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MONEY MEASURES OF WELFARE CHANGE UNDER QUANTITY CONSTRAINTS*

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Working Paper Nº 89

* This paper is a revised version of my Ph.D. dissertation presented at the Faculdade de Economia, Universidade Nova de Lisboa. I would like to thank my superior Diogo Lucena and also Alan Kirman, Pedro Pereira, José Pedro Barosa and the participants to the I.O.Seminar at the University of Pennsylvania for helpfull comments and suggestions. The remaining errors are my own.

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Maio, 1988

ABSTRACT:

The purpose of this paper is to generalize the use of money measures of welfare change to situations in which the consumer faces quantity constraints. The usefulness of this approach lies in its wide scope of application, including situations of rationing (in the strictest sense, such as those imposed during times of war), situations of markert disequilibrium and those in which there are externalities (public goods, for example).

0. Introduction

The purpose of this paper is to generalize the use of money measures of welfare change to situations in which the consumer faces quantity constraints. The usefulness of this approach lies in its wide scope of application, including situations of rationing (in the strictest sense, such as those imposed during times of war), situations of markert disequilibrium and those in which there are externalities (public goods, for example).

The evaluation of the efficiency of any kind of any resource allocation mechanism requires, as a necessary condition, the identification of individual welfare measures. From an abstract viewpoint, an individual welfare measure, corresponds to the concept of utility function.

For Applied Welfare Economics it is desirable to obtain a function whose values have an intuitive counterpart. One way to provide for intuition would be to consider the possibility of expressing such a measure in monetary units. This is because such an expression allows the comparison between costs and benefits and between gains and losses (see, for example, Blinder (1982), Harberger (1971) and McKenzie (1983)). It is customary to identify measures of this class as money metrics (a concept introduced by Samuelson (1974)).

This paper provides a welfare analysis corresponding to the theory of consumer behavior under quantity constraints as presented by Rothbarth (1940), Samuelson (1947), Tobin and Houthakker (1951, 1952), Tobin (1952), Pollack (1969, 1971), Diewert (1978), Latham (1980), Neary and Roberts (1980), Deaton and Muellbauer (1980), Deaton (1981) and Diewert (1986).

A measure of welfare change is no more than a measure of the distance between two indifference surfaces. An interesting property of the first difference in money metrics, as measures of welfare change, is its commensurability, for a given price vector, with variations in income. The possibility of constructing this type of measure given the pre-ordering relation defined by consumer preferences was adressed in the work of Weymark (1985).

When there are quantity constraints, however,the first difference of money metrics ("a la Samuelson-Weymark") ceases to be equivalent to variations in income, thereby loosing some of its intuitive appeal.

The first section of this paper will generalize the work of Weymark (1985) by allowing quantity constraints. The main result of this section will establish the possibility of representing the consumer's preference preorderig using the concept of money metrics (i.e. it will be proven that the money metric is a utility function). As one would suspect this result is proven under stronger conditions on the preference pre-ordering than those needed in Weymark (1985). These aditional restrictions should be recognized and interpreted, especially in applications.

As the money metric "a la Samuelson Weymark" relates to the expenditure function, its generalization by allowing quantity constraints relates to the restricted expenditure function (as defined by Neary and Roberts (1980)).

The use of the first difference of the restricted expenditure function as a measure of welfare change was proposed by King (1983, 1986), Cornwall (1984) and Mäler (1985). The second section of this paper examines the foundations of this proposal according to the contribution of Weymark (1985).

In the third section the problem of computing the money measures of welfare change will be considered. The emphasis will be on the information requirements of such a computation.

The discussion of this problem, in the framework of the traditional model of consumer behavior, follows two paths: the calculation of approximate measures and approximation properties (McKenzie and Pearce (1976), Willig (1976 a, b), Seade (1978) and Mckenzie (1983)); the calculation of exact measures (Zajac (1979), Haussman (1981), Vartia (1983) and McKenzie and Ulph (1986)). The second section of this paper will consider the problem of calculating exact measures of welfare change to a consumer who faces quantity constraints.

The fourth section will deal with applications of the presented methodology.

The first application will illustrate the use of the sum of equivalent variations (Σ EV) as an aggregate measure of welfare (and waste) in an economy with public goods. The relation between Σ EV and Pareto optimality will be studied. It will be shown that Σ EV is an appropriate measure of aggregate welfare losses associated with sub-optimal resource allocation. The importance of an efficiency index in this context was stressed by Cornes and Sandler (1986).

The second application concerns the extension of the economic theory of index numbers by considering the existence of public goods. This extension was suggested by Baye and Black (1986).

The third application, finally, concerns the evaluation of the indidual costs of involuntary unemployment.

1.Money Metric Utility Functions

Let us begin by defining the consumption set as the subset of the n-dimensional Euclidean space, $\mathbb{R}^n$, which corresponds to the set of the phisically possible consumption bundles (see Debreu (1959)). This set will be identified with the non-negative orthant of the n-dimensional euclidean space, $\mathbb{R}^n_+$. This assumption is made only for the sake of simplicity and could, for example, be generalized by considering the possibility of upper bounds on the consumption of some goods. Such generalization would allow the consideration of leisure, the consumption of which is limited by the availability of time.

