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INCREASING RETURNS AND ENDOGENOUS GROWTH: 
MARKET SIZE AND TASTE FOR VARIETY.*

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ABSTRACT

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This paper studies the implications of the existence of Increasing Returns to Scale in the generation of persistent growth in the economy. In a standard neoclassical one-sector model, the optimal growth rate will be proportional to the rate of population growth.

If economies of scale are internal to particular markets and we assume that firms perceive their production functions as CRS, a symmetric equilibrium applied to the model illustrates an example of market failure in promoting its own integration. Steady-state growth may be less than optimal.

This result is not general: if one considers that consumers have taste for variety and a multiproduct economy where there are economies of scale internal to each market, one concludes that the equilibrium growth rate may be higher or lower than optimal. The efficient outcome will lead to higher growth rates for countries with less intense taste for variety and vice-versa.

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1. Introduction.

It is our purpose to consider the possibility of endogenous growth to be generated in the economy due to the existence of Increasing Returns to Scale. The inclusion of such hypothesis in growth models is empirically justifiable: according to Denison's (1974) estimates for the U.S. in the period 1929-1969, economies of scale account for 18-19% of the growth rate of national income per person employed.

Models that depart from utility maximization and generate endogenous growth of per capita consumption in the steady-state do not have necessarily to be based on the existence of IRS. We can generate endogenous growth in a CRS world with a production function for knowledge linear in the human capital index as Uzawa (1965) and Lucas (1985) used, or imposing special structure in the coefficients of as in Martins (1987). However, the approaches e
restrictive assumptions.

Recently, other authors have explored this class of endogenous
growth models, in an attempt to capture some specific features of the
growth process. For example: Stokey (1988) studies the introduction of new
goods; Schmitz (1989), the imitation by firms; Rebelo (1987) and Barro
(1988a and b), the effects of economic policy variables; Grossman and
Helpman (1988), the growth-cum-trade possibilities in a two-country
model.

Arrow (1962) described an economy with IRS where experience
enhances the productivity of physical capital. New capital has embodied all
knowledge available by the time it is produced and posterior experience
does not affect its productive capacity. The model does not depart from
utility maximization. In a steady-state, per capita consumption will grow at
a rate proportional to the rate of population. As we will see, even if we have
a very different framework than his, we get a similar result for our
efficient solution.

Other authors have investigated the consequences of the existence
of Increasing Returns to Scale in dynamic optimization models, for example:
Weitzman (1970), Dixit, Mirrlees and Stern (1977),
authors were concerned with the dynamic evol.
much with the properties of the steady-state (in fact, there seems to be no convergence for one in some of those models). They are concerned only with optimal policies, and do not refer to possible equilibrium outcomes. Dixit et al and Skiba consider one-sector models. Weitzman looks at a world with two types of physical capital where there are substantial economies of scale in the creation of one of the types, 'overhead capital'; he simplifies his world by assuming that this type of capital has no manpower requirements - this allows him to abstract from problems of allocation of labor force between the two sectors.

Romer (1986) devoted some attention to the argument. He tested the hypothesis of the existence of a non-positive trend of the growth rates of several countries using data starting (whenever possible) in the beginning of the eighteenth century. Using a sufficiently long series, the hypothesis is not rejected. But if we use a shorter series, say from 1870 on, then the hypothesis is rejected. One can therefore conclude that we have a path where we observe an increasing growth rate of per capita consumption, tending to a constant value. He proposes to fit that evidence, looking for a model which can generate persistent growth even in the absence of population growth. He uses a production function returns but also exhibiting increasing marginal
Due to that, his model generates the result of an ever increasing rate of
growth of per capita consumption for a certain range of initial conditions of
the state variable. Imposing an exogenous bound for the growth rate of per
capita physical capital, the growth rate of the economy, under those
circumstances, will show an asymptotic tendency to a certain value. Then a
reproduction of the evidence is found.

However, if we allow for population growth other frameworks are
possible to explain such evidence: a one-sector model with decreasing
marginal product of physical capital and IRS would be stable and generate,
for a wide range of initial conditions, the same result without some
exogenous bound necessary to fit last century history. Moreover, plausible
equilibria similar to that proposed by Romer can lead in this context to
(stable) steady-state balanced growth rates not necessarily positively
related to the rate of population growth. To explore such possibilities, we
depart from his framework to generate possible equilibria but rather relate
the features of these equilibria to firm and market size dynamics. We
concentrate on steady-state results, even if some dynamics are considered
in the appendix.

The analysis leads to testable implication

positive correlation between the rate of increase
growth rates of per capita income. If the degree of increasing returns takes the form of an externality associated with the size of the market, the analysis illustrates the effects that barriers to trade may have not only on the levels, but also on the growth rates of a given country.

A further extension of the model allows for an individual utility function that exhibits taste for variety. In static models, the inclusion of such assumption may lead to the non-observance of "excess capacity". Given the features of the utility function chosen below, we derive the undeterminacy of the equilibrium growth rate. If we take into account the controversy on convergence between growth rates or even levels of per capita consumption of different countries - see Baumol (1986) and De Long (1988) for some updated references - , this result seems somewhat applicable. The optimal growth rate, as expected, will be lower the higher the intensity of the taste for variety, meaning we will be more willing to trade growth of total output for growth of the number of diversified products, sacrificing the possibilities that increasing returns offer. An immediate application of the argument refers to the interpretation of productivity slowdown. It may be that it reflects an increased degree of variety. Moreover, it may also be the case that such slowdown is not a negative sign, rather being associated with a better equilibrium solution.
In section II, the efficient solution is described. Features of possible equilibria and influence of firm size and market integration dynamics are analysed in section III. Welfare implications and optimal tax structures will be considered in section IV. The inclusion of taste for variety in the consumption function is delegated for section V. The paper concludes in section VI.
II. Increasing Returns and the Efficient Solution.

Consider that $F(K,L)$, the technology available for the production of the homogeneous tradeable commodity, exhibits Increasing Returns to Scale in $K$ and $L$, respectively, the amount of physical capital and labor employed. Let $c$ denote per capita consumption and $\delta$ the depreciation rate of physical capital. People are infinitely lived. Population, denoted by $L$, grows at rate $\lambda$. Then, the total stock of physical capital, $K$, follows the path:

$$\dot{K} = F(K,L) - \delta K - cL$$  \hspace{1cm} (2.1)

(For simplicity, time subscripts are ignored and for each variable $x$, $\dot{x} = dx/dt$.)

In per capita terms, $\dot{K} = (kL) = kL + k\dot{L}$. We can write therefore:

$$\dot{k} = F/L - c - (\delta + \lambda)k$$  \hspace{1cm} (2.2)

All individuals have per period a time endowment of one unit. The instantaneous individual's utility function is $U(c)$. The planner's problem of this economy is, therefore $^3$: 
Max $\int_{0}^{\infty} e^{-(\rho-\lambda)t} u(c) \, dt$

w.r.t. $c$

s.t.: (2.3) $k = F(K, L) / L - c - (\delta + \lambda) k$

(2.4) $L = \lambda L$

$c \geq 0$

given $k(0)$, and $L(0)$.

