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Theory and Evidence on Self-Fulfilling Sovereign Debt Crises

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Abstract

This work analyzes theoretically and empirically the potential self-fulfilling features of sovereign debt crisis. The theoretical model modifies Cole and Kehoe (1996, 2000) by considering that the default is partial. In the model, there are debt limits within which self-fulfilling crises may occur. The numerical results show that, within the crisis zone, up to an intermediate debt level, the optimal government policy is to run down the debt until it reaches the safe limit to avoid higher borrowing costs. Above a certain amount, however, the government chooses to run up the debt, to avoid sharp reduction in government spending. The empirical investigation assesses the determinants of the probability of default in Portugal. The model builds on Jeanne and Masson (2000) and is brought to Portuguese data using a Markov-switching regime framework. The results show that between 2000-14 two regimes subsisted: a tranquil and a crisis regime. The switch between regimes seems to be unrelated with macroeconomic fundamentals, which is interpreted as self-fulfilling jumps in the beliefs of credit markets.

Keywords: Sovereign debt crisis, Partial Default, Self-fulfilling crisis, Markov-switching regimes

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Abbreviations

CDS	Credit Default Swap
CTP	Constant Transition Probability
EC	European Commission
ECB	European Central Bank
EU	European Union
GDP	Gross Domestic Product
IMF	International Monetary Fund
MSR	Markov Switching Regime
REER	Real Effective Exchange Rate
RVC	Regime Varying Coefficients
TVTP	Time-varying Transition Probability
ULC	Unit Labor Cost

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Introductory note

This thesis addresses *Theory and Evidence on Self-Fulfilling Sovereign Debt Crises.* Its aim is to analyze the potential self-fulfilling features of sovereign debt crises, both theoretically and empirically, the former considering a partial default scenario and the latter observing the performance of the Portuguese economy.

The theoretical background are *second generation crisis* models, which are characterized by their articulation of fundamentals and market expectations, in the sense that crises are ascribed either to the deterioration of economic conditions or to shifts in expectations. These models usually lead to multiple equilibria since, depending on expectations, similar macroeconomic fundamentals may lead the economy to a different equilibrium.

In the first chapter, a Dynamic Stochastic General Equilibrium (DSGE) model is presented. The most significant agent of this theoretical model is a benevolent government in need to rollover the public debt. The model is an extension of Cole and Kehoe (1996, 2000), but it considers the possibility of a partial rather than a total default. Additionally, it postulates that following the partial default the country will maintain its access to international credit markets. This feature contrasts with CK's model, in which the country is banned from international credit markets.

The model analyzes two debt thresholds: a lower safe limit, below which default is never the optimal decision, and a high sustainable limit, above which default is always optimal. In between these thresholds emerges a crisis zone, where self-fulfilling beliefs may condition the government's decision to repay the debt. In this crisis zone, if creditors believe that the government will repay the public debt, they will be willing to lend at a lower price, thus allowing the debt rollover and ultimately helping to meet their initial expectations (of no default). If not, international credit will only be available at a higher price, thus contributing to default, again fulfilling the initial expectations (of default).

The results of the analytic and numerical calculations can be summarized as follows: (i) in the crisis zone, so as to avoid the costs of a default, the government has an incentive to run down debt until it leaves the crisis zone; (ii) however, above a certain intermediate level of the crisis zone, the incentive is to avoid a sharp reduction in government spending, and thus the optimal decision will be to run up the debt; (iii) when the cost of default is low, the crisis zone is narrower, and there is an incentive not to reduce the debt level at all.

The second chapter is an empirical investigation of the determinants of the probability of default in Portugal from January 2000 until December 2014. The model builds on Jeanne and Masson (2000) seminal paper on the self-fulfilling features of currency crisis and is brought to Portuguese data through a Markov-switching regime framework.

The main conclusions are outlined as follows: (i) during the period under analysis two regimes were identified - a tranquil regime, with low expected default rates and low bond yields, where favorable macroeconomic fundamentals are perceived as sustainable and thus self-fulfill low default expectations; and a crisis regime, with high expected default rates and high yields, where macroeconomic fundamentals are unfavorable thus favoring high probability of default; (ii) the switch between regimes seems to be unrelated to macroeconomic fundamentals; (iii) the latter indicates that the debt crisis may have been, to some extent, self-fulfilling; indeed, a regime switch is interpreted as self-fulfilling jumps in the beliefs of creditors.

The two chapters resort to the same theoretically background but present two different approaches to the self-fulfillment hypothesis. The first chapter builds on a DSGE model, in which results are analytically and numerically obtained. The second chapter resorts to a structural approach, applied to the Portuguese data through econometric procedures. Although the two chapters are autonomous and may be independently analyzed, they are theoretically interconnected and the empirical study on the Portuguese scenario represents a natural follow up of the theoretical study.

Chapter 1

Self-fulfilling Sovereign Debt Crises with Partial Default

1.1 Introduction

The recent euro area sovereign debt crisis has attracted renewed interest on the determinants of sovereign debt defaults, in particular on sustainable public debt limits and on the possibility that financial markets' beliefs dominate economic fundamentals. Following Cole and Kehoe (1996, 2000), this work aims at finding circumstances in which sovereign debt crises may be self-fulling. In the model, these circumstances are related to the definition of debt limits, within which a self-fulfilling crisis may occur, and to the lenders' expectations on the probability of default.

A self-fulfilling crisis may arise as a result of lenders beliefs on the government willingness to repay previous debt. Taking into account that public debt is exposed to the financial institutions' disposition to accept government to roll over existing debt that has reached maturity, then if bankers do not lend, the country will default. Within this context, if bankers do not expect government to repay, they will not lend, thus conditioning the government's decision in such a way that it becomes optimal to fulfill those expectations. So, if bankers assign a positive probability to a government default, it may indeed collapse a default on debt that would otherwise be repaid. Such default is expressed by a debt restructuring that results in a loss to creditors, either in the form of an outright default, a loss in face-value (write-off), or even a debt rescheduling.

In the literature, sovereign defaults are commonly modeled as discrete events, where countries either repay or default on the total amount of liabilities. Accordingly, Cole and Kehoe (1996, 2000) assume that the default is on total debt and that whenever a default occurs, the country is permanently banned from international financial markets. However, a partial default is the most common case: one of the findings of the cross-country historical review of public debt data from 1750 by Reinhart and Rogoff (2009, p. 61) is that most defaults are partial because, "even if creditors do not have the leverage (from whatever source) to enforce full repayment, they typically have enough leverage to get at least something back". Therefore, the CK model is modified by introducing the possibility of a partial default and by considering that, when public debt suffers a haircut of $1 - \theta$, in subsequent periods the government is still able to issue new debt, although at a higher cost. The model is a general version of the original one, since if $\theta = 0$ the model becomes a complete default one.

The initial debt level is the crucial variable that determines the occurrence of a default. More specifically, and along the lines of CK, if the initial debt level is low enough, a default never occurs; on the contrary, if the debt level is sufficiently high, a default constitutes the optimal decision. The other possibility is the initial debt level falling in the crisis zone, wherein a self-fulfilling crisis may take place in case international bankers lend at a higher cost: if the country is depending on its ability to roll over debt contracts, such that it becomes vulnerable to an increase in the cost of borrowing, then if creditors lend at a higher price, the country will end up defaulting.

The definition of these debt thresholds is a relevant issue, since empirical literature has shown that defaults occur for almost any initial debt level. Reinhart and Rogoff (2009, p. 25) show that (i) external debt-to-GDP exceeded 100 percent in only 17 percent of the defaults or restructuring episodes; (ii) one half of all defaults occurred at levels below 60 percent; (iii) defaults took place against debt levels that were below 40 percent of GNP also in 17 percent of the cases.

Following Cole and Kehoe (1996, 2000), we construct a DSGE model of a simple economy with three types of agents: households, international bankers and a benevolent government. Households derive utility from the consumption of a private good and from a government-provided public good; they also save in the form of physical capital. International bankers act competitively and decide on the price they pay for public debt. As in the CK model, creditors are international bankers, because empirical evidence suggests that external defaults are more frequent than domestic defaults and that the latter may trigger external defaults (Reinhart and Rogoff, 2009, p. 61). The government decides on the level of spending in the public consumption good, on whether to default, and on the issues of public debt. However, the government lacks commitment to future policy choices, such that it may decide to default even though it has issued new debt. This feature is modeled by assuming that, within each time period, agents make decisions sequentially. As previously mentioned, we modify CK model by allowing a partial default and the country's permanence in credit market, although with a burdensome debt.

After a default, as suggested by empirical evidence, significant downturns in economic activity take place. This feature is included in the model by means of a drop in productivity depending on the haircut fraction. Since private agents' decisions depend on how much they believe productivity will be, the mere possibility of a default leads private agents to lower investment levels. Additionally, a default risk implies an increase in the cost of borrowing, stemming from an increase in the risk premium of bonds. In the model, if the debt level moves from the safe to the crisis zone, there is an increase in the cost of borrowing; after a default occurs, the cost further rises.

The model provides optimal policy decisions for the strategic benevolent government, which is vulnerable to speculative attacks on the part of non-strategic international bankers. The computation of debt thresholds is one of the main features of the model, because agents' optimal decisions depend on which "zone" where the initial debt is included. Moreover, since the model includes partial default to take place, it is possible to relate debt thresholds with the fraction of default. The main findings of the model are obtained through computational methods, using the CK model parameters. The results of the analytical and numerical calculations can be summarized as follows: (i) when in the crisis zone, in order to avoid the costs of a default, the government has an incentive to run down the debt positions until it leaves the crisis zone; (ii) However, above a certain intermediate level the incentive is to run up the debt, avoiding sharp reduction in government spending; (iii) if the cost of default is low , the crisis zone narrows down and there is an incentive not to reduce the debt level at all.

The outline of this chapter is as follows. Section 2 proceeds with empirical observations that inform and motivate our subsequent analysis and reviews the literature related to self-fulfilling crisis and partial defaults. Section 3 includes the characterization of the baseline environment, the definition of a the markov-equilibrium and the computation of the equilibria. Section 4 presents numerical results and its discussion. Finally, Section 5 concludes.

1.2 Facts and background on sovereign defaults

1.2.1 Some evidence on partial defaults

This section provides evidence of some facts related to defaults that support some assumptions of the model. Notwithstanding, the first question that needs to be addressed is what comprises a default. Even though there are several definitions, it is possible to assess a common aspect: a sovereign debt default occurs when, as the result of a change in the contract initial conditions, the creditor incurs in a loss. The most obvious forms of default are outright defaults, when the government repudiates the debt payment, and debt write-off, which corresponds to a reduction in the face-value of debt. Besides these situations, debt rescheduling may also be considered a default, if it results in a loss to creditors, as considered by Reinhart and Rogoff (2009).

A specification of the type of events that constitute a sovereign default is provided by Moody's (2014, p. 4). Accordingly, a default occurs when there is: (i) a missed or delayed disbursement of interest or principal payment; (ii) a distressed exchange whereby (1) an obligor offers creditors a new or restructured debt, or a new package of securities, cash or assets that amount to a diminished financial obligation relative to the original obligation; and (2) the exchange has the effect of allowing the obligor to avoid a payment default in the future; or as defined in credit agreements and indentures; (iii) a change in the payment terms of a credit agreement or indenture imposed by the sovereign that results in a diminished financial obligation, such as a forced currency re-denomination (imposed by the debtor, himself, or his sovereign) or a forced change in some other aspect of the original promise, such as indexation or maturity.

Alternatively, Arellano, Mateos-Planas and Rios-Rull (2013) define a partial default as the fraction of payments missed, which is different from a debt haircut: "Debt haircuts are generally measured as the fraction of the value of debt in arrears that lenders lose after renegotiations. Partial default (...) measures the fraction of bonds in arrears."

To emphasize why the definitions are important, consider the 2012 restructuring of Greek sovereign debt. Greece's haircut did not generate missed payments or non-observation of any other contractual clauses. However, Greece demanded new terms and creditors consented, which was recognized by rating agencies as a default, since the new terms implied losses to investors in excess of 70%.

Herein, we consider that a default occurs whenever the creditor recovers less than the complete amount it was initially agreed. This recovery rate is hardly 0%, which would constitute a complete default. In fact, a partial default is the most common situation and, indeed, several empirical studies find no evidence of complete default. Cruces and Trebesch (2013) construct a database of sovereign debt defaults (haircuts) with foreign banks and bondholders from 1970 until 2010, covering 180 cases in 68 countries. The haircut is computed as the percentage difference between the present values of old and new instruments, discounted at market rates prevailing immediately after the exchange. The authors find that the average sovereign haircut is 37%; that there is a large variation in haircut size (one half of the haircuts are below 23% or above 53%); and that the average haircuts have increased over the last decades. Figure 1.1 depicts how sovereign debt haircuts are distributed over time and countries, showing that no episode of a complete default was found in the sample.

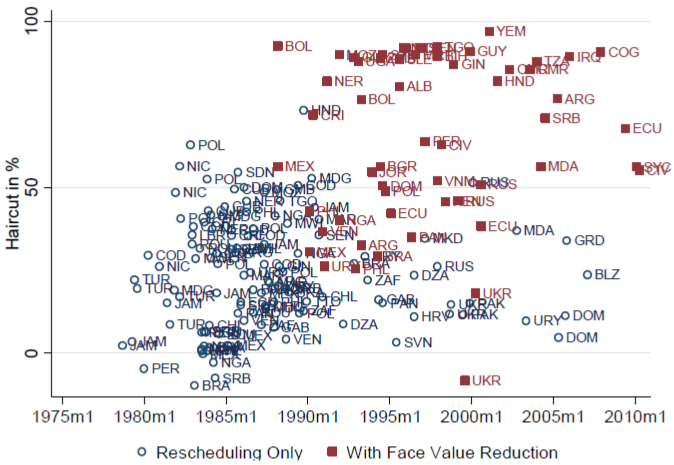


Figure 1.1: Defaults are partial I
Source: Cruces and Trebesch (2013)

Figure 1.1 also shows that the haircut on rescheduling episodes is on average lower than the haircut resulting from debt restructuring. Within the 180 episodes, there were 123 pure rescheduling, with a mean haircut of 24%, while the remaining 57 restructurings also involved face-value reduction and had a much higher mean of 65%.

Arellano, Mateos-Planas and Rios-Rull (2013) corroborate this perspective and, by extending Cruces and Trebesch (2013) panel to 99 developing countries from 1970-2010, they document that countries often have some debt outstanding payments and that default is always partial. The definition of a default here is different: default-to-debt ratio in Figure 1.2 is computed as the ratio of arrears to total debt (arrears plus debt service).

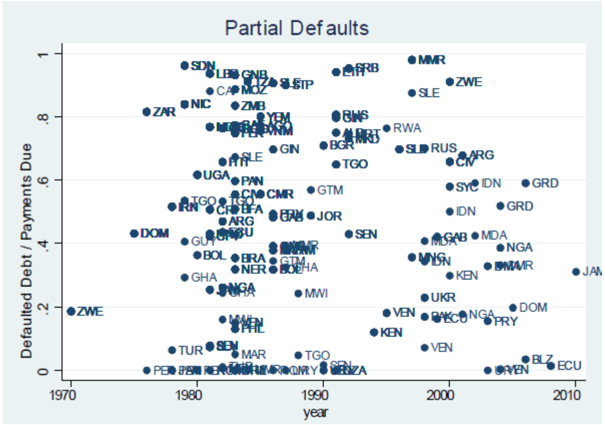


Figure 1.2: Defaults are partial II
Source: Arellano, Mateos-Planas and Rios-Rull (2013)
.

In this study, countries default on average on 22% of total liabilities and they have positive arrears about 53% of the time. In this latter case, the average default increases to 50% and countries continue to service the debt, paying about 4% of output and continue to borrow about 1.7% of output.

Moody's (2014), covering data from 1983-2013 among 124 Moody's-rated sovereigns, provide average recovery rates both issuer- and value-weighted. The distinction between issuer- and value-weighted averages is important because smaller countries are more likely to default, thus potentially inflating issuer-weighted default rates.

Also, they compute the recovery rate as percentage change in the trading price of government bonds as relative to PAR (30-day post-default price or pre-distressed exchange trading price) and as ratio of the present value of cash flows received as a result of the distressed exchange versus those initially promised, discounted using yield to maturity immediately prior to default. As shown in Table 1.1, the historical average recovery rate on sovereign bonds on a value-weighted basis is 26% or 43%, based on the first or on the second measure.

Table 1.1: Default recovery rates, 1983-2013

Year of default	Defaulting Country	Average Trading Price (percent of PAR)	PV Ratio Of Cash Flows (Ratio in percent)
1998	Russia	18	50
1999	Pakistan	52	65
1999	Ecuador	44	60
2000	Ukraine	69	60
2000	Ivory Coast	18	NA
2001	Argentina	27	30
2002	Moldova	60	95
2003	Uruguay	66	85
2003	Nicaragua	NA	50
2004	Grenada	65	NA
2005	Dominican Republic	95	95
2006	Belize	76	NA
2008	Seychelles	30	NA
2008	Ecuador	28	NA
2010	Jamaica	90	80
2012	Greece	24	29
2012	Greece	37	40
2012	Belize	40	65
2013	Jamaica	NA	89
2013	Cyprus	NA	53
Issuer-Weighted Recovery Rates		49	66
Value-Weighted Recovery Rates		26	43

Source: Moody's (2014)

Besides the fact that defaults are partial, Mendoza and Yue (2012) using data from 23 default events in the 1977-2009 period, find three empirical regularities: (i) Default events are associated with deep recessions, (ii) Interest rates on sovereign debt peak at about the same time as output hits its through and defaults occur, and they are negatively correlated with GDP, (iii) External debt as a share of GDP is high on average, and higher when countries default. In relation to output, Peña (2012) shows the movements in output, consumption and investment in Argentina, Ecuador, Mexico, Philippines and Russia around their default episodes of 2001, 1999, 1982, 1983 and 1998, respectively, as presented in Table 1.2.

