Does financial volatility help in explaining and predicting economic activity?

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2520

Direct Research Work Project
Master of Science in Finance

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January, 2017
Abstract

Driven by the difficulty to predict the last financial crisis and possible distortion of predictive power of the conventional financial indicators on economic activity, this thesis provides in-sample and out-of-sample analyses whether financial volatility helps in explaining and forecasting economic activity. Several measures of financial volatility were constructed, such as: realized volatility, volatility following a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) process, common long-run component of volatility estimated by Dynamic Factor Model, Principal Component Analysis and cyclical components of financial volatilities filtered out with Baxter-King filter. I find that statistically there are measures of financial volatility that help in explaining economic activity. Moreover, out-of-sample analysis suggests that the model with term-spread and volatility of financial volatility (volatility of value-weighted returns of market portfolio volatility) performs best in forecasting economic activity. The inclusion of a volatility measure reduces the noise in estimated probabilities of expansions and leads to the lowest number of uncertain periods, i.e. periods for which probability of recession is between 16.86% (percentage of recessions in the sample) and 50%, an event that in some studies is already considered as a recession. Thus, a certain financial volatility measure improves forecasts from the conventional financial indicators, especially during less volatile times. Moreover, the most parsimonious measure of volatility predicts business cycles best. On the other hand, industrial production growth seems to be barely affected by financial volatility measures, which tend to be a better predictor for the direction of the future path of the economy than the actual growth rate.

Keywords: Capital Markets Uncertainty, Macroeconomic Risk, Financial Volatility, Dynamic Factor Model, Baxter King Filter, Business Cycle, Dynamic Binary Choice Models, GARCH models.
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1 Introduction

At the onset of the financial crisis in 2008, everyone was asking the same question: “How come nobody saw it coming?”. Over the past 30 years, economics has been dominated by an “academic orthodoxy” which says economic cycles are driven by players in real economy, i.e. producers and consumers of goods and services, while banks and other financial institutions have been ignored or given little attention.\(^1\) However, financial institutions were mostly responsible for the global crisis of 2008, because of their engagement into high-risk behavior and creation of risky products. One of the critiques focuses on researchers relying on mathematical models to figure out how economic forces will interact, with disregard for human factor and the role it plays in recessions. Most of the models in economics are based on the assumption of human rationality. Still, people behave irrationally in many situations and even if one acts rationally, it is wrong to assume that the whole group of people will react to a given condition as an individual would. This is exactly what one could see during the financial crisis. If people were rational, nobody would take mortgages they could not afford and which in turn, led to the financial crisis. On the other hand, human psychology is a hard-to-measure factor. However, Monash University finance lecturer John Vaz said that over the years, the ability to share the information rapidly and trade with “lightning speed” made markets more susceptible to sudden fluctuations based on human emotions.\(^2\) Given that, it is highly probable that financial volatility contains information about people’s behavior.

The possibility that financial volatility may encode information about real economic activity has important policy implications, and is naturally of immediate concern to corporate decision makers. The correct assessment of the current and, especially, the future economic situation is essential for good policymaking. For years, researchers develop leading indicators that are supposed to signal the movements of the future economic activity before they occur and provide information of the magnitude of these movements. However, the unconventional actions taken by central banks during the recessions have likely distorted the predictive power of the historically most reliable leading indicators – the yield curve and the real monetary base, and made it even harder to predict the recessions (Duncan de Vries, 2015). Furthermore, in the United

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\(^1\) Why Economics Failed to Predict the Financial Crisis., (2009, May 13\(^{th}\)), Knowledge @ Wharton, retrieved online from: http://knowledge.wharton.upenn.edu/article/why-economists-failed-to-predict-the-financial-crisis/

States, the various leading indicators started to give inconclusive signals. For instance, the yield curve and the ratio of the Conference Board’s leading and coincidental indices pointed to a continuing economic expansion in 2015 and 2016, while the real monetary base, corporate profit data and the ratio of Conference Board’s coincidental and lagging indices predicted a recession. Thus, in the event of such problems, this topic is of great relevance since even the simple case where a sustained stock volatility merely anticipates, without affecting, the business cycle might be already informative and important, since policy makers and private agents are more concerned about absolute declines and expansions in activity than in growth cycle measures. Moreover, Figure 1 depicts the annualized volatility of the market portfolio constructed by Kenneth R. French\(^3\) and the annual industrial production growth, and reveals that stock volatility is counter cyclical, i.e. it raises during all dates marked as recessions. It is a first indication that an introduction of such measure may be a missing component in the mathematical predictive models.

\(^3\) Retrieved online from: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
of economic activity. Thus, the purpose of this study is to verify whether financial volatility can improve the forecasts from traditional predictors and contribute to the existing sparse literature on this topic.

Schwert (1989) was the first person to relate capital market uncertainty to future economic fluctuation. He concluded that stock market volatility does not anticipate major financial crises and panics, from 1834 to 1987, but it rather rises after the onset of a crisis. Naturally, recessions may develop independently of financial crises and panics. Moreover, financial turmoil might precede recessions, as for example during the 2007 subprime crisis. More recently, Fornari and Mele (2013) find that financial volatility predicts 30% of post-war economic activity in the United States, and that during the Great Moderation, aggregate stock market volatility explains, alone, up to 55% of real growth. They prove that combining the latter with a term spread factor leads to a predicting block that anticipates the business cycle reasonably well as it would help predict at least the last three recessions with no “false positive” signals. Following, Chauvet, Senyuz and Yoldas (2015) create a simple common factor and find that the stock volatility measures and the common factor significantly improve macroeconomic forecasts of conventional financial indicators, especially over short horizons.

Guided from this motivation, the aim of this research is to use the information that financial volatility encodes about the development of the business cycle, in predicting economic activity including different volatility measures. The study is going to focus mainly on one central issue:

*Does financial volatility help in explaining and predicting economic activity?*

However, several sub-questions are also going to be answered in this thesis.

The literature on the topic focuses on very different volatility measures such as for example realized volatility, common factor estimated with Dynamic Factor Model, classical volatility approaches or principal component analysis. Based on this wide range of volatility measures, one would like to verify whether one of the measures dominates. The second sub-question of this research is whether volatility measures improve forecasts from the conventional indicators. Fornari and Mele (2013) show that the combination of financial volatility with traditional predictors leads to better business cycle predictions. Similarly, Chauvet et al. (2015) combines financial volatilities with autoregressive term, term spread, credit spread and return of
the market portfolio and shows that some of the volatilities seem to have additional information beyond that included in the conventional indicators, over the short time horizon.

To my knowledge, this is one of the first studies including financial volatility in a prediction model conducted for a European country. Thus, since most of the literature on predictive power of financial volatility is based on data for the United States, one could suspect that there might be a difference in the magnitude of predictive power of financial volatility, if any at all. I chose the country of interest based on previous literature. For instance, already Artis, Kostolemis and Osborn (1997) showed that economic cycles of European countries are interconnected and linked to the economy of the United States through Germany. Thus, the German economy is taken as a case study due to its size and significance in the European Union. Therefore, based on the results obtained in this research the last sub-question will be answered, i.e. whether there are any differences in forecasting European and American economies.

The reminder of this thesis is organized as follows. The next section introduces the topic using the relevant empirical literature review and an assessment on the existing relevant methodology. Section 3 presents the hypotheses that might help to explain the predictive power of volatility. Section 4 introduces the data and explains construction of the volatility measures. Section 5 presents the empirical results from several models and compares the in-sample and out-of-sample results of these models. Section 6 contains the discussion and implication for future research, and section 7 concludes.

2 Literature Review

2.1 Empirical literature

An extensive research about the predictive ability of traditional financial variables is available. To introduce into the topic, among other studies, already Stock and Watson included term spread in a construction of their own leading business indicator index in 1989. Later, Estrella and Mishkin (1995), following their work about real rates prediction, investigated different financial variables and verified their predictive accuracy on American recession data. The analysis highlights outperformance of a stock price and other macroeconomic variables in short-term out-of-sample analysis of up to 2 quarters ahead, whereas a yield curve performs best beyond 2 quarters and gives better predictions if taken into analysis alone. Serletis and Krause (1996) investigated a cyclical behavior of money, prices and short-term nominal rates in the US from
1960 to 1993. They showed that short-term interest rates and money are generally strongly procyclical, whereas the price level follows a counter cyclical behavior and thus, can be useful in modeling business cycles. Further, Ang, Piazzesi and Wei (2006) build a dynamic approach for forecasting GDP based on term structure approaches for pricing bonds. They find that the short-term rate has the most predictive power, but lagged GDP value and longest maturity yield should not be left out.

The relationship between uncertainty on capital markets and future economic activity received relatively less attention in the literature compared to the relationship between financial activity and future economic activity. The research so far mostly highlighted the effect of uncertainty on capital markets, such as the stock market. Poterba and Summers (1984) studied shocks to stock market volatility and their influence on stock market’s level. They find that these shocks are not persistent and can be seen on the financial market for not more than 6 months. In turn, neither the volatility shock nor movements in equity risk premia can be responsible for the stock market downturn in 1970’s. Schwert (1989) also raised a similar topic and documented the relations between business cycle, financial crises and stock volatility in the United States in years 1834-1987. He provides evidence that on average stock volatility is higher during recessionary periods, which are shorter than periods of low volatility. Moreover, he shows that even though, major and minor banking crises caused significant changes in the stock market over the last 150 years, the financial volatility does not anticipate the business cycle, but rather follows it. Borio, Furfine and Lowe (2001) raised the same issue and talked about misassessment of risks associated with economic cycles. Importantly, they highlight procyclical attitude towards risk, i.e. it tends to be underestimated during expansion periods and overestimated when the economy goes down, which indicates counter cyclical path of human factor on the stock market. This asymmetric aspect of stock market volatility was further examined by Mele (2007), who developed a theoretical framework about determinants of stock market volatility. On the other hand, Adrian and Rosenberg (2008) decomposed financial volatility into short- and long-run components. They argue that short-run component is related to a market skewness risk, whereas the long-run volatility component is linked to a business cycle risk. Following, Bloom (2009) investigates the impact of uncertainty shocks that increase significantly in a result of political and economic shocks. He simulates the response of three core macroeconomic variables: output, employment and productivity growth, and finds that introduced macro shocks generate a drop,
rebound and then a longer-run overshoot to these variables. Later, based on above conclusion, Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry (2012) introduced uncertainty shocks in business cycle analysis. They find that the shocks explain up to 3% of drop and rebound in quarterly GDP and affect government policies, making these less effective. Authors argue that the impact is substantial and the result suggests the quantitative importance of uncertainty shocks in driving business cycles.

Similarly, there is a limited number of empirical confirmations on the predictive abilities of financial volatility for economic indicators. Hamilton and Lee (1996) extend Schwert (1989) and propose a model that uses the relation between economic recessions and the variance of stock returns to identify and forecast both, a business cycle and stock volatility. Driven by the positive results, Campbell et al. (2001) studied the disaggregated volatility and focused on the idiosyncratic risk at the firm level. They show that all three, volatility of common stocks at the market, industry and firm level, move the same way and their paths are counter cyclical. Moreover, they show signs of leading properties and thus, the inclusion of all components into predictive analysis was expected to increase the accuracy. Indeed, Campbell et al. prove that not only do financial volatility helps in forecasting the economic activity, but also weakens significance of stock index returns. In 2006, Ahn and Lee published a paper about the second moment relationships, between real stock index returns and real output growth, of various forms of generalized autoregressive conditional heteroskedastic models for US, Canada, UK, Japan and Italy. All the volatility models used in their study showed the strong relation between variables. Therefore, high volatility periods of a real output are followed by increased movements in the stock market for all the countries included in the analysis. Moreover, the opposite direction holds also for USA and Italy, but it is weaker in the magnitude. Bakshi, Panayotov and Skoulakis (2011) construct forward variances from option portfolios that help to predict not only a real economic activity, but also various returns such as stock market and Treasury bill returns and changes in variance swap rates. On the other hand, Allen, Bali and Tang (2012) derived a risk measure for financial institutions that improves forecasts of macroeconomic downturns six months ahead. They argue that since banks affect economy, an aggregate risk of their exposure has statistically significant influence on economic slowdowns. Looking from a different perspective Corradi, Distaso and Mele (2013) look for macroeconomic determinants of stock volatilities and volatility premiums. They show that the changes in the volatility and their
magnitude are strongly correlated with business cycles and that there exists an unobserved factor that contributes to around 20% of the overall variation in the volatility. However, macroeconomic factors can explain nearly 75% of the variation of stock volatility. Furthermore, authors argue that actually the volatility of volatility is connected to the business cycle. Driven by this result, Fornari and Mele (2013) conducted extensive study in which they used financial volatility and volatility of financial volatility as predictors of economic activity. Firstly, they showed that financial volatility helps in predicting economic activity and in fact, it alone explains around 30% of the industrial production growth and even up to 55% during the Great Moderation. Secondly, they also try to predict the business cycle indicator adding financial volatility to sets of conventional predictors. Indeed, combining the latter with the term spread makes the predictions more accurate. Authors argue that it is caused by more complete set of information, i.e. term spread encodes information about risk-premiums and is dependent on current macroeconomic policy, whereas financial volatility is seen as uncertainty and therefore has information about the general risks in the economic environment. In 2014, Ferrara, Marsili and Ortega conducted a first study that forecasted economic growth using volatility measures for Europe. They aimed to verify the impact of daily financial volatility, commodity and stock prices on an output growth in the US, UK and France. By using MIDAS approach, which allows different frequency of the data, they prove that the model with daily measure of financial volatility performs better in comparison to the model with just one regressor – the industrial production. In the same year Cesa-Bianchi, Pesaran and Rebucci (2014) conducted the study about interrelation between financial markets volatility and economic activity. They show that the volatility moves counter cyclically and can act as a leading indicator. Moreover, the co-movement of volatilities within the asset classes is stronger than across the classes, suggesting that various volatilities may encode different information about the business cycle. Finally, Chauvet, Senyuz and Yoldas (2015) analyzed the predictive power of different financial volatility measures, such as realized and implied volatilities, and estimated a common long-run component of volatility from both stock and bond markets. They show that inclusion of these measures significantly improves predictions of the traditional explanatory variables such as – the term spread, credit spread and the return on the market portfolio. Moreover, they identify regimes of high and low volatilities, with the former one giving early signals of an upcoming recession.
Few studies focus on interconnections between different economies and their business cycles. Artis, Kostolemis and Osborn (1997) analyzed business cycles in G7 and European Countries to examine the international nature of cyclical movements. They show that it is common that cycles are asymmetric and slopes in recessions are usually larger than in expansion periods. Moreover, their analysis reveals that business cycles of the researched group of European countries are interconnected and linked to the United States economy through Germany. Following this result, Artis and Zhang (1997) investigated whether the introduction of the European Monetary System strengthened the relation between the participating economies. They show that linkages between the latter and the US cycle weakened in a result of the ERM in favor of Germany, whose influence grew by that time. Later, Sensier, Artis, Osborn and Birchenhall (2004) examined the predictive power of the domestic and foreign variables in predicting business cycles in Germany, UK, Italy and France over the period 1970 to 2001. The analysis shows that the domestic variables are able to predict movements in the German economy well in-sample, but poorly in the out-of-sample analysis. Inclusion of two foreign variables: the composite leading indicator and the short-term interest rates in the US, representing impact of US economy on Germany, improves forecasts of the business regimes. However, there is no impact of other European foreign variables on the German economy, but instead, German interest rates seem to lead the other European interest rates (Artis and Zhang, 1998; Barassi et al., 2000).

