STRATEGIC TAX REPORTING

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1. INTRODUCTION

The economic analysis of tax evasion behavior has been centered on extensions of the basic model presented in Allingham and Sandmo (1972). In its simplest version the taxpayer must decide only on which fraction of his income to report. He is an expected utility maximizer, risk averter and his taxable income is exogenously given and known only to him. The tax schedule is linear, and he must pay the unpaid tax plus a fine, if caught cheating. The probability of being audited is independent of his report, and when audited his true income is disclosed to the tax collecting agency.

With this very simple formulation, the problem of the evader can easily be related to: (i) the problem of choice of criminal activity; he faces a given probability of being detected and convicted a fine being applicable in this case; (ii) the problem of optimal portfolio diversification; he chooses to split his income into a safe asset, reported income, which is taxed (and certain), and an asset with stochastic return: unreported income which goes untaxed or is taxed at higher rate depending on him being audited. Most comparative statics results can be easily established by this analogy; and (iii) to the purchase of insurance: the part of the income reported can be seen as providing insurance against the accident of being audited, the price of the insurance provided being the tax rate.

In the last fifteen years there have been numerous contributions that consider extensions of this basic model, allowing for nonlinear tax schedules, differential taxation of labor and capital income, the effects of inflation, endogenous income, etc. Contributions in this line include Ali (1976), Anderson (1977), Cowell (1981, 1985) Fishburn (1981), Koskela (1983),

The probability of auditing and the magnitude of the fine are the instruments available to dissuade tax evasion. One of the major problems of this literature is the assumption that these are exogenously given. The tax collecting agency has no active role in this tax evasion game (see however Polinski and Shavell (1979, 1984) for a discussion on this instruments optimal levels, given a social welfare function).

More recently there have been attempts to model the strategic interaction between the tax authority and the taxpayer in a game-theoretic framework. Contributions in this line include Corchon (1984), Greenberg (1984), Landsberg and Meilijson (1982), Reinganum and Wilde (1985, 1986), Telser (1987) and Sanchez (1988).

The analysis of Greenberg (1984) and Landsberg and Meilijson (1982) are, as stressed by Reinganum and Wilde (1986), applicable to any type of proscribed behavior as they do not incorporate this fundamental feature of tax the tax compliance game.

On the other hand the contributions of Townsend (1979), Baron and Besanko (1983), Reinganum and Wilde (1985), Dye (1986) and Sanchez (1988) share a principal-agent framework. This framework assumes that the tax authority has the possibility to precommit itself to a given policy. Since there does not seem to be possible to enforce such a commitment, credibility problems may arise.
Reinganum and Wilde (1986) analyse the problem without precommitment but focus on separating equilibria while Corchon (1984) and Telser (1987) using a very simple framework emphasize that there may not be equilibria in pure strategies for such games. It must be stressed that neither Corchon (1984) nor Telser (1987) allow for the sequential structure that follows the inclusion of a tax report.

The purpose of this paper is to extend the framework of Corchon (1984) and Telser (1987) to include the sequential structure induced in the game by the requirement of a tax report by the taxpayer. Therefore the informational structure of the model is similar to that of Reinganum and Wilde (1986). We try to discuss this problem in the simplest possible case and focus on the possibility of pooled mixed strategies equilibria.

In section 2 we present the model and basic results. In section 3 we extend the model to include moral hazard and conclude in section 4.

2. TAXGAME BETWEEN THE TAXPAYER AND THE TAXMAN
   2.1. OUTLINE OF THE MODEL

The basic structure of the game is as illustrated in Fig. 1. The purpose of the adopted modelling strategy is on one hand to highlight the interdependence between the taxpayer and the taxman in the tax evasion game and on the other hand to allow explicitly for the sequential structure of their interaction. Namely the fact that a mandatory report on income by the taxpayer precedes the taxman decision on whether to audit or not is a distinctive feature of the tax evasion game.
The idea is to try to model the tax evasion game in the simplest possible framework. To do so we consider the case in which both the taxpayer and the taxman are risk neutral (that is their utility is linear in their payoffs). There are only two possible levels of income that the consumer may have: a high level of income, \( y \), and a low level of income, \( y \).

Let us now describe the sequential structure of the game.

In the first moment of the game nature decides whether he has high or low income. This is private information to which the taxman has no access. Therefore we have a game with asymmetric information.