It is also assumed that the consumer has preferences that define a complete pre-ordering ("R"). A complete pre-ordering is a relation which is complete, transitive and reflexive.

For two consumption bundles x and y belonging to the consumption set ($x, y \in \mathbb{R}^n_+$), "xRy" means that "x is preferred or indifferent to y". The preference pre-ordering gives rise to two more relations: the strict preference relation, "P", and the indifference relation, "I".

Thus, "xPy", "x is strictly preferred to y" if "xRy" but not the reciprocal. On the other hand "xIy", "x is indifferent to y" if "xRy" and "yRx".

We can now define utility function:

DEFINITION: The function:

$$u: \mathbb{R}^n_+ \longrightarrow \mathbb{R}$$

is a utility function for a consumer whose preferences are described by the preference pre-ordering "R" if:

$$u(x) \geq u(y) \Leftrightarrow \text{"xRy"} \qquad (1)$$

The above definition is equivalent to:

$$u(x) > u(y) \Leftrightarrow \text{"xPy"} \qquad (2a)$$

and

$$u(x) = u(y) \Leftrightarrow \text{"xIy"} \qquad (2b)$$

This equivalence can be established noting that it is immediate that (2) => (1) and verifying on the other hand that:

"xPy" $\Leftrightarrow$ "xRy" and not "yRx"

which by (1) is equivalent to:

$u(x) \geq u(y)$ and not $u(y) \geq u(x)$

i.e. (2a). (2b) could be established in an entirely analogous way.

We can now define the set of consumption bundles preferred or indifferent to x as:

$$U_+(x) = \{z \in \mathbb{R}^n_+: \text{"zRx"}\}$$

and the set of consumption bundles to which x is prefered or indiffernt to as:

$$U_-(x) = \{z \in \mathbb{R}^n_+: \text{"xRz"}\}$$

We can also define the following partition for the components of the consumption bundle:

$$x = [x_p, x_a]$$

where x_p is a vector of dimension n_1 and x_a a vector of dimension n_2, $n_1 + n_2 = n$.

Finally we may introduce the following function:

$$M(p^*, \Omega^*; x) = \min_z \{p^* z : z \in U_+(x);\ z_\Omega = \Omega\} \tag{3}$$

in which $p^* = [p_p, p_\Omega] \in \mathbb{R}^n_+$, $p_p \in \mathbb{R}^{n_1}_{++}$ and Ω^* is a non-negative vector of diemnsion n_2.

The interpretation of (3) will be dealt with subsequently. It should be noted, however, that p^* and Ω^* are parameters while x is the argument of the function.

The main result of this section is that, under conditions which will be rigorously specified below, $M(p^*, \Omega^*; x)$ is a utility function. Conditions under which $M(p^*, \Omega^*; x)$ can be interpreted as a money metric will also be presented below.

We will also assume that the preference pre-ordering is continuous i.e. that:

$U_+(x) = \{z \in \mathbb{R}^n_+ : \text{"zRx"}\}$

and

$U_-(x) = \{z \in \mathbb{R}^n_+ : \text{"xRz"}\}$

are closed sets. Then if:

$$\text{"xR } [\emptyset, \Omega^*]\text{" and "yR } [\emptyset, \Omega^*]\text{"} \tag{4a}$$

$$\mathbb{R}^{n_1}_+ \times \{\Omega^*\} \cap U_+(x) \neq \emptyset \tag{4b}$$

$$\mathbb{R}^{n_1}_+ \times \{\Omega^*\} \cap U_+(y) \neq \emptyset \tag{4c}$$

we have:

THEOREM 1:

"xRy" $\Leftrightarrow M(p^*, \Omega^*; x) \geq M(p^*, \Omega^*; y)$

that is in the domain defined by (4) $M(p^*, \Omega^*; x)$ is a utility function.

Proof:

The proof will take place in two steps: in the first it will be demonstrated that problem (3) has a solution. In the second, that $M(p^*, \Omega^*; x)$ verifies the definition of a utility function.

(i) We need to prove that under the above mentioned assumptions there exists a solution to problem (3).

From (4b):

$\mathbb{R}_+^{n_1} \times \{\Omega^*\} \cap U_+(x) \neq \emptyset$

so it contains at least one element, say w. We may consider a consumption bundle z, with $z_i = w_i + \delta$, $i = 1,\dots,n_1$, $z_i = w_i$, $i=n_1+1,\dots,n$. We can now define the set $H^- = \{l \in \mathbb{R}_+^{n_1} \times \{\Omega^*\} : p^* w \geq p^* w\}$. Consider, then, the set $H^- \cap U_+(x)$ which is a non-empty set since it contains w. But $H^- \cap U_+(x)$ is a closed set (intersection of two closed sets) and bounded (H^- is a bounded set). Since $H^- \cap U_+(x)$ is a closed and bounded subset of $\mathbb{R}_+^n$ it is a compact set.

Furthermore, it is clear that the solution to the problem over $H^- \cap U_+(x)$ is identical to the solution over the whole domain.