The most relevant paper for my study is the described above Fornari and Mele (2013) article. They consider two measures of economic activity: a recession indicator and an industrial production growth rate, and forecast those using blocks of traditional predictors and sets of variables adjusted by financial volatility variables and volatility of volatility variables. I follow their framework and extent the research by adding different volatility measures to verify whether there exists a superior definition of financial volatility that contains more predictive information in this area. I include realized volatility, a principal component estimated using Principal Component Analysis, implied volatility, cyclical component filtered using Baxter-King filter, etc. Following Chauvet et al. (2015), among other measures, I also estimate a common factor, a long-run component, which, according to previous research, encodes information about a business cycle (Adrian et al., 2008). However, modeling the European economy is more demanding than forecasting the US business cycle due to its close interconnection with other major economies.
Thus, I further extend the research by adding the so-called “foreign variables”, which link German economy with the US one (Artis et al., 1997; Sensier et al., 2004).

2.2 Methodology

In this thesis, I aim to explain economic activity, which is either expressed by stages of the economy, i.e. recession and expansion periods, or in levels, such as the value of industrial production or its growth rate. I base my research on these two representations of economic activity, with industrial production growth rates as the level of economic activity. The methodology used in this study is based on the results and suggestions obtained from the empirical literature on this topic. My research focuses on the German market, but most of the literature studies the US economy. Hence, I provide a short summary of important developments in this field regarding the methodology of interest, which mostly covers studies for the United States.

Birchenhall, Jessen, Osborn and Simpson (1999) conclude that binary choice models predict the US regimes better than commonly used Markov switching models. In 2008, Kauppi and Saikkonen introduced a dynamic binary choice approach to forecast recessions in the US. They show that dynamic probit model that includes the lagged value of a recession indicator dominates all other approaches. Moreover, the iterative forecast approach performs best in out-of-sample analysis. In 2013, Fornari and Mele estimated a recession indicator with a static probit model and estimated an industrial production growth with a simple linear regression. More advanced econometric measures were introduced by Chauvet et al. (2015), who, among others, estimated a common factor using a Dynamic Factor Model.

Below I summarize traditional models applied in this thesis. Section 2.2.1 reviews methods for business cycle modeling whereas section 2.2.2 focuses on industrial production growth modeling. All the data processing was performed using STATA SE 12.0 and MATLAB R2015a software.

2.2.1 Dynamic binary choice models

Static approach to business cycles modeling is gradually substituted by a dynamic one, which includes information about the current or past state of the economy. For instance, Dueker (1997) and Moneta (2003) extended the probit function by lagged values of recession indicators. Later, Kauppi and Saikkonen (2008) considered four types of models in their analysis: static, dynamic,
autoregressive and dynamic autoregressive probits. Following their approach, this study estimates these four binary choice models.

The static probit model is the well-known book type model (Wooldridge, 2002). For a binary dependent variable, the simplest model is in a form:

\[ p(x_i) = F(\pi_i) \]  

[2.1]

where \( F(\cdot) \) is a cumulative distribution function of a certain distribution. In case of probit models, \( F(\cdot) \) is a cumulative normal distribution function. The probability of success is equal to \( \Phi(\pi_i) \), where \( \Phi(\cdot) \) is a cumulative standardized normal distribution. Consider the latent variable \( y_i^* \), which is a random variable following:

\[ y_i^* = \pi_i + \varepsilon_i \]  

[2.2]

where \( \varepsilon_i \sim N(0,1) \). Then, the relation between the latent variable and observable binary variable is defined:

\[ y_i = \begin{cases} 0 & \text{for } y_i^* \leq 0 \\ 1 & \text{for } y_i^* > 0 \end{cases} \]  

[2.3]

and the probability of a singular event is equal to

\[ \Pr(y_i = 1|x_i) = [1 - \Phi(\pi_i)]^{1-y_i}\Phi(\pi_i)^{y_i} \]  

[2.4]

Maximum Likelihood estimation solves the above specification under the assumption of independence of observations. The purpose of this study is to verify the predictive power of numerous volatility measures. Thus, the actually applied conditional probability is in a form:

\[ \Pr_{t-1}(y_t = 1) = \Phi(\alpha + \beta x_{t-k}) \]  

[2.5]

where \( k \) is the employed lag order of independent variables.

The second approach, dynamic probit is built on a static model by augmenting it with a lagged value of binary variable as an additional regressor. The probability function of the modified model is given below. One can see that the forecast is done based on the information at time \( t - 1 \). The model can also be extended to more lagged values, but for the sake of simplicity, I decided not to employ more regressors in the dynamic probit approach.

\[ \Pr_{t-1}(y_t = 1) = \Phi(\alpha + \delta y_{t-1} + \beta x_{t-k}) \]  

[2.6]

In comparison, the autoregressive model extends the static approach by a lagged value of function \( \pi \) instead of a recession indicator. The conditional probability follows then:

\[ \Pr_{t-1}(y_t = 1) = \Phi(\alpha + \gamma \pi_{t-1} + \beta x_{t-k}) \]  

[2.7]
The last extension of the classical probit model is a combination of two above-mentioned approaches. The most complex model consists of independent variables, lagged value of a business cycle indicator and lagged function $\pi$.

$$\Pr_{t-1}(y_t = 1) = \Phi(\alpha + \delta y_{t-1} + \gamma \pi_{t-1} + \beta x_{t-k})$$  \[2.8\]

Inclusion of the autoregressive parameter in the models above leads to an additional condition that needs to be satisfied, i.e. $\gamma$ needs to take such values that the stationarity condition is satisfied. Otherwise, the crisis would become perpetual, which is counterintuitive. The problem was solved by the implementation of a constrained maximum likelihood estimation method. Moreover, following Candelon, Dumitrescu and Hurlin (2014), I consider the robust covariance matrix to tackle the problem of possible autocorrelation.

The second issue regarding this modeling is a forecast horizon in dynamic models. Notably, one can obtain a “direct” k-step ahead forecast by adjusting the lag value of the dynamic regressors or use an iterative approach in which the lag order does not need to match the forecast horizon. There is no evidence that one method is superior to the other. The latter one makes the computation of forecasts more difficult than the direct approach, but if the model used in iterative forecasts is close to the true-data generating process then the iterative approach outperforms the direct one.

### 2.2.2 Linear Regression

The second part of the research focuses on modeling the industrial production growth with robust linear regression as in Fornari and Mele (2013). The in-sample estimation takes the following form:

$$y_{t\rightarrow t+k} = \alpha + \sum_{j=1}^{n} \beta_j x_{nt} + \epsilon_t$$  \[2.9\]

where $n = \{1,2,\ldots,17\}$, $\beta_j$ are the parameters to be estimated, and $x_{nt}$ is a $n$-th regressor introduced further in the paper. Based on the same study of Fornari and Mele (2013) the out-of-sample estimates are calculated with the simple linear regression following:

$$y_{t\rightarrow t+k} = \alpha + \beta X_t + \epsilon_t$$  \[2.10\]

where $X$ is a matrix of independent variables and $\beta$ is a vector of estimated coefficients. Most of known to me literature, forecasts the growth rate of industrial production k-steps ahead. I follow
the previous studies, forecast only dependent variable and do not focus on future development on input variables.

Due to the nature of the time series data, robust option was used in the estimation process to tackle the possible problems of autocorrelation or heteroskedasticity. Thus, the reported standard errors are robust to any kind of misspecification.

3 Data Analysis

Research is only reliable if the data on which it is based is reliable and uniform. Thus, when retrieving and constructing the database, one has to be careful with choosing official and trustworthy data sources. The data for this research are retrieved from at least six data sources: Bloomberg Professional, FactSet, Kenneth R. French Data library, FRED Economic Data, Deutsche Bundesbank Data Warehouse and OECD Data website. Both, Bloomberg and FactSet, act as financial data vendors: they collect data from different data sources and distribute them though their terminals, which means in reality data were collected from numerous sources. The final sample used in research consists of 261 monthly observations for the period from January 1994 to September 2015.

3.1 Choice of Variables

The aim of the study is twofold. First, I want to verify whether financial volatility helps in predicting recessions using static and dynamic binary choice models. Second, whether it helps in predicting economic activity, i.e. the industrial production growth, using linear regression.

The data used in this thesis is divided into 3 groups presented in table 1 based on the literature on the topic. The independent variables included in this research are traditional predictors of economic activity, such as a term spread, corporate spread, inflation, short-term interest rate, and volatility measures. Among the latter ones, following Fornari and Mele (2013) I distinguish between macroeconomic and financial volatilities. Although, the business cycle analysis is conducted based on only financial volatilities and traditional predictors, whereas forecasting levels of economic activity uses the macroeconomic volatilities, i.e. industrial production growth rates. Then again, the predictors are divided into 9 blocks, as presented in table 2 below, and generated groups are compared in their predictive power. This step is necessary to be able to verify which set of the variables performs best. Furthermore, the blocks
are designed based on the literature and previous research that suggests best predictors of economic activity and they consist only of financial volatility variables and traditional predictors.

In order to construct all the variables mentioned in Table 1 and Table 2, I needed first to download and construct the base variables from which the volatilities and volatilities of volatility were generated.

### Financial volatility
- Stock market volatility
- Volatility of the term spread
- Volatility of the corporate spread
- Volatility of stock market volatility

### Macroeconomic volatility
- Volatility of oil return
- Volatility of industrial production growth
- Volatility of inflation
- Volatility of unemployment rate
- Volatility of metal return

### Traditional predictors
- Term spread
- Corporate spread
- Stock returns
- Oil return
- Growth in composite leading indicator
- Short-term interest rate
- Inflation
- Lagged industrial production growth

*Note: Predictors as in Fornari and Mele (2013)*

<table>
<thead>
<tr>
<th>B0</th>
<th>Lagged industrial production</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>Term spread, corporate spread, 12 month stock market returns</td>
</tr>
<tr>
<td>B2</td>
<td>Term spread, short-term rate</td>
</tr>
<tr>
<td>B3</td>
<td>Term spread volatility, stock market volatility</td>
</tr>
<tr>
<td>B4</td>
<td>Stock market volatility, term spread</td>
</tr>
<tr>
<td>B5</td>
<td>Volatility of stock market volatility, short-term rate</td>
</tr>
<tr>
<td>B6</td>
<td>Volatility of stock market volatility, term spread</td>
</tr>
<tr>
<td>B7</td>
<td>Volatility of stock market volatility, stock market volatility, term spread</td>
</tr>
<tr>
<td>B8</td>
<td>Volatility of stock market volatility, stock market volatility, interaction term, term spread</td>
</tr>
</tbody>
</table>

*Note: interaction term is a product of a volatility of stock volatility and a lagged value of stock market volatility measure. Predictors as in Fornari and Mele (2013)*

### Table 2: Predicting blocks of economic activity.
3.1.1 Recession Indicators

First, I focus simply on predicting recessions using financial volatility measures. To the best of my knowledge, the majority of studies on the topic of impact of financial volatilities on modeling and predicting economic activity are based on the US economy. The literature focuses on the NBER based Recession Indicators for the United States, which is an interpretation of US Business Cycle Expansions and Contractions data. One of its European equivalents is the OECD based Recession Indicator for Germany, which is an interpretation of the OECD Composite Leading Indicators: Reference Turning Points and Component Series data. The indicator is in a form of a dummy variable and it is equal to one when the recessionary period occurs, and zero otherwise. For this time series, the recession begins the first day of the period following a peak and ends on the last day of the period of the trough. The second choice of the recession indicator I have is the Business Cycle Indicator published by Economic Cycle Research Institute. They use NBER-style procedures, i.e. the Bry and Boschan algorithm (1971) to date classical cycle turning points for various countries based on their coincident indexes (defined by production, sales, employment and income data). Thus, one can define the Recession Indicator as follows

\[ Rec_t \equiv I_{R_t=1} \]

where \( R_t \) is either the ECRI-based or the OECD-based Recession Indicator as of month \( t \).

\[R_{ECRI} = 1 \text{ when recession}, \quad R_{OECD} = 1 \text{ when recession} \]

Note: STATA output.