The taxpayer will get the high level of income, \( y \), with probability \( p \) and the low level of income, \( y \), with probability \( 1-p \) (we will consider below the taxpayer ability to influence the probability, \( p \), through an effort variable \( e \)).

In the second moment of the game the taxpayer decides if he is going to report having high income, \( y \), or low income, \( y \). In the third moment of the game the taxman decides, given the received report, whether to audit the taxpayer or not. Finally the player's payoffs are realized.

We assume a linear tax schedule (tax liability - t.y, where \( y \) denotes income and \( t \) the tax rate). If the taxpayer is audited and found to be underreporting he has to pay the tax liability due on his true income plus a fine, \( f \). When the taxpayer is audited he bears a nonpecuniary loss that has an equivalent pecuniary value of \( z \). The tax collecting agency incurs an auditing cost, \( c \). The tax collecting agency net revenue is given by tax revenue plus fines less the cost of auditing.
The game is represented in Fig. 1. The nodes included in the dotted lines are information sets. After nature has chosen the level of income and upon the reception of a tax report, the tax collecting agency can make its decision on whether or not to audit conditional only on this tax report. If the reported income is high (\(\bar{r}\)) the taxpayer is audited with probability \(q_1\) (\(q(1)\) in the figure) and if the report indicates low income this probability is \(q_2\) (\(q(2)\) in the figure). The payoffs of the taxpayer and of the tax collecting agency are shown in the boxes.

2.2. EQUILIBRIUM IN PURE STRATEGIES

It is useful to begin by discussing the taxpayer's problem. Since the taxpayer knows the realization of \(y\) we are able to discuss the two possible cases separately.

When the consumer has high income, \(\bar{y}\), he has to choose the probability of reporting high income, \(\bar{r}\), which will be denoted by \(\bar{q}_1\). Therefore the taxpayer expected payoff has the form:

\[
\bar{q}_1 (q_1((1-t)\bar{y}-A) + (1-q_1)(1-t)\bar{y}) + \\
+ (1-\bar{q}_1) (q_2((1-t)\bar{y}-f-A) + (1-q_2)(\bar{y}-t\bar{y}))
\]

The taxpayer's expected payoff is thus linear in \(\bar{q}_1\) and its coefficient is:

\[-q_1A + q_2(f+A) - (1-q_2)t(\bar{y}-\bar{y})\]
and therefore:

\[
\phi_1 = \begin{cases} 
1 & \text{if } -q_1 \lambda + q_2(t - \lambda) - (1-q_1)t(\bar{y} - y) < 0 \\
? & \text{else} \\
0 & > 0 
\end{cases}
\]

When the taxpayer has low income, \( y \), he also has to choose the probability of reporting high income, \( \bar{r} \), which is denoted as \( \phi_2 \). The taxpayer's expected payoff is:

\[
\phi_2(q_1((1-t)y - \lambda) + (1-q_1)(y - \bar{y}) + (1-\phi_2)(q_2((1-t)y - \lambda) + (1-q_2)(1-t)y))
\]

The taxpayer's expected payoff is thus linear in \( \phi_2 \) and its coefficient is:

\[-q_1 \lambda + q_2 \lambda - (1-q_1) + t(\bar{y} - y)\]

and therefore:

\[
\phi_2 = \begin{cases} 
1 & \text{if } -q_1 \lambda + q_2 \lambda - (1-q_1)t(\bar{y} - y) < 0 \\
? & \text{else} \\
0 & > 0 
\end{cases}
\]

If we assume that \( \lambda < t(\bar{y} - y) \) it is easy to conclude that over the relevant range \( (q_1, q_2 \in [0, 1]) \) \( \phi_2 = 0 \). That is, when the taxpayer has low income it is optimal for him to report that he has low income.

Since \( \phi_2 = 0 \) \( \iff \) \( q_2 \lambda < q_1 \lambda + (1-q_1)t(\bar{y} - y) \) noting that \( q_2 \lambda \leq \lambda \) and \( \lambda = q_1 \lambda + (1-q_1) \lambda < q_1 \lambda + (1-q_1)t(\bar{y} - y) \) which is assured if \( \lambda < t(\bar{y} - y) \).
Now the possibilities of pure strategies are only two: \((\Phi_1 = 0, \Phi_2 = 0)\)
and \((\Phi_1 = 1, \Phi_2 = 0)\).

