Housing Appreciation, Mortgage Interest Rates, and Homeowner Mobility

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Housing Appreciation, Mortgage Interest Rates, and Homeowner Mobility(*)

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1. INTRODUCTION

The objective of this paper is to present a model of intra-metropolitan residential mobility for homeowners which incorporates both the consumption and the investment aspects of homeownership. We focus on how optimal timing of "trading up," i.e. selling one house with the intention of buying a more valuable one, is impacted by macroeconomic variables, especially residential appreciation and interest rates.

The motivation for this paper comes from empirical observations that residential mobility rates for homeowners rose sharply during the 1970's when inflation-adjusted mortgage rates were low and when housing prices were appreciating rapidly, and then fell sharply after 1980 as interest rates soared and residential appreciation slowed. In the same period, mobility rates for renters were relatively stable (See Table 1). Moreover, mobility rates for young homeowners fell dramatically after 1980, though mobility rates for young renters were stable in the same period (See Table 2). At the same time, the ratio of new home value to previous home value for recent home buyers dropped in the early 1980's, compared to the late 1970's (See Table 3). It appears that homeowners were trading up more often and to a more valuable home during the 1970's. One hypothesis is that rapid housing appreciation enables homeowners to accumulate the equity required to trade up to a more valuable house more quickly than when housing prices are stable. The incentive for homeowners to maximize their financial leverage also is enhanced by the expectation of future capital gains. Another hypothesis is that a rise in mortgage interest rates negatively affects the incentive to trading up [See Phillips (1988) for a comprehensive discussion].

Several explanations have been put forward in the literature on residential mobility. For some economists, residential mobility is seen as the consequence of a disequilibrium between a household's desired and actual housing consumption. In Hanushek and Quigley (1978) disequilibrium is measured by the gap between the household's actual housing consumption and an equilibrium value estimated by housing demand models. On the other hand, in Weinberg,
Friedman, and Mayo (1980) disequilibrium is measured by the compensating income variation from moving. However, these models seem to be more applicable to renters than homeowners, in that housing is treated primarily as a consumption good and no aspects essential to homeownership are considered.

Other authors have examined homeowner mobility in relation to housing renovation decisions.\textsuperscript{1} Shears (1983) presents an econometric model of the simultaneous relationship between housing renovation and moving behavior. The basic idea is that, if the welfare benefit from renovation is less than its welfare cost, then households are more likely to move. His paper proceeds to emphasize the effects of life-cycle factors and neighborhood quality on the joint decision. The investment aspects of homeownership are not considered. In particular, mortgage payments and capital gains are not considered. Schwab (1985) focuses on the effect of fixed mortgage interest rates on the decision to renovate or move.\textsuperscript{2} The basic idea is that if the benefits from keeping a low mortgage interest rate are less than the welfare costs of rationing housing inputs that can not be increased through renovation, then households are more likely to move. Once again, the investment aspects of homeownership are not considered.

On the other hand, it has been emphasized that macroeconomic variables, such as inflation, mortgage interest rates, and capital gains, impact housing decisions in relation to housing tax policies. Most analysts agree that high rates of inflation during the late 1970's increased demand for owner-occupied housing units, due to a decline in the user cost of capital caused by the federal tax code [see for example Diamond (1980), Hendershott (1980), Hendershott and Hu (1981), Poterba (1985), and Rosen and Rosen (1980)]. Some analysts argue that there is an negative relationship between inflation and housing demand due to "financial till problem" [see Kearl (1979) and Schwab (1982)]. Moreover, the joint decision of housing demand and tenure choice has been discussed, focusing on relative costs of renting vs owning [see Rosen (1979), Gillingham and Hagemann (1983), and Henderson and Ioannides (1986)]. However, this literature deals with the effects of inflation, mortgage interest rates, and capital gains upon housing demand and tenure choice.
Recently, the investment aspect of housing decisions together with the consumption aspect have been studied within a life-cycle framework [See for example Dolde (1978), Renny (1981), and Wheaton (1985)]. Along this line, Henderson and Ioannides (1983, 1987) develop a two-period model of tenure choice which specifies the differences between the opportunity costs of renting and owning. Under the assumption that households change tenure status from renting to owning within a lifetime, Artle and Varaiya (1978) investigate the relationship between life-cycle consumption and homeownership. However, the timing of the housing tenure transition is not explicitly analyzed. Plaut (1987) extends this model to examine the timing of the housing tenure transition from renting to owning. However, housing decisions on trading up, i.e., from owning to owning, has never been discussed explicitly.

Presumably, household mobility is partially motivated by exogenous factors such as job transfers or the need for additional space to accommodate a growing family. However, these sociological variables can not explain the trend of residential mobility rates in the 1970's and early 1980's. Previous studies on residential mobility focus on the characteristics of housing as a consumption good. However, in the light of the empirical evidence, the study of intrametropolitan residential mobility for homeowners necessitates the consideration of the investment aspect of homeownership. On the other hand, the recent work on the investment aspect of housing decisions has addressed the issue of homeowners mobility and, in particular, the influence of housing appreciation and changes in mortgage interest rates on trading up. This paper is a first step toward bridging the gap between these two bodies of the urban economics literature. It is a unique step in that it examines the effects of housing capital gains and mortgage rates on the timing of trading up and homeowner's welfare within a life-cycle type of model that accommodates both the consumption and the investment aspects of housing decisions.

This paper presents a model of the optimal timing of trade up, which considers consumption and investment motives of homeownership. Households determine the optimal timing of trading up so as to maximize their intertemporal utility of both housing and non-housing consumption. First we consider current homeowners, who already own a house and
expect that they trade up to a more valuable house at some point in the future. Housing appreciation tends to induce an earlier optimal timing of trading up. Moreover, housing appreciation makes current homeowners better off in terms of welfare. However, current homeowners suffer from a rise in mortgage interest rates. Second, we consider first-time home buyers, who have decided to buy a house and expect that they trade up to a more valuable house in the future. Their initial housing consumption is determined by an initial downpayment constraint. In this case, the effect of housing appreciation on the optimal timing of trading up is ambiguous and, unlike current homeowners, first-time home buyers suffer from housing appreciation. Moreover, as current homeowners, first-time home buyers suffer from a rise in mortgage interest rates. Most of the theoretical analytic results are ambiguous. Accordingly, we perform numerical simulations based on the theoretical model in order to determine the most likely comparative effects for a stylized set of parameters.

This paper is organized as follows. Section 2 presents the model of trading up by current homeowners. Section 3 investigates the effect of housing appreciation and mortgage interest rates on the optimal timing of trading up and household's intertemporal welfare for current homeowners. Section 4 extends the analysis to the optimal timing of trade up and welfare to first-time homebuyers. Section 5 develops and presents the numerical simulations. Section 6 summarizes the main results in the paper and provides directions for future work.

2. A MODEL OF TRADING UP

In this section we develop a model of household behavior focusing on the optimal timing of tradeup. The model is deterministic. Therefore, portfolio allocation decisions are ignored. Moreover, our model allows accumulation of a physical asset, i.e., housing, and excludes the possibility of financial saving. In other words, all the money of income net mortgage payment is spent on non-housing consumption in the current period and none is saved/borrowed against the future, except mortgage loan. At time 0, the household purchases a new house for $P_H(0)H_0$. 
where \( H_0 \) represents the number of "standardized housing units" and \( P_H(0) \) represents the price per unit at 0. The household spends all its accumulated savings on the downpayment and borrows \( bP_H(0)H_0 \) at the fixed mortgage interest rate \( r_0 \), where \( b \) is exogenously given as the fraction of the purchase price that is mortgage financed. The mortgage must be paid off in a period of \( T \) years. We assume for simplicity that the time horizon of the model coincides with the full maturity of the mortgage loan.

The constant yearly payment implied by the mortgage loan includes mortgage interest and principal retirement. It is denoted by \( v_0 \) and is given by:

\[
bP_H(0)H_0 = \int_0^T v_0 e^{-r_0 s} ds
\]

or

\[
v_0 = \frac{r_0}{1-e^{-r_0 T}} bP_H(0)H_0 = \frac{r_0}{1-e^{-r_0 T}} bP_H(0)H_0
\]

In turn, the market value at time \( t \) of the first home is denoted by \( A_0(t) \) and evolves according to the following equation of motion:

\[
\dot{A}_0(t) = v_0 - r_0 (P_H(0)H_0 - A_0(t))
\]

with initial condition

\[
A_0(0) = (1-b)P_H(0)H_0
\]

According to equation (2.3), the principal retirement is equal to the difference between the constant mortgage payment and the interest on the unpaid balance. Equation (2.4) indicates that the initial equity is equal to the downpayment for the purchase of the first home. The solutions to the differential equations (2.3) with initial condition (2.4), considering the evolution of housing prices, is given as follows:
\[ A_0(t) = P_H(t)H_0 - \frac{1-e^{-r(T-t)}}{1-e^{-rT}} bP_H(0)H_0 \] (2.5)

If the household remains in the same home for the whole horizon of the model, then \[ A_0(T) = P_H(T)H_0. \] Implicitly, we assume that the housing stock does not depreciate.

