Multi-project R&D competition with free-entry

by

Pedro Pita Barros *

Working Paper n° 201

April 1993

Abstract

We consider a model of innovation where firms are allowed to invest in several projects.

It is shown that free-entry, in the absence of start-up costs, leads to an equilibrium with one project per firm. The presence of start-up costs reintroduces the rationale for parallel projects undertaken by each firm.

In this case, free-entry implies that all adjustment to changes in the gains to innovation occur through the number of firms in the market, with projects per firm remaining constant.

Adresse:

Faculdade de Economia

Universidade Nova de Lisboa

Travessa Estêvão Pinto

P-1000 Lisboa

* I am grateful to António Leite, Paulo Pinho, Amado da Silva, a referee, Peter Swann and specially Luís Cabral for useful comments and suggestions. The usual disclaimers apply.

1 Introduction

Usually, models of R&D find that the number of firms is a fundamental determinant of the level of innovation in a market economy. Nevertheless, the way the number of firms affects the level of R&D depends on the particular model we are dealing with. In these models each firm is, in general, allowed to have only one project of R&D. Recognizing this shortcoming of traditional analysis, Sah and Stiglitz (1987) have presented a model for R&D expenditures where a firm could have more than a single project. They established an invariance result of the market level of expenditures on R&D with respect to the number of firms. Reinganum (1983) also presents a model reaching the same kind of result: firms choose the same level of R&D expenditure in each project, while the number of projects varies across firms. However, Sah and Stiglitz assume an exogenous market structure. If we make the number of firms endogenous, allowing free-entry (exit) in undertaking R&D investments, some different results emerge.

Endogeneity of market structure introduces an additional variable to adjust equilibrium values: the number of firms in the race for the innovation. Free-entry equilibrium characterization reconciles, in the absence of start-up costs, the Sah and Stiglitz model with the traditional hypothesis in the literature of "one firm- one R&D project." Thus, the dependence of R&D on market structure can be viewed not as "an artifact of the arbitrary specification of firms' choices and research technologies [Sah and Stiglitz (1987, page: 102)]" but as an equilibrium result.

More than one project per firm, in a free-entry equilibrium, can be brought up by the presence of a positive start-up cost in R&D activities.

In this case, a different invariance result holds: the number of projects per firm does not depend on the reward to innovation. All the adjustment to changes in the gains from innovation is made by the equilibrium number of firms.

In the second section, we introduce the basic model and allow for positive start-up costs. Next, in the third section, a different invariance result is presented, which holds with free-entry (exit) in R&D activities. In the forth section, a numerical analysis is presented. Section 5 concludes.

2 The basic model

This model elaborates on Sah and Stiglitz's (1987) model. We have a race for an innovation, with uncertainty. If one firm succeeds then a patent (infinitely lived) is given. If more than one firm succeeds then the competition between

the winning firms implies zero profits for all.¹ Firms are symmetric and can have several parallel (and independent) R&D projects of R&D. The only restriction is that they can not share projects. All projects have the same probability function of success, $p(e)$, for an expenditure e (with $0 < p(e) < 1$).

Proposition 1 *In a interior symmetric Nash equilibrium, with free-entry and without start-up (fixed) costs in R&D activities, each firm has only one project of R&D.*

Proof: The expected profit of firm i is

$$V_i = W \left[\prod_{f \neq i; f=1}^N \prod_{j=1}^{m_f} (1 - p(e_{fj})) \right] \left[1 - \prod_{j=1}^{m_i} (1 - p(e_{ij})) \right] - \sum_{j=1}^{m_i} e_{ij} \quad (1)$$

where W is the profit associated with winning the patent, e_{ij} is firm i 's expenditure on project j and m_i is the number of R&D projects taken by firm i . Each firm maximizes expected profits with respect to the expenditure per project, e , and the number of projects taken, m_i . The first order conditions for firm i are:²

$$W \left[\prod_{f \neq i; f=1}^N \prod_{j=1}^{m_f} (1 - p(e_{fj})) \right] p_e(e_{ig}) \prod_{j \neq g; j=1}^{m_i} (1 - p(e_{ij})) = 1, g = 1, \dots, m_i \quad (2)$$

$$W \left[\prod_{f \neq i; f=1}^N \prod_{j=1}^{m_f} (1 - p(e_{fj})) \right] p(e_{im_i}) \prod_{j=1}^{m_i-1} (1 - p(e_{ij})) = e_{im_i} \quad (3)$$

