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STRUCTURAL CHANGE:

A BRIEF REVIEW

by

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1 Introduction

This is a short summary of some aspects of structural change models, and is not intended to be a comprehensive survey or review. Many important works have been left out. Also, the presentation was not intended to be rigorous nor general. It's main purpose is only to focus on some recent developments in this area and motivate the reader to learn more. The interested reader might whish to consult some surveys: Zacks (1983), Kramer and Sonnberger (1986), Tsurumi (1988), Hackl and Westlund (1989), and Brodsky and Darkhovsky (1993).

2 Single Shift Model

2.1 Known Shift Point

$$H_0 : y_t = \mu + e_t, \quad t = 1, 2, \cdots, T$$

$$e_t \text{ i.i.d. } N(0, \sigma^2)$$

Under H_0 the MLE estimator of μ is the sample mean:

$$\hat{\mu} = \bar{y}$$

Let $RSS_r = \sum_{t=1}^{T}(y_t - \bar{y})^2$.

$$\begin{aligned} H_1 : y_t &= \mu_1 + e_t, \quad t = 1, 2, \cdots, k \\ y_t &= \mu_2 + e_t, \quad t = k+1, \cdots, T \end{aligned}$$

$$e_t \text{ i.i.d. } N(0, \sigma^2)$$

Under H_1 the MLE estimators are given by:

$$\hat{\mu}_1 = \bar{y}_1 = \frac{1}{k}\sum_{t=1}^{k} y_t$$

$$\hat{\mu}_2 = \bar{y}_2 = \frac{1}{T-k}\sum_{t=k+1}^{T} y_t$$

Also, let $RSS_u = \sum_{t=1}^{k}(y_t - \bar{y}_1)^2 + \sum_{t=k+1}^{T}(y_t - \bar{y}_2)^2$.

The Chow (1960) or F-test is given by:

$$F = \frac{RSS_r - RSS_u}{RSS_u/(T-2)}$$

If H_0 is true, F has an $F_{(1,T-2)}$ distribution. This test can be shown to be uniformly most powerful invariant.

An equivalent alternative is the LR test which is a monotone transformation of F, and is asymptotically χ_1^2 under H_0. This test is valid even if the errors are non-normal.

Comments:

1. The above results can be extended to multiple regression models. See Kramer and Sonnberger (1986), pp. 44-45.

2. When the variance is also allowed to change, the Chow test is no longer equivalent to the LR test. Kramer and Sonnberger (1986), pp. 46-48.

2.2 Unknown Shift Point

When k is not known, we must estimate it from the data. The MLE of k, denoted by $\hat{k}$, is obtained by minimizing RSS_u over all possible shift points:

$$\min_k RSS_u$$

The same $\hat{k}$ is obtained by selecting the shift point which maximizes the Chow test.

Valid tests are obtained by computing the Chow and the LR test at the estimated shift point, $\hat{k}$. The distribution of these tests under H_0 involves a technical problem because the parameter k is not identified under the null. Only recently has the asymptotic distribution been obtained.

Comments:

1. Once again, these results can be extended to multiple regression models.

2. The proposed tests can also be generalized to non-linear models (Wald, LR and LM tests). Andrews (1993).

3. All the above results have also been extend to the case where the disturbances are not identically nor independently distributed. Andrews (1993).

4. The properties of the estimator $\hat{k}$ have also been discussed. Hinkley (1970), Deshayes and Picard (1986), Bai (1995).

5. The case of multiple shifts has also been considered in the literature. However, the related problem of estimating the number of structural shifts under general conditions is still an open research topic. See Yao (1988), Bai and Perron (1995), Liu, Wu and Zidek (1996).

6. Finally, Andrews, Lee and Ploberger (1996), determine the optimal tests for one or more shifts at unknown times in a multiple linear regression model. They show that the optimal tests are given by exponential averages of the F or Chow tests, and that the usual max-F and the max-LR tests are not optimal in their sense. The average is taken over every possible shift point.

3 Other Models of Structural Change

Several other models of structural change have been proposed in the literature.

3.1 Continuous Structural Change

The standard assumption is that there is a single structural break in the sample. However that shift is usually assumed to occur instantaneously. An alternative is to consider that the transition is smooth over time. The test by Farley et al. (1975) assumes that the parameters changes as a linear function of time. Lin and Terasvirta (1994) allow the change to be non-linear.