Assume the production function is of Generalized Cobb-Douglas form:

(2.5) $F(K, L) = A K^\alpha L^\beta = A k^\alpha L^\alpha \beta$

$0 < \alpha, \beta < 1$

The Hamiltonian of the problem becomes:

(2.6) $H = u(c) + \Theta_1 [ A k^\alpha L^{\alpha+\beta-1} - c - (\delta + \lambda) k ] + \Theta_2 \lambda L =$

$= u(c) + \Theta_1 [ A k^\alpha L^{\beta-1} - c - (\delta + \lambda) k ] + \Theta_2 \lambda L$
Assume $U(c)$ exhibits constant degree of relative risk aversion, i.e., has the form

\[ (2.7) \quad U(c) = c^{(1-\sigma)/(1-\sigma)} \]

Considering the special features of the Cobb-Douglas technology and simplifying notation, the F.O.C. for optimization of the problem include the two state equations, ((2.3) to (2.4) with the form (2.5) for the production function) and the following:

\[ (2.8) \quad \mathbf{H}_c = U_c - \Theta_1 = 0 \]

\[ (2.9) \quad \dot\Theta_1 = \Theta_1 (\rho-\lambda) - \mathbf{H}_K = \Theta_1 (\rho+\delta) - \Theta_1 \alpha A K^{\alpha-1} L^{\alpha+\beta-1} = \]

\[ = \Theta_1 (\rho+\delta) - \Theta_1 \alpha A K^{\alpha-1} L^\beta \]

\[ (2.10) \quad \dot\Theta_2 = \Theta_2 (\rho-\lambda) - \mathbf{H}_L = \Theta_2 (\rho-\lambda) - \Theta_1 (\alpha+\beta-1) A K^{\alpha} L^{\alpha+\beta-2} - \]

\[ - \Theta_2 \lambda = \Theta_2 (\rho-\lambda) - \Theta_1 (\alpha+\beta-1) A K^{\alpha} L^{\beta-2} - \Theta_2 \lambda \]
From (2.8), differentiating with respect to time and dividing by $\theta_1$,

and using (2.7), we can get:

\[
(2.11) \quad \frac{\dot{\theta}_1}{\theta_1} - \frac{U_{cc}}{U_c} \frac{c}{c} - \frac{\sigma}{c}
\]

To derive the balanced growth rates of the problem, we follow the following method. Defining the growth rates of the state and co-state variables, ignoring the growth rate of $\dot{\theta}_2 / \theta_2$, they will be equal to:

\[
(2.12) \quad \dot{k}/k = F/(k \dot{L}) - c/k - (\delta + \lambda) = A \kappa^{-1} \dot{L}^\beta - c/k - (\delta + \lambda)
\]

\[
(2.13) \quad \frac{\dot{\theta}_1}{\theta_1} = -\sigma \frac{c}{c} = \rho + \delta - \alpha A \kappa^{-1} \dot{L}^\beta
\]

From (2.13), constancy of the balanced growth rates imply that $k^{\kappa^{-1}} \dot{L}^\beta$ will be constant. Hence, for $\dot{k}/k$ to be constant, (2.12) imply that $c/k$ will also be constant in the balanced growth path. Thus, denoting $g$ as
the steady-state rate of growth of per capita consumption, we will have

\begin{equation}
(2.14) \quad g = \frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \lambda
\end{equation}

Differentiating $K^{\alpha-1} \lambda^\beta$ with respect to time, and considering the above:

\begin{equation}
(2.15) \quad (\alpha - 1) \frac{\dot{K}}{K} + \beta \lambda = 0
\end{equation}

\begin{equation}
(2.16) \quad \frac{\dot{K}}{K} = \beta \lambda / (1 - \alpha)
\end{equation}

\begin{equation}
(2.17) \quad \frac{\dot{k}}{k} = (\alpha + \beta - 1) \lambda / (1 - \alpha) = g
\end{equation}

**Proposition 1:** In a standard neoclassical growth model with a Cobb-Douglas technology exhibiting increasing returns to scale, the optimal growth rate of per capita consumption is positively related to the rate of population growth, $\lambda$. The rate is positively affected by $\alpha$, and $\beta$. The growth rate will be proportional to the degree of increasing returns, $\alpha + \beta - 1$. 
Transversality conditions would require that

$$\lim_{t \to \infty} e^{-(\rho - \lambda)t} \kappa \Theta_1 = 0$$

which means, $(\rho - \lambda) > g (1 - \sigma)$. The dynamics of the system and the path to the steady-state, and second-order conditions are illustrated in the Appendices.

Even if Arrow's (1962) framework is very different from ours, he also finds proportionality between the two rates. In his model, he considers cumulative production of capital as an index of experience. New capital goods incorporate all the technology available; once built, their productivity is fixed and is not affected by further learning. The underlying production function is therefore positively affected by the accumulated experience, embodied in capital goods, and exhibits increasing returns to scale.

Arrow finds the result of proportionality between the steady-state growth rate and the growth rate of the population "paradoxical". He suggests that "... The explanation seems to be that under full employment, the increasing labor force permits a more rapid introduction of the newer machinery. ...". Subsequent papers that extended Arrow's model - Levhari (1965a and 1965b) do not explain the result either.

Dixit, Mirrlees and Stern (1975) develop a model with increasing
returns to scale. In their concluding remarks, they note that with the homogeneous utility and production technology they use, if population growth were introduced, consumption per head could grow at a (constant) rate proportional to the growth rate of population, the reason advanced being that "... (economies) of scale allow population growth to be amplified".

Romer criticizes the result saying that it has the undesirable property of generating no growth when population is stationary. Moreover, it seems to go against the empirical evidence that LDC's, countries with higher population growth, have lower growth rates.

It seems to me that the explanations offered for the result are incomplete. The missing link to understand the effect of population growth on the steady-state growth rates of income and consumption per capita is the relation between increasing returns and actual possibility of increasing the scale: if population does not increase, the scale of the market does not increase either, and therefore no growth is generated. To the extent that market size increases, then the multiplier effect is higher the larger the degree of increasing returns. Put it in this way, the result seems intuitively correct.

To account for the criticism against the seemingly empirical
contradiction of the model, we advance two remarks. First, one of the economic factors of development for industrialized countries (see Rostow) has been pointed out to be increase in market size. Also, before the 'take-off', population was almost stagnant; increases in its rate of growth occurred near and in the industrialization period. City growth accompanied the phenomena. There have been several empirical inquiries on the relation between population growth and growth of per capita consumption for more recent periods. A negative correlation even in cross-section studies does not seem to appear.