Table 1.2: Changes in output in selected defaulting countries

Country	Default episode (starting date)	Output	Deviation from trend: Consumption	Investment
Argentina	2001Q4	-11.2	-11.4	-44.2
Ecuador	1999Q3	-6.0	-15.1	-25.1
Indonesia	1998Q2	-7.6	-18.1	-16.3
Mexico	1982Q3	-3.1	-4.9	-21.8
Peru	1980Q1	-3.7	-8.7	-25.7
Peru	1983Q1	-8.4	-10.0	-12.9
Philippines	1983Q4	-7.7	-9.6	-39.1
Russia	1998Q4	-10.8	-10.6	-38.3
South Africa	1985Q3	-2.1	-8.1	-8.2
South Africa	1989Q4	1.5	-1.3	6.4
Average		-5.9	-9.8	-22.5

Source: Peña (2012)

All cases are characterized by a decrease in output and consumption, with the largest decline observed in investment dynamics: average output and consumption deviations from trend range roughly between -6% and -10% , respectively, while their investment counterpart varies by as much as 22.5% below trend.

These findings are in line with Mendoza and Yue (2012), who conclude that defaults are associated with an average GDP and consumption fall about 5% below trend and a labor fall to a level about 15% lower than in the three years prior to the defaults.

Another important feature related to debt defaults is its relation to interest rate spreads, since these reflect the probability that investors assign to a crisis. Taking as an example the yields of euro area peripheral countries' sovereign bonds from 2008, shown in Figure 1.3, we observe that these yields started to move apart from German yields, considered risk-free bonds.

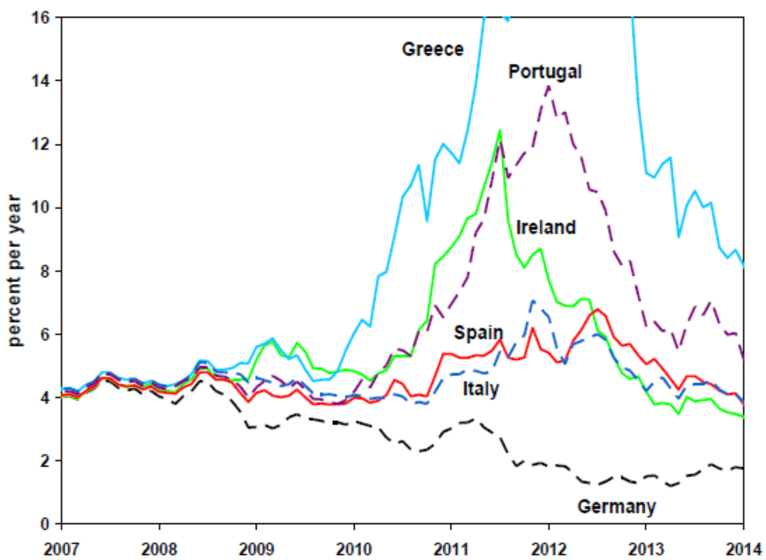


Figure 1.3: Yields on 10-years government bonds for selected European countries
Source: Arellano, Mateos-Planas and Rios-Rull (2013)

In fact, the relation between the haircut fraction $1-\theta$ and the probability of default can be assessed from the Credit Default Swap (CDS) spread. If, for simplicity, we consider a 1-year CDS spread (premium), then the default probability can be estimated as (Deutch Research Bank, 2015):

$$\pi = \frac{CDS\ Spread}{1-\theta}$$

E.g., if the recovery rate is 40%, a spread of 200 bp would translate into an implied probability of default of 3.3%. Even though the implied market probability of default for a sovereign bond can be calculated from yield spreads and a fixed rate of recovery, Blundell-Wignall and Slovik (2011) point out that the fixed rate of recovery assumption may constitute a difficulty, since this changes over time and over countries.

The estimation of the probability of default based on interest rates spreads is the core of empirical literature aiming at analyzing the role of market sentiments in European sovereign debt crisis. Both Bruneau, Delatte and Fouquau (2014) and de Grauwe and Ji (2013), using different econometric procedures, show that both the fundamentals and "animal spirits" ignited the European sovereign crisis.

Finally, with regards to the initial debt level, the crucial variable in the model, that conditions whether the country is vulnerable to a self-fulfilling crisis, there is mixed evidence on its relevance. On the one hand, Mendoza and Yue (2012) identify initial high external indebtedness for countries that default on sovereign debt, on the other hand, Reinhart and Rogoff (2009) show that defaults occur for any debt level, as is shown in Table 1.3.

Table 1.3: External Debt at the Time of Default: Frequency Distribution, 1970-2001

External debt-to-GNP range at the first year of default or restructuring	Percent of defaults or restructuring in middle income countries
Below 40	17
41 to 60	30
61 to 80	23
81 to 100	13
Above 100	17

Source: Reinhart and Rogoff (2009)

To summarize, Aguiar and Amador (2015) survey and list the following key finding regarding empirical facts related to defaults:

i Concerning defaults and its aftermath: defaults happen with regularity throughout history; defaults often occur in bad times, but with exceptions; defaults involve a heterogeneous pattern of "haircuts"; defaults generate a period of lengthy renegotiation;

ii Concerning evidence on bond spreads: on average spreads over US bonds are higher for longer maturity bonds, and while all spreads increase during crises, the short-term bond spread increases relatively more; emerging market bond yields exhibit significant co-movement, suggesting that global factors explain a large fraction of the common variation in spreads;

iii Successful growth episodes are associated with low and declining levels of foreign indebtedness.

1.2.2 Related literature

This work relates to the literature on self-fulfilling debt crisis and to the definition of sustainable debt limits. The term *self-fulfilling belief* captures rational outcomes not related to economic fundamentals, such as preferences, endowments or technology. This feature is modelled by specifying a state of confidence that is assumed to take one of two sunspot-driven values, e.g., normal and high. Typically these models produce multiple equilibria - *sunspots*, and the choice of the equilibrium situation depend on the "random" confidence. Romer (2011, p. 296) identifies a sunspot equilibrium when some variable that has no inherent effect on the economy matters because agents believe that it does. Any model with multiple equilibria has the potential for sunspots: if agents believe that the economy will be at one equilibrium when the extraneous variable takes on a normal value and at another when it takes on high value, then agents' behave in ways that validate this belief.

The incorporation self-fulfilling beliefs in economics literature is attributed to Azariadis (1981), who based on an overlapping generations model and a two-state stationary Markov process over states of the world, find stationary sunspot equilibria built as a randomisation over multiple equilibria. With respect to economic crises literature, self-fulfilling beliefs have long been embedded in currency, debt or bank crises. E.g, Obstfeld (1986) analysed the circumstances under which a balance of payments crisis can be self fulfilling and collapse a fixed exchange regime that would otherwise have been viable. The author finds that such crises reflect, not irrational private behaviour, but rather a multiplicity of equilibria each corresponding to different probabilities of collapse assessed by each agent.

The expansion of self-fulfilling sovereign debt crisis literature [1] has often been motivated by specific episodes. Alesina, Prati and Tabellini (1990) perform an analogy of public debt runs to bank runs based on confidence sentiments and inspired by Italian debt management policies pursued in the 1980s. Building an infinite horizon model, close to the CK model, they find two equilibria outcomes depending on the costs of default: (i) a "good" equilibrium where investors buy new debt and government is able to roll over forever; (ii) a "bad" equilibrium where investors do not lend, because they think other investors will act the same way, and thus the government is forced to default. Moreover, they conclude that if foreign governments or international institutions could act as lenders of last resort, it would increase the chance of a country hit by debt panic to survive without defaulting.

Cole and Kehoe (1996, 2000), motivated by the Mexico's 1994-95 debt crisis, conciliate self-fulling debt crises with debt ceilings' literature, which aims at finding the highest level of debt that potential creditors can lend without triggering a default (Obstfeld and Rogoff, 1996, p. 381-8). Their research builds upon a DSGE model in order to identify the optimal debt policy depending on the initial debt level: below the lower bound the government never defaults even if creditors do not lend; above the upper bound the government defaults even if creditors lend; within those limits lies the crisis zone, where a crisis may occur with positive probability (included as a *proxy* for the market participants' beliefs). In the crisis zone, market participants' beliefs (probability of default) select the equilibrium outcome and, if a default does not occur, the government finds it optimal to reduce debt level so as to leave the zone. These thresholds depend on the level of private capital stock and on the level of government debt, but also on its maturity: even with a low debt to GDP ratio, short-maturity debt plays an important role on whether a crisis may take place. The government optimal decision is to leave the crisis zone, by reducing the debt level or by increasing debt maturity.

It is worth mentioning that this type of crisis results from a coordination failure[2] among private lenders. Suppose that a subset of creditors becomes pessimistic about the possibility of default and decides not to lend. If there exist sufficiently strong strategic complementarities [3] in the economy, then it is in the other creditors' best interest not to lend as well. As a result, borrowers' expectations of a default are validated, which in turn causes an output decrease without any change in economic fundamentals. According to Chamon (2007) this possibility could be eliminated. In fact, at least when there is a single borrower (like the government) such crises could be prevented by mechanisms which allow investors to bid

[1] Tularam and Subramanian (2013) survey financial cries literature and highlight the fact that each model was adapted to specific situations to explain the financial crises faced rather than being visionary or systematic in approach; Aguiar and Amador (2015) review the theoretical literature on sovereign debt; and Tomz and Wright (2013) review the empirical research on sovereign debt and default.

[2] See Romer (2011, chap. 6) on coordination failure models.

[3] The term "strategic complementarity" means that one agent's optimal action is positively correlated with the actions of others (Chauvet and Guo, 2003).

on new bonds contingent on the resulting issue, inducing investors to lend because they expect other investors to so. Moreover, providing that the country satisfies the investors' participation constraint, it can choose any outcome for that issue, being able to select the "good equilibrium".

Araujo, Leon and Santos (2013) extend CK model to discuss alternative currency regimes (dollarization, local currency and common currency) for countries that are highly dependent on international lending. The model does not explicitly model inflation, exchange rate and monetary policy. Instead, the authors summarize the consequences of the currency devaluation in terms of the purchasing power of a currency, which they refer as a partial default, in the sense that, after devaluation, the bond issued in local or in common currency will not have the same value, in real terms, as before. To compare local currency regime with dollarization, they also include domestic debt, thus making it possible to inflate away the domestic debt to avoid the external debt default. Their numerical findings perform a welfare analysis for Argentina and Brazil during the 1998-2001 period, in order to understand the reason why those countries were under different regimes between 1998 and 2001.

A second extension concerns nominal bonds. Aguiar et al. (2013) extend the CK model to nominal debt in an environment in which the government chooses the inflation rate subject to a utility cost of high inflation, which proxies for the government's commitment to low inflation. They show that issuing nominal bonds has an ambiguous effect on the vulnerability to a self-fulfilling debt crisis. Specifically, if the government's commitment to low inflation is high absent a crisis, then nominal bonds have a desirable state-contingent feature; in good times, the real return is high, while in the event of a crisis, the government inflates away part of the real value of the bonds. As creditors prefer partial repayment to outright default, the ability to respond with inflation generates a superior outcome to real bonds. However, if the commitment to low inflation is weak even in good times, the government loses the state-contingency potentially allowed by nominal bonds. In particular, the government has a temptation to inflate ex post even in normal times, and this will be reflected in lower bond prices (or higher interest rates) ex ante, making repayment much more burdensome. This effect may be large enough to dominate, generating a larger crisis zone for nominal bonds. Aguiar et al. (2013) use this fact to rationalize why many emerging markets with weak in stationary regimes issue bonds in foreign currency, while economies like the US, the UK, and Japan issue large amounts of domestic currency bonds at low nominal interest rates and seemingly without risk of self-fulfilling crises.

Da-Rocha, Giménez and Lores (2013) also extend CK framework by combining self fulfilling debt crisis with self fulfilling currency crisis, assuming that debt is denominated in foreign currency, and show that consumers' expectations on devaluation change the government's incentive to default by making the safe level of government debt very low. The model is calibrated to match the Argentinian 2001-2002 crisis.

The euro area debt crisis motivated further research, particularly related to recessions and bailouts. Conesa and Kehoe (2012) extended the original CK model by considering that the economy may be in normal times or in a recession. If the economy is facing a recession with a positive probability of recovery, then governments may find it optimal to keep borrowing, expecting that a recovery will increase its revenues. This "gambling for redemption" behavior leads to increasing debt levels, which, if the recession persists, inevitably causes default. Again, the authors define debt thresholds where self-fulfilling crises are possible, and characterize conditions (i) under which gambling for redemption is the optimal policy, and (ii) under which the optimal policy is to reduce debt level and exit the crisis zone.

Conesa and Kehoe (2014) further extended model by adding a probability of bailout. However, their results indicate that a very large penalty on interest rate is needed to induce the government to run down its debt after a bailout, thus turning the bailout unattractive.

In a related research applied to the euro area sovereign debt crisis, Arellano, Conesa and Kehoe (2012) conclude that under high interest rates on government bonds and high costs of default, governments have incentives to reduce its debt and avoid sovereign default. On the contrary, low interest rates and low costs of default provide incentives for a government to gamble for redemption. In particular, and within the European debt crisis, the authors conclude that the EC, ECB and IMF intervention, by lowering the cost of borrowing and reducing default penalties, are encouraging euro area governments to gamble for redemption.

Adler and Lizarazo (2015) focus on the role of domestic banks as the major governments' borrowing source. Using a small open economy DSGE model, he explores the fact that self-fulfilling crisis may stem from government over borrowing in domestic banks (which borrow abroad), thus linking bank solvency to fiscal solvency and leaving the economy vulnerable to market expectations.

Durdu, Nunes and Sapriza (2013) rationalize self-fulfilling crisis through the incorporation of news. They analyze the extent to which changes in expectations due to policy announcements or news in general matter for macroeconomic fluctuations across countries. The authors find that negative news can lead to a default, even if current output is high. In fact, negative news about future fundamentals increases next period's default risk, which raises the cost of borrowing. As it becomes more onerous to roll over debt, it may trigger a default in the current period.

It is noteworthy that all these models consider a full default, even though data suggest that partial defaults are more likely to happen. The contribution of this paper is the focus on partial default.

1.3 The model

The model includes three types of agents - consumers, government and international bankers - and one type of good, used both for consumption and investment in each period $t = 1, 2, ...$

The framework is Cole and Kehoe (1996, 2000) standard DSGE model with capital accumulation, which includes a benevolent government that provides a valued public good. The provision of this good is financed through a (fixed) tax on household's income and through public debt. Debt matures in period $t + 1$ and the government does not commit to its repayment. We modify CK model by considering the possibility of default being partial $0 < \theta < 1$, instead of always complete, $\theta = 0$.

Consumers are risk neutral and do not access international credit markets, but they save assets in the form of capital, thus they are less willing to substitute consumption over time. The inclusion of capital is related to the fact that a reduction in domestic investment may change the government incentives to honor its debt.

1.3.1 Production function

Production utilizes capital and, implicitly, inelastically supplied labor:

$$y(K_t, \theta_t) = Z^{1-\theta_t} \cdot f(K_t), \tag{1.3.1}$$

where K_t denotes the aggregate or economy-wide stock of capital and $f(K_t)$ is continuously differentiable, concave, monotonically increasing function, satisfying $f(0) = 0$, $f'(0) = \infty$ and $f'(\infty) = 0$. The parameter $0 < Z < 1$ corresponds to the drop in GDP when the country defaults and $\theta_t \in [0, 1]$ represents the fraction of upcoming debt repayed in period t. Hence $1 - \theta_t$ is the haircut imposed on the owners of government debt, such that $\theta = 1$ corresponds to a full repayment and $\theta = 0$ to a full default. In what follows we are considering that the government options are not to default or to default on a pre-determined $\theta = \bar{\theta}$, thus actually $\theta_t \in \{0, \bar{\theta}\}$.

Whenever the country defaults, there is a permanent reduction in output and the higher is the haircut, $1 - \bar{\theta} > 1 - \underline{\theta}$, the lower is the output level, $Z^{1-\bar{\theta}} < Z^{1-\underline{\theta}}$. The assumption that the default penalty is proportional to output is proposed by Sachs (1984) and by Cohen and Sachs (1986) *cit in.* Obstfeld and Rogoff (1996, p. 351). Alternatively, Corsetti and Dedola (2014, p. 7) consider that the default penalty falls on the government budget constraint, in the form of a financial outlay proportional to the size of default. Following Conesa and Kehoe (2012), we assume that the default penalty occurs in the same period as the crisis.

Adam and Grill (2013) show that it is optimal for a fully committed government to occasionally deviate from the legal repayment obligation and to repay debt only partially. This holds true even if default generates significant deadweight costs ex-post.

This default penalty can be rationalized by two reasons. On one hand, a default may be accompanied by "direct punishment" from creditors, which may impose trade sanctions (e.g., confiscation of exports or imports in transit, seizure of trade-related foreign assets or limited access to short-term trade credits) that prevent the country from seizing the gains from trade (Obstfeld and Rogoff, 1996, p. 351). On the other hand, a default involves a reputation damage, which prevents the country from accessing international capital markets, thus generating a credit crunch. Cole and Kehoe (1998) analyse reputation spillover effects.

1.3.2 Consumers

A continuum of measure one, infinitely living consumers derive utility from private consumption, c_t, and from government consumption , g_t. The representative household maximizes the following additively time separable lifetime utility function:

$$U(c_t, g_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left[c_t + v(g_t)\right]. \tag{1.3.2}$$

The utility includes a linear function of private consumption and a function $v(g_t)$ of government spending, that is continuous, differentiable, strictly concave and increasing and $v(0) = -\infty$. Since the consumers are risk neutral in their own consumption, we ignore the possibility of their borrowing from, or lending to, the international bankers. If instead, consumers were risk averse they could be induced by their future income risk to accumulate assets in the form both of capital and, especially, of savings with the international bankers. However, this would substantially complicate the analysis without changing the essential features of the analysis (Cole and Kehoe, 2000, p. 93-4).

Consumers' budget constraint is:

$$c_t + k_{t+1} \leq (1 - \tau) \cdot [y(k_t, \theta_t) - \delta \cdot k_t] + k_t, \tag{1.3.3}$$

where $0 < \tau < 1$ is the constant proportional tax on domestic income after depreciation. Thus, consumers disposable income is a fraction, $1 - \tau$, of domestic income and the amount of capital stock carried out from the previous period, which is used to consume domestically produced goods, c_t, and to invest in domestic capital, k_{t+1}.

