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ABSTRACT

We consider the procurement setting in which a sponsor and a contractor are concerned with the construction of a long term project where the project requires a number of tasks to be completed before the benefit from the project can be realized. The contractor has private, task specific information which is relevant in cost determination. We focus on the case where the sponsor can precommit to compensation per task and where the contractor knows precisely the cost of each task right from the beginning of construction of the project. In this setting we construct the optimal "direct contract" and show that, for each particular firm, the optimal payment consists of two parts: the true cost of the project plus the contractor's reservation level of profit and a premium term which increases with the maximum number of high cost tasks that the sponsor is willing to allow. The sponsor resolves the tradeoff between allocative efficiency and increased ability to extract contractor's rents (which costitute the optimal contract's premium term) on the basis of her own beliefs. Furthermore, we show that the solution to the sponsor's problem is one of underemployment of the existing resources.

1. INTRODUCTION

In this paper we model the relationship between a contractor and a sponsor in the construction of a long term project requiring a number of tasks to be completed before the benefit accruing to the sponsor can be realized. We follow the optimal regulation literature, Baron and Myerson (1982), Baron and Besanko (1984) and (1987), Laffont and Tirole (1986), *et al.*, in assuming that the contract problem contains elements of adverse selection. We also follow the optimal regulation literature in assuming that the sponsor's benefit from completion of the project is publicly known. Other recent papers dealing exclusively with procurement, *e.g.*, Riordan (1986) and Tirole (1986) have focused on the contracting problem under bilateral asymmetric information, where the sponsor's valuation as well as the contractor's cost is uncertain. Because we are not interested in the mechanism design issues *per se*, we take a simpler route in the development of the model by focusing exclusively on cost uncertainty.

Section 2 presents the basic set-up. Some tasks are low cost: development and construction can proceed smoothly at low cost. Other tasks are high cost: there are inherent design and/or operation problems that cause the expected costs of completion to be higher. The contractor has private, task specific information which is relevant in cost determination. In section 3 we analyze the case where the project requires a finite number of tasks, $N>1$, to be completed and assume that the contractor knows the cost of each task right from the begining of construction of the project. As in the dynamic labor contracts models, we also assume that the sponsor has total precommitment power while the contractor is not bound to complete the project and can drop out at any time prior to project completion. We invoke the revelation principle, see Myerson (1979) and Harris and Townsend (1981), to construct the optimal "direct contract". In such a contract, the sponsor induces the the contractor to report the true cost of each task.

In section 4 the general form and basic properties of the optimal payment scheme are examined. The optimal contract can be characterized quite simply. Let k^* be the maximum number of high cost tasks that the sponsor will allow. Then the contractor breaks even when the number of high cost tasks is k^* and earns some rent when the number of high cost tasks is less than k^*, where the amount of rent is decreasing in the number of high cost tasks. This rent generation for the contractor is necessary although the sponsor acts as a monopsonist, because the optimal contract must satisfy self-selection constraints. The structure of the optimal contract is formally demostrated in propositions 1 and 2. The tradeoff between efficiency and increased ability to extract contractor's rents is resolved on the basis of the sponsor's own beliefs regarding the difficulty of tasks. Proposition 3 indicates that the solution to this procurement problem is one of underemployment. Section 5 provides a brief conclusion.

2. THE BASIC MODEL

The contractor's technology is similar to the one presented in Leite (1989)[1]. We consider the relationship between a sponsor and a contractor over the construction of a long term project, which requires $N>1$ tasks for completion. For simplicity assume that each task lasts 1 period. Let i be an index which refers both to the number of tasks and the number of periods remaining till completion of the project, $i=1,\ldots,N$. Let B denote the benefit accruing to the sponsor in the event the project is completed. It is assumed that the benefit to the sponsor is zero in the event that the project is not completed.

