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Epistemic Game Master: A referee for GDL-III Games

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To everyone that helped me in my jorney...
One Master to rule them all.

Source: [One]

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Abstract

General Game Playing is the field of Artificial Intelligence that designs agents that are able to understand game rules written in Game Description Language and use them to play those games effectively. A General Game Playing system uses a Game Master, or referee, to control games and players. With the introduction of the latest extension of GDL, the GDL-III enabled to describe epistemic games. However, the complexity of the state space of these new games became in such way large that is impossible for both the players and the manager to reason precisely about GDL-III games. One way to approach this problem is to use an approximative approach, such as model-sampling.

This dissertation shows a Game Master that is able to understand and control games in GDL-III and its players, by using model-sampling to sample possible game states. With the development of this Game Master, players can be developed to be able to play GDL-III games without human intervention.

Throughout this dissertation, we present details of our developed solution, how we manage to make the Game Master understand a GDL-III game and how we implemented model sampling. Furthermore, we show that our solution, however approximative, has the same capabilities of an non approximative approach while given enough resources. We show how the Game Master timely scales with increasingly bigger epistemic games.

Keywords: GGP; GDL; GDL-II; GDL-III; EGGP; Epistemic Logic; Model-sampling.

Resumo

General Game Playing é o ramo da Inteligência Artificial que desenha agentes que são capazes de perceber regras de um jogo, escritas na Game Description Language e usam-nas para jogar esses jogos efectivamente. Um sistema de General Game Playing usa um Game Master, ou árbitro para controlar os jogos e os jogadores. Com a introdução da última extensão de GDL, o GDL-III deixa permite a descrição de jogos epistémicos. No entanto, o tamanho necessário para manter esses estados todos ficou de certa forma grande, que torna impossível para os jogadores e o árbitro raciocinarem com jogos em GDL-III. Uma forma de contornar este problema é utilizando uma aproximação desse espaço de estados como model-sampling.

Esta dissertação apresenta um Game Master que consegue entender e controlar um jogo em GDL-III e os seus jogadores, usando model-sampling para testar possiveis estados do jogo. Com o desenvolvimento deste Game Master, podem ser desenvolvidos jogadores para jogar jogos em GDL-III sem intervenção humana.

Ao longo desta dissertação, apresentamos detalhes da nossa solução desenvolvida, como conseguimos que o Game Master consiga compreender um jogo em GDL-III e como implementamos o nosso model-sampling. Também, mostramos que a nossa solução de ser aproximada, possui as mesmas capacidades que uma solução que considera todos os estados do jogo, dados recursos suficientes. Mostramos também como este Game Master escala com jogos epistémicos cada vez maiores.

Palavras-chave: GGP; GDL; GDL-II; GDL-III; EGGP; Epistemic Logic; Model-sampling.

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Chapter 1

Introduction

Artificial Inteligence (AI) — the field of computer science that studies intelligent agents, has become essential to the modern world. From medical diagnosis where agents help rendering an image of a tumour, self-driving cars to targeting online advertisements, where an agent tries to evaluate the user preferences to present them with adverts that they might be interested. Everything, in the modern world, uses some AI.

However, modern AI have been focused on solving specific problems that the developed agents lack the ability to perform those tasks under different circumstances. How can an online advertisement agent differ from a Portuguese user preference from a Brazilian? Since they share the same language, but would their tastes be the same? How can a self-driving car adapt to rain, snow or fog? In all of the situations, the best solution might be to reduce speed. However, snow might require different tires.

The fact that these agents perceive their environment and take actions that maximise their success often leads to situations where if we change their environment, even just a little, they will fail. The reason — problem — is that no matter the conditions they are in, they lack the adaptability of a human being. They lack the ability to reason like a human.

There is a field within AI that addresses this problem. This field is called Artificial General Inteligence (AGI). Agents are tested in different sets of environments and are expected behave well on all of them. Notwithstanding, building an agent to test in a real-world scenario can be very expensive, again like self-driving cars, or self-landing rockets. These costly experiments are why a good solution is to develop agents to play games.

Since the Bronze Age, where generals would test and learn new strategic skills by playing Chess, that games where found useful to model different situations. Moreover, games are used as real-life metaphors. E.g. Monopoly, however silly, is a financial metaphor of the real world economics; Battleships, like Chess as a war metaphor.

Indeed, games are so famous for their properties, that since 1713 they have been theoretically studied. Being later formalised as a mathematical discipline called Game Theory (GT), where not only games are studied, but also interactions between agents. GT is nowadays, the centre of the knowledge about games and interactions. It is so essential that more than its scope being economics, where it was born from, it also affects political sciences, psychology, as well as computer science [Mye97].

Despite studying games theoretically, it is necessary to develop agents that apply the theory. General Game Playing for Perfect Information (GGP-I) is the domain, within computer science, that develops agents to play games. The agents, general game players (here merely players), play against each other in a full human free setting. They only realise the game that they are going to play just minutes before it starts. Which means that they are inserted on a new, different environment every time they play a different game. Thus, needing to adapt to any game and without human intervention.

However, to have a human free game it is necessary to have not only players but also a referee that controls the game. In GGP-I, this agent is often called a Game Manager (GM). Without one it would be impossible to have a GGP-I system.

Albeit all, one question remains. How can players understand which the game they are playing? For that was created the Game Description Language (GDL) [GT14]. This language, as the name says, has the objective to describe any game. To specify a game one has to identify and define the set of rules that characterise it. To understand which kind of rules we are referring to, consider a game like the Tic-Tac-Toe. Such game has two players, X and O. An initial state that is the board 3×3 empty. Actions that one can do, i.e. mark the board with the player's particular mark. Middle states that which can be achieved by making a move, e.g. after marking a X the X should always be characterised in that board position. Terminal states and their payoffs, e.g. one draws if the board is filled with X's and O's; or win for the player that completes its' line. These game rules are what is necessary to describe using GDL to define the Tic-Tac-Toe game. Notice that, even though we use a game like the last, GDL allows the description of any *perfect-information*[1] game, e.g. Chess, Checkers.

There are many strategies to play GDL games. The most basic is to assume a pessimistic approach. Since one does not know who is one is playing against, one cannot expect that the adversary will behave rationally. This strategy is called the Minimax. A player tries to maximise their payoff while assuming that others will try anything to minimise it. This game strategy is one of many that work by exploring all possible states of a game to find the best move. So, for a simple case like Tic-Tac-Toe, that has a small number of states, this strategy suffices since it can be computed. However, would it be able to be computed in a game of Go? In this game, the set of possible states is so large[2] that searching the entire game space is impossible. Because of games like this, that have so many

[1] Game theory term that refers to games were each agent, when choosing an action, is perfectly informed of all the events that have occurred previously, and the initial state of the game

[2] A board of Chess has 8×8 and leads to 10^{30} possible states. A board of Go has 19×19 intersecting lines

possible states, those other strategies were developed. One that is worth mentioning is the Monte Carlo strategy. This strategy consists of, instead of searching all possibilities like the Minimax, probing to explore which is the best move.

A problem with GDL is that even though players do not know who are they playing against, they can see the moves that they make. However, much like a human, an agent is expected to conclude what is its best outcome even on tasks where uncertainty is associated. Several games serve as an abstraction to such case. For example, a game of Poker like Texas Hold'em. At the beginning of this game, each player does not see the card of their opponents. A game like this is called *imperfect-information*[3] game and cannot be described using GDL. For that purpose was created an extension, the Game Description Language for games with Imperfect Information (GDL-II) [Thi11a]. We can find many games that belong to this category, e.g. Mastermind (see section 2.3.1), Battle Ships, Blind Tic-Tac-Toe, Blind Chess, as well as most[4] card games.

Under this imperfect information environment, even though previous methods can still work, they are not necessarily good. Because the setting in which players are presented to play in changes. One approach is useful in imperfect-information is the use of an Information Set. This approach comes from the definition of an imperfect information game in GT. More precisely, players consider every possible state where they could be at some time in the game and filter them whenever they obtain new information[5]. Most of the strategies used on GDL-II have as the basis of using an information set. The problem with this is that maintaining the set of all the possible states that a player could be that at some point, might be too large and will just not work. Remember the previous note: a game of Chess has 10^{30} possible states. What about Blind Chess? The set of possible states is even larger. Because of that, it was developed a solution called HyperPlay (HP) [ST15] (see section 2.3.3.1), that involves keeping only a subset of incomplete states[6] instead of the all possible incomplete states of information set. This solution uses a technique called model-sampling, which as the name implies, involves completing the incomplete states and testing them.

However, much like we said before regarding GDL, GDL-II also has a limitation. It does not support the specification of games where the rules depend on the epistemic state of players — the description of an epistemic game. An epistemic state is defined by what the player know at that state, i.e. imagine that we were given a bag that does not show what is inside. This bag contains a ball that can either be red or blue. Unfortunately, while is inside the bag all that we know is that it is a ball. Hence, our epistemic state is defined by that we know that inside the bag is a ball. However, the property of the colour is not part of our epistemic state since we cannot distinguish if it is red or blue, while it is inside the bag.

[3] Game theory term that refers to games where each player, even though they know the initial state of the game, they do not know accurately in which state they are in

[4] Requires at least one: shuffling or not showing the cards to an opponent

[5] Imperfect-information does not mean any information at all

[6] Since, one does not know in which state he is in, it is safe to conclude that one is in an incomplete state

This new extension, the Game Description Language for games with Imperfect Information and Introspection (GDL-III) [Thi16] forces players to reason with their and the opponents' epistemic state. Forces them to be rational. Remember the last example of Blind Chess? Where the state space already enormous? What if we had the players' epistemic state to the mixture? One game state[7] can have many epistemic states. Which means that there is an unmeasurable amount states[8] in the game.

A simple example of an epistemic game is the Russian Cards Problems[Cor+13]. It states that, from a deck of seven distinct cards, Alice and Bob are each dealt three cards, and Cathy is dealt the remaining card. None of the players knows any of the cards of the other players. Using a series of truthful public announcements, Alice and Bob should exchange information about the hands they hold without Cathy being able to deduce the owner of any card other than her own.

This example shows that players will always need to calculate what they know, and their opponents' know. One way to do it is with an Epistemic Logic principle called indistinguishability. This principle says that a player knows something, if and only if it can see that, in all the states in its information set. Some reasoners allow us to reason with epistemic logic, such as Answer Set Programming (ASP). However, such solutions consider all possible states, which as we already stated is impossible to maintain, because the state space is unmeasurable large.

A GM for both GDL and GDL-II only have one state to update — the game instance being played. However, by players having to reason about their knowledge, it is necessary for the GM to be able to deduce if players know something. This means that the GM needs to consider the same game space as the players. Therefore, unlike the previous GM, a referee for GDL-III raises some issues.

The primary objective of this dissertation is to present a new GM for Epistemic General Game Playing (EGGP), which allows players to play games in GDL-III. This GM implements the semantics of GDL-III. We present a GM that samples the game states, since keeping track of all of them is impossible.

The primary beneficiaries of this dissertation are the EGGP community since it allows the community to be able to develop new players to play epistemic games. Players can use parts of our developed solution to be able to play a GDL-III game. Even more, AGI also benefits from this dissertation since it belongs to their scope.

1.1 Structure

The remainder of this document is structured as follows:

- Chapter 2 presents the related work, with GT; GGP-I; General Game Playing for Imperfect Information (GGP-II); EGGP; Epistemic Logic and ASP, already mentioned

[7] For simplicity, let's make a distinction between the state in the game for a player and his epistemic states, even though they are all different states of the game

[8] From now on, we consider that every time the epistemic state changes, it is a new state of the game

in this chapter;

- Chapter 3 describes our implementation of the GM, from a broader perspective into a more detailed one;
- Chapter 4 shows the analysis of our solution;
- Chapter 5 presents our conclusions to this dissertation, as well as, it suggests a set of improvements, that can be developed towards the future.

Chapter 2

Related Work

In this chapter we explore the state of the art concerning the basics of the technologies that relate to our intended work. We start by describing the very basics of game theory necessary for the understanding of our work, like the normal-form, perfect-information extensive-form and imperfect-information extensive-for, as well as, the equilibrium. We then, explore each version of general game playing, going in each version to GDL, the game protocol and strategies used by the players. Moreover we present epistemic logic and a special reasoner for epistemic logic: ASP.

2.1 Game Theory

GT is the mathematical study of interaction among independent self-interested agents [LS08]. It is mainly studied by mathematicians, economists and computer scientists, but is acquiring more interest in other disciplines like political science or sociology. On this subsection we will present the basics of GT necessary for a better understanding of our work.

2.1.1 Normal-Form

The normal-form is the simplest form used to represent a game in GT. On this form, games are assumed to have only one turn and players have to play their actions at the same time. This form is defined as [LS08]:

Definition 1. *A finite n-person normal-form game is a tuple* (N, A, u) *where:*

N *is the finite set of n players, indexed by i;*

$A = A_1 \times ... \times A_n$ *where* A_i *is a finite set of available actions to player i, and every* $a = (a_1, ..., a_n) \in A$ *is called an action profile;*

$u = (u_1, ..., u_n)$ *where* $u_i : A \to \mathbb{R}$ *is the payoff function for player i.*

Because of the characteristics of the games that this form describes, the games representation usually acquires the image of a matrix. Moreover, any game can be described using this form. No matter the kind of game that it is. For a better understanding consider the matrix of the game Matching Pennies. The game states that: "Each of the two players has a penny, and independently chooses to display either heads or tails. The two players then compare their pennies. If they are the same then player 1 pockets both, and otherwise player 2 pockets them."

	Heads	Tails
Heads	$1, -1$	$-1, 1$
Tails	$-1, 1$	$1, -1$

Figure 2.1: Matching Pennies

The rows represent player 1's actions and the columns player 2's. Inside a square the number on the left represents player 1's payoff whereas the number on the right represents player 2's payoff. This game is also a special type of game called a zero-sum game, because in every payoff in a square of the matrix sums up to zero.

Recall, that on definition 1, one can see that games are defined by the possible actions in that game. However, how a player chooses on those actions is up to it. This is called a strategy, and a player can have different strategies for different games. The most simple is a strategy where a player decides on an action and plays it. This is called a *pure strategy* and, because it is choosing an action and playing it can be defined as an action was in definition 1. Likewise for a pure strategy profile, that is a profile where every player chooses a pure strategy.

Another very simple strategy that exists, consists on a player randomizing on his set of possible actions. This strategy is called a *mixed-strategy* and is defined as follows [LS08].

Definition 2. *Let (N, A, u) be a normal-form game as in def. 1, and for any set X let $\Pi(X)$ be the set of all probability distributions over X. Then the set of mixed strategies for player i is defined $S_i = \Pi(A)$ and the set of mixed-strategies profiles is simply the Cartesian product of the individual mixed-strategy sets, $S_1 \times ... \times S_n$*

To have a clear idea of what this definition means consider that, a player can choose between two actions by with a probability of 0.5 between those actions. This means that

half of the time he will play an action, the other half he will play the other. This is mixed strategy.

The objective of a player is to always maximize his payoff. Unfortunately, on a multi-player game, one's payoffs depend of the move of other players, who are also trying to maximize their payoffs. So it is safe to reason that one's best strategy depends on the choices made by others. For the same reason one can simplify the problem. That is, if our best payoff depends on the other player play, then let us assume that he already played. Hence, our player would only have to select the move that maximizes his utility according to what the other player did. In other words it is *his* best response to that play. The definition goes like this [LS08] (Adapted):

Definition 3. *Let $s_{\neg i} = (s_1, ..., s_{i-1}, s_{i+1}, ..., s_n)$ be a strategy profile without i's strategy, thus being able to define $s = (s_i, s_{\neg i})$. Then, player i's best response to the strategy profile $s_{\neg i}$ is a mixed strategy $s_i^* \in S_i$ such that $u_i(s_i^*, s_{\neg i}) \geq u_i(s_i, s_{\neg i})$ for all strategies $s_i \in S_i$*

A best response doesn't have to be unique. And they can be pure strategies or even mixed-strategies.

The definition 3 helps defining the most basic principle in GT for non cooperative games and more specifically zero-sum games, the Nash Equilibrium [LS08].

Definition 4. *A strategy profile $s = (s_1, ..., s_n)$ is a Nash Equilibrium if, for all agents i, s_i is a best response to $s_{\neg i}$.*

This states that, a strategy profile is a Nash Equilibrium if no player wants to change his action regarding what the other player do. Hence, we can call it a stable strategy profile, since everybody is happy with their decision.

For a better understanding of this concept, consider the following normal-form [Bat], representing the known game of the Battle of the Sexes. The game goes like this: "Alice and Bob have a date scheduled, but they didn't decide on what to do. Alice prefers to watch the ballet, while Bob would much rather watch soccer. However, since they are in love, they would both prefer to spend time with each other than to go separately". The numbers square represents the players' payoffs. The first one is Alice's payoff and the second Bob's.

		Bob	
		Ballet	Soccer
Alice	Ballet	2,1	0,0
	Soccer	0,0	1,2

Figure 2.2: Battle of the Sexes

As you can see in the Figure 2.2, the game has two pure Nash equilibria strategies $(Ballet, Ballet)$ and $(Soccer, Soccer)$, because is both better for Alice to choose $Ballet$ when

Bob chooses *Ballet*, since she would receive a payoff of zero instead of two. By the same reasoning Bob should do the same when Alice chooses *Ballet*. The other Nash equilibrium can be justified using the same reasoning for *Soccer*.

There's also mixed Nash equilibria, since Bob could play (*Ballet*, *Soccer*) with a probability of $\frac{1}{3}$ for *Ballet* and $\frac{2}{3}$ for *Soccer*. The probability changes when we look to Alice's side $\frac{2}{3}$ for *Ballet* and $\frac{1}{3}$ for *Soccer*.

Even though, agents can assume something about the opponent strategies, they don't known effectively what's that the opponent are going to do. So sometimes the best strategy is the one that maximizes the worst-case scenario. That strategy is called the *Maxmin strategy* [LS08].

Definition 5. *The maxmin strategy for player i is* $\arg\max_i \min_{\neg i} u_i(s_i, s_{\neg i})$ *and the maxmin value for player i is* $\max_i \min_{\neg i} u_i(s_i, s_{\neg i})$

This strategy can be understood as player i makes the first move and then the other players will try to minimize it. Even if the other players play arbitrarily, i will receive at least his maxmin value.

The interesting part is that, according with the Minimax theorem [VN27], in a finite two player zero-sum game each player receives a payoff that is equal to both his maxmin-value and his minimax-value. And they are in fact a nash equilibrium [LS08].

2.1.2 Perfect-Information Extensive Form

The perfect information extensive form is a representation that doesn't assume that players have to play at the same time. A perfect information game can be interpreted a tree where each node represents a choice of one of the players, an edge represents a possible action, and each leaf represents the terminal node where each player has his payoff function. The formal definition of the perfect information extensive form is [LS08]:

Definition 6. *A perfect-information game in the extensive-form is a tuple $G = (N, A, H, Z, \chi, \rho, \sigma, u)$ where:*

N is the set of players;

A is the single set of actions;

H is the set of nonterminal choice nodes;

Z is the set of terminal nodes (disjoint from H);

$\chi : H \to 2^A$ is the action function that assings to each choice node a set of possible actions;

$\rho : H \to N$ is the player function, which assigns each nonterminal node a player $i \in N$ who chooses an action in that node;

$\sigma : H \times A \to H \cup Z$ is the successor function, which maps a choice node and an action to a new choice or a terminal node such that for all $h_1, h_2 \in H$ and $a_1, a_2 \in A$ if $\sigma(h_1, a_1) = \eta(h_2, a_2)$, then $h_1 = h_2$ and $a_1 = a_2$; and

$u = (u_1, ..., u_n)$ where $u_i : Z \to \mathbb{R}$ is the payoff function for player i on the terminal nodes Z.

It is an extensive form, so every game on this form can be changed into a game in the normal-form described in Definition 1, but not without having redundancy. This representation is much smaller than the normal-form, and can be more natural to reason with. Consider the Figure 2.3. The game is interpreted as player 1 chooses one move between: A, B and the player 2 chooses one move for each node: Y, Z.

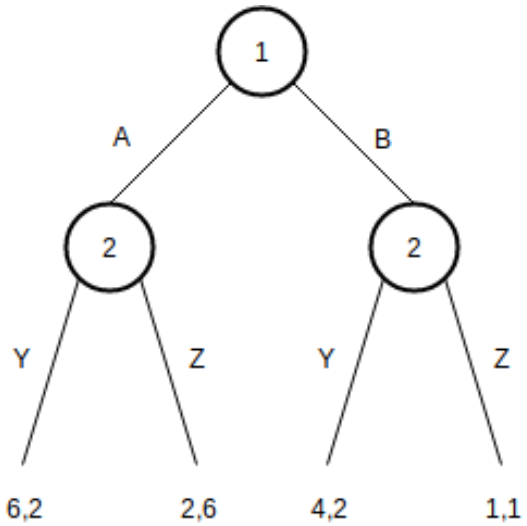


Figure 2.3: Subgame-perfect-equilibria

All the strategies considered in the normal-form work in this new form, but one thing needs to be consistent. That's the equilibria. However, there can be more nash equilibria in this extensive-form, that might not be an optimal strategy profile. Consider the previous Figure shown. It has two nash equilibria: $A,(Z,Z)$ and $B(Z,Y)$. But notice, the $A,(Z,Z)$. Is it really possible that player 2, on his right most node will choose option Z, when Y gives him a better payoff? The answer is no. This is why is necessary to introduce a notion of subgame-perfect equilibrium. The equilibria needs to be consistent with the subgame or subtree after each action. Before defining a subgame-perfect equilibria, one needs to formally define a subgame [LS08]:

Definition 7. *Given a perfect-information extensive-form game G as in def. 6, the subgame of G rooted at node h is a restriction of G to the descendants of h. The set of subgames of G consists of all of subgames of G rooted at some node of G*

Simply speaking, a subgame is the game considered as in begining of the node that the last action took us to. Then one can finally define a subgame-perfect equilibrium [LS08]:

Definition 8. *The subgame-perfect equilibria (SPE) of a game G as in def. 6 are all the strategy profiles s such that for any subgame G' of G, the restriction of s to G' is a Nash equilibrium of G'.*

Intuitively speaking, for a strategy profile be a *SPE* it must be a nash equilibria in every subgame from: the root of the game, passing through all the decision nodes consistent with the strategy profile.

A *SPE* is normally deduced by backward induction from the various outcomes of the game. It eliminates branches which would involve any player making a move that is not viable (because it is not optimal) from that node. A game in which the backward induction solution is well known is Tic-Tac-Toe.

2.1.3 Imperfect-Information Extensive Form

In the previous section, games where in an environment of perfect-information, but there are some situations where players need act with partial or no knowledge of what other players did. This new extensive-form is an extension of the perfect-information extensive form where nodes are splited into information sets. So if, two nodes belong on the same set of the same player, then that player cannot see any difference between them. The imperfect-information extensive-form is defined as follows [LS08]:

Definition 9. *An imperfect-information game in the extensive-form is a tuple* $G = (N, A, H, Z, \chi, \rho, \sigma, u, I)$ *where:*

$G = (N, A, H, Z, \chi, \rho, \sigma, u)$ *is a perfect-information extensive-form game; and*

$I = (I_1, ..., I_n)$, *where* $I_i = (I_{i,1}, ..., I_{i,k_i})$ *is and equivalence relation (i.e. partition of)* $\{h \in H : \rho(h) = i\}$ *with the property that* $\chi(h) = \chi(h')$ *and* $\rho(h) = \rho(h')$ *whenever it exists an* j *for which* $h \in I_{i,j}$ *and* $h' \in I_{i,j}$

As an example of a game in this form, consider the following Figure 2.4, representing a game in the imperfect-information extensive-form:

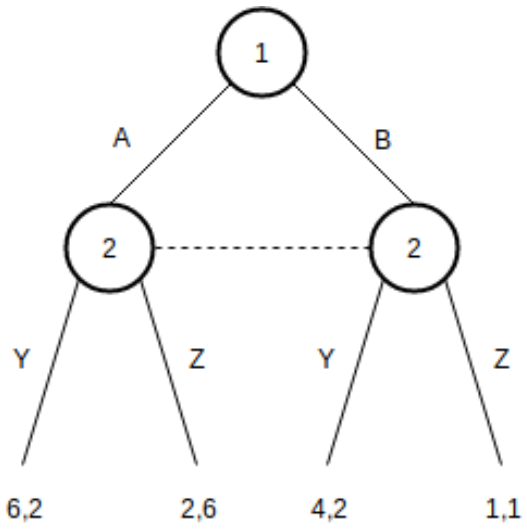


Figure 2.4: Imperfect-information extensive-form

The difference between this form and the last is that the two nodes of player two are in his information set. This means that now he cannot see any difference between them. As we can see, both this form and the last form are very similar, so one might think that SPE would still work. That is not the case anymore. Because, the SPE was a constriction of the possible equilibrias in a game of perfect-information. However, since players don't know where they are, they not only have a game to consider, but several games as well. In other words we have a set of subgames. So, it is necessary to do a relaxation of these possible equilibria. This is called a *Sequential Equilibrium*. It can be defined as [LS08]:

Definition 10. *A strategy profile S is a sequential equilibrium of an extensive-form game G as in def. 9, if there exists probability distribution $\mu(h)$ for each information set h in G, such that the following two conditions hold:*

$(S, \mu) = \lim_{n\to\infty}(S^n, \mu^n)$ for some sequence $(S^1, \mu^1), (S^2, \mu^2), \ldots$ where S^n is fully mixed, and μ^n is consistent with S^n (in fact, since S^n is fully mixed, μ^n is uniquely determined by S^n); and

For any information set h belonging to agent i, and any alternative strategy $S_i^{'}$ of i, we have that

$$u_i(S|h, \mu(h)) \geq u_i((S^{'}, S_{\neg i})|h, \mu(h)).$$

We can think about this equilibrium like this: the first condition consists in the modulation of the players' beliefs about where they are in the tree for every information set; the second condition can be interpreted as: the modulation of the players beliefs' is not contradicted by the actual play of the game and so, the players always best respond to their beliefs.

2.2 General Game Playing for Perfect Information

As said in the Chapter 1, a GGP-I is a system that receives a game description at runtime. This player has to interpret it and play accordingly. But, in order to be able to play a game, more elements are necessary. Specifically a GM and a Game Protocol (GP).

