Taste for Imports and Market Integration

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Preliminary, comments welcome

Abstract

We purport to assess the effects of market integration when preferences for the foreign good differ across domestic consumers. We consider a characteristics line with two locations at the end points (countries). Consumers are in one or in the other country, with zero density otherwise. Preference for the foreign good varies across consumers. We show that while the pre-integration equilibrium had two-way trade, induced by taste differences, the post-integration equilibrium may have two-way trade or one-way trade. The one-way trade runs from the small country to the big economy. Welfare effects of market integration are also addressed.

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1 Introduction

Market integration issues have gained relevance in the recent past. Economic organization of trade flows has witnessed an increasing movement towards market integration. The European Union and the NAFTA deal provide the best examples. Those agreements are expected to bring fundamental changes in trade flows, of both inter-industry and intra-industry type. Intra-industry flows account for a significant share of trade volumes between the countries participating in these arrangements. This is true for the European Union but also for the NAFTA. Although almost all discussions over NAFTA are about US-Mexican relations, trade flows between the countries are smaller than the ones across the US-Canadian border: exports of the United States to Canada and México were worth, in 1992, $bn 90.2 and $bn 40.6, respectively. US imports from Canada and México were $bn 103.9 and $bn 32.6, respectively. The NAFTA is not the first trade liberalizing effort between the United States and Canada. Since 1989 they have their own free-trade agreement. Nevertheless, it is expected that NAFTA will give further impetus to intra-industry commercial flows.

The program of European Union attempts to build up a common market constituted by several countries, with free trade within the whole market. The completion of the European internal market is to be achieved by removal of all barriers to intra-European Union trade.

Abolition of all barriers to trade allows, in principle, arbitrage and prices of products would be equalized, up to transport costs, across countries. Producers will charge the same mill price to all European Union customers. Since the bulk of intra-European trade is already of the intra-industry type, its likely evolution deserves our interest.

Market integration has received both theoretical and empirical attention in the literature. The distinction between segmented and integrated markets has been found to be crucial for several results of strategic trade policy (Markusen and Venables, 1988). The cornerstone of the empirical line of research is the work of Smith and Venables (1988).\(^1\) The theoretical basis has been the demand for variety (Krugman, 1979) and reciprocal dumping models (Brander and Krugman, 1983).\(^2\)

\(^1\)Other applications to the European Single Market can be found in Winters (1992) and Venables and Winters (1991), for example. Examples of studies devoted to the effects of US-Canada free-trade agreements are Brown and Stern (1989), Cox and Harris (1985,1986) and Stern, Tresse and Whalley (1987).

\(^2\)The relevant theoretical literature on strategic trade policy is by now quite extensive.
Arguments on welfare gains from market integration run through increased competition in product differentiated, imperfectly competitive industries and scale economies to be gained. The role of diversity of consumer preferences has been somewhat neglected. Typically, the models assume symmetry across firms and Dixit-Stiglitz (1977) type of preferences for consumers. Those models may specify preference parameters permitting consumers' preferences for domestic over imported goods to differ. Difference in preferences across countries are accommodated easily. It is possible to find, in such setting, a tariff equivalent to taste differences and the standard analysis goes through (Smith and Venables, 1988). In such type of models it is possible to show that integration may raise all equilibrium prices (Haaland and Wooton, 1992). However, the approach is not well suited to analyze diversity across consumers.

Differences of consumer preferences within the country over the imported good may constitute a basis for intra-industry trade. In fact, is easy to find examples of advertising campaigns, by government departments and private institutions, under the generic theme 'buy your own country products.' The aim is clearly to shift trade flows and its existence recognizes the importance of consumer taste diversity.

In the present paper, the effects preferences diversity on the pattern after trade of market integration between two economies of different size are identified.\footnote{Differences in size can be interpreted as a central economy and a peripheral economy: Krugman and Venables (1990).} We consider a imperfectly competitive industry, with horizontal product differentiation and taste diversity across consumers. The motivation for the existence of intra-industry trade is not preference for variety but variety of preferences. The analysis is confined to a partial equilibrium framework.

Market integration means, in our model, that firms cannot price discriminate across markets. The starting point, called market segmentation, has no tariffs and firms can charge a different price in each market. Non-tariff barriers preventing arbitrage and allowing effective market segmentation are in place. The initial situation has no tariffs to concentrate the analysis in the pure gains of not allowing price discrimination. Tariff reductions in a model with horizontal product differentiation were investigated in Schmitt (1990). The implications of diversity in consumers' preferences for the imported good were not addressed though.

It is shown that market integration can give origin to either a two-way trade equilibrium
or to a one-way trade equilibrium. Welfare effects are also discussed. Market integration is always beneficial to the small country. It can benefit or harm the big country, according to whether the two-way trade or the one-way trade equilibrium results. Distributive welfare effects are also different. Consumers benefit and producers lose in the two-way trade equilibrium. The opposite effects prevail in the two-way trade equilibrium.

The paper is organized in the following way. The next section introduces the model. Section 3 characterizes equilibrium outcomes, with and without market integration. Section 4 discusses the welfare effects associated with a move from segmented to integrated markets. Finally, section 5 concludes the paper.

2 The Model

The model employed here is closely related to Garella and Martinez-Giralt (1989) and Neven, Norman and Thisse (1991). There are two countries, with a single producer each, located on the extreme points of a line segment of length $L$. Firms produce at zero constant marginal cost, which is no serious restriction. More important is the assumption of equal marginal costs across firms. Such assumption eliminates the incentive to trade based on comparative advantage. Two cases of pricing strategies are of interest. In the first one, firms set a uniform mill price for each country. That is, market segmentation according to country of destination is possible. In the second case, market integration prevents price discrimination across countries. Firms set the same mill price for both countries.

Consumers are concentrated at the two extremes of the line. Density of consumers in the first country, say country $A$, is restricted, without loss of generality, to $R > 1$ and density of consumers in country $B$ is normalized to one, that is, country $A$ is the big country. The case of $R < 1$ is symmetric.