Prior to the beginning of construction of the project, nature picks a random N-vector, $\tilde{T}$. Each realization of $\tilde{T}$, T, is known by the contractor but not by the sponsor right at the commencement of construction of the project. Let $\tilde{t}_i$ denote the ith component of $\tilde{T}$ and t_i the ith component of T. $t_i \in \{ \underline{c}, \bar{c} \}$, $\underline{c}<\bar{c}$, where $t_i=\underline{c}$ means that the ith task is low cost while $t_i=\bar{c}$ means that the ith task is high cost. Let $\Pr\{ \tilde{t}_i=\bar{c}\}=\theta$. This probability is taken to represent the subjective beliefs of the sponsor concerning the difficulty of task i. θ is taken to be common knowledge to both contractor and sponsor.

The contractor's decision to complete each task is based on π, $\pi\geq 0$, the per period opportunity profit level the contractor can experience elsewhere, and the rents the contractor expects to earn from continuing to participate in the project. The game ends if the contractor drops out. If the contractor has decided to complete task i the contractor then sends the sponsor a perfectly informative signal about the cost of task i, s_i.[2] $c^a(t_i,s_i)$ denotes the actual cost involved in completing task i. The actual cost function is given by:

$$c^a(\underline{c},\underline{c})=\underline{c}, \qquad c^a(\underline{c},\bar{c})=\underline{c}+a,$$
$$c^a(\bar{c},\underline{c})>\bar{c}, \quad \text{and} \qquad c^a(\bar{c},\bar{c})=\bar{c}. \tag{1}$$

This formulation indicates that there is a cost of misrepresenting oneself as a high cost type when one is a low cost type as well as when a high cost type indicates to be a low cost type. In the first case, such cost is given by a constant, a, such that $0<a<\bar{c}-\underline{c}$.

3. THE PRECOMMITMENT N-PERIOD MODEL

3.1. THE PROBLEM

We assume that prior to starting construction of the project, the sponsor offers a contract whose terms will not be changed. In this precommitment case, the contractor may not agree to undertake construction of the project, in which case the project is cancelled. That is, we do not allow for the possibility of contract renegotiation. In case the contractor agrees to undertake construction of the project, the contractor is required to follow the following accounting procedure. At the time task i is completed the contractor sends the sponsor a signal, s_i, indicating whether task i is a low cost task or a high cost task. When the entire project is completed, the history of signals is an

N-vector with an element corresponding to each period of construction. Let $\Gamma=\{\underline{c},\bar{c}\}^N$ be the set of all possible signal histories for a given project.

We assume that both the sponsor and contractor are expected profit maximizers. Since we assume no discounting, the contractor is indifferent to receiving a payment, p_i, immediately upon completion of task i, or receiving p_i upon completion of the entire project. The sponsor is also indifferent to paying p_i at either date, from consideration of the cost of such a payment. However, the sponsor should find that paying p_i upon project completion weakly dominates paying p_i immediately upon completion of task i, because of the incentive effects this has on the contractor. This is so because we assume that the contractor is not bound by the terms of the contract and may drop out of the project at any time. Contractor participation into period i-1 upon completion of task i requires that the value of participating in period i-1, appropriately defined, net of the cost in period i-1, exceeds the contractor's opportunity cost. Note that in determining whether to participate in period i-1, the cost history through period i is irrelevant to the contractor, because these costs are sunk. The payment history through period i is also irrelevant, for the same reason. However, payments for cost incurred through period i deferred until after period i are relevant in the participation decision, because such payments contribute to the value of participating in period i-1, i.e., such payments do not accrue to the contractor if the contractor drops out. Thus, there is no loss in assuming that all payments are made upon project completion.