In the first case \((\Phi_1 = 0, \Phi_2 = 0)\) - in which the taxpayer always reports
having low income - the payoff of the tax authority may be written as:

\[
p(q_2(t\tilde{y}+f-c) + (1-q_2)t\tilde{y}) + (1-p)(q_2(t\tilde{y}-c) + (1-q_2)t\tilde{y})
\]

or:

\[
t\tilde{y} + q_2(p|t(\tilde{y}-y)+f|-c)
\]

The tax collecting agency then chooses \(q_2\) according to:

\[
q_2 = \begin{cases} 
1 & \text{if } p|t(\tilde{y}-y)+f|-c > 0 \\
? & \text{if } p|t(\tilde{y}-y)+f|-c = 0 \\
0 & \text{if } p|t(\tilde{y}-y)+f|-c < 0 
\end{cases}
\]

But \(q_2 = 1\) is inconsistent with \(\Phi_1 = 0\). If everyone who declares low
income is audited \((q_2 = 1)\) a high income taxpayer would never choose to
declare low income for he would be better off by reporting \(\tilde{y}\). To be
consistent with \(\Phi_1 = 0\) we must have, in the pure strategy case, \(q_2 = 0\).

Therefore it is necessary, for an equilibrium to exist that:

\[
p \leq \frac{c}{t(\tilde{y}-y)+f}
\]

If this condition is met there is an equilibrium in which every
taxpayer reports having low income and the taxman audits nobody. This
result is easily interpreted meaning that it does not pay for the taxman to
audit if the expected gain does not cover the costs of auditing. That in turn
happens when \( p \) (the probability of high income) is "small", \( c \) (the cost of auditing) is "large" and \( t(y-x) + f \) (the gain from detecting tax evasion) is "small".

Obviously the tax authority total payoff (tax revenue) in this case is simply \( ty \).

The payoff of the taxpayer is \( y - ty \) if his income is high (which happens with probability \( p \)) and \( (1-t)y \) if his income is low (which happens with probability \( 1-p \)).

This equilibrium is not very interesting since it corresponds to the case in which the taxman is unable to distinguish between high and low income individuals either "ex ante" or "ex post".

In such equilibrium the tax system would be regressive despite a proportional tax schedule due to tax evasion by the high income individuals.

Let us now turn to the second case \( (q_1 = 1, q_2 = 0) \). In this case the high income individuals report high income and the low income individuals report low income.

For this case the tax authority payoff may be written as:

\[
p(q_1(ty-c) + (1-q_1)ty) + (1-p)(q_2(ty-c) + (1-q_2)ty)
\]

which implies that in this case it would be optimal for the taxman to audit nobody \( (q_1 = 0, q_2 = 0) \). This result is intuitive for if every taxpayer is telling the truth it must be optimal to make no audits.
But this situation can not be an equilibrium. The taxpayer who happens to have high income has no incentive to tell the truth (\( \phi_1 - 1 \) would not be optimal). This is also intuitive since if the taxman audits nobody there would not exist any penalty associated with underreporting.

This means that for:

\[
p > \frac{c}{t(\bar{y} - y) + f}
\]

there is no equilibrium in pure strategies. This result is similar to a result attributed by Cowell (1987) to Corchon (1984) and also presented in a different context by Telser (1987).

2.3. EQUILIBRIUM IN MIXED STRATEGIES

To discuss the possibility of equilibria in mixed strategies we can start by discussing the problem of the tax collecting agency.

The taxman has to choose the probability of auditing when receiving a high income report, \( q_1 \), and the probability of auditing when receiving a low income report \( q_2 \). For the first of these cases the tax authority payoff is:

\[
q_1(t\bar{y} - c) + (1-q_1)t\bar{y}.
\]

for we have already established that \( \phi_2 = 0 \). The above expression is equivalent to:

\[
t\bar{y} - q_1 c
\]
which is maximized over the relevant range when $q_1 = 0$. This result shows that it is never optimal for the tax authority to audit those who report high incomes.

We have therefore established that low income taxpayers never lie ($\phi_2 = 0$) and that the tax collecting agency never audits anyone who reports high income ($q_1 = 0$). But high income taxpayers may underreport and to dissuade this behavior the tax collecting agency may audit a fraction of those reporting low income.