Secondly, the result found above is an efficient solution. The equilibrium behind the scenario described could generate a different result. As we will see, the model can accommodate an interpretation of the effect that integration of markets may have on economic growth. It has been observed that higher population density is associated with higher rates of economic growth (see Simon and citations therein); if higher population density is assumed to promote the degree of market integration, a very plausible assumption, then the equilibria described next will be useful for international comparisons.
III. The Role of Size and Possible Equilibria.

We want to consider possible equilibria for this model. We are going to analyse two cases. The features proposed for the equilibria are going to rely on two types of considerations. The first is inspired by Romer (1986). The second comes from the reasonable deduction that an economy will only benefit from the existence of Increasing Returns if the size of the firms or markets does in fact increase over time.

Romer shows that an economy where the technology exhibits IRS may have a suboptimal dynamic equilibrium if the firms behave with respect to a part of the aggregate stock of knowledge as they would with respect to an externality. We want to transpose Romer's concept to our framework. Let each firm perceive output has a CRS function of physical capital and labor employed; then, this economy can have a competitive equilibrium. In such an equilibrium, the number of firms (and, therefore, firm size) is undetermined and each firm makes no economic profits.

At this point, we want to distinguish three types of economies of scale:

1) Economies of scale internal to the firm;

2) Economies of scale external to the firm but internal to a
particular market, e.g., a city or a product:

3) Economies of scale external to the firm and specific markets, internal only to the market economy as a whole.

The three types have different economic implications and provide explanations for different growth patterns. The analysis that follows is going to apply to the first and second types. The third type is the one usually analysed in growth models. As we will see, it can be seen as a special case of the others. This is the form of economies of scale assumed by Romer.

The existence of economies of scale internal to the firm that are not detected by it may seem unrealistic. However, the influence of the aggregate knowledge employed on the production function may be difficult to measure, and we can assume a persistent misperception on the part of the firm with respect to it.

The second type of economies of scale is much more plausible. Moreover, we can interpret the scenario of our problem in terms of degree of integration of markets. This setting can be related, then, with the factors of growth associated with the industrial revolution and, as we will see, can illustrate the effect that lack of market integration may have for LDC's.

Consider the economies of scale are internal to the firm. Let $K_N$ and
Let denote the amount of physical capital and manhours hired by a particular firm. We propose two types of behavior of the firms, which we will call Cases A, and B. Let the externality term be $E$. Our firm maximizes (price of output is fixed to 1)

\[
\begin{align*}
(3.1) \quad \text{Max} & \quad F^N(K_N, L_N, E) - W L_N - r_k K_N \\
\text{w.r.t.} & \quad K_N \text{ and } L_N
\end{align*}
\]

In particular, we consider forms of $F^N(K_N, L_N, E) = F'^N(K_N, L_N) E$

where $F'^N$ has CRS in $K_N$ and $L_N$.

**Case A:** The firm takes $E = L_N^{(\alpha + \beta - 1)}$ as exogenously given:

\[
(3.2) \quad y_N = F'^N(K_N, L_N) E = A K_N^\alpha L_N^{(1 - \alpha)} [L_N^{(\alpha + \beta - 1)}]
\]

**Case B:** The firm takes $E = K_N^{(\alpha + \beta - 1)}$ as constant:
\[ E_n = e^{-\beta N_{n+1}} \]

The choice of the these two cases has to do with different measures of size being taken as an externality. In case A, the externality comes from pure manhours size. In the second, from total physical capital stock. As we will see, under the assumptions of the model, the choice of the type of externality will not affect the equilibrium growth rates of per capita consumption. They will lead to different equilibrium levels of the ratios of per capita consumption to the physical capital intensity.

What we ask ourselves next is what kind of equilibrium can generate a situation where there will be more than one firm in the market. And whether it is possible that such equilibrium will perpetuate itself. Both questions are relevant because different choices of size on the part of the firm will lead to different values of \( E \) actually observed by that firm.

First of all, in an equilibrium where more than one firm is in the market, all firms must be working at the same scale, i.e., we must have a symmetric equilibrium. If not, firms that have a scale smaller than average will have losses and as they have to pay the unique price of the resource, they will be driven out of the market. Firms that have scale higher than optimal, paying the perceived average productivity of the resource, could
have gained, but they would want to produce at infinite scale. Well, once firms are bidding for the resources the same way, see their production function as CRS with respect to labor and capital, a symmetric equilibrium seems a reasonable outcome.

Suppose that newcomers enter the market at a given rate per unit of time - let us say that the rate at which individuals with initiative to open new firms are born is constant. Each potential newcomer looks at the identical firms in the market with the same value of \( E \) - which has been growing at a rate that already accounts for the entry of new firms - and concludes he is going replicate the trend of their behavior. He will compete as an equal for the resources available, and therefore all firms will end up with an equal market share and no profits. The market will support any number of firms.

Notice that next period \( E \) will not be the same. Firms may perceive it as increasing or decreasing. Implicitly we are assuming that they look at it as an indicator of exogenous technical change. A similar analogy can be found in Lucas (1985) when he discusses his equilibrium environment in the presence of an externality.

Given these considerations, assume that
1) All firms are alike in the market, that is, for any firm

\[ L_N = \frac{L}{N} \]  

and

\[ K_N = \frac{K}{N} \]  

where \( N \) is the number of firms in the market.

2) \( N \) grows at a constant rate \( n \):

\[ \dot{N} / N = n, \quad 0 \leq \dot{N} / N \leq \lambda. \]  

Notice that that if we denote by \( n \) the growth rate of firm size measured in manhours employed by the firm, under the previous assumptions (3.6) implies:

\[ \dot{L}_N / L_N = (\dot{L} / N) / (\dot{L} / N) = \dot{m} = \dot{N} / N, \quad 0 \leq \dot{m} \leq \lambda. \]
The requirement that size must grow at a rate smaller or equal to the rate of population growth has to do with the fact that we are bound to have at least 1 firm producing - therefore, in a steady-state $\dot{N}/N < 0$ cannot occur; eventually we would reach the value $N = 1$, after which there cannot be a subsequent decrease in the number of firms in the market.

The assumption that size does not decrease with time, i.e., $m > 0$, is not necessary in the same way the previous assumption is. However, if size decreases with time, then eventually we will achieve a point where $L_N/N = 1$. At that point, one individual is hired, or alternatively, the number of firms is equal to the population. This is not only unrealistic - we do not observe it -, as also the reverse would be very likely: that not all the individual's would have the initiative to establish a firm by themselves. Besides, in our setting, they do not see any financial inconvenient from the fact of working for somebody else. In sum, we want to assume that at least one individual is hired by the firm. Or an individual will not work in more than one firm. Therefore, we only find realism in the situation where $N < L$.

For this to hold dynamically, the size measured in manhours employed must not decrease indefinitely. This implies that $m = 0$. For a steady-state behavior, $m > 0$. 
A richer application of the mechanism described refers to the case where the economies of scale are external to the firm but internal to a particular market. In this case, the behavior of the aggregate number of firms in a certain market can be reproduced by the solution of the problem (3.1) when we take \( K_N \) and \( L_N \) as the amount of physical capital and labor time employed in the market.