At time \( t \) the household receives an exogenous income of \( Y \). With this income, the household pays the fixed mortgage payment \( v_0 \) and \( Y-v_0 \) is spent on non-housing consumption goods. No intertemporal transfers of wealth are allowed.

Suppose that at some \( t \) the household decides to sell the first home in order to purchase a more valuable house. Assume that all the money received by the household upon the sale of the first home is spent on the downpayment on the new home. Also, \( Y \) is assumed to be high enough for the household to afford the mortgage payments associated with the purchase of this new home.

Therefore, the mortgage payment constraint is not binding and the size of the second home is determined solely by the downpayment constraint as follows:

\[ A_0(t) = (1-b)P_H(t)H_1 \] (2.6)

At \( t \), the household borrows \( bP_H(t)H_1 \) at the (new) fixed mortgage interest rate \( r \). The constant mortgage payment \( v_1 \), implied by this second purchase, is given as follows:

\[ v_1 = \frac{r}{1-e^{-rT}} bP_H(t)H_1 = \ell bP_H(t)H_1 \] (2.7)

In turn, the market value of the new home at time \( s \), denoted by \( A_1(s) \), evolves according to:

\[ \dot{A}_1(s) = v_1 - r(P_H(s)H_1 - A_1(s)) \] (2.8)

with initial condition

\[ A_1(t) = (1-b)P_H(t)H_1 \] (2.9)
Accordingly, at T the equity value of the second home is:

\[ A_1(T) = P_H(T)H_1 - \frac{1 - e^{-\gamma T}}{1 - e^{-\gamma T}} b P_H(T)H_1 \]  

(2.10)

Note that \( A_1(T) \) is a function of optimal timing of trading up.

Upon the purchase of the new home, exogenous income is spent on the new fixed mortgage payment \( v_1 \), and the remainder \( Y - v_1 \), is spent on non-housing consumption.

At \( t \) household's utility function is defined as \( U_t(X,H) \), where \( X \) is non-housing consumption. \( U_t(X,H) \) is assumed to be twice continuously differentiable and well behaved in that it satisfies both non-satiation and strict concavity in \( X \) and \( H \). The intertemporal stream of utility is generated by the aggregation of these time-separable, time-invariant utility functions.

At \( T \), the equity value of the housing stock \( A_1(T) \) has a utility value of \( V(A_1(T)) \). The \( V(\cdot) \) function can be interpreted as a sufficient statistic for post-terminal utility that could be financed if the household were to sell its home. \( V(A_1(T)) \) is assumed to be monotonic increasing and strictly concave in \( A_1(T) \).

The decision of the timing of trading up \( t \) is related to the characteristics of housing as an investment good as well as a consumption good. First, households derive utility from the use of housing services, as reflected in their utility function. According to a pure consumption-induced trade-up, households will care only about the trade off between housing and non-housing goods in the horizon of the model. Second, the investment motive is captured by the recognition that at terminal time the value of the housing stock should be consistent with maximized utility in the post-terminal stage. According to a pure investment-nature induced trade-up, households act so as to maximize the utility value of their housing stock at \( T \).
The household decision problem consists of determining when (and if) to trade up. The timing of trading up \( t \) is obtained from the constrained maximization of intertemporal utility. The household's problem may be written as follows:

\[
\max \beta \left[ \int_0^t U(0, H_0) e^{-\delta t} dt + \int_t^T U(X_t, H_t) e^{-\delta t} dt \right] + \beta(1 - \beta) e^{\gamma T} V(A_t(T))
\]

(2.11)

subject to

i) budget constraints:

\[
X_0 = Y - \frac{r_0}{1 - e^{-r_0 T}} bP_H(0) H_0
\]

(2.12)

\[
X_t = Y - \frac{r_t}{1 - e^{-r_t T}} bP_H(t) H_t
\]

(2.13)

ii) downpayment constraint for the purchase of second home:

\[
A_0(t) = (1 - b) P_H(t) H_t
\]

(2.14)

Parameter \( \beta \) indicates the relative degree of importance of the consumption and investment motives. As \( \beta \) approaches one (zero), the consumption (investment) motive tends to dominate the investment (consumption) motive.

The first-order necessary condition for an interior optimal solution is given by:

\[
t: \left[ (U^0 - U^1) e^{-\delta t} + \left( \frac{e^{-\delta t} - e^{-\delta T}}{\delta} \right) \left( \frac{\partial X_t}{\partial t} + \frac{\partial H_t}{\partial t} \right) \right] + \beta(1 - \beta) e^{\gamma T} \frac{\partial A_t(T)}{\partial t} = 0
\]

(2.15)

The optimality of trading up at some point \( t \) in the period 0 to T depends on the concavity of the intertemporal objective function with respect to \( t \). It can be shown that the objective function has the same value at time 0 and T. Then, concavity of the objective function implies the existence of a global interior maximum between 0 and T. It will be optimal to trade up. However, if the objective function is convex in \( t \), then a maximum will occur at a corner either
at 0 or T. The following is a condition for the objective function to be concave in t between 0 and T:

\[ \Delta = B \left[ -\delta e^{-\delta t} (U_0^0 - U_1^0) - 2e^{-\delta t} \left( U_x^1 \frac{\partial X}{\partial t} + U_H^1 \frac{\partial H}{\partial t} \right) + \left( e^{-\theta t} - e^{-\delta t} \right) \left( \frac{\partial X}{\partial t} \frac{\partial X}{\partial t} + U_x^1 \frac{\partial^2 X}{\partial t^2} + U_H^1 \frac{\partial^2 H}{\partial t^2} \right) \right] \\
+ (1-\delta)e^{-\delta T} \left[ V \left( \frac{\partial A(T)}{\partial t} \right)^2 + V^2 \frac{\partial^2 H}{\partial t^2} \right] < 0 \]  

(2.16)

The model cannot capture all the effects of housing appreciation and mortgage interest rates on homeowner mobility. For example, transaction costs are ignored. However, if transaction costs were incorporated in the model, our analysis would not be altered in any essential way. Moreover, only one trading up is allowed during the model horizon. The effect of housing appreciation and mortgage interest rates on the number of moves is not addressed here.

3. OPTIMAL TIMING OF TRADING UP FOR CURRENT HOMEOWNERS

This section investigates how housing appreciation and mortgage interest rates affect both the optimal timing of trading up and household's intertemporal welfare. We shall investigate these effects under three general scenarios. First, we assume that the investment motive dominates (\( \delta = 0 \)); second, that the consumption motive dominates (\( \delta = 1 \)); and finally, in the most general case, that both motives co-exist (\( 0 < \delta < 1 \)).

In each scenario, we focus on how residential appreciation affects the optimal timing of trade up. We assume that housing prices evolve according to \( p_t = e^{nt} p_0 \). Case 1 concentrates on the effects of one discrete permanent increase in housing prices; case 2 analyzes changes in the rate of growth of housing prices; and finally case 3 looks at the effect of changes in mortgage interest rates. The analytic results are summarized in Table 4 and Table 5.
3.1. UPTERPRED INNED BY INVESTMENT MOTIVE (β=0)

If the investment motive dominates, then the household is only concerned with the utility value of the housing stock at T. The household's problem reduces to the maximization of the utility associated with the terminal housing wealth given the assumption that \( H_1 \) is determined by the equity gain from the sale of the first home. The household's problem can be written as:

\[
\max_t \quad V(A_1(T)) \tag{3.1}
\]

subject to

\[
(1 - b)P_H(t)H_1 = A_0(t) \tag{3.2}
\]

The first-order necessary condition for an interior maximum is:

\[
\frac{\partial A_1(T)}{\partial t} v'(\cdot) = 0 \tag{3.3}
\]

or equivalently, given that \( v'(\cdot) > 0 \) by assumption, the first order necessary condition reduces to:

\[
\frac{\partial A_1(T)}{\partial t} = \left( P_H(T) \frac{1 - e^{-rt}}{1 - e^{-rT}} b P_H(t) \right) \frac{\partial H_1}{\partial t} - \frac{r e^{-rt}}{1 - e^{-rT}} b P_H(t) H_1 \frac{1 - e^{-rt}}{1 - e^{-rT}} b \frac{\partial P_H(t)}{\partial t} H_1 = 0 \tag{3.4}
\]

The first term of \( \frac{\partial A_1(T)}{\partial t} \) indicates the marginal benefit from deferring "trading up". The second and third terms represent the marginal costs of the constant mortgage payment and the remaining mortgage loan respectively, from deferring "trading up". The optimal time to "trade up" occurs when the marginal benefit from "trading up" is equal to its marginal cost.