Consumers within the country may differ in their valuation of the good produced by the foreign firm. They agree, on the other hand, on the valuation of the domestic good. A continuum of preference valuations for the foreign good, $\xi < c < \bar{c}$, $\xi < 0$, $\bar{c} > 0$, is introduced, with respect to which consumers are uniformly distributed. The interpretation of $c$ is the following: Consumers value the good in an amount $v$; If the consumer buys the good from the domestic supplier, he gets $v$. Alternatively, if he buys from the foreign supplier, his
valuation is \( v + c. \) The valuation \( v \) is high enough for each consumer to buy one unit of good. Clearly, \( c < 0 \) means that a consumer dislikes the foreign good in the sense that the same good is valued by less if provided by a foreign firm. In contrast, \( c > 0 \) means that a consumer has a "taste for imports."5

Consumers buy one unit of the good from the firm with the lower full price, defined as the sum of the mill price and the transportation cost to deliver the good to consumer's location. The transport cost is a linear function of distance, and is normalized to one. Since consumers are located at the end points of the interval, total transport cost is \( L \), whenever the good is sold to some consumer in another location. So, \( L \) can be interpreted as the product of transport cost and distance.

The following assumption is maintained throughout, to guarantee that with equal prices by both firms some consumers will buy the foreign good:

Assumption 1 \( \bar{c} > L \).

The assumption requires enough diversity of tastes.

## 3 Equilibrium analysis

Firms are price setters. They decide non-cooperatively and simultaneously and the Nash equilibrium concept is used to solve the game. Two different frameworks are considered. In the first one, market segmentation allows firms to price differently in each country. The second case requires firms to charge the same mill price in both countries.

### 3.1 Equilibrium with market segmentation

Market segmentation allows firms to discriminate according to the market where they sell the good. A firm has two prices to choose: domestic and export. Under the assumption of constant marginal costs each country's market can be treated separately. Denote by \( p_{ij} \) the price named by firm \( i \) in market \( j \), \( i, j = a, b \).6 The following proposition characterizes the equilibrium prices under market segmentation.

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4 This formulation is equivalent to have preferences \( v_d \) for the domestic good and preferences \( v_f \) for the foreign good, both uniformly distributed.
5 This approach is similar to the one in Neven, Norman and Thisee (1991).
6 Lowercase letters denote firms and uppercase letters denote countries.
Proposition 1 The Nash equilibrium of the price game is:

\[ p_t^d = (2\bar{c} - \bar{c} - L)/3 \]  \hspace{1cm} (1)

\[ p_t^f = (\bar{c} - 2\bar{c} + L)/3 \]  \hspace{1cm} (2)

The proof of the proposition as well as the properties and comparative statics of the equilibrium can be found in Neven, Norman and Thiss (1991).\(^7\)

The main results of their analysis are the following. Preferences for the domestic over the foreign good must be 'sufficiently widely spread' for trade to exist.\(^8\) The effects of an enlargement of diversity of preferences, keeping the average value constant, is similar to an increase in product differentiation. Both firms benefit from more market power and prices will be higher. It may also imply more imports if preferences are sufficiently biased against the foreign good. In such circumstances, the domestic firm will tend to specialize on those consumers with a high preference for the domestic good, thus leaving room for the foreign firm to increase its market share.

3.2 Equilibrium under market integration

Market integration prevents firms from discriminating consumers on the basis of their location. Still, some elements of differentiation remain: preferences of consumers vary and the different consumers' density gives more local power to one of the firms.

Assume firm a charges a price \( P_a = P_b + L - \bar{c} \). Then, consumers in country A that have the highest relative preference for the domestic good are indifferent between buying it from firm a or firm b. For \( P_a > P_b + L - \bar{c} \), \( \overset{\text{def}}{=} P_a^{\text{max}} \), firm a gets no demand.

For prices lower than \( P_a^{\text{max}} \), firm a starts to serve consumers in its own country. As firm a lowers its price from \( P_a^{\text{max}} \), at some point, consumers of country B will start buying the good in country A. More precisely, for \( P_a < P_a \), \( \overset{\text{def}}{=} P_b + \bar{c} - L \), firm a gets some demand from country B (consumers with high valuation for the imported good).

A further condition is imposed at this point, in addition to Assumption 1:

\(^7\)Our modeling assumptions \( \bar{c} > L \) and \( \bar{c} < 0 \) rule out cases (iii) and (i) of Proposition 1 in Neven, Norman and Thiss (1991). They take the case of symmetric countries. The extension to countries of different size is straightforward.

\(^8\)Our assumptions guarantee it.
Assumption 2 \( \Lambda = 2L - (\bar{c} + \bar{c}) > 0. \)

This is not restrictive if preferences are assumed to be symmetric, that is, \( | \bar{c} | = | \bar{c} |. \) Assumption 2 assures that at price \( \bar{P}_a \) some domestic consumers prefer to buy at country \( B. \) So, there is a range of prices for which two-way trade occurs. Unlike most often used models of trade with imperfect competition and product differentiation, what motivates trade in the model is variety of tastes, not taste for variety. If the condition does not hold, then a region with inelastic demand and no trade results and the analysis follows Garell and Martinez-Giralt (1989).

At a price \( P_a = \bar{P}_a \) \( \overset{def}{=} R P_b + L - \bar{c} \), firm \( a \) gets all consumers from its country. At price \( \bar{P}_a \) the consumer in country \( A \) with the highest valuation for the foreign good is indifferent between buying the good at home or abroad. Finally, if the price of firm \( a \) is low enough, \( P_a \leq P_b - L + \bar{c} \) \( \overset{def}{=} P_a^{\text{min}} \), then firm \( a \) gets all consumers. Its demand consists of the whole market. The demand function for the good produced by firm \( a \) is, thus:

\[
D_a(P_a, \bar{P}_b) = \begin{cases} 
0 & \text{if } P_a \geq P_a^{\text{max}} \\
R(\bar{P}_b - P_a + L - \bar{c}) & \text{if } P_a^{\text{max}} > P_a > \bar{P}_a \\
R(\bar{P}_b - P_a + L - \bar{c}) + \bar{c} - P_a + \bar{P}_b - L & \text{if } \bar{P}_a > P_a > \bar{P}_a \\
(1 + R)(\bar{c} - \bar{c}) & \text{if } P_a^{\text{min}} \geq \bar{P}_a
\end{cases}
\]

The demand faced by firm \( b \) can be obtained simply as \( D_b(P_b, \bar{P}_a) = (1 + R)(\bar{c} - \bar{c}) - D_a(\bar{P}_a, P_b). \)

It is easy to check that demand schedules are continuous.\(^9\) A typical demand schedule is presented in Figure 1.