To illustrate the concept, we can apply a trade example to an environment similar to Arrow's. Take case B: there is a spillover effect, an externality, which comes from a size effect of knowledge. That is, from looking at the goods sold in a given area — the 'market' —, a particular firm established in that area can immediately apply some of the knowledge built in them with no extra use of resources (this is the meaning of the exogenous element E). Therefore, if a certain number of firms communicate with each other in this way, they benefit from an externality arising from that interchange of information which varies directly with the size of the market, i.e., of the amount of output of that market, in this case, the area. The existence of the externality will always lead to an inefficient equilibrium solution. But the welfare loss is even larger if the different areas do not trade with each other, because the size of the externality is.
then, smaller, the firms not benefiting from the knowledge accumulated in foreign products. If the number of areas is fixed and the size of each area grows at a certain uniform rate, we will not observe a dynamic consequence of the situation, that is, the growth rate of per capita income will not be affected. Of course, the level of welfare one could achieve with the same wealth at any point in time if only trade was observed (even if still an inefficient solution) would be larger. If the number of markets grows with time, the size of the whole economy growing faster than that of each area, not only the level but also the growth effect will show up. Moreover, given the importance of the size of the externality as the element driving our results and conclusions (in which interpretation we differ from both Lucas, Romer and Arrow), we illustrate the effect that trade barriers may have not only on levels but also on rates of growth of per capita income of a given country.

The argument can be applied to other examples. An important one is the effects of lack of market integration on growth. This has been pointed as a major cause for LDC's economic failure. Suppose the externality arises from the way financial and monetary institutions operate. For instance, an integrated banking system may provide more rapid access to funding and a more productive allocation of resources (capital) than a desintegrated one.
The argument of the previous paragraph can be applied to the communication between banks and financial institutions and their relation with the general public. Again, the level and growth effect will arise.

Finally, lack of market integration may be a result of a situation exogenous to the model. m above will be in a way the rate of growth of the degree of market integration of the economy. It can be, therefore, a function of possibilities of trade between regions — related, for example, to transportation technology —, of city size, institutional imperfections, etc.. Also it can be a function of population growth; we could accept that m may be bounded to a value, and even that after a point may be negatively related to the rate of growth of the population due to a congestion effect.

The two assumptions above can therefore be transposed to this 'market' setting: we take N as the number of unconnected markets. (The number of firms in one market is independent of the market scale effect, i.e., in a symmetric equilibrium, if $F^N$ is the production function for the market, any firm will see its production function as the function $F^N$ but where the exogenous constant E is $E/M$ and M is the number of firms in the market. In this scenario, the number and size of each firm is irrelevant.) The applicability of the assumptions are justified in similar way as before:
Under the hypothesis of the model, given the CRS assumption of the way firms look at their production function, the market mechanism does not guarantee either that the number of unconnected markets will not change over time. Wherever a firm is going to be installed, it is going to think that she is able to derive the same return for the employed resources whatever size she chooses, once she observes the production function as CRS. As long as any introduction of a market leads to a rearrangement of resources such that all markets are maintained equally sized, such belief is going to be justified ex-post. If not, then it is possible that either the market closes - and this may happen if its size is smaller than average, in which case the firm ends up with negative profits - or it increases drawing resources from all others till the integration is complete. That is, the market has the possibility of rewarding and leading to its own integration for as long as we go away from the situation where new firms enter each market at the same rate and rearrangement of resource use is such that each market size stays equal to the size of any other. However, ex-ante, firms have the same profit prospects for whatever market they plan to enter.

Thus, the possibility that the rate of entry is equal for all markets may be plausible. Furthermore, if the symmetric equilibrium described above may look unstable (even if less unstable than the symmetric
equilibrium usually proposed in static models of imperfect competition, given our assumption of ignorance of the existence of IRS by the firms). In this case of economies scale internal only to the market, it can be seen as having only the meaning of describing the evolution of (using the trade example) closed areas that, for simplicity, are considered equal to each other. Because of lack of communication with each other, they do not benefit from the full extent of the externality.

Having modelled the production side of the market, we describe next the consumer's behavior.

Individuals are infinitely lived. Individual behavior follows the standard neoclassical hypotheses, maximizing accumulated discounted utility:

1) parents share equally with their children the amount of physical capital they possessed – the perceived rate of depreciation of the individual stock of human capital is therefore $\delta + \lambda$;

2) the perceived individual utility function is such that individuals take into consideration their offspring's well-being.

That is, we consider a world where the decision unit is the household, with a finite number of families and where each family values
equally all its members well-being at each point in time. For simplicity, we will talk about a typical individual's problem, the problem 'assigned', so to speak, by the family to each one of its members. Utility is discounted at rate \((r-\lambda)\).

The typical individual's problem is

\[
\text{Max } \int_0^\infty e^{-(r-\lambda)t} U(c) \, dt \\
\text{w.r.t. } c
\]  

subject to:

\[(3.8) \quad k = W + r_k k - c - (\delta + \lambda) k \]

\[c \geq 0\]

The Hamiltonian of the problem is

\[(3.9) \quad \hat{H} = U(c) + \Theta_j [ W + r_k k - c - (\delta + \lambda) k ]\]

From F.O.C. we can write:
(3.10) \[ \dot{k}/k = \left[ w - r_k K \right]/K - c/k - (\delta - \lambda) \]

(3.11) \[ \dot{\theta}_1 / \theta_1 = - \sigma c/c = \rho + \delta - r_k \]

Equilibrium steady state rates can be obtained by requiring equilibrium between supply and demand, replacing optimization conditions of the firm in these expressions, and following the procedure, used above, to find constant rates of change for the state and co-state variables.

Equilibrium between supply and demand of physical capital and labor requires:

(3.12) \[ K = K_N N \]

(3.13) \[ L_N = L/N \]

Optimization from the part of the firms will yield different conditions in the two cases:
Case A:

\[ (3.14) \quad W = (1 - \alpha) \frac{F_N}{L_N} \]

\[ (3.15) \quad r_k = \frac{\alpha F_N}{K_N} \]

Then,

\[ (3.16) \quad \frac{r_k}{W} = \frac{\alpha}{(1 - \alpha)} \]

Using the same type of notation it is easy to show that for the case where \( m = \lambda \), the equilibrium growth rates of the state and co-state variables will be the same as the ones found for the efficient solution.