The second-order sufficient condition for optimality can be written as:
or equivalently, given the first-order condition and again that \( V'(.) > 0 \), \( \frac{\partial^2 A_1(T)}{\partial t^2} \) must be negative. It should be noted that \( \frac{\partial^2 A_1(T)}{\partial t^2} < 0 \) is also sufficient for the concavity of the objective function. In turn, since the objective function has the same value at time 0 and \( T \), concavity of the objective function implies that the optimal timing of trading up is somewhere between 0 and \( T \).

\[ \textbf{Case 1: Effect of a one time change in housing prices} \]

Let us assume that the housing price per unit is constant. Suppose that the housing price per unit increases from \( P_H(0) \) to \( P_H \) soon after the purchase of the first home, so that an increase in the housing price does not affect the initial housing consumption.

The first-order necessary condition for \( t \) reduces to:

\[
\frac{\partial A_1(T)}{\partial t} = \left( P_H \frac{1-e^{-rt}}{1-e^{-rT}bP_H} \right) \frac{\partial H_1}{\partial t} - \frac{re^{-rt}P_H}{1-e^{-rT}bP_H} H_1 = 0
\]

(3.6)

It should be noted that since housing price is independent of \( t \), the marginal cost of the remaining mortgage loan disappears.

In turn, the second-order condition reduces to:

\[
\frac{\partial^2 A_1(T)}{\partial t^2} = \left( P_H \frac{1-e^{-rt}}{1-e^{-rT}bP_H} \right) \frac{\partial^2 H_1}{\partial t^2} - \frac{2re^{-rt}P_H}{1-e^{-rT}bP_H} \frac{\partial H_1}{\partial t} + \frac{r^2e^{-rt}P_H}{1-e^{-rT}bP_H} H_1
\]

(3.7)

where

\[
\frac{\partial H_1}{\partial t} = r_0 \left( \frac{e^{rT}}{1-e^{-rT}} \right) \left( 1-b \left( \frac{P_H(0)}{P_H} \right) \right) H_0 > 0
\]

(3.8)
\[
\frac{\partial^2 H_1}{\partial t^2} = r_b \frac{\partial H_1}{\partial t} > 0 \tag{3.9}
\]

If mortgage interest rates remain constant, then it can be shown that the sign of \(\partial^2 A_1(T)/\partial t^2\) depends on the magnitude of mortgage interest rates and on housing appreciation. In particular,

\[
\text{sign} \left( \frac{\partial^2 A_1(T)}{\partial t^2} \right) = \text{sign} \left[ \frac{b}{1 - e^{-rT}} \frac{P_H(0)}{P_H} \right]
\]

If mortgage interest rates are relatively high and appreciation low, then \(\partial^2 A_1(T)/\partial t^2\) is likely to be positive and, therefore, \(\partial^2 V(A_1(T))/\partial t^2\) is a convex function between 0 and \(T\). In other words, when home values are more or less stable and mortgage interest rate is relatively high, the household is more likely to stay in its current home. If mortgage interest rates are relatively low and housing appreciation high, then \(\partial^2 A_1(T)/\partial t^2\) is likely to be negative and, therefore, \(\partial^2 V(A_1(T))/\partial t^2\) is a concave function between 0 and \(T\). This discussion can be summarized in the following proposition:

**Proposition 1:**

*If the household is motivated solely by investment considerations, then the lower the interest rates and the higher housing appreciation, the more often trading up is expected to occur.*

Let us consider now the effect of a change in housing price \(P_H\).

\[
\frac{\partial t}{\partial P_H} = \frac{1}{\Delta} \left( P_H \frac{1 - e^{-rT}}{1 - e^{-rT}} b_P \frac{\partial H_1}{\partial P_H} - \frac{r e^{-rT}}{1 - e^{-rT}} b_P \frac{\partial H_1}{\partial P_H} \right) \tag{3.10}
\]

where

\[
\frac{\partial H_1}{\partial P_H} = \frac{1 - e^{-r(T-t)}}{1 - e^{-rT}} \left( \frac{b}{1 - b} \left( \frac{P_H(0)}{P_H} \right) \right) H_1 > 0 \tag{3.11}
\]
\[
\frac{\partial^2 H}{\partial \omega \partial P_H} = -\gamma \left( \frac{e^{r(T-t)}}{1-e^{-rT}} \right) \left( 1 - b \right) \left( \frac{P_H(0)}{\beta_H} \right) H_0 < 0
\]  

(3.12)

The first term indicates the effect of equity gains through trading up due to a change in the housing prices, and the second term indicates the effect of the mortgage payment caused by a change in housing size due to an increase in \( P_H \). Considering the first-order condition and substituting equations (3.11) and (3.12) into equation (3.10), we have

\[
\frac{\partial r^*}{\partial P_H} = \frac{1}{\Delta} \left( \frac{r e^{-rT}}{1-e^{-rT}} \right) \left( 1 - b \right) H_0 < 0
\]  

(3.13)

This means that households can accumulate equity more quickly, due to an increase in \( P_H \), than when home values are constant. Therefore, homeowners are more likely to trade up earlier. This effect can be called the "appreciation effect."

Moreover, an increase in housing prices has a positive effect on the maximized intertemporal utility level, i.e., \( V^*(A(T)) \):

\[
\frac{\partial V(A^*(T))}{\partial P_H} = \left( 1 - b \right) \left( \frac{r e^{-rT}}{1-e^{-rT}} \right) \left( H_0 + \frac{\partial H}{\partial P_H} \right) V > 0
\]  

(3.14)

Therefore, we conclude:

**Proposition 2:**

If the household is motivated solely by investment considerations, then a one-time increase in housing prices induces an earlier optimal timing of trading up. Also, housing appreciation increases the maximized intertemporal utility level.
Case 2: Effect of sustained housing appreciation

Suppose that housing price is rising at a constant rate, \( \pi \). The first-order condition for \( t \) reduces to:

\[
\frac{\partial A_1(T)}{\partial t} = \left( P_H(T) \frac{1-e^{-rt}}{1-e^{-rT}} b P_H(t) \right) \frac{\partial H_1}{\partial t} - \frac{r e^{-rt}}{1-e^{-rT}} b P_H(t) H_1 - \frac{1-e^{-rt}}{1-e^{-rT}} b P_H(t) H_1 = 0
\]

(3.15)

i.e., the third term, which is the housing appreciation effect on the remaining mortgage, is added to the first order condition in the previous case.

The second-order sufficient condition, evaluated at the optimal timing, \( t^* \), is assured by the concavity of the objective function. In order for the objective function to be concave in \( t \) between 0 and \( T \), the second derivative of \( A_1(T) \) must be negative:

\[
\frac{\partial^2 A_1(T)}{\partial t^2} = \left( P_H(T) \frac{1-e^{-rt}}{1-e^{-rT}} b P_H(t) \right) \frac{\partial^2 H_1}{\partial t^2} - 2 \frac{r e^{-rt}}{1-e^{-rT}} b P_H(t) \frac{\partial H_1}{\partial t} - 2 \frac{1-e^{-rt}}{1-e^{-rT}} b P_H(t) \frac{\partial H_1}{\partial t}
\]

\[
\times \left( [\pi - r + \pi r] \frac{r e^{-rt}}{1-e^{-rT}} b P_H(t) H_1 - \pi^2 \frac{1-e^{-rt}}{1-e^{-rT}} b P_H(t) H_1 \right)
\]

(3.16)

where,

\[
\frac{\partial H_1}{\partial t} = \left( \pi + (r_0 - \pi) e^{r(T-t)} \right) b \frac{H_0}{1-b} \left( \frac{P_H(0)}{P_H(t)} \right)
\]

(3.17)

\[
\frac{\partial^2 H_1}{\partial t^2} = \left( -\pi^2 + (r_0 - \pi)^2 e^{r(T-t)} \right) b \frac{H_0}{1-b} \left( \frac{P_H(0)}{P_H(t)} \right)
\]

(3.18)

\( \frac{\partial^2 A_1(T)}{\partial t^2} \) is more likely to be negative. In fact, if the rate of housing appreciation is equal to the initial mortgage interest rate, then \( \partial H_1/\partial t > 0 \) and \( \partial^2 H_1/\partial t^2 < 0 \). Therefore, \( \frac{\partial^2 A_1(T)}{\partial t^2} \) is always negative. Hence, \( V(A_1(T)) \) is a concave function between 0 and \( T \), which implies that there exists an optimal timing of trading up between 0 and \( T \).
Proposition 3:

If households are motivated solely by investment considerations, then households are more likely to trade up to a more valuable house when housing prices are rising than when housing prices are stable.

