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TO SEARCH OR NOT TO SEARCH"

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1. Introduction

In many real situations the assumption usually made that all agents are costlessly informed about the prices at which others are willing to trade is clearly non tenable. When information is costly it becomes much more difficult to understand the mechanism by which decentralized price decisions will generate an equilibrium, and analyse its properties. I address here informational problems in markets like the housing market or the "used car" market. Simple observation suggests that in housing markets important informational aspects are involved. The resources that go into "looking for a house" are substantial and an important brokerage industry developed, one of whose main functions can be seen as substituting for this costly search. I will concentrate on price dispersion and assume "sub-markets" of homogeneous houses. Buyers and sellers in this market must spend resources looking for trading opportunities. They may know the relevant price distribution prevailing at each moment in the market, but must engage in costly search in order to locate trading partners. An equilibrium arises characterized by price dispersion, vacancies and unsatisfied demand.

Search models have been extensivelly used to characterize the individual behaviour in these circumstances, and I will assume that both buyers and sellers will follow a reservation price rule, while searching. The emphasis, however, is not in the individual behaviour, but in the market equilibrium. A good understanding of the informational structure of the housing market requires some departure from the usual literature, in which stores and customers face each other. Buyers and sellers are seen as drawn from the same population, and a much more symmetric case arises.

The brokerage industry is modeled as a substitute for the search activity of the agents in the market. Buyers and sellers can use the brokers services, for a price, and avoid the need to search. The brokerage industry has not received much attention and has generaly been analyzed in isolation (Yinger (1981)). Here I claim that the proper understanding of this market requires the simultaneous analysis of the brokers' decision and how they affect and are affected by the equilibrium generated in the search market.

2. Search Equilibrium: An Overview

Buyers and sellers are drawn from the same population, and enter the market at the same rate. The model to be developed addresses the question of price dispersion, but not the price level in this market. We can assume that the average price will adjust in response to excess demand, equilibrating total entrance rates of buyers and sellers.

After entering the market all agents will make the same search effort, which is taken to be constant in time. However, different people will have different costs for this effort. The population is distributed by effort cost according to the cumulative $E(c), c \in [0, c^*]$ (and p.d.f. $e(c) = dE/dC$). The flow of both buyers and sellers entering the market will follow this distribution.

As a result of this search activity a certain number of contacts, per unit of time, is made by each agent. After a while, and according to a rule to be specified below, a contact will be successfull, and both agents leave the market.

Sellers and buyers will stay for a time in the market (i.e., searching). Let $S(B)$ be the number of sellers (buyers), that are currently engaging in search. Sellers with different effort costs will follow different strategies, and will search for a different amount of time. The populaion $S(B)$ is then distributed by effort costs, but following a distribution that differs from $E(.)$. Let $F(c)$ and $f(c)$, $c \in [0, c^*]$ be the cumulative and p.d.f. for S and $G(b)$, $g(b)$, $b \in [0, b^*]$ for B. Simmetry conditions imposed later will make these distributions equal, for much of the analysis.

Although the cost of each unit of search effort is given, for a given agent, the number of contacts he is able to make in each unit of time depends, through the search technology, on the number of agents engaging in search in both sides of the market. The search costs of each agent (i.e. the cost of one contact) are thus endogeneous. This characteristic of the model - in fact a sort of network externality – will have important consequences.

Clearly it makes no sense for a seller to engage in search (e.g. by putting ads on a newspaper) if no potential buyer is also engaging in search (e.g. by answering ads).

The search technology, together with the number of searchers (B,S) and their distribution (F,G) will determine the quit rates. Those must be equal to the entrance rates, E(.), for an equilibrium to obtain. The equilibrium condition will thus determine the values of B and S and the distribution F and G. The equilibrating mechanism is described below.

All sellers are assumed to have the same knowledge about the search market conditions. They face a known distribution of buyers' reservation prices, N(y). Facing this same distribution, sellers with different effort costs will decide on different reservation prices, x. This leads to a distribution of asking prices, M(x). Buyers, facing this distribution, will also decide differently on their reservation prices, generating N(y) in turn.

When a seller and a buyer meet, a bargaining process takes place. If $x > y$ no transaction is possible and both resume search. If $x \leq y$ a transaction will take place at a price $t = \lambda x + (1-\lambda)y$, with $\lambda \in [0,1]$, and both quit the market.

All sellers (buyers) make the same search effort, and thus the same number of contacts. The total number of contacts made in each unit of time is given by the total population currently engaging in search, through a search technology given by:

$$T(B,S) = n_0 S^{\alpha} B^{\alpha} \qquad \alpha > 0 \qquad (2.1)$$

The specific functional form has chosen in such a way that $T(B,0) = T(0,S) = 0$.

The function is symmetric in B and S, admiting implicitly that the same search activities are open to both buyers and sellers.

By appropriate choice of units, I will take $n_0 = 1$, with no loss of generality. Then, if c(b) is the effort cost of a seller (buyer), his search cost is

$$s_c = \frac{c}{S^{\alpha-1}B^{\alpha}} \tag{2.2.a}$$

$$s_b = \frac{b}{S^{\alpha}B^{\alpha-1}} \tag{2.2.b}$$

Search costs become endogeneous to the model, and depend on the total number of buyers and sellers currently searching. Notice that the search cost of a seller with positive c becomes infinite as B goes to zero.