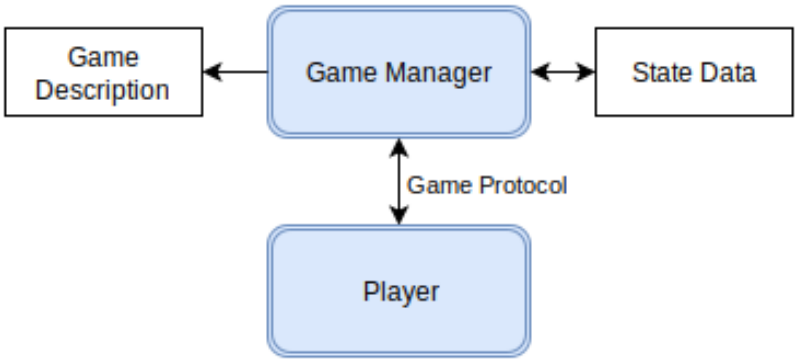


Figure 2.5: Simple GGP sytem

A GGP-I system can be viewed in the Figure 2.5. The GM is the entity that regulates the game, according to the game description. This system conserves the actual state of

the game, decides if the players moves are legal and if not selects one at random. The communication with players is through HTTP connections using the GP. In the Figure, we are only showing one player, but this player represents any player.

2.2.1 GDL

GDL is a language that allows the description of any perfect-information extensive form from GT. It is a pure declarative language that allows: constants, variables, functions and relations. Variables have to begin with an uppercase letter, while constants, functions and relations must start with a lowercase[1].

A GDL description is formed with a set of logical sentences that must be true in every state of the game[Lov+08].

2.2.1.1 Syntax

GDL is an open language in the sense that, the vocabulary can be extended. However, the meaning of these basic vocabulary items is fixed for all games. Those words are:

- **role (R)** — R is a player;
- **init (P)** — P is true in the initial state;
- **true (P)** — P is true in the current state;
- **legal (R, A)** — player R can do action A in the current state;
- **does (R, A)** — player R does action A
- **next (P)** — P is true in the next state;
- **distinct (P1, P2)** — P1 is be different of P2;
- **terminal** — current state is terminal;
- **goal (R, V)** — player R has a payoff V.

GDL also allows the use of 101 numbers. They go from 0 to 100 in decimal base. The 0 is considered the lowest and 100 the highest. These numbers are used to describe payoff **V** on goal rules and usually a step counter representing a turn.

Notice that, in GGP-I every player has to make an action every round, unlike a perfect-information game in GT, which is turn base. However, GDL allows the description of rules where the meaning of the rule is that the players is skipping turn. This rule is often called no-operation, or *noop*.

Recall in the Chapter 1, where we debated about the set of rules that was necessary to describe the Tic-Tac-Toe game. The following is the presentation of a possible game description of those rules, available in [Lov+08]:

[1] For a better understanding of the game rules we will use infix GDL instead of prefix GDL

```
% the roles
role(xplayer)
role(oplayer)

% initial states
init(cell(1, 1, b))
init(cell(1, 2, b))
init(cell(1, 3, b))
init(cell(2, 1, b))
init(cell(2, 2, b))
init(cell(2, 3, b))
init(cell(3, 1, b))
init(cell(3, 2, b))
init(cell(3, 3, b))
init(control(xplayer))

% legal moves for each player if the board is empty
legal(W, mark(X, Y)) :- true(cell(X, Y, b)) & true(control(W))

% legal moves of the player if is the other playing
legal(xplayer, noop) :- true(control(oplayer))
legal(oplayer, noop) :- true(control(xplayer))

% next state the cell has a mark if the player marked a cell and
% that cell was blank
next(cell(M, N, x)) :- does(xplayer, mark(M, N)) & true(cell(M, N, b))
next(cell(M, N, o)) :- does(oplayer, mark(M, N)) & true(cell(M, N, b))

% inertia - all the cells conserve their mark for the next state
next(cell(M, N, W)) :- true(cell(M, N, W)) & distinct(W, b)
next(cell(M, N, b)) :- does(W, mark(J, K)) & true(cell(M, N, b)) & distinct(M, J)
next(cell(M, N, b)) :- does(W, mark(J, K)) & true(cell(M, N, b)) & distinct(N, K)

% players turn switch every round
next(control(xplayer)) :- true(control(oplayer))
next(control(oplayer)) :- true(control(xplayer))

% terminal states
terminal :- line(W)
terminal :- ~open

% goals
goal(xplayer, 100) :- line(x) & ~line(o)
goal(xplayer, 50) :- ~line(x) & ~line(o)
goal(xplayer, 0) :- ~line(x) & line(o)
goal(oplayer, 100) :- ~line(x) & line(o)
goal(oplayer, 50) :- ~line(x) & ~line(o)
goal(oplayer, 0) :- line(x) & ~line(o)
```

```
50  % auxiliar predicates
51  % a row
52  row(M, Z) :- true(cell(M, 1, Z)) & true(cell(M, 2, Z)) & true(cell(M, 3, Z))
53
54  % column
55  column(N, Z) :- true(cell(1, N, Z)) & true(cell(2, N, Z)) & true(cell(3, N, Z))
56
57  % diagonal
58  diagonal(Z) :- true(cell(1, 1, Z)) & true(cell(2, 2, Z)) & true(cell(3, 3, Z))
59  diagonal(Z) :- true(cell(1, 3, Z)) & true(cell(2, 2, Z)) & true(cell(3, 1, Z))
60
61  % line iff one makes a row, a column or a diagonal
62  line(Z) :- row(M, Z)
63  line(Z) :- column(M, Z)
64  line(Z) :- diagonal(Z)
65
66  % game is open iff there is still a cell blank
67  open :- true(cell(M, N, b))
```

2.2.2 General Game Master

In this section we describe the GM available from the GGP-Base package [Ggp]. Figure 2.6 shows it's main components, where arrows represent interactions. For example, the Interpreter interacts with the Game Description, in the sense that, interprets it according to GDL semantics. The State Machine is the module that, as the name suggests, handles games states regarding the Game Description. For instance, to check if a state is terminal is necessary to call the function in the State Machine with the desired state as an argument. This function converts the contents of a state — recall that in GDL the state content is represented with what is with the *true* keyword — and calls the Interpreter to prove the terminal rule. The Server is the module that communicates with the players and controls the state of the game.

The GGP-Base package also provides the implementation for the random player because of games in perfect information games that require a die to be played, e.g. Backgammon.

2.2.3 Game Protocol

The GM and each player communicate through a protocol that has 5 types of messages:

- **info()** — The GM checks if the player is running. To whom the player answers that he is either available or busy;
- **start(id, role, description, startclock, playclock)** — The GM starts a match. He sends a message to the player with the match id; the role that the player will have in the game; the description of the game that is going to be played; the playclock, which is

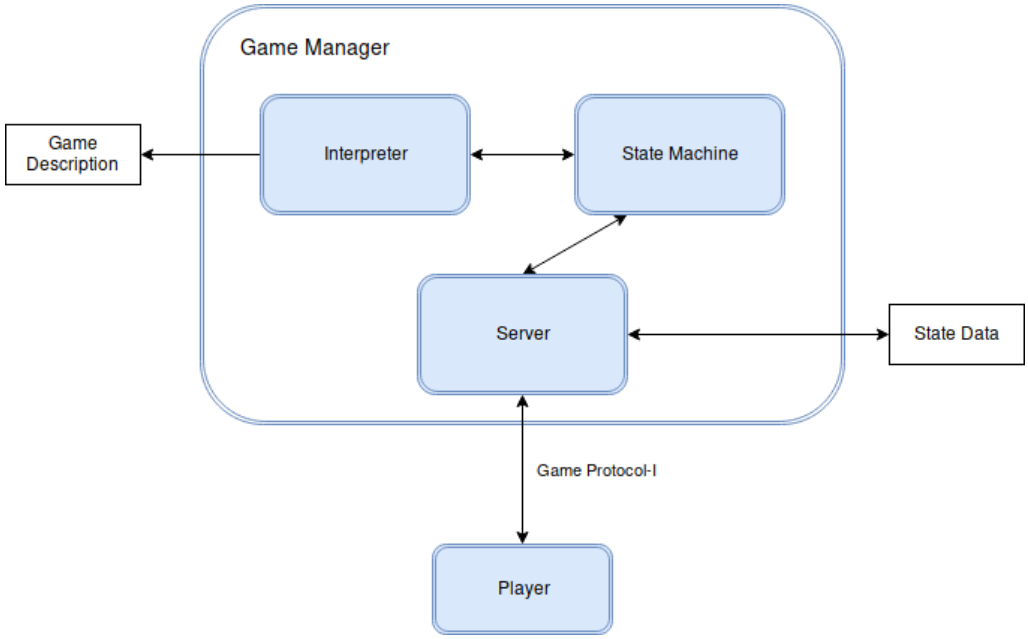


Figure 2.6: Game Manager for GDL-I

the time that the players have to answer back per round; and the startclock that is the time that the GM will wait for the player to answer this message. If the player does not answer in time, the GM assumes that the player is ready;

- **play(id, actions)** — The GM sends a play message to the players with the match id and the actions made by all the players in the last round. The players answer with a legal move. If a player does not answer in time, the GM selects a random legal action for it;

- **stop(id, actions)** — The GM sends a stop message to the players with the match id and the actions made by all the players in the last round. This message represents that the game is over and enables the players to do their after match clean up. The player has should answer back when it's done;

- **abort()** — The GM sends this message if an error occurs. No answer is necessary.

2.2.4 Strategy

There are many strategies that a player can opt in a game. Recall that in section 2.1 we defined some of the basics concepts, that can also be applied in this section.

For simplicity, lets define a perfect-information game as in section 2.1 given by a GDL description.

Definition 11. *Let $G = (R, A, S, T, \eta, \mu)$ be an perfect-information game given by a GDL description, where:*

R is the set of roles in the game;

S is the set of all states in the game;

A is the set of actions in the game; and $L(r,s) \subseteq A$ *is the set of legal moves of role* $r \in R$ *in state* $s \in S$*;*

$T \subseteq S$ *is the set of terminal states;*

$\mu : T \times R \to \mathbb{R}$ *is the payoff function;*

$\eta : S \times A^{|R|} \to T$ *is the successor function;*

Remember that, the backwards induction works by identifying the equilibria at the bottom subgame trees, and assuming those equilibria will be played as one backs up and considers increasingly larger trees. The problem is that when one is in a game with a large state space he doesn't enough time to search the equilibria. And in games like Go players can't evaluate every branch of the tree in the playclock they have to play.

Therefore, sampling algorithms had to be explored. One of them is the Monte-Carlo Search (MCS) [Has70]. This algorithm consists has two phases: an expansion phase, where the player explores the game tree until determined depth; and a probing phase, where the player sends probes at each node at the end of the expansion phase. Figure 2.7 shows an example of MCS execution.

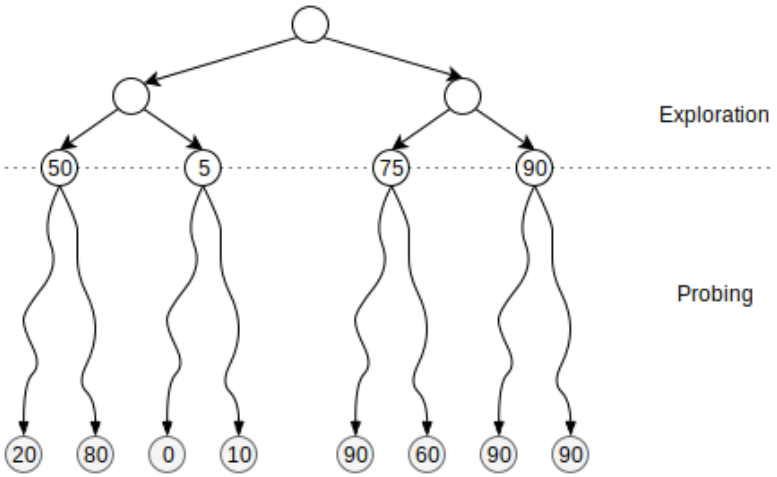


Figure 2.7: Monte Carlo Search

The probing phase is very special, since each probe has the objective of getting to the end of the tree, at each node a random action is selected. The nodes utilities at the probing phase are then calculated according to the following expression:

Definition 12. *Let G be as in def. 11, n be the number of probes to send,* $r_p \in R$ *be the role of the player and* $s_t \in T$ *be a terminal state achieved by randomizing actions.*

$$\frac{1}{n}\sum_{1}^{n}\mu(s_t, r_p)$$

In the probe phase of the MCS, it's not necessary to calculate anything more than just selecting a random action to move forward in the tree. This enables players to create

lots of probes to search each node. Furthermore, this algorithm has been proven that it's solution tends to the subgame-perfect equilibria.

However, with MCS the game tree is equally searched, even when it's clear that it isn't necessary. That leads to an unnecessary search on the sides of the tree that will take us nowhere, when we could be using that time to search the nodes that are promising. The Monte-Carlo Tree Search (MCTS) [Cha+08] was developed with that idea. Instead of two phases, this new method has four phases, that can be seen in the following Figure 2.8. This Figure can be found in[Mon]:

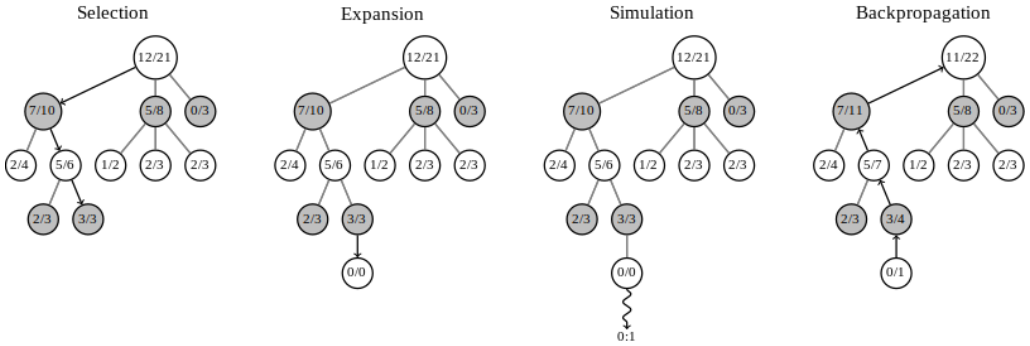


Figure 2.8: Monte Carlo Tree Search Cycle

The first phase is called the selection, where the next node to explore is selected, this selection method uses a policy like the presented in Definition 13. The second phase is the expansion, where the children of the chosen node during the selection phase are added to the tree. The third phase consists on simulating the game starting at the added node in the expansion phase and choosing random actions until a terminal state. The fourth phase is about propagating the value of the terminal state to the root and while passing through the nodes changing the visit counts and the value of each state.

The policy followed in the selection phase is an Upper Confidence Bound (UCT) [KS06], that can be defined as [GT14](Adapted):

Definition 13. *Let G as in def. 11; v_i be the average reward for state $i \in S$; c be an arbitrary constant; n_p be the number of times that the parent of state i was picked; and n_i be the number of times that state i was picked, then the policy is:*

$$v_i + c\sqrt{\frac{\log\ n_p}{n_i}}$$

The main difficulty about selecting the child nodes is to maintain the balance between exploitation and exploration. The formula in the Definition 13 provides an answer to that. The first component corresponds to exploitation — higher for moves with high average win ratio. And the second component corresponds to exploration — higher for moves with few simulations.

Using the MCTS allows the game tree to grow asymmetrically because the method focus on the most promising subtrees. Thus, achieving better results than classical algorithms in games with a higher branching factor.

Moreover, the MCTS can be interrupted at any time and the player can at that moment choose the most promising move. Unlike using a Minimax algorithm, for example.

2.3 General Game Playing for Imperfect Information

GGP-II is the General Game Playing concerned with games for imperfect-information, e.g. Battle Ships, or Poker. The General Game Playing system as in Figure 2.5 remains the same. However, there are some changes regarding the GDL and the protocol, in order to ensure that the players act under imperfect-information.

2.3.1 GDL-II

GDL-II [Thi11a] was introduced due to the impossibility of describing games like the Mastermind, or Poker. In this games it is impossible to describe, for instance, the act of a random event. Moreover, using GDL-II, players are not able to see what other players do. Only information that their' actions might trigger.

2.3.1.1 Syntax

GDL-II extends the syntax of GDL, and the set of keywords is extended:

• **sees R P** — player R sees proposition P in the next state;

• **random** — represents randomness;

The first rule allows the players to see that they successfully triggered an event. Using the description, a player can then see what caused it and use it for his gain. This keyword can also be used to allow communication in a game between players, either public or private. GDL-II is an extension of GDL, which means that any game in GDL can also be described in GDL-II. That's very simple to accomplish, one just needs to add a game rule where a player can always see the moves made by other players, and the game, even though is described in GDL-II it would be a perfect-information game. The keyword random, on the other hand, must be added as a role. This is a game independent role that represents a way to choose legal moves with a uniform probability. For a better understanding of how those keywords can be used, consider the following example — a snapshot of the Mastermind game:

```
1  % the roles
2  role(random)
3  role(player)
4
5  % The random player sets up the colors in the first step.
```

```
6   legal(random, set(C1, C2, C3, C4)) :- true (guess (setup)) & color(C1)
7                                         & color(C2) & color(C3) & color(C4)
8
9   % The player is informed of all colors correctly guessed.
10  sees(player, set(1, C1)) :-  does(player(guessColors(C1, C2, C3, C4))
11                               & true(set(1, C1))
12  sees(player, set(2, C2)) :-  does(player(guessColors(C1, C2, C3, C4))
13                               & true(set(2, C2))
14  sees(player, set(3, C3)) :-  does(player(guessColors(C1, C2, C3, C4))
15                               & true(set(3, C3))
16  sees(player, set(4, C4)) :-  does(player(guessColors(C1, C2, C3, C4))
17                               & true(set(4, C4))
```

On this game, the player random selects the colours so that the player can guess it on other steps of the game. After the player guesses, he will be notified if he correctly guessed a color in the right position. He can use this information to correctly guess the other remaining colours.

2.3.2 Game Protocol

As GDL-II introduces uncertainty, the GP needs the to have the necessary modifications in order for that to happen. Though, it's only necessary to change two methods. The other methods remain the same:

- **play(id, turn, action, percepts)** — the GM sends a play message to the player with the match id, the turn of the game, the action that the player made last turn and the array of percepts, triggered with all the moves received from all the players, according with the game rules. The player has to answer with a legal action. Otherwise, the GM selects a random legal action.

- **stop(id, turn, action, percepts)** — the GM sends a stop message to the player with the match id, the turn of the game, the action that the player made last turn and the array of percepts triggered according with the game rules. The player has to clean the data regarding the match and respond done.

Recall in the previous version of the GP all the players were notified about the adversary move, according to the protocol. In the new modification, the GM sends to the player only his last move and the array of events that were triggered. Notice that, the players don't communicate with each other directly, it always passes trough the GM. Then he warns the players on their array of percepts, as accorded in the game rules.

2.3.3 Strategy

One of the hidden challenges of GDL-II is that the systems need to be able to draw conclusions from partial observations. The most common assumption on this type of games is the use of an Information Set. This strategy is widely utilized because the

initial state is always known to the players. Using that, they can use the description to model the next states, having in consideration, all the possible moves. They also use the percepts that they recieve from the GM to filter that set. Like in the Figure 2.9. Recall the mastermind description from the previous section 2.3.1.1. The "player" random selected 4 colours in a random order that the other player doesn't know. For simplicity, let's assume that the information set of the player is the following.

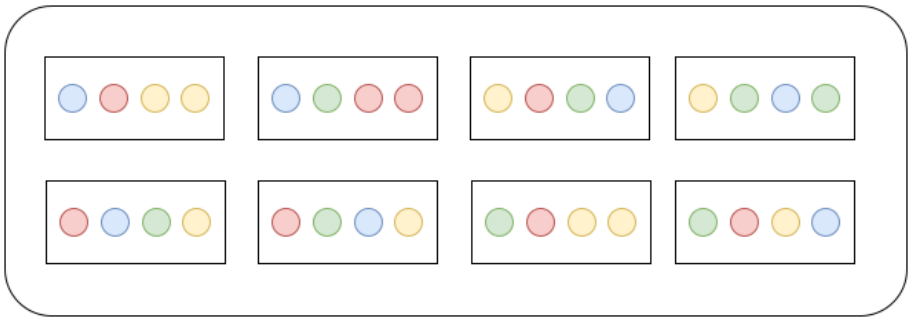


Figure 2.9: Incomplete Mastermind Information Set

Assume that the player guessed the following order: red, red, red, yellow. And that, from that he received two percepts: "set(1,red)" and "set(4,yellow)". The player can use that information to conclude Figure 2.10, where he can now see that in the position 1 has red, the positions 2 and 3 don't have red, and finally the position 4 has yellow. A good player should be, one that uses this to his advantage. He needs to able to acquire information of what he sees, but also, and as much as important, of what he doesn't see.

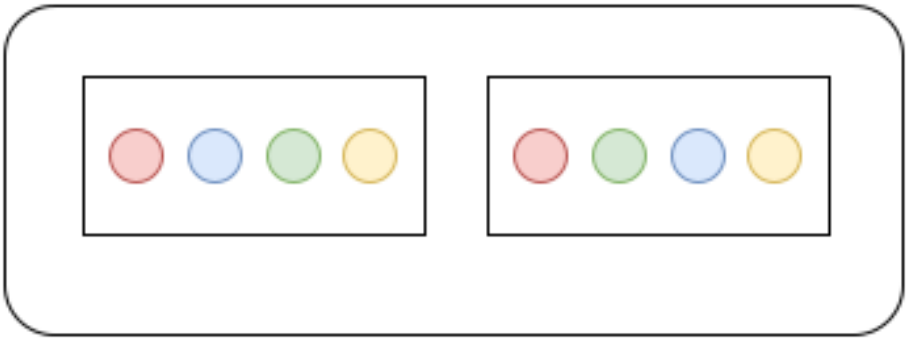


Figure 2.10: Incomplete Mastermind Information Set Filtered

In the Mastermind game, the information set of a player starts with all the possible states at the beginning of the game, and gets smaller after every turn. Notice that, it only happens because of the characteristics of the game and that is not the case for every game. The Information Set can, in fact, grow larger after some turn. That's actually one of the biggest problems of imperfect information games. Since players don't know exactly where they are, there are even more possibilities to consider than in a perfect information game. Thus, making impossible to conserve all possible states in the information set. To have an idea consider a game like Chess, but without players being able to see their

opponents moves; or a game of Poker where 52 cards are shuffled at the beginning of the game. The possibilities are tremendous.

A solution found for this problem is to restrict the computation of all the successors states to just a few, with randomly selected joint moves. This allows to compute only a few elements from the information set. Each of these states can be tested versus the player percept after each round. If the player notices that the state is wrong, then the state is deleted and substituted by other randomly selected possible state. The great thing about random states is that they are independent from each other and the player can parallelize the evaluation of actions per state.

2.3.3.1 Hyper Play

The HP [Sch+12] is a model sample technique that does exactly that. As states can be uniquely identified by the moves and percepts of all the players in the game [Thi11b], a player can use randomness to generate the remaining path of those states[2].

This technique uses a bag of models of the information set. These models are completed states retrieved from the information set of a player. They are filter accordingly and updated based on the actions and the percepts of the player. Then, HP assumes perfect information when testing each completed model.

The HP technique is formally described, but in order to get there, it is necessary to first refer some auxiliar definitions, like a game in GDL-II [ST15].

Definition 14. *Let $G = \langle S, R, A, \sigma, v, do, sees \rangle$ be a imperfect-information game given by a GDL-II description, where:*

S is a set of states, or nodes on the game tree;

R is a set of roles in the game;

A is a set of moves in the game, and $A(s,r) \subset A$ is a set of legal moves, for role $r \in R$ in state $s \in S$;

Σ is a set of percepts in the game, and $\sigma \in \Sigma$ is a percept, given by sees() function bellow;

$v : S \times R \rightarrow \mathbb{R}$ is the payoff function on termination;

$do : S \times A^{|R|} \rightarrow S$ is the successor function; and

$sees : S \times A^{|R|} \rightarrow \Sigma^{|R|}$ is the percept function.

A GDL-II game is composed by the set of roles, the set of moves for all players, the set of states, the set of all percepts in the game given by the new function sees. The payoffs function is on the terminal states and in order for the game continue from state to state one needs a successor function.

It is now necessary to define a move vector and a percept vector, that are necessary to describe the succession of a game [ST15]:

Definition 15. *Let G be a GDL-II game as in def. 14, then:*

[2] The path must be possible according to the game description

$a_r \in A(d,r)$ *is a move for role* $r \in R$ *in state* $d \in D$, *where* $D = S \setminus T$ *is the set of decision states and* T *is the set of termination states;*

$\vec{a} = \langle a_r, a_{\neg r} \rangle$ *is a move vector, one for each role;*

$\langle a_{\neg r}, a_r \rangle = \left\langle a_1 ... a_r ... a_{|R|} \right\rangle$ *is a move vector containing a specific move* a_r *for role* $r \in R$

$\sigma \in \Sigma$ *given by* $sees(d, \vec{a})$ *is a percept for actions* $\vec{a}$ *in state* $d \in D$;

$\vec{\sigma} = \left\langle \sigma_1 ... \sigma_{|R|} \right\rangle$ *is a percept vector; and*

$s = do(d, \vec{a})$ *and* $\vec{\sigma} = sees(d, \vec{a})$ *is the natural progression of the game.*

On this definition it is stated that a move vector is composed by an action of each player in the game. There are special move vectors where a specific action must be made for a player. The percept vector is given by the function sees and it is composed by all the percepts of all the players.

For a game to be played until one reaches the final state it is necessary to define how can one define a move selection policy, or strategy [ST15].

Definition 16. *Let G be a GDL-II game as in def 14, then:*

$\Pi : D \times R \to \phi(A)$ *be a move selection policy expressed as a probability distribution over the set A;*

$\vec{\pi} = \left\langle \vec{\pi_1}, ..., \vec{\pi_R} \right\rangle$ *is a tuple of move selection policies; and*

$play : S \times \Pi^{|R|} \to \phi(T)$ *is the playout of a game to termination according to the given move selection policies.*

This above definition refers that a move selection policy is described by a decision state (recall that it's the set of all the decision states minus the terminal states). A tuple of move selection policies is given by a policy chosen by each player and a playout function is the function that plays the game to a terminal state given a state and a move selection policies tuple.

