Horizontal mergers, entry and international trade*

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Abstract

We use a two-country trade model to analyze an authority’s decision to approve or reject a merger followed by entry, when the entrant can choose where to locate. We show that approval of a merger in the small country followed by timely, likely and sufficient entry may lead to lower consumer welfare than its rejection: when the alternative to such entry is entry into another country that also benefits consumers through trade, then the gains of attracting entry are small. In this context, we discuss differences between optimal decisions by the small country’s authority, large country’s authority and supranational authority.

Key Words: Horizontal mergers, entry, international trade

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1 Introduction

This article analyzes the optimal decision of a competition authority with respect to the approval or rejection of a merger between firms located in a small country, when (i) international trade plays a relevant role and (ii) the merger may affect an entrants’ decision of where to locate.

We use a simple two-country international trade model with post-merger entry to illustrate that the decision to approve a merger followed by likely, timely and sufficient entry may result in lower consumer welfare than the decision to reject it.\(^1\) Although existing guidelines state that when entry is relatively easy, a merger is unlikely to have any significant anti-competitive effects, we show that merger rejection may benefit consumers when the entrant has the possibility of choosing from different entry locations. Our main argument runs as follows. A domestic horizontal merger may lead to entry into a small country that, in its absence, would not have occurred. However, if such merger is rejected, there may be entry into another country instead of no entry at all. With international trade, this alternative form of entry benefits consumers in both countries. Hence, the gains from attracting an entrant may be relatively small, given that entry, although in a different country, would have occurred in any case: the merger merely diverts entry. A forward looking authority should thus compare the effects of the domestic merger followed by entry with those of the alternative outcome in case of merger rejection. When this outcome includes entry abroad and there is international trade, it will benefit domestic consumers.\(^2\) In the absence of trade, there would be no positive effect in one country due to entry into the other one. Therefore, the gains from attracting entry would be larger and a merger that resulted in sufficient entry should be approved.

International trade also implies that the effects of a domestic merger are felt abroad. Thus, other countries’ authorities, or supranational authorities, may also have jurisdiction over the merger.\(^3\) When this is the case, we show that larger countries can be more or less conservative

\(^{1}\)The European Commission (2004) states that "for entry to be considered a sufficient competitive constraint on the merging parties, it must be shown to be likely, timely and sufficient to deter or defeat any potential anti-competitive effects of the merger." According to the European Guidelines, "Entry is normally only considered timely if it occurs within two years. (...) For entry to be likely, it must be sufficiently profitable taking into account the price effects of injecting additional output into the market and the potential responses of the incumbents. (...) Entry must be of sufficient scope and magnitude to deter or defeat the anti-competitive effects of the merger." A similar description can be found in the US merger guidelines.

\(^{2}\)Interestingly, the European Guidelines allow for this possibility: "in some circumstances, the Commission may take into account future changes to the market that can reasonably be predicted. It may, in particular, take into account the likely entry or exit of firms if the merger did not take place when considering what constitutes the relevant comparison".

\(^{3}\)There are several high profile cases of domestic mergers that fall under foreign jurisdiction, such as the 1997 Boeing-McDonnell Douglas merger, an American merger that was subject to the EC remedies. As for supranational authorities, in the European Union for instance, a concentration has a Community dimension, and therefore is analyzed by a supranational authority when some turnover thresholds are found, "unless each
than smaller countries with respect to merger approval, depending on the degree of the countries’ size asymmetry. Finally, we also show that size asymmetry is relevant in determining which decisions by a domestic authority may be safely validated by a supranational authority. Our results suggest that when country asymmetry is small (large), a supranational authority can safely uphold a rejection (approval) decision by the small country’s authority.

As argued by Gowrisankaran (1999) "(...) dynamic determinants of firm behavior (principally entry, exit and investment) are likely to have large effects on which mergers occur in an industry (...)”. Here, we focus on the opposite, i.e. on how mergers affect entry decisions. This article is therefore closely related to the literature on the effects of horizontal mergers in the presence of entry. Werden and Froeb (1998) show that in a symmetric Cournot model entry is unlikely to occur after a merger and also that its expected countervailing price effect is small. Along the same lines, Spector (2003) shows that entry is insufficient to counterweight the negative effects on consumer surplus derived from a price increase as a consequence of the merger. Using a Salop-type approach, Cabral (2003) shows that mergers increase the likelihood of entry, benefitting consumers through lower transportation costs. Contrary to what might be expected, cost synergies may be welfare-detrimental for consumers as they decrease the probability of entry. Pesendorfer (2005) studies merger incentives in a repeated game with gradual entry and shows that a merger for monopoly may not be profitable when firms expect no further mergers in the future and that a merger in a nonconcentrated industry can be profitable when future mergers are expected. Davidson and Mukherjee (2007) analyze the impact of horizontal mergers in the presence of free entry and exit, and show that under free entry the results of Salant et al (1983) and Deneckere and Davidson (1985) no longer hold: mergers with small cost synergies are profitable and insiders always benefit more than outsiders. Despite the fact that much attention has been devoted mergers and entry, none of the above mentioned articles has considered (i) the possibility of alternative types of entry or alternative entry locations and (ii) the effect of the merger decision on the entrant’s choice between these alternatives. This is the main contribution of our article to this strand of literature.

In our analysis, we discuss the differences between the decision taken by a national competition authority and a supranational authority and between the decisions taken by the authorities of small and large countries. These topics have been analyzed, for instance, by Barros and Cabral (1994), Neven and Roller (2003) and Hollis and Yuan (2004). More recently, Breinlich et al (2013) also discuss the possibility of conflicting decisions by the national authorities of two of the undertakings concerned achieves more than two-thirds of its aggregate Community-wide turnover within one and the same Member State."
countries, as well as the effects of having a supranational authority. The domestic country’s authority may be too "lenient" or too "tough" for the neighboring country, depending on the value taken by a domestic bias parameter in the demand function. However, none of these articles considers the possibility of post-merger entry.

This article is also related to the literature on foreign direct investment (FDI) that identifies market concentration as a determinant of investment inflows. By increasing local concentration, a merger may induce entry into a market when it would otherwise not occur. Campa et al (1998), in their work on how strategic interactions between domestic and foreign producers influence the structure of the industry, refer to previous studies that suggest that a high concentration is conducive to more FDI.

The article is organized as follows. Section 2 presents the model and Section 3 presents the game equilibrium, analyzes the results and establishes some policy implications. Conclusions are drawn in Section 4. All proofs are in the appendix.

2 The model

Our results are illustrated with the use of a two-country model, with firms competing à la Cournot. Although quite specific, this type of model is widely used in the literature. See, for example, the recent articles by Chaudhuri and Benchekroun (2012) or Collie (2011), who use similar models to address the effects of trade liberalization on the social desirability of mergers or on the volume of trade and FDI, respectively. Also, Bagwell and Staiger (2012) use the same type of model to study the effects of restraining export subsidies in trade agreements.