3. Seller and Buyer Behaviour

I will assume that $N(y)$, $y \in [p'p'']$ is a continous function and that $n(y) = dN/dy \geq 0$ exists [1]. The searching populations are S and B, and the seller will choose $x^*(c)$ by solving.

$$\max_{x} \pi(x,c) = E\,\{\text{sale price - search cost}\}$$

By a simple argument

$$\pi(x,c) = \lambda x + (1-\lambda)\frac{\bar{y}-Q(x)}{1-N(x)} - \frac{1}{1-N(x)} \tag{3.1}$$

where

$$\bar{y} = \int_{p'}^{p''} y\,dN(y) \text{ and } Q(x) = \int_{p'}^{x} y\,dN(y)$$

A solution $(x^*(c))$ to this problem always exists and $x^*(c)$ is a non-increasing function. If $\pi(x,c)$ is concave in x for any c, then $x^*(c)$ is continuous.

For interior solutions the first-order condition is

$$\lambda + \frac{n(x)}{[1-N(x)]^2}[(1-\lambda)((\bar{y}-Q(x))-x(1-N(x))) - \frac{c}{S^{\alpha-1}B^{\alpha}}] = 0 \tag{3.2}$$

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(1) *For a detailed discussion see Lucena, D. (1978).*

It is in general impossible to get explicitly $x^*(c)$, but it is rather easy to get $c^*(x)$, its inverse.

The value of engaging in this process of search, for a seller with effort cost c is:

$$V(c) = \pi(x^*(c),c) = x^*(c) - \lambda \frac{1-N(x^*(c))}{n(x^*(c))} \tag{3.3}$$

The function $V(.)$ is continuous and decreasing, with $d^2V/dc^2 \geq 0$, whenever this derivative exists.

Similar results can be obtained for the buyer. Assume they face a distribution of sellers reservation prices $m(x)$, $x \in [p',p'']$, with $m(x) = dM\backslash dx \geq 0$. A buyer sets his reservation price in such a way as

$$\max_y \rho(y,b) = E\{\text{transaction price} + \text{search cost}\}$$

Again by a simple argument

$$\rho(y,b) = \lambda \frac{R(y)}{M(y)} + (1-\lambda)y + \frac{b}{S^{\alpha}B^{\alpha-1}} \frac{1}{M(y)} \tag{3.4}$$

where

$$R(y) = \int_{p'}^{y} x \, dM(x)$$

A solution $y^*(b)$ to this problem will always exist, and $y^*(.)$ is a nondecreasing function. If $\rho(y,b)$ is convex in y for any b, $y^*(b)$ will be continuous. For interior solutions the first order condition is

$$(1-\lambda) + \frac{m(y)}{M^2(y)}\left[\lambda(yM(y) - R(y)) - \frac{b}{S^{\alpha}B^{\alpha-1}}\right] = 0 \qquad (3.5)$$

Again it is impossible to get explicitly $y^*(b)$, but it is easy to obtain an analytical expression for $b^*(y)$, its inverse. The value of engaging in search, for a buyer with effort cost b will be

$$Y(b) = \rho(y^*(b),b) = y^*(b) + (1-\lambda)\frac{M(y^*(b))}{m(y^*(b))} \qquad (3.6)$$

The function $y(.)$ is continuous and increasing, with $d^2Y/db^2 \leq 0$.

Sellers will have a limit value below which they will not accept to participate in the market. If $V(c) < \underline{w}$, where $\underline{w}$ can be taken as the value of the next best use for the house, they will simply decide not to sell. Also, buyers will place an upper limit, $\overline{w}$ above which they will not accept to participate in the market. If $Y(b) > \overline{w}$, they will drop out.

4. Equilibrium Price Distributions

In equilibrium, some relations must exist between the price distributions and the distributions of the searching populations by effort cost. It is immediate from the discussion above that

$$\begin{cases} F(c) = 1 - M(x^*(c)) & \text{(4.1.a)} \\ G(b) = N(y^*(b)) & \text{(4.1.b)} \end{cases}$$

or, if $x^*(c)$ and $y^*(b)$ have inverses

$$\begin{cases} M(x) = 1 - F(c^*(x)) & \text{(4.2.a)} \\ N(y) = G(b^*(y)) & \text{(4.2.a)} \end{cases}$$

Where $c^*(x)$ and $b^*(y)$ can easily be obtained from (3.2) and (3.5).

We can look at (4.1.a,b) as a set of integro-differential equations that determine $M(.)$ and $N(.)$, given B, S and the distributions $F(.)$ and $G(.)$. Now, $F(.)$ and $G(.)$ are not the exogeneous variables of the model; the entrance rates are. The quit rates implied by the price distributions can easily be calculated. Take a seller with effort cost c. He sets $x^*(c)$ as his reservation price. He will make, on average, a certain number of contacts, before he strikes a bargain:

$$n(c) = \frac{1}{1 - N(x^*(c))} \qquad (4.3)$$

In each unit of time he makes $S^{\alpha-1}B^{\alpha}$ contacts. The average time he stays in the market is

$$\tau(c) = \frac{1}{S^{\alpha-1}B^{\alpha}} \frac{1}{1-N(x^*(c))} \tag{4.4}$$

The number of sellers with effort costs in $(c,c+dc)$ is $Sf(c)dc$. The quit rates can be written as:

$$q_s(c) = \frac{Sf(c)}{\tau(c)} = S^{\alpha}B^{\alpha}(1-N(x^*(c))\ f(c) \tag{4.5}$$

Similarly the buyers quit rates are:

$$q_b(b) = S^{\alpha}B^{\alpha}\ M(y^*(b))\ g(b) \tag{4.6}$$

If the quit rates are not equal to the exogeneous entrance rates, both the population (B,S) and its distribution (F,G), will change. Associated with those there are changes in the price distributions. The process can be seen as adjusting through this mechanism, driven by the entrance rates.

