumers will be clients of the bank whatever the price charged, provided that is below the reservation price. Decentralization of the pricing decision to the branch level would imply that branches of the same bank compete with each other, thus lowering the banks' aggregate profits. Centralization mitigates price competition among own branches.

Corollary 1 and Proposition 5 provide a rationale to the observed centralized process of price-setting in banking. The strict superiority of centralization over decentralization can be easily established through introduction of uncertainty about exact locations of branches in consumers preferences.

Suppose that the exact location of branches on the circle is determined by consumers' assessments of banks' characteristics. Uncertainty about consumers preferences implies, thus, uncertainty about exact location of branches. A bank cannot identify who are its neighbors in each and every market. It knows, at the best, which banks operate in the same local market and how many branches they have. Only the exact location in the characteristics circle is unknown. Since symmetry of location in each local market is assumed, the uncertainty concerns only the identity of neighbors.

It seems reasonable to assume that a priori a bank has no information concerning preferences of consumers and, probably, about all relevant attributes of local rivals.\(^9\) Prices must be set under some rule for expectations about a neighbor's identity. Assume that a bank takes as equal likely that each of the remaining branches in that local market is its right or left neighbor in a particular branch. Under these assumptions, the following proposition results.

**Proposition 6** Assume that only a multibranch bank exists. In a spatial market where locations are symmetrically distributed, if a bank has various branches then centralization (setting the same price for all branches) is strictly preferred to decentralization (each branch sets its own price) provided there is a positive probability that adjacent branches arise.

\(^9\)This is a reasonable assumption if preferences represented by the circle are derived from some ordering over banks' characteristics or from a ranking over some index composed by benefits of characteristics (Caplin and Nalebuff, 1991). The ordering has to satisfy some conditions to be represented by a circle - see Horstmann and Silvinski (1985).
Proof: To simplify notation, the transport cost is normalized to unity. Banks are assumed to be risk neutral and to care only about expected profit, defined as:

\[ E\Pi^i = \sum_{m=1}^{n} \left( \frac{1}{n} - p_i + \frac{E\rho_{m+1} + E\rho_{m-1}}{2} \right) (p_i - c) \]  \hspace{1cm} (35)

for the multibranch bank and

\[ E\Pi^j = \left( \frac{1}{n} - p_i + \frac{E\rho_{j+1} + E\rho_{j-1}}{2} \right) (p_j - c) \]  \hspace{1cm} (36)

for other banks, where \( E \) denotes the expectation operator. The linearity of demand implies that uncertainty is reflected only on the expected price of neighbors. There are three possible cases: both neighbors of a branch belong to other banks \((m + 1 \notin i, m - 1 \notin i)\); both belong to the same bank \((m + 1 \in i, m - 1 \in i)\); or only one of them is a branch of another bank \((m + 1 \notin i, m - 1 \notin i; \text{ or } m + 1 \notin i, m - 1 \in i)\).

The probabilities of occurrence of each case are:

\[ \phi_0 = \Pr(m - 1 \notin i, m + 1 \notin i) = \frac{n_k - n_{ik}}{n_k - 1} \times \frac{n_k - n_{ik} - 1}{n_k - 2} \]

\[ \phi_1 = \Pr(m - 1 \notin i, m + 1 \in i) = \Pr(m + 1 \notin i, m - 1 \in i) = \frac{n_k - n_{ik}}{n_k - 1} \times \frac{n_{ik} - 1}{n_k - 2} \]

\[ \phi_2 = \Pr(m + 1 \in i, m - 1 \in i) = \frac{n_{ik} - 1}{n_k - 1} \times \frac{n_{ik} - 2}{n_k - 2} \]

and \( \sum_{j=1}^{3} \phi_j = 1 \). The above probabilities are relevant for the multibranch bank only. For one-branch banks, \( \phi_0 = 1 \) and \( \phi_1 = \phi_2 = 1 \). With those probabilities, expected prices of neighbors of each branch can be computed:

\[ E\rho_{m+1} = E\rho_{m-1} = \sum_{j \neq i} \frac{P_j}{n - 1} + \frac{n_i - 1}{n - 1} p_i \]  \hspace{1cm} (37)

for the multibranch bank and

\[ E\rho_{j+1} = E\rho_{j-1} = \sum_{k \neq i, j} \frac{P_k}{n - 1} + \frac{n_j}{n - 1} p_i \]  \hspace{1cm} (38)

for one-branch banks. Expected profits are given by, respectively, for the multibranch bank and one-branch banks:

\[ E\Pi^i = \sum_{m=1}^{n} \left( \frac{1}{n} + \sum_{j \neq i} \frac{P_j}{n - 1} + \frac{n_i - 1}{n - 1} p_i - p_i \right) (p_i - c) \]

\[ E\Pi^j = \left( \frac{1}{n} + \sum_{k \neq i, j} \frac{P_k}{n - 1} + \frac{n_j}{n - 1} p_i - p_j \right) (p_j - c) \]
The corresponding first-order conditions for profit maximization are

\[
\frac{\partial \Pi_i}{\partial p_i} = n_i \left( -\frac{n - n_i}{n - 1} \right) (p_i - c) + n_i \left( \frac{1}{n} + \sum_{j \neq i} \frac{p_j}{n - 1} - \frac{n - n_i}{n - 1} p_i \right) = 0
\]

\[
\frac{\partial \Pi_j}{\partial p_j} = \frac{1}{n} + \sum_{k \neq j \neq i} \frac{p_k}{n - 1} + \frac{n_i}{n - 1} p_i - 2p_j + c = 0
\]

The first-order condition for one-branch banks can be written in a slightly modified way:

\[
\frac{1}{n} + \frac{n_i}{n - 1} p_i + \sum_{j \neq i} \frac{p_j}{n - 1} - \left( 2 + \frac{1}{n - 1} \right) p_j + c = 0
\]  

(39)

Summing the first-order conditions over all \( j \neq i \), and solving for \( \sum_{j \neq i} p_j \), we get

\[
\frac{\sum_{j \neq i} p_j}{n - 1} = \frac{n - n_i}{n + n_i - 1} \left( \frac{1}{n} + \frac{n_i}{n - 1} p_i + c \right)
\]  

(40)

Plugging this solution into the first-order condition of the multibranch it is possible to find its equilibrium price and to calculate equilibrium profits under centralization of decisions. Straightforward substitution gives

\[
p_i = c + \frac{1}{n} \left( \frac{(2n - 1)(n - 1)}{(2n + n_i - 2)(n - n_i)} \right)
\]  

(41)

Equilibrium profits under centralization of price decisions are (for the multibranch bank):

\[
\Pi^i(C) = \left( \frac{1}{n} \right)^2 \left[ \frac{(2n - 1)(n - 1)}{(2n + n_i - 2)(n - n_i)} \right]^2 n_i
\]  

(42)

On the other hand, under decentralization of price decisions, each branch/bank maximizes

\[
E \Pi^i = \left( \frac{1}{n} + \frac{\sum_{j \neq i} p_j}{n - 1} - p_i \right) (p_i - c)
\]  

(43)

The associated equilibrium profits are (for the multibranch bank):

\[
\Pi^i(D) = n_i \left( \frac{1}{n} \right)^2
\]  

(44)

To show that \( \Pi^i(C) > \Pi^i(D) \) it suffices to show that

\[
(2n - 1)(n - 1) > (2n + n_i - 2)(n - n_i)
\]

Rearranging terms,

\[
(n_i - 1)(n + n_i - 1) > 0
\]
which is true since \( n_i > 1 \). ■

One implication of uncertainty about identity of local competitors is that it can give origin to price dispersion across banks, even in a single market context. Some other implications to multibranch-multimarket banks can be discussed. A pure multimarket bank suffers a profit reduction under the constraint of no price discrimination across markets. On the other hand, a pure multibranch bank, under the assumption that a positive probability is assigned to the event of having adjacent locations, chooses to commit to a unique price across branches. From this, a bank operating in several markets with more than one branch in each market would like to choose decentralization of decisions to the market level but not to the branch level.

