Letter

Wireless Systems

New quasi-orthogonal BCH-derived sequences for CDMA applications

Cesaltina N. E. Ricardo¹, Manuel D. Ortigueira¹* and José António B. Gerald²

¹UNINOVA/DEE Campus da FCT da UNL, Quinta da Torre, 2825-114 Monte da Caparica, Portugal
²Instituto Superior Técnico and INESC, R. Alves Redol, 9, 2º 1000-029 Lisboa, Portugal

SUMMARY

Based on two methods recently proposed—the ‘Ranging Criterion’ (RC) and the ‘Generators Ranging Criterion’ (GRC)—new (quasi-orthogonal) even BCH-derived sequences are generated which are very attractive for synchronous or quasi-synchronous Code Division Multiple-Access (CDMA) systems. Numerical results show that the new family of BCH-derived sequences can contain a higher number of quasi-orthogonal sequences with lower correlation values and higher processing gains (PGs) than the spreading sequences typically used in the third generation of mobile communications system, UMTS or in the recent large area synchronised CDMA (LAS-CDMA) technology. It is shown that the even BCH-derived sequences are easily generated by a linear shift register generator, allowing the construction of systems with receiver structures of low complexity as compared with those of quasi-synchronous systems using low correlation zone sequences, as for instance the LAS-CDMA system. Copyright © 2007 John Wiley & Sons, Ltd.

1. INTRODUCTION

In recent years, the Code Division Multiple-Access (CDMA) technology has been used in cellular mobile communication systems (for instance, UMTS/IMT-2000 and IS-95) with two aims: (1) to counterbalance the problem of spectrum overload due to the excessive growth of the number of users; (2) to follow the new tendencies of the telecommunications market in providing a large variety of services, including broadband applications, with identical quality of service of wired networks. These requirements are completely satisfied by the current UMTS system, that uses the wideband CDMA (W-CDMA) technique to provide higher capacity, coverage, and spectral and power efficiencies, than the systems based on TDMA, FDMA or narrowband CDMA (as IS-95) [1–4].

Additionally, the Direct Sequence CDMA (DS-CDMA) form provides some other important features such as resistance against multipath fading and low interference. In these systems, each user is identified by a specific pseudo-noise (PN) code [5–9]. So, the average performance of these systems can degrade significantly when the number of active users increases, due to the increase in the level of multiple-access interference (MAI) in the channel [7, 9–14]. This depends on the (out-of-phase auto and cross) correlation properties of the PN spreading sequences [13, 14]. Hence, reduced values of out-of-phase autocorrelation and cross-correlation should be required, respectively, to make easier the synchronisation and to minimise (or eliminate) the MAI. This issue is very important, because it allows the capacity of CDMA systems to increase [7, 9, 11, 13]. For instance, the well-known

*Correspondence to: Manuel D. Ortigueira, UNINOVA/DEE Campus da FCT da UNL, Quinta da Torre, 2825-114 Monte da Caparica, Portugal. E-mail: mdortigueira@uninova.pt
orthogonal binary Walsh-Hadamard sequences are used to eliminate the MAI in synchronous channels (in-phase channels), like the downlink transmission in mobile communication systems. However, these sequences have the worst values of cross-correlation when compared with other (orthogonal and non-orthogonal) PN sequences usually used in asynchronous (or quasi-synchronous) CDMA systems [14–17].

Recently, quasi-synchronous CDMA (QS-CDMA) systems using sets of orthogonal [18, 19] and quasi-orthogonal [19] spreading sequences have been actively investigated, due to their superior immunity against synchronisation errors [18, 19], higher bandwidth efficiency [20], and consequently, better performance, when compared with those obtained with the traditional spreading sequences [18, 19, 21]—the quasi-orthogonal spreading sequences are characterised by a correlation zone almost zero around the zero-phase shift. We find applications of these results in (quasi-synchronous) large area synchronised CDMA (LAS-CDMA) [19] and multicarrier CDMA (MC-CDMA) [17, 19] systems, which are both strong candidates for the fourth generation of mobile communication systems.