Since consumers' decision depends on the output level, the government decision on whether to default will have an impact on the consumers' problem.

1.3.3 Government

Government is benevolent, in the sense that it maximizes consumers' utility, $U(c_t, g_t)$, subject to its own budget constraint:

$$g_t + \theta_t \cdot B_t \leq \tau \cdot [y(K_t, \theta_t) - \delta \cdot K_t] + q_t \cdot B_{t+1}. \tag{1.3.4}$$

In each period the government sells new debt, B_{t+1}, at price q_t and repays 1 unit for due debt if the decision is not to default or repays θ_t in case it decides to default. As we will see, the price of one-period government bonds depends on the amount of bonds the government offers to sell and on the (arbitrary) probability that a crisis occurs if the economy is in the self-fulfilling crisis zone. If the government defaults on debt repayments, then in subsequent periods there is a drop in output and q_t decreases permanently.

1.3.4 International bankers

A continuum of measure one, risk neutral, infinitely living international bankers derive utility from their private consumption, x_t:

$$U^{IB}(x_t) = E_0 \sum_{t=0}^{\infty} \beta^t x_t, \tag{1.3.5}$$

Bankers' risk neutrality captures the idea that the domestic economy is small compared to world financial markets. Each banker is endowed with $\bar{x}$ in each period and chooses how many new government bonds to purchase. The price that bankers are willing to pay for these bonds, q_t, depend on their expectations regarding government default decision in the next period.

International bankers budget constraint is:

$$x_t + q_t \cdot b_{t+1} \leq \bar{x} + \theta_t \cdot b_t. \tag{1.3.6}$$

The no-Ponzi games condition, $b_{t+1} > -\bar{A}$, does not bind because $\bar{A}$ is large enough.

1.3.5 Debt zones

We consider the existence of three debt zones where the following outcomes are possible: if the current level of debt is in the "no crisis zone", the government never defaults; in the crises zones, a self fulfilling debt crisis may occur, and in the "default zone", where a default has already occurred. These zones are limited by the following debt thresholds:

- $\underline{b}(K_t;\theta_t)$, which corresponds to the highest level of debt where no crisis occurs given K_t. This threshold is called the "upper safe debt limit" or the "lower limit for a no-lending continuation condition". If $B_t \leq \underline{b}$, the government fully repays even if bankers lend at a higher price; alternatively, if $B_t > \underline{b}$, the government defaults if bankers lend at a higher price; otherwise it does not default.

- $\bar{B}(K_t;\pi,\theta_t)$, which defines the lowest level of debt for which default occurs. This threshold is also referred to as "upper sustainable debt limit" or the "upper limit for a participation constraint". If $B_t \leq \bar{B}$, the government repays if bankers lend at a lower price; otherwise it defaults.

The crisis zone lies between those debt levels, where a crisis may occur depending on the confidence that international bankers have on government not defaulting. Cole and Kehoe (2000) showed that in the case $\theta \in \{0,1\}$, then $\underline{b}(K_t) < \bar{B}(K_t;\pi)$. Notice that the limits of the crisis zone depend on B, K and θ, but within the crisis zone the probability of a crisis π is independent on the levels of B, K and θ.

1.3.6 Sunspot variable and market clearing

Self-fulfilling debt models generically have two possible outcomes: a "good" equilibrium, where government sells new debt and repays old debt, and a "bad" equilibrium where the government is unable to roll over the debt and defaults. Within this context, the debt thresholds determine the choice between the "good" and the "bad" equilibrium: below the lower limit, the "good" equilibrium is the only option and the government always repays; above the upper debt limit, the "bad" equilibrium is the optimal choice and the government always defaults. In the crisis zone, the realization of the sunspot variable sets the choice for the "good" or the "bad" equilibrium. So, following Cole and Kehoe (1996, 2000), the model includes an arbitrary and fixed probability, $0 \leq \pi \leq 1$, of a default taking place.

To model the sunspot, we consider that in each period, if the country is in the crisis zone, a random variable ζ is drawn from the uniform distribution $[0, 1]$. The realization of this variable determines the equilibrium outcome:

- If $\zeta < \pi$, then bankers expect a default and lend at a higher price, thus triggering a default and the crisis turns out to be self-fulfilling;
- If $\zeta \geq \pi$, then the risk that bankers assign to a crisis, and a consequential default, doesn't prevent them from buying additional bonds at a lower price and a crisis does not occur.

Because ζ is uniformly distributed on the unit interval, π is both the crucial value of ζ, and the probability that $\zeta < \pi$ (Da-Rocha, Giménez and Lores, 2013, p. 20). It should be noted that the realization of the sunspot variable is only relevant inside the crisis zone. In fact, π corresponds to the probability that the sunspot variable takes on a value that indicates the international creditors to panic if a crisis would be self-fulfilling.

The market-clearing condition for the government debt is $b_{t+1} = B_{t+1}$. For consumer's individual capital stock the condition is $k_{t+1} = K_{t+1}$, where k_{t+1} is the individual decision and K_{t+1} is the aggregate value.

1.3.7 Timing

The timing of actions within each period, and the fact that the government is the only strategic agent, makes it possible to consider the effects of each decision on the future debt price, upcoming government revenue and the level of private consumption. Agents choose their actions sequentially as follows (Cole and Kehoe, 1996, 2000):

1. The sunspot variable, ζ_t, is realized and the aggregate state is $S_t = (B_t, K_t, \theta_{t-1}, \zeta_t)$;
2. The government chooses how much new debt, B_{t+1}, to sell taking bonds' price, $q_t(s_t, B_{t+1})$, as given;
3. Bankers choose whether to purchase new debt, b_{t+1}, at price, q_t;
4. The government decides whether to default, $\theta_t \in \{0, \bar{\theta}\}$, and how much to spend on the public good, g_t;
5. Consumers choose c_t and k_{t+1}.

The importance of the timing of actions is highlighted by Ayres et al. (2014) and Teles (2014), who suggest the existence of multiple equilibria when: (i) creditors move first, whether the borrower chooses current debt or debt at maturity; borrower moves first and chooses current debt (which is the case herein considered). However, if the borrower moves first and chooses debt at maturity, there is a single equilibrium.

It is also worth noting that the price of debt depends on the amount of new debt, B_{t+1}, but the decision on whether to default depend on the amount of debt to be repaid, B_t.

1.3.8 Recursive Markov Equilibrium

The recursive-markov equilibrium here described is characterized by the following features: (i) the government cannot commit itself to repay previous debt or to follow a given debt or spending path; (ii) within each period, agents sequentially choose their actions, in the order previously described.

A recursive competitive equilibrium is characterized by time invariant functions of a limited number of state variables, which summarize the effects of past decisions and current information (Mehra, 2008). These functions include (a) a list of value functions V^c, V^g and V^g, where the superscript denotes the agent; (b) a price function $\psi_q(\cdot)$; (c) policy functions specifying agents' decisions: for consumers $C(\cdot)$, for the government $B(\cdot)$, $G(\cdot)$ and $\Theta(\cdot)$; (d) and a law of motion for capital stock, $\kappa(\cdot)$. The adjective Markov denotes that the equilibrium decision rules depend only on the current values of the state variables, not their histories. Perfect means that the equilibrium is constructed by backward induction and therefore builds in optimizing behaviour for each agent for all conceivable future states (Ljungqvist and Sargent, 2012, p. 185). In fact, backwards induction is the appropriate procedure, because it allows agents to be aware of other agents' optimal decisions. This way, an optimal time-consistent policy can be found to account for the possibility of confidence crises, encouraged through the limited commitment assumption (Cole and Kehoe, 2000, p. 92).

At the beginning of each period, the aggregate state of the economy is defined as $S = (B, K, \theta_{-1}, \zeta)$. Taking into consideration this information, other variables that affect each agent's maximization problem and its state in a future period, determines the current state of an agent. The solution to an agent's maximization can be found with respect to its individual state, and expressed through a value function, maximizing the agent's utility, and a policy function, leading to this maximized utility through the optimal selection of its choice variables. In summary, the following functions are being derived within the recursive equilibrium: consumers value function $V^c(k, S, B', g, \theta)$ and its policy functions $C(k, S, B', g, \theta)$ and $\kappa(k, S, B', g, \theta)$; the government's value function $V^g(S)$ and its policy functions $B'(S)$, $\Theta(S, B', q)$ and $G(S, B', q)$; the bankers' value function $V^b(b, S, B')$ and its policy function $b'(b, S, B')$; the bond price function $\psi^q(S, B')$ and an equation of motion for the aggregate capital stock.

In what follows, subscripts denote the variable with respect to which the derivative is taken, a prime in a variable indicates next-period values, and a prime in a function indicates it is evaluated at next-period variables. Since we use backwards induction, we start with consumers' problem because they move last.

Consumers move last, by choosing c and k', at a time where all other decisions are known. So, consumers know their own capital holdings, k, the government decisions B', g and θ, the price that international bankers pay for new debt, q, and the sunspot, ζ, which is only relevant when the debt level in within the crisis zone. Moreover, in subsequent periods, they expect government to follow the policies $G(\cdot)$, $B(\cdot)$ and $\Theta(\cdot)$, all yet to be defined.

The representative consumer's value function is defined by the functional equation:

$$\begin{aligned} V^c(k,S,B',g,\theta) = &\max_{c,k'}\{U\left(c,g\right) + \beta \cdot EV^c(k',S',B'(S'),g',\theta')\} \\ \text{subject to } & c + k' \leq (1-\tau)\cdot[y(k,\theta) - \delta\cdot k] + k \\ & c,k' \geq 0 \\ & S' = (B',\kappa',\theta,\zeta') \\ & g' = G(S',B'(S'),\psi_q(S',B'(S'))) \\ & \theta' = \Theta(S',B'(S'),\psi_q(S',B'(S'))) \end{aligned} \tag{1.3.7}$$

Since in equilibrium $k = K$, the consumption optimal decision function can be expressed as $c = C(k,S,B',g,\theta)$ and the economy-wide stock of capital is expected to evolve according to $K' = \kappa(k,S,B',g,\theta)$ derived from the budget constraint. The production function is $y(k,\theta) = f(k)$ if $\theta_{-1} = \theta = 1$ and is $y(k,\theta) = Z^{1-\theta}\cdot f(k)$, otherwise.

The **government** makes its decisions at two points in time: at the beginning and at the end of a period. Using backwards induction, we start with this latter decision of choosing whether to default, which affects $y(k,\theta)$, and the level of government spending. At this point in time, the government knows consumers optimizing policies, $C(\cdot)$ and $\kappa(\cdot)$, as well as the price of new bonds, q. So, the decision on how much to consume, $G(\cdot)$, and whether to default, $\Theta(\cdot)$, is the solution to:

$$
\begin{aligned}
V^g(S,g,\theta) = & \max_{g,\theta} \left\{ U\left(c,g\right) + \beta \cdot EV^g(S',g',\theta') \right\} \\
\text{subject to} \quad & g + \theta \cdot B \leq \tau \cdot [y(K,\theta) - \delta \cdot K] + q \cdot B' \\
& c = C(K,S,B',g,\theta) \\
& S' = (B',K',\theta,\zeta') \\
& K' = \kappa(k,S,B',g,\theta) \\
& g \geq 0 \\
& \theta \in \langle 0,1 \rangle
\end{aligned}
\tag{1.3.8}
$$

The resulting optimal public purchases and default policy function can be expressed as $G = G(S,B',q)$ and $\theta = \Theta(S,B',q)$. The default decision will affect the level of output and, through the budget constraint, the level of public spending.

International bankers take as given bond its own holdings, b, and government new debt, B', to solve the functional equation:

$$
\begin{aligned}
V^b(b,S,B') = & \max_{b'} \{ x + \beta \cdot EV^b(b',S',B'(S')) \} \\
\text{subject to} \quad & x + \psi^q(S,B') \cdot b' \leq \bar{x} + \Theta(K,B,\zeta) \cdot b \\
& S' = (B',K',\theta,\zeta') \\
& K' = \kappa(k,S,B',g,\theta) \\
& b \geq -\bar{A}
\end{aligned}
\tag{1.3.9}
$$

Since bankers are risk neutral, and behave competitively, they buy all the bonds offered by the government, as long as $\bar{x}$ is sufficiently large and the price function of new bonds is $\psi_q(S,B')$.

In the beginning of the period, **government** chooses the amount of new debt to issue. At this point in time, the government is aware that the bond's price $\psi^q(S,B)$ depends on the aggregate state of the economy and the new borrowing level. Additionally, the government knows its own later optimizing decisions, $G(\cdot)$ and $\Theta(\cdot)$, as well as its effect on output and consumers decisions. The government's value function is defined by the functional equation:

$$\begin{aligned} V^g(S) = & \max_{B'}\{U\,[c,g] + \beta \cdot EV^g(S')\} \\ \text{subject to} \quad & S' = (B', K', \theta, \zeta') \\ & c = C(k, S, B', g, \theta) \\ & K' = \kappa(k, S, B', g, \theta) \\ & g = G(S, B', \psi_q(S, B')) \\ & \theta = \Theta(S, B', \psi^q(S, B')) \end{aligned} \tag{1.3.10}$$

The solution to this problem is $B' = B(S)$. Notice, however, that the government cannot commit to repay upcoming due debt and if bankers do not lend, a default occurs, even if the government has decided not to default.

This set of individual value functions and the corresponding maximization choices define the recursive equilibrium, in addition to $k_t = K_t$ and $b_t = B_t$. Strategic decisions being made by the government are expressed by the increasing amount of information and dependencies as we work backwards in the proposed timing of actions. Again, this set up is justified by the government's lack of ability to commit to a repayment of its debt at the beginning of the period. The uncertainty prevailing in the market allows for crises of confidence which need to be taken care of through an optimal time-consistent policy (Cole and Kehoe, 2000, p. 92).

1.4 Computation of the equilibria

The derivation of the equilibrium involves severel specificities. The first feature arises from the discontinuity of bonds' prices, caused by the debt thresholds. As no constant price for public debt holds, the government's value function has kinks which directly affect its policy function. The second factor is related to the limited commitment assumption, as the government is not able to commit for future policy choices, the manual derivation of a continuous value function becomes impossible (Madison, 2012, p.29). Moreover, we consider that the default decision is taken in only one period.

The current level of government debt is the crucial determinant of consumers, bankers, and government decisions (Cole and Kehoe, 1996). If the level of debt is low enough so that the government would pay it even if it were unable to sell new debt, then no crisis is possible. Consumers choose a high level of capital; bankers do not demand a risk premium on the return to government bonds; and government maintains a high constant spending level with low constant debt level. If, on the other extreme, the level of debt is high enough that the government would default even if it could sell new debt at a lower price, then consumers choose a low level of capital; bankers buy government debt at a lower price; and the government maintains a low constant spending level with high constant debt level.

In between these two extremes lies the crisis zone. Here self-fulfilling debt crises can occur with fairly arbitrary probability. Consumers choose a level of capital that decreases with higher probability of a crisis; bankers demand a risk premium from the government; and the government may choose to reduce its spending to run down its debt and eventually leave the crisis zone or, if the debt reduction is too costly, it may choose to run up the debt until it defaults, to avoid a sharp reduction in spendings..

1.4.1 Optimal behavior of private agents

The optimal decisions of consumers and bankers depend on their believes about next period output level, which in turn may depend on probability they assign to the government defaulting in the next period. So, in the no-crisis zone private agents expect $\theta = 1$; in the default zone, they expect $\theta = \bar{\theta}$; in the crisis zone, these agents expect $\theta = 1$ with probability π and they expect $\theta = \bar{\theta}$ with probability $1 - \pi$. We now can characterize consumers' and bankers' behavior and then infer the transition function for capital k' and price function $q(B', \theta)$.

1.4.1.1 Consumers

As previously mentioned, consumers do not behave strategically and their decision depends on the expectations for next period output. In this section, to find a stationary steady state, we follow Cole and Kehoe (1996) procedure, which evolves using variational methods[4].

Henceforward, we denote c^n, k^n, c^θ and k^θ as consumption and capital levels contingent on government not defaulting or defaulting in the next period. Moreover, we take into consideration that according to the timing of actions, consumers move after knowing the current government decision, and that the default penalty occurs in the same period of the default decision. In equilibrium, individual choices k coincide with aggregate choices K. The rules henceforth presented are derived in Appendix A.1.

Expectations of a crisis

In the crisis zone, given a constant probability of a crisis taking place, the value of the sunspot variable determines the outcome. Here, if default has not occurred until period t, consumers assign a probability π to a default and a probability $1-\pi$ to no default. The representative consumer, given k_t, k^n_{t+2} and k^θ_{t+2} chooses c_t, k_{t+1} and c^n_{t+1}, c^θ_{t+1}, where the former correspond to the choices of consumption in periods $t+1$ contingent on the government not defaulting or defaulting, respectively. Thus, consumers' solve the variational problem:

$$\begin{aligned}
&\max_{c_t,k_{t+1},c^n_{t+1},c^\theta_{t+1}} \quad c_t + \beta\cdot(1-\pi)\cdot c^n_{t+1} + \beta\cdot\pi\cdot c^\theta_{t+1}\\
&\quad\text{subject to}\\
&\qquad c_t \leq (1-\tau)\cdot[y(k_t,1)-\delta\cdot k_t] + k_t - k_{t+1}\\
&\qquad c^n_{t+1} \leq (1-\tau)\cdot[y(k_{t+1},1)-\delta\cdot k_{t+1}] + k_{t+1} - k^n_{t+2}\\
&\qquad c^\theta_{t+1} \leq (1-\tau)\cdot\left[y(k_{t+1},\bar{\theta})-\delta\cdot k_{t+1}\right] + k_{t+1} - k^\theta_{t+2}\\
&\qquad c_t, c^n_{t+1}, c^\theta_{t+1}, k_{t+1} \geq 0
\end{aligned} \tag{1.4.1}$$

The FOC of the consumers problem imply that investment is stationary at the level k^π determined by the rule:

$$1 = \beta\cdot\left[(1-\tau)\left\{f'(k^\pi)\cdot\left[(1-\pi)+\pi\cdot Z^{1-\theta}\right]-\delta\right\}+1\right] \tag{1.4.2}$$

[4] Variational methods are used to find the stationary state of a dynamic problem. "The basic technique is to assume that the values for the endogenous variables for periods $s-1$ and $s+1$ are given and then maximize the objective equation for the values in period s. The resulting first order conditions must hold in a stationary state along with the condition that the value for each of the variables is the same at times $s-1$, s and $s+1$. These conditions are sufficient to find a stationary state" (McCandless, 2008, p. 34).

