"TRADE IN FACTOR-RELOCATION-REQUIRING
SERVICES: a simple formal model

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TRADE IN FACTOR-RELOCATION-REQUIRING SERVICES: 

a simple formal model*

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International trade in services is becoming an increasingly important topic in trade negotiations. Given the importance of service trade it is somewhat surprising that services, to the extent that they raise issues different from goods, have rarely been analysed in formal economic analysis. In this paper, we develop a formal model of international trade in services with three commodities (two standard goods and a service), two factors, and two countries. Trade in the service requires the locational presence of productive factors, without whose physical presence, the service becomes essentially a non-tradeable. We establish conditions for comparative advantage and analyze the pattern of trade in the service. In this model, trade in all commodities cannot occur without factor mobility and it is clear that restrictions on factor mobility can be used as a policy tool to prevent trade in services.

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1. Introduction

International trade in services has become an increasingly important issue as the level of such trade has greatly risen during the last decade; a rise that has been concurrent with the developed countries, particularly the United States, pushing for further liberalization in the area of services. It is, of course, no coincidence that most of the developed countries which are in favour of trade liberalization in the area of services are at a stage in their development where their economies are highly service oriented. It is a well documented result that there exists a high correlation between a country's level of income and the percentage of this national income that results from the service sector. This has been shown to be the case in both cross-sectional and time series studies 1.

Given the importance that is now attached to international trade in services, it is somewhat surprising that such little formal analysis has been done in this area. Virtually none of the literature on services trade has brought the issue within the theoretical framework of standard international trade analysis. Three exceptions are recent papers by Bhagwati (1984a), Deardorff (1984), and Djajic and Kierzkowski (1986). Bhagwati provides a theoretical basis for the observed phenomenon that services are relatively cheaper in poor countries (i.e. the empirical finding that there exists a positive correlation between per capita GDP and relative service prices). Deardorff analyzes trade in services in a formal trade model and asks whether specific characteristics of services may cause trade, inclusive of services, to violate the law of comparative advantage. Djajic and Kierzkowski provide a formal model for analyzing the importance of what we would call "good services", that is services whose only function is to extend the life of goods. They analyze the production and exchange of services, in a model in which there are two durable goods and one intermediate service whose function is to guarantee the full life of the two goods.

In analysing trade in services, the first task is to make explicit any tangible and potentially important differences between goods and services, (Hindley and Smith, 1984) preferably through formal modelling. Many of these potential differences between goods and services, as well as between different types of services, are outlined by Hill (1977) who stresses the role of time and space in the provision of services.
Following on Hill's conceptual discussion, Bhagwati (1984b) draws an important distinction between services requiring the physical proximity of producer and consumer and those where a "disembodiment effect" operates, resulting in what he has called (1985) "long-distance" services. The former class of services has been the focus of later writings by Bhagwati (1985) and Sampson and Snape (1985). Sampson and Snape develop an important taxonomy, a simplified version of which is utilized by Bhagwati, to highlight different ways in which the necessary producer/consumer proximity requirement can be satisfied. Bhagwati focuses on the further conceptual issue that once a physical proximity requirement is shown to be essential for a service transaction, the concept of "temporary factor relocation requiring" trade must be introduced into formal analysis. His analysis also highlights the need to incorporate this physical-proximity requirement formally into conventional trade theory.²

The above analysis is useful to motivate our paper, which addresses the issue of space or of the specific factor locational requirement of services. We model this requirement in the basic Hecksher-Ohlin-Samuelson (HOS) framework. The outline of the paper is as follows: Section II presents the formal model for both the closed and open economy, Section III analyzes comparative advantage and the pattern of trade, Section IV discusses comparative statics, Section V addresses commercial policy, Section VI focuses on an alternative explanation for pre-trade service relative price differentials, and in Section VII we summarize the results.

II. The Model

The model we develop is in the HOS tradition with the only substantive difference being in the provision of a third commodity which is a service. Thus we have a 3x2x2 model with three commodities, two factors (capital and labour), and two countries.² It would appear that this is the minimum framework necessary to analyse meaningfully the issue of trade in services. A one-good, one-service model is insufficient because if trade is to occur at all, trade in the service is forced upon the system. Our model more realistically allows for the case where trade can take place while still allowing for the service to be a nontradable, policy-imposed or otherwise.

The demand side of the model follows standard trade theory in its assumptions of identical
and homothetic preferences. Technologies are identical across countries. The three commodities (two goods, 1 and 2, and one service, 3) are produced with constant returns to scale (CRS) technology. Factors are available in fixed supply. Production functions are then represented by (1) below:

\[ X_j = f(K_j, L_j) \quad j = 1, 2, 3 \] (1)

and resource constraints by (2.a)-(2.b), respectively:

\[ L_1 + L_2 + L_3 = L \] (2.a)

\[ K_1 + K_2 + K_3 = K \] (2.b)

where \( X_j \) is the output of the \( j \)th commodity, \( K_j \) and \( L_j \) the capital and labour used in the production of the \( j \)th commodity, and \( K \) and \( L \), the total endowments of capital and labour in the economy.

As is shown below, it is the provision of the service that distinguishes our model from other trade models, namely the standard HOS model, the existing transport models in the HOS tradition, and Komiya’s classic nontraded good model. The service, in our model, requires the physical presence of some particular factor (or combination of factors) at the location where it is consumed. If the service is consumed at home, its production function already includes this factor requirement. Thus for the closed economy there are no explicit modelling differences concerning the service and standard neoclassical results obtain. If, however, the service is consumed abroad, it requires the presence of factors from the exporting country. It is this required presence of factors in the location of consumption which distinguishes the service from the two standard goods 1 and 2. We believe that this factor locational requirement, assuming for now that it is some combination of capital and labour, is vital to the provision of many services 6/. We are thinking here, not only of the “haircut type” of services which obviously require the simultaneous presence of both producer and consumer but also of highly advanced services such as banking and insurance; often these can only be effectively sold abroad if the exporting country has some physical presence in the importing country 5/. Medical and educational services also require physical proximity between consumer and producer if they are to be appropriately consumed 5/.

Furthermore, we assume that this factor locational presence, needed for both services
consumed domestically and those exported, necessitates an additional cost when the service is exported; this being for the factors that must be sent abroad. The idea that the costs of providing the service abroad are greater than the costs of producing (and consuming) domestically seems reasonable. One might think of adjustment costs for factors sent abroad or more simply the physical costs of sending them abroad. We assume that these factors must be from the service exporting country, an assumption which is in line with the behaviour of most international banks and service exporting firms which traditionally send at least some experienced workers to service importing countries. We do not, however, in this paper explicitly model the interaction of these mobile factors with foreign factors.

We also assume for simplicity that the amount of additional factors needed for the provision of the service abroad is proportional to the level of service exports. Thus for the exportation of the service we have:

$$E_3 = \min \{ \frac{L^X_3}{\beta_L}, \frac{K^X_3}{\beta_K} \}$$

where $E_3$ are exports of the service and $\beta_i$, $i = K, L$, represents the net additional amounts of capital ($K^X_3 = \beta_K E_3$) and labour ($L^X_3 = \beta_L E_3$) that are needed when exporting one unit of the service. Again, it is only the service that requires such locational presence, a requirement which we assume adds an additional cost to the provision of services abroad. The export requirement provided for in equation (3), in essence, makes our model a hybrid of models where trade in goods occurs without factor mobility and models where factor mobility is allowed to the exclusion of trade in goods. It should be noted that the requirement of physical proximity between production and consumption could, in principle, be satisfied by having the consumer (or receiver) of services move. From an analytical point of view this situation could be handled with some modifications in the model. For many services, however, it would appear that the much more likely occurrence is for the service to be provided abroad rather than domestically to foreign residents. It is for such services that our model is directly relevant.