If the bank can only establish decision processes at the branch level or at the central level, for example, due to high organization costs in creating an intermediate layer of power, it faces a trade-off between the gains of discrimination across markets and the gains of price coordination inside the market.

4 The Empirical Model

Previous sections have shown that multimarket/ multibranch banks committed to a single price across branches can easily generate equilibrium price dispersion. Moreover, commitment to centralization of price setting can be profit maximizing. Those theoretical results have significant implications in market power measurement in the banking industry (and multistore firms in general). The aim is to evaluate how a spatial model can explain price-cost margins differences in deposits across banks. Also of interest is the identification of a general pattern of conduct in the market, that is, how collusive is market equilibrium.

For the empirical implementation, a more general setting is used. Branches are not restricted to be symmetrically located along the circle. Given a distance \( d_{i,j}^m \) of a branch \( m \) to its neighbor \( j, j = m+1, m-1, \) total demand at branch \( m \) can be written as

\[
D^m = \delta_k(m) \left( \frac{d_{m+1}^m + d_{m-1}^m}{2} + \frac{p_{m+1} - p_{m-1} - 2p_m}{2\kappa(m)} \right)
\]  
(45)
Total expected profit is therefore (allowing for differences in marginal costs across banks)

\[
E \Pi_i' = \sum_{m=1}^{n_i} \delta_k(m) \left[ \frac{d_m}{2} + \frac{E_{p_m + 1} + E_{p_m - 1}}{2t_k(m)} - \frac{p_i}{t_k(m)} \right](p_i - c_i).
\]  

(46)

Two benchmark assumptions about conduct are collusive behavior and competitive behavior by banks. The collusive behavioral assumption has all banks setting prices to maximize joint profits. Competitive behavior means that each bank sets the price to maximize its own profits. Prices set by other banks are taken as given.

The collusive outcome is given by joint profit maximization. The corresponding first-order condition is

\[
\frac{\partial \Pi_i}{\partial p_i} = D_i + (p_i - c_i) \frac{\partial D_i}{\partial p_i} + \lambda \sum_{j \neq i} (p_j - c_j) \frac{\partial D_j}{\partial p_i} = 0
\]

(47)

Under Nash-Bertrand behavior, the first-order condition is:

\[
\frac{\partial \Pi_i}{\partial p_i} = D_i + (p_i - c_i) \frac{\partial D_i}{\partial p_i} + \lambda \sum_{j \neq i} (p_j - c_j) \frac{\partial D_j}{\partial p_i} = 0
\]

(48)

where

\[
D_i = \sum_{k} \delta_k(m) d_m + \sum_{j \neq i} n_{ik} \frac{n_{jk}}{n_k - 1} p_j - \left( \sum_{k} \frac{n_{ik} n_k - n_{ik}}{n_k - 1} \right) p_i
\]

(49)

\[
\frac{\partial D_i}{\partial p_i} = -\sum_{k} \frac{n_{ik} n_k - n_{ik}}{n_k - 1}
\]

(50)

\[
\frac{\partial D_j}{\partial p_i} = \sum_{k} \frac{n_{ik} n_{jk}}{n_k - 1} \frac{\delta_k}{t_k}
\]

(51)

where the parameter \( \lambda \) reflects the extent of bank \( i \)'s internalization of the effect of its price changes on others' profits. The critical values for the parameter are \( \lambda = 1 \), where the collusive outcome is duplicated, and \( \lambda = 0 \), which results in the Nash-Bertrand equilibrium.\(^{10}\)

The proposed scheme to encompass a range of possible behaviors between competitive (Nash-Bertrand) conduct and collusive behavior imposes that all banks are equal from the point-of-view of each bank and for all banks. This rules out the possibility that a bank attaches more importance to a particular bank.

Pairwise collusion can also be considered. The more general scheme can be represented by \( \lambda_{ij} \), where \( \lambda \) denotes the extent of bank \( i \)'s consideration of its impact on bank \( j \) marginal net revenue. Under the first scheme, we have restricted \( \lambda_{ij} = \lambda \). Under pairwise collusion, \( \lambda_{ij} = \lambda_{ji} = 1 \) for some

\(^{10}\)Note that this formulation is quite different from a conjectural variations model in prices and in general they are not equivalent.
\(i\) and \(j\). Completely free parameters \(\lambda_{ij}\) render the model impossible to identify. So, it is estimated under different, reasonable, assumptions. The resulting models are then compared with each other.

In the demand equation the parameter \(\alpha_i = \sum_{m=1}^{n} d_m \delta_m\) includes the (unknown to the outside observer) location information of each branch of bank \(i\). This effect is treated as a bank-specific effect.

The empirical application estimates the first-order condition and the demand condition for each bank simultaneously. Joint estimation is necessary to provide estimates for all parameters. Solving for the equilibrium interest rate creates an identification problem. The problem can be easily seen in the simplest case of constant market density and transport costs. In that case,

\[
D^i = \delta_0 \sum_k \frac{n_{ik}}{n_k} - p_i \delta_0 \sum_k \frac{n_{ik} (n_k - n_{ik})}{n_k - 1} + \delta_0 \sum_k n_{ik} \sum_{j \neq i} \frac{n_{jk}}{n_k - 1} p_j + \epsilon_i
\]

Substituting in the first-order condition (under Nash behavior, to simplify matters) and solving for \(r_d^i\), it results

\[
r_d^i = \frac{c}{2} + \left\{ \frac{1}{2\delta_0} \sum_k n_{ik} - \frac{1}{2} \sum_k n_{ik} \sum_{j \neq i} \frac{n_{jk} p_j}{n_k - 1} \right\} \left\{ \sum_k \frac{n_k - n_{ik}}{n_k - 1} \right\}^{-1}
\]

and the parameter \(\delta_0\) cannot be estimated from this equation. The identification problem is solved through the simultaneous estimation of the demand equation and the first-order equation.

It is easy to decompose \(\alpha_i\) in a term reflecting a symmetric equilibrium plus a random term capturing location deviations to the symmetric equilibrium. The last element includes the (unknown to the observer) location information of each branch of bank \(i\):

\[
\alpha_i = \sum_k \frac{\delta_k n_{ik}}{n_k} + \epsilon_i
\]

Additionally, entry will, on average, reduce the distance between branches. Arbitrarily, it is imposed a common mean across banks to this time effect. Random deviations specific to both the bank and the time period are collected in a pure random term.

The unknown parameters of the model are \(\delta_k, t_e\) and conduct parameters \(\lambda_{ij}\). Operationally, take \(\delta_k\) and \(t_e^{-1}\) to be linear functions of market characteristics.

Application to the deposits market requires a slight redefinition of variables. The price is defined as minus the interest rate paid on deposits. A higher interest rate means a lower price. Marginal cost is defined as the real resources marginal cost.

However, deposits will earn some interest, and the relevant rate of interest is given by the money market or government securities market. Profit per unit of deposits is thus given by the money market
rate minus the interest rate on deposits offered by the bank minus the real resources marginal cost of obtaining the deposit. As usual in the literature, banks are assumed to have no market power in the money market (or in the securities market). The interest rate on that market implies a useful separation between the deposits market and the loans market. Demand in the general model is to be read supply of deposits in the application.

