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"TECHNICAL PROGRESS, WAGES AND EMPLOYMENT IN A CONTRACTUAL ECONOMY WITH INCOMPLETE INFORMATION"

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TECHNICAL PROGRESS, WAGES AND EMPLOYMENT IN A CONTRACTUAL ECONOMY WITH INCOMPLETE INFORMATION**

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ABSTRACT

This paper addresses the stylized fact that individual firms, when undertaking major processes of technical innovation, appear to reduce employment and, symultaneously, to offer wage increases to the remaining workers independently of how slack is the labour market. Because this sits unconfortably with the standard labour market stories we develop here an approach based on the theories of idiosyncratic exchange, namely the optimal contracts and the bargaining theories. The obtained results are suggestive that bargaining theories do not offer an explanation for that stylized fact but the contractual model, with incomplete information, under a set of unconventional but realistic assumptions does explain that stylized fact.

1. INTRODUCTION

The general question of the effects of technical progress on employment and wages is gaining an increased importance for both professional economists and the public. One regularly finds in the press statements that technological progress destroys jobs while many policymakers prefer to point the accusatory finger to workers choosing to price themselves out of jobs. These two views need not be incompatible and our goal in this paper is to model circumstances in which, simultaneously, employment falls and wages are increased.

Technical progress has been dealt with in various ways in the literature. Here we will represent it as a reduction in the price of the efficiency units of capital. This modelling strategy was motivated by, among other things, the recent fall in the price of electronic gadgetry.

The "stylized fact" which we address is that individual firms, when undertaking major processes of technical innovation, appear to reduce employment and, simultaneously, to offer wage increases to the remaining workers. Moreover, this seems to happen independently of how slack is the labour market.

One possible explanation which fits into a standard labour market story is that a firm needs higher quality workers when new machines are in place, and hence has to pay higher wages, but this sits unconfortably with the fact that it is (part of) the <u>old</u> work force which is being retained. Given that old workers appear to benefit from the new technology, without reference to the outside labour market, it suggests that one ought to model the process using theories of idiosyncratic exchange - in particular bargaining and contract theories.

In section 3 we will suggest that bargaining theories do <u>not</u> offer a consistent explanation for our "stylized fact" because, under reasonable assumptions, technological innovation should lead not only to an increase in wages but also to an <u>increase</u> in employment. For this reason we developed a more unconventional model based on contract theory with limited information. This model, which is presented in section 2, <u>does</u> explain our stylized fact.

The idea of the model is that wages can only be made contingent on easily verifyable conditions at the sharp end of the productive process. In actual contracts, see e.g. Johnson(1985), one does not find that wages are explicitly made contingent on a variable such as the state of technology available outside. Nor, indeed, are they ever contingent on total employment or capital stock. What <u>does</u> seem critically important are the "working conditions", which we here concretely think of as the capital-labour ratio at the work face. It is commonly observed that when management introduces new equipment and changes the manning levels (perhaps as a consequence of a reduction in its cost) there is serious negotiation over wages and/or "productivity bonuses".

The fact that wages cannot be conditioned directly on the state of technology (i.e. on the price of capital) led us to assume that the price of the capital units is private information of the firm. The simplest motivation for this assumption rests on the oligopolistic nature of the market for capital equipment, tipically a customized good, and the associated strategic behaviour in pricing.

The predictions of our model, namely that in an optimal contract a decrease in the price of capital will lead to higher wages and at the same time the firm will increase the capital-labour ratio and decrease total employment , are robust to various assumptions about the market structure for the firm's output.

The plan of the paper, then, is to present the contract model in section 2 and to discuss the bargaining approach in section 3. A comparison between the main results, and the conclusions, are in section 4. Mathematical proofs are in the appendix.

2.THE CONTRACTUAL ECONOMY

In this section we will analyse a contractual model of wage and employment determination capable of offering a theoretical explanation for the simultaneous reduction in employment and increase in wages for the remaining workers that some firms propose to their labour pool when wishing to introduce major changes in their production techniques. These technological changes are usually seen to imply an increase in the capital intensity of the production process.

More intuitively we are thinking about those situations where the firm wishes to adopt more "automatic" capital units, be it through the introduction of "robots" or otherwise. Also it is well known that the price of the electronic gadgetry that is usually associated with the more automatic machines has been decreasing. This particular development seems to sustain the detectable public feeling that automation is, at least partly, behind the recent decrease in industrial employment observed in many of the more advanced economies.

Reasoning purely on the basis of the neoclassical theory of employment determination the immediate question to ask is what prevents wages or relative wages to adjust in such way as to prevent the emergence of layoffs, in the conditions described above. Moreover what one really observes is not only persistence or rigidity in wages but actually increases in the wages paid to the workers that remain employed after the introduction of the new machines.