In order to be able to evaluate the utility of a state it is necessary a evaluation function that uses the play function already defined [ST15].

Definition 17. *Let G be a GDL-II game as in def. 14, then:*

$\phi(s)$ *is the a priori probability that* $s = s_t$;

$eval : S \times \Pi^{|R|} \times R \times \mathbb{N} \to \mathbb{R}$ *is an evaluation function, where*

$eval(s, \vec{\pi}, r, n) = \frac{1}{n} \sum_1^n \phi(s) \times v(play(s, \vec{\pi}))$ *evaluates the node* $s \in S$ *using the policies in* $\vec{\pi}$, *and sample size n.*

By applying the evaluation function to a move vector $\langle \vec{a_{\neg r}}, a_{ri} \rangle$ *one gets* $eval(do(d, \langle \vec{a_{\neg r}}, a_{ri} \rangle), \vec{\pi}, r, n)$; *and*

$\arg\max_{a_{ri}} \left[eval(do(d, \langle \vec{a_{\neg r}}, a_{ri} \rangle), \vec{\pi}, r, n) \right]$ *is a selection process for making a move choice.*

On this definition, one can understand better the process of evaluation. The first thing to consider, is that by having uncertainty a player doesn't know the state where he is in, though, he must have beliefs of where he is — where is he more to likely to be. Then the eval function has four arguments, where it receives a state, a playout policy vector,

a role and the number of probes to send from that state for the mentioned role. Finally notice that by the eval function to a move vector we get the evaluation of that state and the selection process is therefore the maximization of the utilities.

Before a player can start evaluating the models, it is necessary to identify a state. This consists on a list of play messages, that represent the moves and percepts of all the players in the game. This is defined as follows [ST15]:

Definition 18. *Let G be a GDL-II game as in def. 14, then:*

$p \in P$ *is a play message from the set of all play messages in the game G;*

$p_r = \langle a_r, \sigma_r \rangle$ *is a play message for role* $r \in R$*;*

$\vec{p} = \left\langle p_1, ..., p_{|R|} \right\rangle$ *is a play message vector;*

$\xi : S \rightarrow P^*$ *is a function that extracts the path;*

$\xi(s) = \langle \vec{p_0}, ..., \vec{p_n} \rangle$ *is an ordered list of play messages from* $s_0 \in S$ *(the initial state) to s called the path; and*

$\xi_r(s) = \langle \vec{p_{r1}}, ..., \vec{p_{rn}} \rangle$ *is an ordered list of play messages received by role* $r \in R$

Definition 18 shows a play message for a specific player, that consists on his action and his percepts; a play message vector is a play message for each role. Since we want to identify a state, then we have the necessary tools to define a function that extracts a path of a state. That function returns an ordered list with the complete path of the state, from the initial state until the considered state. Then we can finally define a function that extracts the incomplete path of the state for a role.

The HP technique then constructs the models by grounding the unknown values. This is defined as following [ST15].

Definition 19. *Let G be the GDL-II game as in def. 14, then:*

$hp : P^n \rightarrow 2^{P^n}$ *is the Hyperplay function that completes the path in game G; and*

$h_r \in H_r = hp(\xi_r(s_t))$ *where* s_t *is the true state of the game and* H_r *is the information set of* $r \in R$*.*

The HP function, as aforementioned, returns a completed state or model. Note that, the new path might not be the same as the true path (it only needs to be a legal path). Finally, move selection policy is the one that maximizes the expected utility of actions for the completed states. It is defined as follows [ST15]:

Definition 20. *Let G be a GDL-II game as in def. 14, then:*

$\arg\max_{a_{ri}} \left[\sum_{j=1}^{|H_r|} eval(do(h_{rj}, \langle \vec{a_{\neg r}}, a_{ri} \rangle), \vec{mc}, r, n) \right]$ *is the move selection policy* π_{hp} *where* $\vec{mc}$ *is random.*

2.3.3.2 HyperPlay-II

The problem with the predecessor technique is that it doesn't have in consideration the moves where it's clearly better one can gain more information, since it assumes that it's in a perfect-information environment after completing the states. To solve this problem, the

HyperPlay for Imperfect-Information (HP-II) was developed. This new technique, extends the last one in the sense that it also completes the incomplete path of a state. Moreover, it includes a Incomplete Information Simulation (ISS) to reason with the incomplete information, by exploring the consequences of each action and uses that outcome to make a decision [ST15].

The HP-II can be defined using the definitions of the last subsubsection [ST15]:

Definition 21. *Let G be a GDL-II game as in def.**??**, then:*

$replay : S \times P^n \times \Pi \rightarrow S^{|R|}$ *is the replay of the game consistent with the path of a state;*

$replay(s_0, \xi_i(h_r), \vec{\pi})$ *is the replay of a game, as if* $h_r = s_t$*, and generates information for all roles, such that,* $hp(\xi_i(h_r)) \rightarrow H_i$ *where r is our role and i is any role;*

$ISS : S \times \Pi \times R \times \mathbb{N} \rightarrow \mathbb{R}$ *is the incomplete information simulation; where*

$ISS(h_r, \vec{\pi}_{hp}, r, n)$ *is an evaluation using an incomplete information simulation, and is defined as* $eval(replay(s_0, \xi_i(h_r), \vec{\pi}_{hp}), \vec{\pi}_{hp}, r, n)$.

This technique works by completing the path of a state in the information set of the player, then breaking that path into two different paths and complete those paths again. This makes that from one incomplete state path, we get two complete states paths even though they might not even be in the player's information set. Then the playout is done from the beginning of the game to the new path, having in account that path.

The HP-II technique can finally be defined as [ST15]:

Definition 22. *Let G be a GDL-II game as in def.* **??** *above, then:*

$a_{ri} \in A(h_r, r)$ *is a move to be evaluated by role r;*

$\langle \vec{a_{\neg r}}, a_{ri} \rangle$*, is the move vector containing the move* a_{ri}*;*

$ISS(do(h_r, \langle \vec{a_{\neg r}}, a_{ri} \rangle), \vec{\pi}_{hp}, r, n)$ *is an evaluation; and*

$\arg\max_{a_{ri}} \left[\sum_{j=1}^{|H_r|} ISS(do(h_{rj}, \langle \vec{a_{\neg r}}, a_{ri} \rangle), \vec{\pi}_{hp}, r, n) \right]$ *is the move selection policy* $\vec{\pi}_{hpii}$.

Notice the use of the HP policy on this new policy. This makes the HP-II a nested player. And, because it utilises a nested playout for evaluating move choices, it causes a significant increase in the number of states visited during the analysis [ST16]. However, since we are working with imperfect-information it is necessary to generate paths across other information sets (not only our player), so that one can evaluate what the other players might know. The use of this extended domain and the incomplete information reasoning makes the HP-II more suitable than HP to play in games with imperfect-information.

2.4 Epistemic General Game Playing

EGGP is the category concerned with systems that in human free environments that are able to play epistemic games. This category was created with the introduction of GDL-III.

2.4.1 GDL-III

The GDL-III [Thi17] has the same assumptions as his predecessor, all the described games are in imperfect-information. Though, the difference is that rules depend on the player's knowledge.

2.4.2 Syntax

In order to the players to understand that they need to know something a new keyword was added. This keyword comes in two different forms: [Gdl]

• **knows (R, P)** — player R knows the proposition P in the current state;

• **knows (P)** — proposition P is common knowledge;

This keyword can only be used in the body of the GDL-III rules since players must be able to reason about their own knowledge. The knowledge cannot be defined itself in the rules. There is also other restrictions, such as: there must be a total ordering on all the predicates symbols P so that circular definitions are not allowed.

2.4.3 Semantics

The semantics of GDL-III are characterized by the semantics of GDL-II as pre semantics, followed by the semantics of the knowledge rules. Formally, the semantics of GDL-III are presented in Definition 23, which is an adaptation from [Thi17]. Specifically, the derivable $(role\ r)$ instances represent the players; the state s_0 is represented by the instances $(init\ f)$. All the other keywords use a state (S,K), which S is encoded by using the keyword $true$, and K the keyword $knows$ — the defined in Definition 23. Let $S = \{f_1,\ldots,f_n\}$ be a finite set of ground terms over the signature of G. Moreover, let $S^{true} = \{(true\ f_1)\ldots(true\ f_n)\}$ be an extension of G by the n facts. The legal moves m of player r in position S,K are defined by all instances of $(legal\ r\ m)$ that follow from $G \cup S^{true} \cup K$. Likewise for the $terminal$ and $(goal\ r\ m)$ clauses, that define the terminal states and the goal values relative to the given state S,K. Finally, let M denote a joint move, where players $r_1,\ldots,\ r_k$ make moves $m_1,\ldots,m_k$ and use $M^{does} = \{(does\ r_1\ m_1)\ldots(does\ r_k\ m_k)\}$. The update position is defined by all instances of $(next\ f)$ that follow from $G \cup M^{does} \cup S^{true} \cup K$, and the percepts that the players receive after joint move M, in a given position S,K are all the derivable instances of $(sees\ r\ p)$.

Definition 23. *The semantics of a valid GDL-III game description G is given by*

- $R = \{r : G \models (role\ r)\}$
- $s_0 = \{f : G \models (init\ f)\}$
- $t = \left\{(S,K) : G \cup S^{true} \cup K \models terminal\right\}$

- $l = \left\{(r, m, S, K) : G \cup S^{true} \cup K \models (legal\ r\ m)\right\}$
- $u(M, S, K) = \left\{f : G \cup M^{does} \cup S^{true} \cup K \models (next\ f)\right\}$
- $I = \left\{(r, M, S, K, p) : G \cup M^{does} \cup S^{true} \cup K \models (sees\ r\ p)\right\}$
- $g = \left\{(r, v, S, K) : G \cup S^{true} \cup K \models (goal\ r\ v)\right\}$

Definition 32, copied from [Thi17], shows how the knowledge set K is defined by using indistinguishable legal play sequences, notion that comes from GDL-II. But first, a legal play sequence is a sequence of joint moves $M_1, \ldots M_n$, where $M_i(r)$ is a move — one for each role — at step i, such that there are states $s_0, s_1, \ldots, s_n$ with: $(r, M_i(r), s_{i-1}) \in l$, for all $r \in R$; and, $s_i = u(M_i, s_{i-1})$. We can see at this point how the GDL-II acts as pre semantics of GDL-III, since s_{i-1} represents just the state without the knowledge set K, also valid in a GDL-II description. Moreover, two legal play sequences δ, δ' are indistinguishable ($\delta \sim_r \delta'$) for role $r \in R$ if for all $i \in \{1, \ldots, n\}$: $M_i(r) = M'_i(r)$ and $\{p : (r, M_i, s_{i-1}, p)\} = \{p' : (r, M_i, s_{i-1}, p')\}$. For common knowledge, is defined using a notion of reflexive transitive closure $\sim^+$ of a given family of indistinguishable relations $\sim_r$ (one for every role $r \in R$). Formally [Thi17], $\sim^+$ is the smallest relation such that for all $\delta, \delta', \delta''$:

- $\delta \sim^+ \delta'$
- if $\delta \sim^+ \delta'$ and $\delta' \sim_r \delta''$ for some $r \in R$ then $\delta \sim^+ \delta''$.

Definition 24. *Let G be a game description along with all the sets and relations it describes according to Definition 23.*

- *The play sequence of length 0, denoted by ϵ, is legal and satisfies $\epsilon \sim_r \epsilon$, for all $r \in R$. It results in state s_0 and knowledge state K_ϵ as the smallest set that satisfies*

$$K_\epsilon = \left\{(knows\ r\ p) :\ r \in R, G \cup s_0^{true} \cup K_\epsilon \models p\right\} \cup \left\{(knows\ p) :\ G \cup s_0^{true} \cup K_\epsilon \models p\right\}$$

- *For the inductive definition, let δ be a legal play sequence of length $n \geq 0$ resulting in (s_n, K_n).*

  Sequence δ followed by M, written δM, is a legal play sequence of length $n+1$ if $(M(r), s_n, K_n) \in l$ for all $r \in R$. It results in the state $s_{\delta M} = u(M, s_n, K_n)$ and, as the knowledge state, the smallest $K_{\delta M}$ that satisfies:

$$\begin{aligned} K_{\delta M} = &\left\{(knows\ r\ p) :\ G \cup s^{true}_{\delta' M'} \cup K_{\delta M} \models p\ for\ all\ \delta' M' \sim_r \delta M\right\} \\ &\cup \left\{(knows\ p) :\ G \cup s^{true}_{\delta' M'} \cup K_{\delta M} \models p\ for\ all\ \delta' M' \sim^+ \delta M\right\} \end{aligned}$$

Definition 32 can be understood as follows: relations $\delta M \sim_r \delta' M'$ holds if for role $r \in R$: $\delta \sim_r \delta'$ the legal play sequences are indistinguishable, and after applying the same move $M(r) = M'(r)$, it gets the same percepts $\{p : (r, M, s_n, K_n, p) \in I\} = \{p' : (r, M, s'_n, K'_n, p') \in I\}$.

Then the (*knows r p*) is defined by using this relation to find all the states that are indistinguishable to player *r* and in all of them *p* needs to be derived. For common knowledge (*knows p*) applies the same logic, but with the transitive closure relation $\sim^+$ to find the states.

2.5 Epistemic Logic

Epistemic logic is the logic that reasons about knowledge and belief. It recognizes expressions like *knows that P* or *believes that P* to have properties that are susceptible to formal study. Is exclusively concerned about propositional knowledge. In other words, in knowing that something is true. The semantics of Epistemic logic is very similar to the semantics of GDL-III. Therefore, problems already defined and solved using in Epistemic Logic can be defined in as GDL-III game, such as the Russian Cards Problems, and Muddy Children.

The applications of Epistemic Logic range from computer science e.g. robotics or network security to economics in the study of interactions on different environments.

2.5.1 Syntax

The Epistemic Logic language is presented in the form BNF[VD+07](Adapted):

Definition 25. *Let P be the set of propositions, and $p \in P$ be a proposition, A be the set of all agents, $a \in A$ be an agent and $G \subseteq A$ be a group of agents, then:*

$$\varepsilon ::= p \mid \neg\varepsilon \mid (\varepsilon \wedge \varepsilon) \mid K_a\varepsilon \mid C_G\varepsilon$$

A formula can be build using propositions; formulas with negation; conjunction with other formulas; the knowledge operator for an agent and the group knowledge operator. Finally another operator can be defined using the previous already defined: $\widehat{K}_a\varepsilon := \neg K_a \neg\varepsilon$ that simbolizes that agent a still thinks that ε is possible.

2.5.2 Semantics

The semantics of Epistemic Logic uses a special case of Kripke models. There are two important notions: The notion of *state* and the *indistinguishability*. The following is the definition of a Kripke model [VD+07].

Definition 26. *Let P be the set of propositions, and $p \in P$ be a proposition, A be the finite set of all agents, $a \in A$ be an agent and $G \subseteq A$ be a group of agents, then a Kripke model is a structure $M = \left\langle S, R^A, V^P \right\rangle$, where:*

S is an non-empty set of states;

R^A is a function yielding an accessibility relation $R_a \subseteq S \times S$ for every a;

$V^P : P \to \rho(S)$. V(p) is the subset of the domain where p is true.

A model can be defined using a set of states, a function that connects two states (accessibility relation) for every agent and a function that verifies if a proposition is true. If all the relations in R_a are equivalent relations we write $M = \langle S, \sim, V \rangle$. And M is an epistemic model[3] [VD+07]. An epistemic state is defined as a pair (M, s), where M is an epistemic model and $s \in S$ is a state of on that model. It is necessary to define when a formula is true in the model. We say use the notaion $(M, s) \models p$ to describe that the epistemic state satisfies p or that p is true in the epistemic state (M, s).

The following is the definition of the semantics of Epistemic Logic [VD+07](Adapted).

Definition 27. *Let (M, s) be an epistemic state, and ε be a formula, then:*

$(M, s) \models p$ *iff* $s \in V(p)$

$(M, s) \models \neg\varepsilon$ *iff not* $(M, s) \models \varepsilon$

$(M, s) \models (\varepsilon \wedge \alpha)$ *iff* $(M, s) \models \varepsilon$ *and* $(M, s) \models \alpha$

$(M, s) \models K_a\varepsilon$ *iff for all t:* $R_a st$ *implies* $(M, t) \models \varepsilon$

$(M, s) \models \widehat{K}_a\varepsilon$ *iff there is a t such that* $R_a st$ *implies* $(M, t) \models \varepsilon$

$(M, s) \models C_G\varepsilon$ *iff for all t:* $R_G st$ *implies* $(M, t) \models \varepsilon$, *where* $R_G := (\bigcup_{a \in G} R_a)^+$

This definition states that in order for a proposition to be satisfied in a state, it needs to be true; for a negation of a formula to be satisfied, the state must not satisfy the formula itself; for a conjunction of formulas to be satisfied in the state both of the formulas have to be satisfied in that state; the state satisfies the agent a to know a formula ε if for every accessibility relation from the current state to another state ε is satisfiable in that other state; the state satisfies the possibility of agent a to know the formula ε if there is at least one accessibility relation from the current state to another state and ε is satisfiable in that other state; Common knowledge is like the knowledge operator, but instead of the accessibility relation be only for the agent a, is by all agents in the group, and so, it needs to be satisfiable in all states where there is possible to arrive to.

Let's see with a simple example of what kind of reasoning can be achieved using epistemic logic. Consider a world where there are only two agents: a and b; each agent has a coin, that they flip, see their own result. For clear representation let's assume that agent a only interested in getting heads on his coin and agent b is interested in getting tails on his coin. This model M representation can be found in Figure 2.11. This means that $M = \langle S, \sim, V \rangle$ and every line one can see between the vertexes means that that agent cannot distinguish between those states, i.e. agent a can't distinguish between (h, t) and $(h, \neg t)$.

To be able to reason about their knowledge let's assume that they don't tell anything to each others and that they can't see the other's coin. As you can see in the Figure 2.11, one can argue: $M \models K_a(h \vee \neg h)$, this means that agent a knows his coin. It's verifiable by looking at all possible states whenever h is true, so is it in the state given by equivalence relation. Likewise, when h is false. One can also say $(M, (\neg h, t)) \models K_b t$, in the state $(\neg h, t)$ agent b knows t since between that state and (h, t) that's the only thing b can reason.

[3] Don't confund with the model, from the model-sampling already presented.

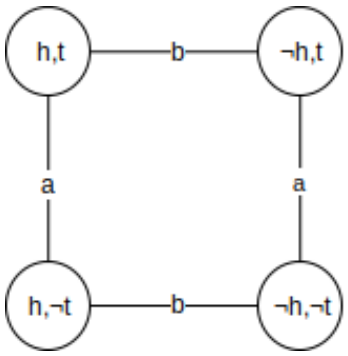


Figure 2.11: Epistemic Model M

Moreover, it is possible to reason about nested knowledge, e.g. $(M,(h,\neg t)) \models K_a K_b t$. At $(h,\neg t)$ one can go to $(h,\neg t)$ through the indistinguishability relation of a and in that state is true that the agent b knows t, so $K_a K_b t$.

Finally, common knowledge it must be achievable in all the states of the model i.e. $M \models C_{ab}(K_a h \vee K_a \neg h)$. It is common knowledge that agent a knows his card. This can be verified by starting at the state (h,t) and verifying that $K_a h$ holds, then proceed to state $(\neg h,t)$ through the accessibility relation of b and verifying that $K_a \neg h$ holds and then to $(\neg h,\neg t)$ with the $\sim_a$, where $K_a \neg h$ holds, and finally to $(h,\neg t)$ with $\sim_b$ and verifying that $K_a h$ holds. As a remark we can say that common knowledge also be interpreted as 'any fool knows p'. And that it must be true in all the states that we can arrive at by at path of links belonging to the agents considered. Hence, defined by the transitive closure.

The principle of indistinguishability important to the way the knowledge is verified, since it is necessary to verify that on all accessible states.

2.6 Answer Set Programing

ASP is a form of declarative programming oriented towards difficult search problems [Geb+12]. Problems are described as a collection of logic clauses like GDL, and solved by finding the minimal model for the specification. To demonstrate a problem written in ASP we present the following example, that can be found in [Gra]:

```
1  % Default
2  #const n = 3.
3
4  % Generate nodes
5  node(1..6).
6
7  % Generate (Directed) Edges
8  edge(1,(2;3;4)).  edge(2,(4;5;6)).  edge(3,(1;4;5)).
9  edge(4,(1;2)).    edge(5,(3;4;6)).  edge(6,(2;3;5)).
10
11 % Generate Colors - one for each node.
12 { color(X,1..n) } = 1 :- node(X).
```

```
13
14  % Remove the colors that have the same color in adjacent nodes
15  :- edge(X,Y), color(X,C), color(Y,C).
```

In line 2 we define the number of colors to use; in the line 5 the number of nodes that our graph will have; line 8 and 9 we build the connections between the nodes; line 12 generates all the colors in the nodes; and in line 15 we remove all the colors of the model so that in each vertex at the end of same the edge, they share the same color. An answer set of this example would be: color(2,2) color(1,3) color(3,2) color(4,1) color(5,3) color(6,1). This solution can be found by running an ASP solver such as clingo [Cli].

Line 15 shows a special type of input formula: clauses without head. They are called constraints, and their purpose is to exclude any model that has a solution where all those clauses are true. This is one characteristic where ASP is differ from GDL.

GDL and ASP are very similar, which allows any GDL description to be converted to ASP. Which makes ASP an optimal tool to play GDL single-player games. Moreover, the semantics of GDL-III can be described in ASP, as shown in Listing 2.6, adapted from [Thi16]. The conversion of a GDL-III description to ASP uses two additional arguments in state dependent predicates:

- $seq(S)$ to identify all legal play sequences;
- $time(T)$ to identify different points in time.

Moreover, it is necessary to replace the predicate $init(P)$ by $true(P,S,0)$ and $next(P)$ by $true(P,S,T+1)$.

```
1   knows_the_number(R, S, T):- seq(S), time(T), role(R), knows(R, num(N), S, T).
2
3   distinguishable(R, S1, S2, N) :- time(N), T<N,
4     does(R, M1, S1, T), does(R,M2,S2,T), M1!=M2.
5   distinguishable(R, S1, S2, N) :- time(T), T<=N,
6     sees(R, P, S1, T), not sees(R, P, S2, T).
7
8   % Indistinguishability between legal play sequences
9   ind(R, S1, S2, N) :- role(R), seq(S1), seq(S2), time(N),
10    not distinguishable(R, S1, S2, N).
11
12  % Transitive Closure application
13  indtrans(S1, S1, N) :- seq(S1), time(N).
14  indtrans(S1, S3, N) :- ind(R, S1, S2, N), indtrans(S2, S3, N).
15
16  % Personal Knowledge
17  knows(R, num(N),S,T) :- num(N , S, T), not np(R, N, S, T).
18  np(R, N ,S, T) :- ind(R, S, S1, T), not num(N, S1, T).
19
20  % Common Knowledge generalized
21  knows(p(x), S, T) :- p(x, S, T), not np(x, S, T).
22  np(x, S, T) :- indtrans(S, S1, T), not p(x, S1, T).
```

Listing 2.6 is a snapshot of the Epistemic Number Guessing, where the remainder of the GDL-III description can be found in Appendix B. Line 1 is the extension of the knowledge predicate used in the goal definition by the sequences and time; Line 3-6 represent the distinguishability between legal play sequences in different time points. We can see that two sequences are distinguishable for player R is he doesn't make the same move, or he sees different percepts at some point in time; Line 9-10 show the definition of indistinguishable legal play sequences; Line 13-14 represent the definition of the transitive closure; Line 17-18 show the semantics of personal knowledge for the Epistemic Number Guessing game. We would have that R knows the $num(N)$ if sequence S is the actual play sequence at time T, $num(N)$ holds and predicate np does not, meaning that is not the case that there is another sequence $S1$ that R cannot distinguish from S where $num(N)$ does not hold; and, finally, Line 21-22 represent a generalized common knowledge rule — since this game does not have a common knowledge rule, because it is a single player game— uses the same logic, using instead the transitive closure predicate to find other legal play sequences $S1$.

This solution, however, as explained in [Thi17], it is very inefficient, since it explicitly requires maintaining the set of relevant legal play sequences. Which is practically viable for short epistemic games or puzzles. But it does not scale for bigger games. In this aspect, a model-sampling is better since it does not require to maintain the set of relevant legal play sequences.

Chapter 3

An Epistemic Game Master

This chapter showcases some of the implementation details of the developed GM for GDL-III. We provide the source code available at: https://gitlab.com/Demofrager/GDL-III_GameMaster. Throughout the chapter, we reaffirm our problem; we explain generally our implemented solution; and then, we detail how the main modules of our GM were implemented and the advantages of our approach over alternatives—as well as some of their limitations. In particular, we briefly introduce the architecture of our implementation; we expose our server implementation that controls the state of the game; we detail how players' knowledge is verified; explain the samplers that were developed; we also introduce a new concept used in this dissertation;

3.1 Problem Description

The introduction of GDL-III raises two problems. This new version extends an general game playing system to allow players to play epistemic games — EGGP. Therefore, the previous iteration of the GM does not have this ability. And secondly, the language forces players to have to reason about what they know. According to GDL-III defined semantics, in order to calculate what a certain player knows, one needs to evaluate all the other states that that player cannot distinguish between.

The GM in all GDL-III extensions always acts under perfect-information, since the initial state is known and receives the players moves at every round. However, by forcing players to reason about their own knowledge, it also forces the master to reason about their knowledge as well. Therefore it is also necessary to consider the state space of the game, in order to be able to consider the states that players cannot distinguish between thus calculating what each one of them knows. Our objective is to make sure that players can play the game e.g. by making sure players can only make certain moves if they know

something, or that the game ends if a certain player knows something. Nonetheless, if one has to consider the state space of the game, then it would have to consider state spaces such as in the Epistemic Number Guessing Game with 9.22×10^{18} possible states with 128 numbers over 9 turns. In this game, for example, the first move is to assign a number that the player needs to know. And, in the following turns, the player asks if the number that he needs to know is smaller than the goal. Such simple game can have a large state space that is necessary to consider.