From (1)-(3), using standard conditions and properties enables us to define the complete resource system of an economy which exports services in the fashion we have outlined:
\[ a_{L1}X_1 + a_{L2}X_2 + (a_{L3}X_3 + \beta_L E_3) = L \]  
\[ a_{K1}X_1 + a_{K2}X_2 + (a_{K3}X_3 + \beta_K E_3) = K \]

where \( a_{ij} \) is the amount of the ith input, \( i = L, K \), used in the production of one unit of the jth good, \( j = 1, 2, 3 \), and \( E_3 \) are exports of the service. In matrix form, we have:

\[ AX + E_3\beta = V \]  

where \( A \) is the 2x3 matrix of input coefficients, \( X \) the output vector, \( V \) the endowments vector, and \( \beta \) the vector of service export-factor requirement coefficients:

\[
A = \begin{bmatrix}
  a_{L1} & a_{L2} & a_{L3} \\
  a_{K1} & a_{K2} & a_{K3}
\end{bmatrix}
\quad X = \begin{bmatrix}
  X_1 \\
  X_2 \\
  X_3
\end{bmatrix}
\quad V = \begin{bmatrix}
  L \\
  K
\end{bmatrix}
\quad \beta = \begin{bmatrix}
  \beta_L \\
  \beta_K
\end{bmatrix}
\]

If the economy does not export the service, the resource system is given simply by (6) below:

\[ AX = V \]  

Assuming for simplicity that domestic factors sent abroad to carry out service exports earn exactly the same return as at home, the pricing system for a service exporting country is established in (7) and (8) below:

\[ wA = p \]  
\[ w[a_3 + 0] = pX_3 \]

where \( w \) is the row vector of factor prices, \( p \) is the row vector of output prices, \( a_3 \) is the column vector of input coefficients in the production of the service, and \( pX_3 \) is the price of the service when it is exported:

\[
w = [w_L \ w_K]
\quad p = [p_1 \ p_2 \ p_3], \quad \text{normalizing } p_1 = 1, \quad p = [1 \ p_2 \ p_3]
\quad a_3 = \begin{bmatrix}
  a_{L3} \\
  a_{K3}
\end{bmatrix}
If the economy does not export the service, the pricing system is given simply by (7) above.

The presence of domestic factors abroad implies a difference between GDP and GNP. For simplicity, we rule out remittances and assume that all income earned by domestic factors abroad will be spent abroad (see Bhagwati and Brecher, 1980).

Given the above assumptions it is clear that the production possibilities frontier (PPF) for the service exporting economy actually shrinks when the economy is opened up to trade and the service is exported; this is due to our assumption that the service will require additional factors and thus necessitates additional costs when used abroad. In Figure 1, the PPF before trade is seen as ABC, but after trade, it shrinks to A'B'C. If the output of the service is not exported but instead entirely consumed domestically, then the production possibility frontiers for the closed and open economy cases are identical.

III. Comparative Advantage and the Pattern of Trade

In the closed economy the model behaves regularly. It is a simple 3-commodity/2-factor neoclassical model whose properties have been fully analysed by Batra and Casas (1973).

In the open economy, if the service is nontradable for exogenous reasons, the model becomes exactly like Komiya's (1967) model. In Komiya's model the nontradability of one good is imposed through the implicit assumption of infinite transport costs for that good and zero transport costs for the traded goods. In our model, however, the nontradability of the service, if it occurs, is policy-imposed either through explicit restrictions on services trade or implicitly through restrictions on factor movements. It is when all commodities are tradable, including the service, that our model becomes relevant to address the issue of trade in factor-relocation-requiring services. The fact that all commodities are tradable, however, does not preclude market forces from ruling out trade in one of them, namely the service.

Even though the international economy under the auspices of the GATT, is not one of liberal "trade" policies with regard to services, we begin with the institutional assumption of free trade, and only later (in section VI) take into account the use of commercial policy measures to restrict service trade; the latter scenario is, of course, much more in line with services trade as it exists in the world economy today.
Let us assume a 2-country world made up of country A (home country) and country B (foreign country). As we would expect from a general equilibrium model, the service may be traded or not depending on the preferences, resources, and technology of the entire economy and not only on the specific characteristics of the service market. In order for the home country (A) to export the service, however, the following two conditions must hold: (i) The service importing country B must allow "partial factor mobility," i.e., it must allow the foreign factors needed in conjunction with the service to enter the country; and (ii) The service exporting country A must have comparative advantage in the service, i.e., $p^x_{3A} \leq p^x_{3B}$.

The first condition above is institutional; it is a direct result of our assumption that a certain amount of factors from the service exporting country A have to be present in the importing country B. Without such factor mobility, trade in the service cannot occur. The second is an efficiency condition; it simply states that for country A to export the service, its export price (inclusive of additional cost factors) must be equal to or smaller than the autarky price of the service in country B. Obviously this implies that the cost of the service in autarky in country A is sufficiently low, so that it can incur the additional cost of selling the service abroad and still maintain a comparative cost advantage. We are now in a position to analyze the consequences of these conditions.

It is important to note that in this model international trade can equalize the prices of the two goods 1 and 2 but not those of the service. If the service is traded, its price will differ across countries by at least the additional cost incurred in exporting it. As a result factor prices can not be equalized.

It is also important to note that the price of the service abroad (the 'exported service') is the relevant price for analyzing comparative advantage and the pattern of trade. In fact, our model provides a direct application of Deardorff’s (1979) analysis. We have more goods than factors (3 goods, 2 factors) and no factor price equalization. Assuming that (a) the goods can be ranked unambiguously in terms of factor intensity, (b) there are no factor intensity reversals between any pair of goods, (c) perfect competition prevails in both countries, and (d) factor prices under free trade are different across countries, Deardorff concludes that the pattern of trade must agree with the ranking of commodities by factor intensity, even in the presence of tariffs or transport costs, in which
case prices inclusive of these additional costs become the relevant prices. In our model, in the absence of direct trade subsidies, the service is exportable only if its price abroad is sufficiently high so as to compensate exporters for the additional cost of the export-factor-requirement. Furthermore, the chain of comparative advantage holds even if the potential importing country does not allow "partial factor mobility".

We now proceed as follows. First, we will identify the relevant capital-labour ratios of the service in order to appropriately measure its factor intensity. Secondly, we will stress the key role of the service export-factor-requirement expressed through coefficients \( \beta_K \) and \( \beta_L \) or the ratio "b", where \( b = \beta_K/\beta_L \). Initially, we assume that the home country has comparative advantage in the service and exports it. We then seek to answer the question of whether the country is in a trade equilibrium. This roundabout approach allows us to immediately show the importance of b, which as we will be shown below is central to determining the pattern of trade with respect to the service. Finally, we analyse the conditions required for comparative advantage in the service and also the implications of our assumptions for the chain of comparative advantage.

Recall that \( k_j = K_j/L_j \) is the jth commodity capital-labour ratio for \( j = 1, 2, 3 \). We define \( k^x_3 = (a_{K3} + \beta_K)/(a_{L3} + \beta_L) \) as the augmented-marginal-capital-labour ratio of the service-exporting country. The corresponding average ratio is \( k^{a,x}_3 = (K_3 + k^x_3)/(L_3 + L^x_3) = (a_{K3}X_3 + \beta KE_3)/(a_{L3}X_3 + \beta LE_3) \). For goods 1 and 2 (as well as the service in the case of no service exports) the corresponding marginal and average capital-labour ratios are identical. The export factor requirement capital-labour ratio or simply the service export ratio is \( b = \beta_K/\beta_L \).