Under this redefinition, profit of bank \( i \) in the deposits market is:

\[
\Pi^i = (r_s(1-m) + m r_0^d - r_c^d - c'_{d}) D^i(r_d)
\]  

(53)

with

\[
D^i = \sum_k \delta_k n_{ik} n_k + r_0^d \sum_k \sum_{l_k} \frac{n_{ik}}{n_k} \left( \frac{n_k - n_{ik}}{n_k - 1} \right) - \sum_k \frac{n_{ik}}{n_k} \sum_{j \neq i} \frac{n_{jk} r_{d}^j}{n_k - 1} + \epsilon_i
\]  

(54)

and the first-order condition is given by

\[
\frac{\partial \Pi^i}{\partial r_d^i} = -D^i + (r_s(1-m) + m r_0^d - r_c^d - c'_{d}) \left( \sum_k \frac{n_{ik} (n_k - n_{ik})}{n_k - 1} \right) - \sum_{j \neq i} \lambda_{ij} (r_s(1-m) + m r_0^d - r_c^d - c'_{d}) \sum_k \frac{n_{jk} n_{ik} \delta_k}{n_k - 1} t_k = 0
\]  

(55)

where \( r_s \) is the money market interest rate, \( m \) is the mandatory level of reserves, \( r_0^d \) stands for remuneration of reserves, \( c'_{d} \) is the real resources marginal cost and \( r_c^d \) is the interest rate charged by bank \( i \). These are the equations to be estimated.

5 Data and Econometric Methods

The Portuguese market is characterized by the existence of significant disparities in size among banks. Some banks have an extensive network of branches and cover, more or less, the whole country. Others have just a few branches concentrated in a small number of markets. We can also find banks with a relatively large network of branches and yet concentrate their activities in only some markets.

The model is estimated with data from a sample of 15 Portuguese banks, including the main institutions, over a two-year period (1991-1992). The sample covers 92% of the deposits market and about 90% of the branches.

It is now necessary to specify the variables determining demand intensity and transport costs. Intensity of demand depends, conceivably, on the number of potential depositors and its needs of

\[\text{See Klein (1971) and Hannan (1991) for presentation of models in this vein.}\]
banking services. Potential depositors are approximated by total population in the market. One measure of the need of banking services commonly used is income per capita (Evanoff, 1988). It is proxied here by income tax payments per capita. The number of firms in the market is also considered, in order to reflect corporate demand for deposits.

With respect to transport costs, everything else held constant, markets with a higher area may imply higher transportation costs to consumers. In order to account for this effect, area of concelhos in km², is included as an explanatory variable for transport costs.

Transport costs are also assumed to differ between rural and urban areas. A dummy variable incorporates this effect in the model. It takes value one if the local market is classified as urban, takes value zero otherwise. Linearity of effects means that

\[ \delta_k = \delta_0 + \delta_1 POP_k + \delta_2 TAX_k + \delta_3 FIRM_k \]  \hspace{1cm} (56)

and

\[ t_{i}^{-1} = t_0 + t_1 URBAN_k + t_2 K M_k^2 \]  \hspace{1cm} (57)

where

- \( POP_k \) — Population of market \( k \);
- \( TAX_k \) — Per capita income tax payments;
- \( FIRM_k \) — Number of firms in the market;
- \( URBAN_k \) — Dummy variable (one if urban market, zero otherwise)
- \( KM_k^2 \) — area of the local market.

The Portuguese banking system is characterized by the existence of links between institutions. Various banks belong to the same economic group. Those banks can, conceivably, coordinate their decisions. A more collusive parameter may result for relations among those banks.\(^{12}\)

Local markets are defined as the Portuguese local jurisdictions, concelhos. Data on the number of branches of a bank operating in each local market is provided by the Portuguese Bankers Association publications.

The variables characterizing local markets can be found in Adiministração Local em Números, published by Direcção Geral da Administração Autárquica (DGAA), a Portuguese government department. Interest rates on a bank basis for 1991 and 1992 are computed from published accounts. For the details, see Amador and Brasão (1993). Definition of urban areas is made according to the classification of Instituto Nacional de Estatística, the Portuguese Bureau of Statistics. A local market

\(^{12}\) A listing of the banks in the sample and the existing groups is presented in the appendix.
is classified as urban is a ten thousand inhabitants threshold is met. This is a crude way of assigning conelhos. Alternatively, an older classification is provided by DGAA for 1982. Unfortunately, recent and unequal evolution of Portuguese regions make it clearly unappropriate.

The real resources marginal cost of deposits is taken from Pinho (1994). For a panel data of banks in the period 1986-1992, estimates from a long-run cost stochastic frontier for real resources emmployed in the Portuguese banking industry suggests that constant returns to scale around 2.5% may not be unreasonable for the banks in our sample. The parameter \( m \) is the mandatory (legal) rate of reserves and takes the value 0.17.

The pair of equations to be estimated is:

\[
D^{jh} = a_h + \sum_k n_{ikh} \delta_{kh} + \rho_d^{jh} \sum_k \delta_k n_{ikh} n_{ikh} + \sum_k \delta_{kh} n_{ikh} \sum_{j \neq h} \frac{n_{ikh}}{n_{kh} - 1} r_d^{jh}, \quad (58)
\]

\[
-\frac{D^{jh}}{r^h} + (r^h (1 - m) + mr_h^j - r_d^{jh} - c_d^{jh}) \sum_k \frac{\delta_k n_{ikh} - n_{ikh}}{n_{kh} - 1} - \sum_{j \neq h} \lambda_{ijh} (r^h (1 - m) + mr_h^j - r_d^{jh} - c_d^{jh}) \sum_k \frac{n_{ikh} n_{ikh} \delta_{kh}}{n_{kh} - 1} l_{kh} + b_h = 0 \quad (59)
\]

\[
h = 1991, 1992; \quad i, j = 1, \ldots, 15; \quad k = 1, \ldots, 304.
\]

The model is to be estimated with a panel data of 15 Portuguese commercial banks over a two-years period. Regression variables must be computed aggregating the local market variables. Aggregation runs over 304 local markets.

The general formulation for the conduct parameter renders the model impossible to estimate. Two particular schemes are considered in order to reduce the number of parameters to be estimated. The first one, called market collusion, has \( \lambda_{ijh} = \lambda_{ijh}^M \), \( \forall i, j \). That is, banks take into account in their choices some effect upon others in a symmetric way. The parameter can vary over time, reflecting changes in the degree of collusion in the market.

The other scheme makes use of outside information on the system and considers collusion only between banks of the same group. That is, \( \lambda_{ijh} = \lambda_{ijh}^G \) for \( i, j \) in the same economic group, \( \lambda_{ijh} = 0 \) otherwise. Again, differences across years in the degree of collusion is allowed. Parameters of economic group collusion have superscript \( G \); superscript \( M \) indicates overall market collusion parameters. The index \( j \) covers the two years in the sample.

Notice also that a whole set of variables must be computed before estimation. For example, the term \( \sum_k n_{ikh} \delta_{kh} / n_{kh} \) involves the calculation of \( \sum_k n_{ikh} / n_{kh}, \sum_k n_{ikh} P O P_{kh} / n_{kh}, \sum_k n_{ikh} T A X_{kh} / n_{kh}, \).
and \( \sum_k n_{kh} FIRM_{kh}/n_{kh} \).