Under these conditions, the existence of a solution to (3) is assured by Weierstrass Theorem (Fleming (1977), Theorem 2.10., pp.61-62 or Sydsæter (1981), Theorem 5.1., p.219).

(ii) We now need to prove that $M(p^*, \Omega^*; x)$ is a utility function.

a. If "xRy" then $U_+(x) \subset U_+(y)$ by transitivity. Then:

$\mathbb{R}_+^{n_1} \times \{\Omega^*\} \cap U_+(x) \subset \mathbb{R}_+^{n_1} \times \{\Omega^*\} \cap U_+(y)$

thus:

$M(p^*, \Omega^*; x) \geq M(p^*, \Omega^*; y)$

which shows that:

"xRy" $\Rightarrow M(p^*, \Omega^*; x) \geq M(p^*, \Omega^*; y)$

b. Let us define $U_{++}(x)$ as:

$U_{++}(x) = \{z \in \mathbb{R}^n_+: \text{"zPx"}\}$

If "yPx" then $U_+(y)$ is strictly contained in $U_{++}(x)$ which, itself, is strictly contained in $U_+(x)$.

But then for any consumption bundle $z \in U_+(y)$ we can define a δ neighborhood, $N_\delta(z)$ (relative to $\mathbb{R}^n_+$) such that $N_\delta(z) \cap \mathbb{R}^{n-1}_+ \times \{\Omega^*\}$ would be contained in $U_{++}(x)$.

Therefore:

$M(p^*, \Omega^*; y) > \inf_z \{p^* z: z \in U_{++}(x); z_\Omega = \Omega\} \geq$

$\geq M(p^*, \Omega^*; x)$

which shows that "yPx" $\Rightarrow M(p^*, \Omega^*; y) > M(p^*, \Omega^*; x)$.

c. We will now use b. to prove the reciprocal of a. by contradiction. We will establish that:

$M(p^*, \Omega^*; x) \geq M(p^*, \Omega^*; y) \Rightarrow$ "xRy"

Let us suppose that:

$M(p^*, \Omega^*; x) \geq M(p^*, \Omega^*; y)$ and "yPx"

That proposition would contradict b. establishing the result.□

It is important to discuss the importance of the assumptions (4) for Theorem 1. Thus (4b) and (4c) assure the existence to the problem that defines $M(p^*, \Omega^*; x)$.

The necessity of (4a) can be established by noting that no asumption of non-satiation was made in the proof of Theorem 1. So in the absence of any regularity hypothesis of the type of (4a), the preferences may assume a configuration such as in Fig.1. In such a case the result would not be valid.

It should be noted that the assumptions needed to establish Theorem 1 are insufficient to assure the continuity of $M(p^*, \Omega^*; x)$. They are, sufficient, however, to assure the existence of a continuous utility function (see Debreu (1954)).

It is convenient to introduce a local and global notion of non-satiation (relative to Ω):

DEFINITION:

(i) A consumption bundle $x \in \mathbb{R}^n_+$ is a point of local satiation relative to Ω if there exists a neighborhood δ of x such that:

$$\{N_\delta(x)\cap\mathbb{R}^{n_1}_+ \times \{\Omega^*\}\} \cap U_{++}(x) = \emptyset.$$

(ii) A consumption bundle $\dot{x} \in \mathbb{R}^n_+$ is a point of global satiation relative to Ω if:

$$\mathbb{R}^{n_1}_+ \times \{\Omega^*\} \cap U_{++}(x) = \emptyset$$

We may now enunciate the following result:

THEOREM 2:

If any point of local satiation relative to Ω^* is also a point of global satiation then $M(p^*, \Omega^*; x)$ is a continuous function.

Proof:

A function is continuous if it is both upper and lower semicontinuous. That is, if the sets:

$$\{x\in\mathbb{R}^n_+ : M(p^*, \Omega^*; x) \geq \pi\}$$

$$\{x\in\mathbb{R}^n_+ : M(p^*, \Omega^*; x) \leq \pi\}$$

were closed sets for any $\pi \in \mathbb{R}$.

Given that $U_+(.)$ and $U_-(.)$ are closed sets it is sufficient to guarantee that the range of the function $M(p^*, \Omega^*; x)$ is a convex set.

Consider then π and π' belonging to the range of $M(p^*,\Omega^*;x)$ with $\pi < \pi'$. Consider π^* as a convex combination of π and π'. We have, by construction, that $\pi < \pi^* < \pi'$.

The idea is to prove the existence of x^* such that:

$$M(p^*,\Omega^*;\ x^*) = \pi^* \qquad (*)$$

It is enough to consider the problem:

$$\max_x\ u(x)$$

s.t.

$$p^* x \leq \pi^*$$

$$x_\Omega = \Omega$$

$$x \in \mathbb{R}^n_+$$

and to verify that its solution x^* satisfies (*) which could be made by contradiction. □

For the interpretation of the results obtained, it is useful to introduce the following identity:

$$M(p^*,\ \Omega^*;\ x) \equiv e(p^*,\ \Omega^*;\ u(x)) \equiv$$

$$\equiv \min_z \{p^* z:\ u(z) \geq u(x),\ z_\Omega = \Omega\} \qquad (5)$$

in which $e(p^*,\ \Omega^*;\ u(x))$ is the restricted expenditure function as defined by Neary and Roberts (1980).