We turn now to a detailed discussion of the role of b, the service export ratio. As will be seen below, b plays an important role in determining the pattern of trade though this is not immediately obvious. Let us assume that the home country is the service exporter. If we assume that (i) a marginal change in service output is entirely accounted for by additional service exports \( dX_3 = dE_3 > 0 \) and (ii) output prices do not change, then by differentiating the full employment conditions stated in (2.a)-(2.b), we have:
\[ a_{L_1}dX_1 + a_{L_2}dX_2 + (a_{L_3} + \beta_L)dE_3 = 0 \quad (9.a) \]
\[ a_{K_1}dX_1 + a_{K_2}dX_2 + (a_{K_3} + \beta_K)dE_3 = 0 \quad (9.b) \]

Solving and rearranging (see Appendix 1) we obtain:

\[ [dX_1/dX_2]_{dX_3=dE_3} = \frac{a_{L_2}(k_3 - k_2)}{a_{L_1}(k_1 - k_3)} \quad (10) \]

The above change can be decomposed into two parts. If a marginal change in service output affects only the domestic market \((dX_3=0, dE_3=0)\) and output prices are constant, we have:

\[ a_{L_1}dX_1 + a_{L_2}dX_2 + a_{L_3}dX_3 = 0 \quad (11.a) \]
\[ a_{K_1}dX_1 + a_{K_2}dX_2 + a_{K_3}dX_3 = 0 \quad (11.b) \]

Solving and rearranging (see Appendix 1) we obtain:

\[ [dX_1/dX_2]_{dE_3=0} = \frac{a_{L_2}(k_3 - k_2)}{a_{L_1}(k_1 - k_3)} \quad (12) \]

If service output and prices are constant, then a marginal change in service exports \((dX_3 = 0, dE_3 > 0)\) implies a shift of service output from the domestic to the foreign market, and we have:

\[ a_{L_1}dX_1 + a_{L_2}dX_2 + \beta_LdE_3 = 0 \quad (13.a) \]
\[ a_{K_1}dX_1 + a_{K_2}dX_2 + \beta_KdE_3 = 0 \quad (13.b) \]

Solving and rearranging (see Appendix 1) we obtain:

\[ [dX_1/dX_2]_{dX_3=0} = \frac{a_{L_2}(b - k_2)}{a_{L_1}(k_1 - b)} \quad b = \beta_K/\beta_L \quad (14) \]

Assume now, without loss of generality, that the home country is labour-abundant, good 1, is capital-intensive, and the service is labour-intensive at all factor price ratios. At this point, we do not need to be specific about good 2. Then it is possible to state:

**Proposition 1:** The home country (labour-abundant) is in equilibrium if the capital-labour ratios \(k_3\) and \(k^x_3\) are equal to the export ratio \(b\). The country will, however, increase (decrease) service exports if \(k_3 > k^x_3 > b\) (\(k_3 < k^x_3 < b\)).
Proof:

Suppose the home country is in the situation where it is indifferent between producing one additional unit for the domestic market and diverting one additional unit from the domestic to the foreign market. This obviously represents a trade equilibrium for the service. Otherwise, the home country will increase or decrease service exports and move towards such an equilibrium. An change in service exports implies changes taken into account by the ratio \( [dX_1/dX_2]_{X=0} \) or \( [dX_1/dX_2]_{E=0} \). Thus the home country will:

- Increase service exports:
- Not change service exports: \[ \frac{dX_1}{dX_2} \bigg|_{X=0} = \frac{dX_1}{dX_2} \bigg|_{E=0} \]
- Decrease service exports:

Substituting from (12) and (13) and solving, we conclude that service exports will be increased (decreased) as long as \( k_3 > (\leq) b \). From \( k^X_3 = b \) (or \( k^X_3 = k_3 \)) by simple substitution from the definitions of \( k_3 \), \( k^X_3 \), and \( b \), we get \( k_3 = b \). Furthermore, since we always have either \( k_3 \geq k^X_3 \geq b \) or \( k_3 \leq k^X_3 \leq b \) it is possible to conclude:

\[
\begin{align*}
\text{if } k_3 > k^X_3 &= b \\
\text{then } k_3 &= k^X_3 &= b \\
\text{if } k_3 < k^X_3 &= b \\
\text{then } k_3 &= k^X_3 < b
\end{align*}
\]

Thus, the intuition for this result is as follows. Input coefficients depend on relative factor prices, \( a_{ij} = a_{ij}(w_L/w_K) \), \( i = K, L, \ j = 1, 2, 3 \). Other things equal, the higher the price of the ith factor, the smaller must be the ith input coefficient \( a_{ij} \); thus the capital-labour ratio \( k_j = a_{Kj}/a_{Lj} \) will adjust accordingly. Exports of the service require additional factors which have to be withdrawn from the production of all goods including the service. If the constant ratio \( b \) at which these additional factors are withdrawn increases the relative labour intensity of the service in the labour abundant country (and thus decreases ratios \( k_3 \) and \( k^X_3 \)), the exports of the service will be increased; the greater the relative impact on the capital-intensive commodity (here good 1), the greater are the potential gains from international specialization and trade. Hence we have the following corollary:
COROLLARY 1.1: The criterion to determine whether the home country will increase service exports relies on whether the export-factor-requirement increases the factor intensity of the service in the direction of the factor abundance of the country. If the service is exported, its capital-labour ratio in equilibrium will be equal to the export ratio $b$.

Figure 2 illustrates both Proposition 1 and Corollary 1.1. The service export ratio $b$ is the same in the initial and the final situation. In fact, it is a technological constant. The initial situation (point M) is not a trading equilibrium. It is characterized by unit iso-cost $C_0C_0$, tangent unit value isoquant $X_0^3 = (1/p_3^0)^0$, and capital-labour ratio $(k_3^0)^0$. In this case, $b$ is to the right of $(k_3^0)^0$ which means that if service exports were increased they would make the service relatively more labour intensive and reduce the amount of labour available for domestic production. As a result, the wage-rental ratio would increase and the capital-labour ratios $k_3$ and $k_3^0$ would both decrease. The final situation (point N), which reflects a trading equilibrium, is characterized by unit iso-cost $C_0C_0$, tangent unit value isoquant $X_0^3 = (1/p_3^0)^0$, and capital-labour ratio $(k_3^0)^0$. In the trading equilibrium the capital-labour ratios of the service $k_3$ and $(k_3^0)^0$ become aligned with the export ratio $b$, i.e. $(k_3^0)^0 = (k_3^0)^0 = b$. Ratio $b$ plays a central role in the adjustment process: factor price changes will force both $k_3$ and $(k_3^0)^0$ to converge to ratio $b$. Only in this way will the country be specializing to the extent called for by comparative cost advantage.

We are now equipped to deal with the issue of comparative advantage in the service. For the home country A to export the service to country B, a necessary but not sufficient condition is $p_{3A}^* = p_{3B}$. From the pricing system (equations (7)-(8)), this implies:

$$p_{3A}^* = w_3^{a_3A} = w_{3B}^{a_{3B}} = p_{3B}, \quad w_n = [w_L, w_K]_n, \quad n = A, B$$

Both the domestic production cost $[w_3^{a_3A}(w_A)]$ of the service and the cost of the export-factor-requirement $[w_A^0]$ are relevant. If condition (15) or its equivalent for country B does not hold, the service is not traded. This will happen in the case where factor endowments and thus
factor prices across countries in the pre-trade situation are not "sufficiently" different. In this case, trade in goods 1 and 2 will suffice to reduce factor price differentials and thus diminish the gap in (15) above; thus there is no economic rationale for trade in the service.