These two expressions give the optimal choice of each firm. Dividing (2) by (3) we get the result of invariance of expenditure per project pointed out by Sah and Stiglitz (1987): $e_{ij} = e, \forall i, j$. Taking market structure as fixed means that conditions (2) and (3) fully characterize the equilibrium and the total number of R&D projects in the market is invariant to the number of firms – this is second part of the invariance result of Sah and Stiglitz, and it crucially relies on market structure being exogenous.

To have an exogenous market structure, entry must be blockaded in some way. Otherwise, there is entry of firms in the race if expected profits are positive. Exit should occur if expected profits are negative. So, in a free-entry equilibrium, expected profits are zero.³

¹This assumption avoids explicit analysis of the R&D investment's appropriability by firms.

²To simplify, we take the first-order condition with respect to m_i as $V(m_i) - V(m_i - 1) = 0$. The true condition is $V(m_i) - V(m_i - 1) \geq 0 \geq V(m_{i+1}) - V(m_i)$. The number of projects is an integer variable.

³The true condition for the free-entry equilibrium is $V_N \geq 0 > V_{N+1}$. For analytical tractability, $V_N = 0$ is assumed.

Since firms are symmetric, $m_i = m, \forall i$ holds in equilibrium. Equating (1) to zero and substituting (3) into (1) we get

$$(1 - p(e))^{1-m} = 1 + (m - 1)p(e) \quad (4)$$

Since $m \geq 1$ (otherwise, the firm is not in the race for the innovation), by convexity on p , $m = 1$ follows. ■

The conventional analysis of R&D with the simple relationship “one firm-one project” emerges in this simple model when we assume away barriers to entry. Such assumption is a crucial one. The existence of barriers to entry, defined as a start-up, fixed cost, changes this result radically.

Proposition 1 is the consequence of the different incentives new firms and firms already in the race have. The expected reward of a R&D project is decreasing in the number of projects a firm undertakes. The incentive for a firm to introduce its first R&D project is always greater than the incentive to add one more project to the current research effort of the firm.

An additional project in the market has some expected value depending on the probability of being the only successful project. But if a firm has already other parallel projects, it must offset to this value the disincentive created by its other research efforts. One extra project has a negative effect on the expected gain of the other projects the firm conducts. In a free-entry equilibrium, new projects are added by new firms. The equilibrium is characterized by each firm having only one R&D project.

The assumption of zero fixed costs common to all projects of R&D taken by a firm is not realistic in many situations. It is usual to see cases where R&D activities demand a significant initial amount of resources, independent of the number of projects that a firm realizes. A natural example of it is a laboratory, where the building is common to several research teams. The cost of the building is the fixed, start-up cost that we are dealing with. The research teams represent the parallel projects. Denote this total fixed cost by C . We are now in position to establish a second result.

Proposition 2 *In a symmetric (and interior) Nash equilibrium, with positive start-up costs, each firm will have more than one R&D project.*

Proof: Firm i 's problem is now to maximize

$$\max_{\{m_i, e_{ij}\}} V_i = W \left[\prod_{f \neq i; f=1}^N \prod_{j=1}^{m_f} (1 - p(e_{fj})) \right] \left\{ 1 - \prod_{j=1}^{m_i} [1 - p(e_{ij})] \right\} - \sum_{j=1}^{m_i} e_{ij} - C \quad (5)$$

First-order conditions in a symmetric and interior Nash-equilibrium are given by

$$T(1 - p(e))^{Nm-1} p_e(e) = 1 \quad (6)$$

$$T(1 - p(e))^{Nm-1} p(e) = e \quad (7)$$

with $T = W \left[\prod_{f \neq i; f=1}^N \prod_{j=1}^m (1 - p(e_{fj})) \right]$, and the zero (expected) profit condition is

$$V_i = T(1 - p(e))^{Nm-m} (1 - (1 - p(e))^m) - me - C = 0 \quad (8)$$

We must now show that for $C > 0$ firm i has negative profits with $m = 1$. From the first-order condition (7) of the symmetric Nash equilibrium, we can establish

$$T(1 - p)^{Nm-m} = \frac{e}{p}(1 - p)^{1-m} \quad (9)$$

Joining this condition with the zero profit condition (implied by free-entry) we get

$$V_i = e \left\{ \frac{(1 - p)^{1-m}}{p} - \frac{1 - p}{p} - m - \frac{C}{e} \right\} \quad (10)$$

