Merger Policy in Open Economies

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Abstract

We develop a number of tests for evaluating the welfare effects of a horizontal merger in the context of an open economy. The analysis is based on the concept of external effect of an infinitesimal merger, introduced by Farrell and Shapiro (1990), and considers three different paradigms of an open economy: a domestic market which is subject to entry by foreign firms, a domestic market which is disciplined by imports at a given price, and a single market made up of a number of national countries.

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1 Introduction

The oligopoly theory literature on merger analysis has, for the most part, considered the case of a closed economy, that is, a well defined geographic market consisting of a number of producers and consumers, all belonging to the same country.¹ Merger analysis then consists of looking at the effects of a proposed merger on total welfare, defined as the sum of consumers' and producers's surpluses (possibly a weighted sum).

This kind of analysis seems inappropriate for the study of economies where a great deal of competition in domestic markets is provided by foreign competition, either in the form of imports or foreign investment. The focus of this paper is precisely the normative analysis of mergers in open economies.

We will consider three different "paradigms" of an open economy and address different questions in each case. First, we consider the case of an economy which is subject to entry by foreign firms (Section 2). The question is then how the rules for approving a merger should be changed if the regulator were only concerned with domestic welfare, which excludes the profits gained by foreign firms.

A second paradigm to be considered is that of a small open economy subject to imports at a given fixed price (Section 3). In this case, we attempt to formalize the idea that imports may act as a disciplining device, therefore reducing or even eliminating the need for the regulation of domestic mergers.

Finally, in Section 4 we present a stylized model of a single market which includes consumers and producers of different countries (all belonging to the same single market). The issue here is to see in what circumstances there will or there will not be a conflict between each country's interest and that of the single market (or economic community) as a whole.

Literature review. Since our analysis is closely related to the recent literature on merger evaluation with theoretical foundations in oligopoly theory, especially the paper by Farrell and Shapiro (1990), we will begin with a brief review of this literature.

It is fair to say that the main theme in most of the papers, as far as nor-

¹Exceptions to this characterization include the work of Landes and Posner (1981), Fisher (1987), and Schmalensee (1987).
mative analysis is concerned, consists of the trade-off between the efficiency gains of a merger and the welfare costs in terms of increased market power.

Some authors, most notably Williamson (1968), have stressed the importance of efficiency gains. According to Williamson (1968), "a merger which yields non-trivial real economies must produce substantial market power and result in relatively large price increase for the net allocative effects to be negative." (p. 23) Now, it can be argued that most horizontal mergers do indeed produce non-trivial real economies. In fact, Salant, Switzer and Reynolds (1983), formalizing an idea due to Stigler (1950), have shown that horizontal mergers in a Cournot model with symmetric firms, linear demand and constant marginal costs are unprofitable to merging firms, unless the merger results in a near-monopoly. Therefore, the evidence that firms are willing to merge should be taken as evidence that there are important economies to be gained.

This view has been challenged by Perry and Porter (1985), who argue that the assumption of a symmetric oligopoly with constant marginal cost is too simplistic. In fact, this assumption implies that the cost function of a merged firm is identical to that of the merging firms. If we instead assume that there exists a scarce resource which enters into each firm’s cost function, then we can model the idea that a firm resulting from a merger is “bigger” than the firms it was originated from; and then it can be shown that a merger can be profitable even if no economies are gained.\footnote{Other papers which depart from the model of Salant, Switzer and Reynolds (1983) include Davidson and Deneckere (1984), Deneckere and Davidson (1985), Kwoka (1989), Daughety (1989) and Levin (1990).}

More recently, Farrell and Shapiro (1990) have put forward a simple equilibrium model which generalizes, and makes more systematic, some of the previous ideas in the literature. Since most of our paper is based on Farrell and Shapiro’s, we will describe some of their results in more detail.

Following what was hinted at in the previous literature, Farrell and Shapiro (1990) take the view that, if a merger is proposed, then it must be profitable for the firms that want to do it. Therefore, it is a sufficient condition for a merger to be welfare enhancing that the “external” effect (the effect on consumers’ surplus plus profits by firms outsider to the merger) be positive.