Figure 2: ECRI-based and OECD-based Recession Indicators

The relatively short sample that was used in this study caused some difficulties in choosing one of the above-mentioned indicators. As can be seen in Figure 2 above, the OECD-
based Recession Indicator indicates recession (with a value of one) in nearly 48% of times, while the ECRI Business Indicator is less reluctant to show recessions and it takes a value of one in only around 24% of cases. Interestingly, not all recession points market by ECRI Business Indicator are marked as recession according to OECD-based Recession Indicator, which may question the validity of available measures. However, the former measure may be considered as too conservative and not able to distinguish between expansion and recession periods very well, indicating more recessions than we observed in reality. Moreover, other research studies on the topic of recession predictions also use the ECRI Business Cycle Indicator as the dependent variables (Sensier et al., 2004). The final choice between these two indicators was made by proving the leading property of financial volatility, which is discussed further in chapter 5.1 of this paper.

3.1.2 Industrial Production

Second, I aim to check whether financial volatility has also predictive power in forecasting quantitative measures of economic activity. In this part, the industrial production growth rate over 3 months plays a role of dependent variable. Industrial production is a measure of output of the industrial sector of the economy and it is very sensitive to interest rates and consumer demand changes. Thus, it is an important tool for forecasting future GDP and economic performance. This measure of economic activity also has one advantage over GDP data. Industrial Production Index is published monthly, in the first half of the succeeding month unlikely to the GDP growth rates, which are published quarterly with approximately one month delay. Define the industrial production growth rate as

$$y_{t \rightarrow t+k} = \ln \left( \frac{IP_t}{IP_{t-k}} \right)$$

[3.2]

$k = 3$, where $IP_t$ is the industrial production index as of month $t$.

3.1.3 Stock Returns

Several different measures of volatility of stock returns are used in this study, but most of them are based on value-weighted returns of the market portfolio for Germany, collected from Kenneth R. French Data library website. The published calculations in this data library are the value and growth portfolios based on raw data from Morgan Stanley Capital International for 1975 to 2006 and from Bloomberg from 2007 to present. The portfolios are formed at the end of December
each year by sorting on one of the four ratios (book-to-market, earnings-price, cash earning to price and dividend yield) and then he computes monthly value-weighted returns for the following 12 months. There are two sets of portfolios. In one, firms are included only if there is data on all four ratios. In the other one, a firm is included in a sort variable’s portfolios if there is data for that variable. The market return for the first set is the value-weighted average of the returns for only firms with all four ratios. The market return for the second set includes all firms with book-to-market data. I decided to use the latter ones in this study due to their better representativeness of market returns.

3.1.4 Term and Corporate Spreads

The most common predictor of economic activity in the literature is the term spread that is proved to be a leading indicator of economic growth worldwide. It is well-known since at least Stock and Watson (1989a), Estrella and Hardouvelis (1991) and Harvey (1991, 1993), who prove that inverted yield curves predict recessions with a lead time of about one to two years. Davis and Fagan (1997) find that the interest rate spread leads to an improvement in forecasting performance of output for around half of the European countries examined, while Galbraith and Tkacz (2000) find this to be the case for all G7 countries apart from Japan. According to Smets and Tsatsaronis (1997), who investigate why the slope of the yield curve predicts future economic activity in Germany and the United States, the monetary policy plays an important role in determining the intensity of the relationship between the term spread and output growth. Two main findings of their work are that firstly, the predictive content of the term spread is not time-invariant and secondly, it is policy dependent. They also show that its leading property is stronger in Germany than in the United States, but half of it is explained by supply shocks. Throughout the literature, the term spread is defined as the difference between the 10-year Treasury note yield and 3-month Treasury bill yield. Due to the short data sample for a 3-month Treasury bill yield for Germany, I define the term spread as the difference between yields of 10-year and 1-year benchmark government bonds.

Corporate spread is another commonly used financial predictor of economic activity that is used in this study. The same as the term spread, corporate spread contains valuable information about the business cycle. Already Stock and Watson (1989) and Bernanke and Blinder (1992) proved that risk premiums contain predictive power due to counter cyclical premiums for long-term investments and the risk of default of corporations. Fornari and Mele (2013) defined it as a
difference between the BAA industrial bond yield and the 10-year Government bond yield. In this study, I define it as the difference between the domestic corporate bond yield and 10-year government bond yield. The yields used to construct term and corporate spreads were downloaded from the time series database published on the Deutsche Bundesbank website.

3.1.5 Short-term Rate

In recent times, Ang, Piazzesi and Wei (2006) argue that the short-term rate has larger marginal power than the term spread. Their finding is based on a model that assumes no arbitrage relations between the yield curve and the growth of GDP. Later, Fornari and Mele (2013) included this variable in their analysis, which was not based on a non-arbitrage model. Indeed, they showed that the combination of short-term rate with term spread performed well but did not outperform significantly other variables in recession predictions and the industrial production growth. In this research, I used the short-term interest rate from the OECD Data website as a short-term rate. It is either the rate at which short-term borrowings are transacted between financial institutions or the rate at which short-term government paper is issued or traded in the market. When available, they are based on three-month money market rates.

3.1.6 Macroeconomic Predictors

Below I define other predictors of economic activity used to model the industrial production growth. Define the inflation rate as \( \text{inflation}_t = \ln \left( \frac{\text{CPI}_t}{\text{CPI}_{t-1}} \right) \), where \( \text{CPI}_t \) is the consumer price index as of month \( t \). I define the lagged industrial production as \( y_t = \ln \left( \frac{\text{IP}_{t-1}}{\text{IP}_{t-2}} \right) \), where \( \text{IP}_{t-1} \) is the industrial production of month \( t-1 \). I calculated oil return based on oil price index and metal return from metals prices in the same way as inflation was calculated. The monthly unemployment rate and growth in the composite leading indicator for Germany were also taken into the analysis.

3.1.7 Foreign Variables

One of the few studies of the links between the levels of economic activity in the countries of the European Union is Artis and Zhang (1997, 1999), who compare the synchronization of business cycles with the US and Germany. Following their work Sensier, Artis, Osborn and Birchenhall (2004) introduced foreign variables in their research about business cycle regimes. They showed that the domestic variables are able to predict recessions well in Germany, but the inclusion of the
composite leading indicator for US and short-term interest rates in US decreases the errors in in-sample analysis. Previously, Canova and De Nicolo (2000) also highlighted the importance of US growth in leading that of Germany. Moreover, a high, nearly 90% and statistically significant, correlation between American S&P500 index and German DAX index suggest that the American indicator can be informative. Thus, two foreign measures of US economy are included in this study, i.e. growth in the composite leading indicator and short-term interest rates in US.

3.2 Financial Volatility Construction

The variable of interest of this research is a financial volatility, which is an unobserved measure. Since it is not directly observed, several methods exist to construct an approximation of it. Below I present common methodologies I followed to construct different measures of volatility.

3.2.1 Realized Volatility of Stock Returns

Realized volatility is one of the measures of historical volatility. Chauvet et al. (2015) define it as follows:

$$RV_t = \frac{1}{2} \ln(\sum_{s \in t} r_{ms}^2)$$

where $t = 1,2,...,T$, $n_t$ is the number of trading days in month $t$, $r_{ms}$ is the daily return and $T$ denotes the total number of months in the sample. I expect that realized volatility measure defined this way would be more informative than the one calculated on monthly returns, as it does not lose information embodied in daily returns if these are independent. On the other hand, daily returns are more volatile than monthly ones, and in turn, volatility obtained this way may be more noisy and unlikely to exhibit any linkage with the business cycles. Due to lack of daily value-weighted returns of the market portfolio, this measure was used for volatility of corporate bond and 10-year government bond.

3.2.2 Volatility of Stock Returns and other variables

The second measure of volatility is defined as a moving average of past absolute returns (Fornari and Mele, 2013). They want to distinguish between short- and long-run components of stock market volatility, because the long-run component is believed to be more informative and in turn, have larger predictive power of the future state of the economy (Adrian and Rosenberg, 2008).
Moving average application aims to smooth the unnecessary isolated episodes of financial turmoil from the returns. Following their approach, I define the volatility of stock market returns:

\[ \sigma^y_t \equiv \sqrt{6\pi} \times \frac{1}{12} \sum_{i=1}^{12} |R_{t+1-i}| \]  

[3.4]

where \( R_t \) is a total return index as of time \( t \).

The volatility of rest of the variables is computed similarly to a stock volatility:

\[ \sigma^y_t \equiv \sqrt{6\pi} \times \frac{1}{12} \sum_{i=1}^{12} |\Delta y_{t+1-i}| \]  

[3.5]

where \( \Delta y_t \) is the monthly variation of the variable of interest \( y_t \). I chose a lag of 12 based on the data frequency and previous literature, especially Fornari and Mele (2013), who argue that this lag order increased the predictive power of financial volatility in their analysis.

### 3.2.3 Volatility of Volatility of Stock Returns

Figure 1 depicts counter cyclicality of volatility of value-weighted returns of the market portfolio. It shows that when the economy deviates, the volatility increases sharply and thus, it is likely that it contains information about the business cycle and future economic activity (Corradi et al., 2013). To further examine this feature, Fornari and Mele (2013) included a volatility of volatility measure, which is defined as follows:

\[ VolVol_{t+l} \equiv \frac{1}{12} \sum_{i=1}^{12} |\sigma_{t+i} - \hat{\sigma}_{t+i}| \]  

[3.6]

\[ \hat{\sigma}_{t+i} \equiv \frac{1}{12} \sum_{i=1}^{12} \sigma_{t+i} \]  

[3.7]

### 3.2.4 Common Factor

One of the estimated volatility measures used in this study is a common factor obtained through Dynamic Factor Model. Following Chauvet et al. (2015) the factor contains information of four stock and bond market financial volatilities: realized volatility of the corporate bond, realized volatility of a 10-year government bond, implied volatility of DAX and volatility of value-weighted returns of the market portfolio calculated as in \([4.4]\). Table 3 below presents the pairwise correlation coefficients between the variables. Interestingly, one can see a negative and

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4 The term \( \sqrt{6\pi} \) arises from Schwert’s (1989) and is later implemented in Fornari and Mele (2013).
significant correlation between the realized volatility of 10-year government bond and unemployment rate. It follows a common pattern that optimistic employment data (lower unemployment rate) usually drives the bond yields up. However, this may be also caused by monetary policies introduced in recessionary periods and resulting from them low interest rate environment that affected the volatility of government bond yields. Moreover, the market portfolio volatility is the only one that is significantly correlated with the industrial production, suggesting that other individual variables will not necessarily alone help in predicting economic activity. Thus, combining the information they all carry could improve the forecasts.

<table>
<thead>
<tr>
<th></th>
<th>RV Corp bond</th>
<th>RV 10Y bond</th>
<th>VDAX</th>
<th>Vol mkt portfolio</th>
<th>UN</th>
<th>IP</th>
</tr>
</thead>
<tbody>
<tr>
<td>RV Corp bond</td>
<td>1.0000</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RV 10Y bond</td>
<td>0.4363**</td>
<td>1.0000</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>VDAX</td>
<td>0.3209**</td>
<td>0.4175**</td>
<td>1.0000</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Vol mkt portfolio</td>
<td>0.2407**</td>
<td>0.1888**</td>
<td>0.7519**</td>
<td>1.0000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>UN</td>
<td>-0.1008</td>
<td>-0.1407**</td>
<td>0.0274</td>
<td>0.2401**</td>
<td>1.0000</td>
<td>-</td>
</tr>
<tr>
<td>IP</td>
<td>0.0425</td>
<td>0.0209</td>
<td>0.0208</td>
<td>-0.2242**</td>
<td>-0.6337**</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Note: This table reports all the pairwise correlation coefficients between the variables. RV Corp bond stands for realized volatility of a corporate bond, RV 10Y bond stands for realized volatility of 10-year government bond, VDAX stands for implied volatility of DAX, Vol mkt portfolio stands for volatility of value-weighted returns of the market portfolio, UN stands for unemployment rate and IP stands for the industrial production. ** Indicate significance at 5% level.

Table 3: Pairwise correlations between financial volatility measure and macroeconomic aggregates.

Dynamic factor models have received a lot of attention in past decades because of their ability to model simultaneously and consistently data sets in which the number of series exceeds the number of time series observations. The premise of the model is that a few latent dynamic factors drive the co-movements of vector of time-series variables. The optimal number of factors in this study is just one common factor that is going to contain information that input variables carry. Let $y_t$ be a vector of independent variables in the order mentioned above, then the simple dynamic factor model of volatility dynamics can be specifies as follows:

$$y_{i,t} = \lambda_i VF_t + u_{i,t},$$

$$u_{i,t} = \varphi_i u_{i,t-q} + \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim NID(0, \sigma^2_i),$$

$$VF_t = \psi VF_{t-p} + \epsilon_t, \quad \epsilon_t \sim NID(0, \tau^2).$$

[3.8]
Where \( VF \) represents a common factor, \( \lambda_i \) is called the dynamic factor loading for the \( i^{th} \) series of \( y \), which shows the degree of correlation between individual volatility series and the common factor, and \( \psi \) is a common factor coefficient. I estimated the model using the first generation factor estimation, which consists of low-dimensional parametric models estimated using Gaussian maximum likelihood estimation and the Kalman filter (Stock and Watson, 2010).\(^5\) The data included in the model were checked for stationarity and standardized.