Issues as the recently observed wage concessions where one notes a shift in the emphasis of contractual clauses towards the protection of employment will be sidestepped. The reason for this is that those concessions seem to be circumscribed not only geographically (mainly to the U.S.A. economy) but also to extreme circumstances where the firm threathens bankruptcy. One could argue that those special circumstances call for a major reapraisal of the implicit contracts while here we intend to analyse the characteristics of the existing implicit contracts.

The modeling strategy consists in summarizing the state of technology by the purchase price (r) of assumedly homogeneous capital units, henceforth called machines (K). For simplicity we shall consider only two technological states. State one, ocurring with probability (a) with associated (r1) and state two with associated (r2). State one will be called the "good" technological state in the sense that

$$r2 > r1$$

The model is based in, among others, the following four crucial assumptions:

ASSUMPTION 1.

The price of the machines (r) is private information of the firm.

The intuition that motivates this assumption rests on the consideration that those machines, often customized goods, are supplied by a number of price discriminating oligopolists engaging in strategic behaviour and, therefore, keen in not revealing the actual discounts offered to each user. Of course, contractual wages cannot be made contingent on the prices of those machines.

This assumption leads to a technical novelty in the paper since the informational asymmetry is introduced on the cost side of the firm's profit equation whereas the common practice before consisted in introducing it on the revenue side of that equation as can be seen in, e.g., Hart's (1983) survey and in the papers presented at the Q.J.E. (1984) symposium.

Another informational assumption is:

ASSUMPTION 2.

The workers cannot observe the "size" of the firm or its total imputs use. All they can observe are the "working conditions", that is, the capital - labour ratio $k=K/L$.

This assumption is in accordance with the fact that actual contracts in general do not make wages contingent on total employment or on the capital stock and only very rarely make them contingent on variables that proxy for the firm size like total output or total revenue. The explanation of this behavioural assumption is clearly out of the scope of the paper but we shall, nevertheless, attempt to motivate it.

One possible motivation rests on the complexity of the productive processes. On one hand the productive processes are frequently divided through many plants sometimes located in different countries and, on the other hand, the complexities of those productive processes may give rise to externalities such that evaluating factor's use through the "number of heads" like machine counting or total employment would render the result meaningless.

However it is only too frequent to observe that in the labour contracts the "working conditions" are made explicit together with the wages. This we proxy by the manning levels formalized as the capital - labour ratio.

Two other unconventional assumptions of different nature were also made:

ASSUMPTION 3.

All reductions in employment can be accomodated through natural wastage.

ASSUMPTION 4.

Eventual new recruits will have to be paid the same wage as the previously employed workers.

These two assumptions basically simplify the analysis while, we feel, reflect the actual labour market institutions. Natural wastage includes not only all the workers that retire or quit for any reason between contract negotiation and the productive run but also all the workers that the firm induces to "quit" through earlier retirements or otherwise. The major consequence of this assumption is that we may ignore the problems of involuntary layoffs and their rationale.

Assumption 4 reflects the fact that firms when hiring a new recruit do pay him according to a wage schedule previously negotiated instead of engaging in direct bargaining. In this paper we consider all workers to be homogeneous and so there will be a single wage rate for both "old" workers and new recruits. A possible motivation for this assumption is the need of avoiding long term job threats on the part of the workers.

Finally we want to make two further assumptions:

ASSUMPTION 5.

All agents are risk neutral.

This assumption is made in order to concentrate the attention on the implications of the informational incompleteness without mixing these with issues of risk sharing.

ASSUMPTION 6.

The revenue function of the firm (F) is homogeneous of degree g, $0<g<1$, with the usual properties of strict quasi concavity.

Formally, if

r, is the unit cost of capital K.

w, is the wage rate of labour L.

$k=K/L$, is the capital-labour ratio.

$F(K,L) = f(k,L)$, is the revenue function of the firm; we can define

$$\pi(k,w,r) = \max_{L} [\, f(k,L) - rkL - wL \,]$$

and let $L = L(k,w,r)$ achieve that maximum.

Now, with the objective of finding k(r), w(r) and implicitly L(r) and w(k), an optimal contract solves:

$$\max_{k_1,k_2,w_1,w_2} \quad Z=a.\pi\ (k1,w1,r1)+(1-a).\pi(k2,w2,r2)$$

subject to the incentive constraints:

$$\pi_{11}=\pi(k1,w1,r1)\geq\pi_{12}=(k2,w2,r1)$$

$$\pi_{22}=\pi(k2,w2,r2)\geq\pi_{21}=(k1,w1,r2)$$

and the utility constraint:

$$(1-q)[a.w1+(1-a)w2] + q.R \ \geq \ U.$$

where q is the probability of "quit" and R is the worker's income after "quiting".