In order to evaluate the external effect of a merger, Farrell and Shapiro
(1990) consider a given merger as a sequence of infinitesimal mergers, each corresponding to a change $dQ$ in total quantity.³ They are able to show that
\[
dW - d\Pi_I = -\frac{\partial P}{\partial Q}(Q_I - \sum_{i \in O} \lambda_i q_i) dQ,
\]
where $W$ denotes welfare and $\Pi$ profits; $I$ is the set of firms "insider" to the merger and $O$ is the set of firms "outsider" to the merger; $q_i$ is firm $i$'s quantity, $Q \equiv \sum q_i$, $P(Q)$ is the inverse demand function, and $C_i(q_i)$ is firm $i$'s cost function.

This equation, together with some stability conditions, implies that a necessary and sufficient condition for the external effect to be positive is that
\[
s_I < \sum_{i \in O} \lambda_i s_i,
\]
where $s_i \equiv q_i/Q$, $s_I \equiv \sum_{i \in I} s_i$, and
\[
\lambda_i \equiv -\frac{P'(Q) + q_i P''(Q)}{C''_i(q_i) - P'(Q)}.
\]

The above inequality (2) provides a useful criterion for merger analysis in the context of a closed economy. In this paper, we attempt to obtain similar formulae for the case of an open economy.

2 The case of foreign firms in the domestic market

The first paradigm we will consider is that of an economy which is closed to imports but open to entry by foreign firms. The absence of imports may result from import restrictions or, more likely, prohibitive transportation costs. Services, or other non-transactionable goods, would provide good examples. The departure from Farrell and Shapiro's (1990) analysis results from considering a regulator who is only concerned with domestic welfare, defined as the

³In most of the analysis, it is assumed that a merger implies an increase in equilibrium price. Farrell and Shapiro (1990) claim that "rather impressive synergies — learning, or economies of scale — are typically necessary for a merger to reduce prices." (p. 114)
sum of consumer's surplus and domestic firms' profits. The external effect of a merger between domestic firms is then given by the increase in domestic outsider firms' profits minus the decrease in consumers' surplus. Let \( I \) be the set of domestic firms insider to the merger, \( O \) the set of domestic firms outsiders to the merger, \( D = I \cup O \), and \( F \) the set of foreign firms.

To simplify notation, \( I, O, D, F \) will denote sets of firms and the number of firms in that set. Thus the total number of firms, \( N \), is given by \( N = D + F = I + O + F \). Inverse demand is given by \( P(Q) \), where \( P \) is price, \( Q \equiv \sum q_i \), and \( q_i \) is firm \( i \)'s output. Domestic welfare is then given by:

\[
W = \int_0^Q P(x)dx - P(Q)Q + P(Q)Q_D - \sum_{i \in D} C_i(q_i)
\]

where \( Q_D \) is total quantity produced by domestic firms: \( Q_D \equiv \sum_{i \in D} q_i \) (a similar notation will be used for the quantity produced by insider, outsider, and foreign firms).

Firm \( i \)'s profits are given by \( \Pi_i = P(Q)q_i - C_i(q_i) \), where \( C_i(q_i) \) is firm \( i \)'s cost function. We assume \( \Pi_i \) to be concave a function for all \( i \). As in Farrell and Shapiro (1990), we assume that a Cournot equilibrium holds both before and after the merger. The first order (sufficient) conditions for profit maximization (from which equilibrium quantities result) are then given by

\[
P(Q) + q_i \frac{\partial P}{\partial Q} - \frac{\partial C_i(q_i)}{\partial q_i} = 0, \quad i = 1, \ldots, N
\]

As we saw in the preceding section, Farrell and Shapiro's (1990) analysis is conducted in terms of the external effect of a merger, that is, the effect on consumers and on firms outsider to the merger. Since we are only concerned with domestic welfare, the external effect now only includes the impact on consumers and on domestic outsider firms. Our main result in this section establishes a necessary and sufficient condition for the external effect of a merger to be positive, and thus a sufficient condition for the impact of the merger on domestic welfare to be positive. (Proof of this and the following results may be found in the Appendix.)

**Proposition 1** The external effect of an infinitesimal merger is positive iff
\[ s_I + s_F < \sum_{i \in O} \lambda_i s_i \]  \hspace{1cm} (6)

with \( s_X = \sum_{i \in X} s_i, X = I, F, s_i = q_i/Q \).

It can be easily shown that the result also applies to the case of a merger between domestic and foreign firms, or between foreign firms only. In the latter case, since the insiders' profits do not enter the domestic welfare function, the condition that the external effect be positive is a necessary and sufficient condition.