We must now consider the case of the tax authority when receiving a low income report. For this it is useful to introduce some additional notation. Call $\tau_1$ the probability of someone who reports low income $r$ to be high income individual; and $\tau_2$ the probability of someone who reports $\tilde{r}$ being of high income type. It is easy to see that:

$$
\tau_1 = \frac{p(1-\phi_1)}{p(1-\phi_1) + (1-p)(1-\phi_2)} \quad 1-\tau_1 = \frac{(1-p)(1-\phi_2)}{p(1-\phi_1) + (1-p)(1-\phi_2)}
$$

$$
\tau_2 = \frac{p\phi_1}{p\phi_1 + (1-p)\phi_2} \quad 1-\tau_2 = \frac{(1-p)\phi_2}{p\phi_1 + (1-p)\phi_2}
$$

Given that $\phi_2 = 0$ we may simplify the above expressions to:

$$
\tau_1 = \frac{p(1-\phi_1)}{p(1-\phi_1) + (1-p)} \quad 1-\tau_1 = \frac{(1-p)}{p(1-\phi_1) + (1-p)}
$$

$$
\tau_2 = 1 \quad 1-\tau_2 = 0
$$

Therefore the expected payoff for the tax collecting agency may now be written (upon receiving a low income report) as:

$$
q_2(\tau_1 (t\tilde{r}+c) + (1-\tau_1)(t\tilde{r}-c)) + (1-q_2)t \tilde{r}
$$
which may be rewritten as:

\[ -q_2 c + q_2 \tau_1 [t(\tilde{y} - y) + f] + \alpha \]

and so we have:

\[ q_2 = \begin{cases} 
1 & \text{if } \tau_1 [t(\tilde{y} - y) + f] - c > 0 \\
0 & \text{if } \tau_1 [t(\tilde{y} - y) + f] - c = 0 \\
? & \text{if } \tau_1 [t(\tilde{y} - y) + f] - c < 0 
\end{cases} \]  \hspace{1cm} (2)

Using (1) and since we have established that \( q_1 = 0 \) the condition that allows taxpayers with high incomes to play mixed strategies becomes:

\[ q_2(f^+ - \lambda) - (1 - q_2)t(\tilde{y} - y) = 0 \]

and so we have:

\[ q_2 = \frac{t(\tilde{y} - y)}{f^+ - \lambda + t(\tilde{y} - y)} \] \hspace{1cm} (3)

On the other hand from (2) above the condition that is compatible with the taxman playing mixed strategies is:

\[ \tau_1 [t(\tilde{y} - y) + f] - c = 0 \]

rearranging and using the fact \( \delta_2 = 0 \) we may write:

\[ \delta_1 = \frac{p[t(\tilde{y} - y) + f] - c}{p[t(\tilde{y} - y) + f - c]} \] \hspace{1cm} (2)

The expected payoff for the tax collecting agency upon the reception of a low income report \((r)\) is:

\[ ty \]
The expected payoff of the tax collecting agency that receives a high income report is:

\[ ty \]

Performing a tedious but simple exercise we may establish that for this case the expected payoff of the tax collecting agency (before the submission of the report) is given by:

\[ ty + \frac{\beta(t - y) + f - c}{t(y - y) + f - c} t(y - y) \]

Finally if \( k = 0 \) the taxpayer has exactly the same payoffs that we would obtain in the absence of asymmetric information, that is:

\[(1-t)y \text{ if he happens to enjoy high income}\]

and \[(1-t)y \text{ if he has low income}.\]

when \( k > 0 \) the payoff is the same for the high income case but equals \((1-t)y - q_k k\) for the low income case. The existence of tax evasion by high income individuals acts as a externality that penalizes the low income individuals.

3. EXTENSION OF THE TAXGAME TO INCLUDE MORAL HAZARD

The basic structure of the game, extended to include moral hazard, is as illustrated in Fig. 2. The notation is exactly the same as in section 2 except for \( e \) which denotes effort and \( g(e) \) which stands for the cost of making the effort \( e \) in monetary equivalent units. The probability of the taxpayer getting high income is now a function of the effort variable \((p - p(e))\).
Fig. 2
The taxpayer has to decide upon the level of effort "ex ante".

We are going to assume that \( e \in [0, +\infty) \) that \( p(e) \in C^2 \), and is a strictly increasing, concave function such that:

\[
p(0) > \frac{c}{t(\bar{y} - y) + f}
\]

and also:

\[
\lim_{e \to \infty} p(e) \leq 1
\]

On the other hand \( g(e) \in C^2 \) and is a strictly increasing, convex function.