2.1 Environment

For simplicity, we assume that there are two countries, $L$ and $S$, that differ in size and are engaged in intra-industry international trade. Firms in the two countries produce a homogeneous product both for the domestic market and for the foreign market (exports). Demand for this product in country $i$ is given by $Q_i = S_i (A - P_i)$, where $Q_i$ denotes total output and $P_i$ denotes price in market $i = L, S$, with $A > 0$ and $\sigma := S_L / S_S > 1$. This means that the difference between country $L$ and country $S$ is merely the size of the demand for the product in question, with $\sigma$ measuring the degree of country asymmetry. In particular, the elasticity of demand is the same in both countries, when prices are equal. Given our assumption that $\sigma > 1$, we refer
to country $L$ as the *large* country and to country $S$ as the *small* country.\footnote{This is the same setting as in Barros and Cabral (2000) who analyze a subsidy game played by the two countries with the purpose of attracting a foreign monopolist.}

At the outset, we assume that there are four active firms, two located in each country, and a single potential entrant. The two firms in the *small* country may decide to merge and the potential entrant may decide to enter either the *small* or the *large* country.\footnote{We will also consider, for comparison purposes, the possibility of a merger between the incumbents located in the *large* country. Cross-border mergers, however, are outside the scope of this article.}

The timing of events is presented in Figure 1. Timing unfolds as follows: First, firms in the *small* country decide whether to merge; Second, the authority approves or rejects this merger; Third, the entrant decides in which country to enter; Fourth, all firms simultaneously choose their quantities for the home market and for export and all markets clear.

Insert Figure 1

### 2.2 Firms

We assume that any firm produces for the domestic market with constant marginal cost $c < A$ and incurs an additional marginal cost of $t > 0$ for exports. Thus, $t$ can be interpreted as the sum of unit transportation costs, tariffs, etc.

In the event of a merger involving any two firms in the same country, the marginal costs of the firm resulting from the merger is assumed to decrease to $c - d \geq 0$.

Let $\delta := d/(A - c)$ and $\tau := t/(A - c)$ denote normalizations of $d$, and $t$, respectively. Parameter $\delta$ is a measure of the magnitude of marginal cost reductions that result from a merger and parameter $\tau$ is a measure of the additional marginal cost each firm incurs when selling abroad. With respect to these model parameters, we make the following assumption, which is assumed to hold in all results:

**Assumption A1:** Let \( i \) \( \tau < 1/4 \) and \( i \) \( 5\sqrt{2}/24 \) \( (3\tau + 1) - \frac{1}{2}\tau - \frac{1}{4} < \delta < 1 - 3\tau \). \hfill \blacksquare

The assumptions that $\tau < 1/4$ and $\delta < 1 - 3\tau$ are necessary and sufficient conditions for all firms to serve both markets in equilibrium.\footnote{In the appendix we show that the assumption that $\tau < 1/4$ ensures that all firm’s equilibrium outputs (both for the domestic market and for export) are positive when no merger has occurred, whereas the assumption that $\delta < 1 - 3\tau$ ensures that all firm’s equilibrium outputs when a merger has occurred.} Additionally, the assumption that $\delta > 5\sqrt{2}/24 \ (3\tau + 1) - \frac{1}{2}\tau - \frac{1}{4}$ is a sufficient condition for merger profitability.\footnote{It should be noted that $5\sqrt{2}/24 \ (3\tau + 1) - \frac{1}{2}\tau - \frac{1}{4} < 1 - 3\tau$ for any $\tau < 1/4$.} This assumption is
included merely because it is helpful in ruling out some trivial cases, like no merger scenarios, or uninteresting cases, such as the case in which mergers are only profitable for parameter values such that entry always occurs in the large country.\footnote{Alternatively, we could assume that $\delta$ can take any value and that the merger profitability arises from the elimination of some fixed costs. It can be showed that it is possible to have a value for fixed costs that is sufficiently high so that the merger is profitable, but low enough to ensure positive profits, even after entry.}

2.3 Competition Authority

Regarding the competition authority, we consider both the case of a decision by a National Competition Authority (NCA) and by a Supranational Authority (SNA). In either case, the authority’s objective is considered to be the maximization of consumer surplus. This assumption is justified by the fact that this is, roughly, the current practice both in the EU and the US. In fact, the European Commission (2004) horizontal merger guidelines explicitly state that "The relevant benchmark in assessing efficiency claims is that consumers will not be worse off as a result of the merger." In the US, Salop (2010) states that "the current antitrust welfare standard is the true consumer welfare standard" and argues that this is, indeed, the best standard for antitrust law.\footnote{Farrell and Katz (2006) illustrate the possibility that the consumer surplus standard may lead to better results than the total surplus standard, even when the objective is to maximize total surplus. For a discussion of consumer surplus versus welfare standards for competition authorities, see also Lyons (2001), Neven and Roller (2005) and Fridolsson (2007).} The NCA considers domestic consumer surplus alone while the SNA considers both countries aggregate consumer surplus. Country $i$’s NCA aims at maximizing

$$CS_i = \int_0^{Q_i} (A - \frac{x}{S_i})dx - (A - \frac{Q_i}{S_i})Q_i = \frac{1}{2} S_i^{-1} Q_i^2$$

while we assume that the SNA aims at maximizing $CS_L + CS_S$. As $CS_i$ is strictly decreasing in $P_i$, some results for the NCA decisions are expressed in terms of prices.

3 Equilibrium Characterization

In this section, we construct the model’s equilibrium by backward induction. Hence, we characterize first the equilibrium of the quantity competition stage, afterwards the decision of the entrant, the authority’s decision, and finally the merging firm’s decision.
3.1 Stage 4: Quantity competition

There are four different possibilities when the quantity competition stage is reached: 1) no merger took place and there was entry in country \( L \); 2) no merger took place and there was entry in country \( S \); 3) there was a merger in country \( S \) followed by entry in country \( L \) and 4) there was a merger in country \( S \) followed by entry in country \( S \).

Let \( \pi^k_l \) denote the total profit accruing to an individual firm \( l \) when case \( k = 1, \ldots, 4 \) occurred and let \( \pi^k_{li} \) denote the profit in country \( i = S, L \) accruing to firm \( l \) when case \( k = 1, \ldots, 4 \) occurred. The entrant and, in the case of a merger, the insider firms are denoted with a subscript \( E \) or \( M \), respectively. Otherwise, firms are simply denoted with the same subscript as the country they are located in, \( S \) or \( L \).

In a game of this type, it is straightforward to show that the equilibrium output for any firm \( l \) destined for country \( i \) is simply given by

\[
q_{li} = S_i \left( \frac{A + C_i}{n + 1} - c_{li} \right) = S_i \left( \frac{A + C_i^{-i} - nc_{li}}{n + 1} \right),
\]

where \( n \) denotes the total number of independent firms selling in both countries, regardless of their location, \( c_{li} \) denotes firm \( l \)'s marginal cost when producing for country \( i \), \( C_i \) denotes the sum of all \( n \) firms’ marginal costs when producing for country \( i \) and \( C_i^{-i} = C_i - c_{li}. \)

As expected, a given firm’s equilibrium output in country \( i \) decreases with its own marginal costs and increases with its rival’s marginal costs.