Lacking knowledge of the exact values of \( \lambda_i \), some information regarding the demand and cost functions might helpful in finding (weaker) sufficient conditions for a positive external effect. If the demand function is sufficiently convex, then \( 0 < \lambda_i < 1 \), and \( s_I + s_F < \sum_{i \in O} s_i \), or simply \( s_I + s_F < 50\% \), becomes a necessary condition. If both the demand and the cost functions are linear, then \( \lambda_i = 1 \) and \( s_I + s_F < 50\% \) is equivalent to the condition in Proposition 1.

It is interesting to note that, contrary to what might at first be expected, foreign competition makes it actually more difficult for a merger to be approved (cf (6)). The idea is that the positive term in the external effect (increase in domestic rival firms' profits) is lower the greater the share of foreign firms in the domestic market is. As we will see in the next section, the idea of foreign competition as a discipline device which makes domestic mergers less harmful is more closely related to competition by imports rather than competition by foreign investment.

As an application of the above results, we consider the case of the Portuguese insurance market. A merger between three major firms, UAP Portugal, Aliança Seguradora and Garantia, is to be soon completed. The merger will include both the life and the non-life lines of business.

The values of the relevant market shares, excluding the competitive fringe, are shown in Table 2. Assuming that the demand and the cost functions are linear, we get \( s_I + s_F < 50\% \) as a necessary and sufficient condition. The values in the third row of Table 2 then suggest that the external effect of the merger would be positive in the case of non-life insurance but negative in the...
Table 1: Relevant market shares in the Portuguese insurance industry (1991–provisional data), excluding firms in the competitive fringe (market share of less than 1%).

<table>
<thead>
<tr>
<th></th>
<th>Life</th>
<th>Non-Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_F$</td>
<td>40.15</td>
<td>27.13</td>
</tr>
<tr>
<td>$s_L$</td>
<td>10.03</td>
<td>11.12</td>
</tr>
<tr>
<td>$s_F + s_L$</td>
<td>50.18</td>
<td>38.25</td>
</tr>
</tbody>
</table>

Notes:
1. Output is measured by total premiums.
2. Market shares are computed with respect to total premiums by major firms, i.e., excluding firms in the competitive fringe.
3. Some firms are only partially owned by foreigners. In some cases, information about the exact share was not available; total ownership was then assumed.
Source: Instituto de Seguros de Portugal (ISP).

other market segment. Closer scrutiny of the merger would thus be needed in the latter case.

3 The case of a small open economy

A second paradigm for merger analysis consists of the case when a domestic oligopoly (made up of national firms) is subject to competition not from foreign firms but from imports from foreign firms. In particular, we will consider the extreme case when the domestic market is small relative to the world market, so that the foreign price is taken as given.4

To make the problem more interesting, we consider the possibility that the domestic and the imported goods are imperfect substitutes. Specifically, consumers have an utility function given by $U(Q, Q^*; \sigma) + m$, where $Q$ is the quantity of domestic good, $Q^*$ is the quantity of imports, $m$ is the quantity of "other goods" and $\sigma$ is the elasticity of substitution between domestic and

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4There is an extensive literature on imports as a discipline device. See, for example, Pugel (1980). An explicit application to domestic merger policy is made by Ross (1983). His first model, which is closer to our paradigm, implies that an increase in foreign competition decreases the price effect of a domestic merger. This is consistent with the results derived below.
imported goods.

From $U(\cdot)$, one can derive demand functions $D(P, P^*; \sigma)$ and $D^*(P^*, P; \sigma)$, where $P$ is the price of domestic good and $P^*$ the price of imports. With perfect substitutibility ($\sigma = \infty$), we have:

$$D(P, P^*; \infty) = \begin{cases} 
0 & P > P^* \\
\frac{D(P)}{P^*} & P \leq P^*
\end{cases}$$

$$D^*(P^*, P; \infty) = \begin{cases} 
0 & P < P^* \\
\frac{D(P) - Q}{D(P)} & P = P^*
\end{cases}$$

We assume both demand functions, $D$ and $D^*$, to be $C^3$ and strictly positive whenever their arguments are finite; and that $D(P, P^*; \sigma)$ converges to $D(P, P^*; \infty)$ as $\sigma \to \infty$, likewise for $D^*(P^*, P; \sigma)$.