3.1.1 Requirements

The requirements of the developing GM for GDL-III are:

- Implementation of the GDL-III semantics as defined;
- Implementation of model-sampling techniques to generate possible game states;
- Cross platform GM capable of being deployed in most commonly used operating systems;

3.2 Solution

We propose the implementation of a GM that uses model-sampling to sample the possible state space of the game, and infer what the players can know while they are playing the game. For a better understanding of our solution consider Figure 3.1. In this example we have a very simple epistemic game tree, that can be interpreted in the following way:

- Turn 0: Player random can select between two numbers: 1;2, and Player $R1$ can only do one operation — no operation: n.
- Turn 1: Player random can only do no operation: n, and Player $R1$ asks if the number selected by random is either 1 or 2.
- Turn 2: The game ends. Player $R1$ wins if we knows the number selected by random. Namely,
  - Green and grey states: $K_{R1}1$ holds;
  - Yellow and blue states: $K_{R1}2$ holds.

The colours in the game tree are only to identify the epistemic state, and not in any way, the knowledge in those states.

This game is very simple, since in all states at turn 2 player $R1$ will always have knowledge of which number was set, therefore he will always win. The objective of using this game as an example is to showcase how our solution was developed. In this game tree, $\langle Rand; R1 \rangle$ represents a legal joint move of player *Random* and $R1$ respectively, and

an indistinguishability relation for player $R1$ at Turn 1. Moreover, the percepts given to Player $R1$ are:

- Turn 0: Nothing;
- Turn 1: Nothing;
- Turn 2: *yes/no* depending on whether the number Player $R1$ asked is the same as random set before.

Our approach to the problem was to model epistemic states at a given turn, since (unlike this example) to consider all epistemic states at a given turn is, in most games, impossible.

The middle of Figure 3.1 showcases our modeling (or sampling) process of possible epistemic game states. We start with a container, that we call Bag (or B) of fixed size N. This sampling, at turn 0, starts with N copies of the initial epistemic state, that we call models. Moreover, at turn 0, one joint move is selected — depending on the used sampler — to update each model in the Bag. We don't show the chosen moves when updating the Bag to keep the image clean, however, one can see which joint move was chosen in the tree on the right.

The sampling process is independent of the way the game proceeds[1] (on the left) if the Random Sampler is used, or partially independent, if used the Perspective Shifting Sampler.

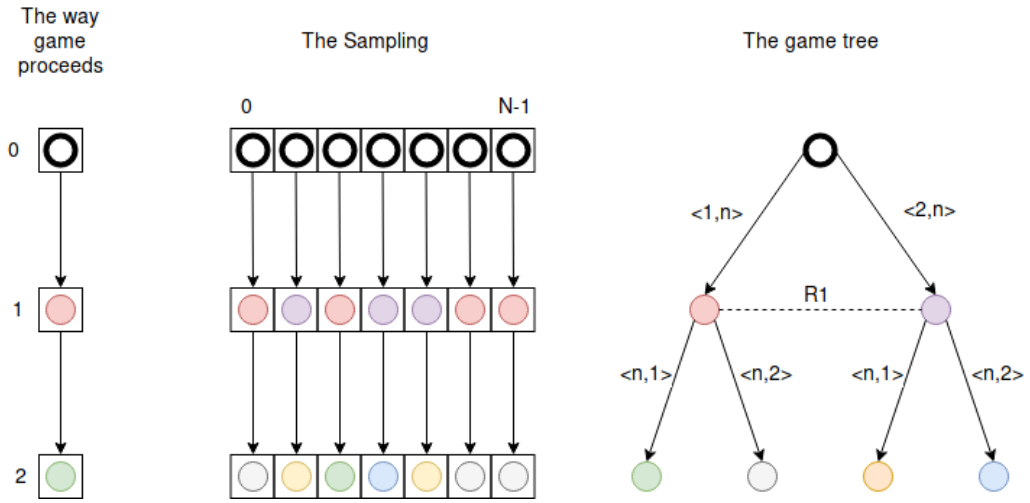


Figure 3.1: Sampling Approach

In order for the GM to know what each player knows (on the left), we use the information that we have about the players in the Bag, to calculate the knowledge in the state on the left — in the same round. Notice that, the process of calculating what each player knows is shown in Figure 3.2, and not here in Figure 3.1, for the sake of clearance.

[1] Also known as the game instance

Figure 3.2 shows our approach regarding the implementation of the GDL-III semantics. As defined in the formalization of GDL-III, we also follow a two step process: a pre-semantics stage, where the all game states do not have the players' knowledge — it is not an epistemic state (with no colours); and a post semantic step where the states have knowledge — it is an epistemic state (with colours). Note that, for instance, at turn 1, in the post-semantics stage, the red state and the purple state have the same knowledge — no knowledge— however, they are also epistemic states.

In the pre-semantics stage the players' indistinguishability relations are built, by comparing the partial sequence of moves that belongs to each player in each state. For instance, in Figure 3.2 at turn 0, in the initial state of the game there are no moves. Therefore all states have the same initial knowledge on the right — no knowledge. At turn 1, after random has made it's move — in the pre semantics, since the same move is made for player $R1$: n, and it does not receive any percept at turn 1 (in either state), and they where indistinguishable for $R1$ beforehand, then they are also indistinguishable at turn 1.

This means that in the post-semantics stage, at turn 2, we will have two epistemic states, where player $R1$ has no knowledge, but we represent them with different colours, since in one, random has chosen 1 to be the number set (red), and in the other was 2 (purple). At turn 2, if players have done the same joint legal move in each state $< n, 1 >$, then player $R1$ will receive different percepts. In the state that was red he will receive *yes*, and in the state that was purple will receive *no*. Therefore they are both distinguishable for player $R1$. At the pre-semantics stage, at turn 2 — as shown in the Figure 3.2 by no relation between the states — when applying the semantics we will have that the red state will become the green state because every state is indistinguishable between itself. The green state will have $K_{R1}1$ since the number set 1 holds in all the indistinguishable states[2], and the state that was purple will update to yellow. We can use the previous explanation to deduce that $K_{R1}2$ will hold.

In order to be able to develop this two step semantics, our Bag was developed to not only to save the state that it contains, but also the sequence of joint moves and percepts that led up to it, in other words, that identify the state in the model.

During the development of our solution we discussed the effects of adding sampling to the application of the GDL-III semantics. This led to the definition of a term that we call *plausibility*. This term can be interpreted as: at a certain point, because we are sampling game states, we might not have all the necessary states to lead to the right conclusions about players' knowledge. E.g. Imagine that in Figure 3.2, from turn 0 to turn 1, the same joint move was selected $< 1, n >$, then the same indistinguishability relation between the states would exist at step 1. However, instead of having two red states — as we should have — we would have two green states: both with $K_{R1}1$. Since the states are the same, and 1 was the number set in both of them, then by GDL-III definition — $K_{R1}1$ holds.

[2]His set of indistinguishable states is composed by itself

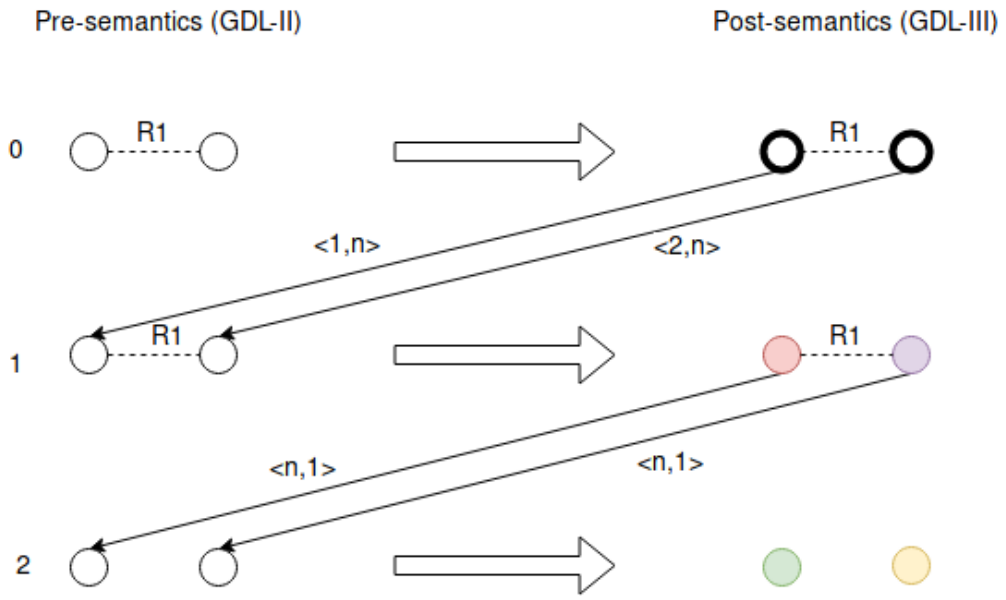


Figure 3.2: Two Stage Semantics Approach

Again, this is due to the sampling, since by using this approach we might not have the state where the number set was 2. Note that this is a very simplistic game, where we only wish to convey the basic understanding of what was developed in this dissertation. Throughout the remainder of this Chapter, we justify some of our decision making by the fact that we want to minimize *plausability* from happening, specially in the way we replace states that are no longer necessary to maintain due to the game instance.

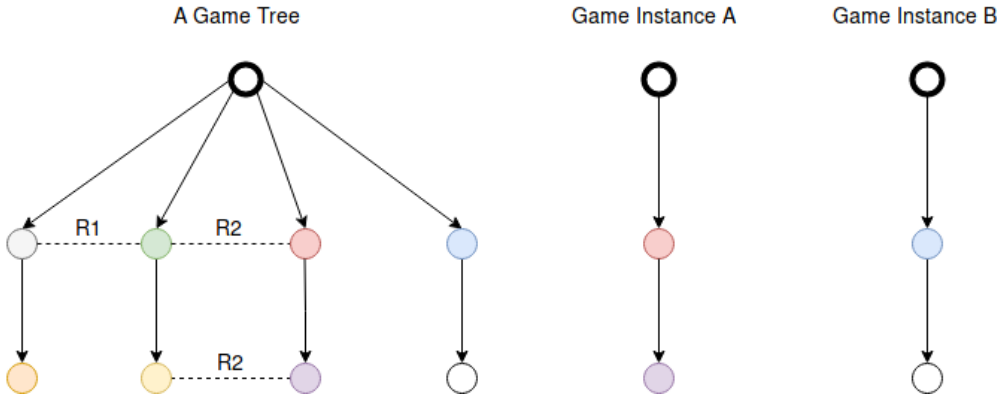


Figure 3.3: An Abstract Game and two game instances

Since we are sampling an epistemic game, it is not necessary to keep all states in our Bag[3], since the players play the game, and move down the game tree. Figure 3.3 shows an abstract game tree with two different Game Instances. In this example, the moves do not need to be considered since we only wish to convey the process over the indistinguishability relations at a certain turn. E.g. At turn 1, if the game proceeds

[3]Game states inside models therefore in the Bag

according to the Game Instance A, which states would be useful to consider having in the Bag? Would epistemic state blue be useful for knowledge calculation at turn 1? Similarly, if the Game Instance were B, at turn 1, would we need to consider the red, green or grey states? The answer is no because every player can distinguish between those states, according to the game instance, respectively.

Moreover, if at turn 1 in the game instance A, would it be better to have in the Bag both states green and grey, since common knowledge can be interpreted as jumping from a state to another if there is a indistinguishability relation for some player — within the game instance, i.e. would it be useful in game instance A, to consider the blue state and any other state eventually connected to it? The answer is also no.

However, still in game instance A at turn 2, is it really necessary to consider the orange state presented in the game tree? The answer is: it depends. Games where legal moves and percepts depend on knowledge, or dynamic epistemic games, it is necessary to consider them in our sampler, though not for knowledge calculation. We detail this further in section 3.7.

3.3 Evolving to an Epistemic Architecture

From the starting point we had either GM for GDL-II with the source code written in Prolog, or another for GDL in Java. Both of them were good starting points. However, since we have more experience with Java, and that by extending from GDL to GDL-II before extending to GDL-III would provide us with detailed knowledge of the starting solution. We chose to continue with the GM in Java.

The function of the GM is to allow players to play a GDL-III game ensuring that they have enough information to deduct their knowledge correctly. Since the state space is too large to be considered, we chose to implement an approximative solution to sample it.

Figure 3.4 shows the modules of our developed solution. As in Figure 2.6 in Chapter 2 arrows represent interactions, and the State Machine and Interpreter are implicit in the Server. Our developed GM is composed by two new modules:

Sampler - where possible states of the game are modeled;

Knowledge - where GDL-III semantics are implemented as in [Thi17];

By looking at Figure 3.4 we can verify that both new modules interact with each other. This happens because we are sampling an epistemic game, where game state models need be updated with knowledge. And we also need those models to calculate the *that* knowledge. We go into a detailed explanation about how we managed to accomplish it in the following sections.

Notice that since we started the development of the new GM with the starting solution for GGP-I, before developing a solution for EGGP, it was necessary to implement the remaining GGP-II features. This means we created *sees* function in the State Machine

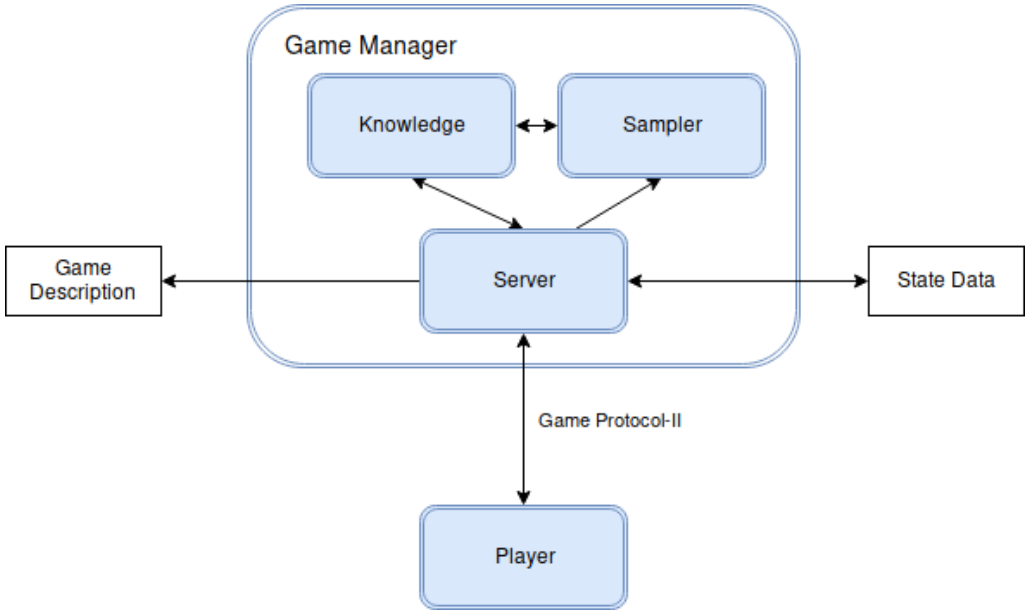


Figure 3.4: Epistemic Game Manager Modules

and the altered the protocol in the Server module. Due to similarities between the *sees*, *legal* and *next* functions it was simply a copy of what was available in the package. For the new protocol it was necessary to change the communication string, since the package already provides the encapsulation over HTTP/TCP/IP.

We chose to leave the communication protocol for EGGP as in GGP-II, because there is no added uncertainty. The difference is that now the players are forced to deduce what they know. Therefore, a GGP-II player can connect to a EGGP GM to play an epistemic game, although their ability to play the game can be questioned.

More details about the architecture of our solution such as the main java classes and their organization can be found in Appendix A.

3.4 Server

Our GM honors the GDL-III formalization where GDL-II acts as pre semantics. This means states are conceptually like GDL-II and updated with the players' knowledge, according with the semantics of indistinguishable play sequences, leading to an epistemic state.

The Server is the module that manages everything. We will present how the Server runs to show the way modules are interconnected to achieve our goals. However, before we proceed, and as we will use this in the following sections, we present some auxiliar definitions.

Definition 28 represents a GDL-III game. We define this function to help present the algorithms of our developed solution. In this definition there is a set of states, or GDL-II states; a set of knowledge conditions given by the GDL-III description; a set of epistemic

states, which is the set of GDL-II states together with the knowledge terms; a set of roles; a set of actions, where at each epistemic state players have legal moves; a set of percepts that are given by the sees function, that takes an epistemic state and a move for each player, or, joint move, and returns the percepts that the players will see in the following round. It also has a goal function that states the payoff of an epistemic state; and the next function that given an epistemic state and the players' joint moves returns the next state.

Definition 28. *Let $G = \{S, K, E, R, A, \Sigma, goal, next, sees\}$ be an incomplete information game with introspection given by a GDL-III game description:*

- *S is the set of all states;*
- *K is the set of k knowledge terms in G, where each k is:*
  - *$K_r p$ for **(knows r p)**, or;*
  - *Cp for **(knows p)**.*
- *$E = (S, K)$ is the set of all epistemic states;*
- *R is the immutable set of roles in the game;*
- *A is the set of moves in the game, and $A(es, r) \subseteq A$ is a set of legal moves, for role $r \in R$ in the epistemic state $es \in E$;*
- *Σ is a set of percepts in the game, and $p \in \Sigma$ is a percept given by the sees function bellow;*
- *$goal : E \times R \mapsto \mathbb{R}$ is the payoff function;*
- *$next : E \times A^{|R|} \mapsto S$ is the successor function;*
- *$sees : E \times A^{|R|} \mapsto \Sigma^{|R|}$ is the percept function.*

In order to implement the knowledge semantics we need to define a legal play sequence. Therefore we need to define moves and percepts available in the game. Definition 29 shows a legal move for a player r in a epistemic state where he makes a move; a joint move, that is a immutable vector with each player's move; a player's percepts; their joint percepts; and the game progression.

Definition 29. *Let G be a GDL-III game defined previously, then:*

- *$a_r \in A(d, r)$ is a move for role $r \in R$ in the decision state $d \in E \backslash T$ where $T \subseteq E$ is the set of terminal epistemic states;*
- *$\vec{a} = \langle a_1, \ldots, a_{|R|} \rangle$ is an immutable move vector, one for each role;*
- *$\sigma \in \Sigma$ given by $sees(d, \vec{a})$ is a percept for actions $\vec{a}$ in the state $d \in E \backslash T$;*
- *$\vec{\sigma} = \langle \sigma_1, \ldots, \sigma_{|R|} \rangle$ is an immutable percept vector;*

- $s = next(d, \vec{a})$ *and* $\vec{\sigma} = sees(d, \vec{a})$ *is the natural progression of a game;*

Finally, we can define a legal play sequence, or in other words, the identification of a state. In any game given by a GDL description, a state can be uniquely identified by the legal play sequence that led to it. Definition 30 shows a legal play sequence and its update. Although the percepts can always be determined by the initial state and the sequence of legal joint moves, we decided to save in a legal play sequence to avoid re computation of the percepts every time that they are necessary.

Definition 30. *Let G be a GDL-III game defined previously, then a legal play sequence P is given by:*

- $P = \{\langle \vec{a_0}, \ldots, \vec{a_t} \rangle, \langle \vec{\sigma_0}, \ldots, \vec{\sigma_t} \rangle\}$ *is a legal play sequence at t turn;*
- $P \oplus \vec{a}, \vec{\sigma} = \{\langle \vec{a_0}, \ldots, \vec{a_t}, \vec{a} \rangle, \langle \vec{\sigma_0}, \ldots, \vec{\sigma_t}, \vec{\sigma} \rangle\}$ *is P's update with* $\vec{a}$ *and* $\vec{\sigma}$.

The GM concept for any general game playing system is that of always control one specific state of the game. The state that players play in, even though they might be uncertain. We refer to this specific state as the true state of the game. This, true state of the game, in a pre semantics stage is a GDL-II state, and after applying the semantics is a GDL-III state, or an epistemic state. The contents of this state similarly with previous extensions, is what is *true*, adding now what is known — *knows*.

Algorithm 1 presents the most important part of the Server. This algorithm can be understood as follows: line 2 represents the Server reading the game description and setting up the possible knowledge terms K of the game; line 3 and 4 represent the players' joint moves and their percepts' initialization; line 5 stands for the initialization of the legal play sequence of the true state of the game ($\mathcal{P} = P$ of def 30); line 7 is the creation of the true initial epistemic state of the game, by applying the semantics of GDL-III[4]; line 8 is the initialization of our sampling of the form $B = \{M_1, \ldots, M_n\}$, where B is a bag with n models and $M = \{P, s\}$ with P as in def 30, and $s \in E$ as an epistemic state.

The Server proceeds always in the same way if the game has not ended. Line 10 represents communication with each player by sending them their previous move and their percepts, and filtering their answer — any move that is submitted by a player that is not legal in es_{true} preserved by the Server is immediately disregarded and is randomly selected a move for that player — line 11 is the interpretation of the game description to retrieve the percepts for the players moves in the current epistemic state; line 12 is the update of the epistemic state to a new state, according to the players moves[5]; line 15 is the update of the bag and their indistinguishably relations between the models; and line 16, similarly to line 7 is the application of GDL-III semantics. Finally, line 18 is the communication with players when the game is over, by sending each one, their last move and percepts.

[4]Since the initial state is unique and known for all the players one just needs to add to the state all the knowledge conditions that are valid in it

[5]The epistemic state update is done in an intermediate manner, that goes from line 12 to 16.

Algorithm 1 Game Master state control

1: **function** RUN
2: $K :=$ setupTargetKnowledge(GDL-III Description)
3: $\vec{m} := \emptyset$
4: $\vec{p} := \emptyset$
5: $\mathcal{P} := \{\langle \vec{m} \rangle, \langle \vec{p} \rangle\}$
6: $t := 0$
7: $es_{true} :=$ s_0 + getTurnKnowledge(s_0, K, t)
8: $B := \{\{\langle \vec{m} \rangle, \langle \vec{p} \rangle, es_{true}\}, \ldots, \{\{\langle \vec{m} \rangle, \langle \vec{p} \rangle\}, es_{true}\}\}$
9: **repeat**
10: $\vec{m} :=$ sendPlayRequests$(\vec{m}, \vec{p}, t)$
11: $\vec{p} :=$ $sees(es_{true}, \vec{m})$
12: $s_{true} :=$ $next(es_{true}, \vec{m})$
13: $\mathcal{P} \oplus \vec{m}, \vec{p}$
14: $t := t + 1$
15: updateTurnKnowledge$(\mathcal{P}, B, \beta, t)$
16: $es_{true} :=$ s_{true} + getTurnKnowledge(s_{true}, K, t)
17: **until** es_{true} is terminal
18: sendStopRequests$(\vec{m}, \vec{p}, t)$
19: **end function**

To develop algorithm 1, we had to implement the *setupTargetKnowledge* method that identifies the new GDL-III - *(knows R Pr)* and *(knows Pr)* - terms and converts them to $K_R Pr$ and CPr. However, GDL allows the use of variables in the description, hence it was necessary to build a specific grounder[6] for those knowledge terms, since they do not appear at any head of a rule. Since Pr can either be a proposition and as such it is grounded, or a relation where it might not be grounded. Nonetheless, being a relation means that it will appear in the head of an auxiliar rule. Therefore it can be grounded. By using an auxiliar tool provided in the GGP-Base package that allows us to find the domain the variables of that relation, we then get our grounded knowledge terms.

We chose to implement sampling management and the calculation of which knowledge terms are possible in two phases. In Algorithm 1 presented as two methods: *updateTurnKnowledge* and *getTurnKnowledge*. The first is presented in Algorithm 2 in this section since it represents the way the Server proceeds in the management of the sampling and structures for knowledge, though we will dive into more necessary details in following sections. The second is shown in section 3.5.

Broadly speaking, the *UpdateTurnKnowledge* starts with the legal play sequence of the true state of the game; the bag of models; a number β that represents a number that will trigger the re sampling and the correspondent game turn that is currently in. This algorithm can be understood as follows: line 2 shows the models update to the following turn with the move selection of the selected Sampler (section 3.6); line 3 shows where the first step into knowledge calculation is made: where legal play sequences are compared

[6] Replacing the variables by their domain

Algorithm 2 Update Turn Knowledge

```
1:  function updateTurnKnowledge(𝒫, B, β, t)                ▷ β ∈ ℕ
2:      moveSelection(m⃗_t, B)                               ▷ m⃗_t is the last joint move in 𝒫
3:      updateAccessibilityRelations(𝒫, B, t)
4:      updateEpistemicState(B, t)
5:      inc := getInconsistent(B, t)
6:      while |inc| > β do
7:          resample(inc, B, t)
8:          inc := getInconsistent(B, t)
9:      end while
10: end function
```

(section 3.5.1) and the information which is saved in matrices — one per player per turn — is available for the subsequent methods; line 4 uses those matrices to update the states inside the models in the bag to epistemic states; and line 5 uses them to check which models are inconsistent(section 3.5.2); line 6 - 9 is the re sampling, which is addressed in (section 3.8), that repeats everything since the start until the current turn.

3.5 Knowledge

In this section we address how the semantics of GDL-III are implemented. More specifically, our procedure to compare legal play sequences; the constraint that we use to be able to figure which models can be resampled; and the algorithm which we apply the semantics for the true state of the game.

3.5.1 Indistinguishable Play Sequences

Given that our objective is to implement the semantics of GDL-III, it is necessary to present the algorithm that compares two play sequences P_1 and P_2 as in def. 30 for a specific player r and until a specific turn t. Before, recall that every state is uniquely identified by his legal play sequence. The algorithm 8 can be understood as follows: if the player's move from the joint move and the player's percepts from the joint percepts of both play sequences are the same at each turn and remain the same until the desired turn, then we have two indistinguishable play sequences at turn t for role r.

We call the legal play sequence belonging to a specific player his partial legal play sequence, and if any two are indistinguishable at turn t for a specific player r we refer to them as $P_1 \sim_{r,t} P_2$. Note that if they are in fact indistinguishable we can also say that any states s_1 and s_2 identified by P_1 and P_2 are indistinguishable: $s_1 \sim_{r,t} s_2$; the same for epistemic states es_1 and es_2. Moreover, we often use the notation that $M_1 \sim_{r,t} M_2$ that intrinsically means that models are indistinguishable. Notice that in this case we are referring to the legal play sequence in the models.