Let us consider the special case in which the rate of housing appreciation is equal to the initial mortgage interest rate. Considering the first-order condition, the effect of an increase in the rate of housing appreciation on \( t^* \) is given by:

\[
\frac{\partial t^*}{\partial \pi} = \frac{1}{\Delta} \left[ (T-t)P_H(T)\frac{\partial H_1}{\partial t} + \left( P_H(T) \frac{1-e^{-\gamma t}}{1-e^{-\gamma T}} bP_H(1) \right) \frac{\partial^2 H_1}{\partial t^2 \partial \pi} \right]
\]

where

\[
\frac{\partial H_1}{\partial \pi} = \left( \frac{1-e^{-\gamma(t-T)}}{1-e^{-\gamma T}} \right) \left( b - \frac{P_H(0)}{P_H(1)} \right) H_0 > 0
\]

\[
\frac{\partial^2 H_1}{\partial t^2 \partial \pi} = \left( \frac{1-e^{-\gamma(t-T)}}{1-e^{-\gamma T}} \right) \left( b - \frac{P_H(0)}{P_H(1)} \right) H_0
\]

(3.19)

The first term shows the direct effect of future equity gain, resulting from deferring trading up, and the second term indicates the indirect effect of an equity gain induced by a change in the housing size due to an increase in the rate of housing appreciation. The third term shows the direct effect of remaining mortgage due to an increase in housing appreciation. Finally, the last term indicates the net indirect effect of financial cost due to an increase in the housing size, resulting from a rise in the rate of housing appreciation. \( \frac{\partial H_1}{\partial \pi} \) is positive and \( \frac{\partial^2 H_1}{\partial t^2 \partial \pi} \) is ambiguous. Therefore, the effect of an increase in the rate of housing appreciation on \( t^* \) also is ambiguous, depending on several conditions such as the equity gain from trading up and the
effects of both constant mortgage payment and remaining mortgage balance. However, if mortgage interest rates remain constant, an increase in the rate of housing appreciation has no effect on the optimal timing of trading up, i.e., \( t^* = T/2 \).

On the other hand, the maximized intertemporal utility level is definitely increased by a rise in the rate of housing appreciation:

\[
\frac{\partial V(A_1(T))}{\partial \pi} = \left[ TP_h(T)H_1 \left[ \frac{1-e^{-rt}}{1-e^{-rT}} bP_h(t)H_1 \right] \left[ P_h(T) \left( \frac{1-e^{-rt}}{1-e^{-rT}} bP_h(t) \right) \frac{\partial H_1}{\partial \pi} \right] \right] V > 0 \tag{3.22}
\]

**Proposition 4:**

If households are motivated solely by investment considerations and if mortgage interest rates remain constant, then an increase in the rate of housing appreciation has no effect on the optimal timing of trading up. Otherwise, the appreciation effect on the optimal timing of trading up is ambiguous. However, an increase in the rate of housing appreciation always has a positive effect on the maximized intertemporal utility level for current homeowners.

**Case 3:** effects of a change in mortgage interest rates

Under the assumption that the housing price per unit is rising at a constant rate, and that \( r_0 \) is equal to \( r \), we investigate the effect of an increase in mortgage interest rates on the optimal timing of trading up. This effect depends on two factors, the remaining mortgage balance and constant mortgage payment as follows:

\[
\frac{\partial t^*}{\partial r} = \frac{1}{A} \left[ \left( \frac{1-e^{-rt}}{1-e^{-rT}} \right) \left( bP_h \frac{\partial H_1}{\partial t} + \pi bP_h H_1 \right) + \left( \frac{e^{-rt}}{1-e^{-rT}} \right) bP_h H_1 \right] \tag{3.23}
\]
where apostrophes denote derivatives with respect to \( r \). If it is further assumed that the rate of housing appreciation is equal to the mortgage interest rate and using the first-order condition, the above result can be written as follows:

\[
\frac{\partial t^*}{\partial r} = \frac{1}{A} \left( \frac{bT - T}{1 - e^{-rT}} \right) p_{H_t}(0) \frac{\partial H_t}{\partial t} + \frac{b p_{H_t}(0) H_t}{1 - e^{-rT}}
\]

(3.24)

The first term of the large bracket indicates remaining mortgage balance effect with an negative sign, while the second term indicates constant mortgage payment effect with a positive sign. Therefore, if the remaining mortgage balance effect dominates the constant mortgage payment effect, then an increase in mortgage interest rates defers the optimal timing of trading up, and vice versa.

However, the maximized intertemporal utility level is definitely decreased due to a rise in \( r \). ³ In fact, using the fact that, if \( r_0 \) is equal to \( r \), then \( t^* = T/2 \), we have:

\[
\frac{\partial V(A_t^*(T))}{\partial r} = \frac{\frac{T}{2} e^\frac{rT}{2} (1 - e^{-\frac{rT}{2}})^2}{(1 - e^{-\frac{rT}{2}})^2} V < 0
\]

(3.25)

**Proposition 5:**

If the remaining mortgage balance effect dominates the constant mortgage effect, then an increase in mortgage interest rates defers the optimal timing of trading up and vice versa. Moreover, the maximized intertemporal utility level is decreased by an increase in mortgage interest rates.
3.2. UPTRADING INDUCED BY CONSUMPTION MOTIVE \((\theta=1)\).

In this section, we concentrate on the consumption motive, and investigate the effects of housing prices and mortgage interest rates on the optimal timing of trading up and homeowner's welfare. If households are solely motivated by consumption considerations, the first-order necessary condition is given by:

\[
(U^0 - U^1)e^{-\theta t}\left(\frac{e^{-\theta t} - e^{-\theta T}}{\theta} \right) \left( U^1 \frac{\partial X}{\partial t} + U^1 \frac{\partial H}{\partial t} \right) = 0
\]

(3.26)

The first term indicates the utility gap, resulting from trading up, while the second term indicates intertemporal utility change. Therefore, the optimal timing of trading up is determined so that the utility gap may be equal to intertemporal utility change.

The second-order sufficient condition is assured by the concavity of the objective function. In order for the objective function to be concave in \( t \) between 0 and T, the second derivative with respect to \( t \) must be negative:

\[
\Delta = -\left(U^0 - U^1\right)e^{-\theta t} - 2\theta e^{-\theta T} \left( U^1 \frac{\partial X}{\partial t} + U^1 \frac{\partial H}{\partial t} \right)
\]

\[
+ \left(\frac{e^{-\theta t} - e^{-\theta T}}{\theta} \right) \left( U^1 \frac{\partial X}{\partial t} \right)^2 + U^1 \frac{\partial^2 X}{\partial t^2} + U^1 \frac{\partial^2 H}{\partial t^2} < 0
\]

(3.27)

If this condition is met and since the objective function has the same value at 0 and T, there exists an optimal timing of trading up between 0 and T.
Case 1: Effects of a one time change in housing prices

The effect of an increase in housing prices on the optimal timing of trading up is given by:

\[
\frac{\partial t^*}{\partial P_H} = \frac{1}{\Delta} \left[ -e^{-\lambda t} \left( \begin{array}{c}
U_X^1 \frac{\partial H}{\partial P_H} + U_X^1 \frac{\partial t^*}{\partial P_H} \\
U_X^1 \frac{\partial H}{\partial P_H} - U_X^1 \frac{\partial X}{\partial P_H} - U_X^1 \frac{\partial H}{\partial P_H} + U_X^1 \frac{\partial t^*}{\partial P_H}
\end{array} \right) \right]
\]

where

\[
\frac{\partial X}{\partial P_H} = \hat{\lambda} \theta \left( H_1 + P_H \frac{\partial H}{\partial P_H} \right) < 0
\]

(3.28)

(3.29)

and \( \frac{\partial H}{\partial t}, \frac{\partial H}{\partial P_H}, \) and \( \frac{\partial^2 H}{\partial t \partial P_H} \) are given by equations (3.8), (3.11), and (3.12). The first term indicates the direct effect which shows how the utility gap is affected due to an increase in housing price. The second and third terms indicate the intertemporal effect which shows how the intertemporal utility is affected by an increase in housing prices. The sign of the first term in the large bracket is indeterminate. However, both second and third terms have a negative sign. If the intertemporal effect resulting from an increase in housing prices is larger than its direct effect, then the optimal timing of trading up is quickened by an increase in housing prices. If the marginal utility of housing consumption is relatively large, i.e., the characteristic of housing as a consumption good plays an important role on consumption behavior, an increase in housing prices quickens the optimal timing of trading up. On the other hand, as marginal utility of housing consumption is less important, the optimal timing is slowed due to an increase in housing prices.
The effect of an increase in housing prices on the maximized intertemporal utility level, denoted by $U^*$, is given by:

$$
\frac{\partial U^*}{\partial p_H} = \left( \frac{e^{-\theta t} \cdot e^{-\gamma t}}{\theta} \right) \left( U_{XH} \frac{\partial X_1}{\partial p_H} + U_{X} \frac{\partial H_1}{\partial p_H} \right)
$$

(3.30)

The first term in the second bracket has a negative sign, implying utility loss, while the second term has a positive sign, implying utility gain from trading up. The net effect is ambiguous.