It is important to stress that the functions $M(p,\ \Omega;\ x)$ and $e(p,\ \Omega;\ u)$ are not identical. In fact the argument of the first is x while the arguments of the second are p, Ω and u.

Nevertheless, given the identity (5), the results above are relevant to the restricted expenditure function. In fact, in the conditions of Theorem 1, which are usually assumed in textbooks and also in aplications, the restricted expenditure function is strictly increasing in its utility argument. This result is stronger than the commonly proven proposition that the restricted expenditure function is not decreasing in utility.

2. Equivalent Variation and Compensating Variation

To introduce the welfare change measures that may be constructed from $M(p^*, \Omega^*; x)$ and to motivate its designation as a money metric it is useful to consider a simple example characterized by:

BASE SITUATION	CURRENT SITUATION
$p \quad \Omega^0 \quad I^0$	$p \quad \Omega^1 \quad I^1$

in which p represents the price vector, Ω the rationing levels which the consumer confronts and I represents income.

The superscript "Ø" identifies the base situation and the superscript "1" the current situation. So the only changes considered in this example are in the rationing levels and income.

If non-satiation is verified (for non rationed goods) we have:

$$e(p, \Omega^0; u(x^0)) = I^0 \qquad\qquad e(p, \Omega^1; u(x^1)) = I^1$$

in which x^i represents the consumption bundle that maximizes consumer's utility in the i-th situation, i=Ø,1.

Notice that the validity of $M(p, \Omega; x)$ above as representative of the consumer' preference pre-ordering does not require the verification of non-satiation. But non-satiation is fundamental to allow the interpretation of $M(p, \Omega; x)$ as a money metric. Non-satiation will be assumed throyghout the remainder of the paper.

We may now define an equivalent variation as:

$$
\begin{aligned}
EV &= \\
&= M(p,\ \Omega^{0};\ x^{1}) - M(p,\ \Omega^{0};\ x^{0}) = \\
&= e(p,\ \Omega^{0};\ u(x^{1})) - e(p,\ \Omega^{0};\ u(x^{0})) = \\
&= e(p,\ \Omega^{0};\ u(x^{1})) - e(p,\ \Omega^{1};\ u(x^{1})) + (I^{1} - I^{0})
\end{aligned}
\tag{6}
$$

This measure determines the income variation that would guarantee to the consumer the welfare change that would occur with the passage from the base to the current situation. Hence the designation of equivalent variation. This measure of welfare change can be interpreted (in an analogous way) as the expenditure variation in non-rationed goods which permits, with given prices and rationing levels, the consumer to experience the welfare change which would occur with the change from the base to the current situation.

The equation (6) also shows that, taking base prices and rationing levels as given, variations in $M(p,\ \Omega;\ x)$ are commensurate with income changes.

The equivalent variation, as defined above is a measure of welfare gain. The legitimacy of this interpretation is guaranteed by Theorem 1: the equivalent variation is positive if and only if there is an increment in consumer's utility. That is:

$$EV > \emptyset \Leftrightarrow u(x^{1}) > u(x^{0})$$

We can now deal with the general case. Allowing for the simultaneous change of prices, rationing levels and income.

So, let us consider:

BASE SITUATION	CURRENT SITUATION
$p^o \quad \Omega^o \quad I^o$	$p^1 \quad \Omega^1 \quad I^1$

As above we may define an equivalent variation as:

$$
\begin{aligned}
EV &= \\
&= M(p^o, \; \Omega^o; \; x^1) - M(p^o, \; \Omega^o; \; x^o) = \\
&= e(p^o, \; \Omega^o; \; u(x^1)) - e(p^o, \; \Omega^o; \; u(x^o)) = \\
&= e(p^o, \; \Omega^o; \; u(x^1)) - e(p^1, \; \Omega^1; \; u(x^1)) + (I^1 - I^o)
\end{aligned}
\tag{7}
$$

The equivalent variation in (7), like the previous one (6), determines the income variation that would guarantee to the consumer the welfare change that would occur with the passage from the base to the current situation. The interpretation of (7) is therefore analogous to that of (6). That is, for the general case, the equivalent variation, as defined in (7), is a legitimate indicator of welfare gain. Formally:

$$EV > \emptyset \Leftrightarrow u(x^1) > u(x^o)$$

A compensating variation could be defined in a similar way as:

$$
\begin{aligned}
CV &= \\
&= M(p^1, \; \Omega^1; \; x^1) - M(p^1, \; \Omega^1; \; x^o) = \\
&= e(p^1, \; \Omega^1; \; u(x^1)) - e(p^1, \; \Omega^1; \; u(x^o)) = \\
&= e(p^o, \; \Omega^o; \; u(x^o)) - e(p^1, \; \Omega^1; \; u(x^o)) + (I^1 - I^o)
\end{aligned}
\tag{8}
$$

The compensating variation determines the amount of income that the consumer would be willing to pay in order to obtain the current situation from the base situation.