As we said in the previous section, we use matrices to save which models are indistinguishable from others, as well as, from the true state of the game at a specific turn. Those

Algorithm 3 Compare Play Sequences

Require: $0 \leq t \leq x$; $0 \leq t \leq w$, $r \in R$

1: **procedure** COMPARE($\{\langle \vec{a}_{1,0}, \ldots, \vec{a}_{1,x} \rangle, \langle \vec{\sigma}_{1,0}, \ldots, \vec{\sigma}_{1,x} \rangle\}, \{\langle \vec{a}_{2,0}, \ldots, \vec{a}_{2,w} \rangle, \langle \vec{\sigma}_{2,0}, \ldots, \vec{\sigma}_{2,w} \rangle\}, r, t$)
2: **for all** $i \in \{0 \ldots t\}$ **do**
3: **if** $a_{1,i,r} \neq a_{2,i,r} \vee \sigma_{1,i,r} \neq \sigma_{2,i,r}$ **then** ▷ $a_{1,i,r} \in \vec{a}_{1,i} = \left\langle a_{1,i,0}, \ldots, a_{1,i,r}, \ldots, a_{1,i,|R|} \right\rangle$
4: **return** 0
5: **end if**
6: **end for**
7: **return** 1
8: **end procedure**

matrices are $N \times N$ boolean matrices, where N represents the number of models in the bag. Since every state — in our solution model — are by themselves indistinguishable, it would render the major diagonal of the matrices useless, since we don't need that information saved to be able to use it. We decided to use this diagonal as the indistinguishability of a model M_i over the true state of the game s_{true}: $s_{true} \sim_{r,t} M_i, i \in N$. Note that because we are using matrices, if $M_i \sim_{r,t} M_j$ then positions i, j and j, i of the matrix will be marked as true.

The method *updateAcessibityRelations* in algorithm 2 is where the all legal play sequences are compared for each role. Notice that, random events are represented in GDL-III as the random role, however, in terms of knowledge, it is not a player. Therefore, we disregard it.

3.5.2 Consistency

Since the core of our solution is to sample game states, it was necessary to find a constraint to decide which samples had become pointless to maintain in the Bag. Between individual and common knowledge in GDL-III, the one with broader constraint is common knowledge, that recurs to the use of a transitive closure operation between indistinguishable play sequences, thus having to consider more play sequences. We refer to a model M_i as consistent at a specific turn, when he is in the transitive closure set of legal play sequences of the true state of the game: $s_{true} \sim_t^+ M_i$. To compute this transitive closure set, we use a depth first search without duplicates over the matrices of the players at the specific round. The inconsistent models able to be resampled.

3.5.3 Semantics

Algorithm 9 shows how we implement the full semantics of GDL-III for the true state of the game: s_{true}. For every knowledge term (either $K_r p$ or Cp) is necessary to verify if proposition p is statisfied, firstly in s_{true} (line 5 or 13), and then in all models according with the semantics of the knowledge term. Notice that, if it enters the second cycle (line 6 or 14), then p needs to be satisfied in the state inside the every model. For simplicity we say $M_j \models p$, where in reality we mean that $s \models p, s \in M_j$.

Recall that, as we have said before, we also need to update the models' states to epistemic states. However, to accomplish it, the algorithm remains logically the same. The only difference is replacing s_{true}, for the model M_i, which is the model we are updating. Method *updateEpistemicState* in Algorithm 2 is where this algorithm is called for every model in the bag.

Algorithm 4 Get Turn Knowledge

```
1:  function getTurnKnowledge(s_true, K, t)
2:      𝒦 = {}
3:      for all k ∈ K do
4:          if k is K_r p then
5:              if s_true ⊨ p then
6:                  for all M_j from s_true ∼_{r,t} M_j do
7:                      if all M_j ⊨ p then
8:                          𝒦 ⊕ k                          ▷ Append k to 𝒦
9:                      end if
10:                 end for
11:             end if
12:         else if k is Cp then
13:             if s_true ⊨ p then
14:                 for all M_j from s_true ∼_t^+ M_j do
15:                     if all M_j ⊨ p then
16:                         𝒦 ⊕ k                          ▷ Append k to 𝒦
17:                     end if
18:                 end for
19:             end if
20:         end if
21:     end for
22:     return 𝒦                                           ▷ 𝒦 ⊆ K
23: end function
```

3.6 Sampling

This section present the samplers that we had to develop in our solution. As we said previously, a bag is composed by a fixed number of models. The developed samplers simply grab each model and, accordingly with the selected sampler chooses the way the models are updated. To simplify the presentation of the algorithm that each sampler has, def. 31 shows how a model is updated. Again, note that whenever a model is updated, it already contains an epistemic state. The update of a model with a joint move is appending the joint move to the list of joint moves of the model, calculate the joint percepts that are necessary to append to the list of joint percepts in the legal play sequence, and calculate the new state to be saved.

Definition 31. *Let the M be a model, then:*

- $\mathcal{M}' = \mathcal{M} \oplus \vec{a} = \{\{\langle \vec{a_0}, \ldots, \vec{a_t} \rangle, \langle \vec{o_0}, \ldots \vec{o_t} \rangle\}, es\} \oplus \vec{a}$ *that is accomplished as:*

- $\mathscr{M}' = \{\{\langle \vec{a}_0, \ldots, \vec{a}_t, \vec{a}\rangle, \langle \vec{\sigma}_0, \ldots, \vec{\sigma}_t, \vec{\sigma} = sees(es, \vec{a})\rangle\}, s' = next(es, \vec{a})\}$ *with* $es \in E$ *and* $s' \in S$.

3.6.1 Random

Algorithm 5 shows how random sampler selects the move to update. Line 3 shows how the move is selected: randomly. Note that, at this point the model has already an epistemic state, then we can easily find the legal moves, including the ones that depend on knowledge, and select one randomly.

Algorithm 5 Random Sampler Move Selection

```
1: function moveSelection(B)
2:     for all M ∈ B do
3:         randomly choose a⃗ ∈ A(M)                ▷ A(es), es ∈ M
4:         M := M ⊕ a⃗
5:     end for
6: end function
```

3.6.2 Perspective Shifting

During the development we realised that the Random Sampler that we had developed was performing poorly timewise on games that we were testing. This sampler was born out of the poor time performance of the Random Sampler, which will be detailed in section 4.2. His objective is to generate consistent indistinguishable models the fastest way possible. We use to our advantage, the fact that in order for a model to be indistinguishable, it needs to have a partial legal play sequence that is indistinguishable from the actual play out of the game $\mathscr{P}$ of another player. Algorithm 6, can be viewed as splitting the models between players (shifting) — except random — and trying to apply the same move they made to update the true state of the game in their' models (perspective). Line 7 - 9 denote possibility that even if the move does not belong in the set of legal moves of the role r_j at a certain point, which means that model is distinguishable for that player. This does not, however, mean that it's necessarily inconsistent, since the transitive closure allows that model to be consistent because of other players.

This approach is different from the HyperPlayer, because it makes use of the perfect-information of the GM. The HyperPlayer, however, only has access to it's own move, having to break the legal play sequence and generating partial possible legal moves sequences for other players. This Sampler can be viewed, in a sense, as an HyperPlayer.

As in the knowledge section, we don't consider applying the random player perspective ($mod\,|R|-1$), since his moves are by definition random. Which are represented both in line 5 — the fixed move is of another player, whereas the rest is randomized — and 8 — everything is randomized.

Algorithm 6 Perspective Shifting Sampler Move Selection

```
1: function moveSelection(m⃗, B)                                  ▷ m⃗ ∈ 𝒫
2:     for all M_i ∈ B do
3:         j := i  mod (|R| − 1)
4:         if m_{r_j} ∈ A(M_i, r_j) then                          ▷ es_true ∼_{r_j} M_i
5:             randomly choose a⃗ ∈ A(M_i), where a_{r_j} = m_{r_j}
6:             M_i := M_i ⊕ a⃗
7:         else                                                   ▷ Consider es_true ∼⁺ M_i
8:             randomly choose a⃗ ∈ A(M_i)
9:             M := M_i ⊕ a⃗
10:        end if
11:    end for
12: end function
```

3.7 Plausibility

In this section we need to address the fact that our solution is by definition approximative. It is necessary to define a new concept, that of *plausibility*. To explain it, we will use an example. Unlike we have been using throughout this chapter, in this section we only consider epistemic states. Notice that, by applying the semantics of GDL-III leads to an epistemic game tree with epistemic states.

Having that said, as you can see in the Figure 3.5, there are two joint game trees of Muddy Children Game with two children (A and B). On the left a complete game tree with all epistemic states, on the right an incomplete tree. In this game children know that at least one of them is muddy, and the objective is for them to know that themselves are muddy. For simplicity, we assume that the children' percepts are received right after a move is made; and that the game finishes after a child says yes. A child can say yes if and only if she knows that she has mud.

As one can see on the tree in the left, in round 0 the random player chooses between the configurations to setup the muddy children. The possibilities are 01 and B is muddy, 11 both A and B are muddy, 10 and A is muddy. The children do nothing, represented as n. Depending on the configuration if is either 01, or 10, at round 1, the child that has mud says yes (y) because she can see the other doesn't have, but if they both have mud, they do no operation — also denoted by n — , symbolizing that they don't know if they have mud. Therefore, at round 2, if they both said no, then both know that they have mud, because they can distinguish between all the possible epistemic states of the game. This is what happens when you can consider all the possible states at a certain turn.

However, since our solution is approximative, such case as in the right tree can happen. Imagine that one doesn't have enough space in the bag for all the possible epistemic states at round 1. Or better, in every game sample, since it is random, the chosen settings were always 11 and 01. One can see that player A can not distinguish between the states 01, and 11, because in both cases he can see that B has mud, but he is still not sure about

himself. However, player B can distinguish between both, since he can see the forehead of player A in both cases. In 01 he sees that player A does not have mud, whereas in 11 he can see that she does not have. Therefore, since there is no epistemic state 10 in round 1, it is impossible to calculate what B knows correctly, and the only legal move that B has at round 1 in the epistemic state 11 will be y, whereas in reality he should do it at round 2, for this epistemic state.

A plausible epistemic state, in Figure 3.5 in red, can only be identified if one has access to the entire game space's turn in the epistemic game tree. However, as we said before, our solution is approximative, therefore plausible epistemic states can appear. In reality, a plausible epistemic state in a model, does not bring any problems into the control of the true state of the game, though it can make it harder when resampling. However, if the true state of the game itself becomes plausible then there is a problem, e.g. it could mean that the game ends before it should. Unfortunately, this is the only drawback of an approximative solution, and there is no solution to solve it. The only thing one can do is minimize it, as we show in chapter 4.

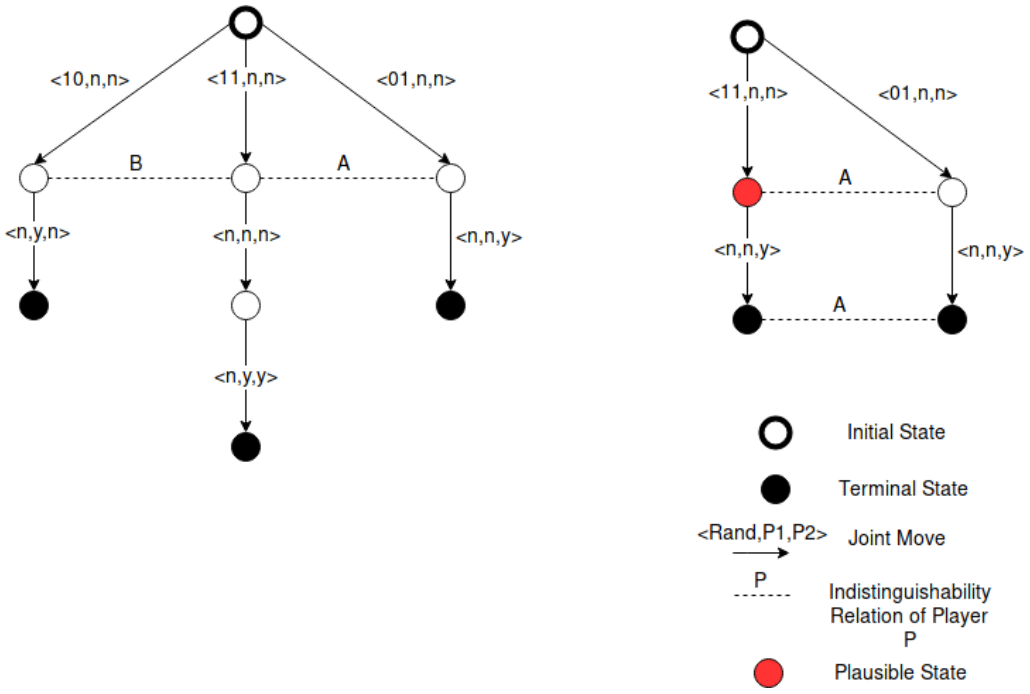


Figure 3.5: Plausibility

Using both game trees in Figure 3.5, we can also show that number β, which is the number of inconsistent models that we allow in the solution, cannot be 0 for games like Muddy Children, where moves legality depends on the knowledge of the players. Imagine that the true epistemic state of the game is 11, and that $\beta = 0$. At round 2, we have that the only possible epistemic state is 11, therefore we need to resample all models that have the epistemic state 10 and 01 and resample them until the epistemic state is 11. However in round 1, as we see in the tree in the right, it is necessary both of them (10 and 01)

so that epistemic state 11 does not become plausible, as well. Then, it is necessary to keep some inconsistent models ($\beta \neq 0$) in these types of dynamic epistemic games. These models, do not introduce problems in the knowledge verification, since they are already inconsistent at round 2. This avoids infinite re sampling, since the plausible epistemic state 11 at round 2 (marked in red in the right tree) would also be inconsistent.

3.8 A Dynamic Epistemic Resample

Whenever models are not in the transitive closure set of the true state of the game, they are able to be removed and replaced by other models with consistent epistemic states. On our re sample approach we considered two options:

- Redo the model from the initial state until the current turn;
- Going back to previous turn and generate a new joint move.

Both approaches could work. However, since our goal with re sampling was to minimize plausibility, we chose the first approach. Our option is due to the fact that the content of an epistemic state is usually determined by the random player's move at round 0.

Algorithm 7 presents our resampling. This algorithm can be viewed as follows: it receives the identification number of the inconsistent models, which is the model position in the bag; the bag of models; and the current turn; line 2-4 shows the replacement of the epistemic state inside the inconsistent model with the true initial epistemic state of the game, and the replacement of it's legal play sequence as empty; After this part is concluded, it starts doing the same thing round by round, until one reaches the given turn t. This means that at every turn it is necessary to apply a joint move with the designated sampler (line 6-8). Followed by a comparison the new legal play sequences until the current turn i with all the other models, and the legal play sequence of the true state of the game (line 9-11). When we have the matrices of the players updated with this new information about their indistinguishability, it is necessary to apply the semantics of GDL-III only to the models' states being resampled (12-14), the others aren't changed.

Another decision we had to consider was the checking of inconsistent models. The options were that, either at each turn i that we complete, or just when we would reach the turn t. In our point of view, both ways would lead to the same end result. But since evaluating the consistency of the bag turn by turn could increase even more time sent on re sampling, so chose the later.

3.9 Information Stealing

A EGGP setting always assumes that the players are truthful about themselves. This means that players should not lie. However, since players' moves can be described by knowledge terms two things can happen. Either a player can submit a move were he

Algorithm 7 Re Sampling

```
1: function RESAMPLE(inc = ⟨0,...,N⟩, B, t)
2:     for all j ∈ inc do
3:         replace(M_j, es_0)
4:     end for
5:     for i ∈ {0,...,t} do
6:         for all j ∈ inc do
7:             moveSelection(M_j ∈ B)
8:         end for
9:         for all j ∈ inc do
10:            updateAccessibilityRelations(P, M_j ∈ B, i)
11:        end for
12:        for all j ∈ inc do
13:            updateEpistemicState(M_j ∈ B)
14:        end for
15:    end for
16: end function
```

thinks he knows something when in fact he does not; or he could use the fact that the GM as access to information of all the players and intentionally submit a move that he is still not sure, just so that the GM answer whether that move was legal or not. Notice that in both cases the player submits a move but, that in the first case the player is being truthful, and in the second it is not. If the situation becomes problematic such as in competitive settings, a game designer should consider the following Listing to ensure that a player thinks twice before submitting these types of moves. The GM does not have the ability to differentiate the situations. He will only make sure players obey the game description.

```
% the move in question
(<= (legal ?r ?m) (knows ?r p))

% copy of the relation since the legal keyword shouldn't appear in the rules' body
(<= (isLegal ?r ?m) (knows ?r p))

% saving for later punish if player ?r's move is not legal
(<= (next (punish ?r)) (does ?r ?m) (not (isLegal ?r ?m)))
(<= (next (punish ?r)) (true (punish ?r)))

(<= (goal ?r 100) (true (someProposition)))
% punishing
(<= (goal ?r 0) (true (someProposition)) (true (punish ?r)))
```

Chapter 4

Analysis

In this chapter, we study the correction of the knowledge semantics implementation, which is based, as aforementioned in the semantics described in [Thi17]. We study the correction of the developed samplers and show that they act as a solution that considers the hole state space, given enough resources. Furthermore, we test the scalability of our developed GM regarding the complexity of games, versus time by turn that it takes to execute to make a consistent bag of models for that turn and calculate the knowledge of players.

4.1 Correction

In this section we show that our developed solution behaves according to what is expected given enough resources.

4.1.1 Knowledge

We now show that our knowledge algorithms are not what induces errors when calculating the knowledge for a specific state. Algorithms 8 and 9 presented in Section 3.5 were copied here in order to be able to make comparisons with Definition 32 which is also copied from Section 2.4.3.

Algorithm 8 shows how two legal play sequences are compared for a specific player. It showcases that for two legal play sequences to be indistinguishable for a player, his partial legal play sequence has to be the same from the initial state until the current state at time t. By starting from the initial turn until the current turn, if at any turn i in the between, if his move or percepts are different then it returns: 0. Otherwise, if it goes out from line 5 and returns: 1 since they are equal. This algorithm is therefore correct in the way it compares any two legal play sequences for a given player at a given turn.

Algorithm 8 Compare Play Sequences

Require: $0 \leq t \leq x$; $0 \leq t \leq w$, $r \in R$

1: **procedure** COMPARE($\{\langle \vec{a}_{1,0}, \ldots, \vec{a}_{1,x} \rangle, \langle \vec{\sigma}_{1,0}, \ldots, \vec{\sigma}_{1,x} \rangle\}, \{\langle \vec{a}_{2,0}, \ldots, \vec{a}_{2,w} \rangle, \langle \vec{\sigma}_{2,0}, \ldots, \vec{\sigma}_{2,w} \rangle\}, r, t$)
2: **for all** $i \in \{0 \ldots t\}$ **do**
3: **if** $a_{1,i,r} \neq a_{2,i,r} \vee \sigma_{1,i,r} \neq \sigma_{2,i,r}$ **then** ▷ $a_{1,i,r} \in \vec{a}_{1,i} = \left\langle a_{1,i,0}, \ldots, a_{1,i,r}, \ldots, a_{1,i,|R|} \right\rangle$
4: **return** 0
5: **end if**
6: **end for**
7: **return** 1
8: **end procedure**

Definition 32. *Let G be a game description along with all the sets and relations it describes according to Definition 23.*

- *The play sequence of length 0, denoted by ϵ, is legal and satisfies $\epsilon \sim_r \epsilon$, for all $r \in R$. It results in state s_0 and knowledge state K_ϵ as the smallest set that satisfies*

$$K_\epsilon = \left\{(knows\ r\ p) :\ r \in R, G \cup s_0^{true} \cup K_\epsilon \models p\right\} \cup \left\{(knows\ p) :\ G \cup s_0^{true} \cup K_\epsilon \models p\right\}$$

- *For the inductive definition, let δ be a legal play sequence of length $n \geq 0$ resulting in (s_n, K_n).*

  Sequence δ followed by M, written δM, is a legal play sequence of length $n+1$ if $(M(r), s_n, K_n) \in l$ for all $r \in R$. It results in the state $s_{\delta M} = u(M, s_n, K_n)$ and, as the knowledge state, the smallest $K_{\delta M}$ that satisfies:

$$\begin{aligned} K_{\delta M} = &\left\{(knows\ r\ p) :\ G \cup s^{true}_{\delta' M'} \cup K_{\delta M} \models p\ for\ all\ \delta' M' \sim_r \delta M\right\} \\ &\cup \left\{(knows\ p) :\ G \cup s^{true}_{\delta' M'} \cup K_{\delta M} \models p\ for\ all\ \delta' M' \sim^+ \delta M\right\} \end{aligned}$$

Algorithm 9 is our implementation of Definition 32. In our algorithm a legal play sequence of any epistemic state is represented as P. In Definition 32 they are represented as δM. Both are composed by a sequence of legal joint moves and percepts. Therefore, both our s_{true}[1] and models $M = \{P, s\}$ can also be understood as s_P, and we can re write them as: $s_{\delta M}$ to match Definition 32.

Our algorithm does the same thing that is defined both in the base case and in the induction case. We identify which knowledge term is going to be tested depending on whether is personal knowledge $(knows\ r\ p)$ — in our algorithm $K_r p$ — or common knowledge $(knows\ p)$ — in our algorithm $K_r p$ — the set $K_{\delta M}$ is calculated. The initial case of Definition 32 is satisfied in the algorithm by lines (5 and 13), where there are only the initial state and no indistinguishability relations. For the induction step of Definition 32, any two states $s_{\delta' M'}$ and $s_{\delta M}$ have already passed in the algorithm to compare there legal play sequences (pre-semantics stage), and the indistinguishability relations $\sim_r$ were

[1] Don't confuse with Def. 32 s^{true}. In our case is the true state of the game. Whereas in the definition is what is fluents that are true in the state s

Algorithm 9 Get Turn Knowledge

```
1:  function getTurnKnowledge(s_true, K, t)
2:      𝒦 = {}
3:      for all k ∈ K do
4:          if k is K_r p then
5:              if s_true ⊨ p then
6:                  for all M_j from s_true ∼_{r,t} M_j do
7:                      if all M_j ⊨ p then
8:                          𝒦 ⊕ k                          ▷ Append k to 𝒦
9:                      end if
10:                 end for
11:             end if
12:         else if k is Cp then
13:             if s_true ⊨ p then
14:                 for all M_j from s_true ∼_t^+ M_j do
15:                     if all M_j ⊨ p then
16:                         𝒦 ⊕ k                          ▷ Append k to 𝒦
17:                     end if
18:                 end for
19:             end if
20:         end if
21:     end for
22:     return 𝒦                                           ▷ 𝒦 ⊆ K
23: end function
```

created correctly. We can then conclude that our algorithm implements what is defined in the induction step of Definition 32. Specifically, line 5-10 shows the implementation of the personal knowledge definition in the induction step. For any state $s_{\delta M}$ that is indistinguishable from $s_{\delta' M'}$ will be verified if p holds. The same case appears in lines 13-18, for any state $s_{\delta M}$ that is in the transitive closure set of indistinguishable relations of $s_{\delta' M'}$ will be verified if p holds. Like in Definition 32 if p holds in all states $s_{\delta M}$ then it is appended to the set K depending on the knowledge term that it is.

We have shown that our algorithm that calculates the knowledge set of a state acts accordingly to what is defined in [Thi17]. Therefore our knowledge implementation it is correct. However, since at some point we might not have all states $s_{\delta M}$ stored then plausible states can appear. We will now show that our developed samplers given enough resources consider all epistemic states, and therefore, again given enough resources, our GM behaves in the same way that of a solution that considers all legal play sequences.

4.1.2 Random Sampler

This section presents that the random sampler does acts as a non-approximative solution, given enough resources such as space and time. To do this, we define another GDL-III game. Definition 33 is different from Definition 28 in the sense that we only consider epistemic states, or states with knowledge. We do this because we already showed that the

algorithm that implements GDL-III semantics acts accordingly to its definition. Therefore it is not necessary to split between a pre-semantics stage and a post-semantics, and we can only consider the post-semantics that lead to the epistemic game tree.

Definition 33. *Let $G = \{S, R, A, \Sigma, goal, next, sees\}$ be an incomplete information game with introspection given by a GDL-III game description:*

- *S is the set of all epistemic states on the game tree;*
- *R is the set of roles in the game;*
- *A is the set of moves in the game, and $A(s,r) \subseteq A$ is a set of legal moves, for role $r \in R$ in the epistemic state $s \in S$;*
- *Σ is a set of percepts in the game, and $p \in \Sigma$ is a percept given by the sees function bellow;*
- *$goal : S \times R \mapsto \mathbb{R}$ is the payoff function;*
- *$next : S \times A^{|R|} \mapsto S$ is the successor function;*
- *$sees : S \times A^{|R|} \mapsto \Sigma^{|R|}$ is the percept function.*

Moreover, let:

- *s_0 be the initial epistemic state;*
- *$S_i \subseteq S$ denote the epistemic state space of G at turn $i \in \mathbb{N}_0$, such that $S_0 = \{s_0\}$ and $S_{i+1} = \left\{s_{i+1} : s_{i+1} = next(s_i, \vec{a})\ for\ all\ s_i, \vec{a} \in A(s_i, r_1) \times ... \times A(s_i, r_{|R|})\right\}$;*

Definition 33 defines an epistemic game, that has the set of all epistemic states in the game; the set of roles in the game; the set of actions; the set of percepts; the payoff function; the state update function; and the percept function. S_i denotes a specific state space at turn i.

Definition 34. *Let G be an epistemic game as defined in def. 33, and:*

- *$Choose : S \mapsto A^{|R|}$ be a stochastic function, and $\vec{a} = Choose(s)$ is the chosen joint move with uniform probability from the set $A(s, r_1) \times ... \times A(s, r_{|R|})$, with $s \in S$;*
- *B_i is a set of ordered pairs of size $N \in \mathbb{N}$ at turn $i \in \mathbb{N}_0$, such that:*
  - *$B_0 = \{(s_0, j) :\ s_0 \in S_0,\ j \in \{1, \ldots, N\}\}$ and,*
  - *$B_{i+1} = \{(s_{i+1}, j) : (s_i, j) \in B_i \wedge s_{i+1} = next(s_i, Choose(s_i))\}$.*

Definition 34 defines the Random Sampler. Throughout the rest of this thesis, we denote by $s_{i,j}$ pairs $(s_i, j) \in B_i$, where j represents the position of state s_i in the bag of models represented by B_i. As one can deduce from the definition above, the position associated with each state never changes, even after applying the function *next*. The only

thing that changes is s_i to s_{i+1}. We have that function *Choose* uses a uniform probability over the domain of the joint legal moves of the players. The set B is defined using an inductive definition where at turn 0, B_0 has copies of the initial state of the game. One for each pair in the set. The following turns are accomplished by applying the *Choose* function to each state in the set B of the previous turn.