**Proposition 6:**

If households are motivated solely by consumption considerations, and if the intertemporal effect due to an increase in housing prices is larger than its direct effect, then an increase in housing prices induces an earlier optimal timing of trading up. The effect of an increase in housing prices on the maximized intertemporal utility level is ambiguous.

**Case 2:** Effects of sustained housing appreciation

The effect of an increase in the rate of housing appreciation on the optimal timing of trading up is given by:

$$
\frac{\partial t^*}{\partial \pi} = \frac{1}{\Delta} \left[ -e^{-\theta t} \left( \left( \frac{U_{XH}}{U_{X} \delta \pi} \right) \frac{\partial X_1}{\partial \pi} + \frac{\partial H_1}{\partial \pi} \right) \right]
\left[ \frac{U_{XH}}{U_{X} \delta \pi} + \frac{U_{X}}{U_{X} \delta \pi} \right] + \left( \frac{e^{-\theta t} \cdot e^{-\gamma t}}{\theta} \right) \left( \frac{X_1}{X_H \delta \pi} + \frac{X_1}{X \delta \pi} \delta \pi \right)
$$

(3.31)

where

$$
\frac{\partial X_1}{\partial \pi} = \gamma X_H \left( H_1 + \frac{\partial H_1}{\partial \pi} \right) < 0
$$

(3.32)
\[
\frac{dX}{dt} = -\hat{d} P_H(t) \left( H_1 + \pi t H_1 + \frac{\partial H_1}{\partial t} + \frac{\partial H_1}{\partial \pi} \frac{\partial^2 H_1}{\partial t \partial \pi} \right) < 0
\] (3.33)

and, \( \partial H_1/\partial t, \partial H_1/\partial \pi, \) and \( \partial^2 H_1/\partial t \partial \pi \) are given by equations (3.17), (3.20), and (3.21), respectively. The sign of the first term in the large bracket of equation (3.31) is ambiguous. However, the second term has an negative sign, and the sign of the third term is also more likely to be negative. If the intertemporal effect resulting from an increase in the rate of housing appreciation is larger than its direct effect, then the optimal timing of trading up is quickened by an increase in the rate of housing appreciation.

Similarly, the effect of an increase in the rate of housing appreciation on the maximized intertemporal utility level is given by:

\[
\frac{\partial U^*}{\partial \pi} = \left( e^{-\delta t} - e^{-T} \right) \left( U_x \frac{\partial X}{\partial \pi} + U_H \frac{\partial H}{\partial \pi} \right)
\] (3.34)

The first term in the second bracket has a negative sign, implying utility loss, while the second term has a positive sign, implying utility gain from trading up. The net effect is ambiguous.

**Proposition 7:**

*If households are motivated solely by consumption considerations, and if the intertemporal effect due to an increase in the rate of housing appreciation is larger than its direct effect, then the optimal timing of trading up is quickened. The effect of an increase in the rate of housing appreciation on the maximized intertemporal utility level is ambiguous.*

**case 3:** effects of a change in mortgage interest rates

Under the assumption that the housing price per unit is rising at a constant rate, the effect of an increase in mortgage interest rates on the optimal timing of trading up reduces to:
\[
\frac{\partial t}{\partial r} = \frac{1}{\Delta} \left[ -e^{-\theta t} \left( U_X \frac{U_X}{n} (U^n - U^i) \right) \frac{\partial X_i}{\partial r} + \left( \frac{e^{-\theta t} - e^{\theta t}}{\theta} \right) \left( U_X \frac{\partial X_i}{\partial r} \right) \right]
\]

where

\[
\frac{\partial X_i}{\partial r} = \frac{\partial^2 X_i}{\partial r^2} \text{ if } H_1 < 0
\]

\[
\frac{\partial^2 X_i}{\partial t \partial r} = \frac{\partial^2 X_i}{\partial r^2} + \frac{\partial H_1}{\partial t} < 0
\]

The first term of the bracket has a positive sign, whereas the second term has an negative sign. Therefore, the net effect of a rise in mortgage interest rates on the optimal timing of trading up is ambiguous.

However, the maximized intertemporal utility level is definitely decreased due to a rise in mortgage interest rates as follows:

\[
\frac{\partial U^*}{\partial r} = \left( \frac{e^{-\theta t} - e^{\theta t}}{\theta} \right) \frac{\partial X_i}{\partial r} < 0
\]

Proposition 8:

If households are motivated solely by consumption considerations, then the effect of a rise in mortgage interest rates on the optimal timing of trading up is ambiguous. However, a rise in mortgage interest rates has a negative effect on the maximized intertemporal utility level.

3.3. HOUSING APPRECIATION AND HOMEOWNER MOBILITY: GENERAL CASE (0 < \beta < 1).

Let us turn to the effect of housing appreciation and mortgage interest rates on the timing of trading up and on household's intertemporal welfare in the most general case where trading up is induced by both consumption and investment motives. The household's problem is discussed in section 2.
Case 1: Effects of a one-time change in housing prices.

The effect of an increase in housing prices on the optimal timing of trading up is given by:

\[
\frac{\partial t^*}{\partial P_H} = \frac{1}{\Lambda} \left\{ \left[ 1 - \frac{e^{\gamma t} - e^{-\gamma T}}{\theta} \right] \left[ \frac{U_H}{P_H} \frac{\partial H}{\partial P_H} - \frac{U}{P_H} \frac{\partial U}{\partial P_H} \right] \frac{\partial V}{\partial P_H} + \frac{\partial V}{\partial P_H} + \frac{\partial^2 A_1(T)}{\partial P_H} \right\}^{\frac{1}{2}}
\]

where

\[
\frac{\partial A_1(T)}{\partial P_H} = \left( \frac{1 - e^{-t}}{1 - e^{-T}} \right) \left( H_1 + P \frac{\partial H_1}{\partial P_H} \right) > 0
\]

\[
\frac{\partial^2 A_1(T)}{\partial P^2 H} = - \left( \frac{e^{-t}}{1 - e^{-T}} \right) \left( b H_1 + b P \frac{\partial H_1}{\partial P_H} \right) < 0
\]

The first and second large brackets show the effects of consumption and investment motives on the optimal timing of trading up, respectively. The sign of the first term in the first large bracket is indeterminate. However, the second term in the first large bracket has a negative sign. If the intertemporal effect resulting from an increase in housing prices is larger than its direct effect, then an increase in housing prices quickens the optimal timing of trading up through consumption motive. The sign of the first term in the second large bracket depends on the sign of \( \frac{\partial A_1(T)}{\partial t} \). As long as the optimal timing in the case of \( \beta = 1 \) is less than that in the case of \( \beta = 0 \), \( \frac{\partial A_1(T)}{\partial t} \) has a positive sign. Therefore, the sign of the second large bracket is negative. That is, the effect of an increase in housing prices through investment motive is
negative. Therefore, an increase in housing prices is more likely to quicken the optimal timing of trading up.

Again, the effect of an increase in housing prices on the maximized intertemporal utility level is ambiguous, though utility from terminal equity is increased by an increase in housing prices.

**Proposition 9:**

If households are motivated by both consumption and investment motives, then an increase in housing prices is more likely to quicken the optimal timing of trading up. Also, the maximized intertemporal utility level is more likely to be increased.

**Case 2: Effects of sustained housing appreciation**

In turn, the effect of an increase in the rate of housing appreciation on the optimal timing of trading up is given by:

\[
\frac{\Delta t^*}{\delta \pi} = \frac{1}{\delta} \left\{ \begin{array}{c}
- e^{-\delta t} \left( \begin{pmatrix} U^T \frac{\partial X}{\partial \pi} + U^T \frac{\partial H}{\partial \pi} \\
\frac{\partial X}{\partial \pi} \end{pmatrix} \right) \\
+ \left( \begin{pmatrix} U^T \frac{\partial H}{\partial \pi} - U^T \frac{\partial X}{\partial \pi} \\
\frac{\partial H}{\partial \pi} \frac{\partial X}{\partial \pi} \end{pmatrix} \right) \right\} \\
+ (1 - \delta) e^{-\delta T} \left[ \begin{pmatrix} V \frac{\partial A(T)}{\partial \pi} - U^T \frac{\partial X}{\partial \pi} \\
V^T \frac{\partial A(T)}{\partial \pi} \end{pmatrix} \right]
\]

(3.42)

where
\[
\frac{\partial A_1(T)}{\partial \pi} = \left( TP_H(T) \frac{1 - e^{-\gamma T}}{1 - e^{-\gamma l}} bP_H(l) \right) H_1 + \left( P_H(T) \frac{1 - e^{-\gamma l}}{1 - e^{-\gamma T}} bP_H(l) \right) \frac{\partial H_1}{\partial \pi} > 0
\]

(3.43)

The first and second large brackets show the effects of consumption and investment motives on the optimal timing of trading up, respectively. The sign of the first term in the first large bracket is indeterminate. However, the second term in the first large bracket is more likely to have an negative sign. If the intertemporal effect resulting from an increase in the rate of housing appreciation is larger than its direct effect, then an increase in the rate of housing appreciation quickens the optimal timing of trading up through consumption motives. The sign of the first term in the second large bracket depends on the sign of \( \partial A_1(T)/\partial \pi \). As long as the optimal timing in the case of \( \beta=1 \) is less than that in the case of \( \beta=0 \), \( \partial A_1(T)/\partial \pi \) has a positive sign. Therefore, though the sign of the second term is ambiguous, the net effect of an increase in the rate of housing appreciation through investment motives is more likely to be negative. Therefore, an increase in the rate of housing appreciation also is more likely to quicken the optimal timing of trading up.

