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ABSTRACT

The purpose of this paper is to consider the use of aggregate money measures of welfare change in economies in which there are public goods. Specifically, the relation between the maximization of the sum of equivalent variations and Pareto optimality will be studied.

We show that any Pareto optimal allocation may be obtained as a result of the maximization of the sum of individual money metrics: for this to happen it is enough to choose the Pareto optimal allocation as the basis for the computation of the money metrics.

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1. INTRODUCTION

The purpose of this paper is to consider the use of aggregate money measures of welfare change in economies in there are public goods Specifically, the relation between the maximization of the sum of equivalent variations and Pareto optimality will be studied. It will be shown that the sum of equivalent variations is an appropriate measure of aggregate welfare losses associated with sub-optimal resource allocation. The importance of an efficiency index was, in this context, stressed by Cornes and Sandler (1986).

When the consumer faces any type of quantity constraints the natural way to define the equivalent variation is to use the first difference of the restricted expenditure function (as defined by Neary and Roberts (1980)). The use of the first difference of the restricted expenditure function as a measure of individual welfare change was proposed by King (1983, 1986), Cornwall (1984) and Mäler (1985), Diewert (1986) and Johansson (1987).

Diewert (1986) uses a different approach to assess the benefits of public goods (specifically his example is infrastructure services). Diewert (1985) discusses, for economies without public goods, two classes of methods to evaluate efficiency and waste in a general equilibrium setting: the quantity oriented method of Allais and Debreu (Allais (1944, 1977), Debreu (1951, 1954) and the price oriented method put forward by Hicks and Boiteaux (Hicks (1956), Boiteaux (1951)).

The purpose of this paper is to present a Hicks-Boiteaux measure of aggregate welfare change in economies with public commodities. King (1986) used a generalization of such a measure to discuss the Ramsey-

Pigou problem of optimal public good provision financed by distortionary taxes.

In the first section of the paper the model will be presented. In the second section the main results will be stated and proved and the third section will conclude.

2. THE MODEL

The economy has H consumers that are identified by the superscript h ($h \in H = \{1,2,...,H\}$). Each consumer cares for three types of goods: private goods (n_1), public goods (n_2) and leisure.

The vector of private goods consumed by h will be denoted by x_p^h, the vector of public goods by x_Q^h, and leisure by ℓ^h. Since the public goods correspond to the definition given by Samuelson (1954) they are consumed in equal amounts by all consumers. Therefore the h superscript may be omited for this case.

To each consumer corresponds a consumption set which will be denoted as X^h. The consumption set will be identified with $\mathbb{R}_+^n \times [0,T]$, where $n = n_1 + n_2$. The consumer preference ordering can, by assumption, be represented by a monotonically increasing, quasi-concave and continuously differentiable of up to the second order ($u \in C^2$) utility function.

Labor is the only production factor in this economy. The technologically feasible and efficient production plans correspond to the production possibilities frontier. The production possibilities frontier may be represented by the function:

$$\ell : \mathbf{R}^n \to \mathbf{R}$$

which associates with each aggregate consumption bundle, of private and public commodities, the quantity of the aggregate labor endowment which remains available for leisure. The function ℓ is assumed to be monotonically decreasing, concave, and continuously differentiable of up to the second order ($l \in C^2$).

3. MAIN RESULT

The purpose of this section is to state and prove the key result of this paper which states that if one uses a Pareto optimal allocation to define the price vector, for private goods, and the quantity vector, for public goods, that identify the basis for the money metric functions, then if one maximizes the sum of equivalent variations one gets the Pareto optimal allocation as a solution.

3.1. INDIVIDUAL MONEY METRICS

We start by recalling that, for every consumer $h \in H$ in an economy without public goods, a money metric at prices $p \in \mathbf{R}_+^{n_1+1}$ is the following function of the consumption bundle x^h, (Weymark (1985)):

$$m^h(p;x^h) = \min \{pw : w \in \mathbf{U}^{h+}(x^h)\}$$

where $\mathbf{U}^{h+}(x^h)$ is the set of bundles which are prefered or indiferent to x^h for consumer $h \in H$. Of course we have $x^h \in \mathbf{U}^{h+}(x^h)$. Note also that for a money metric, as opposed to the "look alike" expenditure function, prices are given and the function argument is the consumption bundle x^h.

When there are quantity constraints, arising from the existence of public goods, the first difference of money metrics ("a la Weymark") ceases to be equivalent, for the consumer, to variations in income thereby loosing some of its intuitive appeal.

We can now consider formally the more general problem of a consumer of private commodities, leisure and public goods.

We shall start by considering the consumption bundle of consumer $h \in H$ as being composed of private goods x_p^h and public goods x_Q^h, and leisure i.e., $x^h = (x_p^h, x_Q^h, \ell^h)$. Price vectors will be divided in a similar way, i.e., $p = (p_p, p_Q, \ell)$ - reflecting the fact that leisure is taken as the numeraire. We are also going to assume that the price of public goods is zero ($p_Q = 0$).