Before proceeding further, we introduce some more notation. Let $c^a(t_i)=c^a(t_i,s_i)$ be the actual cost when the contractor truthfully signals $s_i=t_i$. $c^a(t_i,t_i')$ denotes the actual cost when the contractor signals $s_i=t_i'\neq t_i$. The pattern of tasks faced by the contractor in completing the project is given by an N-vector, say T. C(T) represents the actual cost of building the entire project when the contractor truthfully signals the pattern of tasks T. Let C(T,T') be the actual cost of building the entire project when the contractor faces a pattern of tasks given by T and signals some N-vector T', $T'\in\Gamma$, $T'\neq T$ or $T'=T$. Let $P:\Gamma\to\mathbb{R}_+$ denote the compensation function or contract relative to the entire project. That is, P(T) is the payment received by the contractor upon completion when the signalled cost history is T.

A contractor who has decided to accept the contract chooses the signalled cost history to maximize profit. For a contractor with actual pattern of tasks T^a, this choice problem is given by:

$$\underset{T\in\Gamma}{\text{maximize}}\ P(T)-C(T^a,T). \tag{2}$$

We shall say that P induces truthful reporting for a contractor facing a pattern of tasks given by T^a when

$$P(T^a)-C(T^a,T^a)\geq P(T)-C(T^a,T),\ \forall\ T\in\Gamma. \tag{3}$$

When (3) holds for all $T^a\in\Gamma$ we say that P induces truthful reporting.

Let $\Pi\geq 0$ denote the opportunity cost for the contractor over the lifetime of the project. Suppose the compensation scheme, P, induces truthful reporting. When the contractor's actual pattern of costs is T, the contractor will accept the contract P only if $P(T)-C(T)\geq\Pi$. Let $Q(T,P):\Gamma\rightarrow\{0,1\}$ be defined by

$$Q(T,P)=\begin{cases}1 \text{ if } P(T)-C(T)\geq\Pi \\ 0 \text{ if } P(T)-C(T)<\Pi.\end{cases} \tag{4}$$

Q(T,P) can be interpreted as the probability that a contractor with actual pattern of tasks equal to T accepts the contract P. Note that under this interpretation it is required that the probability of contract acceptance equals one when the contractor is indifferent to accepting or not.

Let the sponsor's beliefs about the actual distribution of tasks be represented by a probability measure Φ over Γ. The problem the sponsor solves in determining the optimal contract is:

$$\underset{P\geq 0}{\text{maximize}} \sum_{T\in\Gamma} \Phi(T)\ Q(T,P)\ [B-P(T)] \tag{5}$$

subject to

(i) P satisfies (3) for all T^a, and

(ii) Q(T,P) is defined by (4).

To avoid the possibility that the project is never undertaken we make the following assumption:

Assumption 1: $B-C(\underline{T})>\Pi$ where $\underline{T}$ is a vector of elements $\underline{t}_i=\underline{c}$ for all $i=1,\ldots,N$.

3.2. EXISTENCE

In order to show that there is a solution to problem (5), we consider the related problem given by:

$$\underset{P,Q}{\text{maximize}} \sum_{T\in\Gamma} \phi(T)\ Q(T,P)\ [B-p(T)] \tag{6a}$$

subject to:

$$Q(T,P)[P(T)-C(T)-\Pi]\geq 0,\ \forall\ T\in\Gamma \tag{6b}$$

$$[1-Q(T,P)][P(T)-C(T)-\Pi]\leq 0,\ \forall\ T\in\Gamma \tag{6c}$$

$$P(T)-C(T)\geq P(T')-C(T,T'),\ \forall\ T,T'\in\Gamma \tag{6d}$$

$$P(T)\geq 0,\ \forall\ T\in\Gamma,\ \text{and} \tag{6e}$$

$$0\leq Q(T,P)\leq 1,\ \forall\ T\in\Gamma, \tag{6f}$$

where the functional dependence on P and Q now appears at (6b) and (6c).