For $m = 1$, it results that

$$V_i = -C < 0 \text{ if } C > 0 \quad (11)$$

This implies that for a positive fixed cost one single project will lead to negative profits. To conclude we must prove the existence of equilibrium for $m > 1$. Manipulating first-order and zero profit conditions we get

$$(1 - p)^{1-m} = 1 + p(m - 1 + \frac{C}{e}) \quad (12)$$

Defining $x(p, m) \equiv (1 - p)^{1-m}$ e $y(p, m, C) \equiv 1 + (m - 1 + \frac{C}{e})p$, it is easy to see that

$$\frac{dx}{dp}(p, m) = (1 - p)^{-m}(m - 1) > m - 1 \quad (13)$$

$$\frac{dx}{dp}(0, m) = m - 1 \quad (14)$$

$$\frac{d^2x}{dp^2}(p, m) = (m - 1)m(1 - p)^{-m-1} > 0 \quad (15)$$

$$\frac{dy}{dp}(p, m, C) = m - 1 + \frac{C}{e} \quad (16)$$

Those features of x and y imply that for each integer m there exists an equilibrium value of $p(e)$. This is a one-to-one relationship, so for each value of p there exists an $m \in N$ (and greater than one) implying

$$V_i(m + 1) \leq 0 \leq V_i(m) \quad (17)$$

■

Intuitively, the existence of fixed costs in R&D activities can be interpreted as generating economies of scope in the number of parallel projects undertaken. The average expenditure per project is decreasing in the number of projects selected by a firm. This start-up cost has a role of public input in the sense of once paid it can be used by any number of R&D parallel projects without any further cost. The adjustment in firms' decisions implied by this fixed cost is made changing the number of projects and not the expenditure done in each one of the projects. In this case, the traditional assumption one firm-one project made in the literature is no longer valid: with start-up costs (and free-entry/ exit) each firm wishes to have more than one project in equilibrium.

Proposition 2 should not be taken too literally. It is a little bit artificial since it crucially depends on how first-order and free-entry equilibrium conditions are imposed in the model. The optimal choice of the number of projects means that some positive marginal profit may accrue to firms in equilibrium. For sufficiently small values of the start-up cost, this positive marginal profit in the first project may allow an overall positive profit. Therefore, the choice of a single project may be optimal even in the presence of (moderate) start-up costs.⁴

3 The number of projects and the gain from innovation

A particular invariance result can be established in the free-entry equilibrium: the number of projects selected by one firm is invariant to innovation gains (no matter how gains are specified) and to the probability of success of other firms.

The endogenous determination of the number of firms in the market is the driving force of the result. Consider a given, fixed, market structure. An increase in innovation gains, W , changes the expected reward of investing in another R&D project. Facing a positive expected profit in extra projects, firms will adjust to the new equilibrium through changes in the number of R&D projects. In a context of free-entry, mobility of firms into and out of the race provides an additional adjustment. In fact, entry is the only variable that adjusts in a free-entry equilibrium in response to a change in the innovation gain.

One characteristic of the original model is, nevertheless, maintained. Like in Sah and Stiglitz (1987), each project's expenditure is determined only by innovation technology and by the project specific fixed cost.

⁴Relaxing the first-order condition on m_i and the free-entry condition to inequalities yields a slightly modified result, where for sufficiently small start-up costs, one project per firm is a free-entry equilibrium. A proof of this claim is available upon request.

Suppose that there are gains for each innovating firm, even in the case where more than one firm are successful. The total profit from innovation is equally shared by all innovating firms. However, it can vary with the number of successful firms.