Note that assumption 3 makes the workers indifferent to q for purposes of contract bargaining. Equivalently one could consider q.R to be an exogenous constant. The utility constraint can, therefore, be simplified to:

$$a.w1+(1-a)w2 \ \geq \ O, \text{ where}$$

$$O=(U-q.R)/(1-q).$$

To see the intuition behind the relevance of the I-C constraints suppose that, in the absence of those constraints,

$$L2=L(k2,w2,r2) > L1=L(k1,w1,r1)$$

Because the workers are risk-neutral increasing w1 and reducing w2 in such a way that expected utility is left unchanged would be a Pareto movement since the firm could gain. In fact it would pay to the firm to twist the wage schedule ($w1,w2$) to make $w1$ "very high" and $w2$ "very low" but, then, it would also pay the firm to cheat by choosing the capital - labour ratio corresponding to the high employment state irrespective of the actual price of capital. Ex- post utility would always fall short of expected utility. The argument, here, applies "mutatis mutandis" to the case where $L1 > L2$ to start with.

We can now establish:

<u>RESULT 1.</u>

Under assumptions 1 to 6, when output prices are constant, an optimum contract has:

$$w1^* > w2^*$$

$$k1^* > k2^*$$

$$L1^* < L2^*$$

Proof: see the appendix.

This result <u>does</u> explain our stylized fact. It should be said that the crux of the proof lies in showing that it is the I-C constraint

$$\pi 11 = \pi(k1,w1,r1) \geq \pi 12 = \pi(k2,w2,r1)$$

that binds.

Intuitively what is happening is that in the good state the firm would like to seize the opportunity to increase production and can do it in two ways. One could be simply to increase employment without any capital deepening (thereby concealing that state one occured) and paying $w2$.

The workers need to make the wages to increase in state one in order not to let the firm to overemploy in that state. Now, if employment cannot increase, the other way for the firm to increase production is through capital deepening with the consequence of revealing that the good state has occured. The precise way to bring about this truthful revelation involves an increase in wages and a decrease in employment.

It is particularly interesting to note that this truth telling mechanism involves the workers somehow "taxing" the firm's investments in new technologies:

$$w=w(k) \text{ with } w'(k)>0.$$

One subsidiary result of the analysis above is that output should be higher in state one than in state two. The fact that output is not constant suggests the extension of the analysis to consider explicitly a negatively sloped demand curve for the firm's output. To keep matters as simple as possible we will now assume that the firm is a monopolist in the market for its output and that the price elasticity of demand is greater or equal to one, as required for a profit maximizing monopolist. Formally:

<u>ASSUMPTION 6'.</u>

The firm's revenue function is

$$F(K,L) \equiv f(k,L) \equiv [h(k,L)]^{(1-b)}$$

where $b = -1/e$ and e, $e < -1$, is the price elasticity of demand.

Note that b is positive and smaller than one.

RESULT 2.

Under assumptions 1 to 5 plus 6' an optimum contract has:

$$w1^* > w2^*$$
$$k1^* > k2^*$$
$$L1^* < L2^*$$

Proof:

The proof is similar to that of result one.[1]

Taken together as limiting cases, results 1 and 2 show that our explanation of the mentioned "stylized fact" is robust with respect to the market structure for the firm's output.

In the next section we will turn our attention to bargaining models as an alternative explanation for our "stylized fact".

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[1] The mathematical conditions for this result are less stringent than before. While with fixed output prices imposing decreasing returns to scale in the revenue is equivalent to imposing them in production with an elastic demand all that is required for the production function is that

$$1 < g < 1/(1-b)$$

3.A BARGAINING MODEL

Apparently, an alternative explanation for our "stylized fact" is that it is the outcome of a bargaining game being played by an union and a firm about the sharing of the rents associated with the introduction of the new technology.

This section only pretends to be suggestive of the contribution of bargaining models for the explanation of that stylized fact. In fact the comparison will not be entirely fair for the bargaining models for two main reasons. The first is that, due to the relative ill development of bargaining theories under asymmetrical information, we will resort to the analysis of models with common information. The second reason results from the inexistence of a generally accepted concept of bargaining solution so that we shall take to analyse a particular one. However it is thought that the analysis below will be suggestive of the likely range of outcomes under this type of model.

Given these considerations and for the sake of clarity we shall keep the analysis as simple as possible. The solution concept we will adopt is the Nash one [2], generalized to non- symmetric bargaining powers. We shall be able, then, to see the influence of changes in those bargaining powers on the results of the model.