In this problem the sponsor chooses P and Q such that the expected value of the sponsor's net benefit is maximized and the resulting payment scheme guarantees self-selection. Condition (6b) establishes that $Q(T,P)\geq 0$ if $P(T)\geq C(T)+\Pi$ and, together with (6f), that $Q(T,P)=0$ if $P(T)<C(T)+\Pi$. Condition (6c) establishes that $Q(T,P)\leq 1$ if $P(T)\geq C(T)+\Pi$ and, together with (6f), that $Q(T,P)=1$ if $P(T)>C(T)+\Pi$. Condition (6d) indicates that the payment scheme part of a solution is required to induce self-selection. We can now state the following result:

LEMMA 1: If (P,Q) solves (6) and $Q(T,P)>0$ then $B-P(T)\geq 0$.

Let (P,Q) denote a solution to (6). That a solution to (6) exists is straightforward since the objective function is continuous and the constraint set is non empty and compact (P is bounded from above).[3] To show that a solution to (5) exists and to see why it is sufficient to consider (6) as the sponsor's problem, let (P,Q) solve (6). Let $P^*(T)=\min(P(T),B)$ and let $Q^*(T,P^*)=1$ if $P^*(T)\geq C(T)+\Pi$, $Q^*(T,P^*)=0$ otherwise. Then (P^*,Q^*) also solves (6), this is shown in the proof of lemma 1, and evidently P^* solves (5).

3.3. A SIMPLIFIED FORMULATION

Leite (1988) shows that in the family of optimal solutions to problem (5) one is symmetric, in which case, if P is the solution to (5) and if for any T and T' we have $C(T)=C(T')$ then $P(T)=P(T')$. Let $p(k)$ represent the payment to the contractor when the contractor signals a pattern of tasks with k high cost tasks. This result implies that the sponsor does not have to pay each contractor based on each contractor's pattern of tasks, T, but only base the payment on the contractor's reported number of high cost tasks and make the payments, $p(.)$, accordingly. This constitutes a simplification of the original problem since it means that we can model the relationship between sponsor and contractor as one in which the contractor does not have to report whether each task is low cost or high cost but simply the total number of high cost tasks faced in order to complete the project. Therefore, the only signal sent to the sponsor is the number of high cost tasks in the project and the payment received by the contractor upon project completion will be a function of this signal alone.

The number of high cost tasks in any vector $T \in \Gamma$ is unknown to the sponsor. Let the sponsor's beliefs about k be represented by the probability mass function, R. R is generated by the assumption that whether task i is high cost is independent of whether task j is high cost, for all $i,j=1,\ldots,N$, $i \neq j$. Since θ denotes the probability that an individual task is high cost, k is a binomial random variable with parameters (N,θ). That is, R is given by

$$R(k)=\binom{N}{k}\ \theta^k(1-\theta)^{N-k} \quad \text{for } k=0,1,\ldots,N. \tag{7}$$

Let h be the number of high cost tasks that the contractor reports to face in order to complete the project, k being the actual number of high cost tasks in the project. When h=k we say that there is truthful revelation of costs; when $h \neq k$ there is misrepresentation. The contractor's actual cost function for the entire project, ψ, can be written as:

$$\psi(k,h)=k\bar{c}+(N-k)\underline{c}+(h-k)a \text{ for } h \geq k \tag{8a}$$

$$\psi(k,h)>k\bar{c}+(N-k)\underline{c} \text{ for } h<k. \tag{8b}$$

In order to ensure that the contractor will always reveal her true costs, the sponsor has to set a payment scheme which is incentive compatible.

Definition 1: A payment scheme is incentive compatible if it is such that a contractor facing k high cost tasks will never report to face h high cost tasks, $h \neq k$, i.e., the payments will be such that $p(k)-\psi(k,k) \geq p(h)-\psi(k,h)$, $\forall$ k, $\forall$ $h \neq k$; $h,k \in \{0,...,N\}$.

Notice that as long as p(.) is nondecereasing in h, $p(.)-\psi(k,h)$ is increasing in h for $h<k$. This implies that the contractor has no incentive to underreport the number of high cost tasks faced in order to complete the project.