The household's Euler equation has the usual interpretation: to attain maximum utility, consumers' must be indifferent between consuming one more unit today and the expected present value of saving and consuming that unit in the future. The additional term $(1-\pi)+\pi \cdot Z^{1-\theta} < 1$ arises because the representative agent internalizes the fact that the possibility of a crisis leads to a lower output. Once the stationary state capital stock is determined, consumers' budget constraint (1.3.3) can be used to determine the stationary value of consumption. In period t, k (chosen in $t-1$) may be different from k^{π}. Thus, consumption, if no default has occurred in t:

$$c^{\pi}(k) = (1-\tau) \cdot [f(k) - \delta \cdot k] + k - k^{\pi}, \tag{1.4.3}$$

As stated by Cole and Kehoe (2000, p. 99) "It is here that the risk neutrality of utility in consumption plays its role: if consumers are risk averse, then optimal investment and consumption are not stationary."

Expectations of default

If consumers expect a default of $\theta = \bar{\theta}$ in the next period, such that they expect the productivity parameter to be equal to $Z^{1-\bar{\theta}}$, then the optimal level of capital accumulation, k^{θ}, is derived from rule:

$$1 = \beta \cdot \left\{(1-\tau) \cdot \left[Z^{1-\bar{\theta}} \cdot f'(k^{\theta}) - \delta\right] + 1\right\}. \tag{1.4.4}$$

The consumption level is set according to:

$$c^{\theta}(k) = (1-\tau) \cdot \left[Z^{1-\bar{\theta}} \cdot f(k) - \delta \cdot k\right] + k - k^{\theta} \tag{1.4.5}$$

In particular, if the debt level is high enough, such that the economy is in the default zone, then a default has already occurred and consumers expect default as the only possible outcome, so the consumption level, $c^{\theta}(k^{\theta})$, evolves according to:

$$c^{\theta}(k^{\theta}) = (1-\tau) \cdot \left[Z \cdot f(k^{\theta}) - \delta \cdot k^{\theta}\right] \tag{1.4.6}$$

Expectations of no default

Analogously, if consumers expect a full repayment, $\theta = 1$, and that the productivity parameter is 1, then next period's capital stock k^{n} satisfies:

$$1 = \beta \cdot \left\{(1-\tau) \cdot \left[f'(k^{n}) - \delta\right] + 1\right\}. \tag{1.4.7}$$

In this case, the consumption level is $c^n(k)$ is:

$$c^n(k) = (1-\tau)\cdot[f(k) - \delta\cdot k] + k - k^n \tag{1.4.8}$$

Notice that the strict concavity of $f(k)$ implies that $k^n > k^\pi > k^\theta$.

1.4.1.2 International bankers

The price function for bonds $q(b', \theta)$ is the solution of the international bankers' dynamic programming problem (1.3.9). The resulting equation defines the price as the risk neutral discount rate of an international banker, β, adjusted by the expected value of a country's default.

$$q(b', \theta) = \beta \cdot E(\theta') \tag{1.4.9}$$

Bankers' risk neutrality and competitive behaviour imply that they are relatively passive, in the sense that, as long as the price of bonds satisfies Eq. (1.4.9), they buy all the debt the government offers. Since the bonds' price depends on the expectations of default, which in turn depend on the zone where the initial deb level is included, then the debt thresholds cause breaks in the price of bonds.

Taking the debt thresholds as given, in the no-crisis zone bankers set $q = \beta$; in the default zone bankers set $q = \beta\cdot\theta$; finally, in the crisis zone, the realization of the sunspot variable rules the lending decision, such that $\zeta > 1-\pi$ indicates a forthcoming crisis, such that bankers as well increase the price they demand for rolling over the government's debt into a future period.

Thus the price of bonds is:

$$q(B', \theta) = \begin{cases} \beta & \text{if } B' \leq \underline{b} \\ \beta\cdot(1-\pi) + \beta\cdot\pi\cdot\theta & \text{if } \underline{b} < B' \leq \bar{B} \\ \beta\cdot\theta & \text{otherwise} \end{cases} \tag{1.4.10}$$

It is visible that prices decrease with an increase in future debt. The model therefore assumes a positive correlation between the current debt level and the risk premium on government debt. The price function in Eq. (1.4.10) is consistent with Cruces and Trebesch (2013) empirical findings: higher haircuts are associated with higher post-restructuring spreads. In fact, a defaulting country that aims to resolve its debt crisis negotiates with creditors not only on the size of the haircut, but also on the level of subsequent risk premium. So, the debtor faces a trade-off: a high haircut implies a large degree of debt reduction now, but is punished by markets tomorrow.

Unlike the CK model, here when a default occurs, bankers pay a lower price for next period's debt, but the country is still allowed to borrow. When we set $\theta = 0$, where a full default takes place, the country is permanently banned from international capital markets because it will not borrow at zero price. Finally, notice that the realization of the sunspot variable is only relevant inside the crises zone.

1.4.2 Government behaviour

In this section, we characterize the government's payoffs and optimal policies in each of the three situations: default, no-default and crisis zone. Herein, we show that for sufficiently low debt levels, it is optimal for the government to repay even if bankers do not lend, and for sufficiently high debt levels, it is optimal to default, even if agents do not expect it.

In what follows, we denote $g^i(B, B', q, k)$ and $V^i(B, k)$, $i = n, \theta$ as the government's spending and payoff functions of not defaulting and defaulting.

1.4.2.1 Default zone

In this situation, we consider that $B_0 \geq \bar{B}(k^\theta)$, such that the initial debt level lie in a zone where a default is the only expected outcome. Thus, $q = \beta \cdot \theta$ and the government solves the variational problem defined in (1.4.11), by choosing g_t, g_{t+1} and B_{t+1}, and taking as given B_t, B_{t+2}, k_t and k_{t+1}. In period $t+1$ the government defaults on its choice for B_{t+1}.

$$\max_{g_t, g_{t+1}, B_{t+1}} \beta^t \cdot v(g_t) + \beta^{t+1} \cdot v(g_{t+1})$$

subject to

$$\begin{aligned} & g_t + B_t \leq \tau \cdot \left[Z^{1-\theta} \cdot f(k_t) - \delta \cdot k_t \right] + \beta \cdot \theta \cdot B_{t+1} \\ & g_{t+1} + \theta \cdot B_{t+1} \leq \tau \cdot \left[Z^{1-\theta} \cdot f(k_{t+1}) - \delta \cdot k_{t+1} \right] + \beta \cdot \theta \cdot B_{t+2} \\ & g_t, g_{t+1} \geq 0 \\ & B_{t+1} \geq \bar{B}(k_t) \\ & \lim_{T \to \infty} (\beta \cdot \theta)^T \cdot B_{t+T} = 0 \end{aligned} \tag{1.4.11}$$

Appendix A.2.1 includes the computation of the general case where $k_0 \neq k^\theta$. Here, we assume that $k_0 = k^\theta$, consumers set $k = k' = k^\theta$ and $c = c^\theta(k^\theta)$, such that the constant debt level is:

$$B^\theta(B_0, k^\theta) = \frac{1}{1 + \beta \cdot (1 - \theta)} \cdot B_0 \tag{1.4.12}$$

Even though there is a constant debt level, there is a reduction in relation to the initial level. If this reduction falls bellow $\bar{B}(k^{\pi})$, as we will see from the policy in the crisis zone, the debt level will turn out to be $\bar{B}(k^{\pi})$.

The government *average* constant spending level is:

$$g^{\theta} = \tau \cdot \left[Z^{1-\theta} \cdot f(k^{\theta}) - \delta \cdot k^{\theta}\right] - \frac{1-\beta \cdot \theta}{1+\beta \cdot (1-\theta)} \cdot B_0 \tag{1.4.13}$$

The government's payoff in this situation is:

$$V^{\theta}(B_0, k^{\theta}) = c^{\theta}(k^{\theta}) + v\left[g^{\theta}(B_0, B^{\theta}, k^{\theta})\right] + \beta \cdot \frac{1}{1-\beta} \cdot \left\{c^{\theta}(k^{\theta}) + v\left[g^{\theta}(B^{\theta}, B^{\theta}, k^{\theta})\right]\right\} \tag{1.4.14}$$

In this zone, the payoff of defaulting is higher than rather not defaulting. In fact, given private agents decisions in this zone ($q = \beta \cdot \theta$, $c = c^{\theta}(k^{\theta})$ and $k' = k^{\theta}$), if the government does not default it sets $B_1 = B_0$, such that:

$$g^{n\theta} = \tau \cdot \left[Z^{1-\theta} \cdot f(k^{\theta}) - \delta \cdot k^{\theta}\right] - (1-\beta \cdot \theta) \cdot B_0 \tag{1.4.15}$$

which is smaller than Eq. (1.4.13).

1.4.2.2 No default zone

Analogously, to derive the government policy in the no-crisis zone, we assume that $B_0 \leq \underline{b}(k_0)$ and that $B_1 \leq \underline{b}(k^n)$, that is, the initial and subsequent debt levels fall in the no-crisis zone, such that since no crisis is possible ($\pi = 0$), then $q = \beta$ and $\theta = 1$.

In this zone, the government solves the variational problem (1.4.16), by choosing g_t, g_{t+1} and B_{t+1}, and taking B_t, B_{t+2}, k_t and k_{t+1} as given.

$$\begin{aligned}
&\max_{g_t, g_{t+1}, B_{t+1}} \beta^t \cdot v(g_t) + \beta^{t+1} \cdot v(g_{t+1}) \\
&\text{subject to} \\
&\qquad g_t + B_t \leq \tau \cdot [f(k_t) - \delta \cdot k_t] + \beta \cdot B_{t+1} \\
&\qquad g_{t+1} + B_{t+1} \leq \tau \cdot [f(k_{t+1}) - \delta \cdot k_{t+1}] + \beta \cdot B_{t+2} \\
&\qquad g_t, g_{t+1} \geq 0 \\
&\qquad B_{t+1} \leq \underline{b}(k_t) \\
&\qquad \lim_{T \to \infty} \beta^T \cdot B_{t+T} = 0
\end{aligned} \tag{1.4.16}$$

As shown on Appendix A.2.2, the optimal government policy is to set $g_{t+1} = g_t$, and hence $B_{t+2} = B_{t+1}$.

When we assume that $k_0 = k^n$ (the general case of $k_0 \neq k^n$ is in Appendix A.2.2), if after bankers have lent the government's decision is not to default, then $B_1 = B_0$ and the government constant spending level is:

$$g^n(B_0, B_0, k^n) = \tau \cdot [f(k^n) - \delta \cdot k^n] - (1-\beta) \cdot B_0 \tag{1.4.17}$$

The government's payoff of not defaulting is:

$$V^n(B_0, k^n) = \frac{1}{1-\beta} \cdot \{c^n(k^n) + v\,[g^n(B_0, B_0, k^n)]\}. \tag{1.4.18}$$

To see that in this zone the payoff of not defaulting is higher than the payoff of defaulting, we characterize an hypothetical situation where the government decides to default on B_0 after bankers have lent B_1. In period t bankers had set $q = \beta$ and consumers set $k = k^n$, $c = c^n(k^n)$ and $k' = k^\theta$. In subsequent periods, and even for low debt levels, the government faces a higher risk premium on debt issuing, $q = \beta \cdot \theta$, and a lower output level, both because of the default penalty and because consumers set capital at a lower level.

The value function of defaulting in the no-default zone $V^{\theta n}$ is:

$$V^{\theta n}(B_0, k^n) = c^\theta(k^n) + v[g^\theta(B_0, B_1, k^n)] + \frac{\beta}{1-\beta} \cdot \left\{c^\theta(k^\theta) + v[g^\theta(B_1, B_1, k^\theta)\right\}$$

$$\begin{aligned} &\text{with} \\ &g^{\theta n}(B_0, B_1, k^n) = \tau \cdot [Z^{1-\theta} \cdot f(k^n) - \delta \cdot k^n] - \theta \cdot B_0 + \beta \cdot B_1 \\ &c^\theta(k^n) = (1-\tau) \cdot \left[Z^{1-\theta} \cdot f(k^n) - \delta \cdot k^n\right] + k^n - k^\theta \\ &g^\theta(B_1, B_1, k^\theta) = \tau \cdot [Z^{1-\theta} \cdot f(k^\theta) - \delta \cdot k^\theta] - (1 - \beta \cdot \theta) \cdot B_1 \\ &c^\theta(k^\theta) = (1-\tau) \cdot \left[Z^{1-\theta} \cdot f(k^\theta) - \delta \cdot k^\theta\right]. \\ &B_1 = \frac{\theta}{1+\beta \cdot (1-\theta)} \cdot B_0 \end{aligned} \tag{1.4.19}$$

Using the same procedure as in Appendix A.2.2 to find the constant debt level, we find:

$$\begin{aligned} B^{\theta n}(B_0, k^n) = &\frac{\theta}{1+\beta \cdot (1-\theta)} \cdot B_0 + \\ &+ \frac{1-\beta}{1+\beta \cdot (1-\theta)} \cdot \tau \cdot \left\{\left[Z^{1-\theta} \cdot f(k^\theta) - \delta \cdot k^\theta\right] - \left[Z^{1-\theta} \cdot f(k^n) - \delta \cdot k^n\right]\right\}. \end{aligned}$$

For low debt levels, $V^n > V^{\theta n}$, such that the optimal decision is not to default.

1.4.2.3 Crisis zone

Herein, we assume that $k_0 = k^\pi$ and $\underline{b}(k^n) < B_0 \leq \bar{B}(k^\pi, \pi)$, that is the participation constraint does not bind, such that the government wants to honor the current lending contract given that it is able to issue new debt. Since $\underline{b}(k^n) > \underline{b}(k^\pi)$ then $B_0 > \underline{b}(k^\pi)$ and the government faces the following choices in period 0:

1. plan to run debt down in T periods to $\underline{b}(k^n)$ or less in T periods if no crisis occurs
2. plan to run debt up in W periods to $\bar{B}(k^\theta, \pi)$ or less in W periods if no crisis occurs
3. never run the debt down and keep a constant debt level;
4. default now and afterwards keep a constant debt level.

After determining the governments' payoff for each of these choices, the optimal decision corresponds to the option that offers the maximum payoff. In this section we find these governments' policies. In section 1.5, we show that the optimal decision up to intermediate debt levels is to run down debt until it reaches the no-crisis zone, but for larger debt levels within the crisis zone, heavier government spending cuts change the incentive to reduce debt towards defaulting, even assuming a future higher borrowing cost.

Run debt down in T periods Under the conjecture that there was no previous default, private agents set $c = c^\pi(k)$, $k' = k^\pi$ and $q = \beta \cdot (1-\pi) + \beta \cdot \pi \cdot \theta$, which we denote by $\hat{\beta}$. The government plans to run debt down to $\underline{b}(k^n)$ in T periods, instead of the standard spending smoothing policy, because it ends up receiving more when it sells less debt, since in the no-crisis zone $q = \beta$. When running down debt in T periods, the level of spending, derived in Appendix A.2.3, is constant at $g = g^T(B_0)$ and set to:

$$g^T(B_0) = \tau \cdot [f(k^\pi) - \delta \cdot k^\pi] + \hat{\beta}^{T-1} \cdot \frac{1-\hat{\beta}}{1-\hat{\beta}^T} \cdot \beta \cdot \underline{b}(k^n) - \frac{1-\hat{\beta}}{1-\hat{\beta}^T} \cdot B_0 \tag{1.4.20}$$

Setting $B_T < \underline{b}(k^n)$ cannot be optimal since it lowers g^T and, by definition of T, $B_{T-1} > \underline{b}(k^n)$ does not eliminate the possibility of a crisis before T (Cole and Kehoe, 2000).

Given these policies, the expected payoff of running down debt in T periods is:

$$
\begin{aligned}
V^T(B_0) = & \frac{1-[(1-\pi)\cdot\beta]^{T-1}}{1-(1-\pi)\cdot\beta}\cdot\{(1-\pi)\cdot[c^\pi(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)]\}+\\
& +[(1-\pi)\cdot\beta]^{T-1}\cdot\{(1-\pi)\cdot[c^n(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)]\}+\\
& +\pi\cdot\beta\cdot\frac{1-[(1-\pi)\cdot\beta]^{T}}{1-(1-\pi)\cdot\beta}\cdot V^\theta(B_0,k^\theta)+[(1-\pi)\cdot\beta]^T\cdot V^n[\underline{b}(k^n)]
\end{aligned}
\tag{1.4.21}
$$

It is worth emphasizing that this expression is an *expected* government payoff, with following interpretation:

1st term The fraction incorporates the cumulative probability of the economy remaining in the crisis zone until period $T-2$, adjusted by the discount factor. That is, the cumulative probability of not defaulting $1-\pi$, otherwise the government would default and change the policy rules. Here $k=k^\pi$, but since there is a possibility that a default takes place, agents set $c=c^\pi$ and $g=g^T(B_0)$, because the economy has not left the crisis zone; or expect a default with probability π and set $c=c^\theta$ and $g^\theta(k^\pi)$.

2nd term Entering period $T-1$, there is still a positive probability for a crisis, but as soon as the sunspot variable indicates an advantageous outcome $\zeta\leq 1-\pi$, the government will announce its future borrowing level of $B_T=\underline{b}(k^n)$ automatically setting $\pi=0$. In this case, according to Eq. (1.4.8), consumers set $c^n(k^\pi)=(1-\tau)\cdot[f(k^\pi)-\delta\cdot k^\pi]+k^\pi-k^n$. So, in period $T-1$, even though a crisis is still possible, if it does not happen, agents know that in the next period the debt level is in a safe zone, thus consumers set $c=c^n$.