We recall that trade in goods in a world without factor mobility is an indirect way of trading factors. In our model, trade in the service provides both a direct and an indirect method for trading factors with the direct avenue arising from the export-factor-requirement. If inequality (15) holds before trade, free trade makes it an equality by reducing the gap between service prices in different countries to the cost of the export-factor-requirement (wB). Trade will reduce factor price differentials and induce adjustments in input coefficients \( a_{L3n} \) and \( a_{K3n} \), \( n = A, B \). This can be accomplished by trade in all three commodities or any subset of them depending on the chain of comparative advantage and the factor endowment differences between the trading partners. Thus we have the following proposition:

PROPOSITION 2: Trade in the service reduces the difference between service prices across countries to the cost of the export-factor-requirement so that \( P_X^A = P_B \) or \( P_X^B = P_A \). However, factor endowments and factor intensities may be such that trade in goods 1 and 2 suffices to reduce factor price differentials and reverse the sign of condition (15) in which case there will be no trade in the service.

At the trading equilibrium, we have seen that the unit value isoquant is tangent to the unit isocost so that \( k_3 = k_X = b \). Since technologies are identical across countries and factor prices are different, applying Deardorff's (1979) analysis it is possible to state the following propositions and corollaries:

PROPOSITION 3: For a country to have comparative advantage in the service, its unit value service isoquant tangent to its unit isocost must lie above and to the right (i.e. it must be the "outwardmost" isoquant) of the corresponding isoquant for its trading partner, along a ray from the origin of slope \( b \), the service export ratio.

Proof:

Figure 3 illustrates this proposition. Since in the trading equilibrium, the capital-labour ratio of the service is equal to the export ratio \( b \) (Proposition 1), comparative advantage in the service
requires comparing unit value isoquants along the ray from the origin which represents ratio b. Suppose, without loss of generality, that under free trade country A (labour abundant) exports the service (labour intensive) to country B (capital abundant). Assume, also without loss of generality, that by a convenient choice of units, \( a_{L3B} = 1 \), and thus \( a_{K3B} = b \). The proposition is proved by showing that if the labour input coefficient of the service in the importing country B is equal to one \( (a_{L3B} = 1) \) at the point of tangency between the unit value isoquant and the unit iso-cost on the ray representing ratio b, then the corresponding augmented labour input coefficient in the exporting country A must be greater than one \( (a_{L3A} > 1) \). In this case, since both unit value isoquants are on the same ray from the origin representing ratio b, we conclude that the isoquant of the exporting country A is above and to the right of the isoquant of the importing country B, which proves that it is the outward most isoquant. Normalizing \( w_K = 1 \) in both countries, from condition (15) we obtain (see Appendix 2):

\[
\frac{(w_A + b)(w_B + b)}{w_A + w_B} = 1/\bar{a}_{L3A} \quad w_n = [w_L/w_K]_n \quad n = A, B
\]

\[
\bar{a}_{L3A} = [a_{L3A} + b_L]
\]

Since country A is labour abundant, we have \( w_A/w_B < 1 \) or adding the constant b to both the numerator and denominator yields \( [(w_A + b)/(w_B + b) < 1] \). Substituting from above we conclude that \( [1/\bar{a}_{L3A} < 1] \). Thus \( \bar{a}_{L3A} > 1 \), which proves Proposition 3.

**PROPOSITION 4:** For the service to be exportable it must be at the "favorable" extreme of the chain of comparative advantage, i.e., it must be the most capital (labour) intensive commodity in the capital (labour) abundant country.

**Proof:**

From the pricing equations (7)-(8) it is possible to derive the following relationship involving capital-labour ratios and output prices (see Appendix 3):

\[
(b - k_2)/a_{L1} + [(k_1 - b)/a_{L2}]p_{X_2} + [(k_2 - k_1)/\bar{a}_{L3}]p_{X_3} = 0
\]

The equilibrium export price of the service is then:
\[ p^X_3 = \left[ \frac{a}{L_3/(k_1 - k_2)} \right] \cdot \left[ \frac{(b - k_2)/a_{L_1} + p^X_2(k_1 - b)/a_{L_2}}{k_1 - k_2} \right] \tag{16} \]

Since prices \( p^X_2 \) and \( p^X_3 \) in (24) must be positive, the capital-labour ratios must be ordered as follows:

- If \( k_1 > k_2 \), then either \( k_2 < k_1 < b \) or \( b < k_2 < k_1 \)
- If \( k_1 < k_2 \), then either \( k_1 < k_2 < b \) or \( b < k_1 < k_2 \)

Therefore, for the service to be exportable, the capital-labour ratios of both goods 1 and 2 should be either greater or smaller than the export ratio of the service \( b \), which in equilibrium, is also the capital-labour ratio of the service. In other words, the service will not be exportable should it be the "intermediate" good in terms of factor intensity (i.e. \( k_1 < b < k_2 \) or \( k_1 > b > k_2 \)).

**COROLLARY 4.1**: The chain of comparative advantage holds even if the service is exportable but actually not traded.

Suppose that the labour abundant country has comparative advantage in the service (labour intensive) but that potential importing countries do not allow "partial factor mobility" and thus the factor locational requirement cannot be fulfilled. Suppose that in the labour abundant country \( k_1 < k_2 < k_3 < k_4 \ldots \) where commodity 2 is the service. If the service is not traded we might be tempted to think that the country could export goods 1 and 3 and import goods 4 and following; the chain of comparative advantage would not be reversed but there would be a jump in the ordering of exports according to their factor intensity. Proposition 4, however, rules out such cases by stating that for the service to be exportable it must be at the favorable extreme of the chain of comparative advantage.

**COROLLARY 4.2**: The issue of indeterminancy is solved in this model: in the case of many standard goods and many services the chain of comparative advantage holds, and we always know what services are exported, imported, or not traded.

Suppose the case of many standard goods, one service, and two factors: the service will only be exportable if it is at the favorable extreme of the chain of comparative advantage (Proposition
4). Suppose the case of two goods, two services, and two factors. If the services are at both extremes of the chain of comparative advantage (Figure 4), the labour intensive service will be exported by the labour abundant country and the capital intensive service will be exported by the capital abundant country. In fact one might think of the Bhagwati (1984a) model where services are nontradable and labour intensive; as Bhagwati (1985) points out the exact same story could be told symmetrically for capital intensive services and indeed for explaining why some services are more expensive in the developed countries. If we follow this framework and allow trade in the service sector then our analysis, which in essences opens up the Bhagwati model to trade, shows that developed countries (capital abundant) would export capital intensive services while developing countries (labour abundant) would export labour intensive services as long as these services occupy the extreme positions of the chain of comparative advantage. If one of the services is not at the extreme position of the chain it will not be traded. Generalizing to the case of many goods, many services, and two factors is straightforward.

IV. Comparative Statics

In terms of comparative statics, three exercises appear to be meaningful; in turn we take up the cases of changes in (i) technology, specifically technical change involving the ratio b, (ii) price changes, at fixed resource levels and finally (iii) changes in resource endowments, at fixed prices.

A. Technology

Changes in export technology or the ratio b are the easiest to handle and can be viewed from either a service trading equilibrium or a nontraded service equilibrium. It is clear that technological improvements in the form of declines in $b_K$ and $b_L$, which could occur without a change in ratio b (i.e. the case of a "neutral" technical change) will necessarily lower export costs and thus diminish the differences in services prices (as well as factor prices) which exist even in a trading equilibrium. Such technological changes would not, however, reverse the pattern of trade in the service.
If the service is traded, as was shown above, it must lie at the extreme end of the chain ranking commodities by factor intensity. Given this, we have two possibilities. If b changes in the direction of factor abundance of the exporting country, (i.e. if the service is labour intensive and \( b = \beta_K/\beta_L \) falls) then the pattern of trade remains unchanged.