Case B:

\[ (3.17) \quad W = \beta \frac{F_N}{L_N} \]
\[ (3.16) \quad r_K = (1 - \beta) \frac{F_N}{K_N} \]

Then,

\[ (3.19) \quad \frac{r_K k}{w} = \frac{(1 - \beta)}{\beta} \]

One can show that:

In the two cases, the growth rates will be the same:

\[ (3.20) \quad g^{eq} = \frac{(\alpha + \beta - 1) m}{(1 - \alpha)} \]

\[ (3.21) \quad \dot{\theta}_1 / \theta_1 = -\sigma \ g^{eq} \]

The wage rate grows at rate \( g^{eq} \). The rental price of physical capital will be fixed. The borrowing rate, the private rate of return, will be:
(3.22) \( r_k^{eq} - (\delta + \lambda) = (\rho - \lambda) + \sigma \gamma^{eq} \)

The social rate of return, the rate at which projects are in fact being discounted, will not, in general be the same as this, and will not be equal for both assets. The net social rate of return of an asset, \( r^S \), is equated to the ratio between the true marginal productivity of a resource and the relative social price of the resource plus the appreciation rate of this price minus the depreciation rate of the asset. That is, in an economy that trades in consumption units, for physical capital, the net rate is:

(3.23) \( r_k^S = F^C_k - (\delta + \lambda) \)

If \( F^C_k > r_k \), then the social rate is higher than the private rate. This happens for case B, where \( F_k / r_k = \beta / (1 - \omega) > 1 \). For the other case, A, the social rate equals the private rate.

**Proposition 2:** Under the assumptions underlined above, the equilibrium growth rates of per capita consumption in the
steady-state will be negatively correlated with the rate of increase of the number of firms (or markets) in the economy. They will be positively correlated with the degree of increasing returns of the technology. The equilibrium growth rates may not differ from the efficient ones (equality will occur if the number of firms or markets does not change over time). However, the proportion of consumption to the total stock of physical will not be efficient even if equality of socially optimal and equilibrium growth rates is observed.

The private and social rates of return of the asset in the economy will not be in general equal. The social rates will be higher when the resource they value is underp
IV. Welfare Analysis and Policy Implications.

The steady state solutions for efficiency and for the equilibrium cases differ in two ways: in terms of growth rates, and in terms of the levels of consumption to the physical capital intensity.

If the number of firms in the economy is fixed over time, i.e., \( m = \lambda \), and economies of scale are internal to the firm, the equilibrium growth rates will be the same as those for the efficient solution. However, this does not mean that equilibrium achieves maximum welfare: departing from given levels of the steady state variables, the efficient allocation would allow a higher accumulated discounted utility stream than the equilibrium ones. Because there are increasing returns to scale, to guarantee efficiency, the planner of this economy has not only to use appropriate fiscal incentives to make the optimal input mix to be chosen, but also make sure that production is concentrated in one plant, i.e., the number of firms in the market is one.

For the cases of economies external to the firm but internal to a market, the planner must guarantee complete integration.

Possible policies leading to the efficient solution for the two cases considered above will be described next.
All of the policies include the requirement that only one firm is left in the market; or a complete unified market. For now, we will assume that for the first scenario (economies of scale internal to the firm), the unique firm behaves competitively with respect to prices.  

Technically, (for each of the A, and B) we want to consider a structure of taxes and subsidies such that the replacement of the F.O.C. of the maximization problem of the firm in the individual's F.O.C. (2.12)-(2.13) reproduce the efficient conditions (3.10)-(3.11). We will consider taxes and subsidies applied to the firm.

Denote

\[ s_j^1 \] proportional subsidy on manhours employed.

\[ s_k^1 \] proportional subsidy on the amount of physical capital employed

\[ T_j \] lump-sum tax applied to the firm (total from all firms).

where \( j = A, B \) for each case respectively.

Then, the firm's problem is:
\[
(4.1) \quad \text{Max } F(K, L, E) = W L (1-s_j^I) - \eta_k K (1-s_k^I)
\]

w.r.t. \( K \) and \( L \).

For each case, \( E \) is as defined before. It is easy to show that for:

**Case A:**

\[
(4.2) \quad s_j^A = \frac{\alpha + \beta - 1}{\beta}
\]

\[
(4.3) \quad s_k^A = 0
\]

\[
(4.4) \quad T^A = s_j^A W L
\]

\( s_j^A \) is constant. \( T^A \) increases at rate \( [\lambda + g] \).

**Case B:**
\[ s_k^{B} = \frac{\alpha + \beta - 1}{\alpha} \]

\[ T^B = s_k^B K \]

\[ s_k^{B} \text{ is constant. } T^B \text{ increases at rate } [\lambda + g]. \]

(Note that these taxes and subsidies are defined for 'now and forever' and not only in the steady-state. As they are written, they will depend on the levels of different variables at each point in time.)

The subsidy rates, will be higher, the higher the degree of returns to scale. In every case, the fiscal policy leaves the firm with no economic profits, which allows us to ignore the market for the shares of the firm.

The last paragraphs of this section will be devoted to comment on possible modifications of the assumptions that can be accounted for in the framework above.)
The physical possibility that only one firm might be in place in the market may be unrealistic. One could assume that, given space limitations—that is, people could not concentrate enough to be able to go to work; or city dispersion necessarily imposes a formidable obstacle—that could not be less than, say, $N^*$ firms in the economy. Or the planner is restricted to the existence of $N^*$ markets in the economy. Then the efficient solutions found above will apply if we replace $L_C$ for $L_N = L / N^*$ and $K$ for $K_N = K / N^*$. The optimal policies would apply with the same substitution.

Also, as already noted, elements external to the model could be assumed to impose a bound on the growth rate of size. That is, if we assume, say that city size for hygienic reasons or due to the pace of transportation technology relative to population growth cannot grow faster than $\lambda$, then the efficient, possible, outcome will not be the one above. In particular, if $m$ cannot be bigger than, say, $m^*$, the results found in the previous sections would apply if we replace $\lambda$ by $m^*$ in the values found for the efficient solutions of $g$, etc. The optimal policies would be the same, only replacing $L$ for $L_N = L / N$ and $K$ for $K_N = K / N$. 
V. Product Differentiation, Economies of Scale, and Optimal Size Growth.

Section III illustrated the case of market failure when there are increasing returns to scale. It was found that the equilibrium growth rate or per capita consumption may be lower than optimal; in a steady-state, it will not exceed the efficient growth rate (equality occurring when \( N^* = 1 \) and \( m^* = \lambda \)). An obvious extension of the situations considered is to analyse the welfare implications of the existence of economies of scale external to the firm but internal to the market of a particular product in a setting were consumers exhibit taste for variety. It will be shown that the equilibrium rate of growth of the economy may then be higher or lower than optimal. Also, the higher the taste for variety, the lower the efficient growth rate.

We will trade-off a part of the potential growth the total use of increasing returns may offer (which happens when \( N=1 \)) for some diversity of the consumption basket; the higher variety is valued, the more we are willing to give up total growth for growth of the number of diversified goods.

Market failure in the determination of product diversity is an old issue. The problem has been widely studied in a static context, both in Chamberlinean and neo-Hotelling spatial models. The problem we consider
differs from those studies in two ways. First, we consider the case in which economies of scale are external to the firm. This allows us to abstract from the monopolistic competition behavior of the firms. Second, our model is dynamic, which raises the question of what is the optimal rate of increase of the number of markets and the size of each market.