Error terms are included in both equations. The random shock in the demand equation is denoted by \( \varepsilon_{1it} \) and the error term in the first-order condition is denoted by \( \varepsilon_{2it} \). There is a gain in efficiency if estimation is made using a system of seemingly unrelated regressions (Zellner, 1962). The coefficients on the independent variables are restricted to be the same across equations. The SUR technique exploits any correlation that may exist for each firm across time.\(^{13}\)

The random residual \( \varepsilon_{fit}, f = 1, 2 \) is assumed to have zero mean and finite variance and to be uncorrelated with any explanatory variable. The residuals can be correlated, for the same moment in time and for the same bank, across equations. Correlation over time for each bank is also allowed. The residual term can thus be written as \( \varepsilon_{fit} = u_{fit} + v_{fit} \), where \( u_{fit} \) is a bank-specific random effect and \( v_{fit} \) is a pure random term. Maximum likelihood estimates are computed under the assumption of normally distributed disturbances.

6 Results for the structural model

The empirical analysis is intended to shed light over two issues: the determinants of demand (transport costs and demand density) and the nature of banks' conduct. All possible combinations of parameters lead to a fairly large number of models. So, the research strategy is the following. Starting from the simplest model, with constant demand intensity and transport costs, variables controlling for differences in conduct and differences in density across markets are introduced each at a time. Only those variables that are jointly and individually significant will be maintained. This search strategy however has to have some starting point. So, prior to investigation of determinants of transport costs and deposits supply intensity, different conduct patterns are investigated in the basic relation. Distinction in conduct across years is allowed for.

The maintained conduct pattern constitutes the basis for the investigation of transport costs and demand density determinants. Afterwards, the same conduct tests are again performed in the selected specification as well as in the more general one. It has the purpose of checking for robustness of results.

Fixed time effects are added to both equations and maintained throughout. There are four conduct parameters to be estimated: collusion common to all banks, which will be termed market collusion, and collusion only with banks in the same economic group, which will be termed group collusion.

\(^{13}\)Fixed effects specific to the firm would consume too many degrees of freedom.
Both types of collusion can differ over the two years of the sample. There are two determinants of transport costs, being a urban or a rural market and geographical space, and three explanatory variables for demand density: population, income per capita, proxied by income taxes, and number of firms. Particular versions of the model are tested against more general nested alternatives using the likelihood ratio test.

### 6.1 Conduct in the basic model

The first set of results respects to market conduct, holding constant transport costs and demand density. Introducing each conduct parameter at a time or all of them jointly produces the same basic pattern: Nash behavior is not rejected. This by itself may have very low predictive content if alternative benchmarks of market and group collusion are also not rejected.

The alternative benchmark oligopoly solutions are characterized by collusion, either at the market level or at the group level. One may argue that intermediate values of the conduct parameter are meaningless in the sense that are not supported by a well defined theoretic structure. To check for robustness of findings, we impose the collusive benchmark values on the conduct parameters. The resulting models are non-nested. Their comparison against Nash behavior through information criteria amounts to comparison of likelihood function values since the various models have the same number of parameters.

Table 1 presents the likelihood ratio tests of each hypothesis relative to the model with no restriction upon conduct parameters. For example, Nash behavior tests the hypothesis that all conduct parameters are jointly zero. The probability of acceptance of the hypothesis is 86%, well above the usual significance level of 5%.\(^{14}\)

From Table 1, one can see that Nash behavior is still a superior description of banks behavior. Also, total collusion (or coordination) among banks in the same economic group has more support from the data that full industry collusion.

Statistically, the test for economic group collusion does not reject the hypothesis. The same is true for Nash behavior of all banks. On the other hand, imposing full collusion for all banks is clearly rejected. The inference is thus of Nash behavior vis-a-vis banks not belonging to the same economic group. Within the group, no definite answer can be given, although the values favor more Nash behavior.

\(^{14}\)The probability of acceptance is defined as the tail of the distribution to the right of the test value. The values of the likelihood function for the estimated models can be found in the appendix.
Table 1: Conduct patterns - likelihood ratio tests

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nash behavior ($\lambda_j^S = 0$)</td>
<td>1.32</td>
<td>0.859</td>
</tr>
<tr>
<td>Market collusion ($\lambda_j^M = 1, \lambda_j^C = 0$)</td>
<td>23.07</td>
<td>0.000</td>
</tr>
<tr>
<td>Group collusion ($\lambda_j^G = 1, \lambda_j^M = 0$)</td>
<td>2.55</td>
<td>0.635</td>
</tr>
<tr>
<td>Market collusion in 1991 ($\lambda_j^{M1} = 1, \lambda_j = 0$)</td>
<td>47.98</td>
<td>0.00</td>
</tr>
<tr>
<td>Market collusion in 1992 ($\lambda_j^{M2} = 1, \lambda_j = 0$)</td>
<td>36.77</td>
<td>0.00</td>
</tr>
<tr>
<td>Group collusion in 1991 ($\lambda_j^{G1} = 1, \lambda_j = 0$)</td>
<td>5.85</td>
<td>0.211</td>
</tr>
<tr>
<td>Group collusion in 1992 ($\lambda_j^{G2} = 1, \lambda_j = 0$)</td>
<td>3.64</td>
<td>0.456</td>
</tr>
</tbody>
</table>

Note: The tests follow a $\chi^2$ distribution with 4 degrees of freedom.

Table 2: Conduct patterns - likelihood ratio tests

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nash behavior ($\lambda_j^S = 0$)</td>
<td>6.81</td>
<td>0.146</td>
</tr>
<tr>
<td>Market collusion ($\lambda_j^M = 1, \lambda_j^C = 0$)</td>
<td>21.31</td>
<td>0.000</td>
</tr>
<tr>
<td>Group collusion ($\lambda_j^G = 1, \lambda_j^M = 0$)</td>
<td>10.20</td>
<td>0.037</td>
</tr>
<tr>
<td>Market collusion in 1991 ($\lambda_j^{M1} = 1, \lambda_j = 0$)</td>
<td>57.15</td>
<td>0.00</td>
</tr>
<tr>
<td>Market collusion in 1992 ($\lambda_j^{M2} = 1, \lambda_j = 0$)</td>
<td>24.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Group collusion in 1991 ($\lambda_j^{G1} = 1, \lambda_j = 0$)</td>
<td>16.59</td>
<td>0.002</td>
</tr>
<tr>
<td>Group collusion in 1992 ($\lambda_j^{G2} = 1, \lambda_j = 0$)</td>
<td>6.22</td>
<td>0.183</td>
</tr>
</tbody>
</table>

Note: The tests follow a $\chi^2$ distribution with 4 degrees of freedom.

over collusion.

Notice that parametrization of the model allows for existence of different degrees of collusion over the years. The distinction is not relevant for market collusion since collusion is non-significant in both years. As to group collusion, total collusion in both years seems to prevail.

To provide more information to choose between global Nash behavior or some intra-economic group collusion, the benchmark cases can also be imposed upon the general specification. The qualitative results are essentially the same. Table 2 presents the associated tests.