Taking domestic welfare to be the sum of consumers' and producers' surpluses, we have:

$$W = U(Q, Q^*; \sigma) - PQ - P^*Q^* + PQ - \sum_{i \in D} C_i(q_i).$$

The condition for a positive external effect of an infinitesimal merger between domestic firms is analogous to the one presented for a closed economy by Farrell and Shapiro (1990), that is:

$$s_I < \sum_{i \in D} \lambda_i s_i$$

In the present case, however, the value of $\lambda_i$ is a function of the values of $P^*$ and $\sigma$.

In particular, we are interested in the case when the domestic and the imported goods are close substitutes, that is, $\sigma$ is close to infinity. Specifically, let $P^C$ and $P^N$ denote the competitive and the equilibrium prices under autarky, respectively, where by autarky we mean the case when $\sigma = \infty$ and $P^* = \infty$.

**Proposition 2** Suppose $\sigma$ is close to infinity.

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5These conditions are satisfied by the well known Dixit-Stiglitz (1977) utility function.
(a) If $P^* > P^N$ then the external effect of a merger is positive if and only if it is positive under autarky.

(b) If $P^C < P^* < P^N$ then the external effect of a merger is positive if and only if $s_f < 1$.

(c) If $P^* < P^C$ then the external effect of a merger is approximately zero.

In case (a), foreign competition is ineffective, or almost ineffective, since the import price is too high relative to the domestic price. Therefore, the domestic market should be treated as a closed market for the purpose of defining merger policy. Case (c) is probably what comes closest to the idea of a small open market. Each domestic firm acts as a price taker and sets its optimal quantity. Imports fill the gap between domestic demand and supply. The external effect is close to zero because neither price nor the quantities produced by other domestic firms change significantly as the result of a merger.

The intermediate case when $P^C < P^* < P^N$ is likely to be the most interesting of the three cases. Here, domestic producers are de facto price makers: if firm $i$ increases its quantity from the equilibrium level, then price goes down. However, the import price acts as a binding constraint, so that firm $i$ is unable to increase the domestic price by decreasing the quantity it supplies. This implies that the external effect of a merger is positive. When two or more domestic firms merge together, the equilibrium quantity supplied by the merged firm is reduced. This slack in domestic supply is taken up by the remaining domestic firms, so that total domestic quantity and price remain unchanged. Therefore, the effect on consumers is approximately zero, whereas the effect on rival firms is positive.

It is interesting to note that case (b) is more favorable to a merger than case (c) is, despite the fact that $P^*$ is lower in the latter case. The intuition is that, in case (c), the gains from import competition are all realized before the merger, whereas in case (b) it is precisely the merger (a reduction in the quantity supplied by a set of firms) which makes import competition active.

It is interesting to note the relationship between case (b) and the model of oligopoly entry deterrence by Gilbert and Vives (1986). In fact, in case (b), "import deterrence" is a public good, and deterring entry entails an "inverse free riding" problem.
Naturally, it is very difficult to find examples of markets that exactly fit the cases listed in Proposition 2. However, in the case of Portugal, cement, mineral and sparkling bottled waters, and most agricultural products (e.g., oranges) seem to be good examples of each of the three cases.

(a) Cement is a fairly homogeneous product, both across domestic producers and between imports and domestic production. However, the (competitive) price of imports would be higher than the equilibrium domestic price due to transportation costs. In fact, imports are insignificant. The number of domestic producers is currently two, which means market power is significant.

(b) The Portuguese mineral and sparkling bottled water industry consists of a small number of firms (25) with a Herfindahl index of 0.143 (1990 values). Domestic production is relatively homogeneous and a good substitute for imports. Imports are less than 5% (and exports are around 3.5%), although casual empiricism suggests that the difference between domestic and international price is not very significant.

(c) Except for seasonal differences and differences in vertical quality, oranges seem to be a fairly homogeneous product. Imports into the Portuguese market are significant, especially from Spain, and import prices are approximately equal to domestic prices.

4 The single market case

In this section we present the third and final paradigm of an open economy, and that is a single market composed of several countries. A good example—in fact, the example which motivates the analysis that follows—is the European Common Market.

Anticipating the creation of the European Single Market, a wave of European mergers, typically between firms of different European countries, has now been going on for a while. However, it is still unclear which role will be played by the European Commission in terms of merger policy (cf Jacquemin, 1990).