The question of existence of equilibrium price distributions can be addressed by looking at equations (4.1 and 4.2) as a mapping of $Y \times Y \rightarrow Y \times Y$, when Y is the space of all cumulative distributions in $[\underline{w},\bar{w}]$. Given the searching populations, B,S and its distributions, F,G, take any (perceived) pair of price distributions, (M_0,N_0). The agents decisions will then generate a pair of distributions (M_1,N_1), defining a continous mapping of $Y \times Y \rightarrow Y \times Y$. This mapping can now take (M_1,N_1) into (M_2,N_2) and so on. The argument now follows the proof developed by Kormendi (1979). Using the supnorm as the metric, the set of all bounded functions is a Banach space. The set of all cumulative functions is a subset of this space which is convex and forms a linear metric space. The closure of Y, $\bar{Y}$, is compact. ($\bar{Y}$ is the set of functions that have the usual properties of cumulatives, but have right limits at the points of discountinuity). As far as our mapping is concerned, Y is equivalent to $\bar{Y}$. So by Schauder theorem a fixed point exists in $\bar{Y} \times \bar{Y} \rightarrow \bar{Y} \times \bar{Y}$ mapping, which is in $Y \times Y$.

5. The Unform Case: No Broker

It is reasonable to assume that, in many of these markets buyers and sellers are drawn from the same population. Entrance rates will then be the same for both sides of the market, and distributed in the same way by effort costs. Call $E(c)$ and $e(c)$ the cumulative and p.d.f. of this distribution. This symmetry suggest a choice of $\lambda = 1/2$. Naturally, in equilibrium, both searching populations will be the same, $B=S=P$, and distribuled in the same way. Call $F(c)$ and $f(c)$ to the cumulative and p.d.f. of this common distribution. Here I will analyse the case where the distribution $E(.)$ is uniform in a given internal $[0,c'']$. This case, although particular, can lead to several interesting outcomes, some of them with non-intuitive results about the functioning of this type of markets. It has the big advantage of having a analytical solution that can be easily computed, allowing for a richness of analysis otherwise impossible. In another paper (Lucena (1988)) I try to assess how general are the results obtained here. For simplicity take $\alpha = 1$ in the search technology.

The total numbers of buyers and sellers entering the market, A, are distributed according to

$$E(c) = \frac{c}{c''} \quad ; \quad e(c) = \frac{1}{c''} \quad , \quad c \in [0,c'']$$

It is very easy to verify that uniform price distributions are consistent with uniform entrance rates. Make

$$M(x) = N(x) = \frac{x-p'}{\Delta} \quad , \quad x \in [p',p'']$$

where $\Delta = p'' - p'$ is the spread of the price distribution. For symmetry reasons we can make

$$\bar{p} = \frac{p'+p''}{2} = \frac{\underline{w}+\bar{w}}{2}$$

Using these distributions to compute optimal pricing $x^*(c), y^*(c)$, and 4.1, we get

$$F(c) = \sqrt{\frac{4}{3}\,\frac{c}{\Delta P}} \tag{5.1}$$

Notice that $F(c'') = 1$. By the equilibrium condition that quit rates of buyers and sellers are equal, and equal to the entrance rates, we get:

$$P^2 = 2A \tag{5.2}$$

$$F(c) = \sqrt{\frac{c}{c''}}\ , \quad f(c) = \frac{1}{2}\sqrt{\frac{1}{cc''}} \qquad c \in [0, c''] \tag{5.3}$$

$$\Delta = \frac{4}{3}\,\frac{c''}{\sqrt{2A}} \tag{5.4}$$

The optimal policy decisions are then:

$$\begin{cases} x^*(c) = \bar{p} + \dfrac{2}{3}\dfrac{c''}{\sqrt{2A}} - \dfrac{4}{3}\dfrac{\sqrt{cc''}}{\sqrt{2A}} \\ \\ y^*(c) = \bar{p} - \dfrac{2}{3}\dfrac{c''}{\sqrt{2A}} + \dfrac{4}{3}\dfrac{\sqrt{cc''}}{\sqrt{2A}} \end{cases} \tag{5.5}$$

and the "value of search":

$$\begin{cases} V(c) = \bar{p} + \dfrac{2}{3}\dfrac{c''}{\sqrt{2A}} - 2\dfrac{\sqrt{cc''}}{\sqrt{2A}} \\ \\ Y(c) = \bar{p} - \dfrac{2}{3}\dfrac{c''}{\sqrt{2A}} + 2\dfrac{\sqrt{cc''}}{\sqrt{2A}} \end{cases} \tag{5.6}$$

We have a flow of A sellers and buyers through the search market, i.e. we have A transactions per unit of time. In each transaction there is a welfare gain of $\bar{w}-\underline{w}=w$. If everybody decide to participate in search the total welfare, generated by the market, in each unit of time is

$$B = Aw - TS \tag{5.7}$$

where TS is the flow of total search costs.

$$TS = \int_0^{c''} 2P\, f(c)c\, dc = \frac{2}{3}\sqrt{2A}\, c'' \tag{5.8}$$

and

$$B = Aw - \frac{2}{3}\sqrt{2A}\,c'' \qquad (5.9)$$

The last participant decides to withdraw from searching if $Y(c'') < \underline{w}$ (or $Y(c'') > w$).