<table>
<thead>
<tr>
<th>Autoregressive process</th>
<th>Autocorrelated errors</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>2500.62</td>
<td>2533.80</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1728.86</td>
<td>1776.79</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1998.85</td>
<td>2046.78</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2275.41</td>
<td>2312.28</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1775.80</td>
<td>1827.41</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1714.31</td>
<td>1780.67</td>
</tr>
</tbody>
</table>


Table 4: Information criteria for alternative specifications estimated by Dynamic Factor Model.

The final model is based on information criteria showed in Table 4 above. According to BIC criterion, unobserved common factor that follows a first-order autoregressive process with autocorrelated errors of order one in the equation for the observables is the optimal model. The BIC criterion is preferred due to its larger punishment for additional estimated parameters compared to the AIC, which is the lowest for the most complex model. Table 5 presents parameter estimates for the dynamic factor model of volatilities. The extracted volatility factor \( \psi \) is highly persistent with an autoregressive coefficient estimate of 0.93. All factor loadings are positive and highly significant indicating co-movement of the volatility measures, especially among standardized realized volatility of corporate bond and 10-year government bond, and implied volatility of DAX. I fail to reject the null hypothesis in Wald test that the coefficients \( \lambda_1, \lambda_2, \lambda_3 \) are equal \( (\chi^2(2) = 0.25, \ p\text{-value} = 0.8832) \). The factor loading of the volatility of the value-weighted returns of the market portfolio is the smallest and therefore, one can see that the factor and volatility of the market portfolio do not necessarily follow the same path.

\(^5\) dfactor command with White estimator was used to estimate Dynamic Factor Model in STATA SE 12.0 software.
Note: This table reports parameter estimates and robust standard errors in parentheses, where $y_t$ = [standardized realized volatility of corporate bond, standardized realized volatility of 10-year government bond, implied volatility of DAX, volatility of value-weighted returns of the market portfolio]. Thus, $\lambda_1$ is a coefficient of standardized realized volatility of corporate bond, $\lambda_2$ is a coefficient of standardized realized volatility of 10-year government bond, $\lambda_3$ is a coefficient of implied volatility of DAX and $\lambda_4$ is a coefficient of volatility of value-weighted returns of the market portfolio. Coefficients $\varphi_i$ and $\sigma_i$ refer to included variables in the same order. * Indicate significance at 10% level, **Indicate significance at 5% level, *** Indicate significance at 1% level.

Table 5: Parameter estimates for the dynamic factor model of volatilities.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>$\psi$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\lambda_3$</th>
<th>$\lambda_4$</th>
<th>$\varphi_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.9334*** (0.0270)</td>
<td>0.2521*** (0.0392)</td>
<td>0.2257*** (0.0472)</td>
<td>0.2092*** (0.0695)</td>
<td>0.0911*** (0.0243)</td>
<td>0.0643 (0.1694)</td>
</tr>
<tr>
<td>$\varphi_2$</td>
<td>0.3630*** (0.1098)</td>
<td>0.9179*** (0.0257)</td>
<td>0.9658*** (0.0167)</td>
<td>0.5068*** (0.0981)</td>
<td>0.5383*** (0.0653)</td>
<td>0.1120*** (0.0237)</td>
</tr>
<tr>
<td>$\varphi_3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varphi_4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: STATA output.

Figure 3: The common factor and standardized realized volatility of corporate bond, standardized realized volatility of 10-year government bond, implied volatility of DAX, and volatility of value-weighted returns of the market portfolio.
Autoregressive coefficients of all idiosyncratic components, with the exception of standardized realized volatility of corporate bond are significant.

Figure 3 depicts paths of standardized independent variables and the estimated common factor. The latter does not seem to follow input volatilities closely, but one may expect that it contains all the information that can enhance the predictive power of financial volatility. Moreover, one can notice that changes in volatilities of the bond market are more rapid and dynamic than in the stock market.

3.2.5 Constructing Principal Components

Another way to decrease the dimension in the analysis and extract information from different volatilities is via principal component analysis - a statistical procedure that uses an orthogonal transformation to convert a number of correlated variables into linearly uncorrelated variables called principal components. In this analysis I decided to use the same input variables as in Dynamic Factor Model, i.e. the realized volatility of the corporate bond, the realized volatility of a 10-year government bond, the implied volatility of DAX and the volatility of value-weighted returns of the market portfolio calculated as in [4.4]. It is important to mention that this procedure is sensitive to scaling of the data. Thus, again, all the variables are stationary and standardized.

In the first step of the analysis, one should check whether the variables are correlated. Table 3 above shows the pairwise correlation coefficients for all included variables. All coefficients between four financial volatilities are statistically significant at 5% significance level. The highest correlation occurs between the implied volatility of DAX index and volatility of value-weighted returns of the market portfolio, which indicates that changes in DAX index explain 75% of the changes that happen in the market. The rest of the variables are not strongly, but significantly correlated.

The optimal number of principal components was chosen based on the aim of this analysis, i.e. to decrease the number of elements with as much information extracted from them as possible, and based on statistical tests and conditions. Moreover, the scree plot of the eigenvalues of a covariance matrix also suggests one to two principal components, but eigenvalue of only one component has a value higher than one. That said, one principal component, that explains 55.84% of financial volatilities included in the model, is being used in further analysis.
Figure 4 depicts relationships between the principal component and standardized independent variables. The realized volatility measures of bond portfolios are lower in their magnitude than changes in the stock market. Moreover, estimated principal component seems to follow closer stock market volatilities – implied volatility of DAX and volatility of the market portfolio.

![Figure 4: Graphs showing relationships between principal component and various realized volatilities](image)

### Note
STATA output.

Figure 4: The principal component and standardized realized volatility of corporate bond, standardized realized volatility of 10-year government bond, implied volatility of DAX, and volatility of value-weighted returns of the market portfolio.

#### 3.2.6 Baxter-King Filters

It is believed that volatility encodes information about a business cycle, but it may also contain information about panic periods that do not necessarily need to be linked to the level of the economy. Thus, there arises a question how to separate the cyclical component from rapidly

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6 Principal Component Analysis was conducted using `pca` command in STATA SE 12.0 software.
varying seasonal or irregular components or slowly evolving secular trends. There are several techniques that could be used to resolve this issue, such as application of moving averages, first differencing, Hodrick-Prescott filter, etc. Baxter and King (1999) present a comparison of methods to isolate the cyclical component from economic data. They show that their filter is more flexible and easier to implement than other measures and what is important, it produces best approximation to the ideal filter. Driven by this result, the Baxter-King band-pass filter was used to separate a time series into trend and cyclical components. The procedure isolates business cycle components using data transformation by applying particular moving averages based on data characteristics. The filter is applied to two volatility measures: the common factor and the principal component estimated in previous steps.\(^7\)

Both time series are checked for stationarity using Dickey-Fuller test for unit root. I reject the null hypothesis that the principal component or the common factor have a unit root (p-value = 0.0003 and p-value = 0.0034, respectively). Thus, the stationary version of a Baxter-King filter is applied for these series with twelve observations in each direction that contribute to each filtered value. For monthly data, Baxter and King (1999) recommend 18 and 96 months of minimum and maximum periodicity included in the filter. They also suggest a symmetric moving average order of 12 for quarterly data. I decided to choose the order of 12 for monthly data, because of the short sample I have and the way the algorithm works. To illustrate, if the order of the symmetric moving average is denoted by q, the estimate for the cyclical component for the \(i^{th}\) observation is based upon the \(2q + 1\) values: \(y_{t-q}, y_{t-q+1}, ..., y_t, y_{t+1}, ..., y_{t+q}\). Thus, by setting q to 12 months, I lose 24 observations – 12 first and 12 last data points. The analysis of the appropriateness if this number is also based on the periodogram – an estimate of the spectral density of a signal (Appendix 2.1). The vertical lines refer to the conventional values for business cycle components introduced by Burns and Mitchell (1946). Ideally, the periodogram should be a flat line at the minimum value of −6 outside the range identified by the vertical lines. One can see that the filter of SMA order 12 removed the stochastic cycles reasonably well.

Figure 5 shows the path of estimated volatility measures – the common factor and the principal component, and their cyclical components from Baxter-King filter. In both graphs, the cyclical component follows the estimated variable reasonably well and does not show signs of

---

\(^7\) Data filtering was conducted in STATA SE 12.0 software using \texttt{tsfilter bk} command with an option for stationary data.
stochastic trend. However, one should notice that the magnitudes of volatility measures estimated by Dynamic Factor Model and Principal Component are different. The estimated common factor exhibits higher volatility than the estimated principal component.

Note: STATA output.
Figure 5: The cyclical component of the principal component and the principal component, and cyclical component of the common factor and the common factor.

3.2.7 GARCH

Other way to estimate volatility is to model it through GARCH(1,1) model. In the first step, the preliminary analysis was conducted to verify whether the returns of the value-weighted returns of the market portfolio have features that would classify them to apply Conditional Heteroskedasticity models – time-variant volatility (conditional heteroskedasticity) and leptokurtic density functions.

From the left part of Figure 6 below, that shows the value-weighted market portfolio returns, one can see a visual evidence of time varying volatility. The ARCH-test verifies this finding, by investigating a time series for the null hypothesis of homoskedasticity within the series of squared returns (Engle, 1982). Based on the conducted test (p-value = 0.0298) one can state that the null hypothesis can be rejected at 5% conventional significance level and thus, it formally confirmed that there is a conditional heteroskedasticity in the data. The distribution of the returns is the second feature that needs clarification. The visual analysis of the histogram presented on the right hand side of Figure 6 justifies the assumption that the distribution is not normal. It displays fatter tails and larger concentration of values around zero compared to the
normal distribution line marked on the graph. It is distinctive of leptokurtic distribution. The formal Jarque-Bera test confirms the expectations, the p-value of 0.0000 allows to reject the null hypothesis that the data is normally distributed.

The formal Jarque-Bera test confirms the expectations, the p-value of 0.0000 allows to reject the null hypothesis that the data is normally distributed.

Note: STATA output.
Figure 6: Value-weighted returns of the market portfolio.

The decision to use GARCH(1,1) instead of ARCH-type models was made by several reasons. Firstly, a number of conducted studies show that GARCH(1,1) model is hardly ever significantly outperformed by other, more complicated models (Hansen and Lunde, 2005). Secondly, I started the analysis with the most parsimonious model that seems to fit data really well, based on results of post estimation tests. Lastly, information criteria also suggest that GARCH(1,1) is superior to alternative models that might have been used in this analysis (Appendix 3.1).

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Constant</th>
<th>Lagged Market Portfolio return</th>
<th>Lagged volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0004**</td>
<td>0.1581***</td>
<td>0.7183***</td>
</tr>
<tr>
<td></td>
<td>(0.0002)</td>
<td>(0.0577)</td>
<td>(0.0816)</td>
</tr>
</tbody>
</table>

Note: This table reports parameter estimates and robust standard errors in parentheses. * Indicate significance at 10% level, **Indicate significance at 5% level, *** Indicate significance at 1% level.

Table 6: Parameter estimates for the GARCH model for market portfolio value-weighted returns.

Table 6 presents the parameter estimates of GARCH(1,1) model.8 All coefficients are statistically significant at 5% significance level and the sum of coefficients of the lagged market portfolio returns and lagged volatility is less than one. Thus, the system is stationary. The post

8 All GARCH estimates were obtained using STATA SE 12.0 software with a command: arch var, arch(1/1) garch(1/1) robust.
estimation Portmanteau test for white noise, suggests that the model is a good fit for the data. The p-value = 0.9282 fails to reject the null hypothesis that there is no serial correlation in the final residual series. Figure 7 depicts the market portfolio volatility calculated as in [4.4] and GARCH(1,1) modeled volatility for the value-weighted market portfolio returns. One can see that the volatility modeled by GARCH process is smaller in its magnitude than volatility of value-weighted returns of market portfolio, but it takes values from nearly the same range as the latter one, with its maximum and minimum being slightly larger and the peak in October 2002. Moreover, the conditional volatility seems to precede the volatility obtained directly from the data and in turn, might have a good leading property in forecasting economic activity.

Figure 7: Volatility of value-weighted returns of the market portfolio and GARCH conditional volatility.

Note: STATA output.

4 Estimation Results

4.1. Stock Volatility as a Leading Indicator

Figure 1 and Figure 8 depict the behavior of volatility of the value-weighted market portfolio returns during the recession and expansion periods. Remarkably, all OECD and ECRI-dated recessions are associated with higher market portfolio volatility. In this section, I present two preliminary analyses confirming the usefulness of financial volatilities in modeling economic activity.
4.1.1 Correlation between variables

The market portfolio volatility is negatively correlated with quarterly GDP growth in Germany. The correlation is -21.64% and it is statistically significant at the conventional 5% significance level. Correlations between other volatility measures and the GDP growth rate are similar in magnitude, also negative and statistically significant. The largest correlation occurs between the implied volatility of DAX and the quarterly GDP growth and is equal to -24.84%. In conclusion, already based on this simple analysis, one may expect that the volatility may have some predictive power in forecasting economic activity.

4.1.2 Linear regression results

Following Fornari and Mele (2013), to further investigate the leading property of stock volatility I estimated the following regression:

\[
\sigma_t = c + \sum_{l \in \{3,12,24,36\}} b_l \sigma_{t-l} + \gamma_1 I_{t \in \{(\text{Recession}_t = 1)\}} + \gamma_2 I_{\text{Recession}_t = 1} + u_t^2 \tag{4.1}
\]

where \(\sigma_t\) is stock market volatility, an indicator function \(I_{t \in \{(\text{Recession}_t = 1)\}}\) is always zero, except during the twelve months preceding any Indicator-dated recession and \(I_{\text{Recession}_t = 1}\) equals one only during Indicator-dated recessions, and zero otherwise, \(u_t^2\) is a residual term.