Merger policy was not explicitly dealt with in the Treaty of Rome. As a result, merger policy has traditionally been guided by a comprehensive interpretation of Articles 85 and 86 of the Treaty. More recently, however, a
new regulation was approved as an attempt to create the basis for a EC-wide merger policy (Regulation No. 4064/89, which became effective in 1990). Unlike the United States, the new EC Merger Guidelines make no explicit reference to concentration levels that trigger closer scrutiny or the interdiction of a proposed merger. Rather, they define a series of thresholds above which a given merger is to be scrutinized by the Commission and below which decisions are delegated to national authorities. In this section, we focus precisely on the role played by the Commission vis-a-vis the role played by the national merger authorities.

Our stylized view of a single market is a measure of consumers whose preferences aggregate into a demand function \( D(p) \) for some given homogeneous good, and a finite set of producers with cost functions given by \( C_i(\cdot) \). What distinguishes this model from the case considered by Farrell and Shapiro (1990) is that both consumers and firms can be of different nationalities, and thus welfare measures will be different depending on whether one takes the perspective of the market as a whole or of a given constituent country in particular. As we will see, this crucially depends on the relation between each country's share in total demand (denoted by \( d_i \)) and in total supply (denoted by \( s_i \), not to be confused with \( s_f \)).

We assume there exists a community-wide merger authority in addition to one in each member country. The central and the national authorities differ, first of all, in their objective functions: the former maximizes total welfare, whereas the latter's objective is national welfare (the sum of consumers' and producers' surplus for national consumers and producers). In addition, we assume that the national authorities are better informed than the central authority: each national authority knows the value of \( \lambda_i \) corresponding to each national firm (and, possibly, the extent of the efficiency gains accruing from a proposed merger).

In this context, the issue of centralizing vs decentralizing merger policy consists of a trade-off between internalizing all of the welfare effects (centralized merger policy) and using better information (decentralized merger policy). A possible compromise would consist of a decision by a central authority based on the evaluation(s) made by the relevant national authority(ies). The following results are then useful.

**Proposition 3** Suppose that country \( i \) is a net importer, that is, \( d_i > s_i \). If
the external effect of a merger between firms in country i is positive from the point of view of country i, then it is also positive from the point of view of the community as a whole.

That is, if country i is a net importer, then the approval of a merger by the national authority should be automatically approved by the central authority. The intuition is that the condition \( d_i > s_i \) ensures that the community-wide external effect is at least as large as the national one. In the limit, if \( d_i = 1 \), then the only effect not accounted for in the external effect from country i’s perspective is the (positive) effect on firms outside of country i. Then, since the external effect on rival firms is always positive, a positive external effect from country i’s perspective clearly implies a positive external effect from the point of view of the community as a whole.

**Proposition 4** Suppose that \( 0 \leq \lambda_i \leq 1 \) (the demand function is sufficiently convex) and that \( 1 + d_i < 2s_i \). If the external effect of a merger between firms in country i is negative from the point of view of country i, then it is also negative from the point of view of the community as a whole.

That is, if the above conditions hold, then a merger that is blockaded by a national authority should also be blockaded by the central authority. The condition \( 1 + d_i < 2s_i \) implies that most of the positive external effects remain in country i (because \( s_i \) is relatively large). In the limit, if \( s_i = 1 \), then the total external effect will necessarily be lower than that from country i’s perspective. Therefore, if the external effect from country i’s perspective is negative, it must also be negative from the community’s perspective.

The conditions presented in the previous propositions leave out a great number of possibilities, arguably most possibilities. Cases of potential conflict between national and supra-national merger guidelines can easily arise. In fact, a good example of this is the recent ruling on the proposed takeover by Aérospatiale SNI-Alenia and Selenia Sp.A of De Havilland, a Canadian aircraft manufacturer owned by Boeing.⁶

⁶Note that, while the previous results considered the case of a merger between firms of a single country, the extension to the case of an arbitrary number of countries is straightforward.
This was the first instance in which a merger or takeover was ruled out by the European Commission since Regulation No. 4064/89 was approved (cf EC Bulletin, Supplement 2/90). The European Commission’s decision was subject to strong criticism by the French and Italian governments. The ruling was based on the analysis of market concentration: the merger would give Aérospatiale-Alenia/De Havilland 67% of the EC market for commuting aircraft and about 50% of the world market. While demand data were not readily available; it is clear that the condition of Proposition 3 does not hold, which justifies the divergence between central and national authorities.