In the event that b changes in the direction opposite to the country's factor abundance, we have three possibilities depending on the magnitude of this change. If the change in b leaves intact the factor intensity ranking of commodities, the pattern of trade remains unchanged. If, however, the change in b moves the service from the extreme position to the intermediate position of the chain, then as was shown above (Proposition 4) the service will not be traded. And finally in the event that the change in b is so large that the service moves to the opposite end of the chain of comparative advantage, the pattern of trade for the service is reversed; that is the country previously exporting (importing) services will now import (export) them.

If we instead start from the assumption that the service is not traded because it is intermediate in the chain of comparative advantage, then, of course, it will become traded if a change in b causes it to move to either extreme of the chain. The pattern of trade in this case will necessarily depend on which end of the chain the service occupies and which country is abundant in this factor.

### B. Prices

Differentiating pricing equations (7)-(8) we get

\[
a_{L1} dw_L + a_{K1} dw_K = 0
\]

\[a_{L2} dw_L + a_{K2} dw_K = dp_x \]

\[(a_{L3} + \beta_L)dw_L + (a_{K3} + \beta_K)dw_K = dp_x\]

In matrix form we have:

\[
\begin{bmatrix}
dw_L \\
dw_K
\end{bmatrix} = \begin{bmatrix}
1/\Delta_{13} & 0 \\
\Delta_{13} & dp_x
\end{bmatrix}
\begin{bmatrix}
(a_{K3} + \beta_K) (a_{L3} + \beta_L) \\
-a_{K1} & a_L
\end{bmatrix}
\]
\[ \begin{bmatrix} dw_L \\ dw_K \end{bmatrix} = (1/\Delta_{12}) \begin{bmatrix} 0 & dp_x^2 \\ \end{bmatrix} \begin{bmatrix} a_{K2} & -a_L \\ -a_{K1} & a_{L1} \end{bmatrix} \]

Hence:

\[ dw_L = \left( a_{K1}/(a_{L1} a_{L3}(b - k_1)) \right) dp_3, \quad a_{L3} = (a_{L3} + \beta_L) \]

\[ dw_K = \left( a_{L1}/(a_{L1} a_{L3}(b - k_1)) \right) dp_3, \]

\[ dw_L = \left( a_{K1}/(a_{L1} a_{L3}(k_2 - k_1)) \right) dp_2 \]

\[ dw_K = \left( a_{L1}/(a_{L1} a_{L3}(k_2 - k_1)) \right) dp_2 \]

Assuming, without loss of generality, that commodity 1 is capital intensive and commodity 3 (the service) is labour intensive, we have: \((k_2 - k_1) < 0\) and \((b - k_1) < 0\). Thus, the signs of changes in factor prices are as follows:

\[ dw_L/dp_3 > 0, \quad dw_K/dp_3 < 0, \quad dw_L/dp_2 > 0, \quad dw_K/dp_2 < 0 \]

Furthermore, differentiating (16) we conclude that:

\[ dp_x^3/dp_x^2 = [a_{L3}/(k_1 - k_2)] [(k_1 - b)/a_{L2}] > 0 \]

Therefore, we have a consistent pricing system, which follows the Stolper-Samuelson theorem:

\[ \begin{bmatrix} dL/dp_x^3 > 0 \\ dK/dp_x^3 < 0 \end{bmatrix} \Rightarrow \begin{bmatrix} dp_x^3 \Rightarrow dp_x^2/dp_x^3 > 0 \Rightarrow \begin{bmatrix} dw_L/dp_x^2 > 0 \\ dw_K/dp_x^2 < 0 \end{bmatrix} \]

C. Resources

If the country exports services and is at a trading equilibrium, differentiating the full employment equations we have:

\[ a_{L1}dX_1 + a_{L2}dX_2 + a_{L3}dX_3 + \beta_L dE_3 = dL \]  \hspace{1cm}  (21.a)

\[ a_{K1}dX_1 + a_{K2}dX_2 + a_{K3}dX_3 + \beta_K dE_3 = dK \]  \hspace{1cm}  (21.b)

From Proposition 1, \( k_3 = k_3 = b \), hence \( a_{L3} = \beta_L, a_{K3} = \beta_K \). Collecting terms above, we get:
\[ a_{L1} dX_1 + a_{L2} dX_2 + b_L (dX_3 + dE_3) = dL \]  
(21.c)

\[ a_{K1} dX_1 + a_{K2} dX_2 + b_K (dX_3 + dE_3) = dK \]  
(21.d)

Solving:

\[
\begin{bmatrix}
dX_1 \\
dZ_3
\end{bmatrix} = (1/\Delta_{13}) \begin{bmatrix}
0_K & -b_L \\
a_{K1} & a_{L1}
\end{bmatrix} \begin{bmatrix}
dL - a_{L2} dX_2 \\
dK - a_{K2} dX_2
\end{bmatrix}
\]

where \( dZ_3 = dX_3 + dE_3 \).

Assuming \( dK = 0 \), an exogenous change in the amount of labour available in the economy \( (dL \neq 0) \) implies the following changes in the output of commodities 1 and 3:

\[ \frac{dX_1}{dL} = (1/\Delta_{13}) [b_K (1 - a_{L2} dX_2/dL) + b_L a_{K2} dX_2/dL] \]  
(22.a)

\[ \frac{dZ_3}{dL} = (1/\Delta_{13}) [-a_{K1} (1 - a_{L2} dX_2/dL) - a_{L1} a_{K2} dX_2/dL] \]  
(22.b)

and simplifying yields:

\[ \frac{dX_1}{dL} = (1/\Delta_{13}) [b_K + a_{L2} b_L (k_2 - b) dX_2/dL] \]  
(22.c)

\[ \frac{dZ_3}{dL} = (1/\Delta_{13}) [a_{K1} + a_{L1} a_{L2} (k_1 - k_2) dX_2/dL] \]  
(22.d)

or:

\[ \frac{dX_1}{dL} = (1/\Delta_{13}) b_L [b + a_{L2} (k_2 - b) dX_2/dL] \]  
(22.e)

\[ \frac{dZ_3}{dL} = (1/\Delta_{13}) a_{L1} (k_1 + a_{L2} (k_1 - k_2) dX_2/dL) \]  
(22.f)

Assuming, without loss of generality, that commodity 1 is capital intensive and commodity 3 (the service) is labour intensive, we have \( k_1 > k_2 > b \) and \( \Delta_{13} < 0 \). The signs in the above equations are as follows:

\[ \frac{dX_1}{dL} = (-)(+)(+)(+) + (+)(+)(+/-) \]

\[ \frac{dZ_3}{dL} = (-)(+)(+)(+) + (+)(+)(+/-) \]
PROPOSITION 5: The Rybczynski theorem holds \((dX_1/dL<0\) and \(dZ_3/dL>0)\) if:
\[-b/a_L2(k_2-b) < \frac{dX_2}{dL} < \frac{1}{a_L2(k_1-k_2)}\]  \hspace{1cm} (23)

Proof:

(i) If \(dX_2/dL > 0\), then:
\(dX_1/dL < 0\) always, and \(dZ_3/dL > 0\) if \(dX_2/dL < \frac{1}{a_L2(k_1-k_2)}\).

(ii) If \(dX_2/dL < 0\), then:
\(dX_1/dL < 0\) if \([-b/a_L2(k_2-b)] < \frac{dX_2}{dL}\), and \(dZ_3/dL > 0\) always.

V. Commercial Policy

Commercial policy aimed at restricting the flow of goods and services can take two major routes: (i) it can impose direct restrictions on trade in goods and services, or alternatively (ii) it can limit the factor mobility needed for the provision of services. This will necessarily limit or eliminate trade in services if such services have an export-factor-requirement as is the case modelled in this paper.