As for aggregate output and price, these are given respectively by

\[
Q_i = S_i \left( \frac{nA - C_i}{n + 1} \right),
\]

\[
P_i = \frac{A + C_i}{n + 1}.
\]

Individual profit is simply \( \Pi_l = \pi_{lL} + \pi_{lS} = S^{-1}_L q_{lL}^2 + S^{-1}_S q_{lS}^2. \)

Naturally, the number of firms and the marginal costs differ across the four cases considered and we have omitted the superscript \( k \) above for simplicity. In the following subsections, we present the entrant’s and the merged firm’s profits as well as the equilibrium prices in both countries in all possible cases.

\[\text{\footnotesize Note that the marginal cost } c_{li} \text{ may take four different values: (i) } c_{li} = c \text{ in the case of production for the home country and no participation in a merger; (ii) } c_{li} = c + t \text{ in the case of production for the foreign country and no participation in a merger; (iii) } c_{li} = c - d \text{ in the case of production for the home country by the participant in a merger; and (iv) } c_{li} = c - d + t \text{ in the case of production for the foreign country by the participant in a merger.}\]
Case 1) No merger and entry in country $L$

When there is no merger and the entrant opts for the large country, there are three symmetric firms in the large country and two symmetric firms in small country. The individual equilibrium profits are given by:

$$
\Pi_L^1 = \Pi_E^1 = \frac{1}{36} S_L (A - c + 2t)^2 + \frac{1}{36} S_S (A - c - 3t)^2,
$$
$$
\Pi_S^1 = \frac{1}{36} S_S (A - c + 3t)^2 + \frac{1}{36} S_L (A - c - 4t)^2.
$$

The equilibrium price in countries $S$ and $L$ are given by

$$
P_S^1 = \frac{1}{6} (A + 5c + 3t),
$$
$$
P_L^1 = \frac{1}{6} (A + 5c + 2t).
$$

Case 2) No merger and entry in country $S$

The case in which there is no merger and there is entry in the small country is similar to the previous one but with three firms in the small country, instead. Hence,

$$
\Pi_L^2 = \frac{1}{36} S_L (A - c + 3t)^2 + \frac{1}{36} S_S (A - c - 4t)^2,
$$
$$
\Pi_S^2 = \Pi_E^2 = \frac{1}{36} S_S (A - c + 2t)^2 + \frac{1}{36} S_L (A - c - 3t)^2.
$$

It can be shown that $\Pi_S^2 < \Pi_L^2$. As for the equilibrium prices, we have

$$
P_S^2 = \frac{1}{6} (A + 5c + 2t),
$$
$$
P_L^2 = \frac{1}{6} (A + 5c + 3t).
$$

Case 3) Merger in country $S$ followed by entry in country $L$

In the event of a merger in the small country followed by entry in the large country, the entrant’s and the insiders’ payoffs are, respectively,

$$
\Pi_E^3 = \frac{1}{25} (A - c - d + t)^2 S_L + \frac{1}{25} (A - c - d - 2t)^2 S_S,
$$
$$
\Pi_M^3 = \frac{1}{25} (A - c + 4d + 3t)^2 S_S + \frac{1}{25} (A - c + 4d - 4t)^2 S_L.
$$
Equilibrium prices are given by

\[ P^3_S = \frac{1}{5} (A + 4c - d + 3t), \]
\[ P^3_L = \frac{1}{5} (A + 4c - d + t). \]

Despite the merger taking place and no entry in market \( S \), prices in this market may be lower due to the increase in competition resulting from entry abroad when compared to the no merger-no entry case. The price in the large country is clearly lower than under the \textit{status quo} because this market is served by the same number of firms, four, but two of them now have lower costs: One foreign competitor "disappeared" but was replaced by a domestic competitor, thus saving on the transportation costs and the other foreign competitor benefitted from the merger related efficiencies, which decrease marginal costs by \( d \).

**Case 4) Merger in country \( S \) followed by entry in country \( S \)**

After a merger followed by entry in the \textit{small} country, the entrant’s and the insiders’ payoffs are respectively given by

\[ \Pi^4_E = \frac{1}{25} (A - c - d + 2t)^2 S_S + \frac{1}{25} (A - c - d - 3t)^2 S_L, \]
\[ \Pi^4_{M} = \frac{1}{25} (A - c + 4d + 2t)^2 S_S + \frac{1}{25} (A - c + 4d - 3t)^2 S_L. \]

Equilibrium prices are given by

\[ P^4_S = P^4_L = \frac{1}{5} (A + 4c - d + 2t). \]

Merger followed by entry in the same market leads to exactly the same number of firms competing in both markets. The firm resulting from the merger, however, has lower marginal costs, meaning that equilibrium prices are lower in both markets when compared with the no merger-no entry situation. Thus, entry in the small country can be considered \textit{sufficient} in the sense that it effectively defeats the exercise of market power. Given the timing in our model, entry is also trivially \textit{timely}. In the next subsection we show under which conditions it is \textit{likely}.

### 3.2 Stage 3: Entry decision

At this stage, the entrant decides where to locate, knowing whether the merger in the small country has been approved or not. Let \( \sigma(\delta, \tau) := \frac{1-\delta}{1-\delta-\tau} \) and note that \( \sigma(\delta, \tau) > 1 \). The
next lemma establishes under which conditions the entrant locates in the large or in the small
country.

**Lemma 1:** If there was no merger in country S, then entry occurs in country L. If there was a
merger in country S, then entry occurs in country S if and only if \( \sigma < \bar{\sigma}(\delta, \tau) \) and entry occurs
in country L if and only if \( \sigma > \bar{\sigma}(\delta, \tau) \).

To understand its entry decision, it is useful to think of the entrant as a firm that, at the
outset, has marginal costs \( c + t \) in both countries but that can choose in which country it will
reduce its marginal cost to \( c \). The entrant selects the large country if and only if the increase in
profit of reducing marginal cost in the large country exceeds the increase in profit of reducing
marginal cost in the small country, i.e. if and only if:

\[
\pi_{EL}(c) - \pi_{EL}(c + t) > \pi_{ES}(c) - \pi_{ES}(c + t)
\]

where \( \pi_{Ej}(x) \) denotes the entrant’s profit in country \( j \) when the entrant has marginal costs \( x \).

This is equivalent to:

\[
- \int_c^{c+t} \frac{\partial \pi_{EL}(x)}{\partial x} dx > - \int_c^{c+t} \frac{\partial \pi_{ES}(x)}{\partial x} dx
\]

(2)

A small reduction in the entrant’s marginal cost impacts its profit from selling in a given
country both directly and strategically. The direct effect is simple: a unit reduction in marginal
cost increases the entrant’s profit by an amount equal to the firm’s equilibrium output. The
strategic effect is as follows: a reduction in the entrant’s marginal cost leads to a lower output
by the incumbents, which increases unit price. The impact on profit corresponds to the price
increase multiplied by the entrant’s output. The overall increase in profit due to the reduction
in marginal cost in a given country \( j \), i.e. the sum of the two effects above, is thus proportional
to the entrant’s output in that country, \( q_{Ej}(x) \).