Global Nash-Bertrand behavior cannot be rejected as a description of the market conduct of banks. As before, global collusion is strongly rejected by the data. With respect to collusion within economic groups, one can reject the hypothesis for the first year and for the two years jointly, but not for the second year. Thus, the evidence in favor of Nash behavior is more strong in the general model. Only in 1992 is the benchmark case of collusion within economic groups significant. The analysis of the benchmark cases qualifies the first set of results on conduct. It suggests that more weight should be given to Nash behavior but group collusion cannot be simply disregarded. The next section will,
Table 3: Deposits supply determinants - likelihood ratio tests

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nash behavior</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_0 : t_1 = 0$</td>
<td>4.41</td>
<td>0.036</td>
</tr>
<tr>
<td>$H_0 : t_2 = 0$</td>
<td>0.95</td>
<td>0.331</td>
</tr>
<tr>
<td>$H_0 : d_1 = 0$</td>
<td>2.25</td>
<td>0.134</td>
</tr>
<tr>
<td>$H_0 : d_2 = 0$</td>
<td>2.65</td>
<td>0.101</td>
</tr>
<tr>
<td>$H_0 : d_3 = 0$</td>
<td>1.79</td>
<td>0.181</td>
</tr>
<tr>
<td>Group collusion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_0 : t_1 = 0$</td>
<td>3.72</td>
<td>0.054</td>
</tr>
<tr>
<td>$H_0 : t_2 = 0$</td>
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<td>0.906</td>
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<tr>
<td>$H_0 : d_1 = 0$</td>
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<td>0.224</td>
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<tr>
<td>$H_0 : d_2 = 0$</td>
<td>2.85</td>
<td>0.091</td>
</tr>
<tr>
<td>$H_0 : d_3 = 0$</td>
<td>1.93</td>
<td>0.165</td>
</tr>
</tbody>
</table>

Note: the tests follow a $\chi^2$ distribution with 1 degree of freedom.

therefore, assume both benchmark cases in turn.

6.2 Deposits supply determinants

The next step in the proposed research strategy is to look for significant variables as determinants of transport costs and deposits supply intensity in each local market. The maintained behavior of banks is Nash, in a first case, and group collusion, in a second case.

The test of joint significance of all variables rejects the null hypothesis under the Nash behavior assumption, but does not reject it under group collusion. In the latter case, the test statistic is however very close to the rejection threshold (the test value under Nash-Bertrand behavior is 12.97 and the probability of acceptance is 2.4%; in the group collusion benchmark, the test value is 10.82, with a probability of acceptance of 5.5%).

Variables are required to be individually not significant in order to be dropped from the specification. The tests can be found in Table 3. The hypothesis $H_0 : t_1 \neq 0$ means that a model with parameter $t_1$ estimated is tested against a model where the coefficient is restricted to be zero, holding all other coefficients of explanatory variables at zero values. Introduction of each variable at a time reveals that only the coefficient associated with the variable $URBAN$ can be rejected the hypothesis of a zero parameter. Also when combinations of four coefficients are tested to be jointly zero, those combinations involving the parameter $t_1$ (variable $URBAN$) reject the null hypothesis.

Taking together all results, the inference to be drawn is that, conditional on Nash conduct, only
transport costs seem to differ across markets. The distinction made is very simple and differentiates only between urban and rural markets, without finer classes of markets within these two broad categories.

Again, to check for robustness of results, the same tests are conducted conditional on collusion between banks in the same economic group. The main conclusions remain valid, although evidence in favor of significance of variables is weaker.

### 6.3 Conduct once again

To keep consistency, the research strategy implies that under the assumption of a non-zero $t_1$ parameter, conduct should be investigated again. The previous pattern of behavior still remains: Nash behavior seems to be the best description although group collusion cannot be discarded on statistical grounds.

Other step to be considered is the joint significance of the variable URBAN with some other variable. Tests are presented in Table 5. The introduction of each variable separately in addition to a non-zero $t_1$ gives the same qualitative results. Namely, the hypothesis of a zero coefficient for the added variable is never rejected, whether Nash or group collusion behavior is considered.

### 6.4 The Maintained Model

The maintained models have only one significant explanatory variable, URBAN. They are differentiated by the underlying behavior assumption: Nash-Bertrand or group collusion. Estimates are presented in Table 6.

Note that both models are included in a more general model, where the group collusive parameters are also estimated. The natural way of choosing between the two alternatives is to test each against the more general specification. Unfortunately, both cases are not rejected, keeping ambiguity in the
Table 5: Deposits supply determinants - likelihood ratio tests (II)

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nash behavior (given that $t_1 \neq 0$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_0 : t_2 = 0$</td>
<td>0.14</td>
<td>0.710</td>
</tr>
<tr>
<td>$H_0 : d_1 = 0$</td>
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<td>0.928</td>
</tr>
<tr>
<td>$H_0 : d_2 = 0$</td>
<td>2.21</td>
<td>0.137</td>
</tr>
<tr>
<td>$H_0 : d_3 = 0$</td>
<td>1.57</td>
<td>0.211</td>
</tr>
<tr>
<td>Group collusion (given that $t_1 \neq 0$)</td>
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<td></td>
</tr>
<tr>
<td>$H_0 : t_2 = 0$</td>
<td>0.39</td>
<td>0.530</td>
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<tr>
<td>$H_0 : d_1 = 0$</td>
<td>0.12</td>
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<td>$H_0 : d_2 = 0$</td>
<td>2.49</td>
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</tr>
<tr>
<td>$H_0 : d_3 = 0$</td>
<td>1.78</td>
<td>0.182</td>
</tr>
<tr>
<td>Nash behavior</td>
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</tr>
<tr>
<td>$H_0 : t_1 = 0; t_2 = 0$</td>
<td>4.55</td>
<td>0.103</td>
</tr>
<tr>
<td>$H_0 : t_1 = 0; d_1 = 0$</td>
<td>4.42</td>
<td>0.110</td>
</tr>
<tr>
<td>$H_0 : t_1 = 0; d_2 = 0$</td>
<td>6.62</td>
<td>0.036</td>
</tr>
<tr>
<td>$H_0 : t_1 = 0; d_3 = 0$</td>
<td>5.98</td>
<td>0.050</td>
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<tr>
<td>Group collusion</td>
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</tr>
<tr>
<td>$H_0 : t_1 = 0; t_2 = 0$</td>
<td>4.12</td>
<td>0.128</td>
</tr>
<tr>
<td>$H_0 : t_1 = 0; d_1 = 0$</td>
<td>3.84</td>
<td>0.147</td>
</tr>
<tr>
<td>$H_0 : t_1 = 0; d_2 = 0$</td>
<td>6.21</td>
<td>0.045</td>
</tr>
<tr>
<td>$H_0 : t_1 = 0; d_3 = 0$</td>
<td>5.51</td>
<td>0.064</td>
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</table>

**small Note:** The tests follow a $\chi^2$ distribution. The first half of the table tests have 1 degree of freedom. The tests in the lower part of the table have 2 degrees of freedom.
Table 6: Estimated Models

<table>
<thead>
<tr>
<th></th>
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<th>$a_{92}$</th>
<th>$b_{91}$</th>
<th>$b_{92}$</th>
<th>$\delta_0$</th>
<th>$t_0$</th>
<th>$\lambda_{M1}$</th>
<th>$\lambda_{M2}$</th>
<th>$\lambda_{G1}$</th>
<th>$\lambda_{G2}$</th>
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<tr>
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<td>-6.860</td>
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<td></td>
<td></td>
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</tr>
</tbody>
</table>

Note: t-statistics within brackets.

choice of the best description of banks' behavior.

Although a precise characterization of market conduct is of interest in itself, the qualitative results do not rely upon the specific assumption maintained about market behavior of banks.

The interpretation of the model must be made carefully. It reflects banks decisions as well as their expectations about supply of deposits at the branch level. Modeling of consumer behavior does not go to the very fundamentals of choice. Also, regression variables involve adjustments for the branch concentration in each market. The exact adjustment to be performed is dictated by the theoretical model. Under other specifications of the structural model, distinct adjustments rules can conceivably arise.