If the prohibition of trade in the service is the desired goal, a country at a cost disadvantage can simply place a quota at zero on either the service or on the movement of the factors required to export it. With direct restrictions on goods and services viewed as particularly non-cooperative from an international point of view, it would appear that the use of restrictions on the movement of factors would be the chosen route. After all, factor quotas are much more easily accepted in the international community than direct trade barriers. Restrictions on factor movements will, however, prevent or diminish trade in services and thus prevent service price differentials from moving closer to the cost of the export-factor-requirement \((w_0)\). We recall that, in our model, service prices will not be equalized internationally even when there is perfect factor mobility due to the assumption of the cost differential \((w_3)\) between services consumed domestically and those consumed abroad. If the use of tariffs is the chosen policy, then taking into account (15), the minimum prohibitive tariff \(t\) that country B could impose on the imports of services from country A is:

\[1 > w_B^{\theta_3B} - w_A^{\theta_3A} \quad \theta_3A = [\theta_3A + \theta]\]  \hspace{1cm} (24)
Choosing units such that $a_{L3A} = 1$ and $a_{K3A} = b$, normalizing $w_K = 1$ in both countries, and rearranging (24) becomes:

$$t > (a_{L3B}w_B - w_A) + (a_{K3B} - b)$$  \hspace{1cm} (24.a)

Alternatively, the same result can be achieved through taxes on foreign factors, where $q$ represents the factor tax vector:

$$[w_A + q]^*[a_{3A} + b] > w_B^B$$  \hspace{1cm} (25)

Choosing units and normalizing as before, we obtain:

$$q_L > (a_{L3B}w_B - w_A)$$  \hspace{1cm} (25.a)

$$q_K > (a_{K3B}/b) - 1 = a_{L3B}(k_B/b) - 1$$  \hspace{1cm} (25.b)

From (24.a) and (25.a)-(25.b) it is possible to establish the following equivalence:

$$t = q_L + bq_K$$  \hspace{1cm} (26)

Therefore, given the service export-factor-requirement, trade in the service can be blocked either through a prohibitive tariff as in (24) or prohibitive taxes on foreign factors as in (25). The equivalence between these alternative measures is given in (26).

VI. Productivity Differentials

The model we have developed can also be used to explain the issue of trade in services when productivity differentials in the traded goods sectors makes the relative prices of services different across countries. This explanation of why services (in general), why non-traded commodities may be relatively cheaper in poor countries has been forwarded by Harrod (1933), Balassa (1964), Samuelson (1964), emphasized by Kravis, Heston, and Summers (1982), and formalized in a general equilibrium framework by Bhagwati (1984a).

Relying heavily on the the Bhagwati analysis, suppose that only goods are tradeable and that a productivity differential of $\mu (\mu > 1)$ exists between the goods sectors in two countries. In this case factor prices are the same in both countries and goods prices are equalized. Figure 5
illustrates this situation. Thus we have

\[ \frac{p_1}{p_2} = \frac{X^0}{X^0_1 A^0 2A} = \frac{\mu X^0}{\mu X^0_1 B^0 2A} = \frac{X^0_1 B^0}{X^0_2 B^0}. \]  

(27)

Suppose further (as figure 5 does implicitly) that there are no productivity differentials in the service sectors. While productivity may be the same in the two service sectors, prices clearly will not be and the following relationship will hold for prices:

\[ p^0_3A = \mu p^0_3B. \]  

(28)

Suppose now that trade in services is allowed in the fashion that we have outlined in this paper, i.e. services can be exported but only with an additional factor relocation cost. In fact we might endogenize service trade in the Bhagwati formalization by positing that non-tradeability is imposed through prohibitions on factor movements: a prohibition which we now remove. The pricing system in this case is given by (7) and (8) above. If trade in services is to occur, we immediately see that only country B can be the service exporter. Country B, however, does not necessarily export services. It will do so only in the case where:

\[ \frac{p_3A}{(p_3B + wB)} > 1 \implies p_3A > p_3B + wB \]  

(29)

But recall that in equilibrium:

\[ p_3A/p_3B = \mu \implies p_3A > \mu p_3B \]  

(30)

Thus trade in the service requires:

\[ p_3B + wB < \mu p_3B \]  

(31.a)

and given that

\[ wB < p_3B (\mu - 1) \]  

(31.b)

we have

\[ wB/p_3B < \mu - 1 \]  

(31.c)

Thus we have the following proposition concerning comparative advantage and the productivity differential:

**PROPOSITION 6:** In the case of productivity differentials in the traded goods sector, for the service to be tradeable, the additional export cost (for factor relocation) per dollar of service exported must be smaller than the productivity differential \((\mu - 1)\) in the traded goods sector.
VII. Conclusion

The model presented here incorporates into standard international trade analysis what we believe to be an important distinction between trade in goods and trade in services, namely the factor-locational-requirement of services. While some services may not need this locational presence, it is clear that other services are indeed appropriately described by such a model. Within the framework of a 3x2x2 model (three commodities, one of which is the service, two factors, and two countries) it is possible to establish conditions for comparative advantage in the service as well as the pattern of trade.

It is seen that service prices will differ between countries because of the assumption of an additional cost in providing services abroad. Thus the service exporter must, in autarky, have a cost advantage greater than this cost factor, for trade to be initiated in the service. Trade will continue until service prices differ only by this cost differential. For the analysis of comparative advantage in the service the relevant price is thus the price of the service abroad. It is shown that in this 3x2x2 model the service would only be traded if it is at the favorable extreme (in terms of factor intensity) of the chain of comparative advantage.

In terms of commercial policy, it is shown that restrictions on factor mobility will have effects analagous to direct intervention in the flow of services. Thus a country wishing to limit service imports can restrict services either directly or indirectly by not allowing or limiting "factor migration;" in our model tariffs and taxes on foreign factors will have similar effects on services trade. As trade in the service cannot occur without factor mobility given the export-factor-requirement, trade in commodities cannot be a substitute for factor mobility and obviously cannot ensure factor price equalization. As restrictions on factor movements will limit service trade as easily as tariffs and quotas, current efforts and negotiations to liberalize international trade in services should take such factor restrictions into consideration. Such an inclusion is necessary if service trade is to be truly liberalized.

If restrictions on factor movements will tend to diminish trade in services, then clearly current U.S. pressures to have GATT applied to trade in services should also include GATT extension to temporary movements of productive factors. If nations were to superficially open to trad
economies to services trade, while simultaneously further restricting factor movements, this could hardly be considered a liberalization for those services for which factor presence is essential.

From a welfare viewpoint, it may well be the case that current restrictions on international trade in services are not optimal and that developing countries should consider liberalization in this area. It is equally clear, however, that developed countries should consider allowing increased factor mobility, especially as it relates to temporary labour movements, for such movements would allow developing nations to exploit services of their comparative advantage. Requests for "rights of establishment" are not on economic grounds inherently different from temporary movements of skilled and unskilled labour. While it is unlikely that many nations, developed or developing, are willing to include immigration policies in future trade negotiations, it is clear from an analytical point of view that these factor movements must be included in international agreements in order to make trade in services free and fair.
Appendices

Appendix 1

Assume that (i) a marginal change in service output is entirely accounted for by additional service exports \(dX_3 = dE_3 > 0\) and (ii) output prices do not change. Then by differentiating the full employment conditions stated in (2.a)-(2.b), we have:

\[
\begin{align*}
\alpha_1 dX_1 + \alpha_2 dX_2 + (\alpha_3 + \beta_L) dE_3 &= 0 \quad (A.1.a) \\
\alpha_K dX_1 + \alpha_K dX_2 + (\alpha_K + \beta_K) dE_3 &= 0 \quad (A.1.b)
\end{align*}
\]

Dividing by \(dE_3\) and rearranging:

\[
\begin{bmatrix}
dX_1/dE_3 \\
-dX_2/dE_3
\end{bmatrix} = - \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_K & \alpha_K \end{bmatrix}^{-1} \begin{bmatrix} \alpha_3 + \beta_L \\ \alpha_K + \beta_K \end{bmatrix}
\]