3.2 Stochastic Parameter Fluctuation

In all the above models, the structural change occuring in the parameters of the model are deterministic. Another strand of literature is concerned about the case in which the alternative to parameter constancy is that the parameters are stochastic and fluctuate according to some time series model.

For example, one possible alternative is that the parameters follow a random walk:

$$\mu_t = \mu_{t-1} + p_t$$

where

$$p_t \text{ i.i.d. } (0, P)$$

and are independent of the disturbance e_t. A test for parameter constancy is a test that $P = 0$, and can be carried out by a LR or an F-test. See Cooley and Prescott (1976), Lamotte and McWhorter (1978), Nyblom (1989).

3.3 Switching Regressions

Standard models of structural change assume that the shift in the parameters is explained only by the time variable. However, in many occasions, the reason for such shifts may be encountered in other relevant variables. The most simple switching regression model is given by (Goldfeld and Quandt, 1973):

$$\begin{aligned} y_t &= x_t'\beta_1 + e_{1t}, \quad t \in I_1 \\ y_t &= x_t'\beta_2 + e_{2t}, \quad t \in I_2 \end{aligned}$$

where I_1 and I_2 are the set of indices for which the regression equations hold. Also, there exists variables z_t such that nature selects regimes 1 and 2 according to whether $\pi' z_t \leq 0$ or > 0 where π is a vector of unknown coefficients. Another possibility is that nature selects regimes 1 and 2 according to some probabilities λ, $1 - \lambda$. There are many other possibilities of what type of mechanism induces the state to switch from one regime to another (for example Hamilton (1994) considers Markov switching).

4 General Tests of Structural Change

4.1 CUSUM and CUSUM-SQ Tests

The most well known procedure to detect structural change is the CUSUM test by Brown, Durbin and Evans (1975). The CUSUM test is based on the (standardized) differences between y_t and the best prediction of y_t based on all data available up to time $t - 1$. These correspond to the recursive residuals:

$$\tilde{u}_t = y_t - \frac{1}{t-1}\sum_{i=1}^{t-1} y_i$$

If μ is constant up to time r then $\tilde{u}_t$ has mean zero for $t \leq r$.

If μ changes after time r then $\tilde{u}_t$ has non-zero mean for $t > r$.

The advantage of recursive residuals over OLS residuals, is that the mean of the latter are affected for all t by a change in the mean μ. Also, recursive residuals are independent, have constant variance, and are normally distributed if the disturbances are normal.

The CUSUM quantity is given by:

$$W_r = \frac{1}{s}\sum_{t=2}^{r} \tilde{u}_t, \qquad r = 2, \cdots, T$$

where s is an estimate for the standard deviation σ.

If, for example, the mean μ, changes at time $r+1$ to $\mu + \delta$ with $\delta > 0$, then the expected value of W_t changes from 0 to some positive values after time $r+1$.

The asymptotic distribution of the CUSUM quantity under the null can be obtained. A possible test procedure rejects the null if the path of W_r, $r = 2, \ldots, T$ crosses two symmetric lines constructed in such a way as to ensure a given significance level.

The CUSUM of squares test relies on successive sums of squares of recursive residuals. This test is preferable when the parameter instability is not systematic and can change signs frequently.

$$S_r = \sum_{t=2}^{r} \tilde{u}_t^2 / \sum_{t=2}^{T} \tilde{u}_t^2, \qquad r = 2, \cdots, T$$

Under the null, S_r has a beta distribution with mean $(r-1)/(T-1)$. A possible test rejects the null whenever S_r crosses one of the lines $(r-1)/(T-1) - c$, $(r-1)/(T-1) + c$. Similar to the CUSUM test, the choice of c determines the significance level.

Comments:

1. The above tests can be extended to multiple regression models with non-identically and non-independently distributed erros. Kramer and Sonnberger (1986).