Formally, for some B, $0 < B < 1$, where B measures the firm's bargaining power, the generalized Nash bargaining solution solves the program

$$\max_{k,w} Z = [\pi(k,w,r)-\hat{\pi}]^{B}.[U(w,L(k,w,r))-\hat{U}]^{(1-B)}$$

where U(w,L) is the union's objective function and $\hat{\pi}$, $\hat{U}$ are the firm's and the union's respective threat points.

The aim of the exercise, here, is to find how the solutions w (r), k (r) and implicitly L (r) vary with r. All variables are as previously defined.

We start the analysis with a special case where

ASSUMPTION 7.

The union's objective function is

$$U(w,L) = w$$

The idea is to consider that the union's objective function is independent of L. This is similar to what was done in the previous section and, again, can be motivated

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[2] The Nash bargaining solution has the advantage of lying on the contract curve and satisfying a set of axioms that however arbitrary may be interpreted as a set of reasonable arbitration rules. For a discussion of the properties of this bargaining solution in conection with economic modelling see, e.g., Binmore et al. (1985).

along the lines already used for the motivation of assumptions 3 and 4 in section two. Another possible motivation results from the modern theories of unions as economic clubs where, e.g., the union menbers are the more "senior" workers whose employment status is not at risk because layoffs are made according to a LIFO rule; see Oswald (1985).

It must be noted that in this case, instead of being subjected to negotiation, the capital- labour ratio is freely chosen by the firm. The reason is simply because there is no conflict of interest between firm and union either about K or about L.

In order to obtain the bargaining solution one needs to specify a functional form for the firm's revenue function. To keep the exercise as simple as possible we restrict our attention to the constant price case and to a sub- class of homogeneous production functions, namely the Cobb- Douglas functional form. Formally we make:

<u>ASSUMPTION 6''.</u>

The firm's revenue function is

$$F(K,L) \equiv c.K^{a}L^{b} \text{ ,with } a,b>0 \text{ and } \gamma=a+b<1.$$

We can now derive:

RESULT 3.1

If 6'' and 7, when r decreases

i) w^B increases

ii) k^B increases

iii) L^B increases iff $b/(1-g) > (1-B)/B$

Proof: see the appendix.

For the symmetric bargaining powers case, where $B=1/2$, condition iii) can be read as meaning iff the share of labour is larger than the share of the fixed factors. Note that here capital is a variable factor since it is only after bargaining that the firm chooses the total input levels and the intensity.

The crucial point to note, here, is that result 3.1 shares with the contractual model a wage schedule making wages an increasing function of the capital- labour ratio but one also finds that employment may actually increase in the good state. This last aspect is the one that is clearly at odds with the stylized fact addressed by the model.

The next question to ask is how sensitive is result 3.1 to the distribution of bargaining powers between the agents. In fact it is found that condition iii) is reinforced if the firm's bargaining strenght increases, as B tends to one. In the limit when B=1 we get the competitive solution with $w=\bar{w}$ and L increases since K and L are complementary factors in production (F12>0).

The intuition behind the behaviour of L across states is simple. The workers do not care about total employment but the firm does and as long as its objectives are given any weight, a Pareto optimal solution will be sensitive to those objectives.

However, towards the case of complete union power, as $\bar{B}$ tends to 0, it is clear from the maximand function itself that the wages will tend to infinity. This suggests that the assumption that natural wastage can accomodate any possible decline in employment should be dropped and L ought to be included as an argument of U(.).

To incorporate this aspect and keep matters as simple as possible we make:

ASSUMPTION 7'.

The union has an utilitarian objective function

$$U(w,L) \equiv L.U(w) + (N-L).U(\bar{w})$$

where $\bar{w}$ is some outside opportunity value and N is the size of the union.

We can now derive:

RESULT 3.2

If B=0, 6'' and 7', when r decreases

i) w^{o} stays unchanged

ii) k^{o} increases

iii) L^{o} increases

Proof: see the appendix.

The other polar case represented by this last result is even further away from the explanation of our stylized fact than result 3.1. Particularly interesting to note is the constancy of wages across states (or the fact that w is independent of r). This results from, in this model, an isoelastic shift in labour demand induced by the change in r.

In this sort of models the f.o.c. for employment and wage determination require the equality of the elasticities of labour demand and utility change with respect to wages. Now, the isoelastic shift of labour demand means that the change in r induces a shift in labour demand such that the wage elasticity of this function is constant for every wage and consequently independent of r. See McDonald and Solow(1981) for a formal discussion of this point.