Inverting the matrix and solving we get expressions for \(dX_j/dE_3, j = 1, 2:\)

\[
\begin{bmatrix}
dX_1/dE_3 \\
-dX_2/dE_3
\end{bmatrix} = - (1/\Delta) \begin{bmatrix} \alpha_K & -\alpha_2 \\ -\alpha_K & \alpha_1 \end{bmatrix} \begin{bmatrix} \alpha_3 + \beta_L \\ \alpha_K + \beta_K \end{bmatrix}
\]

\[
\Delta = \Delta_1 = \alpha_1 \alpha_K - \alpha_2 \alpha_K = \alpha_1 \alpha_2 (k_2 - k_1)
\]

\[
\begin{bmatrix}
dX_1/dE_3 \\
-dX_2/dE_3
\end{bmatrix} = - (1/\Delta) \begin{bmatrix} \alpha_K & -\alpha_2 + \alpha_1 \bar{a}_K3 \\ -\alpha_K & \alpha_1 \bar{a}_L3 + \alpha_2 \bar{a}_K3 \end{bmatrix}
\]

\[
\bar{a}_i3 = a_i3 + \theta_i, \quad i = K, L
\]

\[
\begin{bmatrix}
dX_1/dE_3 \\
-dX_2/dE_3
\end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \alpha_1 \bar{a}_L3 (k_3^x - k_2^x) \\ \alpha_1 \bar{a}_L3 (k_1^x - k_3^x) \end{bmatrix} \quad (A.2.a)
\]

\[
\begin{bmatrix}
dX_1/dE_3 \\
-dX_2/dE_3
\end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \alpha_1 \bar{a}_L3 (k_3^x - k_2^x) \\ \alpha_1 \bar{a}_L3 (k_1^x - k_3^x) \end{bmatrix} \quad (A.2.b)
\]

Dividing (A.2.a) by (A.2.b) and rearranging we get:

\[
\frac{dX_1/dX_2}{dX_3/dE_3} = \frac{\alpha_2 (k_3^x - k_2^x)/\alpha_1 (k_1^x - k_3^x)}{dX_3/dE_3 = a_L2 (k_3^x - k_2^x)/a_L1 (k_1^x - k_3^x)} \quad (A.3)
\]

The above change can be decomposed into two parts. If a marginal change in service output affects only the domestic market \(dX_3 > 0, dE_3 = 0\) and output prices are constant, we have:

\[
\begin{align*}
\alpha_1 dX_1 + \alpha_2 dX_2 + \alpha_3 dX_3 &= 0 \quad (A.4.a) \\
\alpha_K dX_1 + \alpha_K dX_2 + \alpha_K dX_3 &= 0 \quad (A.4.b)
\end{align*}
\]

Solving:
\[
\begin{bmatrix}
\frac{dX_1}{dX_3} \\
\frac{dX_2}{dX_3}
\end{bmatrix} = \begin{cases}
(1/\Lambda) & \text{if } a_L a_{L3}(k_3 - k_2) \\
\frac{a_L a_{L3}(k_3 - k_2)}{a_L a_{L3}(k_1 - k_3)} & \text{if } a_L a_{L3}(k_1 - k_3)
\end{cases}
\]

(A.5.a) \\
(A.5.b)

Dividing (A.5.a) by (A.5.b) we get:

\[
[dX_1/dX_2]_{dE_3=0} = a_L a_{L3}(k_3 - k_2)/a_L a_{L3}(k_1 - k_3)
\]

(A.6)

If service output and prices are constant, then a marginal change in service exports (\(dX_3 = 0, dE_3 > 0\)) implies a shift of service output from the domestic to the foreign market, and we have:

\[
a_L dX_1 + a_L a_{L2} dX_2 + B_L dE_3 = 0
\]

(A.7.a)

\[
a_K dX_1 + a_K a_{K2} dX_2 + B_K dE_3 = 0
\]

(A.7.b)

Solving:

\[
\begin{bmatrix}
\frac{dX_1}{dE_3} \\
\frac{dX_2}{dE_3}
\end{bmatrix} = \begin{cases}
(1/\Lambda) & \text{if } a_L a_{L3}(b - k_2) \\
\frac{a_L a_{L3}(k_1 - b)}{a_L a_{L3}(k_1 - b)} & \text{if } a_L a_{L3}(k_1 - b)
\end{cases}
\]

\(b = B_K/B_L\)

(A.8.a) \\
(A.8.b)

Dividing (A.8.a) by (A.8.b) we get:

\[
[dX_1/dX_2]_{dX3=0} = a_L a_{L3}(b - k_2)/a_L a_{L3}(k_1 - b)
\]

\(b = B_K/B_L\)

(A.9)

Appendix 2

In a trading equilibrium in which the home country A exports the service to country B we have \(p^{x}_{3A} = p_{3B}\). From pricing system (7)-(8) this implies:

\[
p^{x}_{3A} = w_A(a_{3A} + b) = w_B a_{3B} = p_{3B}
\]

(A.10)

or:

\[
w_A(a_{3A} + b_L) + w_K(a_{K3A} + b_K) = w_B a_{3B} + w_K a_{K3B}
\]

(A.10')

Assuming without loss of generality that by a convenient choice of units, \(a_{L3B} = 1\), and thus \(a_{K3B} = b\), and normalizing \(w_K = 1\), condition (A.10') becomes:

\[
w_A(a_{L3A} + b_L) + (a_{K3A} + b_K) = (w_B + b)
\]

Rearranging:

\[
(w_A a_{L3A} + a_{K3A}) + (w_A b_L + b_K) = (w_B + b)
\]
or:
\[ a_{L3A}(w_A + a_{K3A}/a_{L3A}) + b_L(w_A + b_L) = (w_B + b) \]

Taking into account that in equilibrium \( k_3 = k_3^* = b \), we have:

\[ a_{L3A}(w_A + b) + b_L(w_A + b) = (w_B + b) \]

Collecting terms:
\[ (a_{L3A} + b_L)(w_A + b) = (w_B + b) \]

or:
\[ a_{L3A}(w_A + b) = (w_B + b) \]

Hence:
\[ (w_A + b)/(w_B + b) = 1/a_{L3A} \]  \( \text{(A.11)} \)

Appendix 3

From (7) the prices for goods \( X_1 \) and \( X_2 \) are:

\[ 1 = w_L a_{L1} + w_K a_{K1} \]  \( \text{(A.12.a)} \)

\[ p^X_2 = w_L a_{L2} + w_K a_{K2} \]  \( \text{(A.12.b)} \)

Assuming that all changes in service output are export changes, from (8) the marginal price of the service is:

\[ p^X_3 = w_L a_{L3} + w_K a_{K3} \]

\[ \dot{a}_{i3} = a_{i3} + b_i \quad i = K, L \]  \( \text{(A.12.c)} \)

In matrix form we have:

\[
\begin{bmatrix}
1 & p^X_2
\end{bmatrix} =
\begin{bmatrix}
w_L & w_K
\end{bmatrix}
\begin{bmatrix}
a_{L1} & a_{L2} \\
a_{K1} & a_{K2}
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & p^X_3
\end{bmatrix} =
\begin{bmatrix}
w_L & w_K
\end{bmatrix}
\begin{bmatrix}
a_{L1} & \dot{a}_{L3} \\
a_{K1} & \dot{a}_{K3}
\end{bmatrix}
\]