When there are private commodities, public goods and leisure the money metric function may be defined as follows (Gaspar (1988)):

$$M^h(p_p, y_Q, \ell, x^h) = \min \{p_p w_p + \ell : w_Q = y_Q, (w_p, w_Q, \ell) \in U^+(x^h)\}$$

As the money metric function "a la Weymark" relates to the expenditure function, its proposed generalization by allowing for quantity constraints relates to the restricted expenditure function (as defined by Neary and Roberts (1980)).

3.2. AGGREGATE WELFARE MEASURES

Theorem: Let $(x_p^{h*}, x_Q^*, \ell^{h*})$, $h \in H$, be a Pareto optimal allocation and p_p^* the private goods' price vector of the corresponding Lindahl equilibrium. Then we have the following equality:

preferences.

The first order conditions for an interior solution, on the other hand imply that:

$$\frac{\partial M^1}{\partial \ell^1} - \beta = 0$$

$$\lambda^h \cdot \frac{\partial M^h}{\partial \ell^h} - \beta = 0 \qquad h = 2, 3, \ldots, H$$

which allows one to write:

$$\frac{\partial M^1}{\partial \ell^1} = \lambda^h \frac{\partial M^h}{\partial \ell^h} \qquad h = 2, 3, \ldots, H$$

but since we have taken the Pareto optimal allocation to define the basis of the money metric we may write:

$$\frac{\partial M^1}{\partial \ell^1} = \frac{\partial M^h}{\partial \ell^h} = 1 \qquad h = 2, 3, \ldots, H$$

because leisure is taken as the "numeraire". But then $\lambda^h = 1$, $h = 2, 3, \ldots, H$.

This allows us to conclude by noting that (1) leads to the same solution as:

$$\max \Sigma\, M^h (p_p^*, x_Q^*; x_p^h, x_Q^h, \ell^h)$$

s.t.

$$\text{s.t.} \quad x_Q^h = x_Q \text{ for all } h \in H$$

$$\Sigma\, \ell^h = \ell\, (\Sigma\, x_p^h, x_Q)$$

but this shows that the Pareto optimal allocation is also a solution for this problem concluding the proof. ◆

4. CONCLUSION

The theorem presented and proved in section 2 means that the use of monetary measures of efficiency and welfare change is intimately related to Pareto optimality. In fact any Pareto optimal allocation may be obtained as a result of the maximization of the sum of individual money metrics: for this to happen it is enough to choose the Pareto optimal allocation as the basis for the computation of the money metrics.

We may also interpret this result in an analogous way for the sum of first difference in money metrics from a Pareto optimal allocation. The result just discussed would hold true for the first difference in money metrics from such an allocation (i.e. for the sum of equivalent variations).

The result show that monetary measures of welfare change are not value free since they are imposed, implicitly, upon the analysis from the choice of the basis for the money metrics.

The results assure, on the other hand, the possibility of using the sum of equivalent variations, from a Pareto optimal allocation, as an index of efficiency in economies with public goods. Such a measure will, of course, not be unique since it is dependent upon the Pareto optimal allocation that is taken as reference.

Finally Diewert (1985) as proved a similar result for an economy where there are only private commodities. For differentiable economies his result may be established using the approach considered in this paper.

REFERENCES

Boiteaux, M., 1951, Le Revenu Distruable et les Pertes Économiques, Econometrica, 19.

Cornes, R. and T. Sandler, 1986, The Theory of Externalities, Public Goods and Club Goods (Cambridge University Press).

Cornwall, R.R., 1984, Introduction to the Use of General Equilibrium Models (North-Holland).

Debreu, G., 1951, The Coefficient of Resource Utilization, Econometrica, 19.

Diewert, W.E., 1985, The Measurement of Waste and Welfare in Applied General Equilibrium Models, in: J. Piggot and J. Whalley, eds., New Developments in Applied General Equilibrium Analysis (Cambridge University Press).

Diewert, W.E., 1986, The Measurement of the Economic Benefits of Infrastructure Services (Springer-Verlag).

Gaspar, V., 1988, Money Measures of Welfare Change under Quantity Constraints, Working Paper nº 89, Faculdade de Economia, Universidade Nova de Lisboa, Maio.

Hicks, J., 1956, A Revision of Demand Theory (Claredon Press, Oxford).

King, M.A., 1983, Welfare Analysis of Tax Reforms Using Household Data, Journal of Public Economics, 21.

King, M.A., 1986, A Pigovian Rule for the Optimum Provision of Public Goods, Journal of Public Economics, 30, 3, August.

Mäler, K.-G., 1985, Welfare Economics and the Environment, in: A.V. Kneese and J.L. Sweeney, eds., Handbook of Natural Resource and Energy Economics (North-Holland).

Neary, J.P. and K.W.S. Roberts, 1980, The Theory of Household Behavior under Rationing, European Economic Review, 13, 1.

Samuelson, P.A., 1954, The Pure Theory of Public Expenditure, Review of Economics and Statistics, 36.

Weymark, J.A., 1985, "Money Metric Utility Functions, International Economic Review, 26, 1, February.