3rd term Until period $T-1$ there is always a possibility that a default arises with probability π, if it did not happen until the previous period (translated by cumulative probability of not defaulting). In this case, the payoff of defaulting is defined from Eq. (1.4.14).

4th term If the government in fact does not default while running debt down, then from T onwards, it will keep a constant debt level of $\underline{b}(k^n)$ outside the crisis zone. The payoff of no defaulting in the no crisis zone is defined from Eq. (1.4.18)

$$
V^n(\underline{b}(k^n),k^n)=\frac{1}{1-\beta}\cdot\{c^n(k^n)+v\,[g^n(\underline{b}(k^n),\underline{b}(k^n),k^n)]\},
$$

and the constant government level defined from Eq. (1.4.17)

$$
g^n(\underline{b}(k^n),\underline{b}(k^n),k^n)=\tau\cdot[f(k^n)-\delta\cdot k^n]-(1-\beta)\cdot\underline{b}(k^n).
$$

The effect of leaving the crisis zone in period T is that the bonds price change to $q=\beta$, providing the government additional liquidity which results in a rise of consumption and government spending to $c^n(k^n)$ and to $g^n(\underline{b}(k^n), \underline{b}(k^n), k^n)$. At this point in time, the government enters the equilibrium of the "no-crisis zone".

To determine T, we find $max\{V^1(B_0), V^2(B_0), \cdots, V^\infty(B_0)\}$. The existence of such a maximum is guaranteed, since $\lim_{T\to\infty} V^T(B_0) = V^\infty(B_0)$, such that the value function for a debt reduction in an infinite time period is:

$$V^\infty = \frac{1}{1-(1-\pi)\cdot\beta}\cdot \\ \cdot\Big[(1-\pi)\cdot\{c^\pi(k^\pi)+v[g^\infty(k^\pi)]\}+\pi\cdot\{c^\theta(k^\pi)+v[g^\theta(k^\pi)]\}+\pi\cdot\beta\cdot V^\theta(B_0,k^\theta)\Big]$$

with

$$g^\infty = \tau\cdot[f(k^\pi)-\delta\cdot k^\pi]-(1-\hat{\beta})\cdot B_0$$

which actually corresponds to government spending of never running its debt down.

That is, the government responds to the vulnerability to self-fulfilling debt crises by reducing its debt. Cole and Kehoe (2000) show that for higher B_0, the maximizing value function correspond to higher T. However, as stated by Aguiar and Amador (2015) the one caveat to this result is if initial debt is so large that the transition to $\underline{b}$ may be long enough that the government is better off remaining in the crisis zone indefinitely.

Run up debt in W periods Now, we suppose that the government plans to run up debt in W periods, to avoid a sharp reduction government spending, and to default when the debt level reaches the default zone. The government runs up debt until $\bar{B}(k^\theta)$, such that after W, and according to Eq. (1.4.12), it maintains a stationary debt policy. Again we assume that $k_0 = k^\pi$ and the constant government spending is:

$$g^W(B_0) = \tau\cdot[f(k^\pi)-\delta\cdot k^\pi]+\hat{\beta}^{W-1}\cdot\frac{1-\hat{\beta}}{1-\hat{\beta}^W}\cdot\beta\cdot\theta\cdot\bar{B}(k^\theta)-\frac{1-\hat{\beta}}{1-\hat{\beta}^W}\cdot B_0. \quad (1.4.22)$$

Comparing this expression with $g^T(B_0)$, it is possible to conclude that if

$$V^W(B_0) = \frac{1-[(1-\pi)\cdot\beta]^{W-1}}{1-(1-\pi)\cdot\beta}\cdot\{(1-\pi)\cdot[c^\pi(k^\pi)+v(g^W)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)]\}+ \\ +[(1-\pi)\cdot\beta]^{W-1}\cdot\{(1-\pi)\cdot[c^n(k^\pi)+v(g^W)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)]\}+ \\ +\pi\cdot\beta\cdot\frac{1-[(1-\pi)\cdot\beta]^W}{1-(1-\pi)\cdot\beta}\cdot V^\theta(B_0)+[(1-\pi)\cdot\beta]^W\cdot V^\theta[\bar{B}(k^\theta)] \quad (1.4.23)$$

We argue that for a high enough level of B_0, which has a high debt service, it takes to long to reduce debt to a safe limit and thus there is a higher payoff in running up debt to a default level. The effect of entering the crisis zone in period W is that the bonds price increase to $q = \beta \cdot \theta$, but providing the government additional immediate liquidity resulting from not repaying previous debt. At this point in time, the government enters the equilibrium of the "default zone".

To determine W, we find $max\{V^1(B_0), V^2(B_0), \cdots, V^\infty(B_0)\}$.

Moreover, $\lim_{W\to\infty} V^W(B_0) = V^\infty(B_0)$, because it corresponds to keep a constant debt level at B_0.

Notice that in the CK framework with a complete default, the penalization of a default implied that the country would live in autarky and a larger drop in output than in the case of a partial default. This fact may change the incentive to run down debt in the crisis zone.

1.4.3 Characterizing the zones

To define the upper safe debt limit $\underline{b}(k_0)$ and the upper sustainable debt limit $\bar{B}(k_0; \pi)$, requires defining two constraints, which Cole and Kehoe (1996) label the no-lending continuation constraint and the participation constraint, respectively.

1.4.3.1 No-lending continuation condition

The *no-lending continuation condition* builds the fundamentals for making a crisis even possible. When this condition is satisfied, even if the government observes a decrease in the price bankers pay to buy further debt, indicated through a bond price $q = \beta \cdot \theta$, it strictly prefers defaulting instead of repaying (Cole and Kehoe, 1996, 2000). In our framework, this condition is satisfied if $V^\theta(B_0, k) > V^n(B_0, k)$. By equating these payoffs, we find the country's maximum borrowing level for which no default can occur, that is, the upper safe debt limit $\underline{b}(k)$. To characterize this debt limit, we assume that $k_0 = k^n$.

- If the decision is to default on B_0, the payoff is:

$$V^\theta(B_0, k^n) = c^\theta(k^n) + v[g^\theta(B_0, B_1, k^n)] + \frac{\beta}{1-\beta} \cdot \left\{c^\theta(k^\theta) + v[g^\theta(B_1, B_1, k^\theta)\right\}$$

with

$$g^\theta(B_0, B_1, k^n) = \tau \cdot [Z^{1-\theta} \cdot f(k^n) - \delta \cdot k^n] - \theta \cdot B_0 + \beta \cdot \theta \cdot B_1$$

$$g^\theta(B_1, B_1, k^\theta) = \tau \cdot [Z^{1-\theta} \cdot f(k^\theta) - \delta \cdot k^\theta] - (1 - \beta \cdot \theta) \cdot B_1$$

$$B_1 = B^\theta = \frac{1}{1 + \beta \cdot (1-\theta)} \cdot B_0 \qquad \text{Eq. (1.4.12)}$$

- If the decision is not to default, then the government repays its debt liabilities, even though the price of debt decreases ($q = \beta \cdot \theta$). The corresponding payoff is:

$$V^n(B_0, k^n) = c^n(k^n) + v[g^n(B_0, B_1, k^n)] + \frac{\beta}{1-\beta} \cdot \{c^n(k^n) + v[g^n(B_1, B_1, k^n)\}$$

with

$$g^n(B_0, B_1, k^n) = \tau \cdot [f(k^n) - \delta \cdot k^n] - B_0 + \beta \cdot \theta \cdot B1$$

$$g^n(B_1, B_1, k^n) = \tau \cdot [f(k^n) - \delta \cdot k^n] - (1 - \beta \cdot \theta) \cdot B_1$$

$$B_1 = B^n = B_0$$

Equalizing these expressions and solving them after the current debt level, defines the upper safe debt limit $\bar{(}b)(k^n)$. The resulting value is the largest level of debt, where the no-lending continuation constraint is not satisfied, meaning that the government prefers to repay its debt liabilities, even though $q = \beta \cdot \theta$. Below this ceiling, a crisis never occurs. In a complete default setup, Cole and Kehoe (2000) show that $d\bar{b}(k)/dK > 0$.

1.4.3.2 Participation constraint

The *participation constraint* ensures that after issuing new debt, the government's optimal decision is to repay previous debt. This means that the government's payoff of not defaulting is higher than the payoff of defaulting: $V^n(B_0, k, q) \geq V^\theta(B_0, k, q)$. When we set $q = \beta \cdot (1 - \pi) + \beta \cdot \pi \cdot \theta$, this constraint determines the **upper bound** on debt for which positive lending can occur in a sunspot equilibrium, $\bar{B}(k; \pi)$, that is, the highest level of debt for which it is better not to default if bankers lend.

However, as Cole and Kehoe (1996, 2000) note, the characterization of the upper bound on debt is difficult because in the crisis zone optimal government policy will not, in general, be stationary. Still, we may characterize the upper bound on debt under a stationary debt policy B^s, describing the highest level of government debt for which repayment can still be achieved through a stationary debt and expenditures policy, on condition that future debt can be sold at $q = \beta \cdot (1 - \pi) + \beta \cdot \pi \cdot \theta = \hat{\beta}$, and assuming that $k_0 = k^\pi$.

- If the decision is not to default when $q = \beta \cdot (1-\pi) + \beta \cdot \pi \cdot \theta = \hat{\beta}$, the corresponding payoff (derived in Appendix A.2.4) is:

$$V_t^n(\bar{B}, k^\pi) = \frac{1}{1-(1-\pi)\cdot\beta} \cdot \{c^\pi(k^\pi) + v[g^\pi(B,B,k^\pi)]\} + \\ + \pi \cdot \beta \cdot \frac{1}{1-(1-\pi)\cdot\beta} \cdot \left\{c^\theta(k^\pi) + v[g^\theta(B,B,k^\pi)]\right\} + \\ + \pi \cdot \beta \cdot \frac{1}{1-(1-\pi)\cdot\beta} \cdot \frac{\beta}{1-\beta} \cdot \left\{c^\theta(k^\theta) + v[g^\theta(B,B,k^\theta)]\right\}$$

with

$$g^\pi(B,B,k^\pi) = \tau \cdot [f(k^\pi) - \delta \cdot k^\pi] - (1-\hat{\beta}) \cdot B$$

$$g^\theta(B,B,k^\pi) = \tau \cdot [Z^{1-\theta} \cdot f(k^\pi) - \delta \cdot k^\pi] - (\theta - \beta \cdot \theta) \cdot B$$

$$g^\theta(B,B,k^\theta) = \tau \cdot [Z^{1-\theta} \cdot f(k^\theta) - \delta \cdot k^\theta] - (1 - \beta \cdot \theta) \cdot B$$

- If the decision is to default on B, when the government was able to sell new debt at $q = \beta \cdot (1-\pi) + \beta \cdot \pi \cdot \theta = \hat{\beta}$, then after period t the new price of debt drops to $q = \beta \cdot \theta$ and the payoff is:

$$V^\theta(B, k^\pi) = c^\theta(k^\pi) + v[g^\theta(B,B,k^\pi)] + \frac{\beta}{1-\beta} \cdot \left\{c^\theta(k^\theta) + v[g^\theta(B,B,k^\theta)\right\}$$

with

$$g^\theta(B,B,k^\pi) = \tau \cdot [Z^{1-\theta} \cdot f(k^\pi) - \delta \cdot k^\pi] - \theta \cdot B + \hat{\beta} \cdot B$$

$$g^\theta(B,B,k^\theta) = \tau \cdot [Z^{1-\theta} \cdot f(k^\theta) - \delta \cdot k^\theta] - (1 - \beta \cdot \theta) \cdot B$$

The equality of these payoffs allows us to find the upper sustainable debt limit $\bar{B}(K, \pi)$, since it corresponds to the maximum level of debt for which positive lending is possible. However, Cole and Kehoe (2000) notice that, since the government policy within the crisis zone will not be stationary, it is difficult to explicitly characterize this upper bound on debt. To account for this circumstance, an upper stationary debt limit $B^s(K, \pi)$ is introduced, corresponding to the highest level of government debt for which repayment can still be achieved through a stationary debt and expenditures policy, on condition that future debt can be sold for a positive price. Moreover, we assume that $k = k^\pi$.

Above this limit the government would default even if it could sell new debt.

1.4.4 Results

After computing several policy possibilities, depending on the debt level in period t=0, the resulting expected payoffs are summarized as follows:

$$V(B_0,k) = \begin{cases} \frac{1}{1-\beta}\cdot\{c^n(k^n)+v\left[g^n(B_0,B_0,k^n)\right]\} & \text{if } B_0 \leq \underline{b}(k^n) \\ max\{V^{T1}(B_0),V^{T2}(B_0),\cdots,V^{\infty}(B_0)\} & \text{if } \underline{b}(k^n) < B_0 \leq \bar{B}(k^{\pi},\pi) \\ & \text{and } B_0 \leq B^w \text{ and } \zeta \leq 1-\pi \\ max\{V^{W1}(B_0),V^{W2}(B_0),\cdots,V^{\infty}(B_0)\} & \text{if } \underline{b}(k^n) < B_0 \leq \bar{B}(k^{\pi},\pi) \\ & \text{and } B_0 > B^w \text{ and } \zeta \leq 1-\pi \\ c^{\theta}(k^{\theta})+v\left[g^{\theta}(B_0,B^{\theta},k^{\theta})\right]+ & \\ \quad +\beta\cdot\frac{1}{1-\beta}\cdot\left\{c^{\theta}(k^{\theta})+v\left[g^{\theta}(B^{\theta},B^{\theta},k^{\theta})\right]\right\} & \text{otherwise} \end{cases} \tag{1.4.24}$$

The corresponding debt strategies are depicted in Figure 1.4.

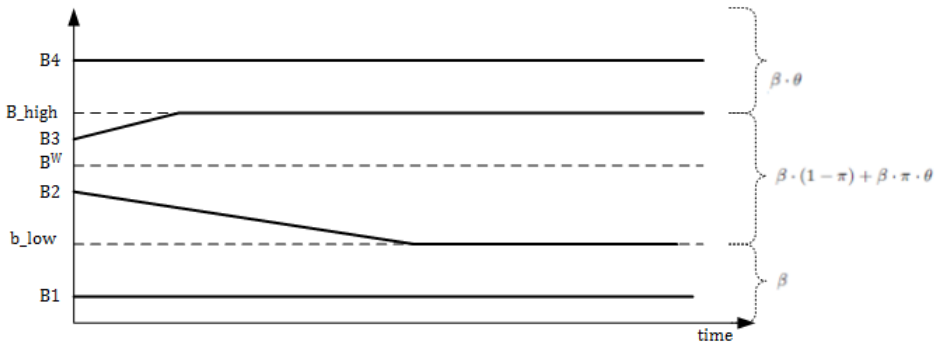


Figure 1.4: Debt paths
Source: Adapted from Cole and Kehoe (2000)

If $B_0 = B1$, the government finds itself in the no-crisis zone in t=0, where the occurrence of a crisis is not possible. In fact, the government wants to avoid the costs of a default by all means and is therefore willing the repay the bonds becoming due, even though this requires a drop in government expenditures. As this behavior can be anticipated by international bankers, they are willing to pay a relatively high price for the government bonds, because there is a low default risk. Moreover, even if the sunspot variable indicates "bad news", it will have no impact on the behavior of the other bankers, neutralizing the danger of an uncoordinated investment stop.

Within the crisis zone, the incentives of the investors change, because there is a positive probability for a future crisis to occur. This probability implies an increase in risk premium, which decreases bond prices. Moreover, there is a reduction in output resulting from a decrease in investment.

Cole and Kehoe (1996, 2000) identify three different strategies (not depicted in Figure 1.4) depending on the level of B_0. A first option if that for a certain $B_0 = B^s$ the optimal decision is to keep a stationary debt limit. A second option considers the case where the initial debt level is lower or equal to the stationary debt limit, and where the government should reduce its foreign borrowing to the upper safe debt limit within T periods, by *smoothing* its expenditures, to avoid painful sharp cuts in expenditures. Therefore, a benevolent government that maximizes the consumers' utility, reduces debt over multiple periods, increasing in B_0. As soon as the debt level reached the upper safe limit and the sunspot has not indicated a crisis, the probability for a default disappears. The third option occurs if the initial level of debt is above the stationary debt limit. In this case, the ability to run down the debt with the help of a standard consumption smoothing policy disappears, demanding for a *sharp* drop of debt in period t=0. Even though the government has an incentive to repay its debt, as new debt still can be sold for a positive price, in order to be able to constantly repay over T periods, B_1 has to abruptly be set below B_0 requiring the government expenditures to be more restricted than in the case of a stationary debt policy. After this sudden reduction in the first period, the government can follow the path of a standard consumption smoothing policy from period t=1 on-wards.

In our case, the second option corresponds to the path of $B2$. However, when $B_0 > B^w$, which corresponds to B_3 path, the required sharp reduction in debt level and thus in government spending is avoided. In fact, in the CK framework the penalty of a complete default is much higher than the penalty of out environment, because a complete default implies a zero bond price and that the country is permanently banned from international credit markets. Under a partial default, even though the government receives a lower price for new debt issuing, the country is still able to borrow. Thus for $B^w < B_0 \leq \bar{B}$, the government maintains a consumption smoothing strategy until it finally defaults and taking advantage of a missed payment of debt, that will later be offset by lower output and bond prices.

Finally, if $B_0 > \bar{B}$ and the government finds itself above the upper sustainable debt limit, the only possible outcome is default and all variables are stationary from period t=1 onwards.

1.5 Numerical computation

This section presents numerical results with the same parameters as Cole and Kehoe (1996), that have been chosen to match the Mexico situation in the late 1994. The results were obtained using Matlab and the code for the calibration of the general model is provided in Appendix A.3.

1.5.1 Functions and parameters

The utility function is $u(c_t) + v(g_t) = c_t + log(g_t)$ and the production function is $f(k_t) = A \cdot k_t^{\alpha}$. The parameters are pooled in Table 1.4. Cole and Kehoe (1996, p. 320) interpret a period as being 2/3 of the year, matching the average maturity of the Mexican government's short term debt.