The compensating variation, as defined above is a measure of welfare gain. The legitimacy of this interpretation is guaranteed by Theorem 1: the compensating variation is positive if and only if there is an increment in consumer's utility. That is:

$$CV > \emptyset \leftrightarrow u(x^1) > u(x^0)$$

These results establish that the compensating and equivalent variations are legitimate measures of welfare change between two situations (i.e. to perform binary comparisons).

On the other hand there is an important difference between the equivalent and the compensating variation. In fact the evaluation of the welfare change between a base and several alternative current situations, using the equivalent variation, is sufficient to the ranking of those situations. This result is a direct consequence of Theorem 1 since the comparison between the base and each of the current situations is made using the same vector of prices and rationing levels as parameters in the money metric .

The same does not hold for the compensating variation, as can easily be established using an example. Take three situations:

BASE SITUATION	CURRENT SITUATION 1
$p^0 \quad \Omega^0 \quad I^0$	$p^1 \quad \Omega^1 \quad I^1$

CURRENT SITUATION 2

$p^2 \quad \Omega^2 \quad I^2$

The compensating variation associated with the passage from the base situation to situation i can be designated as CV^i (i=1,2). Assuming, then, that $p_\Omega^i = \emptyset$, that $\Omega^i = \Omega$ (i=1,2), and also that $(k.p^1, kp^2) = (p^2, I^2)$, $k>\emptyset$, we have:

$$CV^1 = M(p^1, \Omega; x^1) - M(p^1, \Omega; x^0)$$
$$CV^2 = M(p^2, \Omega; x^2) - M(p^2, \Omega; x^0) =$$
$$= M(kp^1, \Omega; x^1) - M(kp^1, \Omega; x^0) =$$
$$= k.CV^1$$

notice, on the other hand that situation 1 and situation 2 are indifferent by construction. But $CV^1 < CV^2$ if $k > 1$ and $CV^1 > CV^2$ if $k < 1$. This proves that the comparison of compensating variations between a base and several alternative situations is insufficient to get the welfare ranking of the alternatives.

It is important to ponder the significance of the results obtained up to this point.

It has been established that knowing the consumer's preference pre-ordering it is possible to construct an utility function whose first difference is commensurate with variations in income, for given prices and rationing levels. Nevertheless the practical relevance of this result is extremelly limited.

This point can be clarified through an argument of Balasko (1988): we may conceive ofexperiments through which sucessive comparison of consumption bundles would allow the determination of consumer preferences to any degree of accuracy desired (assuming that the answers are honest). Therefore there is no theoretical difficulty with the implementation of the concept. Nevertheless the practical realization of such a type of experiment does not seem reasonable.

This last observation serves to introduce the following section.

3. Computing Variations in Money Metrics: the example of Equivalent Variation.

The purpose of this section is the identification of a suitable process of computation for the first difference in money metrics. The idea is to present a process which uses information that one might get from the observation of consumer behavior.

We will show that for the computation of the first difference in money metrics it is sufficient to know the demand system (for non-rationed goods) and the system of marginal benefit functions (for rationed goods).

In order to understand the computation process it is useful to digress a bit into the area of consumer behavior under rationing.

The indirect utility function may be defined (after Pollack (1969)) as:

$$v(p, \Omega; I) = \max_{x} \{u(x): x \in X, px = I, x_{\Omega}=\Omega\} \tag{9}$$

in the formulation of (9) the hypothesis of non-satiation is assumed. We can construct the lagrangean function that corresponds to problem (9):

$$L = u(x) + \lambda(I-px) + \tau(-x_{p}) + \phi(\Omega-x_{\Omega}) \tag{10}$$

Assuming differentiability of $v(p, \Omega; I)$ we have:

$$\frac{\partial v}{\partial p} = -\lambda x \tag{11a}$$

$$\frac{\partial v}{\partial \Omega} = \phi \tag{11b}$$

$$\frac{\partial v}{\partial I} = \lambda \tag{11c}$$

Notice that $x = x(p, \Omega; I)$, $\lambda = \lambda(p, \Omega; I)$ and $\phi = \phi(p, \Omega; I)$. That is, the demand functions $x(p, \Omega; I)$ depend parametrically on the price vector and on income, as well as on the rationing levels.

$\phi_i(p, \Omega; I)$ is the variation in utility that is made possible by a unitary increase in the quantity constraint relative to the "i-th" good. The marginal value of the good, for the consumer; evaluated in monetary units is therefore, $(p_i+(\phi_i/\lambda))$. This function is designated as marginal benefit function (or marginal willingness to pay function) corresponding to commodity i.

ϕ_i may be positive or negative. ϕ_i will be positive (negative) if the marginal benefit corresponding to the good i is more (less) than the respective price.

Equations (7) and (11) provide the fundamental elements for the computation of the equivalent variation. The procedure is analogous to that introduced for the case without rationing by Zajac (1979), Haussman (1981), Stahl II (1983), Vartia (1983) and McKenzie and Ulph (1986).

It must be remembered that the possibility of obtaining individual measures of welfare change, from the Dupuit-Marshall consumer's surplus, is limited in general by its dependence on the path of integration. This point calls attention to the importance of considering restrictions on the path of integration to make it possible to obtain significant measures of welfare change.

The idea is to restrict the path of integration, for the demand and marginal benefit functions, in order to follow a given indifference curve.