In this paper we study a new family of quasi-orthogonal spreading sequences, the even BCH-derived sequences—belonging to the class of pseudo-noise even balanced (PN-EB) sequences discussed in References [22, 23]—having very good correlation properties. This new family of sequences allows a higher performance—measured by the Processing Gain (PG)—in additive white Gaussian noise (AWGN) channels than that provided by the odd ones commonly used in CDMA systems: the Gold sequences [24], the BCH sequences [24], the Gold-Like sequences [24], the m-sequences [24] and the large set of Kasami sequences [24]. The main reason for the choice of these odd PN sequences is due to the existence of a large number of sequences with small (out-of-phase auto and cross) correlation values and their simple generation procedure (the Gold sequences, the m-sequences, the Gold-Like and the large set of Kasami sequences are easily generated by the linear feedback shift register generator [24]).

The numerical results we obtained confirmed that the new family of even BCH-derived sequences contains a high number of (quasi-orthogonal) sequences with low correlation levels, better than those presented by the even sequences of the recent UMTS system [16, 17] and higher PGs, even when compared with the LS sequences [19, 25] of the LAS-CDMA system. These characteristics can make the even BCH-derived sequences very attractive in fading dispersive channels, either in synchronous or quasi-synchronous channels.

The paper outline is as follows. In Section 2, we present the PG definition that we will use and describe the steps for generating the new family of BCH-derived sequences. We give two examples of sequence generation to make easier to understand the procedure. In Section 3, we present some simulation results to illustrate the behaviour of the new sequences. At last we will present some conclusions.

2. ABOUT THE QUASI-ORTHOGONAL BCH-DERIVED SEQUENCES

2.1. The processing gain

Consider the base-band DS-SS communication system of Figure 1, where \(x(t)\) is the information signal, with bit rate \(R_b\), and \(b(t)\) is a periodic PN spread spectrum sequence, with chip rate \(R_c \gg R_b\). The low-pass filter has the same bandwidth \(B_x\) of \(x(t)\). We assume that in each period \(T\) of \(b(t)\), there are \(N\) rectangular pulses of duration \(\tau\) such that \(N\tau = T\) and amplitude \(a_i = \pm 1, \) for \(i = 0, 1, 2, \ldots, N - 1\).

The receiver output signal is the sum of the recovered information signal \(x(t)\) with the filtered spread noise signal \(n_f(t)\). Assume that there is no spreading: we receive a signal that is the sum of the original signal and noise, with powers equal respectively to \(S_o\) and \(N_o\). If we have spreading, we receive again the original signal with a filtered spread noise, with powers equal to \(S_i\) and \(N_i\), respectively. The performance of the system depends only on the spread noise power because, ideally, we do recover the original

---

**Figure 1.** Ideal base-band DS-SS system.
signal \( x(t) \). In this case, the signal power, with \( S_s \) or without \( S_o \) spreading, is the same at the output system, that is \( S_s = S_o \). As the noise output power depends on the spreading, the output signal to noise ratio with, \( S_s/N_s \), and without, \( S_o/N_o \), spreading are different.

These considerations led, Ortigueira et al. [26], to propose the system performance analysis according to a new definition of PG, given as:

\[
P_G = \frac{S_s}{S_o}/N_s/N_o
\]

Using \( S_s = S_o \) in Equation (1), we can obtain the final result:

\[
P_G = \frac{N_o}{N_s}
\]

Remark that this definition of PG is different from the usual, in which the PG is defined as the quotient between the spread and the original signal bandwidths leading, in the case of a binary signal source, to a PG that is the quotient between the spreading chip rate and the source bit rate [5, 6]. To compute the PG defined in Equation (2), we have to analyse the effect of spreading over the noise.