Definition 1': A payment scheme is incentive compatible if it is nondecreasing in h and is such that $p(k)-\psi(k,k) \geq p(h)-\psi(k,h)$, $\forall k, \forall h>k$ with $h,k \in \{0,...,N\}$.

Definition 2: A payment scheme is locally incentive compatible if it is such that a contractor facing k high cost tasks will never report to face k+1 high cost tasks, i.e., the payments are such that $p(k)-\psi(k,k) \geq p(k+1)-\psi(k,k+1)$, $\forall k \in \{0,...,N-1\}$.

The sponsor sets a payment scheme which maximizes the expected value of her net benefits from the project, V, and is incentive compatible. The next result shows that it is sufficient to guarantee that the payment scheme chosen is locally incentive compatible in order to rule out misrepresentation of costs.

LEMMA 2: If a payment scheme, p(.), is locally incentive compatible and nondecreasing in k, then it is also incentive compatible.

This lemma indicates that the sponsor's problem can be reduced to setting a payment scheme that maximizes the sponsor's expected net benefit and is locally incentive compatible. This problem is formally set up in the next section.

4. OPTIMAL INCENTIVE COMPATIBLE CONTRACTS

The sponsor will choose the payment scheme that maximizes the expected value of the sponsor's net benefit from the project and induces the contractor to truthfully report the costs of building the project. Lemma 2 allows us to formally write the sponsor's problem as:

$$\underset{p(k)}{\text{maximize}} \quad \sum_{k=0}^{N} q(k)R(k)[B-p(k)] \tag{9a}$$

subject to:

$$p(k) \geq p(k+1)-a, \forall\ k \in \{0,\ldots,N-1\} \tag{9b}$$

$$q(k)=\begin{cases} 1 & \text{if } p(k)-\psi(k,k) \geq \Pi \\ 0 & \text{otherwise} \end{cases} \tag{9c}$$

$$p(k)>0,\ \forall\ k \in \{0,\ldots,N-1\}, \tag{9d}$$

where $R(k)$ is given by (7) and $q(k)$ represents the probability that the contractor is willing to produce given that there are k high cost tasks. Condition (9d) simply imposes that payments are always strictly positive.

In order to rule out the trivial case where project termination is optimal regardless of the value of $p(.)$, we require the following:

Assumption 2: $B > N\underline{c} + \Pi$.

The (modified) Lagrangean for problem (9) is:

$$L = \sum_{k=0}^{N} q(k)R(k)[B - p(k)] + \sum_{k=0}^{N-1} \mu_k [p(k+1) - p(k) - a] \qquad (10)$$

A vector $\underline{p}$ and $\underline{\mu}$ satisfy the Kuhn-Tucker conditions for the above problem if:

$$p(k) > 0, \ (\partial L^* / \partial p(k)) = 0, \ k = 0, \ldots, N, \qquad (11a)$$

$$\mu_k \geq 0, \ (\partial L^* / \partial \mu_k) \geq 0 \text{ and } \mu_k (\partial L^* / \partial \mu_k) = 0, \ k = 0, \ldots, N-1. \qquad (11b)$$

Let k^* be the maximum number of high cost tasks that the sponsor will optimally allow given the values of the parameters of the model. The following result can now be stated:

PROPOSITION 1:

The optimal payment scheme is one such that

$$p(k^*) = \psi(k^*, k^*) + \Pi \text{ and } q(k) = 1 \text{ for } k \leq k^*.$$

This result simply means that the optimal payment scheme is one such that the contractor breaks even when facing k^* high cost tasks and is always willing to produce if $k < k^*$. The next proposition indicates what the optimal payment is when $k < k^*$.

PROPOSITION 2:

The optimal payment is one such that

$$p(k) = p(k^*) - (k^* - k)a, \ \forall \ k < k^*, \ \forall \ k^* \in \{0, 1, \ldots, N\}.$$

Proposition 3 establishes that the solution to the sponsor's problem is one of underemployment.