Inequality (2) can therefore be written as:

\[
\int_c^{c+t} \frac{2n}{n+1} q_{EL}(x) dx > \int_c^{c+t} \frac{2n}{n+1} q_{ES}(x) dx
\]

In the absence of a merger in the small country, the entrant’s output is always larger in
the large country and, therefore, that is where the entrant prefers to have lower costs. Hence,

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11 Function \( \pi_{Ej}(x) \) also depends on the number of competitors and their marginal cost of selling in country
\( j \). It is thus dependent on whether a merger in the small country took place or not.
12 For market \( i \) the derivative of profit with respect to own output is equal to \( \frac{2n}{n+1} q_i \).
case 2), the no merger and entry in country \( S \) scenario, does not occur in equilibrium for any parameter values. Note that regardless of its entry decision, the entrant always faces two domestic (low cost) rivals and two foreign (high cost) rivals.

After a merger in the small country, the comparison between outputs is less obvious. When entering the large country, demand is larger than when entering the small country. However, demand size is not the only factor affecting a firm’s output. Rival’s costs are also relevant and own output increases with rival’s costs. When entering the large country, two out of three competitors are local and have no transportation costs whereas, when entering the small country, two out of three competitors are foreign and have high marginal costs. Hence, rival’s marginal costs are higher when entering the small country. If country asymmetry is sufficiently small, i.e. \( \sigma < \bar{\sigma}(\delta, \tau) \), the latter effect is dominant and the entrant selects the small country. Thus, the entrant selects the small country after a merger took place only if this country is not excessively small when compared to the large country. Note that

\[
\frac{\partial \sigma}{\partial \delta} = \frac{\tau}{(1 - \tau - \delta)^2} > 0, \\
\frac{\partial \sigma}{\partial \tau} = \frac{1 - \delta}{(1 - \tau - \delta)^2} > 0.
\]

A larger \( \delta \) and/or a larger \( \tau \) increase the range of values for country asymmetry such that entry occurs in the small country, given that a merger has occurred.

From expression (1), one can see that as \( \delta \) increases and, hence, as \( C_l \) decreases, the entrant’s output falls by more in the large country than in the small country because \( S_L > S_S \). As discussed above, this makes it less likely that inequality (2) holds and therefore makes entry into the small country more likely.

As for \( \tau \), an increase in \( \tau \) reinforces the rival’s marginal cost differences between countries. Recall that when choosing the small country the entrant has two rivals with high costs (due to transportation costs) while when selecting the large country there is only one rival incurring in transportation costs. A higher \( \tau \) makes this difference more relevant and favors entry into the small country.

### 3.3 Stage 2: Approval or Rejection Stage

At this stage, the authority decides whether to approve or reject the merger. We present the equilibrium decision for three different cases. First, we assume that the small country’s NCA has jurisdiction. Second, we assume that both countries exert jurisdiction. Finally, we present
the results from the perspective of a supranational competition authority.

**Small country’s NCA has jurisdiction**

The next lemma presents the necessary and sufficient conditions for merger approval by the small country’s NCA.

**Lemma 2A:** If \( \sigma < \bar{\sigma}(\delta, \tau) \), then the small country’s NCA approves the merger if and only if \( \delta > \frac{1-3\tau}{6} \). If \( \sigma > \bar{\sigma}(\delta, \tau) \), then the small country’s NCA approves the merger if and only if \( \delta > \frac{1+3\tau}{6} \).

As expected, the local NCA approves the merger if the cost savings are sufficiently high. This authority requires larger cost savings for merger approval when \( \sigma \) is large, i.e. when the difference in size between the two countries is significant. When this difference is small, the merger diverts entry into the small country and is thus less likely to have anticompetitive effects, as the entrant will have lower costs in the small country. If the transportation costs are extremely small then, from the small country’s perspective, it is irrelevant where entry takes place and the cost savings necessary for merger approval are independent of market size.

In our framework, we did not consider the possibility of firms merging in the large country. We have ruled out this possibility because such mergers do not change the entrant’s decision: entry would always take place in the large country and the merger in the large country would have no effect on the entry decision. However, it may still be interesting to compare the decisions of the local authority in the small and large countries when each is facing a similar domestic merger. Following entry, the prices in the large country are \( P_L = \frac{1}{5} (A + 4c - d + 2t) \) with and \( P_L = \frac{1}{6} (A + 5c + 2t) \) without the merger. Thus, the large country NCA would approve the merger if and only if \( \frac{1}{5} (A + 4c - d + 2t) < \frac{1}{6} (A + 5c + 2t) \) which is equivalent to \( \delta > \frac{1+2\tau}{6} \).

Let \( \sigma > \bar{\sigma}(\delta, \tau) \). Then the small country NCA requires larger cost reductions to approve the merger than the large country’s NCA. This happens because entry always takes place in the large country, regardless of where the merger is proposed. Hence, the pro-competitive effects of entry are smaller in the small country due to the entrant’s cost disadvantage resulting from its transportation costs when exporting to the small country.

When \( \sigma < \bar{\sigma}(\delta, \tau) \) the small country NCA is more likely to approve the merger than the large country NCA because approving the merger has a positive effect in its perspective, which is to divert entry into the home country. For mergers in the large country, entry does not depend on the merger decision, hence this effect does not exist.
Note that we have just compared the decisions by the two countries’ NCAs with respect to a similar, or comparable, domestic merger. The following section presents the large country’s NCA optimal decision with respect to the merger in the small country, which is only relevant when both countries exert jurisdiction.

**Both countries exert jurisdiction**

The next lemma presents the necessary and sufficient conditions for the approval of the merger in the small country by the large country’s NCA.

**Lemma 2B:** If $\sigma < \overline{\sigma}(\delta, \tau)$, then the large country’s NCA approves the merger if and only if $\delta > \frac{1+2\tau}{6}$. If $\sigma > \overline{\sigma}(\delta, \tau)$, then the large country’s NCA approves the merger if and only if $\delta > \frac{1-4\tau}{6}$.

This Lemma is qualitatively similar to Lemma 2A. The large country’s NCA requires larger cost savings for merger approval when $\sigma$ is small, i.e. when the difference in size between the countries is less significant, because in that case the merger diverts entry away from the large country. In section 3.5 the decisions of the two countries with respect to the same merger, i.e. the merger by the two incumbents in the small country, are compared.

**Supranational Authority has jurisdiction**

In this section we present the SNA equilibrium decision. Before proceeding, we must introduce the following notation:

$$
\sigma_1(\delta, \tau) : = \frac{(6\delta - 27\tau + 49)(3\tau + 6\delta - 1)}{(6\delta - 22\tau + 49)(1 + 2\tau - 6\delta)}
$$

$$
\sigma_2(\delta, \tau) : = \frac{(6\delta - 33\tau + 49)(6\delta - 3\tau - 1)}{(6\delta - 16\tau + 49)(1 - 4\tau - 6\delta)}
$$

The next lemma establishes under which conditions the SNA should approve or reject the merger.