It is interesting to note the divergence between the Commission’s criteria for selecting merger cases and what Propositions 3 and 4 would indicate. The Commision’s criteria are based on thresholds. This may be justified on the basis of transactions costs: there are only so many cases which can be analysed by a limited number of staff in a given year. The hypothesis of propositions 3 and 4, however, do not discriminate in favor or against large mergers, or mergers in large countries. Notice, however, that a merger between large firms is more likely a case in which the condition of Proposition 3 does not hold, thus requiring intervention by the central authority.

5 Conclusion

We have developed a number of tests for evaluating the welfare effects of a horizontal merger in the context of an open economy. The analysis is based on the concept of external effect of an infinitesimal merger, introduced by Farrell and Shapiro (1990), and considers three different paradigms of an open economy: a domestic market which is subject to entry by foreign firms, a domestic market which is disciplined by imports at a given price, and a single market made up of a number of national countries.

Recent decisions by the European Commission on cases of mergers and acquisitions have made it clear that the political, administrative, and even economic costs of making decisions on an ad hoc basis are far too high. There is a need for a clear set of rules designating when the central authority should intervene and how it should decide. Our paper is an attempt at contributing to this effort.
Appendix

Proof of Proposition 1: Differentiating \( W \), we get

\[
\begin{align*}
    dW &= -\frac{\partial P}{\partial Q}QdQ + P(Q)dQ_d + \frac{\partial P}{\partial Q}Q_d dQ - \sum_{i \in \mathcal{O}} \frac{\partial C_i}{\partial q_i} dq_i - dC_i
\end{align*}
\]

where \( dC_i \) stands for total cost effect on insiders (including any efficiency gains accruing from the merger).

The effect on the firms insider to the merger is given by:

\[
    d\Pi_I = P(Q)dQ_I + \frac{\partial P}{\partial Q}Q_IdQ - dC_i.
\]

The external effect of an infinitesimal merger is then given by

\[
    dW - d\Pi_I = -\frac{\partial P}{\partial Q}(Q + Q_I - Q_d)dQ + \sum_{i \in \mathcal{O}} \left( P - \frac{\partial C_i}{\partial q_i} \right) dq_i.
\]

From the first-order condition (5) and the implicit function theorem we have

\[
    \frac{\partial q_i}{\partial Q} = -\frac{\partial P/\partial Q + q_i \partial^2 P/\partial Q^2}{\partial P/\partial Q - \partial^2 C_i/\partial q_i^2} = -\lambda_i,
\]

and thus \( dq_i = -\lambda_i dQ \). Furthermore,

\[
    \left( P - \frac{\partial C_i}{\partial q_i} \right) = -\frac{\partial P}{\partial Q} q_i.
\]

Therefore, (11) can then be re-written as

\[
    dW - d\Pi_I = -\frac{\partial P}{\partial Q}(Q_I + Q_F)dQ + \sum_{i \in \mathcal{O}} \frac{\partial P}{\partial Q} \lambda_i q_i dQ
\]

Rearranging terms, we get

\[
    dW - d\Pi_I = -\frac{\partial P}{\partial Q}(Q_I + Q_F - \sum_{i \in \mathcal{O}} \lambda_i q_i) dQ
\]

If we impose the mild stability condition \( \partial P/\partial Q + q_i \partial^2 P/\partial Q^2 < 0 \), together with our assumption on the concavity of \( \Pi_I \), we conclude that a merger
Figure 1: Domestic residual demand with perfect substitutability

![Diagram](image)

implies a reduction in total market output, that is, $dQ < 0$ (cf Dixit, 1986; Seade, 1980). The necessary and sufficient condition for $dW = d\Pi_{f}$ to be positive is then

$$Q_{I} + Q_{F} - \sum_{i \in O} \lambda_{i} q_{i} < 0,$$

from which (6) follows immediately. ■

**Proof of Proposition 2**: Consider first the case when $\sigma = \infty$. In this case, residual demand faced by domestic firms is given by $D(P, P^{*}; \infty)$ in Figure 5.

We begin by considering cases (a) ($P^{*} > P^{N}$) and (c) ($P^{*} < P^{C}$), both of which are depicted in Figure 5.

In case (a), the analysis is clearly identical to the one in autarky since the residual demand curve coincides with the autarky demand curve in the relevant range (recall we are considering the effects of infinitesimal mergers). In case (c), each domestic firm produces up to the point where $P^{*} = \partial C_{i}/\partial q_{i}$. Therefore, neither the equilibrium price nor the rivals' quantities are affected by a merger, which is to say the external effect is null. Cases (a) and (c) in the proposition then follow by continuity.