For example, to compute the equivalent variation it is sufficient to determine $e(p^0, \Omega^0; u^1) - e(p^1, \Omega^1; u^1)$ since (I^1-I^0) is known. So its computation depends only on the variation of the expenditure function along the indifference curve.

To proceed it is useful to differentiate the indirect utility function (using (11)):

$$dv = -\lambda x dp + \phi d\Omega + \lambda dI^*$$

or:

$$\frac{dv}{\lambda} = -\lambda x dp + \phi d\Omega + \lambda dI^* \tag{12}$$

as $\lambda > 0$ (assuming non-satiation), the sign of dv is determined by the sign of the right hand side of (12).

Making $dv = 0$ in (12), in order to dertermine the variation in expenditure that allows the consumer to keep the same utility level, we get:

$$dI^* = xdp - \frac{\phi}{\lambda} d\Omega$$

or, considering an auxilliary variable, t, we have:

$$\frac{dI^*}{dt} = x(p(t), \Omega(t), I(t))dp(t) - \frac{\phi}{\lambda}(p(t), \Omega(t), I(t))d\Omega(t) \tag{13}$$

equation (13) is an expression which, if integrated, allows us to obtain the expenditure variation. Since it is a first order differential equation (correponding to an exact differential) it admits a unique solution.

Therefore:

$$I^*(1) - I^*(\emptyset) = \int_t [x \frac{dp(t)}{dt} - \frac{\phi}{\lambda} \frac{d\Omega(t)}{dt}]dt$$

in which $t \in [\emptyset, 1]$, $p(\emptyset) = p^1$, $p(1) = p^0$, $\Omega(\emptyset) = \Omega^1$, $\Omega(1) = \Omega^0$ and $I^*(\emptyset) = I^1$. We can now present an expression for computing the equivalent variation:

$$\begin{aligned} EV &= \\ &= e(p^0, \ \Omega^0; \ u(x^1)) - e(p^0, \ \Omega^0; \ u(x^0)) = \\ &= e(p^0, \ \Omega^0; \ u(x^1)) - e(p^1, \ \Omega^1; \ u(x^1)) + (I^1 - I^0) = \\ &= \int_t [x \frac{dp(t)}{dt} - \frac{\phi}{\lambda} \frac{d\Omega(t)}{dt}]dt + (I^1 - I^0) \end{aligned} \tag{14}$$

It is now important to comment on the above expression:

(i) In the first place, the computation of the equivalent variation (from (14)) is made entirely using ordinary (i.e. uncompensated) demand functions.

(ii) The considered measure of welfare change is an exact measure (i.e. non-approximated). If the knowledge about demand functions and marginal benefit functions were only approximated, the result would also be only approximated.

(iii) The marginal benefit function for the i-th good $((\phi_i/\lambda) + p_i)$ is not, in general, the inverse of the respective demand function. For such a result to be valid, it would be necessary to assume a convexity hypothesis on preferences. This point is interesting since it shows the economic relevance of marginal benefit functions associated with non-convex preferences.

Therefore the knowledge of the system of marshallean demand functions for all goods is, in general, insufficient to assure the computation of variations in money metrics when there are quantity constraints. Nevertheless if we assume convex preferences this possibility is assured.

(iv) In order to compute the equivalent variation (from (14)) it is necessary to know the demand and marginal benefit functions. However when the quantity constrained goods correspond to public goods or externalities, in general, there is the well known preference revelation problem (see Hurwicz (1986)). This fact is an serious obstacle for the implementation of (14). Note, however that the approach remains, in principle, implementable.

In conclusion we have seen in this section how to compute welfare change measures when demand and marginal benefit functions are known and conform to the restrictions of Consumer's Theory.

If, however, to assure consistency with Consumer's Theory, the estimated functions are derived from a postulated functional form for the indirect utility function or for the expenditure function, the calculation of the welfare change measures becomes immediate.

4. Applications

4.1. Σ EV as an Aggregate Measure of Efficiency and Welfare in an Economy with Public Goods.

This subsection will examine the properties of the sum of the equivalent variations, as an aggegate measure of efficiency and welfare in an economy with public goods.

In particular we will try to establish the relationship between the summation of the equivalent variation (taking a Pareto optimal allocation as the base situation) and Pareto optimallity.

The properties of this measure will be discussed in the framework of a very simplified economy (following, in part, Dierker and Lenninghaus (1986)).

The economy has H consumers who will be identified by the superscript h ($h \in H = \{1,2,\dots, H\}$).

We can define a partition on the h-consumer's consumption bundle:

$$[\ x_p^h,\ x_\alpha^h,\ l^h]$$

where x_p is a private goods vector of dimension n_1, x_α is a public goods vector of dimension n_2 ($n_1+n_2=n$) and l is leisure.

To each consumer corresponds a consumption set which will be denoted as X^h. The consumption set will be identified with $\mathbb{R}_+^n\times[\emptyset,T]$. The consumer preference preordering can, by assumption, be represented by a utility function, smooth and quasi-concave.