To study the band-limited case, we considered a noise signal, \( n(t) \), with bandwidth \( B_n \) smaller than the spread spectrum signal bandwidth, that is narrowband noise. After spreading, the power spectrum of \( n(t) \) is the sum of weighted repetitions of power spectrum of \( n(t) \) located at multiple frequencies of \( 2\pi/T \). Inside the reference band, \( B_s \), the total number of replicas decreases with \( T \), and this decreasing stops when \( T < 1/(B_n + B_s) \). In this last case, there will be only one replica of the power spectrum of \( n(t) \) inside the band \( B_s \) and located at the zero frequency. Hence, the PG defined by Equation (2), will be given by Reference [26]:

\[
P_G = \frac{1}{\left( A(0) \right)^2}
\]

where \( A(n) \) is the Discrete Fourier Transform (DFT) of the sequence \( a_i \) for \( i = 0, 1, 2, \ldots, N - 1 \).

From Equation (3), we can conclude that the PG is not defined if \( A(0) = 0 \). This is an ideal situation that needs a sequence with zero mean value and a strictly band-limited noise. We do not find the second situation in practice but if the noise spectrum assumes very small out of band values, the PG can be made very high by using a spreading signal with a zero mean value. This can be achieved through two distinct ways:

1. Adding one extra zero symbol to the period of a binary \( m \)-sequence [24] or a similar odd-balanced (binary) sequence in order to achieve an equal number of ones and zeros; or,
2. Using Manchester pulses instead of rectangular ones in the odd-balanced (binary) sequences.

This last option was not considered due to the increase in the complexity of the system, corresponding costs and band duplication. So, in the following we shall be interested in constructing zero mean sequences to achieve high PGs.

2.2. The design of even BCH-derived sequences

Taking option (1) of Subsection 2.1 as reference, we will apply two methods—the Ranging Criterion (RC) [22] and the Generators Ranging Criterion (GRC) [23]—to generate new classes of sequences with zero mean value, namely the even (quasi-orthogonal)-BCH-derived sequences obtained from the family of odd-balanced BCH sequences by inserting an extra zero.

Assume we have a set of odd-balanced BCH sequences [24]. To apply the RC we follow the next four steps:

1. Insert the extra zero at the end of sequence;
2. In all those sequences that still present higher out-of-phase autocorrelation levels, move the extra zero into the longest run of zeros;
3. For those sequences without out-of-phase autocorrelation values improvement, place the extra zero into the run of zeros nearest to the \([((N + 1)/2) + 1]\) position;
4. For the cases, in which the steps (2) and (3) did not improve the result of step (1), place the extra zero into the nearest run of zeros of the location in step (1).

In the following example, we illustrate the RC application to one odd-balanced BCH sequence. Example 1. Consider the odd-balanced BCH sequence \( a_1 \) of period 63 generated by the primitive polynomial \( x^6 + x^5 + x^3 + x^2 + 1 \), that is

\[
a_1 = (10110010010000011011100011010100011011001101011010110111000101100100101101)
\]

To generate the new even BCH-derived sequence, the four steps of RC are applied consecutively to the odd-balanced BCH sequence, as described above. For each step, the autocorrelation function of the new sequence is
calculated and compared with that of the next step. At the end of RC, we choose the even sequence with the lowest out-of-phase autocorrelation levels. Thus, applying the step (1) of RC, we obtain the even sequence of length 64,

\[ a_1^{(1)} = (10110010010000011011001010100 \]
\[ 011101100110110010101011001101) \]

with maximum absolute value of out-of-phase (normalised) autocorrelation, \( |R_{a_1^{(1)}}(n)| = 0.375 \).

Next, for step (2), we have,

\[ a_1^{(2)} = (101100100100000011011100011010100 \]
\[ 0111011001101100101010110011011) \]

with maximum absolute value of out-of-phase (normalised) autocorrelation, \( |R_{a_1^{(2)}}(n)| = 0.3125 \). Doing the same for steps (3) and (4), we obtain a maximum value of out-of-phase autocorrelation to the new sequences equal to that of step (2), which is 0.3125. So, comparing all the obtained results, we observe that the even BCH sequence \( a_1^{(2)} \) has the best out-of-phase autocorrelation values.