Most models of product differentiation conclude that in the presence of increasing returns monopolistic competition leads to the appearance of 'excess capacity' or 'excess diversity' \(^9\). Dixit and Stiglitz (1977), assuming that individual's preferences exhibit a taste for variety conclude that the optimal number of firms in the industry need not be smaller than in equilibrium.

We focus here on a structure where there are economies of scale internal to the market of a particular product. We assume a utility function also with the number of products as argument. Unlike Dixit and Stiglitz, we propose an individual utility function of the form:

\[
U(c,N) = U(c) + V(N)
\]

where \(c\) is the total amount of consumption and \(N\) the number of different markets from which the consumer gets that total. For simplicity,
we assume that the utility function is such that the individual must consume equal amounts of each of the products he buys, otherwise he will decrease the number of products purchased (the individual has a taste for 'balanced' variety). Also, as in the previous Section, we treat $N$ as a continuous variable.

We consider:

$$U(c) = ac^{1-\sigma} \quad a = 1/(1-\sigma) \quad \sigma > 1$$

$$V(N) = bN^{1-\sigma_N} \quad b = 1/(1-\sigma_N) \quad \sigma_N > 1$$

$a$, $b$, $\sigma$ and $\sigma_N$ being constants. The assumptions on the size of the two parameters $\sigma_N$ and $\sigma$ guarantee that $N$ does not jump to $+\infty$. Notice that with the two assumptions, the two functions $U$ and $N$ are always negative and bounded above by 0, exhibiting positive (and decreasing) marginal magnitudes. When $N \to +\infty$, because of the existence of $T$, the total production of the economy (which is $N N_f = A K^{\alpha L^{(\beta - 1)} \cdot N^{(1-\alpha-\beta)}}$), and therefore $c$, goes to 0 - this implies that utility $U + V$ goes to $-\infty$, which cannot be optimal 10.

Thus,
\[(5.2) \quad \frac{U_{CC} \cdot c}{U_c} = \sigma \]
\[(5.3) \quad \frac{V_{NN} \cdot N}{U_N} = \sigma_N \]

where \(\sigma\) and \(\sigma_N\) are constant.

The choice of this utility function was not only suggested by mathematical convenience but also because it contains elements for the direct measure of the intensity of the taste for variety which may prove useful in other dynamic models.

Notice that the higher \(\sigma\), the higher will be the rate of decline of the marginal utility derived from the consumption of total output, \(U_c\), for given growth rate of consumption - such rate of decline is \(\sigma \cdot \dot{c}/c\). Analogous considerations can be made with respect to \(V_N, \sigma_N\) and \(\dot{N}/N\). Therefore, the higher the parameter \(\sigma\) relative to \(\sigma_N\), the more we expect a dynamic economy to penalize growth of total output relative to increase in product differentiation. We obtain a result consistent with these arguments.
The technology available is the one described in Section III. There are economies of scale external to each firm but internal to the market of a particular product. Each market produces its own physical capital, and they all use a similar technology (This assumption allows us to abstract from possible complications of the scenario due to the existence of a single sector producing the investment good. Instead, this assumption allows us to look at the markets as identical and use the state equation associated with physical capital in the same manner as in the previous Sections.).

Given the structure of the utility function, it is clear that a symmetric equilibrium in terms of the size of the different markets can occur. Moreover, any symmetric equilibrium will be possible: the individuals, solving the same problem, will buy equal amounts of each of the products, and the prices of all the products will be the same. In a setting like this, the equilibrium number and rate of increase of the number of markets is undetermined. That is, all our conclusions found in Section III apply, for each of the two cases of externalities, A and B.

The efficient solution for the problem will not give, as before, the optimality of the existence of a unique market. We will derive the optimal steady-state growth rate of the size of each market.
As in Section III:

\[ L_N = L / N \]

where \( K \) and \( L \) represent respectively the amount of total physical capital and total manhours in the economy assigned; \( N \) is the number of products or markets in the economy. It will be the case that:

\[ K = K/L \]

We denote also:

\[ N/N = n, \quad 0 \leq N/N \leq \lambda \]

\[ L_N / L_N = (L/N) / (L/N) = m = \lambda - N/N, \]

The planner's problem of this economy is, therefore:
Max \( \int_0^\infty e^{-(\rho - \lambda) t} \left[ U(c) + V(N) \right] \, dt \)

w.r.t. c, N

s.t. (5.8) \( \dot{k} = NFN(k/N, L/N) / L - c - (\delta + \lambda) k \)

(5.9) \( \dot{L} = \lambda L \)

\( c \geq 0 \)

given \( k(0) \), and \( L(0) \).

Assume the production function is of Generalized Cobb-Douglas form expressed by (2.5).

The Hamiltonian of the problem becomes:

(5.10) \( \mathcal{H} = U(c) + V(N) + \)

\[ + \Theta_1 \left[ A K^\alpha L^{\beta-1} N^{1-\alpha-\beta} - c - (\delta + \lambda) k \right] + \Theta_2 \lambda L \]

Considering the special features of the Cobb-Douglas technology and simplifying notation, the F.O.C. for optimization of the problem include:
(5.11) \[ \dot{R}_C = \dot{V}_C - \dot{\Theta}_1 = 0 \]

(5.12) \[ \dot{R}_N = \dot{V}_N - \dot{\Theta}_1 (\alpha + \beta - 1) F^N / L = 0 \]

(5.13) \[ \dot{\Theta}_1 = \Theta_1 (\rho - \lambda) - \dot{R}_K = \Theta_1 (\rho - \delta) - \Theta_1 \alpha F^N / (K/N) \]

From (5.11), differentiating with respect to time and dividing by \( \Theta_1 \), we can reproduce condition (2.11):

\[ \frac{\dot{\Theta}_1}{\Theta_1} = -\sigma \frac{\dot{c}}{c} \]

Recalling the definitions of the capital intensities

(5.15) \[ K = K / L = K^N / L_N = (K/N) / (L/N) \]

To derive the balanced growth rates of the problem, we follow, again, the method of Section II. Defining the growth rates of the state and
co-state variables, ignoring the growth rate of \( \dot{\theta}_2 / \theta_2 \), they will be equal to:

\[
(5.16) \quad \dot{k}/k = N F^N/(k L) - c/k - (\delta + \lambda)
\]

\[
(5.17) \quad \dot{\theta}_1 / \theta_1 = -\sigma c/c = c + \delta - \alpha F^N/(K^N)
\]

Denoting \( g \) as the rate of growth of per capita consumption, constancy of the balance growth rates imply, as in Section III, that

\[
(5.18) \quad g = \dot{k}/k = \dot{K}/K - \lambda = \dot{K}^N/K^N - m
\]

and also that \( g, F^N / K^N \) are constant. Well,

\[
(5.19) \quad F^N / K^N = A K^N (\alpha - 1) (L/N)^\theta
\]