As shown in the previous sections, the effect of an increase in the rate of housing appreciation on the maximized intertemporal utility is ambiguous, though utility from terminal equity is increased by an increase in the rate of housing appreciation.

**Proposition 10:**

*If households are motivated by both consumption and investment motives, then an increase in the rate of housing appreciation is more likely to quicken the optimal timing of trading up. Also, the maximized intertemporal utility level is more likely to be increased.*
Case 3: Effects of a change in mortgage interest rates

Under the assumption that housing prices per unit is rising at a constant rate, we investigate the effect of an increase in mortgage interest rates on the optimal timing of "trading up". This effect is given by:

\[
\frac{\partial t^*}{\partial r} = \frac{1}{\Delta} \left\{ -e^{-\alpha t} \left( U_X^1 - \frac{U_X}{U_X^1}(U_0 - U_1) \right) \frac{\partial X_1}{\partial t} + \frac{e^{\alpha t} e^{\alpha T}}{\theta} \left( \frac{\partial^2 X_1}{\partial t^2} + \frac{U_X^1}{U_X^1} \frac{\partial X_1}{\partial r} \right) \right\} (1 - \beta) e^{-\alpha t} \left[ \left( \frac{V - \partial A_1(T)}{V} \frac{\partial X_1}{\partial r} - \frac{U_X^1}{U_X^1} \frac{\partial X_1}{\partial t} \right) \frac{\partial A_1(T)}{\partial t} + V \frac{\partial^2 A_1(T)}{\partial t \partial r} \right]
\]

(3.44)

where

\[
\frac{\partial^2 X_1}{\partial t \partial r} = \frac{\partial^2}{\partial r^2} b P_H(t) \left( \pi H_1 \frac{\partial H_1}{\partial t} \right) < 0
\]

(3.45)

\[
\frac{\partial A_1(T)}{\partial r} = \left( \frac{1 - e^{-\alpha t}}{1 - e^{-\alpha T}} \right) b P_H(t) H_1
\]

(3.46)

\[
\frac{\partial^2 A_1(T)}{\partial t \partial r} = \left( \frac{1 - e^{-\alpha t}}{1 - e^{-\alpha T}} \right) b P_H(t) \left( \frac{\partial H_1}{\partial t} + \pi b P_H(t) H_1 \right) \left( \frac{e^{-\alpha t}}{1 - e^{-\alpha T}} \right) b P_H(t) H_1
\]

(3.47)

and apostrophes indicate the derivatives with respect to \( r \).

The sign of the first large bracket, which indicates the effect of consumption motive, is indeterminate. That is, the direct effect, resulting from a rise in \( r \), is offset by its intertemporal effect. If households is motivated by both consumption and investment considerations, the optimal timing is earlier than \( t^* = T/2 \), which is obtained under investment considerations only. The signs of both \( \frac{\partial A_1(T)}{\partial r} \) and \( \frac{\partial^2 A_1(T)}{\partial t \partial r} \) in the second large bracket are also indeterminate. In general, we can not determine the sign of the above effect.
Moreover, as shown in the previous scenarios, the effect of a rise in mortgage interest rates on the maximized intertemporal utility is ambiguous, though utility from consumption motive is decreased by an increase in mortgage interest rates.

**Proposition 11:**
If households are motivated by both consumption and investment considerations, then the effect of an increase in housing prices on the optimal timing of trading up is ambiguous. However, the maximized intertemporal utility level is more likely to be decreased.

**4. OPTIMAL TIMING OF TRADING UP BY FIRST-TIME HOME BUYERS**

So far, we assumed that at the beginning of the time horizon the household already owns its house. Therefore, the above analysis is suitable for current homeowners. In this section, we investigate the effect of housing prices and changes in mortgage interest rates on the optimal timing of trading up and household's welfare for first-time home buyers, i.e., households who are usually renters and have decided to buy a house, expecting to trade up to a more valuable home in the future. The initial housing consumption for first-time home buyers is determined by the initial downpayment constraint, and the optimal timing of trading up is chosen so that intertemporal utility may be maximized. We assume that, at time 0, a household spends all its accumulated savings, $S_0$, which is exogenously given, on the downpayment for the first home, and borrows $bP_H(0)H_0$, at the fixed mortgage interest rate, $r$. This allows us to ignore the analysis of accumulation of savings required to pay downpayment for the first home. All the money, which the household will receive upon the sale of its home at time $t$, is assumed to go to the downpayment on the second home. The size of second home is determined by the downpayment constraint.

Note that we ignore the analysis of the timing of tenure conversion from renting to owning. The more general model could consider the effects within a life-cycle framework of
housing appreciation and mortgage effects on the joint decision of both tenure conversion and trading up. The effects of housing appreciation and mortgage interest rates on tenure choice for current renters are very important, since first homebuyers are more likely to face a mortgage payment constraint as well as a downpayment constraint. Nevertheless, it is still important to contrast the effects of housing appreciation and mortgage interest rates on the optimal timing of trading up for current homeowners and those for first-time homebuyers.

The households' problem is given as follows:

\[
\text{max } \int_0^1 U^0(X_0, H_0)e^{-rds} + \int_t^1 U^1(X_t, H_t)e^{-rds} + (1-B)[e^{-\alpha T}V(A_t(T))] \\
\text{subject to}
\]

i) budget constraints:

\[X_0 = Y - \hat{r}_0 bP_H(0)H_0\]
\[X_t = Y - \hat{r}_b P_H(t)H_t\]

ii) downpayment constraint for the purchase of first home.

\[S_0 = (1-b)P_H(0)H_0\]

iii) downpayment constraint for the purchase of second home:

\[A_0(t) = (1-b)P_H(t)H_t\]

In the previous section, a change in housing prices does not affect initial housing consumption. On the other hand, in this section, initial housing consumption is affected by a change in housing prices through the initial downpayment constraint. However, since initial housing consumption is independent of the timing of trading up, both first-order and second-order conditions for first-time home buyers are the same as for current homeowners. In the rest of the section, we investigate the effect of a change in housing price and mortgage interest rates on housing decisions for first-time home buyers. The analytic comparative effects are summarized in Tables 6 and 7.
**Case 1:** Effects of a one time change in housing prices

The housing price per unit is constant over time, which implies that this is a special case of case 2 below. Unlike in Section 3, an increase in housing prices affects the initial housing consumption for first-time homebuyers. The effect of an increase in housing prices on the optimal timing of trading up is now given by:

\[
\frac{\partial t^*}{\partial P_H} = \frac{b}{\lambda} \left( e^{-\beta T} U_p \frac{\partial H_0}{\partial P_H} - U_h \frac{\partial H_1}{\partial P_H} \right) + \frac{e^{-\beta T}}{1-e^{-\beta T}} \left( U_p \frac{\partial H_0}{\partial t} + U_h \frac{\partial H_1}{\partial t} + \frac{\partial^2 H_1}{\partial P_H^2} \right)
\]

where

\[
\frac{\partial H_0}{\partial P_H} > \frac{S_0}{(1-b)P_H} < 0
\]

\[
\frac{\partial H_1}{\partial P_H} = \left( \frac{1-e^{-\beta T}}{1-e^{-\beta T}} \right) \frac{\partial H_0}{\partial P_H} < 0
\]

\[
\frac{\partial^2 H_1}{\partial P_H^2} = \frac{r_0}{(1-e^{-\beta T})^2} \frac{\partial H_0}{\partial P_H} < 0
\]

The first term in the bracket in equation (4.1) shows the direct effect for first-time homebuyers. Since \( \partial X_0/\partial P_H = \partial X_1/\partial P_H = 0 \), this term consists of marginal utilities of housing consumption only due to an increase in housing prices, and its sign is ambiguous. The second term in the bracket indicates intertemporal effect, and its sign also ambiguous. Therefore, if households are motivated by consumption considerations, the effect of an increase in housing prices on the optimal timing of trading up is ambiguous.