PROPOSITION 3:

$$k^* \text{ is such that } B>\psi(k^*,k^*)+\Pi.$$

As this result indicates, it is optimal for the sponsor to trade off allocative efficiency for an increased ability to extract contractor's rents. Underemployment of the existing resources will then occur.

Given the cost structure of this model, we can write the optimal payment scheme as one such that:

$$p(k)=k^*\bar{c}+(N-k^*)\underline{c}-(k^*-k)a+\Pi,\ \forall\ k\leq k^*,\ \forall\ k^*\in[0,N]. \qquad (12)$$

Expression (12) illustrates the tradeoff faced by the sponsor. Since $\bar{c}-\underline{c}-a>0$, $p(.)$ is increasing in k^*. Therefore, the sponsor has an incentive to pick a small k^*, since this implies making small payments for $k\leq k^*$. On the other hand, the smaller the k^* picked by the sponsor, the less often the project will be built, driving the solution away from allocative efficiency (first best solution).

To further illustrate this point we can rearrange expression (12) and write the optimal scheme as one such that

$$p(k)=k\bar{c}+(N-k)\underline{c}+(k^*-k)(\bar{c}-\underline{c}-a)+\Pi,\ \forall\ k\leq k^*,\ \forall\ k^*\in[0,N]. \qquad (13)$$

This shows that, for each particular firm, the optimal payment consists of two parts. The first one is the true cost of the project plus the contractor's reservation level of profit. The second part can be seen as a premium depending upon the difference (k^*-k). The premium that the sponsor has to pay to each firm increases with k^*. However, bigger premia will drive the sponsor closer to achieving production efficiency.

The solution to this tradeoff is resolved on the basis of the sponsor's beliefs. If the sponsor views it likely that the project will have a large number of low cost tasks then the sponsor will opt for extracting relatively more rents in each period and be willing to forego the loss of surplus in some cases where the project proves to have many high cost tasks. On the other hand, if the sponsor deems it likely that the project will have many high cost tasks then the risk of contractor drop out becomes too great, in which case the compensation function rises to ensure contractor participation.[4]

5. CONCLUSION

In this paper we focused on the case where the sponsor can precommit to compensation per task, where the contractor is not bound to complete the project, and where the contractor knows precisely the cost of each task right from the beginning of construction of the project. Given this, we constructed the optimal "direct contract" and showed that , for each particular firm, the optimal payment contains a premium term which increases with the maximum number of high costs tasks that the sponsor is willing to allow, k^*. The sponsor has an incentive to pick a small k^*, since this implies making small payments to the contractors that are willing to participate; however, bigger premia will drive sponsor closer to achieving production efficiency. This tradeoff is solved on the basis of her beliefs. Nevertheless, the optimal solution always entails underemployment of the existing resources.

NOTES

[1]There are, however, some major differences between the two set-ups which will become clear in the description of the model. Here, the contractor is assumed to know precisely the cost of each task right from the beginning of construction of the project. In Leite (1989), the contractor acquires the information pertaining to the cost of each task only upon task completion. Furthermore, it is now assumed that the sponsor can precommit to the entire payment stream.

[2]In a separating equilibrium this turns out to be a perfectly informative signal since there is no noise added to s_i.

[3]See proof of lemma 1.

[4]Leite (1988) provides a detailed comparative statics analysis of this problem.