We use this Definition 34 to define set $\mathscr{B}_i$ in the Lemma 4.1.1, that represents the set of all possible sets B_i at turn i, to be able to mathematically show that our samplers behave as a solution that considers all states given enough resources. Lemma 4.1.1 proves that the random sampler behaves accordingly. By defining the function that chooses randomly an action to update a state, we can show that if set B's size is infinite, then it will always have all the epistemic states belonging to a game at that round.

Lemma 4.1.1. *Let G be as in def. 33, B_i as in def. 34 and:*

- *$\mathscr{B}_i$ denote the set of all possible B_i at turn i.*

Then,

$$\forall_{s\in S_i, B_i\in\mathscr{B}_i} \lim_{N\to+\infty} P(s \in B_i) = 1.$$

Proof Skecth. By induction on the turn i of game G.

- Base case ($i = 0$): Let $s \in S_0$ be an epistemic state, and $B_0 \in \mathscr{B}_0$ be a possible set as described previously. With $i = 0$, we have that $S_0 = \{s_0\}$, therefore $s = s_0$. Since, by definition, B_0 contains N ordered pairs (s_0, j) (for $j \in \{0,\dots,N\}$, corresponding to the N copies of the initial state s_0), it follows:

$$P(s \in B_0) = 1.$$

- Induction step ($i > 0$): Let $s_i \in S_i$ be an epistemic state at turn i, where $s_{i,j} \in B_i$ means that s_i is at position j of B_i. And, $s_{i+1} \in S_{i+1}$ be a state that follows from unequivocally choosing $\vec{a}$ from $A(s_{i,j}, r_1) \times \dots \times A(s_{i,j}, r_{|R|})$ and applying the function $s_{i+1} = next(s_i, \vec{a})$ as defined in def. 33.

  Then, let: E_j be the event of choosing an action $\vec{a}$ for the state $s_{i,j}$, and E_{j+1} be the event of choosing an action $\vec{a}$ for the state $s_{i,j+1}$. Therefore, since by definition E_j and E_{j+1} are chosen with a uniform probability. And, furthermore, both E_j and E_{j+1} are independent events, since there is no correlation between choosing $\vec{a}$ for state $s_{i,j}$ or $s_{i,j+1}$. Then, we can say that:

$$\sum_{j=1}^{\infty} P(E_j) = \infty$$

  Using the Second Borel-Cantelli Lemma we have that the probability of a non null independent event to happen when the number of times the same event occurs is infinite is:

$$\lim_{j\to\infty} P(E_j) = 1.$$

Then we can conclude since the probability of the event E_j, when $j \to \infty$ is one, then for sure $\vec{a}$ will be selected, which will follow that $s_{i+1,j} = next(s_{i,j}, \vec{a})$, we can conclude that:

$$P(s_{i+1} \in B_{i+1}) = 1.$$

□

Note that if our solution would have infinite space, then we do not need to resample models since the probability of having all the epistemic states of the game at any round in the bag is one. Moreover, we can also conclude that if we have enough space for all states, then there are no plausible states since they only happen if we do not consider all the necessary states.

4.1.3 Perspective Sampler

As in the previous section, we will show why this sampler is correct, in the sense that considers all possible epistemic states.

Definition 35. *Let G be as in Definition 33, and:*

- $s_{true,i} \in S_i$ *- the true state of the game at turn i;*
- a_r *is a specific move for role* $r \in R$ *in joint move* $\vec{a} \in A^{|R|}$*;*
- $\vec{m}$ *is the real joint move that updated* $s_{true,i+1} = next(s_{true,i}, \vec{m})$*, with* $\vec{m} \in A(s_{true,i}, r_1) \times ... \times A(s_{true,i}, r_{|R|})$.

Definition 35 shows the definition of the true state of the game, which the state that is being updated by the players. Likewise, the real joint move is the move that updates the true state of the game. Definition 36 shows the definition of the Perspective Sampler.

Definition 36. *Let G be as in def. 33 and m as in def. 35, then:*

B_i *is a set of ordered pairs of size* $N \in \mathbb{N}$ *at turn* $i \in \mathbb{N}_0$*, such that:*

- $B_0 = \{(s_0, j) : \ s_0 \in S_0, \ j \in \{1, \ldots, N\}\}$ *and,*
- $B_{i+1} = \{(s_{i+1}, j) : (s_i, j) \in B_i \wedge s_{i+1} = next(s_i, Choose(s_i, j, \vec{m}))\}$*, where:*
  - $Choose : S \times \mathbb{N} \times A^{|R|} \mapsto A^{|R|}$ *is a stochastic function chooses with a uniform probability from the set* $M \subseteq A^{|R|}$*, such that* $\vec{a} = Choose(s_i, j, \vec{m})$*:*

$$\begin{cases} M = A(s_i, r_1) \times \cdots \times \{m_{r_z}\} \times \cdots \times A(s_i, r_{|R|}), & \text{if } m_{r_z} \in A(s_i, r_z) \\ M = A(s_i, r_1) \times ... \times A(s_i, r_{|R|}), & \text{otherwise,} \end{cases}$$

  where $z = j \mod (|R| - 1)$.

Like our previous sampler we have that set B at turn 0, or B_0, starts with N copies of the initial state of the game. The update of the set is also like the previous sampler. However, *Choose* function in this sampler takes an extra argument, a joint move $\vec{m}$. We use the position of the state in the set B_i to determine a role r_z. Then we calculate a new domain for the legal joint moves of r_z. This new domain is built from the given role, where the specific move of that role is fixed, if the role r_z can make that move in that state, or in other words, $m_{r_z} \in A(s_{i,j}, r_z)$. Otherwise, the domain of legal moves of r_z remains the same. All the sets of legal joint moves for the other players continue the same.

We will now present the conjecture that this sampler is correct and therefore considers all epistemic states of a game G at a particular turn for games where $|R| > 1$. Notice that also that if $|R| = 1$ then there is only one player, which is either random (which is pointless to describe an epistemic game with only the nature). Or that, there is only a player, but there is no uncertainty since the player is the only one in the game[2].

Conjecture 4.1.1. *Let G be as in Definition 33 for $|R| > 1$ and B_i as defined in Definition 36:*

- *$\mathcal{B}_i$ denote the set of all possible B_i at turn i.*

Then:

$$\forall_{s \in S_i, B_i \in \mathcal{B}_i} \lim_{N \to +\infty} P(s \in B_i) = 1.$$

We argue why we believe this sampler considers all the possible states at turn i. At turn 0 we have that B_0 starts with N copies of s_0, therefore we can say that, since $S_0 = \{s_0\}$ then $P(s_0 \in B_0) = 1$. As for the following turns, we have $\vec{a} \in M$, where $\vec{a}$ is chosen with a uniform probability over the set M. M is determined by two conditions: the "otherwise" condition, where M is as in the previous sampler, which it is proven as correct in Lemma 4.1.1 proof sketch. Or the first condition, where move m_{r_z} is fixed. It will always enter this condition if player r_z is uncertain about that state. However, if it's certain that it is not in that state, it will always go to the "otherwise" condition. The probability of r_z being certain about that state will be one at a certain turn since the objective of an uncertain game reduce the number of uncertain states for each player. Therefore it will always jump onto the "otherwise" condition.

Another reason why we believe that the conjecture is true is that since a move is fixed at a state $s_{i,j}$ in the set B_i — because of j — and that if the size of B_i tends to infinity, then there will be infinite s_i, ∞, were will be fixed each player move infinite times. With the fact that the random player's move is always random and that this is represented by its set of legal moves always remains untouched. We can say that even though r_z is fixed in $s_{i,j}$, it will not be fixed in $s_{i,j+1}$, because even though is the same state, the fixed move will be of another player, except if it is a single player game (note that a single-player game is a game with 2 roles: random and the player). In a single-player game to explore the other moves of the player are unnecessary since both the personal knowledge set and

[2]Single-player epistemic games should be described using two players: the player and random

the common knowledge set are the same. For the other cases, we can say that the union of the update the function *Chooses* infinite times also gives all states, using the Second Borel-Cantelli Lemma, since choosing over the set M is always independent and with a probability different than 0.

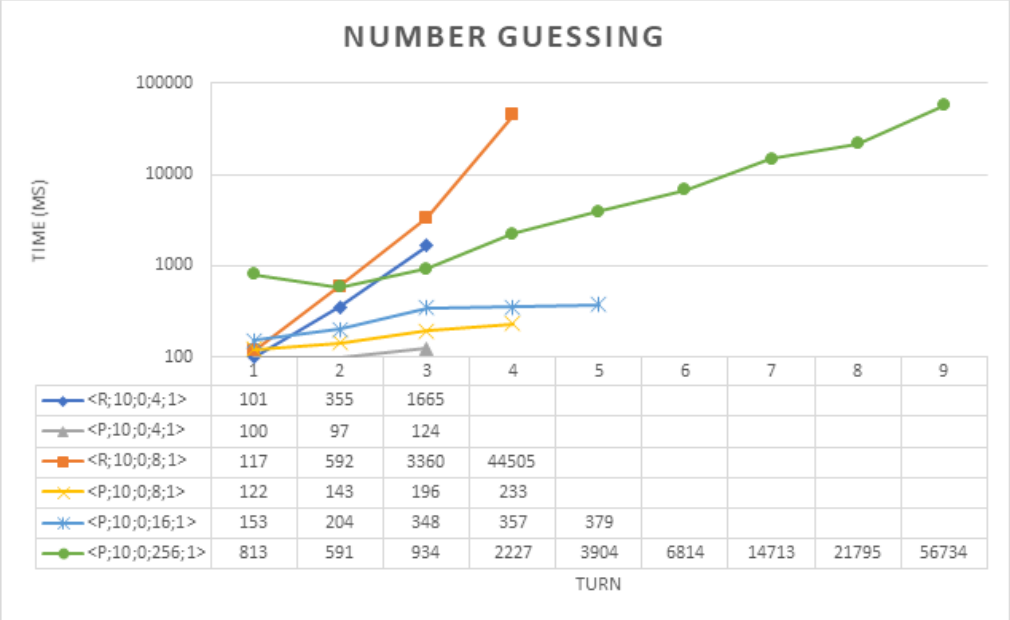
4.2 Scalability

In this section shows how our solution scales timely by increasing the size of the game. We measure the time it takes into updating models, making sure these are consistent and calculating knowledge in between turns. In some plots, we use a logarithmic scale in order to be able show all the information on it. We also present the results in a table so that the reader can understand them clearly. Each line in a plot is represented as $\langle Sampler; N; \beta; K; R \rangle$, where N means the total number os models; β the number of inconsistent models allowed; K the number of rules that the game has; and R the number of roles in the game. Results are presented turn by turn, by the time it took execute do line 15 and 16 of algorithm 1. All the tests were executed 20 times in a localhost environment between the GM and the players, in an Intel® Core™ i7-3537U CPU @ 2.00GHz and 4 GB of RAM machine. Tries where the game instance became plausible were disregarded. Games used to analyse the scalability can be found in Appendix B. We have developed more games, such as the hats puzzle problem that is presented in Appendix C, but because this game is similar to the Muddy Children we do not analyse the GM scalability with it. Furthermore, we also build games, that are presented in Appendix D, however these games were developed to test if the GM would act accordingly to what it should, and not to measure scalability.

4.2.1 Number Guessing Epistemic

This game is a variation of a GDL-II game, where a player wants to know the secret number. This secret number is set by the nature by the first move. After that, nature does always no operation, whereas the player can always ask if the secret number is less than another. This game has a branching factor of the numbers of the game. The tests procedure were: nature selected always the same number, and the selected move was always the same at the same round. It was applied in order to divide the set consistent epistemic states of the player by half. The game has a branching factor of the possible numbers in the game. The game was intentionally designed to end when the player knows the number. That is the reason why a game with 4 numbers has only measured 3 turns. At turn 3, player knows the number set by random. Therefore for games with more numbers: 8, 16 and 256, games end at turn 4, 5 and 9, respectively.

Figure 4.1 shows the time scalability by using the different samplers. Recall that each line in the plot represents a different setting and that the horizontal scale is in



	1	2	3	4	5	6	7	8	9
<R;10;0;4;1>	101	355	1665						
<P;10;0;4;1>	100	97	124						
<R;10;0;8;1>	117	592	3360	44505					
<P;10;0;8;1>	122	143	196	233					
<P;10;0;16;1>	153	204	348	357	379				
<P;10;0;256;1>	813	591	934	2227	3904	6814	14713	21795	56734

Figure 4.1: Number Guessing

milliseconds in a logarithmic of base 10. For details, we also present the values in a table below the plot, with the setting and turn, also in milliseconds.

In this plot, we can see that by increasing the number of knowledge terms possible in the game the time increases in the first round. We can say this because the re sample does not trigger since all possible epistemic states are indistinguishable for the player. In the second round, however, since the move is to shorten the player's epistemic states, the resample is triggered and we can see the difference between the samplers starting to notice. Since, the random move selection is always random takes more time than the other sampler. Differences increase when another round is played. In both settings, $\langle R;10;0;4;1\rangle$ and $\langle R;10;0;8;1\rangle$, we can see that take effectively longer than $\langle P;10;0;4;1\rangle$ and $\langle P;10;0;8;1\rangle$, respectively. Notice that, by increasing the number of possible terms it also increases the time spent to achieve that round. However, we don't consider this relevant, since the major cause to take more time is the sampler's move selection, as we can see between the referred settings. In the fourth round, with 8 numbers it is shown that the Random Sampler is largely inefficient since it takes 44 seconds for a branching factor of 8 at turn 4. The Perspective Shifting Sampler, on the other hand, manages it perfectly since with the same game setting takes only 233 milliseconds in the same round. With 16 numbers and 16 knowledge terms, the Perspective Shifting Sampler vastly outperforms the opposing sampler in every test setting after the second turn. Taking only more time to setup in the first round due to number of knowledge terms.

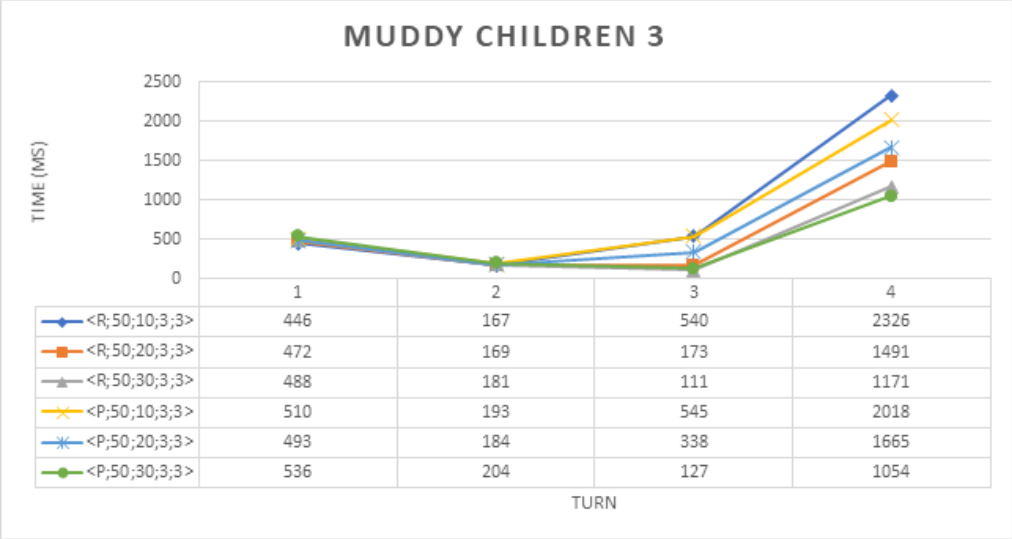
We were not able to test more numbers since with 16 numbers the Random Sampler wasn't able to stop, and therefore impossible to measure. The Perspective Shifting Sampler, however, supported a game with at least 256 numbers before starting to slow down

at later turns.

4.2.2 Muddy Children

This game diverges from the game presented in section 3.7 in the sense that players are not warned immediately of their percepts in the first turn, only in the second. Moreover, players only do no operation until the game ends. The selected setting was always all children have mud. The game ends when the children realise they have mud, namely at turn 4, for a game with 3 children, or 5 for a game with 4 children.

Figure 4.2 shows how our solutions react by increasing the number of inconsistent models. We can see that due to all epistemic states being indistinguishable for all children at turn 1 no resample is necessary to turn 2, since they only receive their percepts, which justifies the reduction in time from the previous round. By increasing the number of inconsistent models it is shown that it also has optimization effect. Specifically to turn 3 and to turn 4. Since by having a larger number of inconsistent models allowed we either don't trigger re sampling $\langle R; 50; 30; 3; 3\rangle$ and $\langle P; 50; 30; 3; 3\rangle$ at turn 3, or if it is triggered it takes less time than with a smaller inconsistent number which can be seen with the same settings at turn 4. Neither of the samplers shows considerably superiority over the other in this test, as predicted since there is only one possible joint move.

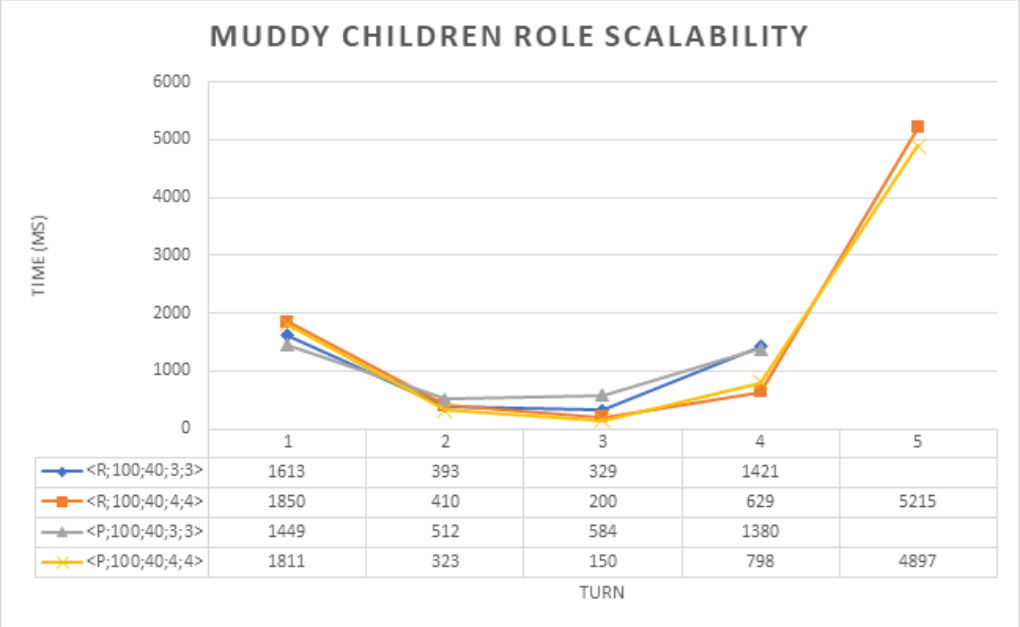


	1	2	3	4
<R;50;10;3;3>	446	167	540	2326
<R;50;20;3;3>	472	169	173	1491
<R;50;30;3;3>	488	181	111	1171
<P;50;10;3;3>	510	193	545	2018
<P;50;20;3;3>	493	184	338	1665
<P;50;30;3;3>	536	204	127	1054

Figure 4.2: Muddy Children 3

Figure 4.3 shows a different test. It shows how our solutions behave by increasing the number of roles in a game — doing this adds one more knowledge term. As one can see in the plot, with four roles it takes more time than with 3 roles at round 1, for both samplers. This happens because there is one more player's matrix to manage, and one more term. To update to turn 2 no resample is necessary. To turn 3 we see a small difference in time between games. Whereas for 100 models at turn 3 with a game with 3 players there is only 4 epistemic states where at least 2 of the children have mud, with 4 children there

are more epistemic states with at least two children with mud. This is the reason why for the settings with 3 players it takes more time than with 4. The same thing happens to turn 4, since with 3 children there is only one consistent epistemic state — the one with three children with mud. With 4 children, however, there are 5 consistent epistemic states with at least three children having mud. The game with 3 children ends at round 4, while the one with 4 children takes one more turn. This turn is heavy on resample for both Random and Perspective. Both take equal time, because there is ony one possible epistemic state. We see with this plot that with a branching factor of 1 both samplers behave the same way.



	1	2	3	4	5
<R;100;40;3;3>	1613	393	329	1421	
<R;100;40;4;4>	1850	410	200	629	5215
<P;100;40;3;3>	1449	512	584	1380	
<P;100;40;4;4>	1811	323	150	798	4897

Figure 4.3: Role Scalability

4.2.3 Russian Cards Games

The standard The Russian Cards Game is a public announcement game described in [Rus] has three players: Alice; Bob and Cath. Cards are dealt in a setting 3; 3 and 1, respectively. Alice is able to repeatedly say a triplet of cards, and Bob being able to say Cath's card once he knows it. However, this game as proven to have too many possible moves for Alice therefore it couldn't be tested with either sampler. We proceeded to develop a variation of the game also to be able to test common knowledge terms, since the previous games were only individual knowledge. We developed two variations: In both the players are dealt 1 card, and the objective of the game is for Cath's card to be common knowledge. Unlike the standard game, both Alice and Bob can make public announcements of triplet of cards. The difference is that in the first version, Cath doesn't do anything, whereas in the second Cath is shown the players cards and she is able to switch with one of the players

without the other being warned. We show the second varitation's results to present how the system reacts to one of the players adding uncertainty to others.

Figure 4.4: Russian Cards Games

Figure 4.4 shows the results of the test. Again, as in section 4.2.1, we were forced to use a logarithmic scale to be able to present the results of the game. So, we present the time in a table in the plot so the reader can understand it. We want to show which setting is better, by having the number of consistent models ($N - \beta$) to be the same, if it would be better to have that number with a small number of total models, or with a bigger number. Results show, again that by having this difference with a bigger number of total models can improve the time performance of the solution. Comparing the developed samplers shows that the Perspective Shifting Sampler in either setting, vastly outperforms the Random Sampler in the last turn — which is the most important. We weren't able to provide results for $\langle R;10;0;3;3\rangle$ because it took longer than $\langle R;50;40;3;3\rangle$. We can see that in the first turn, where the random chooses the distribution of cards in the game, the settings with more models justify themselves by taking more time to update. For turn 2 is where Alice said her first triplet of moves, it takes relatively the same time between the two samplers, however we can see that $\langle P;10;0;3;3\rangle$ is worse than $\langle 50;40;3;3\rangle$ since it takes more time making the solution ready for that round. The reason is it is necessary to resample, and with more than 40 models to be able to become a consistent epistemic state it is faster than having less than 10. On the third turn, the Random Sampler takes more time than the Perspective in either setting. We can see the difference of the branching factor having in account. For the fourth turn, since it is Cath's move, where she switches with Bob, we can see that no resample is necessary, since it takes less than 100 miliseconds in either test setting. Since for this turn Cath can only do 3 possible moves, which 2 of

them will continue to make the models consistent, no resample is necessary. Therefore the test settings which takes more time are the ones with more models. Althouh the difference is barely noticeble. We start to see meaningful differences after the uncertainty, when Alice and Bob start to do their own move to restrict the number of consistent models. For the fifth turn, Alice has made her move, we can see that the resample is triggered. Again more inconsistent models acts as an optimization. Notice, that at this point only one of the samplers with a specific setting is acceptable: $\langle P; 50; 40; 3; 3\rangle$, since it takes around 4 seconds. On others settings, the Perspective Shifting with 10 total models is inefficient, since it takes 25 seconds and the Random Sampler is even worse because it takes 300 seconds or, 5 minutes. For the last round, all the samplers take way to much time, being the best of them the Perspective Shifting with $N = 50; \beta = 40$ which took 57 seconds. The other Perspective with $N = 10, \beta = 0$ took 1911 seconds which is essentially 31 minutes. The Random is the worst of the samplers taking 8452 seconds approximatly, which is 140 minutes on that round. We can conclude with this test that by increasing N and β accordingly can provide games to be able to be played. Although the more we go deep into the joint game tree, the harder it is for the GM to be able to making sure that the bag is consistent and calculating the knowledge of the players at that round using either of the samplers.

Chapter 5

Conclusion

The work done in the context of this thesis resulted in a functional Game Master for EGGP that successfully implements the semantics of GDL-III. This GM was developed with model sampling due to the fact that solutions that consider all possible states of a game in GDL-III are impossible to maintain thanks to games' state space complexity. We have shown that, because we are not considering every state, using a sampling method can cause errors in the way the GM controls a game.

Our work started from a disappointingly undocumented GM package that was, nevertheless, very modular and easy to work with. The end result was a functional GM that may be deployed in any Operating System thanks to the technology on which it was developed: Java Ant [Apa] (section E). We enjoyed working and developing games for GDL-III. Our efforts to optimize the developed GM resulted in a second sampler, the Perspective Shifting Sampler, that is able to perform its job in a less time consuming manner. Yet, while we formally show that with the Random Sampler the GM is able to reach every possible state, for the Perspective Shifting Sample we only provide an intuition on why it should also consider all epistemic states.

As shown in this document, when a game has many rounds, calculating what each player knows becomes very time consuming: this happens not because of the knowledge that the GM has to calculate, but because of the time that it takes to resample to the round in question (since game trees can be very large). However, we show how this may be optimized by increasing the number of total models and the number of inconsistent models. Moreover, we show that for dynamic epistemic games we must allow some inconsistent models so that the resampling may finish.

In an EGGP system, while players only have fifteen seconds to decide a move, there is no defined time limit for the GM: this is because until GDL-III the GM's job was straightforward (only having to update the true state of the game and not having to consider more

states than that one). Because the situation changes in GDL-III, it is necessary to calculate what players know. With that in mind, the GM needs to consider the possible states of the players. This significantly increases the time complexity of the GM. Whereas players in a GGP-I system only have 15 seconds to calculate everything they need, it would only be fair that the GM has some kind of constraint as well.

Finally, to conclude this dissertation, we recall an example brought up in Chapter 1 about automated cars. By successfully developing this GM, we allow game designers to build GDL-III games where, for instance, in order for an agent (car) to pass another agent (car), he must know that there is no risk in doing so. The risk can then be defined as an auxiliar term in the game to represent conditions such as the type of the road, weather, etc. These agents can then be tested in different games, with different risk conditions, and find the best solution for their risks. This way, stakeholders can practice real world problems without having to spend funds on cars: since the car would be the agent playing the game controlled by our solution.