Define the following domains for prices at the two countries \((i = a, b)\):

\[
D_{i1} = \{ P_i | P_i^{\text{max}} \geq P_i \geq \bar{P}_i \} \\
D_{i2} = \{ P_i | \bar{P}_i \geq P_i \geq \bar{P}_i \} \\
D_{i3} = \{ P_i | \bar{P}_i \geq P_i \geq P_i^{\text{min}} \}
\]

Lemma 1 If a non-cooperative price equilibrium exists, it is such that either

(a) \( P^*_a \in D_{a1} \) and \( P^*_b \in D_{b3} \) or

\(^9\) \( P_i^{\text{min}} \) and \( \bar{P}_j, j = a, b \) can be negative.
(b) \( P_a^* \in D_{a2} \) and \( P_b^* \in D_{b2} \)

**Proof:** See the appendix. □

The following proposition characterizes the possible equilibria of the game.

**Proposition 2** The unique Nash equilibrium of the price game is:

(a) For \( d = (\bar{c} - L)/(\bar{c} - \underline{c}) \in [0.2, 0.5] \) the equilibrium prices are:

\[
\begin{align*}
  P_a^* &= \frac{(2 + R)\bar{c} - (2R + 1)\underline{c} + (R - 1)L}{3(1 + R)} \\
  P_b^* &= \frac{(2R + 1)\bar{c} - (2 + R)\underline{c} - (R - 1)L}{3(1 + R)}
\end{align*}
\]

(3) For \( d \in [0, 0.2] \) and

(a) \( d \leq \left(6 + 4R - R^2 - 3\sqrt{4(1 + R) + 3R^2(1 - R)}\right)/(12R + R^2) \) for \( 1 \leq R \leq 1.54 \);

(b) \( d \leq (2 + 3R - 2R^2)/(8R + 2R^2) \) for \( 1.54 \leq R \leq 1.83 \);

(c) \( d \leq \left(4 + 6R + 11R^2 - (6 + 9R)\sqrt{R(R + 1)}\right)/(10R^2 - 8R) \) for \( 1.83 \leq R \).

the equilibrium prices are:

\[
\begin{align*}
  P_a^* &= \frac{(2(L - \underline{c}) + (\bar{c} - \underline{c})/R + \bar{c} - L)}{3} \\
  P_b^* &= \frac{(L - \underline{c} + 2(\bar{c} - \underline{c})/R + 2(\bar{c} - L))}{3}
\end{align*}
\]
Proof: See the appendix. ■

Conditions (a) to (c) in Proposition 2 guarantee the existence of the $\beta$-equilibrium and are necessary to prevent profitable deviations to other price domains. They essentially require that preferences spread is high relative to the net preference gain of buying the foreign good for the consumer who prefers it the most.

The parameter $d$ is ratio of the range of preferences for which consumers would buy the foreign good under equal prices over the total spread of preferences. A low value of $d$ can be brought up by a relatively high spread of preferences, high transport costs or low preference for the imported good over the domestic one. Notice that Assumption 1 implies $d < 0.5$.

The distinctive feature between the two types of equilibria is the existence of two-way trade in the $\alpha$-equilibrium and the collapse to one-way trade under the $\beta$-equilibrium. Thus, one important result of the model is that one-way trade may result from market integration and always implies that the small country exports to the big country. For ease of reference, the $\alpha$-equilibrium will be named two-way trade equilibrium and the $\beta$-equilibrium one-way trade equilibrium. Some comments on the equilibrium characteristics are warranted.

Volume of trade effects can be investigated by inspection of critical levels of preference for the foreign good. Consumers buy the imported good if their preference for the foreign good over the domestically produced one compensates transport costs plus any price differential. Define $c_i$, $i = 1, b$ as the critical level of preferences such that for $c \geq c_i$ the consumer in country $i$ buys the imported good. $c_i = P_j - P_i + L - z_i, i, j = a, b; i \neq j$. An increase in $c_i, i = a, b$ means that less of the foreign good is imported by country $i$, so trade volume is lower.

Corollary 1  Market integration decreases trade volumes.

Proof: See the appendix. ■
That market integration decreases trade volume should be of no surprise in our model. Market segmentation makes more attractive a price cut in exports, since domestic price remain unchanged. The possibility of practicing a lower mill price on the export market motivates some trade. The removal of this possibility by market integration makes more costly to the firm to export.

One-way trade may result if transport costs/length of the interval are not high relative to preferences for the foreign good, and if consumers' density in the country $A$ is sufficiently large. The conditions for existence of the two-way trade equilibrium are related to the range of tastes for imports ($\bar{\epsilon} - \underline{\epsilon}$), namely $d$ must be low. A small value for $d$ makes more likely to result the equilibrium. This means that increasing the bias in preferences towards the home produced good ($\epsilon$ decreases; $\bar{\epsilon}$ increases) raises the probability that with market integration, firm $a$ will choose to sell only to consumers in its country.

Intuitively, a low value of $d$ corresponds to a higher degree of product differentiation, which gives more market power to both firms. The one-way trade equilibrium emerges because with market integration the firm in the big country prefers to specialize on serving consumers with low preference for imports. To keep exports to country $B$, firm $a$ would have to lower its price in the domestic market also. The higher density of demand in country $A$ implies that costs of selling in the foreign market, in terms of foregone domestic profits, are higher for firm $a$ than for firm $b$. In case of a sufficiently important domestic base of consumers, firm $a$ achieves profit maximization with a high price in the domestic market and no exports. Small values of $d$ are associated with a more important base of 'loyal' domestic consumers and therefore make more likely that firm $a$ neglects the foreign market. Note that changes in the spread, keeping density constant, also brings more consumers to the market. An increase in $\bar{\epsilon}$, decrease in $\underline{\epsilon}$, reinforce the domestic base of consumers through a stronger bias towards the domestically produced good and through augmenting the total number of consumers in the country.

The transition from the segmented market equilibrium to the two-way trade equilibrium under market integration has an intuitive effect on prices: each firm sets its price between

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10 Neither is the finding specific to intra-industry trade. The same type of result has been obtained for inter-industry trade volumes by Norman (1990).

11 Under symmetry of preferences, the exact condition is $L = 3E/5$, that is, the overall transport cost must be close to the highest preference for the foreign good.
the two prices previously charged. The adjustment implies that consumers in a country face opposite price movements. The home good price decreases, while the price of the foreign good increases. The change in prices shifts demand away from the foreign good towards the domestic good. The intuition for the result is that market integration makes reciprocal dumping less attractive because any price cut in the foreign market also affects the own-country price of the firm.