On the other hand, since \( \partial A_1(T)/\partial P_H = \partial^2 A_1(T)/\partial t \partial P_H = 0 \), the effect of terminal wealth on the optimal timing of trading up also disappears. Therefore, if households are motivated by consumption considerations, an increase in housing prices has no effect on the optimal timing of
trading up. In fact, it can be shown that $t^* = T/2$. If both motives coexist, then the effect of an increase in housing prices on the optimal timing of trading up is ambiguous.

Moreover, using equations (4.2) and (4.3), the effect of an increase in housing prices on the maximized intertemporal utility is given by:

$$\frac{\partial U^*}{\partial P_H} = \left(1 - e^{-at}\right) \frac{e^{\beta H_0}}{P_H} + \left(e^{-aT} - e^{-at}\right) \frac{e^{\beta H_1}}{P_H} < 0$$

(4.5)

Note that the effect through investment motive disappears. Therefore, if households are motivated by investment considerations, an increase in housing prices has no effect on the maximized intertemporal utility level. In general, the maximized intertemporal utility for first-time home buyers is definitely decreased due to an increase in housing prices.

**Proposition 12:**

For first-time home buyers, the effect of an increase in housing price on the optimal timing of trading up is ambiguous. However, the intertemporal utility level is definitely decreased by an increase in housing prices. Moreover, if investment motives dominate ($\beta = 0$), an increase in housing prices has no effect on either the optimal timing of trading up or the maximized intertemporal utility level.

**Case 2: Effects of sustained housing appreciation**

We consider the effect of an increase in the rate of housing appreciation on the optimal timing of trading up. Since both initial non-housing consumption and housing consumption are determined independently of the rate of housing appreciation, the effect of an increase in the rate of housing appreciation can be expressed in the same form as in Section 3. Therefore, the
argument on the effect of an increase in the rate of housing appreciation, which was made in Section 3, still holds.

**Proposition 13:**

An increase in the rate of housing appreciation has the same effect on the optimal timing of trading up for both current homeowners and first-time home buyers.

**Case 3: Effects of a change in mortgage interest rates**

The effect of a rise in mortgage interest rates on the optimal timing of trading up is given by:

\[
\frac{dT}{dt} = \beta \left\{ -e^{-\theta T} \left( \frac{\partial X_0}{\partial t} + \left( \frac{U_x}{U_x} - \sum_{U_x}^{U_x} (U^{n}_x - U^{1}_x) \frac{\partial X_1}{\partial t} + U^{n}_x \frac{\partial H_1}{\partial t} \right) \right) \\
\quad + \frac{1}{V} \left( \frac{e^{-\theta t} e^{-\gamma t}}{\lambda} \right) \frac{U'_x}{U'_x} \frac{\partial H_1}{\partial t} - \frac{e^{-\theta t}}{\lambda} \frac{U'_x}{U'_x} \frac{\partial H_1}{\partial t} + \frac{e^{-\theta t} e^{-\gamma t}}{\lambda} \frac{U'_x}{U'_x} \frac{\partial H_1}{\partial t} \right) \\
\quad + (1-\beta) e^{-\theta T} \left[ \left( 1 - \frac{\partial A_1(T)}{\partial t} \right) \frac{\partial X_1}{\partial t} + \frac{\partial A_1(T)}{\partial t} \right] \right\}
\]

where

\[
\frac{\partial X_0}{\partial t} = \frac{\partial f}{\partial t} bP_H(0)H_0 < 0 \tag{4.6}
\]
\[
\frac{\partial X_1}{\partial t} = \left( \frac{\partial g}{\partial t} H_1 + \int \frac{\partial h}{\partial t} bP_H(1) \right) < 0 \tag{4.7}
\]
\[
\frac{\partial A_1(T)}{\partial t} = \left( P_H(T) - \frac{1 - e^{-\gamma T}}{1 - e^{-\gamma T}} bP_H(1) \right) \frac{\partial H_1}{\partial t} - \left( \frac{1 - e^{-\gamma T}}{1 - e^{-\gamma T}} bP_H(1) \right) \frac{\partial H_1}{\partial t} \tag{4.8}
\]
\[
\frac{\partial^2 A_1(T)}{\partial t^2} = \left( \frac{1 - e^{-\gamma t}}{1 - e^{-\gamma t}} bP_H(1) + \int bP_H(1) \right) - \left( \frac{re^{-\gamma T}}{1 - e^{-\gamma T}} bP_H(1) \right) \frac{\partial H_1}{\partial t} \tag{4.9}
\]

The first term in the first large bracket shows the direct effect for first-time homebuyers. Unlike for current homeowners, an increase in mortgage interest rates affects the initial non-
housing consumption and the size of the second home. This term has either sign. The second and third terms in the same bracket indicate intertemporal effect. Both terms also have either sign. Therefore, if households are motivated by consumption considerations, the net effect of an increase in housing prices on the optimal timing of trading up is ambiguous.

On the other hand, the second large bracket reflects the investment motive. If households are motivated by investment considerations, an increase in mortgage interest rates has no effect on the optimal timing of trading up. In fact, it can be shown that \( t^* = T/2 \). In general, however, the sign of this term is ambiguous. If both motives coexist, then the overall effect of an increase in housing prices on the optimal timing of trading up is ambiguous.

The effect of an increase in mortgage interest rates on the maximized intertemporal utility level is expressed as follows:

\[
\frac{\partial U^*}{\partial r} = \beta \left( \frac{1-e^{-rt}}{\theta} \frac{\partial X_0}{\partial r} + \frac{e^{-rt} - e^{-rt'}}{\theta} \left( U_t^* \frac{\partial X_1}{\partial r} + U_{Ht}^* \frac{\partial H}{\partial r} \right) \right) + (1-\beta) \left[ e^{-rt} \frac{\partial A(t)}{\partial r} \right]
\] (4.11)

Though the sign of \( \partial H/\partial r \) is ambiguous, a rise in mortgage interest rates is more likely to decrease the maximized intertemporal utility level. However, if households are motivated by investment considerations, then the maximized intertemporal utility is definitely decreased by a rise in mortgage interest rates.

**Proposition 14:**

For first-time home buyers, an increase in mortgage interest rates has an ambiguous effect on the optimal timing of trading up, and is more likely to decrease the maximized intertemporal utility level. However, if first-time home buyers are motivated by investment considerations, a rise in mortgage interest rates has no effect on the optimal timing of trading up, and definitely decreases the maximized intertemporal utility level.
5. SIMULATION ANALYSIS

As shown in the previous sections, some of the comparative statics results are ambiguous. In this section, we develop a simulation model to obtain numerical results on how housing appreciation and mortgage interest rates affect the optimal timing of trading up and homeowners’ welfare. Simulations are based on the theoretical model developed in Section 3 and 4. The household’s yearly income is $50,000, which is assumed to be constant over time. The household’s initial saving is $10,000, which is assumed to be spent on the downpayment for the first home. The initial housing price is $50,000. The real mortgage interest rate is 3% and the loan-to-value ratio is .8. The rate of time preference also is set at 3%. The utility function is the same form as is used by Dolde (1978), Schwab (1982), and Alm and Follain (1982). The results in the base cases are obtained under the assumptions above.

First, we shall look at the results for current homeowners. Simulation results are reported in Tables 8, 9 and Figures 1,2. The results in the first columns of Tables 8 and 9 are obtained under the assumption that the price of housing rises from \( P_H(0)=50,000 \) to \( P_H=55,000 \). From the investment point of view, an increase in housing prices enables current homeowners to accumulate equity necessary to trade up to a more valuable house, and is more likely to quicken the timing of trading up. From the consumption point of view, the effect of an increase in housing appreciation consists of two elements: direct effect and intertemporal effect. The simulation result tells us that, since intertemporal effect is larger than direct effect, an increase in housing appreciation quickens the optimal timing of trading up. Also, the maximized intertemporal utility level is increased by an increase in housing prices, regardless of whether consumption motive dominates investment motive. These effects can be called the "appreciation effect."

The results in the fourth columns are obtained under the assumption that the mortgage interest rate increases from 3% to 5%. An increase in mortgage interest rates has the opposite effect to the above. From an investment perspective, an increase in mortgage interest rates
quicksens the optimal timing of trading up; however, the maximized intertemporal utility level is decreased. On the other hand, from a consumption perspective, an increase in mortgage interest rates defers the optimal timing of trading up and also decreases the maximized intertemporal utility level. Therefore, if transaction costs are substantial, homeowners are more likely to stay at their current homes. This effect can be called the "lock-in effect."

The results in the third columns are obtained under the assumption that the rate of housing appreciation rises from \(\pi=0\%) to \(\pi=2\%)\). From the investment point of view, an increase in the rate of housing appreciation does not affect the optimal timing of "trading up". From the consumption point of view, an increase in the rate of housing appreciation quickens the optimal timing of "trading up". The simulation result tells us that the intertemporal effect is larger than direct effect. The maximized intertemporal utility level is increased, due to an increase in housing prices, whether or not consumption motives dominate. That is, an increase in the rate of housing appreciation enables current homeowners to accumulate the equity necessary to trade up to a more valuable house and is more likely to quicken the timing of trading up.