APPENDIX

Proof of Lemma 1: Let $P^*(T)=\min(P(T),B)$. Then P^*satisfies (6d). To see this note that when $P(T)\le B$ the left hand side of (6d) is unchanged while the right hand side does not increase since $P^*(T')\le P(T')$. When $P(T)>B$, $P^*(T)=\max_{T'\in\Gamma} P^*(T')$. $C(T)=\min_{T'\in\Gamma} C(T,T')$. Hence (6d) is satisfied in this case as well. Under P^* the sponsor does not make a loss. Thus, there is no loss in assuming that $Q^*=1$ as long as the contractor is willing to participate, i.e., $Q^*(T,P^*)=1$ if $P^*(T)\ge C(T)+\Pi$ and $Q^*(T,P^*)=0$ if $P^*(T)<C(T)+\Pi$. By construction (P^*,Q^*) satisfies (6b)-(6e).

$$[B-P(T)]Q(T,P)\le[B-P^*(T)]Q^*(T,P^*);$$

since (P,Q) is optimal this weak inequality necessarily holds as an equality for all $T\in\Gamma$. Hence, if $Q(T,P)>0$ then $B-P(T)\ge0$. ■

Proof of Lemma 2: According to definition 2, a payment scheme is locally incentive compatible if

$$p(k)-\psi(k,k)\ge p(h)-\psi(k,h) \text{ for } h=k+1,\ \forall\ k\in\{0,\dots,N-1\}.$$

This condition is equivalent to

$$p(k)\ge p(k+1)-a,\ \forall\ k\in\{0,\dots,N-1\}.$$

If we add both the left hand side and right hand side over j of these constraints,

$$p(k+l)\ge p(k+l+1)-a,\ l\le j,\ j\in\{1,\dots,N-k\},$$

and cancel the common terms, we obtain

$$p(k)\ge p(k+j)-ja,\ k=0,\dots,N-1 \text{ and } j=1,\dots,N-k;$$

or, equivalently,

$$p(k)-\psi(k,k)\ge p(k+j)-\psi(k,k+j)$$

for all $k\in[0,N-1]$ and $j=1,\dots,N-k$. When p(.) is nondecreasing in k, this condition indicates that the payment scheme is incentive compatible according to definition 1. ∎

Proof of Proposition 1: As long as p(.) is nondecreasing in k, the self-selection constraints simply impose that:

$$p(k)-\psi(k,k)\geq p(h)-\psi(k,h),\ \forall\ k,\ \forall\ h\in]k,k^{*}]. \tag{A1}$$

We can write $\psi(k^{*},k^{*})=k^{*}\bar{c}+(N-k^{*})\underline{c}$ as $\psi(k^{*},k^{*})=\psi(k,k)+(k^{*}-k)(\bar{c}-\underline{c})$.

This condition can be rewritten as:

$$\psi(k,k)=\psi(k^{*},k^{*})-(k^{*}-k)(\bar{c}-\underline{c}). \tag{A2}$$

We also have that:

$$\psi(k,k^{*})=\psi(k,k)+(k^{*}-k)a. \tag{A3}$$

Substituting (A2) into (A3) leads to

$$\psi(k,k^{*})=\psi(k^{*},k^{*})-(k^{*}-k)(\bar{c}-\underline{c})+(k^{*}-k)a. \tag{A4}$$

Substituting (A4) into (A1) for the case in which $h=k^{*}$, we obtain:

$$p(k)-\psi(k,k)\geq$$

$$p(k^{*})-\psi(k^{*},k^{*})+(k^{*}-k)(\bar{c}-\underline{c}-a),\ \forall\ k<k^{*}. \tag{A1'}$$

Since $k^{*}-k>0$ and $\bar{c}-\underline{c}-a>0$, $p(k)-\psi(k,k)\geq\Pi$, $\forall$ $k<k^{*}$ is certainly satisfied when $p(k^{*})-\psi(k^{*},k^{*})\geq\Pi$. Therefore we have that $q(k)=1$, $\forall$ $k<k^{*}$ when $p(k^{*})-\psi(k^{*},k^{*})\geq\Pi$.