More interesting are the price effects associated with the one-way trade equilibrium. The price charged by firm a increases with market integration. Firm b increases its mill price of exports and it may increase its domestic price. This effect runs against what can be termed 'conventional wisdom': prices go down as a result of market integration. The underlying argument is that firms have reduced market power under market integration.

However, if the firm in the big country chooses not to export, it can increase the price charged in the domestic market. The price no longer needs to compensate for transportation costs of foreign consumers. In turn, firm b has increased market power over consumers of country B. Due to market integration, the exercise of market power by firm B raises the price named in both markets. So, an equilibrium with higher prices may result.

4 Welfare effects

As important as equilibrium characterization are the welfare effects of market integration. The previous section has shown that market integration can change the type of equilibrium. From intra-industry trade before market integration, a pattern with one-way trade may result afterwards. It is, thus, natural to evaluate the associated welfare effects. The model assumes unit demand schedules. More generally, with elastic demands, price discrimination is beneficial in social terms because it increases quantity relative to the situation with no discrimination when firms have market power. The assumption of inelastic demand has the advantage of eliminating this welfare effect and thus makes more clear other welfare effects. Since consumers buy one unit of the good and there are no economies of scale to be gained in production, total welfare can change due to transport costs and preferences for the foreign good. Its distribution across countries and between consumers and firms (within the country) is also of interest.
National welfare is made up of profits from selling the good in both markets, consumers' surplus of buying the home good and consumers' surplus of buying the foreign good. Total welfare for country A is given by

\[ W_{LA} = R P_a^c (c_a - \bar{c}) + P_a^c (\bar{c} - c_b) + R \int_{c_a}^{c_b} (v - P_a^c) \, dc' + R \int_{c_a}^{\bar{c}} (v - P_b^c - L + c') \, dc' \]  

(5)

where \( c_a \) denotes the indifferent consumer in country A between buying the home and the foreign good. It is defined by \( c_a = P_b^c - P_a^c - L \) and \( c_b = P_b^c - P_a^c - L \). Similarly, total welfare of country B is

\[ W_{LB} = P_b^c (c_b - \bar{c}) + R P_b^c (\bar{c} - c_a) + \int_{c_b}^{c_a} (v - P_b^c) \, dc' + \int_{c_b}^{\bar{c}} (v - P_a^c - L + c') \, dc' \]  

(6)

Total welfare is defined as the sum of the two countries' individual welfare:

\[ TW = (1 + R) v (\bar{c} - c) + \frac{1}{2} R \bar{c} - \frac{R}{2} c_a^2 - \frac{1}{2} c_b^2 - L ((1 + R) \bar{c} - R c_a - c_b) \]  

(7)

The existence of international trade \( (c_b < \bar{c}) \) has two opposite effects upon total welfare. On the one hand, international trade increases transport costs, a welfare decreasing effect. On the other hand, variety of tastes makes total welfare increase with the consumption of the foreign good. The optimal level of international trade can be determined by straightforward maximization of expression (7). It implies equal mill prices in both markets (although its exact level is indeterminate). Neither the segmented market nor the integrated market structure achieves this. If somewhat more weight is attached to consumers' surplus than to profits of firms, the optimal prices are zero (equal to marginal cost).

The two type of integrated market equilibria entail different welfare implications. Table 1 summarizes the relevant effects.\(^{12}\)

The movement to market integration is beneficial to country B whatever the equilibrium prevailing afterwards. However, the distribution of the gains within the country depends on whether the two-way trade or the one-way trade equilibrium results. The emergence of one or the other type of equilibrium depends, intuitively, on the market power of firm a in its country. One-way trade equilibrium appears if firm a chooses not to export and exploits domestic consumers. Under the two-way trade equilibrium, such market power is eroded.

\(^{12}\)The expressions of the derivatives can be found in appendix.
Table 1: Welfare effects of market integration

<table>
<thead>
<tr>
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<th>$\alpha$-equilibrium (two-way trade)</th>
<th>$\beta$-equilibrium (one-way trade)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumers' surplus in A</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Consumers' surplus in B</td>
<td>+</td>
<td>-</td>
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<tr>
<td>Profits of firm A</td>
<td>-</td>
<td>+</td>
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<tr>
<td>Profits of firm B</td>
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<td>Welfare of country A</td>
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<td>-</td>
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<tr>
<td>Welfare of country B</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Total welfare</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

It is therefore natural that welfare effects differ for consumers and firms under two-way or one-way trade equilibrium. In the former case, consumers gain and producers loose, but by less. In the latter case, producers gain more than consumers loose. Consumers' welfare is reduced despite the possibility of lower prices of firm b due to the preference bias.

The gains that accrue to consumers are positive in the two-way trade equilibrium and negative in the other equilibrium. It comes from the rise in the equilibrium prices associated with the one-way trade equilibrium, namely in country A.

Market integration is beneficial to both countries if the two-way trade equilibrium results. Otherwise, country A looses overall, even if the profits of its firm increase. Total welfare gains are also positive in the $\beta$-equilibrium. The gains to country B are enough to compensate the losses of country A. The welfare gains, from a global point-of-view, accrue from lower trade costs. They are higher, in absolute value, than pure preference losses arising from reduced trade volumes. Market integration is superior to market segmentation in our framework, no matter how biased are the preferences for the foreign good or the transport cost (of course, the magnitude of welfare effects hinges on those elements).

The distributive effects are, however, more uncertain and vary according to the type of equilibrium that characterizes the market integration case. An equilibrium with one-way trade favors firms, while an equilibrium with two-way trade benefits consumers.

5 Final remarks

The effects of a move towards integrated markets have received attention from the economics profession in recent years. The European Union program and the NAFTA agreement are two
prominent examples of the relevance of the issue.

Market integration may drastically change trade patterns. In industries characterized by imperfect competition in national markets, expected gains of integration come from increased competition. Nevertheless, some literature has found that it may not be the case (Haaland and Wooton, 1992).

The role of consumers' preferences on market integration effects is addressed in a partial equilibrium model of horizontal product differentiation. Consumers differ within the country in their preference for the foreign good. Economic integration can occur between countries of different size. To cope with this characteristic, differences in country sizes are also incorporated.