Second, we shall look at the simulation results for first-time home buyers. Simulation results are reported in Table 10, 11 and Figures 3, 4. The results in the first column of Tables 10 and 11 are obtained under the assumption that the price of housing rises from \(P_H(0)\)=50,000 to \(P_H\)=55,000. From the consumption point of view, an increase in housing price slows the optimal timing of trading up and decreases the maximized intertemporal utility level. From the investment point of view, however, an increase in housing price affects neither the optimal timing of trading up nor the maximized intertemporal utility level. Unlike current homeowners, first-time home buyers suffer from an increase in housing price. First-time home buyers can not obtain the equity gain from a one time increase in housing prices.

The results in the fourth columns are obtained under the assumption that mortgage interest rates increase from 3% to 5%. From the investment point of view, an increase in mortgage interest rates does not affect the optimal timing of trading up, and the maximized
intertemporal utility level is decreased much more than that of current homeowners. On the other hand, from the consumption point of view, an increase in mortgage interest rates defers the optimal timing of trading up, implying that direct effect is larger than intertemporal effect, and the maximized intertemporal utility level also is decreased much more than that of current homeowners. Therefore, if transaction costs are substantial, first-time home buyers are more likely to stay at a new home.

6. SUMMARY

Previous theoretical and empirical literature on homeowner mobility has focused on life-cycle factors (including job change), the gap between actual and desired housing consumption, and changes in neighborhood quality. However, recent observations on homeowner mobility suggest a crucial role for macroeconomic variables, especially residential appreciation and mortgage interest rates, and that the investment aspect of homeownership must be emphasized. On the other hand, the recent work on the investment component of housing decisions has examined tenure choice and the timing of tenure conversion from renting to owning, but omitted the issue of homeowners' mobility and in particular the effect of housing appreciation and changes in mortgage interest rates in the phenomena of homeowner mobility.

This paper develops a model of trading up, which focuses on both consumption and investment components of housing decisions. Households determine the optimal timing of trading up so as to maximize their intertemporal utility of both housing and non-housing consumption. The analytic and numerical results can be summarized as follows:

1. If current homeowners are motivated by investment considerations, then a one time increase in housing prices induces an earlier optimal timing of trading up, and current homeowners are better off than when housing prices are stable. If mortgage interest rates remain constant, an increase in the rate of housing appreciation has no effect on the optimal timing of trading up, although current homeowners are better off. On the other hand, the effect
of a rise in mortgage interest rates on the optimal timing of trading up is ambiguous and current homeowners are worse off than when mortgage interest rates are constant.

(2) If current homeowners are motivated by consumption considerations, housing appreciation tends to induce an earlier optimal timing of trading up, although welfare effects are ambiguous. On the other hand, the effect of a rise in mortgage interest rates on the optimal timing of trading up is ambiguous and current homeowners are worse off. The simulation analysis suggests that housing appreciation increases the maximized intertemporal utility level and that a rise in mortgage interest rates defers the optimal timing of trading up.

(3) If first-time home buyers are motivated by investment considerations, the effect of an increase in housing prices has no effect on either the optimal timing of trading up or the maximized intertemporal utility level. A rise in mortgage interest rates does not affect the optimal timing of trading up and first-time homebuyers are worse-off.

(4) If first-time homebuyers are motivated by consumption considerations, the effect of an increase in housing prices on the optimal timing of trading up is ambiguous although first-time homebuyers are worse off. An increase in the rate of housing appreciation tends to induce an earlier optimal timing of trading up and welfare effect also is ambiguous. On the other hand, the effect of a rise in mortgage interest rates on the optimal timing of trading up is ambiguous and first-time homebuyers are worse off. The simulation analysis suggests that an increase in housing prices and a rise in the rate of housing appreciation defers and quickens the optimal timing of trading up, respectively, and that a rise in mortgage interest rates also defers the optimal timing of trading up.

As it is apparent, the model captures the recent observations on homeowner mobility and suggests that macroeconomic variables such as housing appreciation and mortgage interest rates effect the optimal timing of trading up and homeowner's welfare. Nevertheless, our model has several shortcomings. First, transaction costs are ignored. If transaction costs are incorporated, the "lock-in effect" from a rise in mortgage interest rates is well explained. However, the general analysis above is not altered in any essential way. Second, multiple moves
are not considered in this model. Therefore, we concentrate on the timing of one trade up as opposed to the timing and frequency of trading up.

In a different vein, it would be interesting to test empirically the importance of the effects of macroeconomic variables on trading up by using micro-data.
FOOTNOTES

(1) This paper ignores homeowners' maintenance problems. However, the joint decision of
maintenance and mobility has never been discussed within an intertemporal framework. See for
example Dildine and Massey (1974) and Mendelsohn (1977) for homeowners, and Mayer
(1981), Margolis (1981), and Arnott et al. (1983) for landlords.

(2) The mortgage lock-in effect resulting from an increase in real interest rates is examined
empirically by Quigley (1986), in which the effect of capital gains on homeowner mobility is
also ignored.

(3) If transaction cost is incorporated in the model, then intertemporal utility function
becomes discontinuous at the terminal time. Therefore, it is more likely that households are
staying at current homes.
REFERENCES


TABLE 1
MOBILITY RATES FOR U.S. HOUSEHOLDS: 1973-1983

<table>
<thead>
<tr>
<th>YEAR</th>
<th>ALL HOUSEHOLDS</th>
<th>OWNERS</th>
<th>RENTERS</th>
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<tr>
<td>1973</td>
<td>19.2</td>
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</tr>
<tr>
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</tr>
<tr>
<td>1983</td>
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TABLE 2
MOBILITY RATES FOR YOUNG HOUSEHOLDS

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<th>YEAR</th>
<th>PREVIOUS OWNERS</th>
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<td>1981</td>
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<tr>
<td>1983</td>
<td>14.7</td>
<td>33.2</td>
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TABLE 3
RATIO OF NEW HOME VALUE TO PREVIOUS HOME VALUE FOR RECENT HOMEBUYERS, 1973-83

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<th>PREVIOUS</th>
<th>RATIO</th>
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</tr>
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<td>1983</td>
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### TABLE 4
EFFECTS ON OPTIMAL TIMING FOR CURRENT HOMEOWNERS
ANALYTIC RESULTS

<table>
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<th>Increase in $\pi$</th>
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<td>more likely</td>
<td>?</td>
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<td>more likely</td>
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<td>more likely</td>
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</table>

### TABLE 5
EFFECTS ON UTILITY LEVEL FOR CURRENT HOMEOWNERS
ANALYTIC RESULTS

<table>
<thead>
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<th></th>
<th>Increase in $P_H$</th>
<th>Increase in $\pi$</th>
<th>Increase in $r$</th>
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</thead>
<tbody>
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<td>?</td>
<td>-</td>
</tr>
<tr>
<td>$0 &lt; \beta &lt; 1$</td>
<td>more likely</td>
<td>more likely</td>
<td>more likely</td>
</tr>
<tr>
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<td>+</td>
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**TABLE 6**
EFFECTS ON OPTIMAL TIMING FOR FIRST-TIME HOME BUYERS
ANALYTIC RESULTS

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<tr>
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<td>?</td>
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**TABLE 7**
EFFECTS ON UTILITY LEVEL FOR FIRST HOME BUYERS
ANALYTIC RESULTS

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<th>Increase in $r$</th>
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<tr>
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<td>-</td>
</tr>
<tr>
<td>$0&lt;\beta&lt;1$</td>
<td>-</td>
<td>+ more likely</td>
<td>- more likely</td>
</tr>
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<td>+</td>
<td>-</td>
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**TABLE 8**
EFFECTS ON OPTIMAL TIMING FOR CURRENT HOMEOWNERS
SIMULATION RESULTS

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**TABLE 9**
EFFECTS ON UTILITY LEVEL FOR CURRENT HOMEOWNERS
SIMULATION RESULTS

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<tr>
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### Table 10
Effects on Optimal Timing for First-Time Home Buyers
Simulation Results

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### Table 11
Effects on Utility Level for First-Time Home Buyers
Simulation Results

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<tr>
<td>$\beta=1$</td>
<td>BASE</td>
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SIMULATION RESULTS FOR CURRENT HOMEOWNERS

FIGURE 1
UTILITY LEVEL FOR DIFFERENT TRADING-UP TIMING (β=1)

FIGURE 2
TERMINAL EQUITY FOR DIFFERENT TRADING-UP TIMING (β=0)
SIMULATION RESULTS FOR FIRST-TIME HOME BUYERS

FIGURE 3
UTILITY LEVEL FOR DIFFERENT TRADING-UP TIMING (β=1)

FIGURE 4
TERMINAL EQUITY FOR DIFFERENT TRADING-UP TIMING (β=0)