Now it is left to be shown that $p(k^{*})-\psi(k^{*},k^{*})>\Pi$ is not optimal. When p(.) is such that $p(k^{*})-\psi(k^{*},k^{*})>\Pi$ we can construct an alternative payment scheme, $\hat{p}(.)$, such that

$$\hat{p}(k^{*})=p(k^{*})-[p(k^{*})-\psi(k^{*},k^{*})]+\Pi$$

and

$$\hat{p}(k)=p(k)-[p(k^{*})-\psi(k^{*},k^{*})]+\Pi,\ \forall\ k<k^{*}.$$

Since the contractor would incur in a loss for $k>k^{*}$ with p(.), she will certainly incur in a loss for $k>k^{*}$ if she is paid according to $\hat{p}(.)$. Clearly, $\hat{p}(k^{*})-\psi(k^{*},k^{*})=\Pi$. If we subtract

$p(k^*)-\psi(k^*,k^*)$ from both sides of (A1'), we obtain:

$$\hat{p}(k)-\psi(k,k) \geq (k^*-k)(\bar{c}-\underline{c}-a)+\Pi, \quad \forall\ k<k^*. \tag{A1"}$$

Since $\bar{c}-\underline{c}-a>0$ and $k<k^*$ we have that $\hat{p}(k)>\psi(k,k)+\Pi$, $\forall$ $k<k^*$ and therefore that $q(k)=1$ for $k<k^*$. If we subtract $p(k^*)-\psi(k^*,k^*)$ from both sides of (A1) we have that

$$\hat{p}(k)-\psi(k,k) \geq \hat{p}(h)-\psi(k,h), \quad \forall\ k,\ \forall\ h \in]k,k^*].$$

This shows that if we have that $p(k^*)-\psi(k^*,k^*)>\Pi$, we can always construct an alternative payment scheme involving a smaller cost for the sponsor, such that the conditions of problem (9) are satisfied. Therfore, the optimal payment scheme is one such that $p(k^*)=\psi(k^*,k^*)+\Pi$. ■

Proof of Proposition 2: Pick any $k<k^*$. The local self-selection constraints indicate that $p(k) \geq p(k+1)-a$. Hence,

$$p(k) \geq \psi(k^*,k^*)+\Pi-(k^*-k)a\ , \quad \forall\ k<k^* \tag{A5}$$

since $p(k^*)=\psi(k^*,k^*)+\Pi$. If we take (A5) as an equality and set $p(k)=p(k^*)-(k^*-k)a$, $\forall$ $k<k^*$, we have that $q(k)=1$, $\forall$ $k<k^*$, since

$$\begin{aligned} p(k^*)-(k^*-k)a &= \psi(k^*,k^*)+\Pi-(k^*-k)a \\ &= \psi(k,k)+(k^*-k)(\bar{c}-\underline{c}-a)+\Pi \\ &> \psi(k,k)+\Pi. \end{aligned}$$

This is the optimal payment scheme since these are the minimum payments satisfying the self-selection constraints given by (A5). ■

Proof of Proposition 3: If $B<\psi(k^*,k^*)+\Pi$, the sponsor will not tolerate participation of a contractor that truthfully signals k^*. If $B=\psi(k^*,k^*)+\Pi$, (but $B>\psi(k^*-1,k^*-1)+\Pi$) the contractor which signals k^* adds nothing to the expected value of the sponsor's net benefit. Suppose that initially payments were made such that all contractors facing $k \le k^*$ were induced to participate. The sponsor will increase the expected value of her net benefits by equally lowering payments to all contractors signalling $k \le k^*$, in a way such that all contractors facing $k \le k^*-1$ high cost tasks will still participate. The self-selection constraints, given by

$$p(k)-\psi(k,k) \ge p(k+j)-\psi(k,k+j), \ \forall \ k \in [0,N-1], \ j=1,...,N-k$$

will still be satisfied, because both $p(k)$ and $p(k+j)$ will be reduced by the exact same amount. The contractor signalling k^* will no longer produce. Therefore it must be that $B > \psi(k^*,k^*)+\Pi$. ■

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