Differentiating the right hand-side with respect to time, and considering the above:
\[(5.20) \quad -(1 - \alpha)(g + m) + \beta m = 0\]

The problem has the same structure as the one described in section 31.5 and the growth rate is related to \(n\) in the same way. From (5.20),

\[(5.21) \quad g = \frac{(\alpha + \beta - 1) m}{(1 - \alpha)}\]

The next step is to derive the optimal growth rate of the size of each market. This value, \(m^*\), can be replaced in the expressions above to obtain the steady-state optimal growth rates. From (5.11) and (5.12):

\[(5.22) \quad V_N / U_c = (\alpha + \beta - 1) F^{N/L}\]

Multiplying both sides by \(N/k\):

\[(5.23) \quad V_N N / (U_c k) = (\alpha + \beta - 1) N F^{N/(k L)}\]
The right-hand side is constant in the steady-state. Therefore \( V_N N \)

\((U_{ck})\) will also be constant, which implies:

\[
(5.24) \quad \frac{V_{NN}^N}{V_N} (\lambda - m) + \sigma g + (\lambda - m) - g = 0
\]

Manipulating:

\[
(5.25) \quad g (\sigma - 1) = (\sigma - 1) (\lambda - m)
\]

Using also (5.21), we derive:

\[
(5.26) \quad m^* = \frac{(1 - \alpha) \lambda}{(\alpha + \beta - 1) (\sigma - 1)/(\sigma - 1)}
\]

The optimal number of markets will increase at rate \( n^* = \lambda - m^* \):

\[
(5.27) \quad n^* = \frac{(\alpha + \beta - 1) \lambda}{(\alpha + \beta - 1) + (1 - \alpha) (\sigma - 1)/(\sigma - 1)}
\]
we see that

\[(5.28) \quad 0 < m^*, n^* < \lambda\]

This means that the number of firms in the market will not be fixed, nor will it increase to the bound where each new individual creates a new market. Also

\[(5.29) \quad \frac{\partial m^*}{\partial \sigma} > 0 \quad \text{and} \quad \frac{\partial m^*}{\partial \sigma} < 0\]

\[(5.30) \quad \frac{\partial n^*}{\partial \sigma_N} < 0 \quad \text{and} \quad \frac{\partial m^*}{\partial \sigma_N} > 0\]

(Recall that $\sigma, \sigma_N > 1$).

or more specifically:

\[(5.31) \quad \frac{\partial m^*}{\partial [\sigma_N^{-1}/(\sigma-1)]} > 0 \quad \text{and} \quad \frac{\partial m^*}{\partial [\sigma_N^{-1}/(\sigma-1)]} < 0\]

\[(5.32) \quad \frac{\partial n^*}{\partial [\sigma_N^{-1}/(\sigma-1)]} < 0\]

Given the structure of the utility function chosen, $[(\sigma_N^{-1})(\sigma-1)]$
behaves as a measure of the intensity of preferences for total output relative to diversified consumption (recall that σ and σN were related to the rate of change of the marginal utilities of total consumption and diversity respectively).

The optimal growth of market size will be proportional to the rate of population growth. To the extent that taste for variety exists, the proportionality coefficient will be less than one: we will not use to the full extent the potential market increase to generate growth of total (per capita) consumption.

Replacing m* in the expression (5.21) we can derive the expression of the optimal growth rates of per capita consumption. Given that any m can be achieved for any of the three equilibria considered, we conclude that the equilibrium growth rate may be higher or lower than the optimal, g*.

\[ (5.33) \quad g^* = \frac{(\alpha + \beta - 1) \lambda}{(\alpha + \beta - 1)(\sigma - 1)/(\sigma_N - 1) + (1 - \alpha)} \]

The optimal growth rate of the economy will be lower than those applying to a situation where there is no taste for variety. g approximates
the value found in Section II if \( \sigma_N \to +\infty \), in which case \( m^* \to \lambda \) (there is no growth in variety). If \( \sigma \to +\infty \), then \( m^* \) and \( g \to 0 \); all the growth possibilities offered by the existence of IR are used in increasing variety of the consumer's basket.

**Proposition 3:** The higher the relative preference for total output relative to diversity, the lower the optimal growth rate of the number of differentiated markets and the higher the optimal growth rate of the size of each market.

Even if the problem raised here concerning the consequences of taste for variety may exist, it may be difficult to recognize empirically. The problem was, nevertheless, stated as an example of how higher than optimal growth may be a possibility for an economy. The issue is more than academic: assume taste for variety is related to a preference towards a more disperse urban development and there are increasing returns, associated with some externalities as seen above, in the way a city may generate income \(^{11}\). Then the subject treated here can deal with some of the reasons and consequences of our cities growing at a higher than optimal
Moreover, the so much debated productivity slowdown \(^1\(^2\), may as well be a reflex of approaching a better equilibrium solution rather than a tragic accident: more and faster variety would necessarily lead to a slowdown in observed growth rates of per capita consumption and could mean a welfare improvement.
VI. Summary and Conclusions.

A one-sector model of economic growth was presented. Three main subjects were addressed:

1. Existence of increasing returns to Scale as a source of economic growth.

2. Possibility of divergence between equilibrium and efficient solutions due to economies of scale external to the firm but internal to particular markets.

3. Consequences of the introduction of taste for variety by consumers.

The main conclusions of the paper can be summarized in the following way:

1. In the presence of Increasing Returns to Scale, endogenous growth can be generated in the economy. The optimal rate of growth of per capita consumption is proportional to the rate of population growth; we can only take dynamic advantage of increasing returns if the size of the market can actually increase.

2. If economies of scale are external to a firm but internal to a
particular market or area, any symmetric equilibrium is possible. In this case, the steady-state growth rates of the economy are undetermined. Nevertheless, one concludes that observed growth rates will be positively related to the rate of increase of market size, not necessarily with the rate of population growth.

Optimal policies to achieve the efficient solution are considered. Given the nature of the problem, taxes and subsidies are not enough; total market integration must also be ensured.

3. In a setting where there are economies of scale internal to the market of each product (an industry) and the individual's utility function exhibits taste for variety, equilibrium growth may be higher or smaller than optimal. The efficient growth rate of diversity in the economy, i.e., of the number of differentiated products, will be higher - and the efficient growth rate of total output will be lower - , the higher the preference for diversification relative to total output. Both rates will be higher, the higher the population growth rate of the economy.
Footnotes

1 See Dixit and Stiglitz (1978). Also, see Lancaster (1986) and Ohta (1977) for a survey of the subject.

2 See Romer (1987a) for a discussion of the subject and relevant literature.

3 \((\rho-\lambda)\) is sometimes called the social rate of discounting. See Cass (1965) for a derivation of this result.