Case (b) is somewhat more complicated since neither $\partial P / \partial Q$ nor $\partial P^{2} / \partial Q^{2}$
are defined at the equilibrium when $\sigma = \infty$. In fact, one can show that if $P^c < P^* < P^N$ then the equilibrium will occur at the point where the demand function has a kink (cf Figure 5).

Notice however that the marginal revenue function is discontinuous at $Q_D$ (for $\sigma = \infty$). Hence, the slope of marginal revenue at $Q_D$ tends to minus infinity as $\sigma \to \infty$, which implies $\partial P^2/\partial Q^2 \to -\infty$ as $\sigma \to \infty$. From the first-order condition for each firm, we known that, for $\sigma < \infty$, $\partial P/\partial Q < 0$ at the equilibrium quantity. We thus conclude that $\lambda_i \to +\infty$, which implies that condition (8) is satisfied for any $\lambda_i < 1$. \hfill \Box

**Proof of Proposition 3:** Assuming that consumers are homogeneous across the single market, country $i$'s welfare is given by

$$W_i = d_i \left( \int_0^Q P(x)dx - P(Q)Q \right) + P(Q)Q_i - \sum_{j \in i} C_j(q_j). \tag{17}$$

Differentiating, we get

$$dW_i = d_i \left( \frac{\partial P}{\partial Q} \right) dQ + PdQ_i - \frac{\partial P}{\partial Q} Q_i dQ - \sum_{j \in O_i} \frac{\partial C_j}{\partial q_j} dq_j - dC_i, \tag{18}$$

where $O_i$ is the set of country $i$'s firms outsider to the merger. Since
\[ d\Pi_l = PdQ_l + Q_l \frac{\partial P}{\partial Q} dQ - dC_l \]  
\[ \text{(19)} \]

and

\[ dq_i = -\lambda_i dQ. \]  
\[ \text{(20)} \]

we get

\[ dW_i - d\Pi_l = -\frac{\partial P}{\partial Q} Q \left( d_i - \sum_{j \in O_i} (1 + \lambda_j) s_j \right) dQ. \]  
\[ \text{(21)} \]

Now suppose that the external effect from country $i$'s perspective is positive, that is,

\[ d_i - \sum_{j \in O_i} (1 + \lambda_j) s_j < 0. \]  
\[ \text{(22)} \]

Since $d_i > s_i$, by hypothesis, we have

\[ s_i < \sum_{j \in O_i} (1 + \lambda_j) s_j, \]  
\[ \text{(23)} \]

which implies that

\[ s_i < \sum_{j \in O_i} (1 + \lambda_j) s_j + \sum_{j \in O_{-i}} \eta_j s_j, \]  
\[ \text{(24)} \]
(assuming \(\lambda_j \geq 0\)) which in turn is equivalent to

\[
s_i = \sum_{j \in O_i} s_j = s_f < \sum_{j \in O} \lambda_j s_j,
\]

which is the condition for a positive global external effect. \(\blacksquare\)

**Proof of Proposition 4:** Suppose that the external effect from country \(i\)'s perspective is negative. From the proof of Proposition 3, we know this is equivalent to

\[
d_i - \sum_{j \in O_i} (1 + \lambda_j) s_j > 0.
\]

Since \(1 + d_i < 2s_i\), by hypothesis, we have

\[
2s_i > 1 + \sum_{j \in O_i} (1 + \lambda_j) s_j,
\]

or simply

\[
s_i = \sum_{j \in O_i} s_j = s_f > 1 - s_i + \sum_{j \in O_i} \lambda_j s_j.
\]

Now, if the demand function is sufficiently convex, then \(0 < \lambda_j < 1\). Since, by definition, \(1 - s_i = \sum_{j \in O_{-i}} s_j\), we have

\[
1 - s_i > \sum_{j \in O_{-i}} \lambda_j s_j.
\]

Therefore, (28) can be re-written as

\[
s_f > \sum_{j \in O_{-i}} \lambda_j s_j + \sum_{j \in O_i} \lambda_j s_j = \sum_{j \in O} \lambda_j s_j,
\]

which is the condition for a negative global external effect. \(\blacksquare\)
References