The existence of intra-trade is motivated by preference diversity of consumers. The integrated market equilibrium will have less trade than the segmented market equilibrium. Welfare gains from lower trade costs are sufficient for a welfare increase to be associated with market integration. The integrated market equilibrium can be of one of two types: two-way trade or one-way trade, running from the small to the big country. In the first type of equilibrium, global welfare rises, consumers benefit and firms lose. Welfare goes up in both countries. In contrast, in the equilibrium with one-way trade, the total surplus of country A goes down. Due to its increased exports to the big country, the small economy improves its position. The gains and losses are, nevertheless, distributed in such a way that consumers loose and producers gain (by more than consumers' losses). The results show that the distribution of welfare effects crucially depend on the type of equilibrium that emerges in the integrated market, which in turn hinges upon the market power of the firm in the big country. In industries where it is high and diversity of consumer preferences matter, the one-way trade case is more likely to arise. On the other hand, low market power in the domestic market of the big country or a small spread in preferences will bring a two-way trade equilibrium.

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13 A qualification must be made. The result may not be robust to relocation of economic activities from peripheral (small) country to the central (big) country. See Krugman and Venables (1991).
Appendix

Proof of Proposition 1

There are nine possible combinations of price domains. By construction, except for sets $D_{i1}$ and $D_{j3}$ and $D_{i2}$ and $D_{j2}$, $i,j = a,b$, $i \neq j$, all other six combinations of prices interior to the specified domains lead to contradictions:

(i) $P_*^a \in D_{a1}, P_*^b \in D_{b1}$. One obtains $P_*^a > P_*^b + \bar{c} - L > P_*^b$ and simultaneously $P_*^b > P_*^a + \bar{c} - L > P_*^a$, a contradiction.

(ii) $P_*^a \in D_{a3}, P_*^b \in D_{b4}$. One has again $P_*^a > P_*^b$ and, at the same time, $P_*^b > P_*^a$, a contradiction.

(iii) $P_*^a \in D_{a1}, P_*^b \in D_{b2}$. One obtains $P_*^a < P_*^b + L - \xi$ and $P_*^a < P_*^a + \bar{c} - L$. By Assumption 2, $\bar{c} - L < L - \xi$ and it follows $P_*^a < P_*^b + \bar{c} - L$. On the other hand, is also required that $P_*^a > P_*^b + \bar{c} - L$, a contradiction.

(iv) $P_*^a \in D_{a2}, P_*^b \in D_{b1}$. It implies at the same time $P_*^a > P_*^b + L - \bar{c}$ and $P_*^a < P_*^b + L - \bar{c}$, a contradiction.

(v) $P_*^a \in D_{a2}, P_*^b \in D_{b3}$. One gets $P_*^a < P_*^b + \bar{c} - L$ and also $P_*^b + \bar{c} - L < P_*^a$, a contradiction.

(vi) $P_*^a \in D_{a3}, P_*^b \in D_{b2}$. One obtains $P_*^a < P_*^b + L - \bar{c}$ and also $P_*^a > P_*^b + L - \bar{c}$, a contradiction.

In the remaining three cases, price domains for firm $a$ and firm $b$ are compatible. It is, nevertheless, discard one more case. $R > 1$ implies that $P_a \in D_{a3}$ and $P_b \in D_{b1}$ cannot be an equilibrium. The proof this claim is done by contradiction. Suppose that $R > 1$ and that $P_a \in D_{a3}$ and $P_b \in D_{b1}$ is an equilibrium. The associated equilibrium prices, obtained by profit maximization of firms in the appropriate domains, are:

\[ P_*^a = \frac{(2(R+1)\bar{c} - 2(R+1)\xi - L)/3}{P_*^b = \frac{(R+1)\bar{c} - (R+2)\xi + L)/3}{3} \]

Both prices are positive, so it remains to check the boundary conditions. Take the condition $P_*^a \leq \bar{P}_a, P_* = P_*^b + L - \bar{c}$. Substituting $P_*^a$ and $P_*^b$ on this inequality, straightforward
manipulations result in

$$R < 1 - \frac{5(\bar{c} - L)}{\bar{c} - \bar{c}}$$

(8)

which is a contradiction to $R > 1$ under Assumption 1.

Proof of Proposition 2

By Lemma 1, attention can be restricted to two cases only. We begin by showing that a $\alpha$-equilibrium is possible. If the boundary conditions are satisfied, the equilibrium prices are determined by profit maximization in the domains $D_{a2}$ and $D_{b2}$:

$$P^*_a = \frac{2 + R}{1 + R} \bar{c} - \frac{2R + 1}{1 + R} \bar{c} + \frac{R - 1}{1 + R} L / 3$$

$$P^*_b = \frac{2R + 1}{1 + R} \bar{c} - \frac{2 + R}{1 + R} \bar{c} - \frac{R - 1}{1 + R} L / 3$$

Note that prices are positive. It is now necessary to check that $\bar{P}_a \leq P^*_a \leq \bar{P}_a$ and similarly for $P^*_b$. The boundary conditions can be subsumed in

$$dP = P^*_a - P^*_b \leq \bar{c} - L$$

(9)

and

$$dP = P^*_a - P^*_b \geq L - \bar{c}$$

(10)

Straightforward computations show that

$$dP = \frac{1}{3} \frac{R - 1}{1 + R} (2L - \bar{c} - L) > 0$$

(11)

The positive sign follows from Assumption 2 and $R > 1$. Let's first check condition (9). Solving the inequality for $R$, $R(5L - 4\bar{c} - L) < 2\bar{c} - \bar{c} - L$ and if the term in brackets in the left-hand side is negative, then condition (9) holds. If it is positive, then it holds whenever

$$R < R(\alpha) = \frac{2\bar{c} - \bar{c} - L}{5L - 4\bar{c} - L} = \frac{\bar{c} - L + \bar{c} - L - 5(\bar{c} - L)}{\bar{c} - L - 5(\bar{c} - L)} > 1$$

(12)

As to condition (10), it can be rewritten as $R(-L + 2\bar{c} - L) > 5L - 4\bar{c} - L$. By Assumption 1, if the right-hand side is negative, condition (10) holds. If it is positive, is required that

$$R > \frac{5L - 4\bar{c} - L}{-L + 2\bar{c} - L} = 1 - 6 \frac{\bar{c} - L}{2\bar{c} - L - L}$$