4 See Simon (1986) and literature there cited for a survey.

5 Notice that goods are purchased by consumers - only if the consumers decide to import can the firms observe the foreign goods. Given that areas produce the same good, in the plausible case of a symmetric equilibrium there is no reason why consumers of one area should try to buy from any other (especially if we think that there may be transportation costs which, for simplicity we ignore).

   Described in this way, we could ask ourselves why would not the firms, by themselves, try to observe the goods produced in other areas. Firstly, recall that the situation which we address is characterized (as in Romer's equilibrium) by a misperception with respect to the existence of the economies of scale associated with the externality. Even if we could think that firms ignorance of the effect is not complete, the scenario would apply as an extreme example.

   Secondly, one can think that inspection of only one good of another area by the firm is not enough to benefit from the whole 'external' knowledge of that area - the size effect is crucial in our model - and only when a sizeable proportion of the market is flooded with the foreign good can that happen. Barriers to trade or communication of various kinds could prevent it.

   Finally, we could think that the externality arises from the fact that individuals' human capital becomes more productive when the workers, as consumers, are subject to foreign goods.
Lucas cites Anne Krueger and Arnold C. Harberger's surveys of growth experiences of poor countries, where the authors identify inefficient barriers to trade as a limitation on growth, and their removal as a key explanation of several growth episodes. Lucas points out that removal of trade barriers is on neoclassical grounds a level effect, not a growth effect and therefore he finds the facts collected by the two authors paradoxical. Our framework provides the means to understand the evidence found and to distinguish the two effects that the lifting of trade barriers may have. Moreover, given the 'knife-edge' nature of the endogenous growth mechanics of Lucas model, (noted above and in Martins (1987)), we conclude that Lucas' human capital externality is more appropriate to explain level than growth effects in the neoclassical point of view.

It would be possible to assume that such firm, being a monopolist-monopsonist position, would take advantage of it. The firm could be assumed to maximize accumulated discounted profits with respect to K, L, and r; subject to the F.O.C. of the typical individual problem. The reason why we do not pursue this argument and rather assume a competitive behavior of the firm is because the problem can become mathematically cumbersome. See Romer (1987b) for the discussion of a model with monopolistic competition behavior of firms.

See, for example, Lancaster (1986) for a survey.


See Dixit, Mirrlees and Stern (1975) for a discussion of a similar problem.

Lucas cites Jane Jacobs (1970) and makes an analogy between the effect of his externality and her description of the advantages of population concentration in urban centers. Again, it seems that our interpretation and the dependence of our result upon the actual size of the externality would seem even more appropriate for the comparison, once it allows for city growth to play a direct role in the development of nations.

See footnote 2.
Appendix 1. Growth Dynamics:

The Path to the Steady-State.

This appendix is devoted to explain the dynamics of the system and the path to the steady-state. Let us depart from the two equations (2.12) and (2.13). Replacing the production function we have, thus,

(A.1) \( \dot{k}/k = AK^{\alpha}L^{\beta} / (Lk) - c/k - (\delta + \lambda) = AK^{(\alpha-1)}L^{\beta} - c/k - (\delta + \lambda) \)

(A.2) \( \tilde{\Theta}_f / \Theta_f = -\sigma \dot{c}/c = \rho + \delta - \alpha AK^{(\alpha-1)}L^{\beta} \)

Constancy of \( g \) implies constancy of \( K^{(\alpha-1)}L^{\beta} \). Given (2.17), we can therefore write for the steady-state:

(A.3) \( (K^{(\alpha-1)}L^{\beta})_t = \frac{\rho + \delta + \sigma \dot{c}/c}{\alpha A} = \frac{(\rho + \delta)(1 - \alpha) + (\alpha + \beta - 1)}{\alpha A(1 - \alpha)} \)

Thus, constancy of \( \dot{k}/k \) in the steady-state and of the above, implies from (A.1) that \( c/k \) will be constant in the steady-state and equal to:
\[(A-4) \quad \frac{c}{k} = \frac{(\rho - \delta)(1 - \alpha) + (\alpha + \beta - 1)}{\alpha (1 - \omega)} - (\delta + \lambda) \]

Notice also that

\[(A-5) \quad \frac{\dot{c}}{c} - \frac{\dot{k}}{k} = \alpha \frac{A^{(\alpha - 1)} L^{\beta} - (\rho - \delta)}{A^{(\alpha - 1)} L^{\beta} + \gamma} - \frac{A^{(\alpha - 1)} L^{\beta}}{A^{(\alpha - 1)} L^{\beta} + \gamma} \frac{c}{k} - (\delta + \lambda) \]

We can therefore interpret the dynamics of the system in a phase diagram in space \((K^{(\alpha - 1)} L^{\beta}, c/k)\). (One can easily show stability of the equilibrium in these two variables, implying a saddle-path for the dynamics of the problem.)

If \(K^{(\alpha - 1)} L^{\beta} > (K^{(\alpha - 1)} L^{\beta})^*\), then, from (A.2) we see that \(\dot{\theta}_1 / \theta_1\) must be growing more slowly than in the steady-state and \(c/k\) must be higher than the steady-state growth rate \(g\).

Also, in the saddle-path, if \(c/k > (c/k)^*\), then \(c/k\) must be decreasing, that is \(\dot{c}/c - \dot{k}/k < 0\). From (A.5), we conclude that if \(K^{(\alpha - 1)} L^{\beta} > (K^{(\alpha - 1)} L^{\beta})^*\), then the first term of \(\dot{c}/c - \dot{k}/k\) is smaller than in the steady-state. If \(c/k < (c/k)^*\) the whole expression would be negative and \(c/k\) would be decreasing \(\dot{c}/c - \dot{k}/k < 0\); therefore, we must
have a saddle-path for which if $K^{(x-1)} L^\beta > (K^{(x-1)} L^\beta)^*$, then $c/k > (c/k)^*$ and decreasing.

Similarly, if $K^{(x-1)} L^\beta < (K^{(x-1)} L^\beta)^*$, then $c/k < (c/k)^*$ and increasing.

Both movements are depicted in Fig. 1.
Appendix 2. Second Order Conditions

Following Uzawa (1964), let us introduce auxiliary variable (Lagrange multipliers) \( q_1(t) \) and \( q_2(t) \) corresponding to the restraints (2.3) and (2.4). If we can find auxiliary variables \( q_1(t) \) and \( q_2(t) \) for which \( c \), maximizing the unconstrained problem:

\[
\int_0^\infty (U(c) - q_1(t) [k - F/(Lk) - c - (\delta + \lambda) k] + q_2(t) [L - \lambda L]) e^{-\rho t} \, dt
\]

satisfy the feasibility conditions (2.6)-(2.10), then the path of \( c \) found is an optimal path. The expression above, given the assumptions on \( F \) and \( U \), is concave in \( c \). Then our optimum problem is reduced to maximization of the unconstrained problem above for a given set of auxiliary variables. And this is a concave problem in the calculus of variations.
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