Combining the implications of results 3.1 and 3.2 our interpretation is that the bargaining model does not offer an explanation for the stylized fact that motivated this paper. This is so, because, in general, as r decreases, wages and the capital-labour ratio increase but <u>employment also increases</u> .

4. CONCLUSIONS

In this paper we offered a theoretical explanation for the "stylized fact" that firms, when engaged in processes of technological innovation, do increase the wages and the capital intensity of production while decreasing employment.

Starting with an approach based on theories of idiosyncratic exchange, namely optimal contracts and bargaining, we could derive results suggesting that the first of these does offer a consistent and reasonably general explanation while the later does not. In this respect the bargaining model while showing a wage schedule with the desired properties predicts an increase in employment that is at odds with the stylized fact it wants to explain.

To develop the contractual model we made a set of unconventional assumptions which we think are reflective of the real world. One of these is the form taken by the technological progress. The specificity of this assumption leaves an agenda for future research to see how robust are our conclusions to the form taken by the technological progress.

APPENDIX

1) Contractual model: proof of result 1.

$f(k,L) \equiv h(L)g(k)$

change in r, such that $r_1 < r_2$

$$\underset{k,L,w}{\mathrm{Max}\,\pi} = \alpha[h(L_1)g(k_1)-r_1k_1L_1-w_1L_1]+(1-\alpha)[h(L_2)g(k_2)-r_2k_2L_2-w_2L_2]$$

$$\text{s. a}: \alpha w_1+(1-\alpha)w_2 \geq \bar{U}$$

$$h(L_1)g(k_1)-r_1k_1L_1-w_1L_1 \geq h(\hat{L})g(k_2)-r_1k_2\hat{L}-w_2\hat{L} \qquad (*)$$

$$\pounds=\alpha[h(L_1)g(k_1)-r_1k_1L_1-w_1L_1]+(1-\alpha)[h(L_2)g(k_2)-r_2k_2L_2-w_2L_2]+$$

$$+\lambda[\alpha w_1+(1-\alpha)w_2-\bar{U}]+\mu\{[h(L_1)g(k_1)-r_1k_1L_1-w_1L_1]-$$

$$-[h(\hat{L})g(k_2)-r_1k_2\hat{L}-w_2\hat{L}]\}$$

$$\frac{\partial \pounds}{\partial L_1} = (\alpha+\mu)\ [f_2(k_1,L_1)-r_1k_1-w_1] = 0 \qquad (1)$$

$$\frac{\partial \pounds}{\partial L_2} = (1-\alpha)\ [f_2(k_2,L_2)-r_2k_2-w_2] = 0 \qquad (2)$$

$$\frac{\partial \pounds}{\partial \hat{L}} = -\mu\ [f_2(k_2,\hat{L})-r_1k_2-w_2] = 0 \qquad (3)$$

$$\frac{\partial \pounds}{\partial w_1} = -\alpha\ L_1 + \lambda\alpha - \mu L_1 = 0 \qquad (4)$$

$$\frac{\partial \pounds}{\partial w_2} = -(1-\alpha)\ L_2 + \lambda\ (1-\alpha) + \mu\hat{L} = 0 \qquad (5)$$

$$\frac{\partial \pounds}{k_1} = (\alpha + \mu)\ [f_1(k_1,L_1) - r_1L_1] = 0 \qquad (6)$$

$$\frac{\partial \pounds}{k_2} = (1-\alpha)\ [f_1(k_2,L_2)-r_2L_2]-\mu[f_1(k_2,\hat{L})-r_1\hat{L}] \quad = 0 \tag{7}$$

$$\frac{\partial \pounds}{\partial \mu} = f(k_1,L_1)-f(k_2,\hat{L})-[r_1k_1L_1+w_1L_1-r_1k_2\hat{L}-w_2\hat{L}] \quad = 0 \tag{8}$$

$$\frac{\partial \pounds}{\partial \lambda} = \alpha w_1+(1-\alpha)w_2-\bar{U} = 0 \tag{9}$$

From expressions (6) and (1), one can note that in state one, the first-best solution, $k_1=k_1^*$ and $L_1=L_1^*$ may occur.

Expressions (2) and (3) imply, respectively:

$$f_2(k_2,L_2) = r_2k_2+w_2$$

$$f_2(k_2,\hat{L}) = r_1k_2+w_2$$

and since, by assumption, $r_1 < r_2$, the two expressions above imply:

$$f_2(k_2,L_2) > f_2(k_2,\hat{L}) \text{ so,}$$

$$L_2 \ \frac{>}{<}\ \hat{L} \quad \text{as} \quad f_{22}\ \frac{>}{<}\ 0$$

This last condition is particularly interesting in the case of homogeneous (and homotetic) revenue functions because it can be interpret as stating that the firm would like (and would do it if allowed) to "overemploy" in state one, where its revenue is subjected to decreasing returns to scale.