Denoting the gain per firm by $W(N^*)$, where N^* is the number of successful firms, firm i expected profits can be written as

$$V_i = \left[1 - \prod_{j=1}^{m_i} (1 - p(e_{ij})) \right] \left\{ - \sum_{t=0; t \neq i}^N W(t+1) \binom{N-1}{t} r_f s_g \right\} - \sum_{j=1}^{m_i} e_{ij} - C \quad (18)$$

where f denotes successful firms and g unsuccessful firms and r_f and s_g are given by

$$r_f = \left(1 - \prod_{j=1}^{m_f} (1 - p(e_{fj})) \right)^t \quad \text{and} \quad s_g = \left(\prod_{j=1}^{m_g} (1 - p(e_{gj})) \right)^{N-t-1} \quad (19)$$

First-order conditions for profit maximization of firm i , with respect to the decision variables (expenditure by project and number of projects), are, in a symmetric and interior Nash equilibrium,

$$(1-p)^{m-1} \left\{ \sum_{t=0}^{N-1} W(t+1) (1 - (1-p)^m)^t ((1-p)^m)^{N-1-t} \binom{N-1}{t} \right\} p_e = 1 \quad (20)$$

$$(1-p)^{m-1} \left\{ \sum_{t=0}^{N-1} W(t+1) (1 - (1-p)^m)^t ((1-p)^m)^{N-1-t} \binom{N-1}{t} \right\} p = e \quad (21)$$

Once again, the zero (expected) profit condition associated to the assumption of free-entry (exit) is

$$\left\{ \sum_{t=0}^{N-1} W(t+1) (1 - (1-p)^m)^t ((1-p)^m)^{N-1-t} \binom{N-1}{t} \right\} \times \\ \times (1 - (1-p)^m) - me - C = 0 \quad (22)$$

Proposition 3 *The number of R&D projects undertaken by each firm in an interior and symmetric Nash equilibrium does not depend on the profit gained by the firm in the case of success.*

Proof: Substituting (21) in (22) we get exactly expression (12). Since expenditure per project, e , is independent of W , (12) gives the value of m no matter what value W takes. ■

The analysis shows that the innovation gain does not affect neither the number of projects selected nor the expenditure per project made. Any increase (or decrease) in the profit of an innovation affects only the number of firms in the market. This characteristic is a direct consequence of the free-entry assumption.

Table 1: Numerical simulations

m/p	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13
2	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.09	0.10	0.11	0.12	0.14	0.15
3	0.03	0.06	0.09	0.13	0.16	0.20	0.23	0.27	0.31	0.35	0.39	0.43	0.47
4	0.06	0.12	0.19	0.26	0.33	0.40	0.47	0.55	0.63	0.72	0.80	0.90	0.99
5	0.10	0.21	0.32	0.43	0.55	0.68	0.81	0.95	1.09	1.24	1.40	1.56	1.73
6	0.15	0.31	0.48	0.66	0.85	1.04	1.25	1.47	1.69	1.94	2.19	2.46	2.74
7	0.22	0.44	0.68	0.94	1.21	1.49	1.79	2.11	2.46	2.82	3.20	3.61	4.05
8	0.29	0.6	0.92	1.27	1.64	2.03	2.46	2.91	3.39	3.91	4.46	5.06	5.70
9	0.37	0.77	1.20	1.66	2.15	2.68	3.24	3.86	4.52	5.23	6.00	6.84	7.74
10	0.47	0.97	1.51	2.10	2.73	3.42	4.17	4.97	5.85	6.81	7.86	9.00	10.2

m/p	0.14	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23	0.24	0.25	0.30
2	0.16	0.18	0.19	0.20	0.22	0.23	0.25	0.27	0.28	0.30	0.32	0.33	0.43
3	0.51	0.56	0.61	0.66	0.71	0.76	0.81	0.87	0.93	0.99	1.05	1.11	1.47
4	1.09	1.19	1.29	1.41	1.52	1.64	1.77	1.90	2.03	2.18	2.33	2.48	3.38
5	1.92	2.10	2.30	2.51	2.73	2.96	3.21	3.46	3.73	4.02	4.32	4.64	6.55
6	3.04	3.36	3.69	4.05	4.43	4.83	5.26	5.71	6.20	6.71	7.27	7.86	11.5
7	4.51	5.01	5.54	6.11	6.72	7.37	8.07	8.83	9.64	10.5			
8	6.39	7.13	7.93	8.79	9.73	10.7							
9	8.73	9.80	10.9										
10	11.6												

m/p	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
2	0.54	0.67	0.82	1.00	1.22	1.50	1.84	2.33	3.00	4.00	5.67	9.00	19.0
3	1.91	2.44	3.12	4.00	5.16	6.75	9.02	12.4					
4	4.55	6.07	8.13	11.0									
5	9.15	12.8											
6	16.8												