**Lemma 2C:** If $\sigma < \overline{\sigma}(\delta, \tau)$, then the SNA approves the merger if and only if (i) $\delta > \frac{1+2\tau}{6}$ or (ii) $\frac{1-3\tau}{6} < \delta < \frac{1+2\tau}{6}$ and $\sigma < \sigma_1(\delta, \tau)$. If $\sigma > \overline{\sigma}(\delta, \tau)$, then the SNA approves the merger if and only if (i) $\delta > \frac{1+3\tau}{6}$ or (ii) $\frac{1-4\tau}{6} < \delta < \frac{1+3\tau}{6}$ and $\sigma > \sigma_2(\delta, \tau)$.

---

13These two mergers are comparable because insider market shares in the domestic market are the same in the two cases.
The objective function of the SNA is the sum of the objective functions of each country. Therefore, whenever both countries’ decision is to approve or to reject the merger, the SNA makes the same decision. When the two countries have conflicting positions, the SNA’s decision depends on country asymmetry. When country asymmetry is sufficiently low (high), the SNA sides with the small (large) country’s NCA.

3.4 Stage 1: Merging decision

Here, we assume that there are no costs of making a merger proposal. If $\sigma < \sigma(\delta, \tau)$ the insiders decide to merge if and only if $\Pi_M^4 > 2\Pi_S^1$ and if $\sigma > \sigma(\delta, \tau)$ the insiders decide to merge if and only if $\Pi_M^3 > 2\Pi_S^1$.

Lemma 3: The two firms in the small country always propose to merge.

This follows directly from the assumption that $\sigma > \frac{5\sqrt{7}}{24}(3\tau + 1) - \frac{1}{2}\tau - \frac{1}{4}$ which implies that cost reductions are sufficiently large to ensure merger profitability.

3.5 Equilibrium of the whole game

This section presents the equilibrium of the whole game for the case of a small country NCA, a large country NCA and a SNA. The following proposition summarizes our results for the case of a small country NCA.

Proposition 1A: Let $A1$ hold and consider the case of a small country’s NCA:

a) Let $\sigma < \sigma(\delta, \tau)$. If $\delta < \frac{1-3\tau}{6}$, then the merger is rejected and there is entry in the large country. If $\delta > \frac{1-3\tau}{6}$, then the merger is approved and there is entry in the small country.

b) Let $\sigma > \sigma(\delta, \tau)$. If $\delta < \frac{1+3\tau}{6}$, then the merger is rejected and there is entry in the large country. If $\delta > \frac{1+3\tau}{6}$, then the merger is approved and there is entry in the large country.

The following Corollary highlights an interesting possibility that results from Proposition 1A.

Corollary 1: Assume that $\sigma < \sigma(\delta, \tau)$ and $\delta < \frac{1-3\tau}{6}$. Then, a small country NCA that would approve a merger in the case of timely, likely and sufficient entry would not maximize domestic consumer surplus.
With $\sigma < \sigma(\delta, \tau)$, from Lemma 1, the merger leads to entry in the small country. This is sufficient to lower price and increase consumer surplus for any level of cost savings. After the merger and subsequent entry, the number of competitors is the same as before the merger but the firm resulting from the merger has lower costs, which translates into a lower equilibrium price in both countries. Why, then, should the authority reject such merger? In this case, approval of the merger changes the entrant’s decision. Due to the merger, entry does not take place in the large country but instead is diverted to the small country. When assessing the effects of the merger on price, the authority compares the price after the merger followed by entry to the price that would prevail with no merger and, hence, with entry in the large country. International trade ensures that, after entry into this country, prices are also lower in the small one. Hence, in order to maximize consumer surplus, the authority requires larger cost reductions for merger approval ($\delta > \frac{1-3\tau}{6}$) than those it would require if it overlooked the possibility of alternative forms of entry ($\delta > 0$). In other words, the small country authority compares the post-merger price with the price after entry in the large country, instead of comparing the post-merger price with the pre-merger and pre-entry price. As, due to international trade, price in the small country after entry in the large country is lower than pre-merger and pre-entry prices, it takes larger cost reductions for the merger to be approved.

The policy implications of Corollary 1 are straightforward. To maximize consumer surplus, in general, it is not sufficient to approve a merger after establishing that after the operation there will be entry into the country in question that would otherwise not take place. The alternatives that are open to the entrant are relevant for this decision. If, for some reason, the entrant has no other entry alternative but the country in which the merger occurs, then such merger can be safely approved. However, if the merger merely diverts entry into the country and, in its absence, entry would have occurred into another country, a more careful analysis is required. In particular, it is important to assess whether the entrant would, from the alternative location, export to the country in question and at what cost.

Insert Figure 2a and Figure 2b

Figures 2a and 2b present the different authorities equilibrium decisions with respect to the merger. The lower case letters $a$ and $r$ in Figures 2a and 2b represent the small country’s NCA decision of approval or rejection in the $(\delta, \tau)$-space for the cases of $\sigma < \sigma(\delta, \tau)$ and $\sigma > \sigma(\delta, \tau)$, respectively. Large cost reductions result in merger approval for any $\sigma$, whereas small cost reductions result in its rejection, despite the fact that the merger may lower price. For
intermediate values of cost reductions, the equilibrium decision depends on the size asymmetry parameter, $\sigma$. If the size asymmetry is small, the merger diverts entry into the small country and the optimal decision is to approve the merger. If it is large, the optimal decision is to reject the merger. The optimal decision thus depends on how small the small country is.

We now turn to the case of a large country’s NCA.

**Proposition 1B.** Let $A1$ hold and consider the case of a large country’s NCA:

a) Let $\sigma < \bar{\sigma}(\delta, \tau)$. If $\delta < \frac{1+2\tau}{6}$, then the merger is rejected and there is entry in the large country. If $\delta > \frac{1+2\tau}{6}$, then the merger is approved and there is entry in the small country.

b) Let $\sigma > \bar{\sigma}(\delta, \tau)$. If $\delta < \frac{1-4\tau}{6}$, then the merger is rejected and there is entry in the large country. If $\delta > \frac{1-4\tau}{6}$, then the merger is approved and there is entry in the large country.

If, after the merger, entry occurs in the small country, the merger should be approved by the large country’s NCA if and only if $P^1_L > P^4_L \Leftrightarrow \delta > \frac{1+2\tau}{6}$. This threshold should be compared to $\delta > \frac{1-3\tau}{6}$ and, in this case, the large country has a bias towards the rejection of the merger. This happens because, in addition to the reduction in the number of firms, the merger, by shifting the entrant’s optimal location from the large country to the small country results in higher costs for the entrant when selling in the large country and hence to a higher price.

If entry always occurs in the large country the merger should be approved if and only if $P^1_L > P^3_L \Leftrightarrow \delta > \frac{1-4\tau}{6}$ which can be compared to $\delta > \frac{1+3\tau}{6}$, the condition for approval by the small country’s NCA. Thus, the large country has a bias towards merger approval when compared to the small country. The merger does not change the entrant’s decision and its potentially negative effect is larger in the small country, where it takes place, than in the large country where the insiders have, ex-ante, lower market shares due to transportation costs. The following corollary compares Propositions 1A and 1B and summarizes the discussion above.

**Corollary 2:** If $\sigma < \bar{\sigma}(\delta, \tau)$, then the small country NCA is more likely to approve the merger than the large country NCA. If $\sigma > \bar{\sigma}(\delta, \tau)$, then the large country NCA is more likely to approve the merger than the small country NCA.