This one is, in fact, the most interesting situation because it includes the situations of decreasing returns to scale in revenue (DRSR) both by technological reasons (due to the production function) and those brought about by a combination of a finitely elastic demand and constant or even increasing returns to scale in the technology.

It should also be noted that in the case of constant returns to scale in the revenue (CRSR), $\bar{L}=L_2$ and employment is constant. In this particular case, the firm has no incentive to "cheat" in this dimension and, again, the first-best solution may occur; see equations (2), (3) and (7).

In the following we will concentrate the attention in the case where $f_{22} < 0$, DRSR, when $L > L_2$. This shows the existence of I-C problems in state 1 and the relevant I-C constraint is $\pi_{11} \geq \pi_{12}$, the profit of revealing that state two occured when the true state is state one, doesn't pay.

From (4) and (5) one can see that:

$$\mu = \frac{\alpha(1-\alpha)(L_2-L_1)}{(1-\alpha)L_1+\alpha\bar{L}}$$

and since $\mu \geq 0$, we obtain $\bar{L} > L_2 > L_1$.

The relationship between the k's, across states, can now be checked. Substituting (1) and (3), into (8) we have

$$[f(k_1,L_1)-f(k_2,\bar{L})] - [L_1 f_2(k_1,L_1) - \bar{L}f_2(k_2,\bar{L})] = 0$$

or

$$f(k_1,L_1) - L_1 f_2(k_1,L_1) = f(k_2,\bar{L}) - \bar{L}f_2(k_2,\bar{L}] \qquad (8')$$

Define

$f(k,L) - Lf_2(k,L)$ as $\Delta(k,L)$

Then,

$$\Delta_1(k,L) = f_1 - Lf_{12} \qquad (a)$$

$$\Delta_2(k,L) = -Lf_{22} \qquad (b)$$

where $b > 0$, results from the previous assumption that $f_{22} < 0$.

A sufficient, but not necessary, condition for $f_{22} < 0$ is that F_{12} and F_{11} (derivatives of the original revenue function) are both negative. This is the case for an homogeneous revenue function with a degree of homogeneity smaller than one.

In this case $\Delta(k_1,L_1) = \Delta(k_2,\bar{L})$ implies $k_1 > k_2$ because $\bar{L} > L_1$.

Consider, now, a particular case of the revenue function:

$f(k,L) = L^{\gamma} \cdot g(k)$, where γ is the degree of homogeneity.

In this case $f_2(k,L) = \gamma L^{-1} f(k,L)$ or $Lf_2(k,L) = \gamma f(k,L)$ so that (8') can be written as

$$(1-\gamma)f(k_1,L_1) = (1-\gamma)f(k_2,\bar{L})$$

This shows that the I-C constrains the firm in such a way that its revenue in the "bad" state do not exceed that of the "good" state.

The question to analise next consists in asking which wage structure should the contract incorporate to verify the I-C constraint. To see this:

<u>Suppose</u> $w_2 \geq w_1$.

(8), then, implies:

$$-f(k_1,L_1) + r_1k_1L_1 + w_1L_1 \geq -f(k_2,\bar{L}) + r_1k_2\bar{L} + w_1\bar{L}$$

and <u>define</u> $T(k,L) = r_1kL + w_1L - f(k,L)$ to answer the question:

- How does $T(k,L)$ changes with the move from (k_1,L_1) to $(k_2,\bar{L})$?

We can define $k(L)$, implicitly, from $\Delta(k,L) \equiv \bar{\Delta}$, a constant and $d\,\Delta(k,L) = 0$, implies:

$(f_1-Lf_{12})dk-Lf_{22}dL = 0$ so that

$$k'(L) = \frac{Lf_{22}}{f_1 - Lf_{12}} < 0.$$

Now,

$$\frac{d}{dL}\, T[k(L),L] =$$

$$= r_1k + w_1 - f_2(k,L) + [r_1L - f_1(k,L)]\,\frac{Lf_{22}}{f_1-Lf_{12}} =$$

$$= \frac{1}{f_1-Lf_{12}}\,\{[r_1k + w_1 - f_2(k,L)](f_1-Lf_{12}) + Lf_{22}(r_1L-f_1)\}$$

To answer the previous question one needs to know the sign of the expression above in the point (k_1,L_1). Using the first-order conditions (1) and (6) it can be shown that that derivative is zero in that point and we need to look at the second derivative.