Full participation will take place if

$$\frac{9}{32}A\left(\frac{w}{c''}\right)^2 > 1 \qquad (5.10)$$

For the moment just assume this condition to hold. The decision to participate will be more carefully analised together with the introduction of a broker.

It is interesting to make some comparative statics in the two parameters that define the situation: A and c".

Start with an increase in A. Notice first that $\partial P/\partial A > 0$, and so the total searching population also increases in equilibrium. But F(.) does not depend on A (nor does it depend on α) being solely determined by E(.).

As a consequence of P increasing the search cost will decrease for everyone. Simultaneously, is $\partial\Delta/\partial A < 0$: the price distribution will become more concentrated around p. A simple interpretation of what is going on is that the zero search cost people, who set the extreme reservation prices lose some of their "monopoly power" over the searchers with highest costs of the other side.

Individual welfare then decreases for low search cost people ($c < c''/9$)

and increases for high search cost ones ($c > c''/9$). Simultaneously optimal pricing policies change. For the seller's side, searchers with low cost decrease this asking price ($c < c''/4$) and the other increase the asking price.

The flow of total search costs increases as expected, $\partial TS/\partial A > 0$. But average search costs decrease $\partial(TS/A)/\partial A < 0$. So there is a welfare gain ($\partial B/\partial A>0$), which is to be expected, but also a gain on average welfare $\partial(B/A)/\partial A>0$. This is the result of the positive externality that an increase in P introduces by lowering everyone search costs.

Notice however that $\partial(TS/P)/\partial A = 0$, i.e., when the average is taken for all market participants average search cost do not change. This shows how carefull one must be when taking averages in applied work. For welfare analysis the average must be taken in the flow and not in the stock.

The average time searching $\bar{\tau} = P/A = \sqrt{27A}$. Then $\partial\bar{\tau}/\partial A < 0$ and, on average, people stay less time in the market. Actually this is true also for everybody, for $\tau(c) = (1/\sqrt{2A})\sqrt{c''/c}$ is the average searching time as function of c, and $\partial\tau(c)/\partial A < 0$. This is at a first sight counterintuitive. Search costs do decrease for everyone, but average time searching also decrease. The reason is that the total (P) and average (P/A) number of contacts per unit of time also increase, while the total number of contacts before striking a bargaining ($n(c) = \sqrt{c''/c}$) does not change.

Individual welfare of a seller, as a function of c, can be decomposed as $V(c)=\bar{p}(c)-T(c)$, where $\bar{p}(c)$ is expected sale price and $T(c)= \tau(c).c=\sqrt{cc''}/P$ is total expected search costs. Clearly $\partial T(c)/\partial A < 0$ for everybody. So $\bar{p}(c)$ must decrease for low search cost sellers. Actually $\partial\bar{p}(c)/\partial A < 0$ iff $c < 4/9\, c''$.

The limiting case, as $A \rightarrow \infty$ leads to $\Delta = 0$, all the transactions at $\bar{p}$, and $V(c)=Y(c)=\bar{p}$. Individual search costs ($T(c)$) go to zero, but total search costs don't because A and P grow also indefinetly. However the benefits grow faster and total welfare is unbounded.

An increase in c", without changing A, has no effect on P. The total number of searchers depends only on total entrance rates, and not on their distribution. Now the spread of the price distribution increases: $\partial\Delta/\partial c" > 0$. Zero effort cost people are now able to use their increased "monopoly power" over the highest effort cost people of the other side of the market.

Only the low effort cost sellers will be better off ($c < 4/9\, c"$), while for a fixed c, the asking price will always increase ($\partial x^*(c)/\partial c" > 0$). Notice however that at the "new" c", the asking price is smaller than it has at the "old" c"($dx^*(c")/dc" < 0$), as needed to be consistent with zero effort cost people being better off. Total welfare decreases, as total search costs per unit of time increase ($\partial TS/\partial c" > 0$) while total number of sales is the same (A), with the same gross benefit (w).

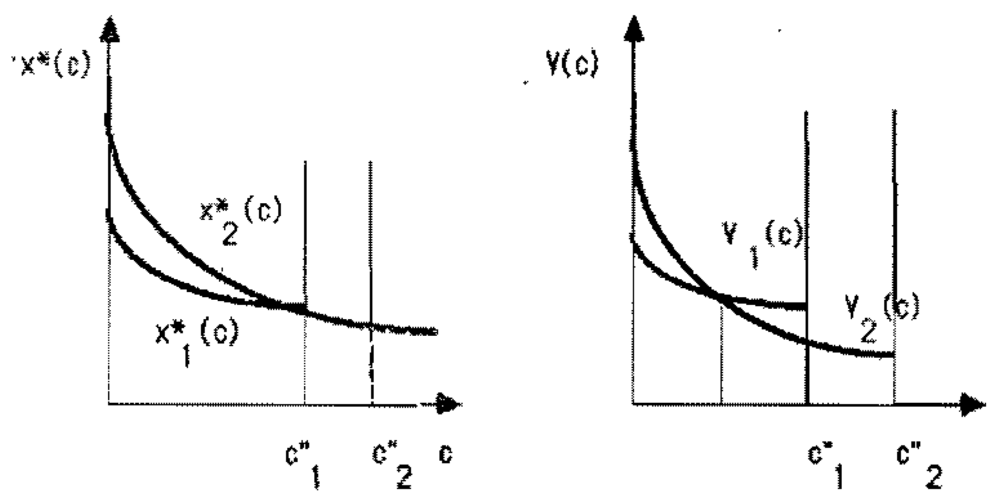


Fig. 1: An Increase in c"

Average time in the market $\bar{\tau}$ does not change. However, for fixed c, $d\tau(c)/dc">0$. "Old timers" stay longer in the market, while the highest effort cost will stay the same one period as before ($d\tau(c")/dc" = 0$). The two facts are made compatible by a change in the equilibrium distribution, F(c), that decreases the relative wheight of low search cost people.