For such a case we know from the previous section that the only possible equilibrium is in mixed strategies.

The taxpayer payoffs, in mixed strategies, may be written as:

\[
\psi(e) = p(e)(1-t)\bar{y} + (1-p(e))[(1-t)y - q_2 x] - g(e)
\]

where \( q_2 \) is given by (3).

The first order condition for an interior solution for the taxpayer problem may be rearranged as:

\[
\frac{g'(e)}{p'(e)} = [(1-t)(\bar{y} - y) + q_2 x]
\]

We may now define \( \psi(e) \) as:

\[
\psi(e) = \frac{g'(e)}{p'(e)}
\]
which, given the assumptions made is an increasing function of the effort variable \( e \).

The first order conditions may then be written as:

\[ \psi(e) = [(1-t)(\bar{y}-y) + q_2 \lambda] \]

Substituting equation (3) for \( q_2 \) this may be rewritten as:

\[ \psi(e) = \left[ (1-t) (\bar{y}-y) + \frac{t(\bar{y}-y)}{1+t(\bar{y}-y)} \right] \]

(4)

If there are no nonpecuniary losses associated with being audited \( \lambda = 0 \). The optimal level of effort for the taxpayer, in the taxgame \( e^* \), is exactly the same as for the case in which the income level is costlessly observed by the tax collecting agency. In this particular case the existence of the possibility of tax evasion does not change the taxpayer's effort level.

When there are non-pecuniary costs associated with the event of being audited the effort level chosen by the taxpayer is going to be higher than the effort level that would be optimal were the income level to be public information.

A little reflection suggests that this result follows from the fact that the taxpayer will not be audited if he declares high income. As the probability of getting a high income is an increasing function of the effort level, the taxpayer trying to avoid the nonpecuniary costs of being audited is induced to increase the effort level.

This result constrasts with what happens when there is an increase in the fine. One could think that since there are pecuniary costs when caugh
cheating and non-pecuniary costs from being audited, both would have the same effect on the effort level. Actually what happens is that an increase in the fine decreases the effort level (see (4)). This result becomes intuitive if one remembers that only the high income taxpayer that cheat can be fined. So reducing the effort level the taxpayer actually deceases the probability of being fined.

Both these results and equation (4) are illustrated in Fig. 3.

4. Conclusion

One question that naturally arises is: how to interpret mixed strategies? One persuasive interpretation was provided by Harsanyi (1973) and presented for example in Harsanyi and Selten (1988).

One of the reasons for difficulty in interpreting mixed strategies derives from the fact that neither player has any incentive to play his equilibrium mixed strategy and not any of his pure strategies. That is true even if the other player plays his equilibrium strategy. In general this problem is called the instability problem.

Harsanyi's point is that the instability is only apparent. Actually, if arbitrarily small fluctuations were allowed in the payoff functions (what Harsanyi calls irreductible uncertainty), it is possible to show that the game will only admit equilibria in pure strategies.

But what is even more interesting is that the joint fluctuation of both players' payoffs interact in such a fashion as to assure that each pure strategy is played in a proportion that "almost" equals the corresponding mixed strategies' probabilities (for proofs see Harsanyi (1973)).
This argument allows a plausible interpretation of mixed strategies equilibrium.

Another interesting feature of the equilibrium is that there is pooling. To be specific when the taxman receives a low income report he does not know whether it is a report of a low income taxpayer that is telling the truth or the report of a high income individual trying to evade taxes. We think that this non-revelating feature of equilibrium is intuitive. It is also interesting to point out that some recent contributions to the literature focus on revealing-separating equilibria (see e.g. Reinganum and Wilde (1985, 1986) and Sanchez (1988)).

We have already pointed out that the possibility of tax evasion by high income individuals penalizes the honest low income ones, if there is some disutility from being audited. This point is conceptually similar to the externality imposed by high risk individuals in the form of incomplete insurance coverage in Rothschild and Stiglitz (1976) paper on equilibrium in competitive insurance markets.

Also the extension of the model to include moral hazard does not change the basic flavor of these results.

Finally we have investigated elsewhere the possibility of commitment by the tax collecting agency. One of the most interesting results that emerges is related to the problem of optimal plan's inconsistency: after having committed himself the taxman would find it profitable to default. That possibility makes, of course, his commitment a non-credible one.
REFERENCES