5.1 Future Work

The following paragraphs present some of the possible features and improvements:

Nested Knowledge Rules Support — GDL-III supports the use of nested knowledge keywords through the use of auxiliar rules. However, the GM does not support this functionality. Like the GDL-III nested knowledge rule is defined as:

```
1  (a (knows r2 p))
2  (<= (goal r1 100) (knows r1 a))
```

This could be achieved by creating our representation rules that follow the same type: $K_{r1}a$ and $K_{r2}p$, with $K_{r1}a$ points to $K_{r2}p$. When applying the semantics of the nested knowledge rule will start with $K_{r2}p$ and then proceed to apply the outwards rule $K_{r1}a$;

Timed Resample — Our developed resample strategy can adapt to a timely resample, since it can be stopped whenever the re sampling turn matches the turn of the game. We will add a time constraint so that the resample can stop even if the bag of models it is not completely consistent;

Knowledge Terms Grounder — The developed knowledge terms grounder has a limitation, were it only creates knowledge rules that have one variable. We will use the same intuition when explained the intuition behind it. Moreover, since the semantics of GDL-III and ASP are similar, this can also be achieved by using an ASP Solver such as Clingo to ground the knowledge rules;

Comparing Legal Play Sequences — The methods that compare the legal play sequences may be optimized by not comparing all play sequences. Since we allow some models to be inconsistent, we can adapt the solution to not keep comparing consistent legal play sequences with inconsistent ones;

Prespective Shifting Sampler — We can optimize this Sampler by whenever the move we are trying to apply is not legal, to try to apply legal move of another player;

Dynamic Epistemic Sampling — A dynamic solution can be achieved by increasing, or decreasing, the number of inconsistent models throughout the rounds of a game;

Developing Games — As GDL-III is a recent language extension, there are not many games developed. It would be interesting to implement more games specially public and private announcement games;

Player Implementation — By using parts of our developed prototype a player that can reason with in epistemic games can be built. This can be achieved by using our Knowledge module and the Random Sampler;

User Game Interface — The base package that we started our solution from allows the description of an interface for the game to be displayed. This can be accomplished by adapting those interface classes from the base package to allow GDL-III games to be displayed on screen so that viewers can see the game being played out;

Optimization for Single Player Games In case of a single player game there is no motive to use the common knowledge as the constraint to trigger resample, since in those games it is the knowledge set is the same because there is only one player. Instead of recurring to the transitive closure algorithm to calculate if states are inconsistent, one can redirect it in order to only calculate the models that are indistinguishable for that player;

Plausibility Comparison between Samplers This can be achieved by calculating the number of expected plausible states at a certain round for both Samplers.

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Appendix A

Class Diagrams

This chapter we show the class diagrams of our developed prototype.

A.1 Overview

The class that deals with the knowledge of the players at a certain round of the game is called Knowledge State Machine. This abstract class was developed with the intention to implement any kind of logic semantics to calculate the knowledge of the players. We extended this class and implement their methods to use model sampling with GDL-III semantics. This class is called Dynamic Epistemic State Machine (It is an intermediate class that in Chapter 3 we assume in the Server module). This class only deals with a Machine State - the true state of the game. It uses the classes presented in the figure A.1. Specifically, those classes are:

KnowledgeRule java representation that we said previously. It is necessary at least one of this objects, otherwise it is not a GDL-III game;

DynamicEpistemicSampler entity that manages the models;

AccessibilityRelationManager entity that manages the structures - where we save the information about the players (pre semantics);

KnowledgeVerifier entity that implements the semantics of the GDL-III (post-semantics).

A.2 Terms Processor

Our implementation only recognizes KnowledgeRules. Those can be one of the two classes:

- CommonKnowledge - equivalent to the GDL-III keyword: (knows P), where P is a proposition.
- PersonalKnowledge - equivalent to the GDL-III keyword: (knows R P), where R is a role and P is a proposition.

The Knowledge Terms Processor, which can be found in the figure A.2, is the entity that parses the terms from a GDL-III description into Personal Knowledge or Common Knowledge.

A.3 Dynamic Epistemic Sampler

The Dynamic Epistemic Sampler is an extension of a Sampler. The Sampler is a simple class that saves the models of the game that we have in a Bag. As we can see the in the figure A.3, the most important methods for Dynamic Epistemic Sampler are:

getSamplerRound Returns the round that the sampler is currently on;

sample Updates all the models into the next round by selecting a legal joint move;

updateMState Epistemic updates the mstate of all the models inside the bag, according to what the players can't distinguish;

updateMState Epistemic updates the mstate of a specific number of models, given by their ID, and according to what the players can't distinguish;

resample Resamples a specific number of models, given by their ID, in a specific round according, or not, to their moves. Essentially, the same as the **sample** but for a specific round;

setTargetKnowsRules Similar to what was described in section **??**

To develop a Dynamic Epistemic Sampler it is only necessary to implement the referred abstract methods. There are two types of Dynamic Epistemic Samplers:

1. Random Sampler;
2. Perspective Shifting Sampler;

A.4 Accessibility Relation Manager

This class manages the accessibility relations necessary to the calculation of knowledge of the players. For that effect, we use a Accessibility Relation Structure per player. This object uses a boolean matrix that is $N \times N$, where N is the number of models presented in the Sampler.

From the figure A.4 shows the ARS of a specific round is called a Krike structure, and the Kripke structures of all the rounds is called a Kripke history. The methods in this figure are:

updateCurrentAccessibilityRelation updates the current ARS. To be used after the sample of the Sampler;

getCurrentStructures returns the Kripke structures of the current round;

getStructures returns the Kripke structure of a specific round;

updateAcessibilityRelation updates the ARS of a specific round and only for the modelID passed as argument. It is used in the re sampling;

saveCurrentStructure saves the Kripke structure of the current round;

A.5 Knowledge Verifier

The knowledge verifier is the class where the semantics of the GDL-III are implemented. Whereas the Accessibility Relation Manager as the function of updating the ARS, this class has the objective of verify if the Knowledge Rules are true in the models' state that the players can't distinguish between.

The methods of the Knowledge Verifier are show in Figure A.5. In this class, the Knowledge Rules are verified changes from whether we are verifying a model or the actual state of the game:

- Personal Knowledge

  State Verify if the target is true in all the useful states of the player's models;

  Model Verify if the target is true in all the indistinguishable states of the player's models;

- Common Knowledge

  State Check all the models that are transitive closure useful. Then, verify if the target is true in all of the states of those models;

  Model Verify the target in all the transitive closure set of the given model.

A.6 Dynamic Epistemic Knowledge Machine

This class implements the Knowledge State Machine. It uses all the classes described to calculate the knowledge of the players in the state of the game at a certain round. As we can see from the figure A.6, it is necessary a Dynamic Epistemic Sampler and an

Accessibility Relations Manager. The Knowledge Verifier is used in the method getTurnKnowledge, since it is the one that returns which knowledge terms are satisfied in the current round.

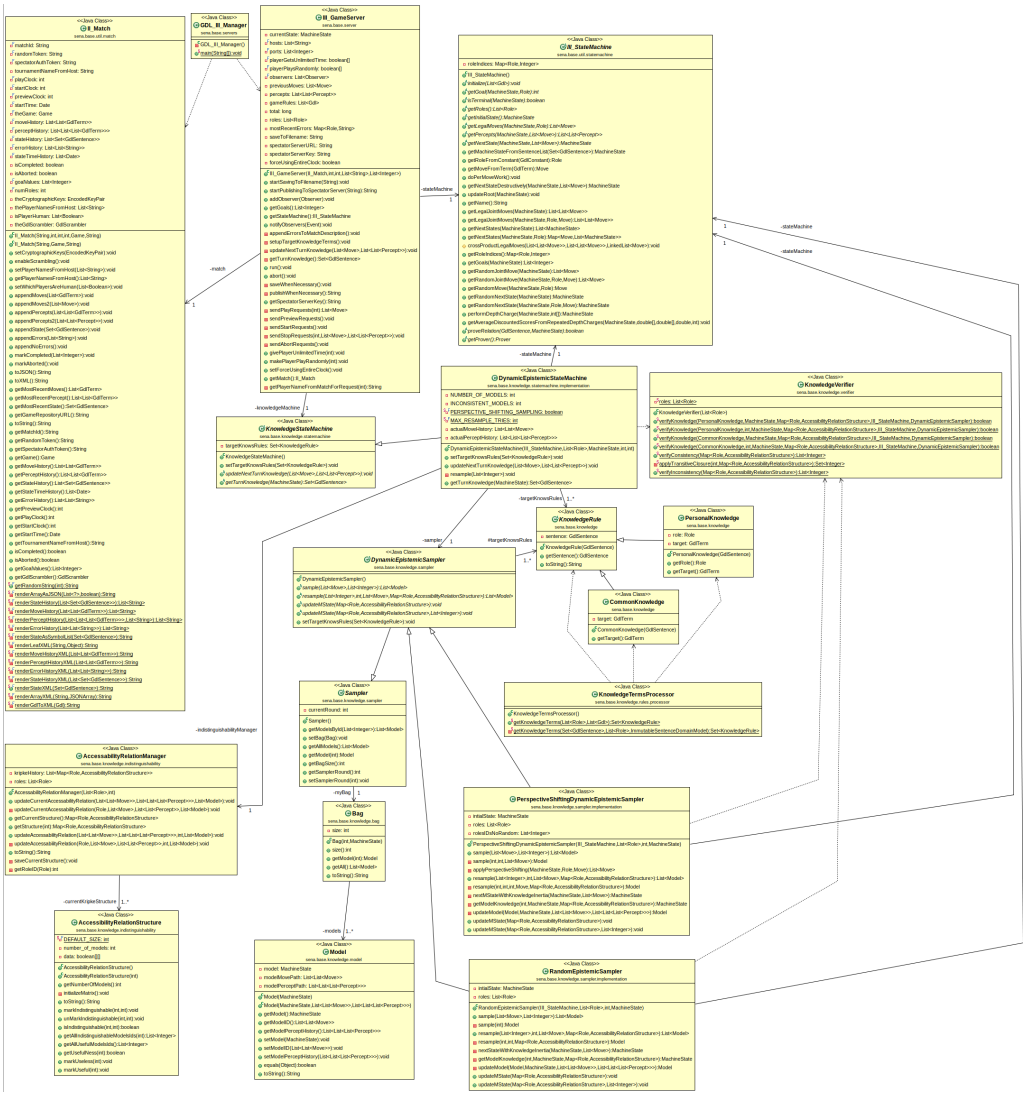

Figure A.1: Overview

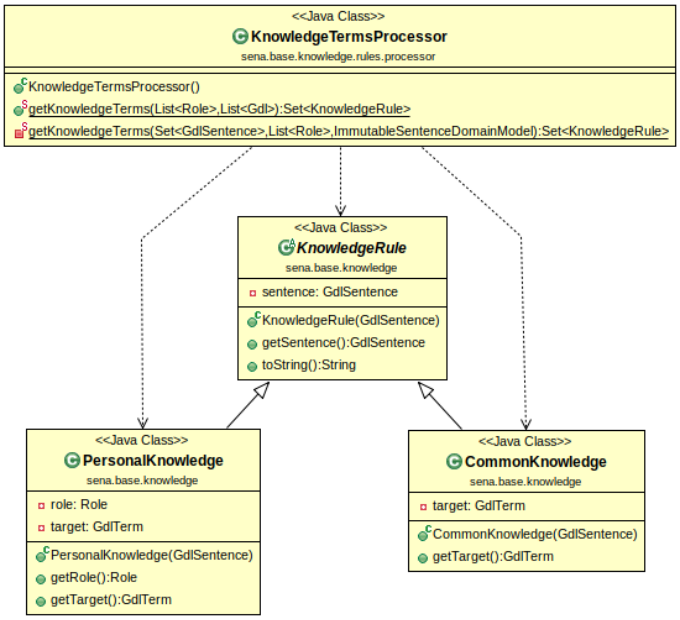

Figure A.2: Rules Processor

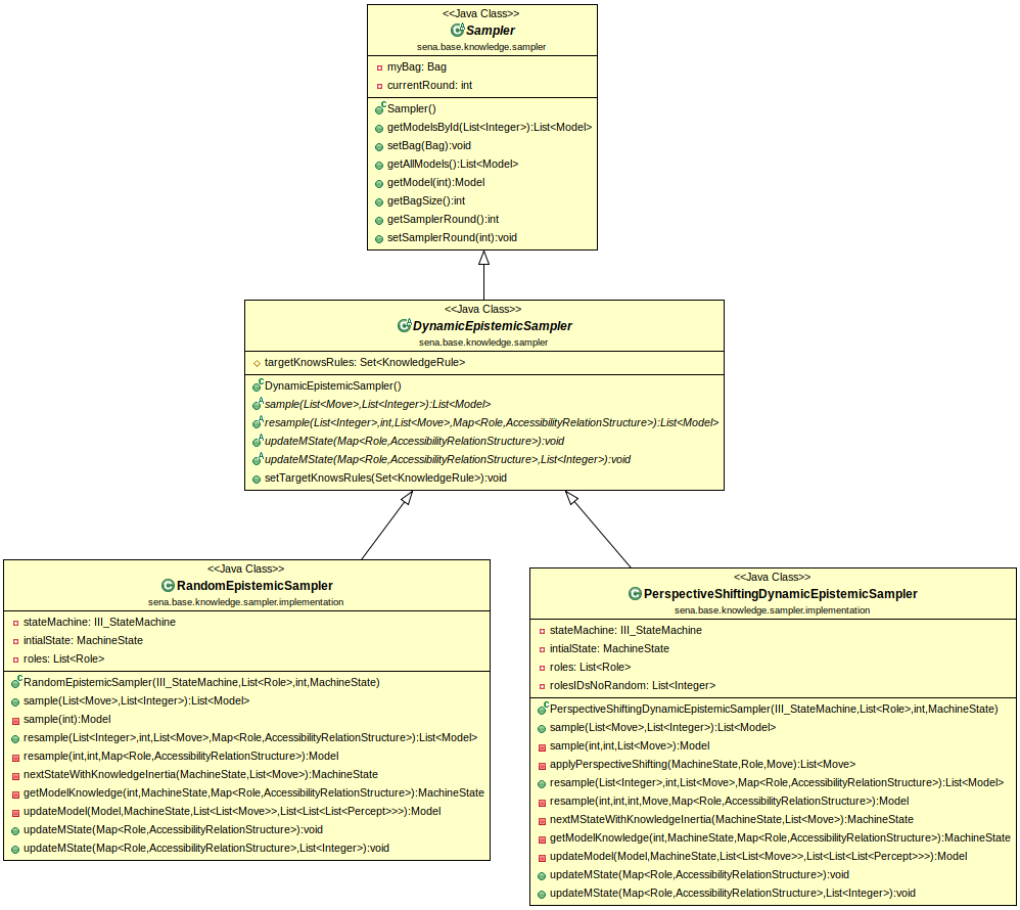

Figure A.3: Sampler

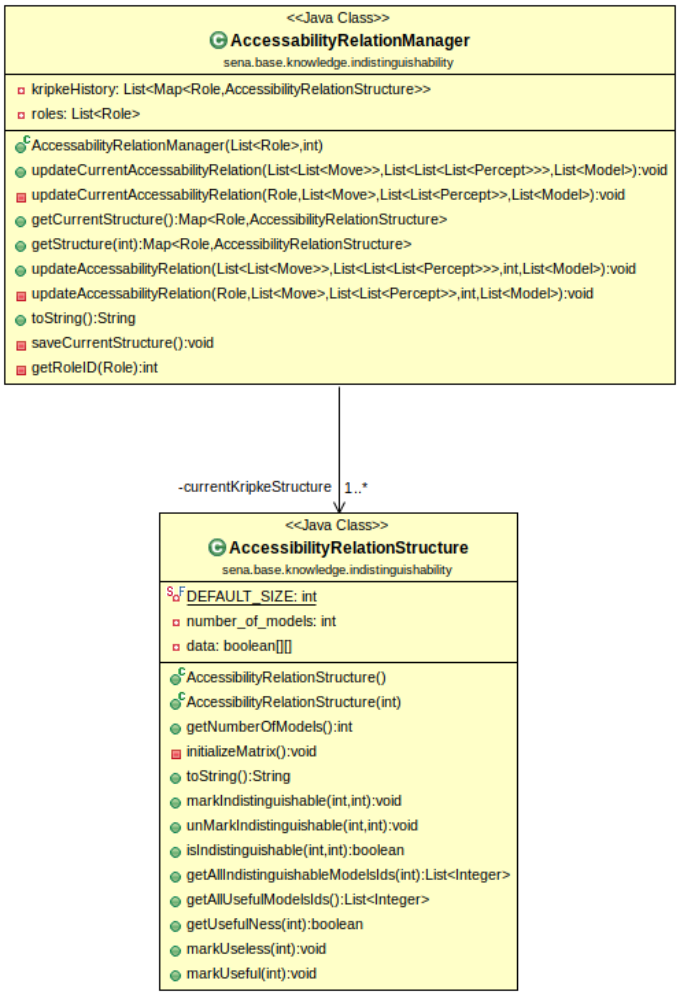

Figure A.4: Accessibility Relation Manager

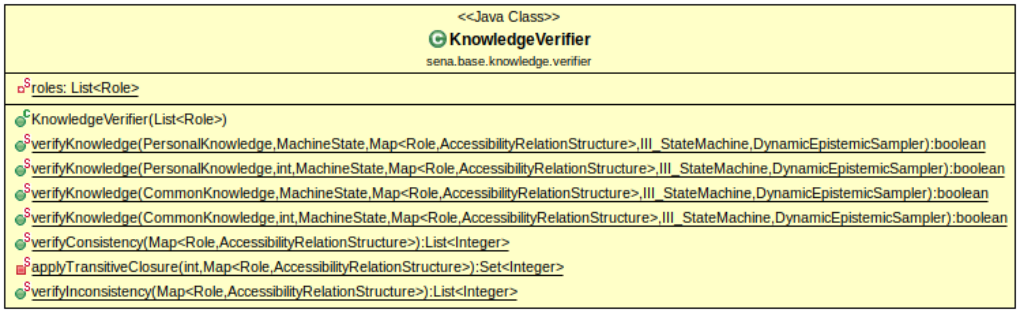

Figure A.5: Knowledge Verifier

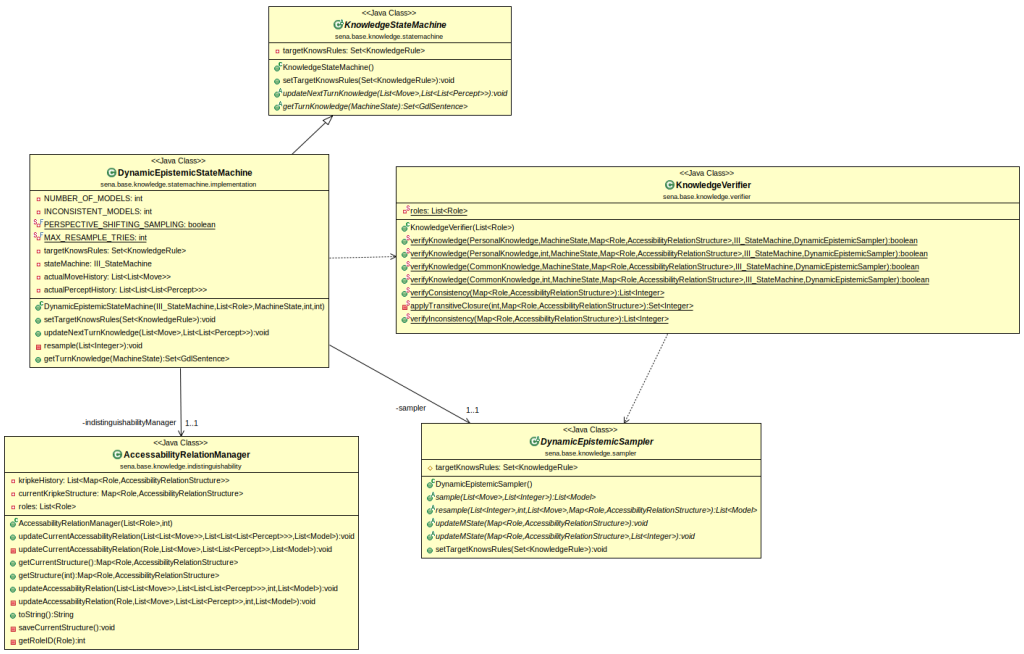

Figure A.6: Dynamic Epistemic Knowledge Machine

Appendix B

Games used for Analysis

This section shows the games used for time analysis of our referee.

B.1 Number Guessing Epistemic

This is the description of the GDL-III game of the number guessing epistemic. The game only ends if a player knows the secret number set by the random player at step zero. Every other round a player can ask if the secret is strictly less than an number, and if it is, it receives *yes* as percepts. Otherwise, the player doesn't receive anything and it should consider as a no, or that is bigger or equal to the number that he asked.

```
1  %%% ROLES
2  (role random)
3  (role player)
4
5  %%% INITIAL STATE
6  (init (step 0))
7
8  %%% LEGAL MOVES
9  % Random chooses a number in the first step
10 (<= (legal random (choose ?n))
11    (number ?n)
12    (true (step 0))
13    )
14
15 % Random does nothing else in the other steps
16 (<= (legal random noop)
17    (not (true (step 0)))
18    )
19
20 % Player does noop in the initial step
```

```
(<= (legal player noop)
  (true (step 0))
  )

% Player asks if less a number in the next turns
(<= (legal player (ask_if_less ?n))
  (number ?n)
  (not (true (step 0)))
  )

%%% STATE UPDATE
% When random chooses the number it stays a secret
(<= (next (secret ?n))
  (does random (choose ?n))
  )

% Inertia for secret
(<= (next (secret ?n))
  (true (secret ?n))
  )

% Increase the step of the game
(<= (next (step ?n))
  (true (step ?m))
  (succ ?m ?n)
  )

%%% PERCEPTS
% player sees yes if the secret is smaller than the number he said
(<= (sees player yes)
  (does player (ask_if_less ?n))
  (true (secret ?m))
  (less ?m ?n)
  )

%%% TERMINAL CONDITIONS
(<= terminal
  (knows_the_number player)
  )

(<= terminal
  (true (step 12))
  )

%%% GOAL CONDITIONS
(goal random 100)

(<= (goal player 100)
    (knows_the_number player)
    )
```

```
71
72 (<= (goal player 0)
73     (not (knows_the_number player))
74     )
75
76 %%% AUXILIARS
77 % knows the number
78 (<= (knows_the_number ?r)
79     (role ?r) (knows ?r (num ?n))
80     )
81
82 % less predicate
83 (<= (less ?m ?n) (succ ?m ?n))
84 (<= (less ?m ?n) (succ ?m ?l) (less ?l ?n))
85
86 % num is the secret
87 (<= (num ?n) (true (secret ?n)))
88
89 (number 1)
90 (number 2)
91 (number 3)
92 (number 4)
93 (number 5)
94 ...
95 (number 127)
96 (number 128)
97
98 (succ 0 1)
99 (succ 1 2)
100 (succ 2 3)
101 (succ 3 4)
102 (succ 4 5)
103 ...
104 (succ 127 128)
```

B.2 Muddy Children

This section shows the used Muddy Children game with 3 children used in Section 4.2.2. At round zero the random role sets the children with mud. They can only see their percepts at round 2. Whenever they know they have mud the game ends. So if only one has mud, the game ends at round 2. Children can do *noop* if they don't know they have mud, or say *yes* if they do know. However, with this description they will never say it.

```
1 %%% ROLES
2 (role random)
3 (role ann)
4 (role bob)
5 (role cat)
```

```
6
7  %%% INITIAL STATE
8  (init (round 0))
9
10 %%% LEGAL MOVES
11 (<= (legal random noop)
12     (not (true (round 0)))
13     )
14
15 (<= (legal random (muddy ?a ?b ?c))
16     (true (round 0))
17     (yesno ?a)
18     (yesno ?b)
19     (yesno ?c)
20     (not (allzero ?a ?b ?c))
21     )
22
23 (<= (legal ?c sayYes)
24     (knows ?c (isMuddy ?c))
25     )
26
27 (<= (legal ?c noop)
28     (role ?c)
29     (distinct ?c random)
30     (not (knows ?c (isMuddy ?c)))
31     )
32
33 %%% STATE UPDATE
34 (<= (next (has ann mud))
35     (does random (muddy 1 ?b ?c))
36     )
37
38 (<= (next (has bob mud))
39     (does random (muddy ?a 1 ?c))
40     )
41
42 (<= (next (has cat mud))
43     (does random (muddy ?a ?b 1))
44     )
45
46 (<= (next (has ?c mud))
47     (true (has ?c mud))
48     )
49
50 %%% PERCEPTS
51 (<= (sees ?c (dirt ?d))
52     (role ?c)
53     (role ?d)
54     (true (has ?d mud))
55     (distinct ?c ?d)
```

```
56      )
57
58  (<= (sees ?c (Yes ?d))
59      (role ?c)
60      (does ?d sayYes)
61      )
62
63  %%% TERMINAL CONDITIONS
64  (<= terminal
65      (knows ?c (isMuddy ?c))
66      )
67
68  %%% GOAL CONDITIONS
69  (<= (goal ?c 100)
70      (role ?c)
71      (knows ?c (isMuddy ?c))
72      )
73
74  (<= (goal ?c 0)
75      (role ?c)
76      (not (knows ?c (isMuddy ?c)))
77      )
78
79  (goal random 0)
80
81  %%% AUXILIARS
82  (yesno 0)
83  (yesno 1)
84
85  (allzero 0 0 0)
86
87  (<= (isMuddy ?c)
88      (true (has ?c mud))
89      )
```

B.3 Russian Cards Games

This game is an adaptation of the Russian Cards Problems to an 1-1-1 setting. There are 4 different numbers. In this game, numbers are dealt in a matter of 1 for Alice, 1 for Bob and 1 for Trudy. Bob and Alice then exchange public announcements of their cards in order to figure out Trudy's card. It is a cooperation game between Alice and Bob but also a zero-sum game with Trudy. Trudy, can add uncertainty in the round 3, if the game doesn't end by then. She is shown the setting of the cards in the begining so that at round 3 she can swap her card with one of the players without the other one sees it. The depth of this game is at maximum 6 rounds. The objective of the game is:

Trudy - disturb the other players so that they can't figure out her cards in the total rounds

of the game.