For any seller total expected search cost increases ($dT(c)/dc" > 0$). The expected sale price increase for any fixed c, although the new c" searchers have a smaller expected sale price.

For "old timers" on the seller side the expected sale price increases ($\partial\bar{p}(c)/\partial c" > 0$), but for the new "last seller" it is now smaller than it was before ($\partial\bar{p}(c")/\partial c" < 0$, as should be expected, for this seller makes only one contact and faces now a worse situation.

6. The Broker

The broker is an agent that offers to buy and sell houses at given prices. Using the broker's services avoids the need to use costly search.

In a symmetric equilibrium the broker must offer buying and selling prices that are symmetric in relation to $\bar{p}$. Otherwise he will get a bigger flow of clients on one of the sides, which is not possible as long term solution.

Call s the price at which the broker buys houses. In each transaction he makes $2(\bar{p}-s)$. If the total number of transactions going through the broker in n, in each unit of time, his profit is

$$\phi(n) = 2(\bar{p}-s)n - c(n) \tag{6.1}$$

In an equilibrium where search and the broker exist, the high effort cost agents will be the ones using the broker. Call $c_e(s)$ the limiting effort cost:

$$n = \int_{c_e(s)}^{c''} A\, e(c)dc = A[1 - E(c_e(s))] \tag{6.2}$$

The flow of people entering the search market is now given by $AE(c_e)$. And they are distributed according to $E^*(c)=E(c)/E(c_e)$. The search equilibrium generated by these agents must be such that $V(c_e(s)) = s$ which is the equation defining $c_e(s)$.

To study the equilibrium with the broker we must first understand how the search equilibrium changes when c_e changes. And then to analyse how introducing of the broker, changing its costs or of the structure of the brokerage industry does affect the way people will split among searchers and broker users.

7. Search Equilibria

Search equilibria will be analysed, parametric on the value of c_e. By changing c_e we change simultaneously the total number of entrants in the search market and their distributions.

Take the entrance rates as uniform in $[0,c'']$. And call $A = e\,c''$ the total number of entrants in each side of the market. Then $E(c) = c/c''$ and $e(c) = 1/c''$. If the limiting effort cost, separating those that engage in search from those that don't, is $c_e \le c''$, then the entrants in the search market are given by $AE(c_e) = e\,c_e$. The relevant distribution becomes $E^*(c) = c/c_e$, and $e^*(c) = 1/c_e$. The distribution of current searchers by effort costs is $F^*(c) = \sqrt{c/c_e}$ and the total population of current searcher is $P = \sqrt{2AE(c_e)} = \sqrt{2A}\sqrt{c_e/c''} = \sqrt{2ec_e}$.

Price distributions are still uniform, centered at $\bar{p}$, with a spread given by $\Delta = 4/3\sqrt{c_e c''/2A} = 4/3\sqrt{c_e/2e}$. For the sellers side the optimal pricing policy and the value of search function become:

$$x^*(c) = p + \frac{2}{3}\sqrt{\frac{c_e c''}{2A}} - \frac{4}{3}\sqrt{\frac{cc''}{2A}} = p + \frac{2}{3}\sqrt{\frac{c_e}{2e}} - \frac{4}{3}\sqrt{\frac{c}{2e}}$$

and

$$V(c) = \bar{p} + \frac{2}{3}\sqrt{\frac{c''c_e}{2A}} - 2\sqrt{\frac{cc''}{2A}} = p + \frac{2}{3}\sqrt{\frac{c_e}{2e}} - 2\sqrt{\frac{c}{2e}}$$

The welfare benefits generated by the existence of the market are, per unit of time, $B(c_e) = AE(c_e)w = ewc_e$; total search costs, also per unit of time are

$$TS(c_e) = \int_0^{c_e} 2P(c_e)cf^*(c)dc = \frac{2}{3}\sqrt{\frac{2A}{c^*}}\,c_e^{3/2} = \frac{2}{3}\sqrt{2e}\,c_e^{3/2}$$

Total welfare gains is given by β-TS

$$B(c_e) - TS(c_e) = ewc_e - \frac{2\sqrt{2}}{3}\sqrt{e}\,c_e^{3/2}$$

For any given value of c_e the comparative static results obtained before still apply, with the same interpretations. The interesting analysis now is to make compartative statics in c_e. Notice this is quite a different exercise from the one made before by changing c^*: both total number of entrants and their distribution change simultaneously in a very specific way.

Take an increase in c_e. The key result is that $\partial\Delta/\partial c_e > 0$. This means that low effort cost sellers will raise their asking price and be better-off now. Actually all the "old-timers" will increase their asking price and be better-off than before. The new entrants cause always a positive externality for all those already in the market. The new "last-seller" is however worse off than the "old last seller", and will set a lower asking price.