To apply the GRC to the odd-balanced BCH sequences, we follow the steps:

1. Insert the extra zero at the end of sequence;
2. In all those sequences that still present higher out-of-phase autocorrelation levels, move the extra zero into the longest run of zeros;
3. For those sequences that did not improve their autocorrelation levels with the previous two steps, the extra zero should be placed at the nearest phase defined by the first or the second large run of zeros of one of the two generating sequences of the odd-balanced BCH sequence, as indicated in the Example 2 below.

To note that the steps (1) and (2) of GRC are equivalent, respectively, to the steps (1) and (2) of RC.

Next, we give an example of GRC.

**Example 2.** Consider the odd-balanced BCH sequence \( a_1 \) of Example 1.

In this case, the steps (1) and (2) of GRC give the same results of steps (1) and (2) of RC of Example 1. Hence, we do not repeat them here.

To apply the step (3) of GRC, we consider \( a_1 \) decomposed on its generating sequences \( a_{11} \) and \( a_{12} \), such that \( a_1 = a_{11} \oplus a_{12} \), where \( \oplus \) is the modulo-2 addition. Let us apply the step (3), for instance, to the sequence \( a_1^{(i)} \) (of length 63), given by:

\[ a_{11} = (1011010110011001010011000001110100 \]
\[ 11001001010000011010011001001010) \]

We will obtain the even sequence (of length 64)

\[ a_{11}^{(3)} = (10110101101100110001100011100000 \]
\[ 1001111110101001100111000111011011010) \]

Let \( a_{12}^{(3)} \) be the sequence of length 64, resultant of \( a_{12} \) by inserting one zero at the end of sequence, that is

\[ a_{12}^{(3)} = (00000110100110010010100000110100 \]
\[ 1100100100000110100110011000110001100100) \]

Now, doing the modulo-2 addition of \( a_{11}^{(3)} \) and \( a_{12}^{(3)} \) sequences, we obtain the even BCH-derived sequence

\[ a_1^{(3)} = (10110101001000001101000011101000101000 \]
\[ 01001001101001001101101101011010001100011000) \]

with maximum absolute value of out-of-phase (normalised) autocorrelation function, \( |R_{a_1^{(3)}}(n)| = 0.25 \).

We can note that \( a_1^{(3)} \) has a lower value of out-of-phase autocorrelation peak than that provided by steps (1) and (2) of GRC, which are, respectively, 0.375 and 0.3125.

### 3. NUMERICAL RESULTS

Table 1 presents the numerical results of out-of-phase (normalised) autocorrelation function of the new even BCH-derived sequences obtained with RC and GRC criteria.

<table>
<thead>
<tr>
<th>Number of sequences by correlation level</th>
<th>0.1875</th>
<th>0.25</th>
<th>0.3125</th>
<th>0.375</th>
<th>0.4375</th>
<th>0.5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even BCH</td>
<td>1</td>
<td>49</td>
<td>37</td>
<td>7</td>
<td>1</td>
<td>1</td>
<td>96</td>
</tr>
<tr>
<td>BCH with RC</td>
<td>9</td>
<td>73</td>
<td>14</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>96</td>
</tr>
<tr>
<td>BCH with GRC</td>
<td>12</td>
<td>78</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>96</td>
</tr>
</tbody>
</table>

Table 1. (Periodic) out-of-phase (normalised) autocorrelation function results with RC and GRC criteria applied to odd-balanced BCH sequences of period 63.
sequences can make impractical the CDMA communications in presence of highly dispersive channels due to the highest interference levels. By the above described reasons, the (quasi-orthogonal) even BCH-derived sequences can be a good alternative to the LS and LA sequences in very dispersive channels.