Table 7 below reports the estimates and p-values, computed through heteroskedasticity and autocorrelation consistent standard errors, for the parameters \(c, b_l, \gamma_l\).\(^9\) Over the whole sample, the ECRI Business Cycle estimate of \(\gamma_2\) is positive and highly significant, while estimate of \(\gamma_1\) is not different from zero. For the OECD-based Recession Indicator none of the \(\gamma_l\) is statistically significant – the financial volatility measure is not affected by the OECD-based Recession Indicator. Moreover, lagged values of financial volatility matter only until 12 past months and are statistically not different from zero for longer lags for any indicator. Given that, I decided to estimate the equation [5.1] with only 3 and 12 months lagged values of volatility. Then, the coefficient \(\gamma_1\) became positive and statistically significant at 10% significance level for the ECRI-based Recession Indicator (p-value = 0.0870), but it did not change for the OECD-based Recession Indicator and it was still statistically not different from zero.\(^{10}\)

---

\(^9\) Estimated using STATA SE 12.0 software.

\(^{10}\) For the sake of brevity, the numerical results are not displayed in this paper, but available from the author upon request.
Figure 1 and Figure 8 show the volatility of value-weighted returns of market portfolio constructed by K. R. French and periods marked as recessions by both ECRI-based Recession Indicator and OECD-based Recession Indicator, respectively. In both cases, it is clearly visible that the magnitude of volatility is higher during the recession-marked periods and in case of Figure 1 also at times before recession.

Thus, based on the preliminary analysis, the value-weighted returns of the market portfolio volatility is counter cyclical, since the volatility increases around 12 months before the recessions. Furthermore, it also anticipates economic activity, because the impact of the business cycle indicator is statistically significant over the sample. Hence, the ECRI-based Recession Indicator is used further in the study as the recession indicator. The second part of this chapter presents obtained results, followed by their discussion and conclusion of this thesis.

Note: Graph obtained using MS Excel 2010.
Figure 8: Volatility of value-weighted returns of the market portfolio and the OECD-based Business Cycle Indicator
Table 7: Parameter estimates for the linear regression verifying the leading property of financial volatility.

<table>
<thead>
<tr>
<th></th>
<th>c</th>
<th>$b_3$</th>
<th>$b_{12}$</th>
<th>$b_{24}$</th>
<th>$b_{36}$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ECRI Estimate</strong></td>
<td>0.0031***</td>
<td>0.9657***</td>
<td>-0.2238***</td>
<td>0.0398</td>
<td>-0.0467</td>
<td>0.0004</td>
<td>0.0008***</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td>(0.0293)</td>
<td>(0.0344)</td>
<td>(0.0266)</td>
<td>(0.0261)</td>
<td>(0.0004)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td><strong>OECD Estimate</strong></td>
<td>0.0029***</td>
<td>0.9853***</td>
<td>-0.2142***</td>
<td>0.0591</td>
<td>-0.0463</td>
<td>-0.0004</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>(0.0006)</td>
<td>(0.0301)</td>
<td>(0.0336)</td>
<td>(0.2820)</td>
<td>(0.0275)</td>
<td>(0.0003)</td>
<td>(0.0003)</td>
</tr>
</tbody>
</table>

Note: This table reports parameter estimates and standard errors computed through heteroskedasticity and autocorrelation consistent standard errors for the linear regression presented in [5.1]. * Indicate significance at 10% level, **Indicate significance at 5% level, *** Indicate significance at 1% level.

4.2. In-sample analysis of recession indicator

All the estimations are based on two versions of defined sets of blocks. First one consists of just the variables which were introduced in Table 2, whereas the second one is extended by two foreign variables - growth in the composite leading indicator and short-term interest rates in US, which are added to each block. The analysis considers 3-months-ahead prediction models presented below. Three-months are preferred because it allows for realistic time lags in the availability of data and for lags in the response of agents to economic information, both domestic and international.

The conclusions are drawn from several characteristics. In-sample analysis is based on information criteria, Mean Squared Error (MSE), Mean Absolute Error (MAE), Area Under Curve (AUC)\(^\text{11}\) measure and the number of classified events. In terms of the information criteria, Schwarz Information Criterion\(^\text{12}\) is preferred in this study, because as mentioned in Swanson and White (1996) it focuses on out-of-sample forecasting and is valid even for misspecified models, when it asymptotically selects the best model from the choice set (Sin and White, 1995). Secondly, lately, researchers showed that AUC is quite noisy and has some other significant problems in model comparison. Thus, it is used with caution in this research and is treated just as an indicator, but no final decision is based just on its value. Thirdly, each event can be classified as correctly estimated, recession or expansion, if the estimation is higher or lower than 0.5, and the observed event was either one or zero, respectively. Furthermore, following Birchenhall, Jessen, Osborn and Simpson (1991) and Sensier et al. (2004) I also identify number of

\(^{11}\) AUC is the area under the receiver-operating characteristic used as model comparison tool. It is a credit-scoring criterion that reveals the predictive abilities of the model by relying on all the values of the cut-off, i.e. the threshold used to compute crisis forecasts.

\(^{12}\) In the form $SBC = \frac{-2\log L + k \log T}{T}$, where $L$ is the likelihood value and $T$ is the number of observations.
“uncertain” events, i.e. months for which the probability is lower than 0.5, but higher than 16.86%, which is the percentage of recessions in the sample. This characteristic is used in case other two measures suggesting the accuracy of predictions are equal or nearly equal for different models. That said, it is considered as noise in the analysis. The analysis presented below was conducted using MATLAB R2015a software.

4.2.1 Static Probit Model Results

Block 1 (term spread, corporate spread, 12 month stock market returns) performed best among the basic version of blocks in the static probit estimation. The AUC is the highest for this block and equal to 89.40%, where 100% is the perfect classifier. Also, the highest $R^2$ of 34.81% suggests that this model fits the dependent variable best. Moreover, information criteria and calculated errors are lowest for B1. The frequency of correctly classified recessions and expansions is one of the highest. Block number 8, which consists among others of the volatility of the value-weighted returns of the market portfolio, performs as well as the best block in this area, but its other characteristics are not as satisfying. Importantly, the best statistical model predicts recessions successfully in 31.82% of times, which is the highest rate for this type of estimation approach. It is very important since one can assume that the cost of an unpredicted crisis is higher than cost of an unpredicted expansion.

The inclusion of the foreign variables into blocks improved estimates significantly. The best model among these is B2 (the term spread and short-term rate), which also dominates other models in every other characteristic. Notably, the frequency of successful predictions of recessions is nearly twice larger than in the best block without the foreign variables and equal to 68.18%. AUC (97.18%) and pseudo-$R^2$ are also higher than in any model estimated by static probit. Moreover, it has the lowest AIC and SBC among all the models. Thus, statistically it is the best fit and it also predicts recessions best – 68.18% of recessions were predicted successfully. On the other hand, the accuracy of correct expansion estimations is lower than in the same block without the US indicators, but still larger than in the considered best B1.

Formal t-student test\textsuperscript{13} was conducted to verify whether the differences between obtained characteristics with and without the foreign variables are statistically significant. The frequency of successfully predicted recessions is statistically smaller after the inclusion of foreign variables

\textsuperscript{13} Two tailed test with the null hypothesis that the difference between means is equal to zero.
(p-value = 0.0228), but the difference was statistically not significant for expansion predictions (p-value = 0.5346). MAE and MSE are also statistically smaller for the dataset with the foreign variables, with p-values equal to 0.0434 and 0.0361, respectively. Provided that, the foreign variables representing the US influence on German market significantly affect this analysis for the static probit. Furthermore, the traditional economic activity predictors dominate in the static approach.

Figure 9 below presents estimated probabilities for both models. Block 1, with just the domestic variables, gives quite a few false signals during expansion periods. Moreover, what is even more alarming, the probability of recession is low during 2001 and 2002. Thus, despite being the best among other blocks with just domestic variables, it does not predict the recessions good enough. On the other hand, block 2, with the domestic and foreign variables, predicts expansions much clearer. Its probabilities in 2001 clearly indicate recession, although the model was late in predicting it. However, it predicted a crisis of 2008 reasonably well and early despite its poor performance in the previous recession. This improvement might be due to the more international character of the last crisis.

![Graph](image-url)  

*Note: Graph obtained using MS Excel 2010.*

Figure 9: Estimated probabilities from block 1 with the domestic variables and block 2 with the domestic and the foreign variables.

### 4.2.2 Dynamic Probit Model Results

Inclusion of a lagged value of the business cycle indicator dominates the results for every block regardless whether these consist of only the domestic or also the foreign variables. The frequency of correctly forecasted recessions and expansions is the same for each model and equal to 95.45% and 98.62%, respectively. Nevertheless, block 4, which comprises of stock market volatility and
term spread is the most appropriate model from statistical perspective, i.e. the AIC and SBC criteria are lowest. Moreover, in this case, the foreign variables are not as important as in static analysis. Despite the same number of successfully predicted recessions and expansions, its information criteria are larger and pseudo $R^2$ measure is lower and equal to 80.62% compared to 81.88% for the model with only the domestic variables. Moreover, based on the t-student test, one cannot see any statistical difference in errors performance between these two datasets (p-value = 0.7933, p-value = 0.6853 for MAE and MSE errors, respectively).

Figure 10 below depicts the probabilities estimated by the statistically best models – blocks 4. One can see that the estimations are nearly the same although, the model with just domestic variables gives a bit higher probabilities of recession during expansion periods, but it does not influence the estimated state of the economy. In conclusion, the exactly same number of correctly estimated recessions and expansions suggest the predictive dominance of lagged component and superior performance of this model over the static approach. Nonetheless, the models with financial volatility variables perform better on the statistical ground, i.e. the information criteria are lower for these models, which indicates that even though the percentage of correctly estimated recessions and expansions is the same regardless of the inclusion of the variables the model may perform better in future estimates.

![Graph obtained using MS Excel 2010.](image)

**Note:** Graph obtained using MS Excel 2010.

Figure 10: Estimated probabilities from block 4 with the domestic variables and block 4 with the domestic and the foreign variables.

### 4.2.3 Autoregressive Probit Model Results

Autoregressive model includes the lagged value of the function $\pi$ instead of the recession indicator. The best model with the domestic variables is block 6 (the volatility of stock market
volatility and term spread) with the value of AUC equal to 96.22% and the lowest SBC criterion. It predicts recessions and expansions reasonably well – 90.91% and 95.85% of cases, respectively. Obtained value of pseudo-$R^2$ of McKelvey and Zavoiny suggests the good fit of the model (60.04%). Looking at the dynamics of the errors for blocks with and without financial volatility variables, one can see that the latter ones are smaller, compared to the errors obtained for blocks with the classical predictors. The same pattern is visible for the blocks with the foreign variables, but the difference is not statistically significant in either of the cases. There is also no statistical difference between the errors nor correctly predicted recessions and expansions between the blocks with the domestic and foreign variables.

Then again, as in the case of static approach, the inclusion of the foreign variables positively affects the prediction accuracy. Block 8 (the volatility of stock market volatility, stock market volatility, interaction term and term spread), which is the best among the blocks with the foreign variables, predicts 90.91% of recessions and 97.24% of expansions correctly. Pseudo $R^2$ based goodness of fit suggests that this model fits best in this approach, with $R^2$ equal to 75.49%. Moreover, the smallest information criteria indicate that statistically this model performs best, also compared to B6 with the domestic variables. Even though, the statistical tests do not prove the statistical difference between the errors of these two versions of blocks, the calculated errors for block 8 are nearly twice smaller than the ones for block 6.

One can see from Figure 11 that the estimated by B8 probabilities are more accurate and do not give as many false signals as probabilities of B6. Its better performance is possible due to the inclusion of the foreign variables and the international character of the last crises. The over 60% probability of recession in 1994 should not be considered, because of the beginning of the sample. Furthermore, B8 is the most complex model, containing information from not only the stock market volatility, but also the volatility of volatility, term spread and interaction term, which again contains information of volatility and volatility of volatility with the former one being one period lagged. Thus, all these measures are significant in modeling the business cycle. Overall, the autoregressive approach performs reasonably well in predicting business cycles and significantly, the financial volatilities play an important role in this modeling.
Note: Graph obtained using MS Excel 2010.

Figure 11: Estimated probabilities from block 6 with the domestic variables and block 8 with the domestic and the foreign variables.

4.2.4 Autoregressive Dynamic Probit Model Results

The most complex – an autoregressive dynamic probit is a combination of two above described models. Again, as in the case of dynamic probit, the lagged value of the business cycle indicator dominates the other variables. Regardless of the domestic or the foreign variables in the datasets, the number of correctly estimated recessions and expansions is the same and, what is interesting, the frequency of successfully predicted recessions is smaller than in the dynamic approach and equal to 93.18%. Thus, one can conclude that the lagged function adds some noise into analysis that lowers the predictive power of the model. The best block with the domestic variables is block 6, which is also the best model overall based on information criteria and pseudo-R².

Figure 12 below depicts paths of probabilities estimated by B6 with just the domestic variables and B4 with the domestic and foreign variables. One can see that these do not differ significantly. The percentage of correctly estimated recessions and expansions is the same for these two. However, B6 performed slightly better in modeling recession in 2001-2003, and importantly, this model is more parsimonious in comparison to the model with domestic and foreign variables are thus, it is preferred.
In conclusion, the dynamic approach performs best in in-sample analysis. It predicts recessions and expansions most accurately, has the highest AUC and pseudo-R² and the lowest IC for block 4, which consists of the most common traditional predictor – term spread, and the stock market volatility. Nonetheless, to focus on the purpose of this study the autoregressive model for block 8 with the foreign variables is taken further into the analysis to check whether the other measures of financial volatility perform better.