Thus, for some values of parameter $\delta$ there may be a potential source of conflict between the two NCA’s.\textsuperscript{14} This is illustrated in Figures 2a and 2b, where the upper case $A$ and $R$

\textsuperscript{14}For other sources of conflict, in particular for those related to the definition of the relevant market, see
represent the large country’s NCA decision of approval or rejection in the \((\delta, \tau)-\)space. If size asymmetry is small (Figure 2a) and the \(\delta\) takes intermediate values, the large country’s NCA may prefer to reject the merger when the small country NCA decision is to approve it. For large size asymmetries, however, the opposite occurs (Figure 2b). Interestingly, we cannot tell if larger countries tend, in general, to be more or less conservative than smaller countries as the result depends on the size asymmetry between countries being too large or too small.

Finally, we turn to the equilibrium of the game when the authority is a Supranational Competition Authority.

**Proposition 1C.** Let A1 hold and consider the case of a SNA:

a) Let \(\sigma < \overline{\sigma}(\delta, \tau)\). If (i) \(\delta < \frac{1-3\tau}{6}\) or (ii) \(\frac{1-3\tau}{6} < \delta < \frac{1+2\tau}{6}\) and \(\sigma > \sigma_1(\delta, \tau)\), then the merger is rejected and there is entry in the large country. If (i) \(\delta > \frac{1+2\tau}{6}\) or (ii) \(\frac{1-3\tau}{6} < \delta < \frac{1+2\tau}{6}\) and \(\sigma < \sigma_1(\delta, \tau)\), then the merger is approved and there is entry in the small country.

b) Let \(\sigma > \overline{\sigma}(\delta, \tau)\). If (i) \(\delta < \frac{1-4\tau}{6}\) or (ii) \(\frac{1-4\tau}{6} < \delta < \frac{1+3\tau}{6}\) and \(\sigma < \sigma_2(\delta, \tau)\), then the merger is rejected and there is entry in the large country. If (i) \(\delta > \frac{1+3\tau}{6}\) or (ii) \(\frac{1-4\tau}{6} < \delta < \frac{1+3\tau}{6}\) and \(\sigma > \sigma_2(\delta, \tau)\), then the merger is approved and there is entry in the large country.

The following corollary compares the optimal decision from the perspective of both country’s local authorities with the one from the perspective of the supranational authority.

**Corollary 3:** If \(\sigma < \overline{\sigma}(\delta, \tau)\), then the small country NCA is more likely to approve the merger than the SNA, which is more likely to approve the merger than the large country NCA. If \(\sigma > \overline{\sigma}(\delta, \tau)\), then the large country NCA is more likely to approve the merger than the SNA, which is more likely to approve the merger than the small country NCA.

Again, this is illustrated in Figures 2a and 2b where the SNA’s optimal decision of whether to approve or reject the merger, \(A\) or \(R\), is presented in a circle.

In terms of policy implications, we establish that if the size asymmetry is sufficiently small, the SNA could safely uphold rejection decisions by local authorities in the small country but should investigate their approval decisions. On the contrary, if size asymmetry is large, the SNA could safely validate approval decisions by the small country’s NCA. The intuition is simple: When \(\sigma\) is small, the merger changes the entrant’s decision and increases the entrant’s

---

Neven and Roller (2003).
cost of selling in the large country. This affects consumers in the large country negatively, an irrelevant effect from the small country NCA’s perspective. However, the SNA considers the welfare of all consumers, hence it is less prone to approve the merger. When $\sigma$ is large, the merger does not affect the entrant’s decision. Additionally, any negative impact of the merger on consumers located in the large country is smaller than the impact on consumers located in the small country. As mentioned above, this results from the fact that one of the implications of the merger is the elimination of a firm that is both a low cost producer in the small country and a high cost producer in the large country. Therefore, when all consumers are taken into account, the merger is less harmful than when only those located in the small country are considered. Hence, the SNA is not as demanding as the small country NCA in terms of the efficiencies required for merger approval.

4 Conclusions

In this article we analyze the optimal decision regarding a domestic horizontal merger by several types of authorities, concerned with consumer welfare. We consider a merger that results in marginal cost reductions for the insiders in a small country and which may divert entry into the domestic market. This merger may result in lower prices due to entry and it may be established that entry into the small country would not have taken place had the merger not taken place. Although a competition authority could approve this merger on the grounds of correctly establishing the timeliness, likeliness and sufficiency of entry, we argue that there could be basis for its rejection. The main argument is that entry by a firm into a given country may preclude its entry into another one. The increase in concentration in the smaller country that results from the merger may be sufficient to make the entrant divert to that country instead of selecting another one. To the extent that there is international trade between these countries, there is an "opportunity cost" associated with entry into one country. This cost corresponds to the consumer benefits that would result from the alternative entry possibility. Thus, by changing the entrant’s decision, the merger is attracting entry into its country, but at the same time it is preventing an alternative entry from taking place. This means that the gains from attracting entry may be relatively small, given that entry, although in another location, would occur in any case. Therefore, unless there are no other entry alternatives, it is not sufficient to show that entry is timely, likely and sufficient for the optimal decision to be the approval of a given merger.

We illustrated our reasoning with an application of the Cournot model that is widely used
in international trade analysis. However, the key features in our analysis would hold under different modeling assumptions. These key features are: (i) a merger makes a market more attractive to an entrant, (ii) entry into a given country reduces prices in all countries connected by international trade. Additionally, we assumed the existence of a single potential entrant but we believe that considering more entrants would have similar qualitative effects. The majority of entrants would have selected the larger country in the absence of the merger, but after the merger in the small country, some would change their decision and our main argument would apply.

We also contribute to the identification of potential conflicts between the NCA’s of different countries and between a NCA and a supranational authority. We conclude that when two countries have jurisdiction over a given merger that may affect entry decisions, we cannot tell if larger countries tend to be more or less conservative than smaller ones as the result depends on whether the size asymmetry between countries is too large or too small. Finally, we conclude that when size asymmetry is sufficiently small, a SNA could safely uphold any rejection decision by the local authority of a small country but should investigate its approval decision more thoroughly. The opposite occurs for large size asymmetry.

**Appendix**

We start by presenting the expressions for the equilibrium outputs, followed by the proofs of all results. The following Table presents the equilibrium outputs for the cases in which there is no merger.