2. In the case of several explanatory variables, the sign of the expected value of recursive residuals is not obvious. It depends on the signs and magnitudes of the changes in each of the parameters. It is possible for some combinations of parameter

changes that the expected value remains zero even after the change has occured. In such cases the CUSUM test will have no power: The folllowing example illustrates this point. Consider the following model:

$$y_t = \beta_{2t}(-1)^t + 1/2 + e_t$$

where $\beta_{2t} = 1/2$ for $t \leq k$, and $\beta_{2t} = 3/2$ for $t \geq k+1$. The mean regressor is given by $\lim_{T\to\infty} \sum x_t/T = (0, 1/2)'$. The structural shift in the parameter vector is given by $(1, 0)'$. In this case the structural shift and the mean regressor are orthogonal and the CUSUM power will have no power. For example, if k is odd, $(-1)^{(k+1)} = 1$, and the recursive residual will be $\tilde{u}_{k+1} = 1$. At time $k+2$, the recursive residual will become $\tilde{u}_{k+2} = -1$. The successive recursive residuals cancel each other and the CUSUM quantities cannot accumulate.

4.2 Modifications of the CUSUM and CUSUM-SQ Tests

The following modifications of the CUSUM and CUSUM-SQ tests have been proposed in the literature:

1. If the shift occurs late in the sample, both tests don't have enough time to acumulate the effects of the shift. The power will be low in such cases.

One possible solution is to revert the order of the observations.

A second solution computes the recursive residuals in the usual way but cumulates these residuals backwards. Schweder (1976).

2. Although the estimator s is well behaved under the null, depending on the type of structural change, it is possible for s^2 to overestimate σ^2, resulting in a loss of power of the CUSUM test. Alternative estimators of σ^2 which are not affected by structural changes, although less efficient under ther null, might improve the power. See Harvey (1975).

3. The distribution of the CUSUM-SQ test is heavily dependent on the assumption of normality of the disturbances. In particular, the kurtosis enters the distribution of the test statistic. One solution is to "studentize" the CUSUM-SQ by multiplying it by a factor that corrects the deviation from normality, and ensures the same asymptotic distribution whether the disturbances are normal or not. See Kramer and Sonnberger (1986).

4.3 OLS-CUSUM and OLS-CUSUM-SQ Tests

Since OLS residuals are much more easily obtained, a modification of the CUSUM and CUSUM-SQ tests can be proposed with OLS rather than recursive residuals. As discussed above, the OLS residuals do not have the simple distributional properties of recursive residuals. For this reason, the distribution of the OLS-CUSUM and OLS-CUSUM-SQ was harder to obtain. However, Ploberger and Kramer (1992), show that while the usual CUSUM quantities converge to a Brownian motion, the OLS-CUSUM quantities converge to a Brownian Bridge. They also show that the OLS-CUSUM has higher (local) power for certain types of structural change than the CUSUM test, but no test is uniformly superior.

4.4 MOSUM Tests

The MOSUM quantities are given by:

$$M_t = \frac{1}{s} \sum_{i=t-G+1}^{t} \tilde{u}_i, \quad t = 1 + G, \cdots . T$$

where G is a fixed number of terms in each moving sum.

The advantage of this quantity is that the relative importance of recursive residuals that due to a zero mean do not contribute to a significant test is automatically limited. There is in general no rule to choose G.

The MOSUM-SQ quantities are given by:

$$MS_t = \sum_{i=t-G+1}^{t} \tilde{u}_i^2 / \sum_{i=t-G+1}^{T} \tilde{u}_i^2, \quad t = 1 + G, \cdots , T$$

For a discussion of the MOSUM test see Hackl (1980) and Chu, Hornik and Kuan (1993).

4.5 Fluctuation Test

Instead of looking at recursive residuals, it is possible to test for parameter constancy by looking at the fluctuations of the recursive parameter estimates $\hat{\mu}^t = \sum_{i=1}^{t} y_i / t$. Since the true parameter against which to compare the fluctuations of the recursive

estimates is not known, the full sample estimate $\hat{\mu}^T$, is used. The test statitic is given by:

$$F^T = \max_{t=1,\ldots,T} F_t^T$$

where

$$F_t^T = \frac{t-1}{s(T-1)}\sqrt{T} \mid \hat{\mu}^t - \hat{\mu}^T \mid$$

Notice that $F_1^T = F_T^T = 0$. The test rejects the null when F^T exceeds some critical value.