4.2.5 In-sample Results for Different Financial Volatility Measures

Table 8 presents characteristics of block 8 with the foreign variables and different measures of financial volatility estimated by the autoregressive probit. Based on the percentage of correctly classified estimated recessions and expansions, the earlier estimated common factor performs best among the various volatility measures. Despite the same number of predicted recessions as by the market portfolio volatility, the number of successfully estimated expansions increased to 98.16%. However, this model does not perform better than the earlier models on the statistical ground – its information criteria are larger than in the other models.

Figure 13 depicts the probabilities estimated by the different volatility measures included in block 8. The market portfolio volatility performed well in predicting the last two crises, and did not show any “false positive” signals, except at the beginning of the sample. Similarly, the implied volatility of DAX predicted all the recessions perfectly, but it also did introduce more noise in the analysis, i.e. uncertain periods. Already the beginning of the sample could be
considered as possible recession, just like the small peak after the last crisis. The GARCH modeled volatility performed alike. It forecasted the probabilities of recession reasonably well, but it introduced noise into expansion periods. For example, the probability of recession after the crisis of 2001 stays high longer than necessary. The same situation takes place after the 2008 financial crisis. Although there is a significant peak before the last crisis, which could indicate changes on the markets, then again it decreases and yet again goes sharply up at the onset of the financial crisis in Germany. The best model, with the common factor, performs very well in predicting expansions and recessions. Compared to the market portfolio volatility, this measure does not give a peak in 1994, at the beginning of the sample, and therefore, outperforms the latter in predictions accuracy. The estimated cyclical component of the common factor produces very smooth estimates. Its probability of recessions decreases gradually in 2004. Moreover, again, this model gives a warning signal of the last recession. Lastly, the estimates of the cyclical component of principal component are also smoother than the principal component’s probabilities of recession. However, they estimate the same peaks, just in slightly different magnitude.

<table>
<thead>
<tr>
<th>Block 8 (foreign)</th>
<th>Market portfolio</th>
<th>VDAX</th>
<th>GARCH</th>
<th>Factor</th>
<th>Factor BK</th>
<th>Principal Component</th>
<th>PC BK</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUC</td>
<td>98.94%</td>
<td>97.59%</td>
<td>95.90%</td>
<td>95.00%</td>
<td>96.50%</td>
<td>96.54%</td>
<td>97.89%</td>
</tr>
<tr>
<td># recessions</td>
<td>40</td>
<td>39</td>
<td>35</td>
<td>40</td>
<td>39</td>
<td>31</td>
<td>34</td>
</tr>
<tr>
<td># expansions</td>
<td>211</td>
<td>216</td>
<td>211</td>
<td>213</td>
<td>213</td>
<td>210</td>
<td>215</td>
</tr>
<tr>
<td># uncertain</td>
<td>9</td>
<td>13</td>
<td>26</td>
<td>9</td>
<td>11</td>
<td>28</td>
<td>25</td>
</tr>
<tr>
<td>AIC</td>
<td>52.05</td>
<td>63.88</td>
<td>90.87</td>
<td>80.37</td>
<td>68.10</td>
<td>92.73</td>
<td>72.08</td>
</tr>
<tr>
<td>SBC</td>
<td>88.56</td>
<td>100.40</td>
<td>127.39</td>
<td>116.89</td>
<td>104.62</td>
<td>129.25</td>
<td>108.60</td>
</tr>
<tr>
<td>$R^2$</td>
<td>75.49%</td>
<td>70.49%</td>
<td>59.09%</td>
<td>63.52%</td>
<td>68.71%</td>
<td>58.30%</td>
<td>67.02%</td>
</tr>
<tr>
<td>MAE</td>
<td>5.49%</td>
<td>5.35%</td>
<td>9.50%</td>
<td>7.41%</td>
<td>6.31%</td>
<td>10.50%</td>
<td>7.54%</td>
</tr>
<tr>
<td>MSE</td>
<td>2.69%</td>
<td>2.43%</td>
<td>4.32%</td>
<td>3.10%</td>
<td>2.69%</td>
<td>5.33%</td>
<td>3.74%</td>
</tr>
<tr>
<td>freq recession</td>
<td>90.91%</td>
<td>88.64%</td>
<td>79.55%</td>
<td>90.91%</td>
<td>88.64%</td>
<td>70.45%</td>
<td>77.27%</td>
</tr>
<tr>
<td>freq expansion</td>
<td>97.24%</td>
<td>99.54%</td>
<td>97.24%</td>
<td>98.16%</td>
<td>98.16%</td>
<td>96.77%</td>
<td>99.08%</td>
</tr>
</tbody>
</table>

Note: The best model in bold. Market portfolio stands for volatility of value-weighted returns of the market portfolio, VDAX stands for implied volatility of DAX, GARCH stands for volatility following the GARCH(1,1) process, Factor is the estimated common factor, Factor BK is a cyclical component of the common factor, Principal Component is the estimated principal component, PC BK stands for cyclical component of the principal component.

Table 8: In-sample characteristics of block 8 with the domestic and the foreign variables estimated with autoregressive approach.
Note: Graph obtained using MS Excel 2010.

Figure 13: In-sample estimated probabilities of recessions from block 8 with the domestic and the foreign variables and different volatility measures.

4.3. Out-of-sample Forecasting Results of Recession Indicator

The out-of-sample recursive analysis is done to calculate 3-months-ahead forecasts by using an iterative approach where needed. This approach is preferred to multi-step ahead probabilities of recession as it gave significantly better results in the preliminary analysis. The out-of-sample analysis is more of the interest of this study, because it will signal the best prediction model. Due
to the recursive approach, neither information criteria nor a pseudo-$R^2$ was registered. Thus, this analysis is solely based on models’ predictive performance measured in AUC value, the percentage of correctly identified recessions and expansions, and the number of uncertain events. This part of the research is structured the same as the in-sample analysis and the results are obtained using MATLAB R2015a software.

4.3.1. Out-of-sample Results for Value-weighted Returns of the Market Portfolio Volatility

The best models to predict the business cycle indicators using a static approach are again B1 and B2, which perform the same in the percentage of successfully predicted expansions and recessions (Appendix 4.1). The set of only domestic variables predicted correctly 100% of recessions and 84.52% of expansions, whereas in the second set the percentage of predicted expansions is lower and equal to 82.14%. Thus, again, in the most simple and static approach, the financial volatility does not help in predicting the recession indicator and it is outperformed by the traditional predictors. However, Figure 14 shows that even though they may perform well compared to the other models, they still perform poorly. Both blocks give noisy predictions, with probabilities of recessions being higher than 50% very often. Looking at Figure 14, one may suspect that the estimated probabilities are random and thus, cannot be trusted.

Surprisingly, the best approach in-sample performs poorly out-of-sample. The dynamic probit fails in predicting recessions regardless of the version of neither datasets nor the blocks of variables. B1 and B2 predict the highest percentage of recessions that amounts to 66.67%,
whereas their power in forecasting expansions is stable and equal to 95.24%. Similarly, the dynamic autoregressive model performs poorly out-of-sample, with its results being nearly the same, but slightly worse than the ones obtained from the dynamic approach (Appendix 4.2).

In this analysis, the autoregressive approach gives the best recession estimates. Table 9 presents the models’ characteristics. Firstly, three of the models estimated on the domestic variables predict 100% of recessions successfully, i.e. block 1, 2 and 5. Nevertheless, blocks 1 and 2 predict expansions with a higher accuracy than block 5, thus they are superior. Secondly, the inclusion of the foreign variables increases the number of correctly estimated recessions. It mostly affects the blocks, which among others consist of the financial volatility measure. Although, the difference is not statistically significant according to t-student test (p-value = 0.1808), one can see that the percentage of correctly classified recessions is higher in the lower part of the Table 9. Nonetheless, the best model with the foreign variables does not outperform B1 and B2 with the domestic variables in terms of the percentages of correctly classified events. The number of predicted recessions and expansions is the same for these blocks. However, the number of observations classified as uncertain is lower in block 6 with the foreign variables. Furthermore, this block has also lower MAE and MSE, which indicates that the probabilities are closer to either zero or one than in the case of models B1 and B2 with the domestic variables. Thus, based on the models’ predictive performance and calculated errors block 6 with the domestic and foreign variables is considered the best.

Figure 15 presents estimated probabilities for the best model with the domestic and foreign variables. The first estimated “false positive” signal of crisis was around October 2007. However, it does not necessarily need to be considered as false since in April 2007 the leading subprime mortgage lender, New Century Financial Corp in the US, filed for Chapter 11 bankruptcy protection, and since August that year Fed cut the discount rate four times until the end of 2007, by a total 1.5 percentage points. Thus, the inclusion of two foreign variables – the growth in the composite leading indicator and short-term interest rates in the US, most likely affected the probability estimation significantly, giving a sign of an upcoming recession. The model predicts the 2008 recession reasonably well, but according to the encoded information, the estimated crisis lasts longer than the recession indicated by the business cycle indicator. The estimated probability can be justified with, currently historical, data. In 2009, the annual percentage change in the real GDP of Germany was -5.1% whereas the total employment change
Variables as in Table X

<table>
<thead>
<tr>
<th>Block</th>
<th>B0</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>B5</th>
<th>B6</th>
<th>B7</th>
<th>B8</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUC</td>
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<td>93.25%</td>
<td>93.25%</td>
<td>28.31%</td>
<td>80.42%</td>
<td>85.58%</td>
<td>80.42%</td>
<td>82.80%</td>
<td>85.71%</td>
</tr>
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<td>9</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
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<td>73</td>
<td>84</td>
<td>66</td>
<td>79</td>
<td>65</td>
<td>64</td>
<td>65</td>
</tr>
<tr>
<td># uncertain</td>
<td>51</td>
<td>12</td>
<td>12</td>
<td>27</td>
<td>2</td>
<td>6</td>
<td>4</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>MAE</td>
<td>28.01%</td>
<td>15.85%</td>
<td>15.85%</td>
<td>22.42%</td>
<td>20.67%</td>
<td>13.94%</td>
<td>21.17%</td>
<td>23.87%</td>
<td>22.79%</td>
</tr>
<tr>
<td>MSE</td>
<td>13.34%</td>
<td>11.45%</td>
<td>11.45%</td>
<td>11.18%</td>
<td>16.08%</td>
<td>11.83%</td>
<td>15.48%</td>
<td>18.89%</td>
<td>18.50%</td>
</tr>
<tr>
<td>freq reession</td>
<td>11.11%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>100.00%</td>
<td>0.00%</td>
<td>66.67%</td>
<td>66.67%</td>
<td>66.67%</td>
</tr>
<tr>
<td>freq expansion</td>
<td>92.86%</td>
<td>86.90%</td>
<td>86.90%</td>
<td>100.00%</td>
<td>78.57%</td>
<td>94.05%</td>
<td>77.38%</td>
<td>76.19%</td>
<td>77.38%</td>
</tr>
</tbody>
</table>

Variables as in Table X and two foreign variables

<table>
<thead>
<tr>
<th>Block</th>
<th>B0</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>B5</th>
<th>B6</th>
<th>B7</th>
<th>B8</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUC</td>
<td>59.13%</td>
<td>93.92%</td>
<td>93.92%</td>
<td>50.00%</td>
<td>93.65%</td>
<td>77.91%</td>
<td>92.72%</td>
<td>97.22%</td>
<td>91.14%</td>
</tr>
<tr>
<td># recessions</td>
<td>3</td>
<td>9</td>
<td>9</td>
<td>4</td>
<td>8</td>
<td>6</td>
<td>9</td>
<td>9</td>
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<td>14</td>
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<td>20</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>MAE</td>
<td>37.92%</td>
<td>20.05%</td>
<td>20.05%</td>
<td>28.23%</td>
<td>20.55%</td>
<td>13.16%</td>
<td>14.29%</td>
<td>14.96%</td>
<td>16.95%</td>
</tr>
<tr>
<td>MSE</td>
<td>20.28%</td>
<td>12.77%</td>
<td>12.77%</td>
<td>17.44%</td>
<td>12.58%</td>
<td>10.70%</td>
<td>10.73%</td>
<td>11.39%</td>
<td>14.39%</td>
</tr>
<tr>
<td>freq reession</td>
<td>33.33%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>44.44%</td>
<td>88.89%</td>
<td>66.67%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>88.89%</td>
</tr>
<tr>
<td>freq expansion</td>
<td>79.76%</td>
<td>83.33%</td>
<td>83.33%</td>
<td>89.29%</td>
<td>84.52%</td>
<td>90.48%</td>
<td>86.90%</td>
<td>83.33%</td>
<td>82.14%</td>
</tr>
</tbody>
</table>

*Note:* The best model in bold.

Table 9: Out-of-sample characteristics of models estimated with autoregressive approach.

![Observed crisis and Block 6 (foreign variables)](image)

*Note:* Graph obtained using MS Excel 2010.

Figure 15: Estimated probabilities from block 6 with domestic and the foreign variables.

was 0%. This was the largest fall of the real GDP in post-war Germany. Finally, the peak at the beginning of 2012 can also be explained by the slowdown of German economy, such as the industrial production shrinkage by 1.8% in September 2011 after already declining 0.4% the
month before. Even though, the 2011 was a good year, with the real GDP growth of 3%, the expected growth rate fallen to 0.8% in 2012.

### 4.3.2. Out-of-sample Results for Different Financial Volatility Measures

Table 10 presents characteristics of block 6 with the domestic and foreign variables and the different volatility measures included in the analysis. Based on the percentage of correctly predicted recessions and expansions, the basic estimated model with the volatility of value-weighted returns of the market portfolio volatility performs best among all the measures. It successfully predicts 100% of recessions and 86.90% of expansions out-of-sample. The second best model is the GARCH model that is again estimated out of the value-weighted market portfolio returns. That said, the broad market return helps in predicting the economy activity best.