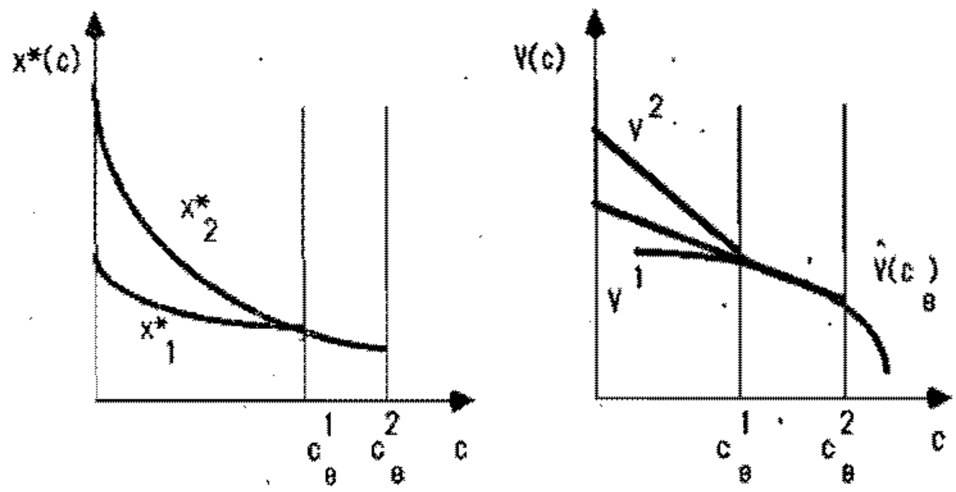


Fig. 2: A Change in c_e

This last result, that $\hat{V}(c_e)$ is a decreasing function, means that the demand for the broker's services is well behaved, in the sense that depends negatively on the margin he sets.

In this case, when c_e increases the price policies are adjusted in such a way that the number of contacts each agent makes to get a sale increases $(n(c)=\sqrt{c_e/c})$. But, given the increase in P, it is easier to make a contact, and the average searching time does not change $(\tau(c) = 1/\sqrt{2ec})$. As a consequence the total expected cost of search, for any agent does not change $(T(c)=\tau(c).c=\sqrt{c/2e})$. The individual welfare gain from participating in the market is $V(c)=\bar{p}(c)-T(c)$, where $\bar{p}(c)$ is the expected sale price. The increase in $V(c)$ when c_e increases is exclusively due to an increase in $\bar{p}(c)$. However $\partial\hat{p}(c_e)/\partial c_e < 0$, which is consistent with $V(c_e)$ being a decreasing function.

When one introduce the broker the value of c_e will become endogeneous. Each agent will decide wether to search or use the broker. But he can also decide not to enter the market at all because the transaction costs are too high. It is conceivable that some very high search cost sellers will decide not ot participate in the search market but are willing to use the brokers services at the price he sets.

8. Equilibrium and Optimal Participation

A seller (buyer) will participate in the market, and engage in search, only if the expected price he gets, net of expected search costs, is bigger (smaller) than his reservation value $\underline{w}(\bar{w})$:

$$\hat{Y}(c_e) = \bar{p} - \Delta > \underline{w}$$

$$\hat{Y}(c_e) = \bar{p} + \Delta < \bar{w} \qquad (8.1)$$

Making as before $w = \bar{w} - \underline{w}$, we get the equivalent condition $\Delta(c_e) \leq w/2$, which has as solution $c_e/c" \leq 9/32\ A(w/c")^2$.

Then:

(i) if $9/32\ A(w/c")^2 < 1$, the equilibrium participation is partial and $c_e/c" = 9/32\ A(w/c")^2$

(ii) if $9/32\ A(w/c") \geq 1$, the equilibrium participation is full participation and $c_e = c"$.

New entrants, as we have seen, will cause an externality in those already participating. There is then a reason to think that the optimal level of participation may no coincide with the equilibrium.

The total welfare, net of search costs, is given by

$$B - TS = e\,w\,c_e - \frac{2\sqrt{2}}{3}\sqrt{e}\,c_e^{3/2}$$

The optimal participation is found by making

$$\frac{\partial(B-TS)}{\partial c_e} = 0 \text{ , or}$$

$$\frac{c^*_e}{c''} = \frac{A}{2}\left(\frac{w}{c''}\right)^2$$

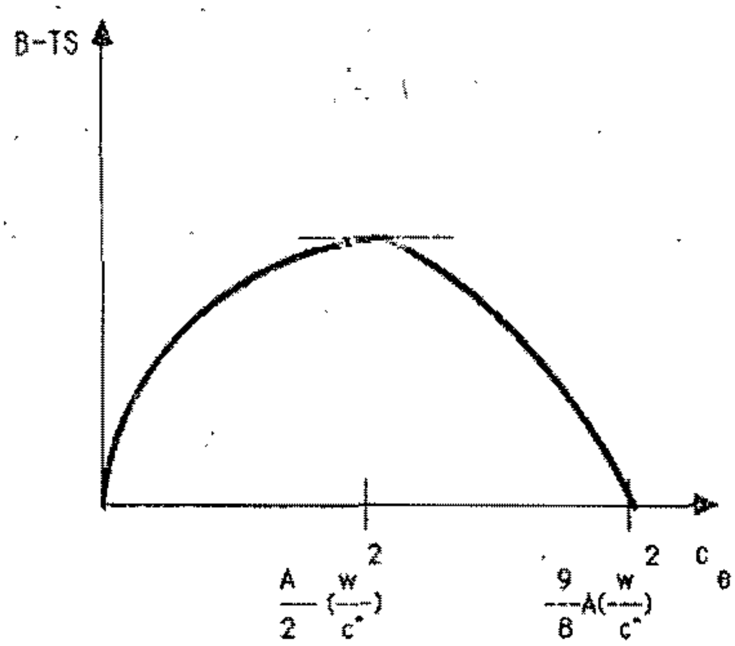
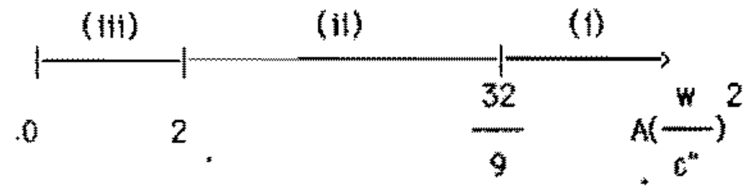


Fig. 3: Welfare as a Function of c_e.