Remembering the particularization of the revenue function of the firm:

$$f(k,L) = L^{\gamma}.g(k)$$

from which one gets $f_1 = L^{\gamma}g'(k)$; $f_2 = \gamma L^{\gamma-1}g(k)$ and

$k' = -\frac{\gamma g(k)}{g'(k)L}$, we can write

$$\frac{dT}{dL} = r_1k + w_1 - \gamma L^{\gamma-1}g(k) + [r_1L-L^{\gamma}g'(k)][-\frac{\gamma g(k)}{g'(k)L}] =$$

$$= r_1k + w_1 - \frac{r_1\gamma g(k)}{g'(k)}$$

e, $$\frac{d^2T}{dL^2} = r_1k' - \frac{r_1\gamma g'^2(k)k'-g''(k)k'r_1\gamma g(k)}{g'^2(k)}$$

$$= r_1k' - \frac{r_1\gamma k'[g'^2(k)-g''(k)g(k)]}{g'^2(k)}$$

$$= r_1k'(1-\gamma\frac{g'^2(k)-g''(k)g(k)}{g'^2(k)})$$

$$= r_1k'(1-\gamma+\frac{\gamma g''(k)g(k)}{g'^2(k)}) = A$$

The sign of the expression A depends on the sign of the expression within brackets because $r_1k' < 0$.

In fact, $A > 0$ iff $(1-\gamma+\frac{g''(k)g(k)}{g'^2(k)}) < 0$,

or iff $$(\gamma-1) > \gamma \frac{g''(k)g(k)}{g'^2(k)} \qquad (B).$$

What does the condition B mean? This expression is the condition of strict convexity of the isoquants of the production function what can be see differentiating in order to k (because, for an homogeneous production function, the marginal rate of substitution can be written in terms of the factor ratio) the slope of those isoquants. Starting from the original production function:

$$F(K,L) \equiv L^{\gamma}g(\frac{K}{L})$$

$$F_K = L^{\gamma-1}.g'(k)$$

$$F_L = L^{\gamma-1}g(k) - kL^{\gamma-1}g'(k) = L^{\gamma-1}[\gamma g(k) - kg'(k)]$$

and $\frac{F_L}{F_K} = \gamma \frac{g(k)}{g'(k)} - k$ and so

$$\frac{\partial}{\partial k}(\frac{F_L}{F_K}) = \gamma \frac{g'^2(k)-g''(k)g(k)}{g'^2(k)} - 1$$

$$= \gamma(1-\frac{g''(k)g(k)}{g'^2(k)}) - 1$$

$$= (\gamma-1) - \gamma\frac{g''(k)g(k)}{g'^2(k)}$$

from which $\frac{\partial}{\partial k}(\frac{F_L}{F_K}) > 0$ implies

$$(\gamma-1) > \gamma\frac{g''(k)g(k)}{g'^2(k)} \text{ that is (B)}$$

This result <u>contradicts</u> the assumption that $w_2 \geq w_1$, requiring $A < 0$, thereby prooving Result 1.

B) Bargaining Model

For a Cobb-Douglas technology, with fixed output price, the profit equation can be written as

$$\pi = cK^aL^b - rK - wL$$

and the corresponding demand functions for capital and labour

$$K = a(\frac{c}{w} \quad b^b a^a)^{\frac{1}{1-a-b}} (\frac{r}{w})^{\frac{b-1}{1-a-b}}$$

$$L = \frac{b(cb^b a^a)^{\frac{1}{1-a-b}}}{w(w^g r^h)}$$

where $g = \frac{b}{1-a-b}$ and $h = \frac{a}{1-a-b}$

implying $k = \frac{aw}{br}$

One can, then, write the profit function as:

$$\pi = \frac{\Psi}{(w^b r^a)^{\frac{1}{1-a-b}}} \quad \text{with } \Psi = (1-a-b)(cb^b a^a)^{\frac{1}{1-a-b}}$$

B1) Proof of result 3.1

The Nash bargaining solution solves

$$\underset{w}{\text{Max}}\ Z = [\frac{1}{w^g r^h} - \bar{\pi}]^B [w-\bar{w}]^{(1-B)}$$

i) Results immediately from the nature of the Nash bargaining solution consisting in a share of rents such that $\pi > \bar{\pi}$ and $w > \bar{w}$. The wage has to increase because after the introduction of the new technology, decreasing r originates the sharing of a bigger price.

ii) Results from the expression for k that increases because of the decrease in r and because of the increase in w.

iii) To prove this aspect of the result it is necessary to look at the wages resulting from the game above.