On the other hand, the estimated common factor and the principal component that encodes information from not only the stock market but also the bond market volatilities significantly underperform the measures that are more parsimonious. With this in mind, one may suspect that the bond volatility does not necessarily contain information about business cycle in Germany.

<table>
<thead>
<tr>
<th>Block 6 (foreign)</th>
<th>Market portfolio</th>
<th>VDAX</th>
<th>GARCH</th>
<th>Factor</th>
<th>Factor BK</th>
<th>Principal Component</th>
<th>PC BK</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUC</td>
<td>92,72%</td>
<td>94,12%</td>
<td>90,85%</td>
<td>87,19%</td>
<td>88,24%</td>
<td>89,02%</td>
<td>88,76%</td>
</tr>
<tr>
<td># recessions</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td># expansions</td>
<td>73</td>
<td>50</td>
<td>63</td>
<td>51</td>
<td>59</td>
<td>54</td>
<td>53</td>
</tr>
<tr>
<td># uncertain</td>
<td>3</td>
<td>11</td>
<td>16</td>
<td>13</td>
<td>7</td>
<td>20</td>
<td>14</td>
</tr>
<tr>
<td>MAE</td>
<td>14,29%</td>
<td>35,10%</td>
<td>25,01%</td>
<td>36,69%</td>
<td>27,54%</td>
<td>34,05%</td>
<td>32,95%</td>
</tr>
<tr>
<td>MSE</td>
<td>10,73%</td>
<td>27,63%</td>
<td>16,84%</td>
<td>28,49%</td>
<td>23,19%</td>
<td>22,83%</td>
<td>23,62%</td>
</tr>
<tr>
<td>freq recession</td>
<td>100,00%</td>
<td>100,00%</td>
<td>100,00%</td>
<td>100,00%</td>
<td>100,00%</td>
<td>100,00%</td>
<td>100,00%</td>
</tr>
<tr>
<td>freq expansion</td>
<td>86,90%</td>
<td>59,52%</td>
<td>75,00%</td>
<td>60,71%</td>
<td>70,24%</td>
<td>64,29%</td>
<td>63,10%</td>
</tr>
</tbody>
</table>

*Note: The best model in bold. Market portfolio stands for volatility of value-weighted returns of the market portfolio, VDAX stands for implied volatility of DAX, GARCH stands for volatility following the GARCH(1,1) process, Factor is the estimated common factor, Factor BK is a cyclical component of the common factor, Principal Component is the estimated principal component, PC BK stands for cyclical component of the principal component.*

Table 10: Out-of-sample characteristics of block 6 with domestic and the foreign variables estimated with autoregressive approach. Although, more extensive analysis should be done to prove this point. Chauvet et al. (2015) shows that the bond market volatility measures are less useful than the stock market volatility indexes, but they contain information about the development of a business cycle. In conclusion, the broad market portfolio volatility predicts the recessions best compared to the other volatility measures.
Note: Graph obtained using MS Excel 2010.

Figure 16: Out-of-sample estimated probabilities of recessions from block 6 with domestic and the foreign variables and different volatility measures.

Figure 16 depicts the estimated probabilities and the observed crisis according to the ECRI Business Cycle indicator. Interestingly, all of the measures, but the basic one, indicate recessions in the first half of 2011, then beginning of 2012 and some of them even in 2013. Indeed, it was a very turmoil time in the European Economy with the European debt crisis that according to the ECRI indicator is not marked as recessionary period, whereas the OECD-based recession indicator classifies mid-2011 to the end of 2012 as a recession. Moreover, according to Sensier et al. (2004) the money market is an important indicator in explaining business cycles. Thus, high
probabilities in that period, does not need to be necessarily classified as “false positive” signals. With this in mind, a different model could be more appropriate for this data.

4.4. Industrial Production growth Estimates – In-sample Analysis

The in-sample analysis is based on the linear regression that includes three types of predictors listed in table 1. The first regression includes the stock market volatility; the second regression includes the stock market volatility and the volatility of the term spread as regressors. The last regression has all the regressors listed in the mentioned table. The Granger causality test statistic was calculated for each of the regressions. The null hypothesis assumes that the coefficient of the stock market volatility variable is zero. All of the tests failed to reject the null, with p-values significantly higher than the conventional significance level.

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Market portfolio</th>
<th>VDAX</th>
<th>GARCH</th>
<th>Factor BK</th>
<th>Principal Comp.</th>
<th>PC BK</th>
</tr>
</thead>
<tbody>
<tr>
<td>stock market volatility</td>
<td>0,00%</td>
<td>0,61%</td>
<td>0,20%</td>
<td>-0,05%</td>
<td>0,03%</td>
<td>0,28%</td>
</tr>
<tr>
<td>vol of the term spread</td>
<td>-0,33%</td>
<td>0,36%</td>
<td>-0,11%</td>
<td>-0,35%</td>
<td>-0,24%</td>
<td>0,00%</td>
</tr>
<tr>
<td>vol of the corp. spread</td>
<td>-0,25%</td>
<td>0,40%</td>
<td>-0,07%</td>
<td>0,69%</td>
<td>1,32%</td>
<td>0,35%</td>
</tr>
<tr>
<td>vol of stock market vol</td>
<td>-0,26%</td>
<td>1,91%</td>
<td>0,97%</td>
<td>1,32%</td>
<td>2,51%</td>
<td>0,03%</td>
</tr>
<tr>
<td>vol of oil return</td>
<td>-0,63%</td>
<td>1,67%</td>
<td>0,55%</td>
<td>0,92%</td>
<td>2,21%</td>
<td>-0,18%</td>
</tr>
<tr>
<td>vol of industrial production index</td>
<td>0,52%</td>
<td>2,28%</td>
<td>1,58%</td>
<td>3,04%</td>
<td>3,62%</td>
<td>0,78%</td>
</tr>
<tr>
<td>vol of inflation</td>
<td>0,81%</td>
<td>2,78%</td>
<td>1,89%</td>
<td>2,92%</td>
<td>3,53%</td>
<td>0,96%</td>
</tr>
<tr>
<td>vol of unempl. rate</td>
<td>1,66%</td>
<td>3,40%</td>
<td>3,24%</td>
<td>3,32%</td>
<td>3,88%</td>
<td>1,87%</td>
</tr>
<tr>
<td>vol of metal return</td>
<td>1,21%</td>
<td>2,91%</td>
<td>2,77%</td>
<td>2,83%</td>
<td>3,37%</td>
<td>1,40%</td>
</tr>
<tr>
<td>all regressors</td>
<td>17,44%</td>
<td>18,67%</td>
<td>21,32%</td>
<td>17,51%</td>
<td>18,00%</td>
<td>18,56%</td>
</tr>
</tbody>
</table>

Note: The best model in bold. Market portfolio stands for volatility of value-weighted returns of the market portfolio, VDAX stands for implied volatility of DAX, GARCH stands for volatility following the GARCH(1,1) process, Factor is the estimated common factor, Factor BK is a cyclical component of the common factor, Principal Component is the estimated principal component, PC BK stands for cyclical component of the principal component.

Table 11: Adjusted R² for in-sample regressions on predictors of economic activity with different volatility measures.

Table 11 reports the cumulative adjusted R². The broad market portfolio volatility seems to perform poorly among other measures. The best individual predictor is the implied volatility that alone explains 0.61% of the industrial production growth. First four predictors, which are based on the financial volatility, predict only at most 2.21%, which is significantly less than the Fornari and Mele’s (2013) estimate of 14%. This is the first indication that the financial volatility may not be as useful in predicting the European economy activity as it is in the American one.
Overall, the model with the GARCH volatility explains most of the industrial production growth when all the variables are taken into the analysis. Unexpectedly, the set of variables with the broad market portfolio volatility underperforms all the other models.

4.5. Industrial Production growth Estimates – Out-of-sample Analysis

The same as in the out-of-sample analysis for the binary choice models, this analysis is based only on the calculated errors of the estimates since the recursive approach does not allow comparison of information criteria and adjusted \( R^2 \).

Table 12 reports the Mean Absolute and Mean Squared Errors for 9 blocks with the domestic and foreign variables. One can see that the errors are of the same magnitude for every block regardless of the inclusion of the foreign variables. However, the latter ones keep the errors slightly lower, except for block 1, but the difference is statistically insignificant. Among all the blocks included in the analysis B1, which consists of just the conventional predictors of economic activity, with only the domestic variables performs best in terms of the magnitude of errors and outperforms all the models with the financial volatility measures. Nevertheless, block 5 with the foreign variables has the second lowest MAE and MSE. Moreover, Giacomini and White (2006) conditional predictive accuracy test suggests that either B5 or B2 perform best and the difference between them is statistically insignificant (p-value = 0.1171) although the test statistic points into B5 as a better model (Appendix 5.1). Thus, based on these characteristics, model B1 performs best in forecasting levels of economic activity in terms of the industrial production growth rate over the 3 months horizon. However, comparing Figure 17 and Figure 18, one can see that even though the blocks with the financial volatility variables have larger errors in the analysis, they seem to perform better in giving early signals about the direction of the path of economy. Furthermore, in order to verify the results obtained with the mean absolute and squared errors, I conducted additional analysis on demeaned data.\(^{14}\) Obtained estimated suggest the same conclusion as the analysis above. In terms of the calculated errors, block 1 with only the domestic variables performs best followed by block 5 with the domestic and foreign variables. Thus, considering it and the relative good performance of model B5 with the volatility of stock market volatility and the foreign variables, I decided to include it further into the analysis to verify whether any other volatility measure outperforms the traditional predictors.

\(^{14}\) For the sake of brevity, the numerical results are not displayed in this paper, but available from the author upon request.
Table 12: Mean average and mean squared errors for different blocks of predictors.

<table>
<thead>
<tr>
<th>Block</th>
<th>B0</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>B5</th>
<th>B6</th>
<th>B7</th>
<th>B8</th>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variables as in Table X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAE</td>
<td>2,78%</td>
<td>2,70%</td>
<td>2,89%</td>
<td>2,81%</td>
<td>2,79%</td>
<td>2,95%</td>
<td>2,80%</td>
<td>2,79%</td>
<td>2,79%</td>
</tr>
<tr>
<td>MSE</td>
<td>0,18%</td>
<td>0,14%</td>
<td>0,17%</td>
<td>0,18%</td>
<td>0,18%</td>
<td>0,17%</td>
<td>0,17%</td>
<td>0,17%</td>
<td>0,17%</td>
</tr>
<tr>
<td>Variables as in Table X and two foreign variables</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAE</td>
<td>2,77%</td>
<td>2,82%</td>
<td>2,77%</td>
<td>2,77%</td>
<td>2,78%</td>
<td>2,75%</td>
<td>2,76%</td>
<td>2,77%</td>
<td>2,78%</td>
</tr>
<tr>
<td>MSE</td>
<td>0,15%</td>
<td>0,15%</td>
<td>0,15%</td>
<td>0,15%</td>
<td>0,15%</td>
<td>0,15%</td>
<td>0,15%</td>
<td>0,15%</td>
<td>0,15%</td>
</tr>
</tbody>
</table>

Note: Graph obtained using MS Excel 2010.

Figure 17: Actual (red, right hand side axis) and forecasted 3-months ahead industrial production growth (blue, left hand side axis) by B1.

4.5.1 Comparison of Different Volatility Measures

Sadly, no volatility measure dominates the other based on the MAE and MSE, and Giacomini-White conditional tests. Table 13 presents the values of calculated errors. There is no statistical nor simple predictive difference between the different definitions of volatilities. Moreover, looking at Figure 18, one can say that there is nearly no difference between the estimated industrial production growth rates. They all seem to follow the same path and predict more the direction in which the industrial production will go than the actual value of it. This result is consistent with the binary analysis described above, in which the volatility matters in reducing the noise in predictions of the business cycle indicator. Thus, no measure is considered best in predicting the industrial production, although, based on the smallest errors, the cyclical component of the common factor might be recognized relatively better.
Note: Graph obtained using MS Excel 2010.

Figure 18: Actual (red, right hand side axis) and forecasted 3-months ahead industrial production growth (blue, left hand side axis).
### Table 13: Mean average and mean squared errors for different volatility measures.

<table>
<thead>
<tr>
<th>Block</th>
<th>Market portfolio</th>
<th>VDAX</th>
<th>GARCH</th>
<th>Factor</th>
<th>Factor BK</th>
<th>Principal Component</th>
<th>PC BK</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAE</td>
<td>2.75%</td>
<td>2.73%</td>
<td>2.74%</td>
<td>2.72%</td>
<td>2.72%</td>
<td>2.76%</td>
<td>2.74%</td>
</tr>
<tr>
<td>MSE</td>
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<td>0.15%</td>
<td>0.15%</td>
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</tr>
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</table>

*Note: Market portfolio stands for volatility of value-weighted returns of the market portfolio, VDAX stands for implied volatility of DAX, GARCH stands for volatility following the GARCH(1,1) process, Factor is the estimated common factor, Factor BK is a cyclical component of the common factor, Principal Component is the estimated principal component, PC BK stands for cyclical component of the principal component.*

5 Discussion of Results and Implications for Future Research

The connection between financial volatility, German business cycle and the industrial production growth is not as strong as it can be found in the studies conducted for the United States. The in-sample analysis focuses on answering the question whether financial volatility helps in explaining economic activity. In case of the in-sample binary choice models, the dynamic approach with the lagged recession indicator dominates the other models with the block 4 that consists of the stock market volatility and the term spread as the best statistical fit. It is not surprising since very often, when testing naive forecasts against experts in the financial industry, the naive forecast wins and beating it is more complicated than most people assume. However, when looking at the forecasts without the dynamic component, the block 8 with the domestic and foreign variables estimated by the autoregressive approach performs best in the percentage of predicted recessions and expansions, the number of uncertain events and the information criteria. This finding suggests that the financial volatility indeed helps in explaining business cycles, but it is dominated by the lagged probability of recession. Comparing different volatility measures it seems that the estimated common factor adds more information than any other measure, even though the model is not statistically superior. The common factor and the principal component, two of the measures that were estimated from the stock and bond markets volatilities, increase the probability of correctly estimated expansions, which suggests that the bond market volatility may contain more information about expansionary periods. However, the stock market volatilities encode all the necessary information which improve forecasts of recessions. Moreover, two blocks with just traditional predictors of economic activity and the foreign variables dominate the static approach as shown in Sensier et al. (2004), but these perform poorer than B8 estimated by the autoregressive probit. Thus, the inclusion of financial volatility improves the explanatory
power of the conventional financial indicators for business cycles in the in-sample analysis. Furthermore, the foreign variables play an important role in explaining the state of economy, unless the dynamic model is used. In this case, the past probability of recession is all that matters.