In what concerns to the construction procedure, the even BCH-derived sequences have lower complexity of hardware implementation than the GMW-derived sequences [27] or the LS and LA sequences. (The even BCH sequences can be easily generated by a linear shift register generator or by a cyclically read memory table.) In the special case of CDMA systems which use sequences with zero correlation zones (as the LS and LA sequences), the mechanisms of dispreading and code acquisition are more involved and difficult than that of the conventional DS-CDMA systems [29]. This complexity can still be larger when the parameter $Z$ increases (to accommodate the highest delay dispersion of the channel), once a new construction of zero correlation zone sequences will be required [30]. This can be a disadvantage when compared, for instance, with the optimised even BCH-derived sequences which do not need of a new construction of sequences for variations of $Z$.

Figures 2 and 3 show the results we obtained for the PG as a function of the period, for odd sequences (LS [19, 25], Gold, large set of Kasami, BCH, Gold-like and $m$-sequences [24]) and even sequences (even BCH-derived, Walsh/Hadamard [6, 31] and $m$-sequences with Manchester pulses), respectively.

In these simulations, the base-band DS-SS system of Figure 1 and a PN code with a chip rate $R_c = 400$Kchips/s were used. It was also assumed a base-band noise signal (obtained by low-pass filtering) with bandwidth $B_n$ equal to the signal bandwidth, that is, $B_n = B_s = 1$KHz. (Remark that the noise bandwidth is narrower than that of the spread signal.) The PG was calculated using Equation (2). In these conditions, the aliasing occurs when the sequence period has a number of chips greater than 200. Figure 2 shows that the LS\textsuperscript{1} sequences can provide a higher PG (for lengths less than 150) than the (balanced) Gold, Kasami, $m$-sequences (all with identical gains), and the non-balanced BCH and Gold-like sequences. We expect similar results to balanced sequences in the PG when the BCH and the Gold-like sequences are balanced as the Gold sequences.

\textsuperscript{1}The odd and even LS sequences have identical PG and non-zero mean value.
In Figure 3, we can observe that all the sequences with zero mean value, including the (quasi-orthogonal) even BCH-derived sequences generated by the GRC method, present a very high PG when compared with the odd sequences, in the case of no aliasing. This confirms our previous statements in Section 2. These results also show that the (quasi-orthogonal) even BCH-derived sequences have better resistance against the narrowband interference.
than the LS sequences. This last result is due to the lowest spectral content of even BCH-derived sequences around the low frequencies when compared with that of LS sequences.

Another result, not illustrated in these figures, is that the optimised even BCH-derived sequences present a slightly smaller gain (a few dBs) than the non-optimised even BCH sequences (with the extra zero placed at the end sequences period). This is justified by the fact that the non-optimised even BCH sequences presented a lower spectral content at low frequencies when compared with the same optimised even BCH-derived sequences. However, the improvement in the correlation results largely justifies the use of the optimised ones.

4. CONCLUSIONS

In this paper, we presented and studied a family of sequences that due to the fact of presenting good correlation properties (excepting the Hadamard sequences) and existing in high number make them useful in CDMA systems. These new sequences are easily generated from pre-existing odd BCH sequences through two similar criteria: the RC and the GRC. The steps of both procedures were described and exemplified.

We illustrated the characteristics of the sequences, in terms of the PG and the cross-correlation levels they assume. The new (quasi-orthogonal) optimised BCH-derived sequences, allow a very high PG in selective noise condition, which can be important in fading channels, where the traditional odd sequences present smaller efficiency. This subject deserves further research in future. Nevertheless, the results already obtained confirm the theoretical predictions and represent a high motivation for the study and development of new even spreading sequences suitable for SS systems.

It was also shown that the RC and GRC criteria can provide families of even PN sequences presenting a high number of quasi-orthogonal sequences with reduced out-of-phase correlation levels. This feature can make them attractive for quasi-synchronous systems.

REFERENCES