Writing the game, equivalently, as:

$$\max_w Z' = B \log \left(\frac{1}{w^g r^h} - \bar{\pi}\right) + (1-B) \log (w-\bar{w}) \quad \text{and}$$

$$\frac{\partial}{\partial w}(Z') = \frac{-Bg/w^{g+1} r^h}{\left(\frac{1}{w^g r^h}\right) - \bar{\pi}} + \frac{1-B}{w-\bar{w}} = 0$$

That is

$$\frac{1}{w^g r^h}[1 - \frac{Bg}{1-B}(1 - \frac{\bar{w}}{w}) = \bar{\pi} \text{ , or}$$

$$\frac{1}{w^g r^h} = \frac{\bar{\pi}}{[1 - \frac{Bg}{1-B}(1 - \frac{\bar{w}}{w})]}$$

and substituting this expression in that for L, that can be written in the form

$$L = \phi \frac{1}{w} \frac{1}{w^g r^h} \quad \text{where} \quad \phi = b(cb^b a^a)^{\frac{1}{1-a-b}}$$

one gets:

$$L = \phi \frac{1}{w} \frac{\bar{\pi}}{[1 - \frac{Bg}{1-B}(1 - \frac{\bar{w}}{w})]}$$

and now:

$$\frac{\partial L}{\partial w} = - \Phi \frac{\bar{\pi} \{[1 - \frac{Bg}{1-B}(1 - \frac{\bar{w}}{w})] - w \frac{(1-B)Bg\bar{w}}{[(1-B)w]^2}\}}{w^2[1 - \frac{Bg}{1-B}(1 - \frac{\bar{w}}{w})]^2}$$

from which $\frac{\partial L}{\partial w} > 0$ iff $1 - \frac{Bg}{1-B}(1 - \frac{\bar{w}}{w}) - \frac{B\bar{w}}{(1-B)w} < 0$

That is $1 - \frac{Bg}{1-B} < 0$, or $g > \frac{1-B}{B}$, Q.E.D.

B2) Proof of Result 3.2

If $B = 0$, the union fixes k and the firm chooses L to maximize the profits:

$$\pi = cK^aL^b - r\,k\,L - wL$$

$$\frac{\partial}{\partial L}\pi = b\,c\,k^aL^{b-1} - rk - w = 0$$

$$L = [\frac{b\,c\,k^a}{rk+w}]^{\frac{1}{1-b}}$$

The problem of the union consists, then, in choosing w and k in such a way as to:

$$\max_{w,k} H = [\frac{b\,c\,k^a}{rk+w}]^{\frac{1}{1-b}} [U(w) - U(\tilde{w})]$$

$$\frac{\partial}{\partial k} H = \frac{a\,b\,c\,k^{a-1}}{rk+w} - \frac{r\,b\,c\,k^a}{(rk+w)^2} = 0 \text{ , that is:}$$

$$\frac{k^{a-1}}{(rk+w)^2}[(rk + w)\,a\,b\,c - r\,b\,c\,k] = 0$$

$$k\ r\ b\ c(1-a) = w\ a\ b\ c$$, from where

$$k = \frac{aw}{r(1-a)}$$ and,

$$\frac{\partial}{\partial w} H = \frac{1}{1-b}\left[\frac{b\ c\ k^a}{rk+w}\right]^{\frac{1}{1-b} - 1} \left(-\frac{b\ c\ k^a}{rk+w}\right)^2 [U(w) - U(\bar{w})] +$$

$$+ U'(w)\left[-\frac{b\ c\ k^a}{rk+w}\right]^{\frac{1}{1-b}} = 0$$

$$\left[\frac{b\ c\ k^a}{rk+w}\right]^{\frac{1}{1-b}} \left\{-\frac{1}{1-b}\,\frac{1}{rk+w}[U(w) - U(\bar{w})] + U'(w)\right\} = 0$$

$$U'(w) = \frac{U(w)-U(\bar{w})}{(1-b)(rk+w)}$$ and, substituting in this the expression for k:

$$U'(w) = \frac{U(w) - U(\bar{w})}{(1-b)\left(\frac{aw}{1-a} + w\right)} = \frac{U(w) - U(\bar{w})}{\frac{(1-b)\ w}{1-a}}$$

$$U'(w) = \frac{(1-a)[U(w) - U(\bar{w})]}{(1-b)w}$$

$$\frac{(1-b)U'(w)}{U(w)-U(\bar{w})} = \frac{1-a}{w}$$, which shows that w is independent of r.

The steps of the proof show that when r decreases:

w^o is constant
k^o increases
L^o increases

Q.E.D.

(*) This formalization of the incentive-compatibility constraint (I-C), through the introduction of an artificial variable $\mathcal{L}$, results from the necessity of assuring the I-C for every possible employment level , in the absence of that same constraint.

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