Inverting we get factor prices:

\[ [w^X_L \quad w^X_K] = (1/\Delta_{12}) \begin{bmatrix} 1 & p^X_2 \end{bmatrix} \begin{bmatrix} \dot{a}_{K2} & \dot{a}_{L2} \\
\dot{a}_{K1} & a_{L1}
\end{bmatrix} \]  \( \text{(A.13.a)} \)

\[ [w^X_L \quad w^X_K] = (1/\Delta_{13}) \begin{bmatrix} 1 & p^X_3 \end{bmatrix} \begin{bmatrix} \dot{a}_{K3} & \dot{a}_{L3} \\
\dot{a}_{K1} & a_{L1}
\end{bmatrix} \]  \( \text{(A.13.b)} \)

\[ \Delta_{12} = a_{L1} a_{K2} - a_{L2} a_{K1} = a_{L1} a_{L2} (k_2 - k_1) \]
\[ \Delta_{13} = a_{L1} \bar{a}_3 k_3 - \bar{a}_{L3} a_{K1} = a_{L1} \bar{a}_{L3} (k_3^x - k_1) \]

Equating the values for \( w_k \) from (A.13.a) and (A.13.b):

\[ (-a_{L2} + p_x^{a_{L1}})/\Delta_{12} = (-\bar{a}_{L3} + p_x^{a_{L1}})/\Delta_{13} \]

Substituting for the values of \( \Delta_{12} \) and \( \Delta_{13} \) and rearranging:

\[ -1/a_{L1}(k_2 - k_1) + p_x^{a_{L2}}/a_{L2}(k_2 - k_1) = -1/a_{L1}(k_3^{x_3} - k_1) + p_x^{a_{L3}}/\bar{a}_{L3}(k_3^{x_3} - k_1) \]

\[ (-k_3^{x_3} + k_1 + k_2 - k_1)/a_{L1}(k_2 - k_1)(k_3^{x_3} - k_1) + p_x^{a_{L2}}/a_{L2}(k_2 - k_1) - p_x^{a_{L3}}/\bar{a}_{L3}(k_3^{x_3} - k_1) = 0 \]

\[ (k_3^{x_3} - k_2)/a_{L1}(k_2 - k_1)(k_1 - k_3^{x_3}) + p_x^{a_{L2}}/a_{L2}(k_2 - k_1) + p_x^{a_{L3}}/\bar{a}_{L3}(k_1 - k_3^{x_3}) = 0 \]

\[ (k_3^{x_3} - k_2)/a_{L1} + [(k_1 - k_3^{x_3})/a_{L2}]p_x^{a_{L2}} + [(k_2 - k_1)/\bar{a}_{L3}]p_x^{a_{L3}} = 0 \]

Since in equilibrium \( k_3 = k_3^{x_3} = b \) we obtain:

\[ (b - k_2)/a_{L1} + [(k_1 - b)/a_{L2}]p_x^{a_{L2}} + [(k_2 - k_1)/\bar{a}_{L3}]p_x^{a_{L3}} = 0 \]  \hspace{1cm} (A.14)
Figure 5

$X_1^0 = \mu X_{1B}^0 = \frac{1}{P_1^0}$

$X_2^0 = \mu X_{2B}^0 = \frac{1}{P_2^0}$

$X_3^0 = \mu X_{3B}^0 = \frac{1}{P_3^0}$

$X_3^0 = \frac{1}{P_3^0}$

$X_3^0 = \frac{1}{\mu P_3^0}$
1/ See in particular the work of Chenery (1960) and Chenery and Syrquin (1975).

2/ Deardorff (1984) does analyze this issue of factor presence in his discussion of traded services and comparative advantage. He analyzes three particular features of services: (i) that traded services often arise as a by-product of goods trade; i.e. "trade services" such as shipping and freight insurance are necessary only to aid trade in goods [For a comprehensive treatment of "trade services" see Cunha (1985) where such services in the form of transport costs are handled in a 3x2x2 model] (ii) that services may require the provider of services to send factors abroad, especially in the form of direct foreign investment, to deliver the service; i.e. producer presence in the importing country and (iii) that while services may need to be consumed where they are produced, production can take place without the presence of all factors; i.e. some factors can contribute to production from a distance. In essence, Deardorff is interested only in whether these characteristics can cause the law of comparative advantage to be violated. He finds that it is case (iii), (analyzed in a one-good, one-service model) where some factors can be physically distant (while others are necessary 'on site') from the production and consumption process that poses the most difficulty for comparative advantage theory while cases (i) and (ii) can be handled relatively easily.

3/ We will use the terminology "commodity" where this embraces both goods and services.

4/ Indeed Sampson and Snape (1985) cite the importance of this proximity specifically for international trade. They write: "In a great number of cases, services require the supplier of the service and the receivers to be in physical proximity..... For many purposes the distinction between those services which require the physical proximity of the supplier of the service and the receiver of a
and those services (like goods) which do not is unimportant. In trade across national frontiers, however, it can carry major policy implications." (p. 172)

5/ In terms of banking one might argue that international banks wishing to export banking services can simply use correspondent banks. While correspondent banks can indeed provide some of the services offered by international banks, physical presence or the ability to 'have one's own people there' seems important. In numerous conversations with international bankers, the importance of being able to set up shop abroad directly was emphasized to us. Presence is required for 'explaining' the wide variety of services which are offered and also for attracting new customers or to get a 'feel' for local business opportunities. In some cases this local presence might be thought of as an overseas skilled sales force which is not available in the service importing country. While we might expect that technology would allow 'long distancing' to operate, the opposite appears to be the case: Deepak Nayyar(1986) writes: "...technological change has substantively enhanced the possibilities of trade in what have always been traded services. Financial services such as banking and insurance, which can be disembodied from the producer and transported to the consumer, have entered into this category. Ironically enough, in these cases, the impact of technical change is such that the physical proximity of the producer and the consumer, not necessary so far, is now eagerly sought because it enlarges the size of the market and increases the profitability for producers of services." (p. 7)

6/ While Bhagwatl(1984b) and (1985) points out a number of areas in which technology has made 'long distancing' possible, it would appear medical care and education still fall outside of this realm. Thus medical advice can be transmitted across countries, though medical services of this type cannot take into account the importance of patient/doctor relationships. The same holds for education where exporting textbooks or even videotapes of a prestigious professor are an imperfect substitute for the 'real thing.'

7/ This is clearly a simplification if one thinks of the additional exports costs as some form of learning costs; in this case we would expect these additional costs of doing business abroad to fall over
time. We could even model some type of learning curve with declining costs over time, though this would be at great loss in terms of the model's simplicity.

8/ In a forthcoming paper we model the interaction of these mobile factors explicitly. For example, this allows for the service exporting country to send 'management' abroad while hiring labour domestic labour in the importing country. This is clearly a more realistic view of the way in which service exporter companies operate.

9/ From a formal point of view, this export factor requirement can be seen as a "transport cost" of the "melting-ice" variety. However, the assumption that it is only the service which requires an additional cost when exported rather than all traded goods, distinguishes our model from the standard melting-ice model of Samuelson (1952) and a fortiori from the more recent full-fledged transport model of Cunha (1985).

10/ The prohibition of services occurs for a variety of reasons, often dealing with the protection of certain "infrastructure" services, especially banking and telecommunications. Where specific sectors are nationalized, prohibition or very limited access is virtually a certainty.

11/ Following Bhagwati (1964), if two countries A and B produce two commodities $X_1$ and $X_2$ using two factors, K and L, let $X_A = F_A(K_{X1}, L_{X1})$ and $X_B = \mu F_A(K_{X1}, L_{X1})$. If $\mu > 1$, then country B has Ricardian style neutral productivity advantage in the production of $X_1$.

12/ For recent discussions on trade negotiations with special emphasis on trade in services see Bhagwati (1985, 1986), and Hindley (1986).
REFERENCES


Bhagwati, Jagdish N., 1984a, "Why are Services Cheaper in the Poor Countries," Economic Journal, 94 (June) 279-286.