<table>
<thead>
<tr>
<th>Firm located in L</th>
<th>Firm located in S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry in L</td>
<td></td>
</tr>
<tr>
<td>$q_{IL} = S_L \left( \frac{A+5c+2t}{5+1} - c \right)$</td>
<td>$q_{IS} = S_S \left( \frac{A+5c+3t}{5+1} - c \right)$</td>
</tr>
<tr>
<td>$q_{IL} = S_L \left( \frac{A+5c+3t}{5+1} - c - t \right)$</td>
<td>$q_{IL} = S_L \left( \frac{A+5c+2t}{5+1} - c - t \right)$</td>
</tr>
<tr>
<td>Entry in S</td>
<td></td>
</tr>
<tr>
<td>$q_{IL} = S_L \left( \frac{A+5c+3t}{5+1} - c \right)$</td>
<td>$q_{IL} = S_L \left( \frac{A+5c+2t}{5+1} - c \right)$</td>
</tr>
<tr>
<td>$q_{IS} = S_S \left( \frac{A+5c+2t}{5+1} - c - t \right)$</td>
<td>$q_{IS} = S_S \left( \frac{A+5c+3t}{5+1} - c - t \right)$</td>
</tr>
</tbody>
</table>

Let $x_{li} = \frac{q_{li}}{S_i(A-c)}$. Clearly, $q_{li} > 0$ if and only if $x_{li} > 0$. Then,

<table>
<thead>
<tr>
<th>Firm located in L</th>
<th>Firm located in S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry in L</td>
<td></td>
</tr>
<tr>
<td>$x_{IL} = \frac{1+2r}{6}$</td>
<td>$x_{IS} = \frac{1+3r}{6}$</td>
</tr>
<tr>
<td>$x_{IS} = \frac{1-3r}{6}$</td>
<td>$x_{IL} = \frac{1-4r}{6}$</td>
</tr>
<tr>
<td>Entry in S</td>
<td></td>
</tr>
<tr>
<td>$x_{IL} = \frac{1+3r}{6}$</td>
<td>$x_{IS} = \frac{1+2r}{6}$</td>
</tr>
<tr>
<td>$x_{IS} = \frac{1-4r}{6}$</td>
<td>$x_{IL} = \frac{1-3r}{6}$</td>
</tr>
</tbody>
</table>
Inspection of the expressions above shows that the smallest output by any firm is $\frac{1-4\tau}{6}$ which is positive if and only if $\tau < 1/4$.

The following Table presents the equilibrium outputs for the cases in which there is a merger in the small country.

<table>
<thead>
<tr>
<th>Firm l located in L</th>
<th>Firm l located in S</th>
<th>Firm M located in S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry in L $q_{lL} = S_L \left( \frac{A + c + 1 - d}{4 + 1} - c \right)$</td>
<td>$q_{lS} = S_S \left( \frac{A + c + 3t - d}{4 + 1} - c - t \right)$</td>
<td>$q_{lL} = S_L \left( \frac{A + 3t - d}{4 + 1} - c + d \right)$</td>
</tr>
<tr>
<td>Entry in S $q_{lS} = S_S \left( \frac{A + c + 3t - d}{4 + 1} - c - t \right)$</td>
<td>$q_{lS} = S_S \left( \frac{A + 4c + 2t - d}{4 + 1} - c \right)$</td>
<td>$q_{lL} = S_L \left( \frac{A + c + 2t - d}{4 + 1} - c + d \right)$</td>
</tr>
</tbody>
</table>

or,

<table>
<thead>
<tr>
<th>Firm l in L</th>
<th>Firm l in S</th>
<th>Firm M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry in L $x_{lL} = \frac{1-\delta + \tau}{5}$</td>
<td>$x_{lS} = \frac{1+\delta + 3\tau}{5}$</td>
<td>$x_{lL} = \frac{1+\delta + 4\tau}{5}$</td>
</tr>
<tr>
<td>$x_{lS} = \frac{1-\delta - 2\tau}{5}$</td>
<td>$x_{lS} = \frac{1-\delta + 2\tau}{5}$</td>
<td>$x_{lL} = \frac{1+\delta - 2\tau}{5}$</td>
</tr>
<tr>
<td>Entry in S $x_{lL} = \frac{1-\delta + 2\tau}{5}$</td>
<td>$x_{lS} = \frac{1-\delta + 2\tau}{5}$</td>
<td>$x_{lS} = \frac{1+\delta + 2\tau}{5}$</td>
</tr>
<tr>
<td>$x_{lS} = \frac{1-\delta - 3\tau}{5}$</td>
<td>$x_{lL} = \frac{1-\delta - 3\tau}{5}$</td>
<td>$x_{lL} = \frac{1+\delta - 3\tau}{5}$</td>
</tr>
</tbody>
</table>

Inspection of the expressions above shows that all outputs by firm M are positive if $\tau < 1/4$ and that the smallest output by any firm l is $\frac{1-\delta - 3\tau}{5}$ which is positive if and only if $\delta < 1 - 3\tau$.

**Proof of Lemma 1**: Recall that, by assumption, all individual profits are positive. Assume initially that no merger occurred. Then, entry occurs in country L if and only if $\Pi_E^1 > \Pi_E^2$. This is equivalent to

$$\frac{5}{36} (A - c) t (S_L - S_S) (2 - \tau) > 0$$

which is always true.

Consider now that the merger in country S has taken place. Then, entry occurs in country L if and only if $\Pi_E^3 > \Pi_E^4$. This is equivalent to

$$\frac{8}{25} (A - c) t S_S (\sigma (1 - \delta - \tau) - (1 - \delta)) > 0 \Leftrightarrow \sigma > \sigma(\delta, \tau).$$

Otherwise, it occurs in country S.

**Proof of Lemma 2A**: Assume initially that $\sigma < \sigma(\delta, \tau)$. Then, if there is a merger, entry occurs in country S. Otherwise, it occurs in country L. The small country’s authority.
approves this merger if and only if $P^4_S < P^1_S$ which is equivalent to $\delta > \frac{1+3\tau}{6}$. Assume now that $\sigma > \bar{\sigma}(\delta, \tau)$. Then, entry always occurs in country $L$. The small country’s authority approves this merger if and only if $P^3_S < P^1_S$ which is equivalent to $\delta > \frac{1+3\tau}{6}$.

Proof of Lemma 2B: Assume initially that $\sigma < \bar{\sigma}(\delta, \tau)$. Then, if there is a merger, entry occurs in country $S$. Otherwise, it occurs in country $L$. The large country’s authority approves this merger if and only if $P^4_L < P^1_L$ which is equivalent to $\delta > \frac{1+3\tau}{6}$. Assume now that $\sigma > \bar{\sigma}(\delta, \tau)$. Then, entry always occurs in country $L$. The large country’s authority approves this merger if and only if $P^3_L < P^1_L$ which is equivalent to $\delta > \frac{1+3\tau}{6}$.

Proof of Lemma 2C: Assume initially that $\sigma < \bar{\sigma}(\delta, \tau)$. Then, the merger increases aggregate consumer surplus if and only if $CS^4_L + CS^4_S > CS^1_L + CS^1_S$:

$$\frac{1}{2} S^{-1}_L \left( S_L \left( A - \frac{1}{5} (A + 4c - d + 2t) \right) \right)^2 + \frac{1}{2} S^{-1}_S \left( S_S \left( A - \frac{1}{5} (A + 4c - d + 2t) \right) \right)^2$$

$$> \frac{1}{2} S^{-1}_L \left( S_L \left( A - \frac{1}{6} (A + 5c + 2t) \right) \right)^2 + \frac{1}{2} S^{-1}_S \left( S_S \left( A - \frac{1}{6} (A + 5c + 3t) \right) \right)^2$$

which can be simplified to

$$\frac{1}{25} (4 + \delta - 2\tau)^2 (\sigma + 1) > \frac{1}{36} (\sigma (5 - 2\tau)^2 + (5 - 3\tau)^2) \iff (3)$$

$$\sigma (6\delta - 2\tau - 1) > -\frac{(6\delta - 2\tau - 1)(3\tau + 6\delta - 1)}{(6\delta - 2\tau + 49)} \iff (4)$$

If $\delta > \frac{1+2\tau}{6}$, (4) is equivalent to $\sigma > \sigma_1(\delta, \tau)$ which is always true because $\sigma_1 < 0$.