(13)

which is true by $R > 1$. 16
Let's now look at the $\beta$-equilibrium. Firms maximize profits in the domains $D_{a1}$ and $D_{b1}$. Simultaneous resolution of first-order conditions yields the equilibrium prices:

$$P_a^* = \left(2(L - \bar{c}) + \frac{\bar{c} - c}{R} + \bar{v} - L\right)/3$$

$$P_b^* = \left(L - \bar{c} + 2\frac{\bar{c} - c}{R} + 2(\bar{v} - L)\right)/3$$

Inspection of equilibrium prices reveals that they are positive. It remains to check the boundary conditions: $\hat{P}_a(P_b^*) \leq P_a^* \leq \hat{P}_a^{\text{max}}(P_b^*)$ and $\hat{P}_b^{\text{min}}(P_a^*) \leq P_b^* \leq \hat{P}_b(P_a^*)$. Stating boundary conditions in terms of equilibrium price differences:

$$dP > \bar{c} - L$$ (14)

and

$$dP < L - \bar{c}$$ (15)

where $dP$ is now

$$dP = \frac{2LR - \bar{c}(1 + R) - \bar{c}(R - 1)}{3R}$$ (16)

Take first condition (14). It implies $R(5L - 4\bar{c} - \bar{c}) > \bar{c} - \bar{c}$. If the bracketed term in the left hand side is negative, the condition never holds. If it is positive, it also requires

$$R > R(\beta) = \frac{\bar{c} - \bar{c}}{5L - 4\bar{c} - \bar{c}} = \frac{\bar{c} - \bar{c}}{\bar{c} - \bar{c} - 5(\bar{c} - L)}$$ (17)

Assumption 2 implies $R(\beta) < 1$ and this condition is always satisfied. As to condition (15), it is easy to show that is equivalent to

$$R(-L - \bar{c} + 2\bar{c}) < \bar{c} - \bar{c}$$ (18)

Since the expression between brackets is always negative under our assumptions, condition (15) holds.

Note now that there are two conditions on the value of $R$ in order to the proposed equilibrium fulfill domain boundary conditions: condition (12) for the $\alpha$-equilibrium and condition (17) for the $\beta$-equilibrium, under the case of $\bar{c} - \bar{c} > 5(\bar{c} - L)$. From inspection of expressions, $R(\alpha) > R(\beta)$. This means that exists an overlapping region of values for $R$ such that both types of equilibria satisfy the appropriate boundary conditions. However, profits in the
\( \alpha \)-equilibrium are lower than the ones in the \( \beta \)-equilibrium in the overlapping region of \( R \), whatever the price of the other firm. At the \( \alpha \)-equilibrium candidate pair of prices, each firm would like to deviate to a price in domain \( D_a \) or \( D_b \) for firm \( a \) or firm \( b \), respectively. So, the \( \alpha \)-equilibrium prevails for \( R < R(\beta) \) and the \( \beta \)-equilibrium arises otherwise. Computation of equilibrium profits, for firm \( a \), reveals that

\[
\Pi_a^*(\beta) - \Pi_a^*(\alpha) = (1 + 2R)^2(\bar{c} - \underline{c})^2 - 4R(1 + 2R)(\bar{c} - \underline{c})(\bar{c} - L) + R^2(\bar{c} - L)^2 > 0
\]

(19)

since by Assumption 2,

\[
(1 + 2R)(\bar{c} - \underline{c}) > (1 + 2R)2(\bar{c} - L) > 4R(\bar{c} - L)
\]

(20)

Also, for firm \( b \), \( \Pi_b^*(\beta) > \Pi_b^*(\alpha) \).

To complete the characterization of equilibria, it is necessary to establish the conditions for its existence. That is, it must be checked that, for the \( \alpha \)-equilibrium,

\[
\Pi_a(P_a^*(\alpha), P_b^*(\alpha)) \geq \Pi_a(P_a', P_b^*(\alpha)), P_a' \in D_a
\]

\[
\Pi_a(P_a^*(\alpha), P_b^*(\alpha)) \geq \Pi_a(P_a', P_b^*(\alpha)), P_a' \in D_a
\]

\[
\Pi_b(P_a^*(\alpha), P_b^*(\alpha)) \geq \Pi_b(P_a^*(\alpha), P_b'), P_b' \in D_b
\]

\[
\Pi_b(P_a^*(\alpha), P_b^*(\alpha)) \geq \Pi_b(P_a^*(\alpha), P_b'), P_b' \in D_b
\]

For the \( \beta \)-equilibrium, the relevant deviations to check are

\[
\Pi_a(P_a^*(\beta), P_b^*(\beta)) \geq \Pi_a(P_a', P_b^*(\beta)), P_a' \in D_a
\]

\[
\Pi_a(P_a^*(\beta), P_b^*(\beta)) \geq \Pi_a(P_a', P_b^*(\beta)), P_a' \in D_a
\]

\[
\Pi_b(P_a^*(\beta), P_b^*(\beta)) \geq \Pi_b(P_a^*(\beta), P_b'), P_b' \in D_b
\]

\[
\Pi_b(P_a^*(\beta), P_b^*(\beta)) \geq \Pi_b(P_a^*(\beta), P_b'), P_b' \in D_b
\]

We now proceed to an analysis of deviations to the Nash equilibrium on a case by case basis. Consider first the \( \alpha \)-equilibrium. For it to be an equilibrium outcome, it must be the case that \( 2(\bar{c} - L) < (\bar{c} - \underline{c}) < 5(\bar{c} - L) \), that is, \( 0.2 < d < 0.5 \), where \( d = (\bar{c} - L)/(\bar{c} - \underline{c}) \).

Take the deviation of firm \( a \) in the domain \( D_a \). The resulting deviating price is \( P_a'(P_b) = (P_b - \underline{c} + L)/2 \). It is required that \( P_a' > P_a(P_b(\alpha)) \). Computations yield \( P_a' - P_a(\bar{c} - \underline{c}) - \).
$8(\bar{\epsilon} - L) + 2R[(\bar{\epsilon} - \omega - 5(\bar{\epsilon} - L)] < 0$, so $P'_a = \hat{P}_a$ under the conditions for existence of the $\alpha$-equilibrium.

Comparison of profits gives:

$$\Pi'_a(P'_a(\alpha), P'_b(\alpha)) - \Pi'_a(\hat{P}_a, P'_b(\alpha)) = \frac{((\bar{\epsilon} - \omega)(R - 1) - (\bar{\epsilon} - L)(5R + 1))^2}{9(1 + R)} > 0$$

Firm $a$ can also deviate to a price in domain $D_a$. In this case, the deviating price is $P'_a(\bar{P}_b) = ((1 + R)(\bar{\epsilon} - \omega) + (\bar{\epsilon} - L) + P_b)/2$.