The Fluctuation test was first proposed by Ploberger (1983). For further discussion see Kramer and Sonnberger (1986), and Ploberger, Kramer and Kontrus (1989).

5 Asymptotic Distributions Under the Null

5.1 FCLT and the CMT

Consider the null:

$$H_0 : y_t = \mu + e_t, \quad t = 1, 2, \cdots, T$$

$$e_t \text{ i.i.d. } (0, 1)$$

Consider the following sums (random-walk):

$$S_k = \sum_{i=1}^{k} e_i, \quad k = 1, 2, \cdots, T$$

We have that: $E(S_k) = 0$, $\text{var}(S_k) = k$, $\text{var}(S_T) = T$.

Consider now the normalized sums:

$$\tilde{S}_k = S_k/\sqrt{T}, \quad k = 1, 2, \cdots, T$$

We have that: $E(\tilde{S}_k) = 0$, $\text{var}(\tilde{S}_k) = k/T$, $\text{var}(\tilde{S}_T) = 1$.

Finally, consider

$$S_T(r) = \tilde{S}_{[Tr]}, \quad r \in [0, 1]$$

We have that: $E(S_T(r)) = 0$, $\text{var}(S_T(r)) = r$, $\text{var}(S_T(1)) = 1$.

Also, by the FCLT: $S_T(r) \Rightarrow W(r), \quad k \in [0,1]$, as $T \to \infty$, where $W(r)$ is the standard Wiener process or Brownian motion.

Another very useful theorem is the CMT which states that if g is a continuous functional, then $g(S_T(r)) \Rightarrow g(W(r)), \quad k \in [0,1]$, as $T \to \infty$.

It can be shown that the fundamental FCLT holds under more general conditions of the disturbance term. It allows non-identically and non-independently distributed errors.

5.2 Examples

1. It is easy to show that the LR test (more precisely, -2 times the log-likelihood ratio) is given by:

$$LR = \max_k k \frac{T}{T-k} (\bar{y}_1 - \bar{y})^2$$

Under the null it follows that

$$\begin{aligned} LR &= \max_k k \frac{T}{T-k} \left(\frac{1}{k}\sum_{t=1}^{k} e_t - \frac{1}{T}\sum_{t=1}^{T} e_t\right)^2 \\ &= \max_k \frac{T}{k} \frac{T}{T-k} \left(\tilde{S}_k - \frac{k}{T}\tilde{S}_T\right)^2 \\ &= \max_r \frac{T}{[Tr]} \frac{T}{T-[Tr]} \left(S_T(r) - \frac{[Tr]}{T} S_T(1)\right)^2 \\ &\Rightarrow \max_r \frac{1}{r} \frac{1}{1-r} (W(r) - rW(1))^2 \end{aligned}$$

To ensure a non-degenerate distribution, the maximum over r has to be taken over a compact strict subset of $[0,1]$. Tables with the distribution of this random variable can be consulted to provide the appropriate critical values for the LR test. It can be shown that the Max-Chow test has the same asymptotic distribution. The results for general multiple regression models and non-i.i.d. errors are presented in Andrews(1993).

2. The asymptotic distribution of the CUSUM quantities can also be derived. The CUSUM test is based on the quantities, $\frac{1}{\sqrt{T}} \sum_{t=2}^{r} \tilde{u}_t$. Since the recursive residuals are i.i.d. under the null we can apply the FCLT to obtain that the CUSUM quantities

converges to the Brownian motion. The CUSUM test follows by applying known results of boundary crossing probabilities of the Brownian motion.

3. The Fluctuation Test can be written as

$$\begin{aligned} F^T &= \max_{k=1,\ldots,T} \frac{k}{\sqrt{T}} \mid \hat{\mu}^k - \hat{\mu}^T \mid \\ &= \max_r \mid S_T(r) - \frac{[Tr]}{T} S_T(1) \mid \\ &\Rightarrow \max_r \mid W(r) - rW(1) \mid \end{aligned}$$

4. The CMT allows us to obtain the distribution of many other tests based on different functionals. Examples of other such tests are the range of recursive estimates test, the range of the recursive residuals test, or the average F-test. See Kuan and Hornik (1993), and Andrews, Lee and Ploberger (1996).