The most striking feature of the model is the independence of deposits supply intensity of the branch level from population and its income per capita. Also, the level of economic activity, measured in a very crude way by the number of firms in the market, seems to have no bearing upon deposits supply density.

The interpretation of this finding is that exists a strong core of deposits associated with each branch and remaining differences across branches, banks and markets are not related to market characteristics.
An additional explanation for insignificance of other effects is a statistical one. There is a high correlation among explanatory variables after adjusting for the markets a bank operates. The variation introduced by the aggregation over markets dominates in the transformed variables, making them highly colinear. The structure imposed by the theoretical model can be too demanding on the data. It seems desirable to allow for a structure somewhat more flexible to assess the robustness of findings. This will be taken up in the next section.

Nevertheless, the result stresses the importance of localized competition in deposits in the sense that volume of deposits is mainly associated with the network of branches.

The other main result of the model is that transport costs matter. Urban markets are associated with higher transport costs. Transport costs in rural areas are roughly 40% of the ones in urban markets.

Moreover, transport costs are in no way related to the geographical size of the market. This may be explained by the fact that banks tend to cluster on the centre of towns and that our definition of market is too broad. If the relevant market is at the town level, differences in size across administrative jurisdictions should not matter much.

7 A simpler model

One justification for the apparently odd results of the structural model in previous sections may be a too strong straitjacket of theory. This section explores an alternative empirical formulation, based on a looser theoretical basis, namely on the supply of deposits.\textsuperscript{15} Specifically, consider the supply of deposits for each branch in market \( k \) to be of the form:

\[
D_{ik} = f(X_k \beta, r^i_d, r^{-ik}_d) \tag{60}
\]

where \( X_k \) is a matrix of market characteristics and \( r^{-ik}_d \) is an index of prices of other banks operating in market \( k \). In particular, for tractability, assume a linear functional form:

\[
D_{ik} = X_k \beta + \gamma_0 r^i_d + \gamma_1 r^{-ik}_d \tag{61}
\]

The profit function of bank \( i \) is given by

\[
\Pi^i = \sum_{k=1}^{n_i} n_{ik} D_{ik}(r_c(1-m) + m r^i_d - r^i_d - c^i_d) \tag{62}
\]

\textsuperscript{15}Unlike the initial sections, we treat the deposits market directly in this section.
Maximization of profits with respect to own price gives the following first-order condition:

$$\frac{\partial \Pi^i}{\partial r_d^i} = -\sum_{i=1}^{n_i} n_i (X_k \beta + \gamma_0 r_d^i + \gamma_1 r_d^{-ik}) + (r_s (1 - m) + mr_d^i - r_d^i - c_d^i)(\sum_{k=1}^{n_i} \gamma_0) +$$

$$+ \sum_{j \neq i} \sum_{k=1}^{n_j} n_j \left\{ \lambda_i j n_j k \gamma_1 \frac{\partial r_d^{-jk}}{\partial r_d^i} (r_s (1 - m) + mr_d^j - r_d^j - c_d^j) \right\}$$

where \( \lambda \) reflects the degree of collusive behavior. Since no explicit theoretical model is employed to derive the deposits supply function, it is assumed that the price index is simply

$$r_d^{-ik} = \sum_{j \neq i} \frac{n_j}{n_k - n_i} r_d^j$$  \hspace{1cm} (64)

The variables included in the vector \( X_k \) are the ones used earlier: population, income taxes per capita, number of firms, urban/rural classification and local market area. An additional variable, controlling for the size of the bank in the local market is introduced: the market share defined in terms of branches for each market.

Like in the previous model, resolution of the first-order condition raises an identification problem. Of course, it is possible to estimate a reduced form for the equilibrium interest rate. Nevertheless, the previous procedure of joint estimation of the deposits supply equation and the first-order condition is followed also here because it identifies all structural parameters. The equations to be estimated are:

$$\frac{D_{d}^{i}}{n_{i}^{h}} = a_{h} + a_{1} r_{d}^{i} \frac{D_{d}^{i}}{n_{i}^{h}} + a_{2} r_{d}^{-i} \frac{D_{d}^{i}}{n_{i}^{h}} + \sum_{i=3}^{9} a_{i} X_{k}$$

$$r_{d}^{i} = b_{h} + r_s - \frac{1}{a_1} \frac{D_{d}^{i}}{n_{i}^{h}} + a_{2} \frac{r_{d}^{i}}{a_1} \sum_{j \neq i} n_{j} k \left( \frac{r_{d}^{j} - r_{d}^{i}}{n_{j} k} \right) \sum_{h} \frac{\partial r_{d}^{j h}}{\partial r_{d}^{i h}} \frac{n_{i} k}{n_{i} k}$$

where

$$r_{d}^{j h} = r_s^h (1 - m) + mr_d^{j h} - c_d^j$$

\( h = 1991, 1992; \quad i, j = 1, \ldots, 15; \quad k = 1, \ldots, 304; \)

and \( n_i \) is the total number of branches of bank \( i \). For simplicity, total deposits are defined in per branch terms. The error structure of the previous model is applied and maximum likelihood estimates are computed. The variables included in the deposits supply relation are
Table 7: Conduct tests in the simpler model

<table>
<thead>
<tr>
<th>Description</th>
<th>d.f.</th>
<th>test value</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_k^i = 0$</td>
<td>4</td>
<td>1.87</td>
<td>0.759</td>
</tr>
<tr>
<td>$\lambda_k^i = 1, \lambda_j^c = 0$</td>
<td>4</td>
<td>28.31</td>
<td>0.000</td>
</tr>
<tr>
<td>$\lambda_k^i = 0, \lambda_j^c = 1$</td>
<td>4</td>
<td>29.42</td>
<td>0.000</td>
</tr>
<tr>
<td>$a_{91} = a_{92}$</td>
<td>1</td>
<td>5.79</td>
<td>0.016</td>
</tr>
<tr>
<td>$b_{91} = b_{92}$</td>
<td>1</td>
<td>8.80</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Note: d.f. stands for degrees of freedom. The tests follow a $\chi^2$ distribution.

Estimation of the simpler model yields distinct results from the structural model. Deposits supply determinants at the local market level are found to be relevant, contrary to previous section findings. On the other hand, Nash conduct emerges as a robust piece of evidence. The test of Nash-Bertrand behavior for all banks in the market is not rejected, while collusion, either market-wide or at the economic group level, is clearly rejected (see Table 7). This is in line with the previous inference. Nash conduct was favored, although economic group collusion was not statistically eliminated.16

As to deposits supply determinants, all variables specified are found to be highly significant (see Table 8). Unfortunately, some of the estimated coefficients contradict intuition. The first problematic parameter is the one associated with supply sensitivity to other banks interest rates. The positive sign means that an increase in another bank’s interest rate leads to increased deposits into bank $i$. This is hardly convincing. However, since the index is based on the number of branches in each market, the two effects, branching and pricing, may be mixed up, thus justifying the ‘wrong sign’ of the coefficient.