Table 1.4: Parameters for Mexico at the end of 1994

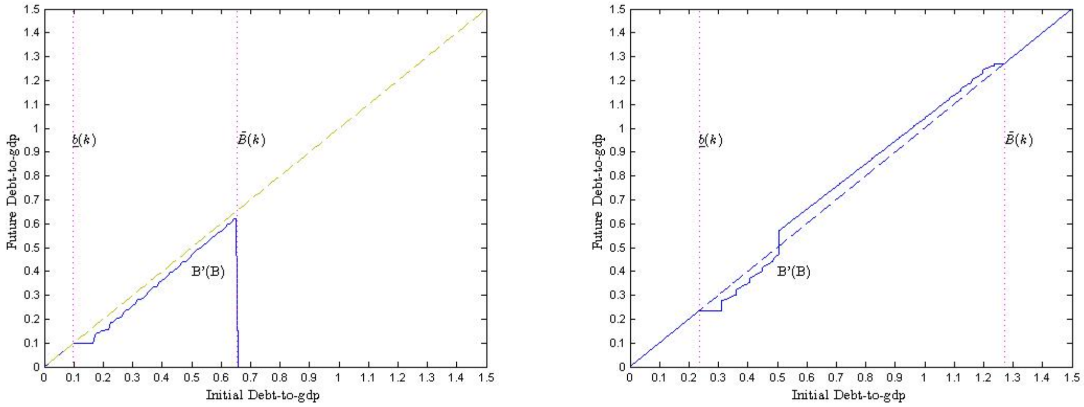
	Description	Value	Comments
β	Discount factor	0.97	Yearly discount factor of $0.955 = 0.97^{3/2}$, which implies a yearly yield of $0.047 = 0.955^{-1} - 1$ on risk free bonds as measured by U.S. treasury bills.
A	Technology in production function	2	Scaling factor
Z	Default penalty	0.95	
π	Probability of default	0.02	
α	Capital share	0.4	Estimate of capital share in national income
δ	Depreciation rate	0.05	Corresponds to a yearly depreciation rate of $0.074 = 1 - 0.95^{3/2}$
τ	Tax rate	0.2	Government revenues as a share of output

Source: Cole and Kehoe (1996)

1.5.2 Numerical results

Figure 1.5 plots the upper safe limit, $\underline{b}(k^n)$, and the upper sustainable debt limit, $\bar{B}(k^\pi)$, as ratios to GDP, as well as the government's optimal debt policy function $B'(B)$, denoting the optimal future borrowing amount, given the initial level of indebtedness. We also draw the 45° degree line in order to facilitate the assessment of running down debt position whenever the optimal policy is below this line or of running up debt when the optimal policy is above. Assuming that no crisis ever happened in the crisis zone, the optimal government debt policy considering the possibility of a full default (CK model) is depicted in Figure 1.5a and considering a recovery of $\theta = 90\%$ is plotted in Figure 1.5b.

In the benchmark case produced by the CK model, the upper safe limit is a debt level corresponding to 10% of GDP and the upper sustainable limit is a debt-to-GDP ratio of 65%. For a current borrowing level below the upper safe debt limit ("no-crisis zone"), a stationary



(a) $\theta = 0\%$ (b) $\theta = 90\%$

Figure 1.5: Government debt policy function for full and partial default of 10%
Source: Author's calculations

debt policy is optimal. When the country roll-over amount exceeds a debt-to-GDP of 10%, it enters the crisis zone and exposes itself to the decisions of the international bankers. However, if the debt exceeds the upper sustainable debt limit, fixed at 65% debt to GDP, an immediate default is the optimal option. If the country is in the crisis zone, it becomes vulnerable to an uncoordinated investment stop by international bankers and the increased interest rate on bonds provide an incentive to run down the debt position to the upper safe debt limit. This decrease is done step by step, reflecting that for low initial debt (inside the crisis zone), the number of periods T to run debt down is small. In fact, for debt-to-GDP ratios from 10% until approximately 17%, the reduction to the upper safe limit is achieved in one period. Once the upper safe limit is reached, the level of indebtedness proceeds to a stationary expenditures at that debt limit and at a lower price for government, independently of the realization of the sunspot variable.

Cole and Kehoe (1996) assess the validity of the model in the light of the Mexican 1994-95 debt crisis, and find that since Mexico has a very short maturity structure of debt, then the crisis zone is quite large and starts at a low debt-to-GDP ratio. In fact, an increase in maturity reduces the vulnerability to self-fulfilling crises, because the roll over amounts decrease, which in turn increases the upper safe debt limit, $\underline{b}$.

When we consider a partial default of 10% or a recovery rate of 90% , plotted in Figure 1.5b, the upper safe and sustainable debt limits increase to 24% and 129%, respectively. This means that the country is safe at higher debt-to-GDP ratios when compared to the case of a complete default and a permanent banning from international credit markets. In fact, now the country is able to borrow, even though at a lower price. Figure 1.6 shows the bonds' prices in each zone.

Within this framework, the bonds' price is constant in each zone. However, in the crisis

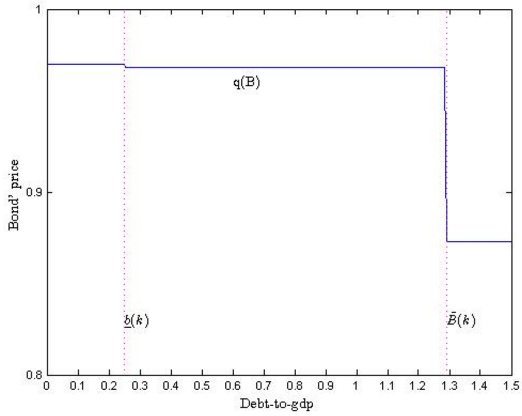
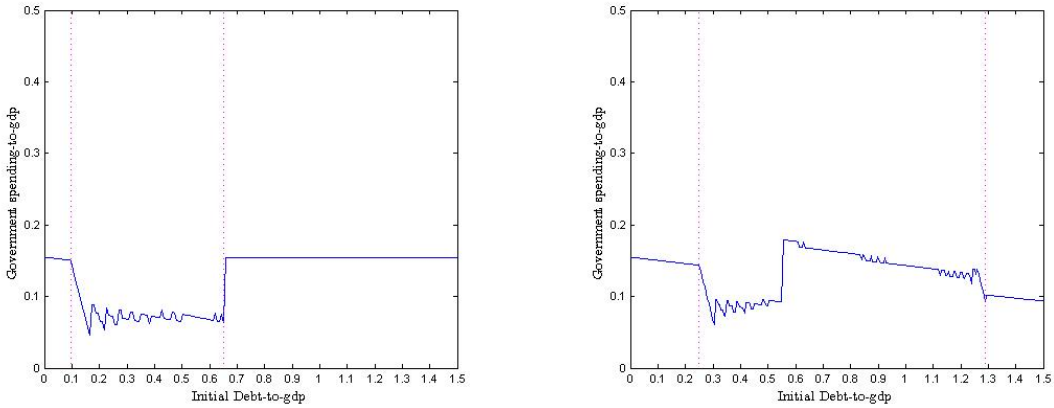


Figure 1.6: Bonds' price in different zones
Source: Author's calculations

zone, a topic that deserves further research is considering the relation between the probability of default and the debt level, such that the price of bonds is increasing with the debt reduction.

Comparing Figures 1.5a and 1.5b, the most important difference in the results when we consider a partial default is the change in the optimal debt policy. Figure 1.5b shows that when the country finds itself vulnerable to a self-fulfilling crisis, there are two different policies. Until an intermediate roll over amount up to 55% of GDP, the maximization of utility implies running down future debt to the upper safe debt limit, as the payment of increased interest rates wants to be avoided. During this process, the costs of rolling over the debt positions decline and the additional liquidity allows the government to run down its debt positions.

For amounts higher than 55%, the incentives change and the government starts running up the debt positions until reaching the upper sustainable debt limit $\bar{B}$. The trigger for this result is that the lower price received by issuing new debt associated with a higher debt level requires issuing higher debt to avoid a large reduction in public spending. Once the upper sustainable debt level is reached, the government defaults and afterwards it follows a stationary debt level, thus smoothing public consumption, even though at a higher cost. As seen in Figure 1.7, since the default penalization in our model is less burdensome, starting from an intermediate debt level, the government optimally chooses to keep increasing the debt level.



(a) $\theta = 0\%$ (b) $\theta = 90\%$

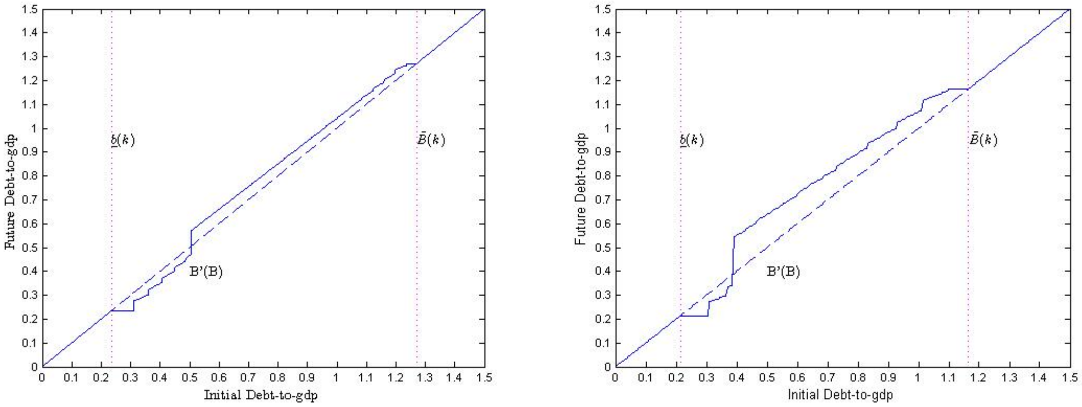
Figure 1.7: Government spending as ratio to GDP for full and partial default of 10%
Source: Author's calculations

This change in strategy only depends on the initial debt level, thus two countries vulnerable to the same probability for a panic and with the same potential loss in revenue, may decide to run contrary strategies. However, here we do not address the question of how the country accumulated that debt.

The results discussed above have shown how the possibility of a partial default instead of a full default changes the optimal policy path of a country. The main basis of decision-making is the current debt level: is the debt level still low, a country has an incentive to run down its debt positions in order to avoid being vulnerable to self-fulfilling movements on the part of the international bankers. However, is the country facing a high indebtedness, running up the debt proves to be the optimal strategy. Blundell-Wignall and Slovik (2011) corroborates this result, by stating that a debt restructuring (partial default) is more likely the larger the initial stock of debt as a share of GDP. The larger the initial share, the more likely that the debt service burden in perpetuity will be too high; this is a permanent burden on taxpayers. Moreover, when a significant amount of debt is held by foreigners, this represents a real transfer abroad (and a widening gap between GDP and GNP).

Numerical result 1 *A higher probability of default (π) decreases both thresholds and implies a faster debt reduction/increase (policy function farther from 45° line).*

A change in the probability of default lowers the debt level from which it is optimal to start running up the debt, but not whether to follow the same strategy. Figure 1.8 shows the government debt policy function when we increase π from 2% to 20%, where both the upper safe and upper sustainable limits decrease to 21% and 116%, respectively. In fact, as π increases, the no-crisis zone shrinks and the default zone widens. The policy function $B'(B)$ is farther away from the 45° line, which means that the government has an incentive to run down/up the debt faster.



(a) $\pi = 2\%$ (b) $\pi = 20\%$

Figure 1.8: Government debt policy function for different probabilities of default
Source: Author's calculations

By plotting such a large increase in π, allows a clearer analysis of the effect of this change. An increase in the probability of default while the debt level is in the crisis zone, increases the gain of reducing debt level in fewer periods, since the likelihood of a crisis taking place during more periods increases. At the same time, above the intermediate debt level, it increases the gain of running up debt rapidly because there is an approximation between the cost of debt in the crisis zone and in the default zone (Figure 1.9), which also results in a decrease in the upper sustainable limit.

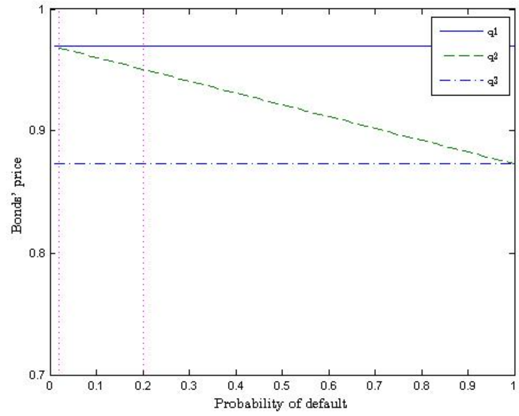


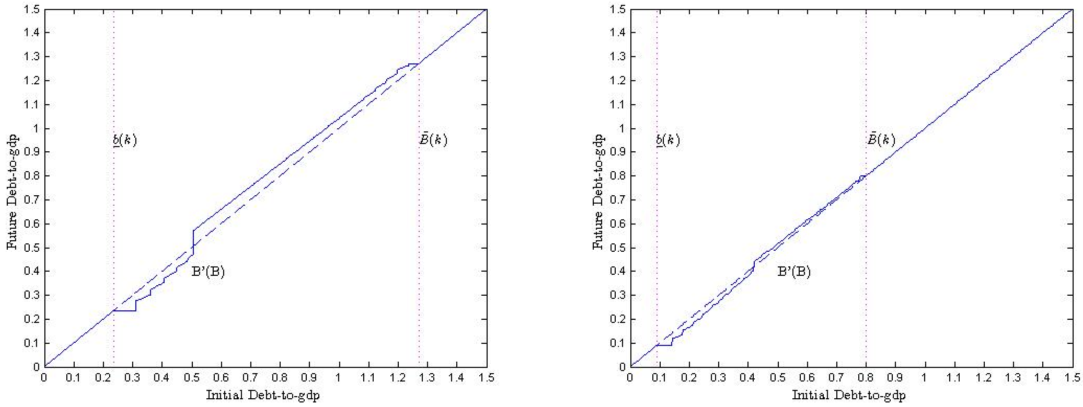
Figure 1.9: Relation between bonds' price and the probability of default with $\theta = 90\%$
Source: Author's calculations

This result may *partially* justify the lack of effectiveness of bailout programs to induce a reduction in sovereign debt level. In fact, Jorra (2012) explores empirically how the adoption of IMF programs affects sovereign risk over the medium term and find that those programs significantly increase the probability of subsequent sovereign defaults by approximately 1.52 percentage points.

Numerical result 2 *A smaller recovery rate lowers both the upper safe and the upper sustainable limits and slows the debt reduction/increase.*

In the CK framework, $\underline{b}^n$ corresponds to the highest level of debt below which the payoff of repaying is higher then the payoff of defaulting and afterwards living in autarky. Instead, in our framework, it is the highest level below which it is better to repay then to assume a higher price, but still being able to access international credit markets. Since in our model staying in the crisis zone is not so penalizing, for some recovery rates the upper safe limit may be even lower then under a full default.

A higher haircut decreases the price investors pay for new bonds, such that a debt level that is safe with $\theta = 90\%$ makes the country vulnerable to crisis at $\theta = 80\%$ (Figure 1.10). However, the intermediate debt level until which it's better to run down the debt is now relatively higher, revealing a strong willingness to leave the crisis zone.



(a) $\theta = 90\%$ (b) $\theta = 80\%$

Figure 1.10: Government debt policy function for different recovery rates
Source: Author's calculations

Again, the price of bonds plays an important role, since a lower recovery rate, for a given probability of default, widens the gap between the price in the crisis zone and in the default zone, as seen in Figure 1.11.

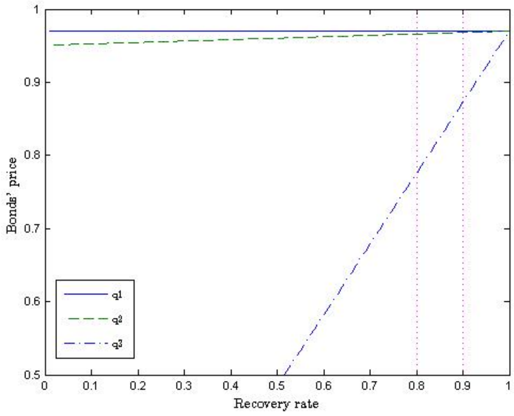
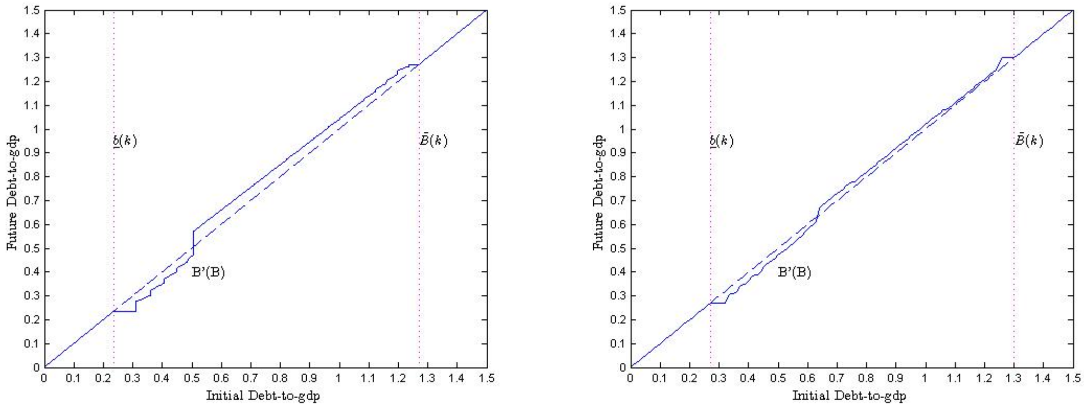


Figure 1.11: Relation between bonds' price and the recovery rate with $\pi = 2\%$
Source: Author's calculations

Numerical result 3 *A higher default cost (lower Z) increases the upper safe limit and implies a slower debt "reduction".*

The default penalty is crucial for the decision over the future debt positions because it causes a drop in the GDP that is available for government expenditures, private consumption, and repayment of debt. So a higher penalization in case a default occurs, increases the incentive of a government to undertake cuts in expenditures. In this case, the upper safe limit increases and the policy to running down the debt position and exiting the crisis zone persists until higher debt levels (Figure 1.12), in order to avoid the penalization of suffering from self-fulfilling debt crisis. E.g., a debt-to-GDP ratio of 60% with a low default penalty would lead to running up debt until a default, but with a higher default penalty it is optimal to run down the debt until leaving the crisis zone.