5. These tools also allow the computation of the asymptotic power of the tests considered here. It is possible to compare different tests in terms of power and local power. Local power refers to the case where the magnitude of the break diminishes as the sample size increases. It is possible, for example, to prove that the Fluctuation and the LR (or max-F) tests have non-trivial power against local alternatives of a single shift, as well as other local alternatives. However the CUSUM-SQ has only trivial local power against single-shift local alternatives. Also, the CUSUM has local power not less that the CUSUM-SQ test against single-shift local alternatives.

6 Simulations

Several authors have conducted simulation experiments in order to compare the performance in terms of power of different tests for structural change. The results may be summarized as follows.

1. Single Shift Model

The CUSUM test performs better when the shift occurs early in the sample. Also, the power of the CUSUM test is heavily dependent on the angle between the structural change and the mean regressor.

The Fluctuation test is more powerfull when the shift occurs in the latter half of the sample than when it occurs in the earlier half.

The max-F test has power that is invariant with respect to the direction of time and is more powerfull at the middle of the sample.

The MOSUM tests are dominated by the Fluctuation test.

The Range of moving estimates is the best at exactly mid-sample.

The average-F test performs very well for a wide range of locations of the shift.

2. Double Shift Model

When there are two shifts in the mean, the possible combinations of location and magnitude of the shifts is very large. The available results are mixed. It is no longer possible to say which test is better in which situation. It seems better to base any conclusions on the results of several tests. See Kuan and Hornik (1993).

7 Alternative Approaches

We have left out some other important approaches to structural change. There is an extensive literature on Bayesian and Non-parametric approaches. Tsurumi (1988) contains a survey of Bayesian methods, and Brodsky and Darkhovsky (1993) a review of non-parametric methods.

Another important approach to structural change is the sequential detection of structural change

7.1 Monitoring Structural Change

Consider the following problem. We are given an historical data set

$$\{X_t, \ t = 1, \ldots, m-1, m\}$$

Suppose that based on this data set a well formulated parametric model has been estimated. This model can be used for decision or policy analysis.

With the arrival of new data, the following questions emerges: Is yesterday's model capable of explaining today's data? Is yesterday's policy still valid?

The objective is to monitor parameter constancy after the model has been estimated.

Consider the following sequential post-sample F-test monitoring procedure.

Wait for 5 observations to arrive, then perform an $F(5, m-k)$-test. If the F-test is passed, update the model by including the 5 observations and wait for another 5 days. If the F-test fails, signal inadequacy.

Consider the case where no structural change occurs. Simulation results show that 30 periods later, the F-test test will have failed with a proability of 33%. 100 periods later the probability increases to 70%, and 300 periods later, the probability is already more than 95%.

Consider now the following setting.

Y_t i.i.d. $(0, 1)$

We want to monitor if the mean of Y_t stays at zero or not. Consider the Fluctuation test:

$$FT_n = \max_{k \leq n} \sqrt{n}(k/n) \mid \bar{Y}_k \mid$$

where $\bar{Y}_k = k^{-1} S_k$, and $S_k = \sum_{t=1}^{k} Y_t$. The following decision rule is adopted at time n:

(a)If $\mid S_n \mid \geq \sqrt{n} c$ signal a structural change,

(b)If not, mantain the model and keep on monitoring.

Here, c, is a constant that ensures a given significance level.

However the Law of the Iterated Logarithm implies that under the assumption of no structural change,

$$Prob\{\mid S_n \mid < \sqrt{n} c, \text{ for every } n \geq 1\} = 0.$$

So, this sequential monitoring procedure eventually rejects the null even though the null is correct. The probability of type I error is one.

The important point made by these examples is that one should find a boundary function such that the path of our test statistic crosses this boundary with a required probability under the null.

Once again, the choice of these boundary functions, g, can be based on the FCLT:

$$\lim_{n \to \infty} Prob\{S_n \geq \sqrt{m} g(n/m), \text{ for some } n \geq 1\}$$

$$= Prob\{W(t) \geq g(t), \text{ for some } t \geq 0\}$$

It is possible to approximate the boundary crossing probability of a normalized partial sum process by the boundary crossing probability of a Brownian motion. Chu, Stinchcombe and White (1996), develop boundary functions under very general assumptions on the disturbance terms.

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