Two other problematic parameters are the negative effects of income per capita and the number of firms in the markets on deposits supply. In the case of income per capita, it can be argued that what is captured is a nonlinear relationship between the supply of deposits and population. As to the negative

16Likelihood values for estimated supply models can be found in the appendix.
Table 8: Estimates for the simpler model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate 1</th>
<th>Estimate 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_01$</td>
<td>-22.264</td>
<td>-22.363</td>
</tr>
<tr>
<td></td>
<td>(-18.47)</td>
<td>(-22.61)</td>
</tr>
<tr>
<td>$a_02$</td>
<td>-22.698</td>
<td>-22.830</td>
</tr>
<tr>
<td></td>
<td>(-19.96)</td>
<td>(-26.65)</td>
</tr>
<tr>
<td>$b_{91}$</td>
<td>0.008</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>(1.24)</td>
<td>(0.98)</td>
</tr>
<tr>
<td>$b_{92}$</td>
<td>0.014</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>(2.12)</td>
<td>(1.97)</td>
</tr>
<tr>
<td>$a_1 = \partial(D^i/n^i)/\partial r_d^i$</td>
<td>128.725</td>
<td>136.380</td>
</tr>
<tr>
<td></td>
<td>(30.74)</td>
<td>(44.63)</td>
</tr>
<tr>
<td>$a_2 = \partial(D^i/n^i)/\partial r_{d}^i$</td>
<td>82.660</td>
<td>75.460</td>
</tr>
<tr>
<td></td>
<td>(12.71)</td>
<td>(15.45)</td>
</tr>
<tr>
<td>$a_3 = \partial(D^i/n^i)/\partial SC^i$</td>
<td>3.385</td>
<td>3.270</td>
</tr>
<tr>
<td></td>
<td>(2.37)</td>
<td>(2.68)</td>
</tr>
<tr>
<td>$a_4 = \partial(D^i/n^i)/\partial URBAN^i$</td>
<td>2.192</td>
<td>2.275</td>
</tr>
<tr>
<td></td>
<td>(7.75)</td>
<td>(7.37)</td>
</tr>
<tr>
<td>$a_5 = \partial(D^i/n^i)/\partial POP^i$</td>
<td>22.844</td>
<td>24.330</td>
</tr>
<tr>
<td></td>
<td>(9.06)</td>
<td>(9.43)</td>
</tr>
<tr>
<td>$a_6 = \partial(D^i/n^i)/\partial TAX^i$</td>
<td>-21.737</td>
<td>-23.632</td>
</tr>
<tr>
<td></td>
<td>(-8.15)</td>
<td>(-9.27)</td>
</tr>
<tr>
<td>$a_7 = \partial(D^i/n^i)/\partial TOUR^i$</td>
<td>1.906</td>
<td>1.900</td>
</tr>
<tr>
<td></td>
<td>(15.92)</td>
<td>(15.39)</td>
</tr>
<tr>
<td>$a_8 = \partial(D^i/n^i)/\partial KM^2$</td>
<td>0.214</td>
<td>0.293</td>
</tr>
<tr>
<td></td>
<td>(2.08)</td>
<td>(2.76)</td>
</tr>
<tr>
<td>$a_9 = \partial D^i/n^i/\partial FIRM^i$</td>
<td>-0.203</td>
<td>-0.216</td>
</tr>
<tr>
<td></td>
<td>(-8.59)</td>
<td>(-9.02)</td>
</tr>
<tr>
<td>$\lambda^M_{91}$</td>
<td>0.001</td>
<td>(0.37)</td>
</tr>
<tr>
<td>$\lambda^M_{92}$</td>
<td>0.000</td>
<td>(0.06)</td>
</tr>
<tr>
<td>$\lambda^G_{91}$</td>
<td>-0.013</td>
<td>(-0.78)</td>
</tr>
<tr>
<td>$\lambda^G_{92}$</td>
<td>-0.013</td>
<td>(-0.66)</td>
</tr>
</tbody>
</table>

Log Likelihood: 113.844, 112.908

Note: $t-$statistics within brackets.
positions. The empirical specification treats deviations to the symmetric equilibrium as the pure random effects.

Second, it was assumed one banking product only. In reality, banks have different types of deposits and credits and their interaction can affect the equilibrium outcome. Data availability prevents such analysis. Extension of the model to incorporate multiproduct considerations is straightforward, but demands unavailable data in the empirical application.

Third, an inelastic demand of the good is assumed. Consumers can either buy one or zero units of the good. Intermediate adjustments in the quantity purchased in response to changes in prices other than buy or not are not allowed.

Having put forth the main caveats of the model it should be noted that even very simplistic models can shed some light over oligopolistic interaction in product differentiated industries.

In this paper the multilocation nature of the banking activities was explicitly considered in market power measurement in the context of a spatial (product differentiation) model. It was shown that centralization of price decisions over branches is profit maximizing. The absence of price discrimination across branches and local markets generates equilibrium price dispersion.

Confrontation of the theoretical framework with data for Portuguese commercial banking was reported. Accounting for the multilocation nature of banks provides a reasonable description of the oligopolistic interaction between banks.

Two benchmark patterns of behavior that received some support from the data were overall Nash behavior and collusion among banks belonging to the same economic group. Market collusion among all banks is clearly rejected by the data. Although the evidence in favor of Nash behavior is stronger, it is not possible to discard group collusion.

It can be argued that spatial models have too much localized competition. The structure of the model would then be biased against collusive behavior. Also, the implied structure of the model may be too rigid to fit to the data. This remark suggested a reformulation of the deposits supply structure. Instead of the rigid structure of the circular city model, a more simple linear supply deposits function was specified. The main feature of the model, aggregation over local markets, as retained as supply of deposits is defined at the branch level.

Estimation of this simpler model revealed that, on the one hand, differences in market characteristics do influence supply of deposits. It constitutes a contrary finding to the structural model results. On the
impact of the number of firms on the volume of deposits, no convincing explanation is available.

Turning now to the remaining variables, it is possible to see that supply of deposits reacts positively to own bank’s interest rate. In fact, this sensitivity is quite high. The elasticity of deposits supply per branch, evaluated at the sample mean for each year, is roughly about 3 (more precisely, 3.14 in 1991 and 3.52 in 1992). Given that banks expanded their networks of branches in 1992, the rise in deposits may also be due to an increase in the number of branches. In that sense, the elasticity of deposits per branch can be seen as a more reliable indicator of interest rate sensitivity. Nevertheless, the surprisingly high value computed reinforces the need of a more careful analysis of the structure of deposits supply.

Lower competition at the local level increases, naturally, the volume of deposits per branch. Also, intensity of tourism and population have a positive effect on deposits. Urban areas are associated with a higher volume of deposits per branch, reflecting the greater need of banking services relative to rural markets.

Taking together these results, it is clear that future investigation of supply deposits determinants and the functional form of the relationship is desired. It is worthwhile to note that in spite of the not-so-clear interpretation of the effect of some variables on the supply of deposits, the inference on the conduct pattern seems fairly robust.

8 Final Remarks

Market power measurement for multilocation firms was addressed. An application to the banking sector was developed. The banking industry is one of the best examples of industries where local market competition matters. Moreover, activities of a branch typically span over several branches and markets.

A theoretical model of the multibranch–multimarket competition, based on the circular city model of Salop (1979) was presented. The model is highly simplified and overlooks several aspects that may be relevant. Some characteristics of the model are noteworthy by their potential restrictiveness. First, a symmetric equilibrium in each local market is always assumed. This means that entry of a new bank implies relocation of all other banks in the circle. This is clearly an untenable assumption if the circle is interpreted strictly in a geographic space sense and location-specific costs exist. With an interpretation based on consumers’ preferences, such relocation may be due to reassessment by consumers of banks’
other hand, Nash-Bertrand behavior was not rejected, reinforcing, therefore, the inference on conduct. However the specified linear deposits supply function yielded estimates with little economic content. Future research in the modeling of deposits supply is thus called for.

The main conclusion of the work is that market power measurement and explanation of margins in the banking industry should take into account the local market nature of the activity. Differences across markets can originate equilibrium interest rate dispersion, without implying different conduct by banks or different marginal cost structures.