The upper sustainable limit also increases. A government that faces a higher default penalty runs the same strategy, but the decision to file bankruptcy only dominates at higher debt to GDP levels. In relation to the slower debt increase, notice that a higher penalty in case of a default decreases the gains of running up the debt in more periods.



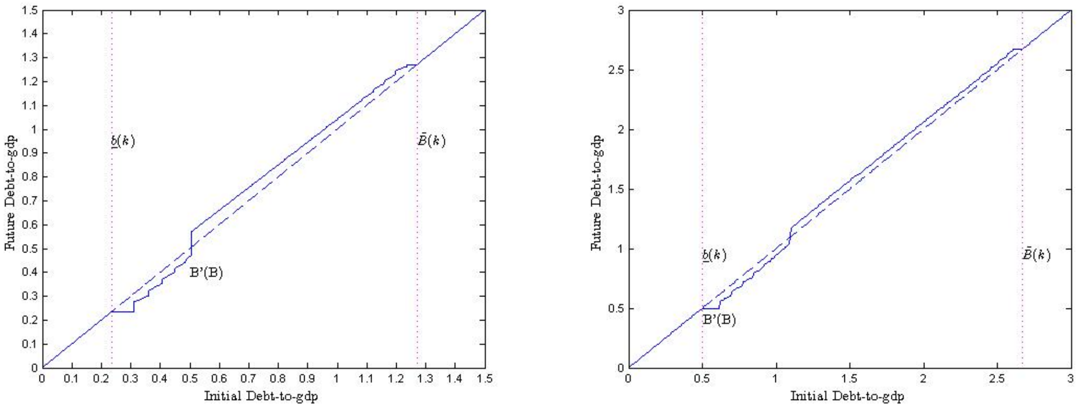
(a) $Z = 0.95$ (b) $Z = 0.90$

Figure 1.12: Government debt policy function for different default penalties (Z)
Source: Author's calculations

The increase in the cost of default (lower Z) is often perceived as a mean to increase the credibility of commitment, such that a country be given the incentive to run down its debt positions in situations, where running up the borrowing level would have been optimal in the first place. If we consider that rescue packages increase commitment, then in fact, depending on the initial debt level, they may lead to a sovereign debt reduction. This is the contrary result of the one obtained by Arellano, Conesa and Kehoe (2012) when the country is facing a recession. They conclude that policy interventions taken by the EC, ECB and IMF, by inducing lower borrowing costs and lower default penalties, have encouraged governments to gamble for redemption.

Numerical result 4 *An increase in fixed tax rate increase both the upper safe and upper sustainable debt limits.*

Another significant effect is provided through a change in the tax rate, since it is an indicator for the dependence on sovereign debt. If the revenue share generated through taxation is high (Figure 1.13b) the country is in a no-crisis zone for higher debt level. Moreover, in the crisis zone, with a lower need further borrowing, there is an incentive to running down the future debt even for high levels of indebtedness.



(a) $\tau = 20\%$ (b) $\tau = 40\%$

Figure 1.13: Government debt policy function for different tax rates (τ)
Source: Author's calculations

An alternative perspective is provided by Blundell-Wignall and Slovik (2011), as they consider that debt restructuring is more likely the smaller the primary deficit. In fact, a relatively small primary deficit indicates the government has already taken significant steps to eliminate most or all of the primary deficit it is living within its means and going any further is likely to produce unpopular economic hardship.

Numerical result 5 *For each initial debt level, below a certain fixed default rate the decision is never to default; above that fixed rate the "optimal" default rate tends to be total.*

The model was run for selected recovery rates (1− default rate), from 0.9 until 0.2, and for selected initial debt levels, 60%, 80%, 100% and 120%. The value function of not defaulting, V(B), and of defaulting, VDef(B), was then retrieved.

Figure 1.14 plots the evolution of the *maximum* of V and VDef relative to each recovery rate. This procedure was entailed for each initial debt to GDP ratio.

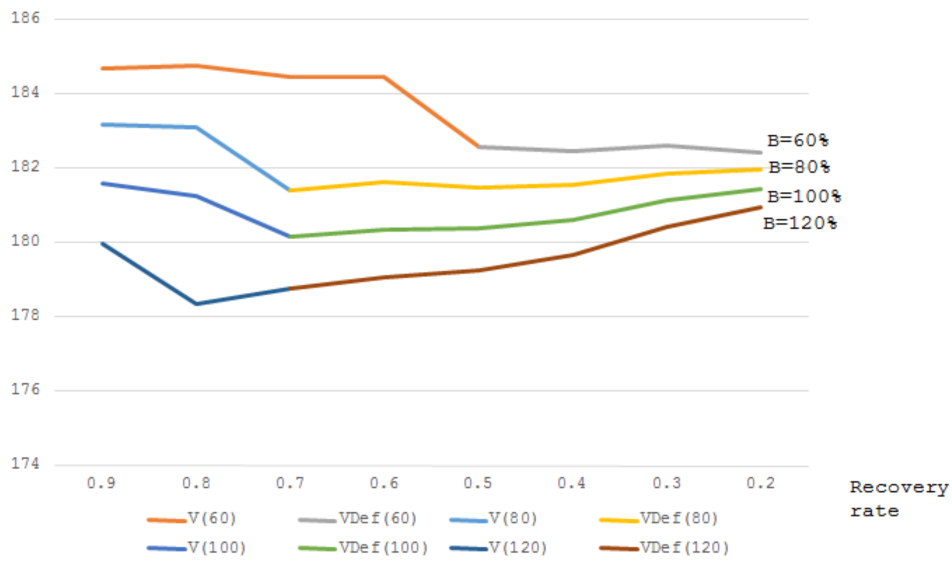


Figure 1.14: Evolution of the value function and the recovery rate

Considering the initial debt level of 60% GDP the model indicates that it is always optimal not to default ($Max\{V, VDef\} = V$) above a benchmark recovery of $\theta = 0.5$. This indicates that if the country considers a partial default up to 0.5, the government will disregard the recovery rate since the optimal choice will be not to default. This reasoning applies to any of the selected initial debt levels, used as examples. Moreover, it should also be noted that higher debt levels relate to lower benchmark default rates. For instance, for $\theta = 0.6$ (default rate of 40%), if the initial debt to GDP ratio is 60% the country will not default, but if the debt to GDP ratio is 120% then default becomes the optimal decision.

However, for the same initial 60% debt to GDP, if the recovery rate is below 0.5, the value of defaulting exceeds the value of not defaulting ($Max\{V, VDef\} = VDef$). As the recovery rate continues to decrease (which means that the default rate is increasing),the benefit of defaulting rises. Again, this applies for any given initial debt level: when default becomes the optimal decision, a total repudiation of the debt seems to be a better choice than any positive recovery rate. This may occur since the government is still able to access credit markets, even after the default, which would show that it is not being thoroughly penalized for its default decision.

1.5.3 Limitations of the results

One of the limitations of these results is the relation established between some of the parameters. For example, a higher probability of default can be retrieved from a lower recovery rate. Additionally, the tax rate is related with the probability of default, since the collateral for a government bond is the good standing of the issuer based on its ability to tax private agents and service its loans. This means that an increased tax rate may imply a reduction in the probability of default. While the essence of the *ceteris paribus* condition is not present, it still allowed to investigate the relationship between key variables, which could provide some qualitative and quantitative insights.

Another issue is related to bonds that are assumed to have a maturity of one period, such that the government has a need to roll over its debt period by period. This circumscribes the applicability of the model to emerging market economies, that typically issue short term bonds. In order to account for this fact, it is possible to re-scale a period length to match the average maturity of debt, even though this does not directly reflect the fraction of debt becoming due, since it depends on the composition of the total portfolio. To overcome this problem, and at the sake of further complicating the analysis, it would be possible to include in the model short-term and longer-term bonds, and find debt limits for each type of bonds.

1.6 Conclusion

Throughout history, countries have been exposed to the financial institutions' willingness to roll over sovereign debt. Increasing risk premia and the threat of a country's default through a self-fulfilling crisis induce governments to apply different policies. The purpose of this paper was to deliver an explanation for such policies through the application of a dynamic stochastic general equilibrium model, providing the optimal debt policy for a country being vulnerable to self-fulfilling financial markets beliefs, in a partial default setup.

A self-fulfilling crisis may occur within a crisis zone, that is delimited by a lower debt limit (called the upper safe limit) and a higher debt limit (designated the upper sustainable limit). To assess if the country is in the crisis zone, the crucial variable is the initial debt level. If the initial debt is bellow the upper safe limit, the government keeps a constant debt level, a high constant spending path and consumers set capital a high level. In this case, no default ever happens. If the initial debt is above the upper sustainable limit, a partial default is the optimal decision. After the default, government keeps a constant debt level, a constant low spending path, a consumers set capital a low level. In between those limit is the crisis zone, where up to a certain intermediate level the government wants to avoid a crisis, such that it runs down the debt level until it reaches the safe limit. However, above that intermediate level, debt reduction is too painful and to avoid sharp spending reductions the benevolent government starts running up debt until the upper sustainable limit. The reason why this limit is not exceeded, since anyway debt is heading for a default, is to avoid a further government spending reduction, once the country enters the default zone.

The original Cole and Kehoe (1996, 2000) models with complete default only produced the running down debt policy. However, taking into consideration recent European empirical evidence that suggested that countries in the crisis zone were increasing debt levels, Conesa and Kehoe (2012, 2014) introduced the possibility of "gambling for redemption", where governments of countries facing a recession keep increasing debt levels hoping for a recovery. By considering the possibility of defaults being partial (and thus not so penalizing), our models produces that same result of running up the debt within the crisis zone.

The numerical results aim at finding the implications of changes in selected parameters. In particular, we find that overall a higher probability of default, a lower recovery rate, a lower cost of default and a lower tax rate induce a decrease in the upper safe limit and a decrease in the upper sustainable limit. Moreover, changes in those parameters do not change the government strategy within the crisis zone, but change the debt level from which it is optimal to start running up debt and change the speed of debt reduction/increase.

Recent events in Europe suggest that evaluating whether a country's debt is vulnerable to a crisis is an important issue. Specially the distinction between a confidence crisis that may induce a possible default and a crisis stemming from a default being the optimal decision, even if the government is able to roll over debt. Moreover, it is relevant to find out what type of third party interventions should be provided. In fact, by lowering risk premia and decreasing default penalties diminishes the costs of an upcoming default, inducing a government to take additional risk, leading to an increase in indebtedness. So the model gives some insights on countries' persistent budget deficits, as well as the incentive to display opportunistic behavior caused by moral hazard, triggered by the awareness of a potential bailout in turbulent times. To explore the type of interventions that could induce the government to leave the crisis zone is a topic that deserves further investigation.

Our model only considers external debt. However, taking into consideration that an increase portion of debt is domestic debt (Reinhart and Rogoff, 2009), it would be interesting to find out the effects on debt thresholds and government policies if we allow for domestic public debt and domestic consumers to save in the form of bonds.

Finally, in our model and within the crisis zone, a default is triggered exogenously by a sunspot variable. To better understand what drives panics, it would be desirable to find whether investor's beliefs may be linked to fundamentals, which is the purpose of the following Chapter.

Chapter 2

Market beliefs and fundamentals in the Portuguese sovereign debt crisis

2.1 Introduction

The euro area sovereign debt crisis has drawn renewed attention to the possibility of crises episodes being driven by markets' sentiments, rather than solely by fundamentals, thus highlighting the potential self-fulfilling features of crises. The aim of this chapter is to verify whether the recent Portuguese sovereign debt crisis, which ultimately led to the 2011 EC, ECB and IMF intervention, was driven by economic fundamentals, like weak economic growth or high debt to GDP ratio, or if financial markets' beliefs dominated. The estimation results suggest that both the fundamentals and expectations ignited the Portuguese sovereign debt crisis and that the impact of fundamentals in the probability of default depend on whether the economy is going through a crisis or not.

Motivation Considering that a sovereign debt crisis may be defined as economic and financial problems caused by the (perceived) inability of a country to pay its public debt, then it is reflected in the increase of the probability of default, which is mirrored in the widening of the spread of sovereign bonds relatively to a risk free sovereign bond. Since 2008, the difference between Portuguese and German sovereign bond yields suffered a large increase, indicating that financial markets perceive Portuguese debt as much riskier the German debt, that is to say, with a higher probability of default. Figure 2.1 plots the evolution of these spreads from 2000 until 2014, along with selected events that occurred during the same period. These selected events are far from portraying the complete situation, as done by Lane (2012), who describes the European debt crisis since its origins, related to the design of the euro, until its propagation mechanisms.

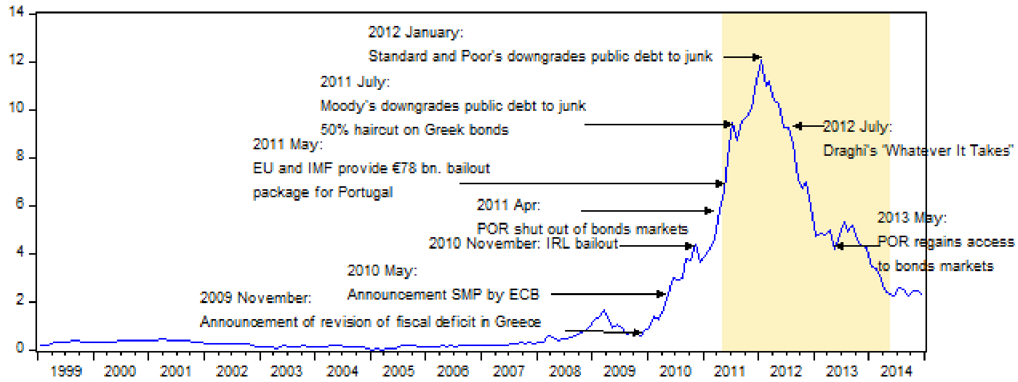


Figure 2.1: 10-year Portuguese government bond spreads relative to German bonds
Shaded area corresponds to the bailout period: May 2011 until May 2014
Source: Eurostat (2015*a*)

The sequence of events started with the 2008 financial crisis and proceed with Greece difficulties to repay its debt and ultimately being bailed out by the EU and the IMF in May 2010. Greek problems fostered the fear about the fate of other European economies, especially heavily indebted countries such as Portugal, Ireland, Italy and Spain. Eventually, the EU and the IMF agreed on a second bailout package for Greece (September 2010) and bailout packages for Ireland (November 2010) and Portugal (May 2011). However, these bailouts did not make the risk disappear, as seen in the figure.

Research has shown that market expectations can contribute to the occurrence of a crisis that would not have taken place if such expectations could have been prevented at first place. In this context, it is pertinent to find whether the changes in the probability of default have been determined by economic fundamentals, by market sentiments or both.

Method To test which features have played a role in the Portuguese case, this work builds on Jeanne and Masson (2000) seminal paper, which was applied to verify the presence of self-fulfilling beliefs in French franc devaluation episode between 1987 and 1993. The idea is to use a multiple equilibria model, where switches between an "optimistic" and a "pessimistic" equilibrium are allowed through a Markov-switching regimes (MSR) framework. The possibility of self fulfilling beliefs is considered in this model since the change from a tranquil to a crisis regime may be unrelated to economic fundamentals. The analysis of the dynamics of the Portuguese sovereign bond spreads was carried out from January 2000, well before the start of the global financial crisis, until December 2014.

The empirical strategy is as follows. First, a purely fundamental-based model is estimated, that is, a single regime is considered. Afterwards, a MSR model is estimated considering constant transition probabilities (CTP) between regimes. Then, the model is estimated by considering that one of the fundamental variables may be governing the transition probability, that is, we consider time-varying transition probabilities (TVTP). Finally, the model allows for regime-varying coefficients (RVC) both with CTP and TVTP.

Main findings The results indicate that a MRS model preforms better than a single-regime model, thus confirming the existence of two regimes. So, a model allowing for sunspots performs better than a purely fundamental-based model. Additionally, the CTP model with regime-varying coefficients performs better than the other models, suggesting that the fundamental variables are not determining the probability of a regime change, which is taken as evidence of the presence of self-fulfilling features. Moreover, the economic fundamentals have a different impact on the spreads, whether the economy is in the no-crisis or crisis regime.

One potential explanation for preferring a CTP model for the switch from low to high probability of default, unrelated to economic fundamentals, is that markets are forward looking, not pricing entirely on current fundamentals but on expected further deterioration in future economic fundamentals. Another possible interpretation is that the economy switched to a "pessimistic" self-fulfilling expectations equilibrium. In fact, the results are consistent with multiple equilibria with an abrupt switch from a "good" (tranquil) expectations equilibrium with low expected default rates and narrow spreads when economic fundamentals are favorable to a "bad" (crisis) expectations equilibrium with high expected default rates and wide spreads when fundamental positions are unfavorable. Given turbulent conditions during the crisis period and if apparently markets did not focus on current fiscal fundamentals, then the markets may have simply overreacted and mispriced risk of default (Aizenman, Hutchison and Jinjarak, 2013) .

Contributions This work can contribute to the large empirical literature that studies the determinants of sovereign default probability in several manners. First, it considers an adaptation of an existing model of self-fulfilling speculation to obtain a structural approach to assess the nature of the Portuguese sovereign debt crisis. Second, since the model is estimated with a single-country data, it has the advantage of paying greater attention to country-specific issues by using the corresponding control variables and focusing on relevant sample periods. Third, we adopt a sequential empirical strategy that provides a more complete picture in capturing the dynamics of investors' beliefs during the crisis.