If $\delta < \frac{1+2\tau}{6}$, (4) is equivalent to $\sigma < \sigma_1(\delta, \tau)$. Note that if $(3\tau + 6\delta - 1) < 0 \iff \delta < \frac{1-3\tau}{6}$ this is impossible because $\sigma_1(\delta, \tau) < 0$ when $\delta < \frac{1+2\tau}{6}$.

Assume now that $\sigma > \bar{\sigma}(\delta, \tau)$. Then, the merger increases aggregate consumer surplus if and only if $CS^3_L + CS^3_S > CS^1_L + CS^1_S$:

$$\frac{1}{2} S^{-1}_L \left( S_L \left( A - \frac{1}{5} (A + 4c - d + t) \right) \right)^2 + \frac{1}{2} S^{-1}_S \left( S_S \left( A - \frac{1}{5} (A + 4c - d + 3t) \right) \right)^2$$

$$> \frac{1}{2} S^{-1}_L \left( S_L \left( A - \frac{1}{6} (A + 5c + 2t) \right) \right)^2 + \frac{1}{2} S^{-1}_S \left( S_S \left( A - \frac{1}{6} (A + 5c + 3t) \right) \right)^2$$

If $\sigma > \frac{1+3\tau}{6}$, (4) is equivalent to $\sigma > \sigma_1(\delta, \tau)$ which is always true because $\sigma_1 < 0$.

If $\sigma < \frac{1+3\tau}{6}$, (4) is equivalent to $\sigma < \sigma_1(\delta, \tau)$. Note that if $(3\tau + 6\delta - 1) < 0 \iff \delta < \frac{1-3\tau}{6}$ this is impossible because $\sigma_1(\delta, \tau) < 0$ when $\delta < \frac{1+3\tau}{6}$.
which can be simplified to
\[
\frac{1}{25} ((4 + \delta - 3\tau)^2 + \sigma(4 + \delta - \tau)^2) > \frac{1}{36} (\sigma(5 - 2\tau)^2 + (5 - 3\tau)^2) \iff \\
\sigma (4\tau + 6\delta - 1) > -\frac{(6\delta - 33\tau + 49)}{(6\delta - 16\tau + 49)} (6\delta - 3\tau - 1) \tag{5}
\]

If \(\delta > \frac{1-4\tau}{6}\), (5) is equivalent to \(\sigma > \sigma_2(\delta, \tau)\). If \((6\delta - 3\tau - 1) > 0 \iff \delta > \frac{1+3\tau}{6}\) this is always true because \(\sigma_2 < 0\).

If \(\delta < \frac{1-4\tau}{6}\), (5) is equivalent to \(\sigma < \sigma_2(\delta, \tau)\) which is impossible because \(\sigma_2(\delta, \tau) < 0\) when \(\delta < \frac{1-4\tau}{6}\).

**Proof of Lemma 3**: Consider initially that \(\sigma < \bar{\sigma}(\delta, \tau)\). The insiders decide to merge if and only if \(\Pi_{M}^4 > 2\Pi_{S}^3\). This is equivalent to
\[
\left(\frac{1}{25} (1 + 4\delta + 2\tau)^2 - \frac{2}{36} (1 + 3\tau)^2\right) + \sigma\left(\frac{1}{25} (1 + 4\delta - 3\tau)^2 - \frac{2}{36} (1 - 4\tau)^2\right) > 0.
\]

It is easy to check that \(\frac{1}{25} (1 + 4\delta - 3\tau)^2 - \frac{2}{36} (1 - 4\tau)^2\) is positive if \(\delta > \frac{3}{4}\tau + \frac{5\sqrt{2}}{24} (1 - 4\tau) - \frac{1}{4}\) and that \(\frac{1}{25} (1 + 4\delta + 2\tau)^2 - \frac{2}{36} (1 + 3\tau)^2\) is positive if \(\delta > \frac{1}{4}\tau^2 - \frac{1}{4} > \frac{3}{4}\tau + \frac{5\sqrt{2}}{24} (1 - 4\tau) - \frac{1}{4}\).

Consider now that \(\sigma > \sigma(\delta, \tau)\) the insiders decide to merge if and only if \(\Pi_{M}^3 > 2\Pi_{S}^1\). This is equivalent to
\[
\left(\frac{1}{25} (1 + 4\delta + 3\tau)^2 - \frac{2}{36} (1 + 3\tau)^2\right) + \sigma\left(\frac{1}{25} (1 + 4\delta - 4\tau)^2 - \frac{2}{36} (1 - 4\tau)^2\right) > 0.
\]

It is easy to check that \(\left(\frac{1}{25} (1 + 4\delta - 4\tau)^2 - \frac{2}{36} (1 - 4\tau)^2\right)\) is positive if \(\delta > \tau + \frac{5\sqrt{2}}{24} (1 - 4\tau) - \frac{1}{4}\) and that \(\left(\frac{1}{25} (1 + 4\delta + 3\tau)^2 - \frac{2}{36} (1 + 3\tau)^2\right)\) is positive if \(\delta > \frac{5\sqrt{2}}{24} (3\tau + 1) - \frac{3}{4}\tau - \frac{1}{4} > \tau + \frac{5\sqrt{2}}{24} (1 - 4\tau) - \frac{1}{4}\).

Hence, the merger is profitable regardless of where entry takes place if (but not only if)
\[
\delta > \max\left\{\frac{5\sqrt{2}}{24} (3\tau + 1) - \frac{1}{2}\tau - \frac{1}{4}, \frac{5\sqrt{2}}{24} (3\tau + 1) - \frac{3}{4}\tau - \frac{1}{4}\right\} = \frac{5\sqrt{2}}{24} (3\tau + 1) - \frac{1}{2}\tau - \frac{1}{4}\).
\]

**Proof of Propositions 1A to 1C**: The proofs follow directly from Lemmas 1, 2A-C and 3.


References


Figures

Figure 1: Sequence of decisions

Figure 2a: Equilibrium decisions by the small country NCA (lower case), by the large country NCA (upper case) and by the SNA (circled) when $\sigma < \sigma(\delta, \tau)$: In Fig. 2a and 2b, "a" or "A" denotes approval, "r" or "R" denotes rejection, $\tau = 1/4$, $\delta = \frac{5\sqrt{3}}{24} (3\tau + 1) - \frac{1}{2} \tau - \frac{1}{4}$ and $\bar{\delta} = 1 - 3\tau$. 
Figure 2b: Equilibrium decisions by the small country NCA (lower case), by the large country NCA (upper case) and by the SNA (circled) when $\sigma > \overline{\sigma}(\delta, \tau)$. 