For $P'_a$ to belong to this domain, $P'_a < \hat{P}_a(P'_b(\alpha))$. Calculations reveal that

$$P'_a - \hat{P}_a = \frac{(\bar{\epsilon} - \omega)(2 + 2R + 3R^2) + (\bar{\epsilon} - L)(10 + 8R)}{6(1 + R)} > 0$$

This means that $P'_a = \hat{P}_a$. Comparison of profits gives

$$\Pi'_a(P'_a, P'_b(\alpha)) - \Pi'_a(\hat{P}_a(P'_b(\alpha)), P'_b(\alpha)) = \frac{((\bar{\epsilon} - \omega)(1 + R) - (\bar{\epsilon} - L)(5 + R))^2}{9(1 + R)} > 0$$

Neither deviation to $D_a$ nor to $D_a$ is profitable for firm $a$.

As to firm $b$, deviation to domain $D_b$ has a price $P'_b(\bar{P}_a) = (P_a + L - \omega)/2$. The boundary condition $P'_b > \hat{P}_b(P'_a(\alpha))$ requires

$$(\bar{\epsilon} - \omega)(2 + R) - (\bar{\epsilon} - L)(10 + 8R) > 0 \quad (21)$$

However, since $5(\bar{\epsilon} - L) > (\bar{\epsilon} - \omega)$ in the $\alpha$-equilibrium this condition is never satisfied. It results $P'_b = \hat{P}_b$ and it follows that

$$\Pi'_b(P'_a(\alpha), P'_b(\alpha)) > \Pi'_b(P'_a(\alpha), \hat{P}_b).$$

Finally, the deviating price of firm $b$ to the domain $D_b$ is

$$P'_b(P_a) = \frac{1}{2}P_a + \frac{\bar{\epsilon} - \omega + R(\bar{\epsilon} - L)}{R}$$

The relevant boundary condition is

$$P'_b < \hat{P}_b = \frac{(\bar{\epsilon} - \omega)(3 + 2R) + (\bar{\epsilon} - L)8R + 2R(5(\bar{\epsilon} - L) - (\bar{\epsilon} - \omega))}{6R(1 + R)} > 0$$

because $5(\bar{\epsilon} - L) > (\bar{\epsilon} - \omega)$ in the $\alpha$-equilibrium. So, $P'_b = \hat{P}_b$. Computation of profits gives

$$\Pi'_b(P'_a(\alpha), P'_b(\alpha)) - \Pi'_b(P'_a(\alpha), \hat{P}_b) = \frac{-((\bar{\epsilon} - L)(1 + 5R) + (\bar{\epsilon} - \omega)(R - 1))^2}{9(1 + R)} > 0$$

19
Also for firm \( b \), no deviation from the equilibrium price is profitable. The equilibrium holds without further requirements on parameter values.

Consider now the \( \beta \)-equilibrium. Suppose a deviation of firm \( a \) to domain \( D_{a2} \). First of all, the deviating price must belong to the assumed domain of prices:

\[
\tilde{P}_a(P_a^*(\beta)) \leq P'_a(P_a^*(\beta)) \leq \hat{P}_a(P_a^*(\beta))
\]

If the computed deviating price is greater than \( \hat{P}_a \) then \( P'_a = \hat{P}_a \) and, by definitions,

\[
\Pi'_a(P_a^*(\beta), \hat{P}_a^*(\beta)) > \Pi_{a2}(\hat{P}_a, \hat{P}_a^*(\beta))
\]

and the deviation is non-profitable to the firm. Take again \( d = (\bar{c} - L)/(\bar{c} - \bar{q}) \). Notice that \( d < 0.2 \) in the \( \beta \)-equilibrium. The condition for \( P'_a < \hat{P}_a \) can be stated as

\[
d > \frac{2R^2 - 2 - 3R}{R(4 + 10R)}
\]  

(22)

Notice that it never holds for \( R < 2 \). The other boundary condition on the deviating price \( (P'_a > \hat{P}_a) \) requires

\[
d > \frac{2 + 3R - 2R^2}{8R + 2R^2}
\]  

(23)

If the deviating price is interior to the domain, the requirement of Nash equilibrium amounts to

\[
\Pi'_a(P_a^*(\beta), P_a^*(\beta)) > \Pi_a(P'_a, P_a^*(\beta)), P_a \in D_{a2}
\]

This is equivalent to

\[
\gamma_0 + \gamma_1 d + \gamma_2 d^2 \geq 0
\]  

(24)

where

\[
\gamma_0 = -4 - 8R - 5R^2 + 8R^3
\]

\[
\gamma_1 = -4R(4 + 6R + 11R^2)
\]

\[
\gamma_2 = 4R^2(-4 + 5R)
\]

Deviations to the \( \beta \)-equilibrium are non-profitable if

\[
d < \frac{4 + 6R + 11R^2 - (6 + 9R)\sqrt{R(1 + R)}}{10R^2 - 8R}
\]  

(25)
Take now the case of $P_a' < \hat{P}_a$ or condition (23) not being satisfied. For the equilibrium to hold it is necessary that $\Pi_1(P_a^*(\beta), P_b^*(\beta)) \geq \Pi_1(\hat{P}_a, P_b^*(\beta))$. The condition is equivalent to

$$
(R^2 + 12R)d^3 + (2R^2 - 8R - 12)d + (R - 1)^2 \geq 0
$$

(26)

and to guarantee it, for $R < 2.577$.

$$
d < \frac{-R^2 + 4R + 6 + 3\sqrt{4(1 + R) + (3 - 2R)R^2}}{(12 + R)R}
$$

(27)

For values of $R$ greater than 2.577 the deviation is never profitable.

Let's look now at deviations of firm $a$ to the domain $D_a$. The boundary conditions to be satisfied in this case are

$$
P_a^\text{min}(P_a^*(\beta)) \leq P_a'(P_a^*(\beta)) \leq \hat{P}_a(P_a^*(\beta))
$$

The condition $P_a' \geq P_a^\text{min}$ implies $d \leq (2R^2 + 5R - 2)/4R$, which always holds. On the other hand, $P_a' \leq \hat{P}_a$ requires $d \leq (R + 2 - 3R^2)/8R \leq 0$, which holds only for $R = 1$ ($d = 0$). The analysis falls in the previous case.