Alice and Bob - share information about their cards in order to find out Trudy's card (the description is according to common knowledge about Trudy's cards, because according to the way the game is described, if Alice knows Bob's cards and Bob Alice's, then Trudy's cards will always be common knowledge.

```
1  %%% ROLES
2  (role random)
3  (role alice)
4  (role bob)
5  (role trudy)
6
7  %%% INITIAL STATE
8  (init (round 0))
9
10 %%% LEGAL MOVES
11 % Random deals cards in the initial round
12 (<= (legal random (deal ?a1 ?b1 ?t1))
13     (true (round 0))
14     (number ?a1) (number ?b1) (number ?t1)
15     (distinct ?a1 ?b1) (distinct ?a1 ?t1) (distinct ?b1 ?t1)
16     )
17
18 % Random does nothing if it is not the first round
19 (<= (legal random noop)
20     (not (true (round 0)))
21     )
22
23 % Trudy can do normal noop
24 (<= (legal trudy noop)
25     (not (true (turn trudy)))
26     )
27
28 % Trudy can do a different noop if is her turn to play
29 (<= (legal trudy noop1)
30     (true (turn trudy))
31     )
32
33 % Trudy can switch a card with alice player
34 (<= (legal trudy (switchalice ?t1 ?a1))
35     (true (turn trudy))
36     (true (has trudy ?t1))
37     (true (has alice ?a1))
38     )
39
40 % Trudy can switch a card with bob player
41 (<= (legal trudy (switchbob ?t1 ?b1))
42     (true (turn trudy))
43     (true (has trudy ?t1))
```

```
44      (true (has bob ?b1))
45      )
46
47
48  % Bob and Alice make public announcements about their card, in a triplet,
49  % where one card is for sure theirs, but the others can be any subset of the
50  % Cartesian product of the remaining numbers
51  (<= (legal ?r (have ?n1 ?n2 ?n3))
52      (true (turn ?r)) (number ?n2) (number ?n3) (role ?r) (distinct ?r trudy)
53      (true (has ?r ?n1)) (not (true (has ?r ?n2))) (not (true (has ?r ?n3)))
54      )
55  (<= (legal ?r (have ?n1 ?n2 ?n3))
56      (true (turn ?r)) (number ?n1) (number ?n3) (role ?r) (distinct ?r trudy)
57      (not (true (has ?r ?n1)))  (true (has ?r ?n2)) (not (true (has ?r ?n3)))
58      )
59
60  (<= (legal ?r (have ?n1 ?n2 ?n3))
61      (true (turn ?r)) (number ?n1) (number ?n2) (role ?r) (distinct ?r trudy)
62      (not (true (has ?r ?n1))) (not (true (has ?r ?n2)))  (true (has ?r ?n3))
63      )
64
65  (<= (legal alice noop)
66      (not (true (turn alice)))
67      )
68
69  (<= (legal bob noop)
70      (not (true (turn bob)))
71      )
72
73  %%% STATE UPDATE
74  % Increase the round of the game
75  (<= (next (round ?n)) (true (round ?m)) (succ ?m ?n))
76
77  % Players have the numbers as soon as random deals them
78  (<= (next (has alice ?a1))
79      (does random (deal ?a1 ?b1 ?t1))
80      )
81
82  (<= (next (has bob ?b1))
83      (does random (deal ?a1 ?b1 ?t1))
84      )
85
86  (<= (next (has trudy ?t1))
87      (does random (deal ?a1 ?b1 ?t1))
88      )
89
90  % Players keep their numbers if is not trudy's turn or she does noop1 at her turn
91  (<= (next (has ?r ?n))
92      (true (has ?r ?n))
93      (not (true (turn trudy)))
```

```
94      )
95  (<= (next (has ?r ?n))
96      (true (has ?r ?n))
97      (true (turn trudy))
98      (does trudy noop1)
99      )
100
101 % At Trudy's turn can privately switch a card
102 (<= (next (has alice ?a1))
103     (true (has alice ?a1))
104     (true (turn trudy))
105     (does trudy (switchbob ?t1 ?b1))
106     )
107 (<= (next (has bob ?b1))
108     (true (has bob ?b1))
109     (true (turn trudy))
110     (does trudy (switchalice ?t1 ?a1))
111     )
112
113 % Players only keep the switched card that Trudy gave them
114 (<= (next (has alice ?t1))
115     (true (turn trudy))
116     (does trudy (switchalice ?t1 ?a1))
117     (true (has alice ?a1))
118     )
119
120 (<= (next (has bob ?t1))
121     (true (turn trudy))
122     (does trudy (switchbob ?t1 ?b1))
123     (true (has bob ?b1))
124     )
125
126 (<= (next (has trudy ?n1))
127     (true (turn trudy))
128     (does trudy (switchalice ?t1 ?n1))
129     (true (has alice ?n1))
130     )
131
132 (<= (next (has trudy ?n1))
133     (true (turn trudy))
134     (does trudy (switchbob ?t1 ?n1))
135     (true (has bob ?n1))
136     )
137
138 % Turn iteration
139 (<= (next (turn alice))
140     (or
141         (true (turn trudy)) (true (round 0))
142         )
143     )
```

```
144
145 (<= (next (turn bob))
146     (true (turn alice))
147     )
148
149 (<= (next (turn trudy))
150     (true (turn bob))
151     )
152
153 %%% PERCEPTS
154 % NOTE - at beggining of turn 4 if Trudy switches the card with someone,
155 % that someone is going to see 2 percepts: one that cooresponds to the card that
156 % he currently owns and the other, that represents the swap action of trudy.
157 % Meaning that "You currently own this card, but trudy switched it.
158 % So in the next turn the only percept you'll recieve is the new card".
159
160 % Alice sees her card
161 (<= (sees alice (cardA ?a1))
162     (or (true (has alice ?a1)) (does random (deal ?a1 ?b1 ?t1)) )
163     )
164
165 % Bob sees his card
166 (<= (sees bob (cardB ?b1))
167      (or (true (has bob ?b1)) (does random (deal ?a1 ?b1 ?t1)) )
168     )
169
170 % Trudy sees her card
171 (<= (sees trudy (cardT ?t1))
172     (or (true (has trudy ?t1)) (does random (deal ?a1 ?b1 ?t1)) )
173     )
174
175 (<= (sees trudy (deal ?a1 ?b1 ?t1))
176     (does random (deal ?a1 ?b1 ?t1))
177     )
178
179 % All the players are warned about the public announcement (except random)
180 (<= (sees ?o (haveOne ?r ?x ?y ?z))
181     (role ?o) (does ?r (have ?x ?y ?z)) (distinct ?o random))
182
183 % Whomever trudy switchs her card with, gets warned
184 (<= (sees alice (swap ?trudyC ?rC))
185     (does trudy (switchalice ?trudyC ?rC))
186     )
187 (<= (sees bob (swap ?trudyC ?rC))
188     (does trudy (switchbob ?trudyC ?rC))
189     )
190
191 % Of course trudy as well
192 (<= (sees trudy (swap ?trudyC ?rC))
193     (does trudy (switchalice ?trudyC ?rC))
```

```
194       )
195  (<= (sees trudy (swap ?trudyC ?rC))
196       (does trudy (switchbob ?trudyC ?rC))
197       )
198
199  %%% TERMINALS
200  % Game ends at round 6
201  % or is public knowledge one of trudy's card (doesn't matter because
202  % the way the game is described
203  % when is public knowledge one of her cards, it is public knowledge the other)
204
205  (<= terminal
206       (true (round 6))
207       )
208
209  (<= terminal
210       (commonKnowledge)
211       )
212
213  %%% GOALS
214  % The objective of the game is for trudy to disrupt Alice and Bob's reasoning.
215  % And for alice and bob to know Trudy's card.
216
217  (goal random 100)
218
219  (<= (goal trudy 100)
220       (true (round 6))
221       )
222
223  (<= (goal trudy 0)
224       (not (true (round 6)))
225       )
226
227  (<= (goal alice 100)
228       (commonKnowledge)
229       )
230  (<= (goal alice 0)
231       (not (commonKnowledge))
232       )
233
234  (<= (goal bob 100)
235       (commonKnowledge)
236       )
237
238  (<= (goal bob 0)
239       (not (commonKnowledge))
240       )
241
242  %%% AUXILIARS
243  (<= (hasTrudy ?t1)
```

```
244     (true (has trudy ?t1))
245     )
246
247 (<= (commonKnowledge)
248     (knows (hasTrudy ?t1))
249     )
250
251 (number 0)
252 (number 1)
253 (number 2)
254
255 (succ 0 1)
256 (succ 1 2)
257 (succ 2 3)
258 (succ 3 4)
259 (succ 4 5)
260 (succ 5 6)
```

Appendix C

Hats Puzzle

This game, is an adaptation of the hats puzzle [SHDT08], where there are four players and all of them are assigned a hat. There are 2 blue hats and 2 red ones. Three of the players are put in a stair in such way that they can only see the colour of the players' hats in front of them (down the stairs). But they can't see the ones behind them (up the stairs).

The game is designed in the following way:

P1 - is not in the stairs. Thus none of the players can see his hat (neither does he);

P2 - is situated at the lowest stair. Can't see any of the hats;

P3 - above P2. Thus can see the P2's hat;

P4 - above P3. Can see both P2 and P3' hats.

The game doesn't have a winning condition, since it's only to test if the players make a right move. Moreover not all of them can win the game. It depends on the assigned setting. Players can publicly announce the colour of their hat if they know it. Otherwise *noop*.

```
1  %%% ROLES
2  (role random)
3  (role prisoner1)
4  (role prisoner2)
5  (role prisoner3)
6  (role prisoner4)
7
8
9  %%% INITIAL STATE
10 (init (round 0))
11
12 %%% LEGAL MOVES
```

```
13 (<= (legal random noop)
14     (not (true (round 0)))
15     )
16
17 (<= (legal random (putHat ?a ?b ?c ?d))
18     (true (round 0))
19     (yesno ?a)
20     (yesno ?b)
21     (yesno ?c)
22     (yesno ?d)
23     (splitHats ?a ?b ?c ?d)
24     )
25
26 (<= (legal ?r haveRed)
27     (knows ?r (hasHatRed ?r))
28     )
29
30 (<= (legal ?r haveBlue)
31     (knows ?r (hasHatBlue ?r))
32     )
33
34 (<= (legal ?r noop)
35     (role ?r)
36     (distinct ?r random)
37     (not (knows ?r (hasHatRed ?r)))
38     (not (knows ?r (hasHatBlue ?r)))
39     )
40
41 %%% STATE TRANSITION
42 (<= (next (has prisoner1 red))
43     (does random (putHat 1 ?b ?c ?d)))
44 (<= (next (has prisoner2 red))
45     (does random (putHat ?a 1 ?c ?d)))
46 (<= (next (has prisoner3 red))
47     (does random (putHat ?a ?b 1 ?d)))
48 (<= (next (has prisoner4 red))
49     (does random (putHat ?a ?b ?c 1)))
50 (<= (next (has prisoner1 blue))
51     (does random (putHat 0 ?b ?c ?d)))
52 (<= (next (has prisoner2 blue))
53     (does random (putHat ?a 0 ?c ?d)))
54 (<= (next (has prisoner3 blue))
55     (does random (putHat ?a ?b 0 ?d)))
56 (<= (next (has prisoner4 blue))
57     (does random (putHat ?a ?b ?c 0)))
58
59 (<= (next (has ?r red))
60     (true (has ?r red)))
61 (<= (next (has ?r blue))
62     (true (has ?r blue)))
```

```
63
64 ; Increase the step of the game
65 (<= (next (round ?n)) (true (round ?m)) (succ ?m ?n))
66
67 %%% PERCEPTS
68 (<= (sees prisoner3 (color2 ?color))
69     (role prisoner3)
70     (role prisoner2)
71     (true (has prisoner2 ?color)))
72
73 (<= (sees prisoner4 (color2 ?color))
74     (role prisoner4)
75     (role prisoner2)
76     (true (has prisoner2 ?color)))
77
78 (<= (sees prisoner4 (color3 ?color))
79     (role prisoner4)
80     (role prisoner3)
81     (true (has prisoner3 ?color)))
82
83 (<= (sees ?c (saysHasRed ?d))
84     (role ?c)
85     (does ?d haveRed))
86 (<= (sees ?c (saysHasBlue ?d))
87     (role ?c)
88     (does ?d haveBlue))
89
90 %%% TERMINALS
91 (<= terminal (true (round 5)))
92
93 %%% GOALS
94 (<= (goal ?r 0) (role ?r) )
95 (goal random 0)
96
97 %%% AUXILIARS
98 (yesno 0)
99 (yesno 1)
100
101 (splitHats 1 1 0 0)
102 (splitHats 1 0 1 0)
103 (splitHats 1 0 0 1)
104 (splitHats 0 0 1 1)
105 (splitHats 0 1 0 1)
106 (splitHats 0 1 1 0)
107
108 (<= (hasHatRed ?r)
109     (true (has ?r red)))
110 (<= (hasHatBlue ?r)
111     (true (has ?r blue)))
112
```

```
113 (succ 0 1)
114 (succ 1 2)
115 (succ 2 3)
116 (succ 3 4)
117 (succ 4 5)
```

Appendix D

Tested Problems

More games that were tested in the GM to see if the knowledge and the semantics of GDL were applied correctly.

D.1 Player Knowledge uncertainty without percepts

With this description we were testing if at round 2 a player that doesn't know the number that he owns, since he can't see it. Then he can select a new one, and he should know which number he has. This game should end after the player switches his number.

```
1  %%% ROLES
2  (role random)
3  (role player1)
4
5  %%% INITIAL STATE
6  (init (round 0))
7
8  %%% LEGAL MOVES
9  % Random deals cards in the initial round
10 (<= (legal random (deal ?a1))
11     (true (round 0))
12     (number ?a1)
13     )
14
15 % Random does nothing if it is not the first round
16 (<= (legal random noop)
17     (not (true (round 0)))
18     )
19
20 % Random does nothing if it is not the first round
21 (<= (legal player1 (switchTo ?n1))
```

```
22      (number ?n1)
23      (true (has player1 ?n2))
24      (true (round 2))
25      )
26
27  (<= (legal player1 noop)
28      (not (true (round 2)))
29      )
30
31  %%% STATE UPDATE
32  (<= (next (has player1 ?a1))
33      (does random (deal ?a1))
34      )
35
36  (<= (next (has player1 ?a1))
37      (true (has player1 ?a1))
38      (not (true (round 2)))
39      )
40
41  (<= (next (has player1 ?a1))
42      (does player1 (switchTo ?a1))
43      )
44
45
46  % Increase the round of the game
47  (<= (next (round ?n)) (true (round ?m)) (succ ?m ?n))
48
49
50  %%% TERMINAL
51  (<= terminal (true (round 5)))
52  (<= terminal
53      (knows (hasPlayer ?t1))
54      )
55
56  %%% GOALS
57  (goal random 100)
58  (<= (goal player1 100)
59      (knows (hasPlayer ?t1))
60      )
61
62  (<= (goal player1 0)
63      (true (round 5))
64      )
65
66  %%% AUXILIARS
67  (<= (hasPlayer ?a1)
68      (true (has player1 ?a1))
69      )
70
71  (number 0)
```

```
(number 1)
(number 2)
(number 3)
(number 4)
(number 5)
(number 6)
(number 7)



(succ 0 1)
(succ 1 2)
(succ 2 3)
(succ 3 4)
(succ 4 5)
(succ 5 6)
```

D.2 Player Knowledge without percepts

This is a variation of the previous example. The player should be able to switch the number that he has if he knows a proposition that is set since the initial state. Every round he switches the number because the proposition is always true.

```
%%% ROLES
(role random)
(role player1)

%%% INITIAL STATE
(init (round 0))
(init (alpha 1))
(init (has player1 1))

%%% LEGAL MOVES
% Random does nothing if it is not the first round
(legal random noop)

(<= (legal player1 (switchTo ?n1))
    (number ?n1)
    (knows player1 (beta 1))
    )

(<= (legal player1 noop)
    (not (knows player1 (beta 1)))
    )

%%% STATE UPDATE
(<= (next (has player1 ?a1))
    (does player1 (switchTo ?a1))
    )
```

```
27
28 (<= (next (has player1 ?a1))
29     (true (has player1 ?a1))
30     (does player1 noop)
31     )
32
33 (<= (next (alpha 1))
34     (does player1 (switchTo ?a1))
35     )
36
37 % Increase the round of the game
38 (<= (next (round ?n)) (true (round ?m)) (succ ?m ?n))
39
40 %%% PERCEPTS
41 (<= (sees player1 a) (does player1 noop) (true (has player1 ?a1)) )
42 (<= (sees player1 (b ?n1)) (does player1 (switchTo ?n1)))
43
44 %%% TERMINALS
45 (<= terminal (true (round 5)))
46
47 %%% GOALS
48 (goal player1 100)
49 (goal random 100)
50
51 %%% AUXILIARS
52 (<= (beta 1)
53     (true (alpha 1))
54     )
55
56 (number 0)
57 (number 1)
58 (number 2)
59 (number 3)
60 (number 4)
61 (number 5)
62 (number 6)
63 (number 7)
64
65 (succ 0 1)
66 (succ 1 2)
67 (succ 2 3)
68 (succ 3 4)
69 (succ 4 5)
70 (succ 5 6)
```

D.3 Steal the Ruby

In this description we tested: $K_r p$ and $K_r \neg p$. At the initial state player Alice has a ruby, that she can drop, or not at round 1. At round 2, player Bob can as if she has or not the ruby. And at round 3 player Bob has different moves. Try to steal the ruby knowing that Alice has it, or try to steal it without having it. The same thing about not stealing it. Either doesn't steal if he knows that she doesn't have, or not.

```
%%% ROLES
(role alice)
(role bob)

%%% INITIAL STATE
(init (round 0))
(init (has alice ruby))

%%% LEGAL MOVES
(<= (legal alice noop)
    (not (true (round 1))))

(<= (legal bob noop)
    (true (round 0))
    )

(<= (legal bob noop)
    (true (round 1))
    )

(<= (legal alice drop)
    (true (round 1))
    )
(<= (legal alice noop)
    (true (round 1))
    )

(<= (legal bob ask)
    (true (round 2))
    )
(<= (legal bob noop)
    (true (round 2))
    )

(<= (legal bob mugKnowingly)
    (true (round 3))
    (knows bob (hasRuby alice))
    )

(<= (legal bob noopKnowingly)
    (true (round 3))
    (knows bob (doesnotHaveRuby alice))
```

```
43      )
44
45  (<= (legal bob steal)
46      (true (round 3))
47      )
48
49  %%% STATE UPDATE
50  (<= (next (has alice ruby))
51      (true (has alice ruby)) (not (true (round 1)))
52      )
53
54  (<= (next (has alice ruby))
55      (true (has alice ruby))  (true (round 1)) (does alice noop)
56      )
57
58  (<= (next (didBob ?move)) (does bob ?move) (true (round 3)) )
59
60  % Increase the round of the game
61  (<= (next (round ?n)) (true (round ?m)) (succ ?m ?n))
62
63  %%% PERCEPTS
64  (<= (sees bob have) (true (has alice ruby)) (does bob ask))
65  (<= (sees bob nHave) (not (true (has alice ruby))) (does bob ask) )
66
67  %%% TERMINALS
68  (<= terminal (true (round 4)))
69
70  %%% GOALS
71  (goal alice 100)
72
73  (<= (goal bob 100)
74      (true (didBob steal)) (true (has alice ruby)) )
75  (<= (goal bob 0)
76      (true (didBob steal)) (not (true (has alice ruby))) )
77  (<= (goal bob 80)
78      (true (didBob mugKnowingly)) (true (has alice ruby)))
79  (<= (goal bob 80)
80      (true (didBob noopKnowingly)) (not (true (has alice ruby))))
81
82  %%% AUXILIARS
83  (<= (hasRuby alice)
84      (true (has alice ruby))
85      )
86
87  (<= (doesnotHaveRuby alice)
88      (not (true (has alice ruby)))
89      )
90
91  (succ 0 1)
92  (succ 1 2)
```

```
(succ 2 3)
(succ 3 4)
(succ 4 5)
(succ 5 6)
```

D.4 Conjunction

With this description we wanted to evaluate if the semantics of a knowledge conjunction were applied. It this description the terminal state is defined as a conjunction of rules, where player one knows his card and the opponent. But he is only shown his. Therefore the game will always end with the other condition and it will never be true that he knows both his and the opponent's card.

```

%%% ROLES
(role random)
(role player1)
(role player2)

%%% INITIAL STATE
(init (round 0))

%%% LEGAL MOVES
% Random deals cards in the initial round
(<= (legal random (deal ?a1 ?a2))
    (true (round 0))
    (number ?a1)
    (number ?a2)
    )

% Random does nothing if it is not the first round
(<= (legal random noop)
    (not (true (round 0)))
    )

(legal player1 noop)
(legal player2 noop)


%%% STATE UPDATE
(<= (next (has player1 ?a1))
    (does random (deal ?a1 ?a2))
    )

(<= (next (has player2 ?a2))
    (does random (deal ?a1 ?a2))
    )

```

```
(<= (next (has player1 ?a1))
    (true (has player1 ?a1))
    )

(<= (next (has player2 ?a2))
    (true (has player2 ?a2))
    )

% Increase the round of the game
(<= (next (round ?n)) (true (round ?m)) (succ ?m ?n))

%%% PERCEPTS
(<= (sees player1 ?a1)
    (does random (deal ?a1 ?a2)))

(<= (sees player1 ?a1)
    (true (has player1 ?a1)))

%%% TERMINALS
(<= terminal (true (round 5)))
(<= terminal
    (knows player1 (bla1 0))
    (knows player1 (bla2 1))
    )

(<= terminal
    (knows player1 (bla1 1))
    (knows player1 (bla2 0))
    )

%%% GOALS
(goal random 100)
(goal player1 0)
(goal player2 0)

%%% AUXILIARS
(<= (bla1 ?a1)
    (true (has player1 ?a1))
    )

(<= (bla2 ?a2)
    (true (has player2 ?a2))
    )

(number 0)
(number 1)

(succ 0 1)
(succ 1 2)
(succ 2 3)
```

```
86 (succ 3 4)
87 (succ 4 5)
88 (succ 5 6)
```

Appendix E

Run the Game Master

This Project is an extension of the ggp-base-package that has two additional servers. The sena package follows the same idea than the ggp-base package. You can find the main classes servers in sena.base.servers; For knowledge you can find under sena.base.knowledge; and sampling: sena.base.knowledge.samplers.

Users can use apache-ant to build the project. For that simply go to the main directory of the project and:

- Connect each player to the desired address and port;
- run: ant buildName -Darg0=TounamentName -Darg1=game -Darg2=StartTime -Darg3=RoundTime -Darg4=N -Darg5=β -Darg6=Player1Address -Darg7=Player1Port -Darg8=Player1Name -Darg9=Player2Address -Darg10=Player2Port -Darg11=Player2Name ...

Where N is the total number of models and β is the number of inconsistent models allowed.

We have created several different builds:

GDL_III_Manager_1_Player for a single player game. Note that this setting needs to have two players as argument. The random, and the single-player;

GDL_III_Manager_2_Players for a multi-player game with two players;

GDL_III_Manager_3_Players for a multi-player game with three players;

GDL_III_Manager_4_Players for a multi-player game with four players;

GDL_III_Manager_5_Players for a multi-player game with five players;

The following are examples of how to run the Game Master.

```
For a Single-Player game:
ant GDL_III_Manager_1_Player -Darg0=myTournament
  -Darg1=numberGuessingEpistemic32 -Darg2=60 -Darg3=15 -Darg4=50 -Darg5=40
  -Darg6=127.0.0.1 -Darg7=9999 -Darg8=RandomPlayer -Darg9=127.0.0.1
  -Darg10=4001 -Darg11=PlayerOne -Darg12=127.0.0.1 -Darg13=4002
  -Darg14=PlayerTwo

For a 2 Multi-Player game:
ant GDL_III_Manager_2_Players -Darg0=myTournament -Darg1=muddyChildren2
-Darg2=60 -Darg3=15 -Darg4=50 -Darg5=40 -Darg6=127.0.0.1 -Darg7=9999
-Darg8=RandomPlayer -Darg9=127.0.0.1 -Darg10=4001 -Darg11=PlayerOne
-Darg12=127.0.0.1 -Darg13=4002 -Darg14=PlayerTwo

For a 3 Multi-Player game:
ant GDL_III_Manager_3_Players -Darg0=myTournament -Darg1=muddyChildren3
-Darg2=60 -Darg3=15 -Darg4=50 -Darg5=40 -Darg6=127.0.0.1 -Darg7=9999
-Darg8=RandomPlayer -Darg9=127.0.0.1 -Darg10=4001 -Darg11=PlayerOne
-Darg12=127.0.0.1 -Darg13=4002 -Darg14=PlayerTwo -Darg15=127.0.0.1
-Darg16=4003 -Darg17=PlayerThree

For a 4 Multi-Player game:
ant GDL_III_Manager_4_Players -Darg0=myTournament -Darg1=muddyChildren4
-Darg2=60 -Darg3=15 -Darg4=50 -Darg5=40 -Darg6=127.0.0.1 -Darg7=9999
-Darg8=RandomPlayer -Darg9=127.0.0.1 -Darg10=4001 -Darg11=PlayerOne
-Darg12=127.0.0.1 -Darg13=4002 -Darg14=PlayerTwo -Darg15=127.0.0.1
-Darg16=4003 -Darg17=PlayerThree -Darg18=127.0.0.1 -Darg19=4004 -Darg20=PlayerFour

For a 5 Multi-Player game:
ant GDL_III_Manager_5_Players -Darg0=myTournament -Darg1=muddyChildren5
-Darg2=60 -Darg3=15 -Darg4=50 -Darg5=40 -Darg6=127.0.0.1 -Darg7=9999
-Darg8=RandomPlayer -Darg9=127.0.0.1 -Darg10=4001 -Darg11=PlayerOne
-Darg12=127.0.0.1 -Darg13=4002 -Darg14=PlayerTwo -Darg15=127.0.0.1
-Darg16=4003 -Darg17=PlayerThree -Darg18=127.0.0.1 -Darg19=4004
-Darg20=PlayerFour -Darg21=127.0.0.1 -Darg22=4005 -Darg20=PlayerFive
```