Labor is the only production factor in this economy. The technologically feasible and efficient production plans correspond to the production possibilities frontier. The production possibilities frontier may be represented by the function:

$$l: \mathbb{R}^n \longrightarrow \mathbb{R}$$

which associates with each consumption bundle, of private and public goods, the quantity of the labor endowment which remains available for leisure. The function l is assumed to be smooth and concave. A resource allocation is possible if it belongs to the set, P, of feasible resource allocations:

$$P = \{ [x_p^h, x_Q^h, l^h]_{h\in H} \in \prod\{\mathbb{R}_+^n \times [\emptyset, T]\}: \Sigma l^h \leq l(\Sigma x_p^h, \Sigma x_Q^h) \}$$

the set P is, by assumption, convex. This property may be readily illustrated in which there is only one consumer and to goods: a consumption good and leisure.

We may now present a simple maximization problem in which the the maximand is, precisely, the sum of the equivalent variations (Σ EV). The problem will be exactly the "first best" problem of optimal provision of public goods formulated and solved by Samuelson (1954).

The problem may be formulated as:

$$\max \Sigma e^h(p_p^o, x_Q^o; u^h) - e^h(p_p^o, x_Q^o; u^{ho})$$

s.t.

$$x_Q^h = x_Q$$

$$\Sigma x_p^h = x_p$$

$$\Sigma l^h = l(x_p, x_Q)$$

The corresponding lagrangean function may be written:

$$\Sigma \; e^h(p_p^o,\; x_Q^o;\; u^h) + \Sigma\, \lambda_h(x_Q - x_Q^h) + \phi(x_p - \Sigma\, x_p^h) + \\ + \theta\, (l(x_p,\; x_Q) - \Sigma\, l^h)$$

in which the price of labor (the opportunity cost of leisure) is ommited in the definition of the expenditure function since leisure is taken as numeraire.

Rearranging the first order conditions for an interior solution we can get the following:

$$\frac{\partial e^h}{\partial u^h}\,\frac{\partial u^h}{\partial x^h_{jp}} = \frac{\partial e^H}{\partial u^H}\,\frac{\partial u^H}{\partial x^H_{jp}} \quad \text{for any } h \in H,$$

$$\frac{\partial e^h}{\partial u^h}\,\frac{\partial u^h}{\partial l^h} = \frac{\partial e^H}{\partial u^H}\,\frac{\partial u^H}{\partial l^H} \quad \text{for any } h \in H \qquad (19)$$

$$\sum \frac{\dfrac{\partial u^h}{\partial x_{iQ}}}{\dfrac{\partial u^h}{\partial x^h_{jp}}} = \frac{\dfrac{\partial l}{\partial x_{iQ}}}{\dfrac{\partial l}{\partial x^h_{jp}}} \qquad (20)$$

in which the first condition has the form of an interpersonal equity condition which can be ennunciated as assuring that the marginal utility of any private good divided by the marginal utility of income is the same for all consumers.

The second is just Samuelson's condition for the optimal provision of public goods. This condition requires that the sum of the marginal rates of subsititution (between public and private goods) equals the respective marginal rate of transformation.

There are additional conditions that reproduce the definition of feasible allocation.

Consider now any Pareto optimal resource allocation:

$$[\, x_p^{h*},\ x_Q^{h*},\ l^{h*} \,]_{h\in H}$$

which will be taken by hypothesis to be interior.

By a argument similar to the one used for establishing the Second Fundamental Theorem of Welfare Economics (Nikaido (1968)) we can show that associated with any Pareto optimal allocation, there is a price vector for the private goods that, contingent on the provision level for the public goods,permits its descentralization.

Consider now the following problem:

$$\max \sum e^h(p_p^*,\ x_Q^*;\ u^h) - e^h(p_p^*,\ x_Q^*;\ u^{h*})$$

s.t.

$$x_Q^h = x_Q$$

$$\sum x_p^h = x_p$$

$$\sum l^h = l(\ x_p,\ x_Q)$$

It is easy to verify that:

$$[\, x_p^{h*},\ x_Q^{h*},\ l^{h*} \,]_{h\in H}$$

is a solution to the above problem. That is, the maximization of the $\sum$ EV leads to the Pareto optimal allocation used as its basis.

These result as an important corollary. If $\{u^h\}_{h\in H}$ is the utility vector associated with an allocation which is not Pareto optimal, then:

$$\Sigma\, e^h(p_p^*,\ x_Q^*;\ u^h) - e^h(p_p^*,\ x_Q^*;\ u^{h*}) < 0$$

but then ($-\Sigma$ EV) can be used as an index of inefficiency in the allocation of resources.

Note further that this inefficiency index is not, in general, null when comparing Pareto optimal allocations. If $\{u^h\}_{h\in H}$ is the utility vector associated with a resource allocation which is not Pareto optimal then:

$$\Sigma\, e^h(p_p^*,\ x_Q^*;\ u^h) - e^h(p_p^*,\ x_Q^*;\ u^{h*}) \leq 0$$

This result shows that Σ EV is not a pure inefficiency index implying (implicitly) distributional judgements.

These results are analogous to those derived by Diewert (1985) for a private goods economy. Diewert (1986) proposes the use of efficiency measures in the tradition of Debreu (1951) for an economy with public goods. In the same line, there are suggestions by Cornes and Sandler (1986). The measure presented in this subsection is in the tradition of Hicks (1943) and Boiteaux (1953).