Deviations of firm $b$ are considered next. Take first the deviation $P_b' \in D_b$. The deviating price must be lower than $\hat{P}_b(P_a^*(\beta))$ and greater than $\hat{P}_b(P_a^*(\beta))$. The former requirement is fulfilled but the latter requires

$$
d > \frac{1 + 2R^2}{4R + 10R^2}
$$

(28)

and $\Pi_2(P_a^*(\beta), P_b^*(\beta)) > \Pi_2(P_a^*(\beta), \hat{P}_b)$ implies

$$
d < \frac{27 + R - 8R^2 - 4R^3 + (-7 + 6R + 4R^2)\sqrt{R(R + 1)}}{2R(-4 + 5R)}
$$

(29)

If condition (28) is not satisfied, $P_b' = \hat{P}_b(P_a^*(\beta))$ and, by construction

$$
\Pi_2(P_a^*(\beta), \hat{P}_b(\beta)) > \Pi_2(P_a^*(\beta), \hat{P}_b) = \Pi_2(P_a^*(\beta), \hat{P}_b)
$$

The final deviation to consider for firm $b$ is to price in domain $D_b$. For the deviating price to belong to the specified domain, $P_b' \geq \hat{P}_b$ which implies $d < (R - 1)/8R$.

Take the case of $d > (R - 1)/8R$. Deviation is not profitable if

$$
\Pi_2(P_a^*(\beta), P_b^*(\beta)) > \Pi_2(P_a^*(\beta), \hat{P}_b)
$$
or, in an equivalent condition,

\[ 3R + 6R^2 + (R - 2)^2 - 2R(6R + 7)d + R(12 + R)d^2 \geq 0 \] (30)

The roots of the polynomial (for \( d \)) are always greater than 0.2, the maximum value for \( d \) allowed in the \( \beta \)-equilibrium, meaning that the condition holds for all \( R \) greater than one.

It remains the case \( d < (R - 1)/8R \). The condition to be satisfied in this case is

\[ \Pi^*_b(P^*_a(\beta), P^*_b(\beta)) \geq \Pi_b(P^*_a(\beta), P^*_b(\beta)) \]

which can be written as

\[ (R - 1 - 2Rd)(2 + R + Rd) \geq 0 \] (31)

or

\[ d \leq \frac{R - 1}{2R} \] (32)

Since the condition to be in the interior of the price domain \( D_b \) is stronger, deviation is not profitable. For firm \( b \), there is no profitable deviation to the price prescribed by the \( \beta \)-equilibrium in the \( D_b \).

Second-order conditions are trivially satisfied in both equilibria. Given the concavity of the profit function, under the stated conditions, the equilibrium is also unique.

Proof of Corollary 1

Analysis of trade volumes amounts to the computation of the change in the critical level of the preference for the foreign good. In the \( \alpha \)-equilibrium, straightforward calculations give

\[ \Delta c_t = \frac{2(2 - \epsilon - 2L)}{3(1 + R)} > 0, \ i = a, b \]

The changes in the \( \beta \)-equilibrium are

\[ \Delta c_a = \frac{\zeta - \epsilon}{3R} > 0 \]
\[ \Delta c_b = \frac{2\epsilon - \zeta - L}{3} > 0 \]

In the two-way trade equilibrium, \( c'_t, i = a, b \), increases for both countries by Assumption 2. Assumption 1 signs without ambiguity \( \Delta c_i \). Imports decrease after market integration, whatever the type of equilibrium that emerges.
Derivation of welfare effects

The welfare definition of country A can be simplified to (a similar simplification can be done for country B):

\[ WL_A = Rv(\bar{c} - \zeta) + \frac{R}{2} \bar{c}^2 + P_d(\bar{c} - c_A) - (P_d + L)(\bar{c} - c_A) - \frac{R}{2} \bar{c}_A^2 \]  \hspace{1cm} (33)

The relevant derivatives can be found through substitution of equilibrium values in welfare expressions and straightforward simplifications. In the \( \alpha \)-equilibrium,

\[
\begin{align*}
    dCS_A &= \frac{R(3 + R)}{9(1 + R)^2} \bar{c}^2 > 0 \\
    dCS_B &= \frac{R(1 + 3R)}{9(1 + R)^2} \bar{c}^2 > 0 \\
    d\Pi_A &= -\frac{R}{9(1 + R)} \bar{c}^2 < 0 \\
    d\Pi_B &= -\frac{R}{9(1 + R)} \bar{c}^2 < 0 \\
    dWL_A &= \frac{2R}{9(1 + R)^2} \bar{c}^2 > 0 \\
    dWL_B &= \frac{2R^2}{9(1 + R)^2} \bar{c}^2 > 0 \\
    dTW &= \frac{2R}{9(1 + R)^2} \bar{c}^2 > 0
\end{align*}
\]

As to the \( \beta \)-equilibrium,

\[
\begin{align*}
    dCS_A &= \frac{(\bar{c} - \zeta)^2(1 - 8R) - 2R(\bar{c} - \zeta)(\bar{c} - L)}{18R} < 0 \\
    dCS_B &= \frac{(5R - 12)(\bar{c} - \zeta)^2 - R(\bar{c} - L)^2 - 14R(\bar{c} - L)(\bar{c} - \zeta)}{18R} < 0 \\
    d\Pi_A &= \frac{(1 + 3R)(\bar{c} - \zeta)^2 - 4R(\bar{c} - \zeta)(\bar{c} - L) - R(\bar{c} - L)^2}{18R} > 0 \\
    d\Pi_B &= \frac{4(\bar{c} - \zeta)^2 - R(\bar{c} - L)^2 + 8R(\bar{c} - L)(\bar{c} - \zeta)}{18R} > 0 \\
    dWL_A &= \frac{(3 - 2R)(\bar{c} - \zeta)^2 - 2R(\bar{c} - L)^2 - 10R(\bar{c} - L)(\bar{c} - \zeta)}{18R} < 0 \\
    dWL_B &= \frac{(5R - 4)(\bar{c} - \zeta)^2 - 3R(\bar{c} - L)^2 + 2R(\bar{c} - L)(\bar{c} - \zeta)}{18R} > 0 \\
    dTW &= \frac{(3R - 1)(\bar{c} - \zeta)^2 - 5R(\bar{c} - L)^2 - 8R(\bar{c} - L)(\bar{c} - \zeta)}{18R} > 0
\end{align*}
\]

To sign appropriately the derivatives in the \( \beta \)-equilibrium, we make use of \( \bar{c} - \zeta > 5(\bar{c} - L) \).
References