The in-sample analysis of the industrial production growth is based on the percentage of the industrial production growth that is explained by predictors. Unfortunately, the magnitude of these is significantly smaller than in the study conducted by Fornari and Mele (2013) in which the first four predictors, based on the financial volatility, explain about 30% of the industrial production growth. In my study, the value is at most 2.51% for the cyclical component of the common factor as a financial volatility measure. After including all the regressors in the analysis, the economic activity is explained in, at most, 21.32% for the GARCH volatility. Furthermore, the obtained results suggest that the financial volatility measures introduced in this thesis do not influence the economic activity. In light of the in-sample results for the industrial production, the financial volatility barely explains the economic activity in terms of the industrial production growth.

The out-of-sample forecasts were performed to verify the predictive power of the financial volatility. Fornari and Mele (2013) and Chauvet et al. (2015) show that the inclusion of the volatility measures positively affects forecasts of recessions and expansions. The same as in the in-sample analysis, the static approach estimated for the domestic conventional predictors performs relatively well with 100% of correctly estimated recessions and 84.52% of correctly estimated expansions. The autoregressive probit model gives the most accurate results out-of-sample. In terms of the number of successfully predicted recessions and expansions, B1 and B2 with the domestic variables perform best as well as B6 with the domestic and foreign variables. That said, one can conclude that the financial volatility does not help in predicting business cycles. However, even though there are measures suggesting superiority of B1 and B2 models, the block that among others consists of the volatility of stock market volatility gives the least number of events considered as uncertain. The number is four times lower than in the case of the blocks with the domestic traditional predictors. This indicates that the volatility of financial volatility reduces the so-called noise in the analysis, and makes it clearer, since the uncertain events may be already considered as recessionary periods. Furthermore, this result confirms the suggestion of Corradi et al. (2013) that volatility of stock volatility is what affects business cycles. Moreover, comparing the other volatility measures with the broad market volatility gives
no better measure than the most parsimonious one used in the first step of this analysis. Thus, the broad market portfolio and the volatility of value-weighted returns of it seem to perform best in predicting business cycles. Although, it is essential to mention the estimated by this model high probability of recession in 2012. The result indicates a recession period, when in reality according to the indicator no recession took place. Similarly, other volatility measures (Figure 16) gave “false signals” first in 2011, then 2012 and some of them even in 2013 even though these periods were classified as expansions by the ECRI. Contrary, the same periods are recessionary in the OECD-based recession indicator data. Moreover, looking at the economic situation at that time, the European economy faced many problems with the sovereign debt crisis that worsened in the second half of 2011 and caused the rapidly cooling economy in the euro area. The steps taken by the euro zone countries to reach an orderly sovereign debt workout for Greece were met with continued financial market turbulence and concerns of debt default in some of the larger economies in the euro area. Despite monetary policies remained accommodative with the use of various unconventional measures, the continuing financial sector fragility and persistent high unemployment made them ineffective. Furthermore, the German economy also experienced a slowdown in the second half of 2011, with a significant shrinkage in the industrial production, which also affected next year’s forecasts. Indeed, in 2012 the German economy grew by 0.7%, and continued the sharp slowdown compared to the previous year. On top of that, the United States struggled with a slowdown in economic growth in the first half of 2011, which was expected to weaken further in 2012, the high total public debt equal to 100% of GDP and historically low yields on government bonds. All things considered, the estimated high probabilities of recessions can be explained by the economic situation in Europe and the United States and the fact that the economies are interconnected.

The financial volatility does not seem to help much in predicting the industrial production growth. Included foreign effects, measured as the effect of the American economy on the German one, improved the forecasts in terms of obtained errors. However, it is also difficult to say that blocks with the financial volatility measures perform better than the traditional predictors. Block 1 with traditional predictors of economic activity characterized by the smallest errors, but its predicted growth seems to follow the actual industrial production growth, whereas block 5 with the domestic and foreign variables gave early signals about the drop in industrial production. Thus, it was chosen further into comparison analysis. Unfortunately, no other volatility measure
was more informative about the economic growth as they all follow nearly the same path and anticipate more the direction of the industrial production growth than its value. In conclusion, all blocks performed similarly and poorly in predicting the economic activity.

All things considered, there is a significant difference between modeling European, on the example of Germany, and American economic activity with financial volatilities. Firstly, in general the inclusion of two foreign effects, representing situation of the American economy, in this thesis helps in explaining and predicting the economic activity. Thus, as expected, the largest European economy is interconnected to the American one, and the other economies in the European Union are expected to be connected more between each other (Sensier et al., 2004). Provided that, modeling such economies is more complicated due to the problem of capturing the connection between economies. Secondly, the influence of the financial volatility on the German economy seems to be significantly smaller than on the American one. This may be caused by several reasons, for example because of the abovementioned interconnections or different policies that influence the regressors. Above all, there seem to be more structural differences between these two countries.

Conducted study suggests that the connection between the financial volatility and the German business cycle in Europe is weaker than expected. Nevertheless, it does not imply that researchers should not use financial volatilities in future studies about the business cycle in Europe. Although this study was prepared carefully, it has several limitations. Firstly, the lack of certain data or access to it led to exclusion of variables that might have predictive power, and based on the obtained results, they very likely do. Namely, no daily value-weighted returns of a market portfolio for Europe are available to calculate the realized volatility of the market portfolio, which was dominant in this analysis. Moreover, to the best of my knowledge, no dataset with daily value-weighted returns of all firms in the industries exists to construct idiosyncratic firm level volatility. I believe this data could increase the predictive power of the financial volatility. The “gross profit ratio”\textsuperscript{15} of non-financial corporations in the euro area is between 40-42\% since 2006, indicating that firm-level data could encode information about the real economic activity. The second significant limitation of the study is the lack of the ideal recession indicator. The choice of the ECRI-based business cycle indicator was made based on

\textsuperscript{15} The “gross profit ratio” is calculated in percentages as gross operating surplus divided by gross value added. Provided by Eurostat.
the previous research, methodology used for its construction and preliminary analysis that suggested it as a better choice. However, as mentioned above, this indicator does not classify 2011-2012 as a recession even though one can argue that during that time Germany observed an economic slowdown. In turn, the performance based model verification was not necessarily optimal, since it compared the estimated probabilities with observed values according to the chosen recession indicator. Nonetheless, there was no better option to consider. The OECD-based recession indicator was not diversified enough in terms of the values, and the model did not have enough calm periods to learn. Thirdly, the interconnection between German and American economies may have been modelled incompletely. I decided to use the same two foreign variables as in research done by Sensier et al. (2004). However, the paper is relatively outdated and does not include any financial volatility measures. Thus, possibly, the inclusion of international or foreign financial volatilities might have increased the predictive performance of the financial volatility. Fourthly, the data sample was relatively short and consisted of 261 monthly observations. Longer time series would allow extending the analysis to other financial crises and verifying whether the prediction power of the volatility changed over time. Usually, models estimated on larger datasets (in terms of number of observations) perform better. Lastly, due to lack of the computation power, only 3-months ahead iterative forecasts were calculated. However, Fornari and Mele (2013) show that certain blocks with financial volatilities and volatilities of financial volatility perform better in longer forecast horizons. In the view of the mentioned limitations, future research about this topic with the inclusion of the above-mentioned issues is needed to verify the fully captured predictive magnitude of the financial volatility and its inclusion in widely used prediction models.

6 Conclusion

The counter-cyclicality of the financial volatility is believed to encode information about the business cycle, since volatility seems to be larger during recessions and smaller during expansions. Challenging economic environment of global economy, various policy measures taken by central banks that affected and possibly distorted the predictive power of the most conventional economic indicators, and the critique turned against researchers that failed to predict the last severe recession, which led to a search for new leading indicators that would improve forecasting of the real economic activity. Thus, financial volatility was one of the solutions to the problem.
This work organized the previous models and summarized the basic findings in this field. The purpose of this thesis was to verify whether financial volatility can act as predictor of the economic activity, and if so, whether any measures of volatility perform better in this task, and help to improve forecasts from the conventional financial indicators. To my knowledge, it is one of the first studies including the volatility measures conducted for Europe, on the example of German economy, which was chosen based on its size and impact on other European countries. The expectations were verified by both the in-sample and out-of-sample analysis using the dynamic binary choice models to predict the business cycle and linear regressions to forecast the industrial production growth.

The obtained results suggest that financial volatility does not explain the business cycle in the magnitude that it does in the United States. However, it seems to contain more information about the expansionary periods and makes the estimates less noisy, even though it does not improve the forecasts in terms of the percentages of correctly identified recessions and expansions. Moreover, the study confirmed that the interconnection between the US and German economies is significant and important in modeling German business cycles. In-sample analysis reveals that no financial variable is significant, if the lagged probability of the recession indicator is taken into analysis. Although, statistically the model with the financial volatility fits best, i.e. the SBC is lowest. Nonetheless, for the autoregressive probit model, which performs reasonably well, the model with the volatility of financial volatility outperforms the other blocks of variables. The most parsimonious volatility measure of the broad market portfolio encodes most of the information needed for the prediction analysis. On the other hand, explaining and forecasting the industrial production growth seems to be a more difficult task. None of the volatility measures perform well in that part, they all seem to predict the direction in which the economy will go, but not the actual values.

The topic is of great concern for policy and corporate decision makers, because the correct assessment of the current and, especially, future economic situation is essential for good policymaking. This study can be considered as an introduction and indication for future research for Europe. Especially, in light of the limitations of this paper, more research is needed to further verify whether the financial volatility helps in predicting economic activity and should be included in mathematical models.
References


Moneta, F. (2003, December). Does the Yield Spread Predict Recessions in the Euro Area?. *ECB working paper, no. 294*


Appendix 1  Principal Component Analysis

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A1.1: Scree plot of eigenvalues after principal component analysis, and the characteristic of chosen principal component.

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A1.2: Eigenvalues and proportion explained of components.

Appendix 2  Baxter-King filter

A2.1: Spectral density functions for the principal component and the common factor.
Appendix 3  
GARCH modeling

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A3.1: Information criteria for alternative models.

Appendix 4  
Out-of-sample binary choice models estimates

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A4.1: Characteristics of static probit estimates of the business cycle indicator.
### A4.2: Characteristics of dynamic probit estimates of the business cycle indicator.

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### A4.4: Characteristics of dynamic autoregressive probit estimates of the business cycle indicator.

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<td>8.75%</td>
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| Variables as in Table X and two foreign variables |       |      |      |      |      |      |      |      |      |
| AUC   | 61.11% | 59.66% | 59.66% | 59.79% | 94.44% | 58.73% | 72.62% | 68.39% | 71.36% |
| # recessions | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| # expansions | 80 | 80 | 80 | 80 | 80 | 80 | 80 | 80 | 80 |
| # uncertain | 0 | 0 | 0 | 0 | 1 | 2 | 0 | 0 | 0 |
| MAE   | 8.83% | 8.75% | 8.75% | 8.63% | 9.01% | 9.08% | 8.74% | 8.68% | 8.67% |
| MSE   | 8.60% | 8.61% | 8.61% | 8.60% | 7.90% | 8.66% | 7.92% | 8.60% | 8.60% |
| freq reession | 55.56% | 55.56% | 55.56% | 55.56% | 55.56% | 55.56% | 55.56% | 55.56% | 55.56% |
| freq expansion | 95.24% | 95.24% | 95.24% | 95.24% | 95.24% | 95.24% | 95.24% | 95.24% | 95.24% |

| Variables as in Table X and two foreign variables |       |      |      |      |      |      |      |      |      |
| AUC   | 61.24% | 57.54% | 57.54% | 60.32% | 76.19% | 58.53% | 61.97% | 65.41% | 66.27% |
| # recessions | 5 | 5 | 5 | 5 | 6 | 5 | 5 | 5 | 5 |
| # expansions | 80 | 80 | 80 | 80 | 78 | 80 | 80 | 80 | 80 |
| # uncertain | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| MAE   | 8.90% | 8.72% | 8.72% | 8.62% | 9.79% | 9.18% | 9.12% | 8.63% | 8.64% |
| MSE   | 8.61% | 8.60% | 8.60% | 8.60% | 9.68% | 8.68% | 8.68% | 8.60% | 8.60% |
| freq reession | 55.56% | 55.56% | 55.56% | 55.56% | 66.67% | 55.56% | 55.56% | 55.56% | 55.56% |
| freq expansion | 95.24% | 95.24% | 95.24% | 95.24% | 92.86% | 95.24% | 95.24% | 95.24% | 95.24% |
### Appendix 5  Giacomini-White conditional test statistics

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Note: In bold statistics significant at 5% significance level.

**A5.1:** Test statistics for models with the domestic and foreign variables, estimated probabilities of recession out-of-sample by autoregressive probit.