Without this assumption the number of firms (market structure) must be specified exogenously and the number of projects undertaken by each firm is affected by the innovation's profit. The free-entry hypothesis is then an important one. It is not difficult to verify that the change in the total number of projects in the market after an increase in the innovation gain is the same whether the adjustment is made through changes in the number of firms or through changes in the number of projects per firm. The existence of a start-up, fixed cost in R&D activities has a primary role in justifying more than one project per firm (in fact, what matters is the relative size of the fixed cost to the specific expenditure in one project). If we have zero fixed costs then each firm would have only one project (Proposition 1 applies). On the other hand, free-entry has another strong implication – the number of firms in the race for the innovation is the only variable that adjusts in response to an increase in innovation gains.

4 The number of R&D projects - simulation results

In order to provide a better idea of the practical relevance of the previous results some numerical simulations are now presented. Consider again expression (12). We can re-write it as

$$\frac{C}{e} = \frac{(1-p)^{1-m}}{p} - \frac{1}{p} + (1-m) \quad (23)$$

This expression gives values of C/e which are, in equilibrium, compatible with a probability p of success and m projects. For a higher of C/e , the representative firm has negative profits. Table 1. presents critical values of C/e for different pairs (p, m) . Figure 1 depicts those critical values.

As one can see from numerical simulations, for a success probability of 10 % in each R&D project, a firm would have two R&D projects if the ratio of the total

Figure 1: Numerical Simulations

fixed cost to project specific cost was less than 11 %.

Both the numbers and the figure suggest that when the project's probability of success is low and/or global fixed costs are sufficiently higher than project specific costs one should expect several projects per firm in equilibrium.

As an example, consider the following data on R&D expenditures of Portuguese firms in selected industries.

Industry (code)	Number of Firms	Total Number of Projects	Projects per Firm	Capital R&D spending ratio per project
Food (311)	7	10	1.43	0.27
Chemical Products (352)	21	27	1.28	0.024
Electric Machinery (383)	19	22	1.15	0.25
Communications (720)	3	46	15.3	4.64

Source: JNICT R&D survey 1988

Values in the above table reveal that in those industries where entry in R&D activities seem to be easier firms tend to have one, or at most, two R&D projects—*Chemical Products* and *Electric Machinery*, for example. On the other hand, on the *Communications* industry new technologies are prominent and entry in R&D activities is expected to be more difficult than in mature industries. There is a much lower number of firms engaging in R&D activities in the *Communications* industry, but each of them has a well diversified portfolio of projects. Those characteristics fit well the model predictions.

5 Final remarks

We have considered a static model of innovation with parallel R&D efforts by each firm. Free-entry and start-up costs (fixed costs common to all of the firm's

parallel R&D projects) are important determinants of market R&D efforts. The existence of a positive start-up cost induces firms to have, in equilibrium, more than one project. An invariance result holds in this model: the number of R&D projects of each firm does not depend on the reward to innovation. Numerical simulations reveal that the existence of several projects per firm is more likely when the project's probability of success is low and/or when global fixed costs are high relative to individual project cost.

The results presented seem to call for further research on the sensitivity of traditional R&D results to the hypothesis of the one firm- one project assumption.

References

- [1] Junta Nacional de Investigação Científica e Tecnológica - JNICT (1990) *Inquérito ao Potencial Científico e Tecnológico Nacional - 1988*, Lisbon, November.
- [2] Reinganum, Jennifer (1983) "Uncertain Innovation and the Persistence of Monopoly," *American Economic Review*, 73(4):741-749, September.
- [3] Sah, Raj and Joseph Stiglitz (1987) "The Invariance of Market Innovation to the Number of Firms," *Rand Journal of Economics*, 18(1): 98-108, Spring.