The finding of Nash behavior in the application to the Portuguese commercial banking sector provides further understanding of the results in Barros and Leite (1994). There, the deposits and the loans market were investigated jointly. That study detected substantial variation in price-cost margins across banks in both markets. The present work shows that such variation is not incompatible with Nash behavior. Another finding of Barros and Leite (1994) was an increase in competition in deposits, as measured by the level of price-cost margins. That effect was attributed in a large extent to growth strategies through branching. The characterization of Nash-Bertrand for both years in the setting of our model reinforces that interpretation. Once the branch structure over local markets is taken into account, no change in behavior is found. The more competitive market allocation is, then, due to branching, which in turn implies a change in pricing strategies of banks.

Some work on the theoretical foundations of branching alone has been carried out in Cabral and Majure (1993), where an application to the Portuguese banking sector is presented. Their empirical framework is developed at a higher aggregation level. Portuguese regions (18) are the unit of reference and results are not directly comparable.

A deeper understanding of branching strategies and their interactions with price policies is necessary and should motivate further research.
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Tables of likelihood values

The circular market model

Table 9: Log likelihood values: conduct patterns

<table>
<thead>
<tr>
<th></th>
<th>Basic</th>
<th>General</th>
<th>Maintained (t_1 \neq 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free</td>
<td>-80.2699</td>
<td>-71.0349</td>
<td>-77.5158</td>
</tr>
<tr>
<td>\lambda_j = 0</td>
<td>-80.9274</td>
<td>-74.4411</td>
<td>-78.7222</td>
</tr>
<tr>
<td>\lambda_j \neq 0</td>
<td>-80.9126</td>
<td>-73.9775</td>
<td>-78.6467</td>
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<td>\lambda_j \neq 0</td>
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<td>-73.6898</td>
<td>-78.2620</td>
</tr>
<tr>
<td>\lambda_j \neq 0</td>
<td>-81.9266</td>
<td>-73.7934</td>
<td>-78.3531</td>
</tr>
<tr>
<td>\lambda_j = 1</td>
<td>-104.259</td>
<td>-99.6077</td>
<td>-104.025</td>
</tr>
<tr>
<td>\lambda_j = 1</td>
<td>-98.6528</td>
<td>-83.0437</td>
<td>-98.6170</td>
</tr>
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<td>-81.6892</td>
<td>-89.4967</td>
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<td>-79.3317</td>
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<tr>
<td>\lambda_j = 1</td>
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</tr>
<tr>
<td>\lambda_j = \lambda_j = 1</td>
<td>-81.5456</td>
<td>-76.1371</td>
<td>-79.6844</td>
</tr>
</tbody>
</table>

Table 10: Log likelihood values: deposits supply determinants

<table>
<thead>
<tr>
<th></th>
<th>Basic (\lambda_j = 0)</th>
<th>General (\lambda_j \neq 0)</th>
<th>Maintained (\lambda_j = 1, \lambda_j = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free</td>
<td>-74.4411</td>
<td>-71.0349</td>
<td>-76.1371</td>
</tr>
<tr>
<td>t_1 \neq 0</td>
<td>-78.7222</td>
<td>-77.5158</td>
<td>-79.6843</td>
</tr>
<tr>
<td>t_2 \neq 0</td>
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<td>-77.6997</td>
<td>-81.5387</td>
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<tr>
<td>d_1 \neq 0</td>
<td>-79.8042</td>
<td>-78.2141</td>
<td>-80.8048</td>
</tr>
<tr>
<td>d_2 \neq 0</td>
<td>-79.5835</td>
<td>-78.0583</td>
<td>-80.1182</td>
</tr>
<tr>
<td>d_3 \neq 0</td>
<td>-80.0341</td>
<td>-78.9956</td>
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</tr>
<tr>
<td>all = 0</td>
<td>-80.9274</td>
<td>-80.2700</td>
<td>-81.5456</td>
</tr>
</tbody>
</table>

Table 11:

<table>
<thead>
<tr>
<th></th>
<th>Basic (\lambda_j = 0)</th>
<th>Maintained (\lambda_j = 1, \lambda_j = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t_1 \neq 0; t_2 \neq 0</td>
<td>-78.6531</td>
<td>-79.4873</td>
</tr>
<tr>
<td>t_1 \neq 0; d_1 \neq 0</td>
<td>-78.7181</td>
<td>-79.6261</td>
</tr>
<tr>
<td>t_1 \neq 0; d_2 \neq 0</td>
<td>-77.6160</td>
<td>-78.4395</td>
</tr>
<tr>
<td>t_1 \neq 0; d_3 \neq 0</td>
<td>-77.9392</td>
<td>-78.7926</td>
</tr>
</tbody>
</table>
The simpler model

<table>
<thead>
<tr>
<th>Description</th>
<th>Log likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free</td>
<td>113.844</td>
</tr>
<tr>
<td>$\lambda_j = 0$</td>
<td>112.908</td>
</tr>
<tr>
<td>$\lambda_j = 1, \lambda_j^a = 0$</td>
<td>99.6851</td>
</tr>
<tr>
<td>$\lambda_j = 1, \lambda_j^b = 0$</td>
<td>99.1360</td>
</tr>
<tr>
<td>$\alpha_2 = 0(\lambda_j = 0)$</td>
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</tr>
<tr>
<td>$\alpha_3 = 0(\lambda_j = 0)$</td>
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</tr>
<tr>
<td>$\alpha_4 = 0(\lambda_j = 0)$</td>
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</tr>
<tr>
<td>$\alpha_2 = 0(\lambda_j = 0)$</td>
<td>94.0812</td>
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<td>$\alpha_2 = 0(\lambda_j = 0)$</td>
<td>97.2804</td>
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<tr>
<td>$\alpha_2 = 0(\lambda_j = 0) = 91.5838$</td>
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<tr>
<td>$\alpha_2 = 0(\lambda_j = 0)$</td>
<td>109.459</td>
</tr>
<tr>
<td>$\alpha_2 = 0(\lambda_j = 0)$</td>
<td>96.3902</td>
</tr>
<tr>
<td>$a_{91} = a_{92}; b_{91} = b_{92}$</td>
<td>108.911</td>
</tr>
<tr>
<td>$\lambda_j = 0, a_{91} = a_{92}; b_{91} = b_{92}$</td>
<td>106.616</td>
</tr>
<tr>
<td>$b_{91} = b_{92}$</td>
<td>106.826</td>
</tr>
<tr>
<td>$a_{91} = a_{92}$</td>
<td>110.949</td>
</tr>
<tr>
<td>$a_{91} = a_{92}, \lambda_j = 0$</td>
<td>110.116</td>
</tr>
</tbody>
</table>

Note: the restriction upon the free model is shown under description

Sample Information

The banks included in the sample are: Banco de Fomento e Exterior; Banco Pinto e SottoMayor; União de Bancos Portugueses; Banco Comercial dos Açores; Banco Comercial Português; Banco Espírito Santo; Banco Totta & Açores; Banco Fonseca's & Burnay; Banco Nacional Ultramarino; Crédito Predial Português; Banco de Comércio e Indústria; Banco Comercial de Macau; Banco Borges e Irmão; Banco Português do Atlântico; Caixa Geral de Depósitos.

The Portuguese banking sector has witnessed the emergence of economic groups. In the sample of this study, the relevant associations are $BFE$ and $BB1; UBPP, BCM$ and $BPA; BTA$ and $CPP$; and $BNV$ and $CGD$.  

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