So, if $A/2(w/c'')^2 < 1$ the optimal participation is partial and given by $c_e/c'' = A/2(w/c'')^2$. Otherwise the optimum is full participation and $c_e/c'' = 1$.

Three cases are relevant



CASES	PARTICIPATION Equilibrium	PARTICIPATION Optimum	WELFARE Equilibrium	WELFARE Optimum
(i)	FULL	FULL	$\sqrt{\frac{8A}{9}}\, c''[2\sqrt{\frac{9}{32}A(\frac{w}{c''})^2} - 1]$	
(ii)	PARTIAL	FULL	$\frac{9}{64}A^2(\frac{w}{c''})^2 w$	(same as above)
(iii)	PARTIAL (<)	PARTIAL	$\frac{9}{64}A^2(\frac{w}{c''})^2 w$	$\frac{1}{6}A^2(\frac{w}{c''})^2 w$

9. Equilibrium with the Broker

The broker will decide on its pricing policy, given by s. For any given value of s, the effort cost separating searchers from brokers's users is given by $\hat{V}(c_{\theta}) = s$.

In this case $\hat{V}(c_{\theta}) = \bar{p} - \Delta = s$, and this leads to $c_{\theta} = 9/8\ e(\bar{p}-s)^2$. The total number of entrants in the search market is given by $A(c_{\theta}) = A\ c_{\theta}/c'' = 9/8\ e^2(\bar{p}-s)^2$, and the total search population, in equilibrium is $P=3/2\ e(\bar{p}-s)$. Assuming that the pricing policy of the broker is such that no one leaves the market, the demand for the broker's services, in number of transaction per unit of time is

$$D(s) = A - \frac{9}{8} e^2(\bar{p}-s)^2 \qquad (9.1)$$

Simple computations establish that the total search costs per unit of time, by all participants in the search market, are given by

$$TS = \frac{9}{8} e^2(\bar{p}-s)^3 \qquad (9.2)$$

To isolate the pure informational aspects, when dealing with the organization of brokerage industry, I will assume a very simple cost structure for this activity: a constant unit cost γ, per transaction.

Then the profits of the industry are

$$\pi(s) = D(s)[2(\bar{p}-s) - \gamma]$$

The extreme cases, where the industry is competitive or fully monopolized, are discussed below. The case of a welfare-maximizing broker is also studied, and is used as a bench-mark.

The competitive equilibrium in the brokerage industry will lead to

$$\bar{p} - s_F = \frac{\gamma}{2} \tag{9.3}$$

The welfare maximizing broker will has as an objective

$$\min_{s} \gamma D(s) + TS(s)$$

when $\gamma D(s)$ is total costs of the brokerage industry and $TS(s)$ total search costs of those engaging in search (we keep assuming $2(\bar{p}-s) < w$). The solution is

$$\bar{p} - s_W = \frac{2}{3}\gamma \tag{9.4}$$

The monopolist broker will maximize his profits against the demand $D(s)$:

$$\max_{s} D(s)[2(\bar{p}-s) - \gamma]$$

The solution is

$$\bar{p} - S_M = \frac{\gamma}{3} + \sqrt{\frac{\gamma^2}{9} + \frac{32}{27}\frac{A}{c^2}} \geq \frac{2}{3}\gamma \tag{9.5}$$

So it will always be

$$\bar{p} - S_M > \bar{p} - S_W > \bar{p} - S_F \tag{9.6}$$

When the brokers margin increases, starting from $\bar{p} - S_F$, some people will switch from the broker to the search market. They will be in worse situation than before and so will be those staying with the broker. But both the brokers profits and the "old time searchers" welfare will increase. Initially these two effects will more than compensate the losses. The optimal degree of competition for the brokerage industry will depend on the network externality of the search market, which is, after all, a market performing the same job as the market for broker's services. As the brokers price (margin) gets bigger the relative size of the effects will be altered but the monopoly situation is also not optimal.

The total welfare, in each case, can be easily computed by noticing that it must be

$$TW = Aw - \gamma D(s) - TS(s)$$

This leads to

$$TW_F = A(w-\gamma) + \frac{9}{64}\theta^2\gamma^3 \tag{9.7}$$

$$TW_W = A(w-\gamma) + \frac{1}{6}\theta^2\gamma^3 \qquad (9.8)$$

$$TW_M = A(w-\gamma) + \left[\frac{1}{4}\theta^2\gamma^2 - \frac{A}{3}\right](\bar{p}-S_M) \qquad (9.9)$$

It is not possible to get, in a simple way, TW_M as function only of the exogeneous parameters. But it is not difficult to see that $TW_F > TW_M$ if γ is small and that $TW_M > TW_F$ if γ is big. An increase in the net cost of the brokerage industry weakens the case for competition. This creates a definite link between technological progress and the organization of industry. If, associated with new technologies, there is a presumption of decreasing costs, then it will be optimal to allow a certain degree of cartelization.

We are now in a position to evaluate the welfare impact of the brokerage industry (opening up a new market), when there was already a search market operating. That impact will depend on the conditions prevailing before the broker makes its appearence and on the organization of the industry.