The Σ EV fits into the scope of the measures defended by Harberger (1971) for Cost-Benefit Analysis.

4.2. The Provision of Public Goods and the Economic Theory of Index Numbers.

The extension of the economic theory of index numbers to the consideration of the provision of public goods was proposed by Baye and Black (1986). They proposed a simple generalization of the Konüs Index:

$$I(p^1,\ p^o,\ x^1_\Omega,\ x^o_\Omega,\ u) = \frac{e(p^1, x^1_\Omega, u)}{e(p^o, x^o_\Omega, u)}$$

the properties of which will be summarily presented.

We are going to consider the index I(.) in which $u = u^1$. Therefore, if the expenditure level is invariant between the base and the current period $I^1=I^o$, we can write:

$$u^1 \geq u^o \Leftrightarrow e(p^o,\ \Omega^o;\ u^1) \leq e(p^o,\ \Omega^o;\ u^o)$$
$$\Leftrightarrow I(p^1,\ p^o,\ x^1_\Omega,\ x^o_\Omega,\ u) \leq 1$$

this, because $I(p^1,\ p^o,\ x^1_\Omega,\ x^o_\Omega,\ u) \leq 1$ is, in the stated conditions, equivalent to $EV \geq \emptyset$.

For the more general case in which I^1 may be different from I^o, the result is:

$$u^1 \geq u^o \Leftrightarrow I(p^1,\ p^o,\ x^1_\Omega,\ x^o_\Omega,\ u) \times (I^o/I^1) \leq 1$$

We will show that the second term is equivalent to $EV \geq \emptyset$:

$$\frac{e(p^1,x_Q^1,u)}{e(p^0,x_Q^0,u)}\ \frac{I^0}{I^1} \leq 1$$

$$\frac{e(p^1,x_Q^1,u)}{e(p^0,x_Q^0,u)} \leq \frac{I^1}{I^0}$$

$$\frac{e(p^1,x_Q^1,u)}{e(p^0,x_Q^0,u)} - \frac{I^0}{I^1} \geq \emptyset \qquad \text{given that } I^1>\emptyset$$

$$e(p^0,\ \Omega^0;\ u^1) - I^0 \geq \emptyset$$

$$e(p^0,\ \Omega^0;\ u^1) - e(p^0,\ \Omega^0;\ u(x^0)) \geq \emptyset$$

So we have, finally:

$$EV \geq \emptyset$$

This type of result is the basis for the generalization of the economic theory of index numbers by the inclusion of state provision of public goods. The provision of public goods by the state is modelized here as corresponding, for the consumer, to a quantitative restriction.

4.3. Individual Costs of Involuntary Unemployment.

Given the classical definition of involuntary unemployment by Keynes (1936, p.15), we will take it as implying that the consumer finds himself facing quantity constraints in the job market, in the sense that, given relative prices, he is forced to consume more leisure than he would like to. Thus, we may apply the methodology of the first three sections of the paper to the determination of the individual costs of involuntary unemployment.

To be specific, we will take the simple case of a consumer of two goods: leisure, l, and a composite consumption good, C. We will also assume that the consumer preference pre-ordering can be represented by a utility function quasi-concave, smooth, and monotone.

On the production side, we will assume that labor is the only production factor and that there are constant returns to scale. Note that these hypotheses are consistent with the existence of a linear technology.

Consider further that, in this economy, the relative price of goods in terms of leisure is one. The consumer has an endowment of time, T and of consumption goods, θ.

The expenditure function for this case is defined as:

$$e(\ . \ ; u) = \min \{ C + l_a : u(c, l) \geq u\}$$

We will take the situation in which the consumer is unemployed as the base situation. In such a situation the consumer enjoys his endowment of time, T, as leisure and consumes his endowment of consumption goods, θ.

We can define the individual cost of involuntary unemployment as the equivalent variation associated with the passage from the base situation to a situation in which there the quantity constraint on the consumption of leisure would not exist. This situation would be characterized by a consumption $\bar{l}^1$ of leisure and, therefore, of $\theta + (T - l^1)$ of the consumption good.

Thus the individual cost of unemployment can be formally defined as:

$$ICU = EV =$$
$$e(1, T; u(\theta + (T - l^1), l^1)) - e(1, T; u(\theta, T)) \quad (21)$$

Given that:

$$e(1, T; u(\theta, T)) = e(1, l^1; u(\theta + (T - l^1), l^1)) =$$
$$= \theta + T$$

(21) can be re-written as:

$$IEU = e(1, T; u(\theta + (T - l^1), l^1)) -$$
$$- e(1, l^1; u(\theta + (T - l^1), l^1))$$

which, denoting $\partial e/\delta l_a$ as $-\beta$, we have:

$$IEU = \int_{l^1}^{T} (-\beta)\, dt \quad (22)$$

This type of approach allows us to question the idea suggested in some macroeconomic textbooks that the cost associated with unemployment could be measured through the associated losses in production.

For an interpretation of unemployment as a rationing phenomenon, see Ashenfelter (1980).

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