First take the competitive case, and assume that $9/32\ A(w/c'')^2 > 1$, so that the equilibrium before the broker entailed full participation.

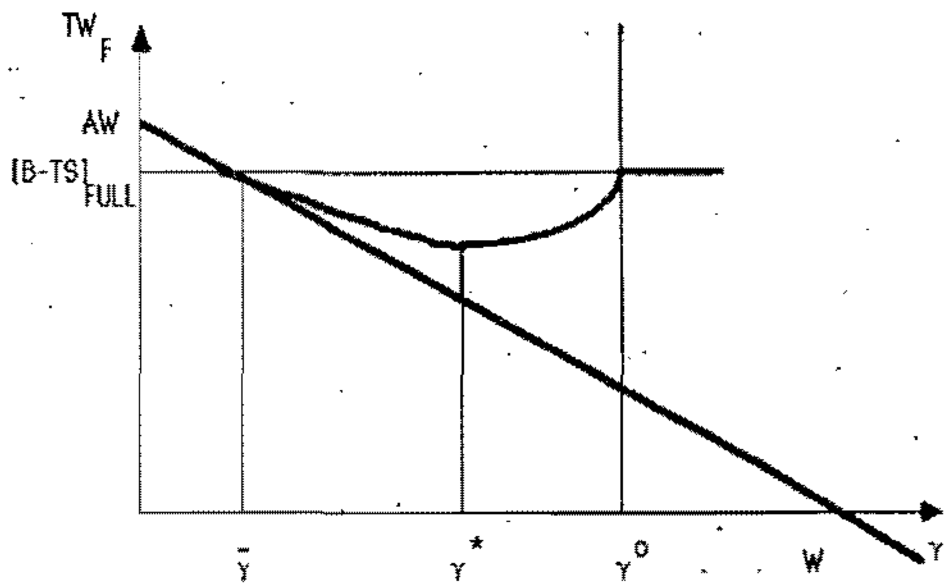


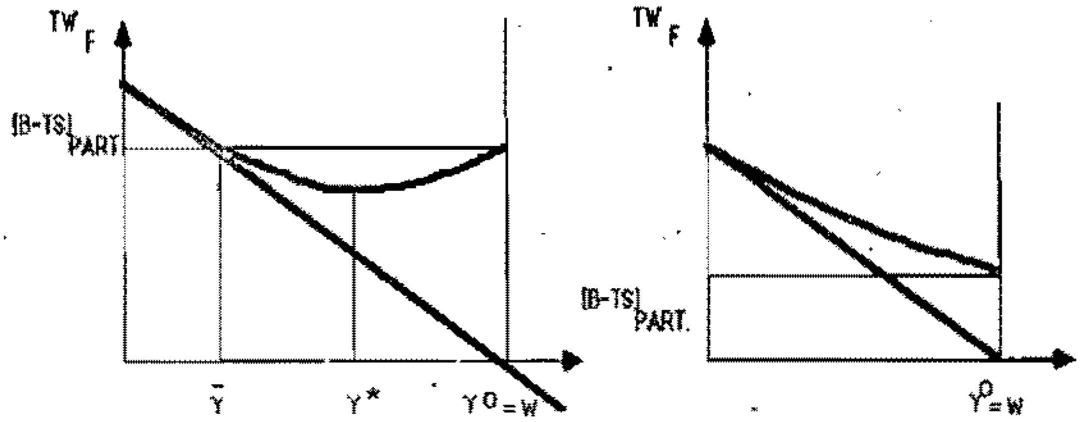
Fig. 4: Welfare with Competitive Broker

$TW_F(\gamma)$ has a minimum at $\gamma^* = (8/3\sqrt{3})(c''/\sqrt{A})$. Broker's demand becomes zero at $\gamma^0 = (4\sqrt{2}/3)(c''/\sqrt{A})$, and so at this point $TW_F = [B-TS]_{FULL}$, which is the welfare without the broker. For $\gamma > \gamma^0$ there is no place for the broker. The other point at which $TW_F = [B-TS]_{FULL}$ is given by $\bar{\gamma}=(\sqrt{4/3(\sqrt{10}-\sqrt{2}})(c''/\sqrt{A})$.

Some interesting points come out of this analysis. For one thing, introducing a brokerage industry, even if it is competitive, may be welfare decreasing ($\gamma > \bar{\gamma}$). Increasing the number of possible "channels of communication" among buyers and sellers, is not always a good thing. Also somewhat paradoxically, an increase in the unit costs may be welfare increasing ($\gamma^* < \gamma < \gamma^0$).

If w is sufficiently small, it may happen that people will leave the market altogether, rather than use the broker. This will happen if $\gamma > w$, in the competitive case.

The other two relevant cases (see page 25) are depicted below.



$\frac{2}{3} < \frac{9}{32} A(\frac{w}{c^*})^2 < 1$ $\qquad$ $\frac{9}{32} A(\frac{w}{c^*})^2 < \frac{2}{3}$

Fig. 5: Welfare and the Competitive Broker

In both cases the equilibrium is partial, when the broker is absent. When γ gets to γ^0 no one will ever use the broker. For values of $\gamma < \gamma^0$, there is a split of the total population among searchers and broker's users. In the second figure the equilibrium before the broker is a very bad outcome and it is always better to have a broker as long as $\gamma < w$.

A welfare maximizing broker will always increase welfare, as could be anticipated. The three value $\bar{\gamma}$, γ^*, γ^0 will coincide and the counter-intuitive effects do not exist.

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