The rest of the chapter is organized as follows. Section 2 surveys empirical literature related to the determinants of spreads in the context of the euro area debt crisis. Section 3 describes the model as an adaptation of Jeanne and Masson (2000) framework. Section 4 describes the variables, its statistical characteristics and selects the most relevant ones. Section 5 presents the econometric specifications, its results and discusses the empirical results. Section 6 establishes a discussion of the most relevant results and, finally, the last section concludes.

2.2 Related literature

Recently a substantial number of empirical papers have addressed issues related to sovereign risk in the euro area. This paper builds upon the theoretical contribution made by Jeanne and Masson (2000) and on the empirical contributions provided by several papers aiming at finding the determinants of the sovereign probability of default. Although recognizing the extensive body of empirical literature, recently surveyed by Tomz and Wright (2013), this section focus on the recent studies related to euro area countries.

In what follows, the literature is categorized in two strands: literature that searches for relationships between a country's sovereign spreads and other countries' spreads or between the same country's sovereign and private debt spreads; and literature that aims at finding the relation of sovereign debt spreads with economic fundamentals and expectations. In addition, and in the context of self-fulfilling crises, this section concludes with a brief summary of the econometric techniques used to test the aforementioned type of crisis.

2.2.1 Spillover effects

One strand of empirical literature studies spillover effects of sovereign debt spreads. These spillovers may occur between countries (contagion effects), or for individual countries considering, for example, the relationship between sovereign CDS and sovereign bond spreads or between sovereign CDS and bank CDS spreads.

Contagion between countries Most studies consider that contagion occurs when the cross-country correlations of spreads increase during "crisis times" relative to correlations during "tranquil times". The empirical literature on the European sovereign debt crisis shows mixed results regarding contagion effects, even though the majority of conclusions favors the presence of such spillover effects among countries.

Kalbaska and Gtkowski (2012) examine sovereign risks and the occurrence of financial contagion among peripheral and core countries (France, Germany and the UK), since the latter bought a large share of the debt of peripheral countries. The authors analyze the co-movements of CDS spreads of different countries, in the period spanning from August 2005 until September 2010. The paper shows that there were several waves of contagion and correlations increased already after the credit crunch in August 2007. They also validate the role of the global financial crisis in triggering sovereign risk and find that peripheral countries have lower capacity to trigger contagion than core EU countries. Moreover, Portugal is the most vulnerable country to shocks, whereas the UK is the most immune.

Metiu (2012) also finds evidence of significant contagion effects among long-term bond yield premia between January-2008 and February-2012. Italy was hit by contagion from Spain and Portugal while these two countries, in turn, were "importers of risk" from Greece. Moreover, he finds that contagion from Spain to Italy is significant both statistically and economically: more than half of the unexpected increases in the Spanish spread are transmitted to the Italian spread, even if they are unrelated to Italian fundamentals. Identically, Gómez-Puig and Sosvilla-Rivero (2014) find evidence of contagion in the European sovereign debt crisis. Using the yields on 10-year government bonds issued by 11 EMU countries, their results provide evidence of contagion in the aftermath of the crisis, since the data show 41 new causality patterns, which appeared for the first time in the crisis period, and the intensification of causality recorded in 70% of the cases.

Broto and Pérez-Quirós (2015) decompose the sovereign CDS spreads of ten OECD economies into three components: a common factor, a second factor driven by European peripheral countries and an idiosyncratic component. According to their findings, since the onset of the sovereign debt crisis, contagion has played a non-negligible role in the European peripheral countries, which confirms the existence of significant financial linkages between these economies.

Fourie and Botha (2015) establish that sovereign rating contagion existed between euro area countries during the two recent crises: the Lehman and sovereign debt crisis. Their results suggest that during both crises, as opposite to tranquil periods, there is a higher vulnerability to shocks and a higher degree of connection. Moreover, they find that the sovereign debt crisis experienced a more pronounced degree of contagion than the Lehman crisis period did, specially between peripheral countries.

Contrarily, Beirne and Fratzscher (2013) suggest that a deterioration in countries' fundamentals and fundamentals contagion (identified as a sharp rise in the sensitivity of financial markets to fundamentals) are the main explanations for the rise in sovereign yield spreads and CDS spreads during the crisis, not only for euro area countries but globally. By contrast, regional spillovers and contagion have been less important, including for euro area countries.

Identically, Pragidis et al. (2015) findings do not confirm any contagious effects stemming from the 10-year Greek sovereign bond to bond yields of France, Germany, Greece, Ireland, Italy, Portugal, and Spain from July 2006-July 2012.

Blatt, Candelon and Manner (2015) show that immediate contagion from Greece does not take place, but the dynamic spillovers are shown to increase after controlling for breaks in the different model parameters. For other countries, they find evidence of both contagion and flight-to-quality mechanisms.

Finally, Beirne and Fratzscher (2013) conciliates contagion effects with fundamentals. The paper analyzes the drivers of sovereign risk for 31 advanced and emerging economies during the European sovereign debt crisis. They show that a deterioration in countries' fundamentals and fundamentals contagion, defined as a sharp rise in the sensitivity of financial markets to fundamentals, are the main explanations for the rise in sovereign yield spreads and CDS spreads during the crisis, not only for euro area countries but globally. By contrast, regional spillovers and contagion have been less important, including for euro area countries. The paper also finds evidence for herding contagion, that is, sharp and simultaneous increases in sovereign yields across countries, but this contagion has been concentrated in time and among a few markets.

Sovereign CDS and bond spreads Calice, Chen and Williams (2013) examine liquidity spillovers between sovereign bond and CDS spreads. The idea is that the manipulation of the CDS market by speculative investors may have played a crucial role in exacerbating the liquidity dry up in the market for Greek, Irish, Portuguese and Spanish sovereign debt, driving upwards the sovereign bond yields on these countries during the first half of 2010. The authors find that (i) explosive trends did appear during the sovereign crisis and that the CDS market does appear to have been a driver in most cases; (ii) a positive and significant lagged transmission from the liquidity spread of the CDS market (*proxied* by the bid-ask spread) to the credit spread in the bond market; (iii) the sovereign CDS market has a substantial time-varying influence on sovereign bond credit spreads over the 2007-2010 period .

Fontana and Scheicher (2010) identify the determinants of the bond and CDS spreads of ten Euro area countries in the period 2006-10 and explain which factors drive the differences in pricing between the two markets. The authors suggest that since September 2008, (i) the repricing of sovereign credit risk in the CDS and sovereign bonds markets seems mostly due to common factors; (ii) CDS spreads have on average exceeded bond spreads, which may have been due to "flight to liquidity" effects and limits to arbitrage; (iii) market integration for bonds and CDS varies across countries: in half of the sample countries, price discovery takes place in the CDS market and in the other half, price discovery is observed in the bond market.

Similarly, Delatte, Gex and López-Villavicencio (2012) analyze the influence of CDS premia on underlying bond spreads for peripheral countries and five core European countries during the 2008-10 period. They find that: (i) linearity tests clearly reject the null hypothesis of a linear transmission mechanisms between the bond and the CDS markets; (ii) market distress alters the mutual influence and (iii) the higher the distress, the more the CDS market dominates the information transmission between CDS and bond markets.

Sovereign and banks CDS spreads Another strand of empirical research focuses on the dynamics between the sovereign and bank CDS market. Within the context of the European sovereign debt crisis, this literature looks for differences before and after government interventions, by comparing sovereign CDS of EU countries and the CDS of their banks.

Within this line, Alter and Schuler (2012), using a sample from June 2007 until May 2010, find that before the government rescue interventions, there was contagion from the banking sector to the sovereign CDS market, whereas after the interventions sovereign CDS spreads largely determine the price of banks' CDS. The authors also observe a short-term impact of the financial sector on sovereign CDS spreads, but an insignificant long run impact.

Identically, Dieckmann and Plank (2012), covering the period January 2007 to April 2010, also find evidence for a private-to-public risk transfer in the countries with government interventions. Moreover, the authors argue that this transfer is larger for the EMU countries, that are more sensitive to the health of the financial system, than non-EMU states.

Similary, Ejsing and Lemke (2011) examine the period from January 2008 to June 2009 and document that the government rescue packages led to a decrease in the CDS spreads of the banking sector at the cost of the increase in the price of sovereign CDSs. Furthermore, the bailout schemes made sovereign CDSs even more sensitive to any future shocks.

Likewise, for the period of 2007-2011, Acharya, Drechsler and Schnabl (2014) find that the announcement of financial sector bailouts was associated with an immediate widening of sovereign CDS spreads and narrowing of bank CDS spreads. However, in the post-bailout period there emerged a significant co-movement between bank and sovereign CDS, which is consistent with an effect of the quality of sovereign guarantees on bank credit risk.

While a consensus is forming around the idea that contagion may be an important determinant of the increase in sovereign risk premia in some countries, di Cesare et al. (2012) note that there still lacks a theoretical framework to understand contagion. In fact, they find that for several countries the spread has increased to levels that are well above those that could be justified on the basis of fiscal and macroeconomic fundamentals, possibly due to a perceived risk of a break up of the euro area.

2.2.2 Fundamentals and expectations

Another strand of the recent empirical literature focuses on the relationship between the sovereign default risk and economic fundamentals and expectations. In order to analyze the role of market sentiments in euro area sovereign debt crisis, Bruneau, Delatte and Fouquau (2014) estimate the probability of default (*proxied* by the sovereign debt spread) using panel data of five European peripheral countries from January 2006 to September 2011. Their results suggest that i) both the fundamentals and "animal spirits" ignited the European sovereign crisis; ii) the sovereign Credit Default Swap (CDS) market, the rating agencies and the CDS of the banking sector have played dominant roles in driving market sentiments. The results suggest the existence of self-fulfilling mechanisms during the European crisis, such that the progressive deterioration of the market sentiment about peripheral sovereigns has been validated by an *ex-post* increase in these countries' spreads.

Similarly, de Grauwe and Ji (2013) test for self-fulfilling sentiments in the European sovereign debt crisis. They estimate the probability of default using panel data on macroeconomic explanatory variables for 3 peripheral euro area countries and 14 "stand alone" countries. They find evidence that a significant part of the increase in the spreads of the peripheral euro area countries during 2010-11 was more related to negative investors' expectations, than to the deterioration of debt and fiscal positions, thus supporting the existence of a self-fulfilling crisis. They also find evidence that belonging to the EMU aggravated the process. In fact, because members of a monetary union cannot issue their money, they are unable to guarantee debt repayment and, as a result, such countries are more vulnerable to negative market sentiments that, in a self-fulfilling way, can create a liquidity crisis.

Aizenman, Hutchison and Jinjarak (2013) compare the market pricing of CDSs in the euro area and in peripheral euro area countries with the pricing of risk in countries with similar fundamentals. They find evidence that in 2010 the CDSs of the peripheral countries are priced much higher than CDSs of other similar countries. As a possible interpretation the authors suggest negative expectations of the market about the future fundamentals of peripheral countries and their exchange rate inflexibility.

Santos (2011) also seeks to understand whether quotes in the CDS market are related to fundamentals. Different methodologies are used, which overall, support the conclusion that the domestic savings rate is lenders' main concern. Economies with worse saving habits are penalized both in the CDS and sovereign bonds markets. Notwithstanding, for countries on the top quantiles of the implied default probabilities, higher public and external debt increase the likelihood of higher insurance premium in the derivatives market. Considering that Portugal had the lowest net savings rate in the EU27 in 2008, the author suggests that public policies that fail to take savings into proper account shall always be deemed to fail.

2.2.3 Empirical methods

The sovereign debt crises empirical literature is reviewed by Tomz and Wright (2013). This paper surveys emphasizis parallel developments by theorists and recommend steps to improve the correspondence between theory and data. In the context of the euro area crisis, the empirical research is particularly extensive, for this reason the literature is summarized in Appendix B.1, while not claiming to be fully comprehensive. Table B.1 summarizes the most recent empirical work on the determinants of the government bond yield spreads in the euro area and identifies the econometric procedures that have been used. This review is an updated and extended version of Giordano, Linciano and Soccorso (2012).

The focus herein is on single-country, even though the vast majority of empirical literature uses panel data. This approach allows to concentrate attention on Portugal's specific issues (even though not necessarily disconnected from other countries) and to focus on a relevant sample period. In contrast, the advantage of the panel data studies is that they improve the statistical inference by expanding the cross-sectional dimension of the data. However, their disadvantage is the implicit assumption that sovereign spreads in all countries respond to changes in economic fundamentals similarly (Poghosyan, 2014).

The empirical strategy used in this work is borrowed from the currency crises literature, particularly from Jeanne and Masson (2000) seminal work in applying a Markov switching regime (MSR) model with constant transition probabilities (CTP) to a crisis situation. Afterwards, several authors have been using regime-switching approaches that define a crisis endogenously, that is, that simultaneously identify the crisis and the determinants of switching from a normal to a crisis regime. Among several advantages attributed to MSR models, Hamilton (1994), points out that this approach is appropriate to model time series that have suffered a change in regime that may again occur in the future.

Jeanne and Masson (2000) framework has also been the adopted by Tamgac (2011) to assess the role of expectations in the 1994 and 2000-2001 Turkish currency crisis. Lopes and Nunes (2012) extend the seminal paper by considering time-varying transition probabilities (TVTP) in the context of currency crises and contagion, since the authors allow for shifting correlations that capture contagion effects between Portuguese escudo and Spanish peseta exchange rates while in the European Monetary System 1992 crisis. In the context of private bonds, Chun, Dionne and François (2014) apply a MSR model to private credit spreads, concluding that the introduction of two regimes to explain changes in credit spread enhances the explanatory power of the well-known theoretical determinants. Moreover, they find that regime based models cannot be improved by simply adding an endogenously dummy variable (the one characterizing the regime), as the dummy variable for the regimes is never statistically significant. Therefore, they conclude that the marginal contribution of the regime model does not stem from adding the prevailing cycle as an explanatory variable but from accounting for changes in explanatory variable effects on credit spreads across regime stances.

The next section presents the model that justifies the use of MSR in detecting self-fulfilling crisis based on shifts in expectations, but where fundamentals also play an important role.

2.3 The model

The theoretical model follows Jeanne and Masson (2000), but adapted to the context of a sovereign debt crisis (identified as an increase in the probability of default), rather than a currency crisis (increase in the probability of a devaluation). The model is a reduced form representation of the governments' decision on whether to default on previous debt and has the advantage of straightforwardly being brought to the data.

2.3.1 Assumptions and setup

In each period, the government may decide to repay previous debt or to exercise an escape clause and default, depending on the net benefits of repaying: if the latter is negative, then the government defaults. In such a case, the (absolute) value of the net benefit of repaying may be interpreted as the benefit of defaulting, which occurs due to the reduction of interest and principal payments. Analogously to Jeanne and Masson (2000), the net benefit of repaying at time t is written in a reduced form as:

$$B(\phi_t, \pi_t) \tag{2.3.1}$$

where ϕ_t stands for the *exogenous* economic fundamentals and π_t is the *endogenous* probability of default. This continuously differentiable function is increasing in the fundamentals ($B_1 > 0$), but decreasing in the probability of default ($B_2 < 0$). In fact, a higher expectation of default increases the risk premium on sovereign debt, thus increasing interest rates, which in turn makes the repayment more costly, thus decreasing the net benefit of government not defaulting. That is, higher default expectations increase the government's benefit from defaulting.

It is also assumed that, for each level of default probability, there is a level of fundamentals that makes the government indifferent between repaying or defaulting, $\forall \pi, \exists \phi, B(\phi, \pi) = 0$.

Regarding the exogenous fundamental variable, ϕ, it reflects the economic factors that influence the government's decision on whether to default. As assumed by Jeanne and Masson (2000), it is a stochastic variable governed by a Markov process with the following *transition cumulative distribution function*:

$$F(\phi, \phi') = Prob\left[\phi_{t+1} < \phi' | \phi_t = \phi\right] \tag{2.3.2}$$

To ensure that the fundamentals are not negatively correlated, $F_1 \leq 0$, that is, an increase in the current value of the fundamentals shifts the conditional cumulative distribution function of the next period fundamental in the same direction.

In what concerns the endogenous probability of default, π, it is an estimate formed by a continuum of atomistic creditors $i \in [0,1]$, and corresponds to the average estimate at t of the probability of default in period $t+1$: $\pi_t = \int_0^1 \pi_t^i di$. The expectations of the creditors depend on their own beliefs about the future level of fundamentals and also on their own beliefs about the beliefs of other creditors. Each atomistic creditor, i, will form its expectations by considering that π^i is the probability that the net benefit of repaying will be negative in the next period, conditional on the current level of the fundamental variable:

$$\pi_t^i = Prob\left[B(\phi_{t+1}, \pi_{t+1}) < 0 | \phi_t\right] \tag{2.3.3}$$

Jeanne and Masson (2000) emphasize the importance of this equation in the self-fulfilling features of the model: rational agents' expectations are forward looking and depend on its beliefs (i) about future fundamentals and (ii) about other agents' future beliefs. This is because each agent realizes that the expectations of other creditors will affect next period's cost of not defaulting and thus the objective probability of defaulting.

Considering that creditors are rational and that they share the same information set, then the default probability of a representative agents is:

$$\pi_t = Prob\left[B(\phi_{t+1}, \pi_{t+1}) < 0 | \phi_t\right] \tag{2.3.4}$$

The characterization of the equilibrium defaulting expectations corresponds to determining the stochastic processes π that are a solution to Eq. (2.3.4) for a given exogenous process of the fundamentals, ϕ.

2.3.2 Fundamental-based equilibria

In this type of equilibrium, the fundamental variable, ϕ determines the state of the economy. To characterize this equilibrium, Jeanne and Masson (2000) define two extra parameters: ϕ^{*e} and ϕ^*, which correspond, respectively, to the creditors' threshold for the fundamentals below which they believe the government will default and to the government's threshold for the fundamentals below which the optimal decision is to default.

Taking these cut-off levels into consideration, on the one hand, each creditor estimates the probability of default as the probability of ϕ being below ϕ^{*e}, conditional on the current value of ϕ. Using Eq. (2.3.2):

$$\pi_t = F(\phi, \phi^{*e}) = Prob\left[\phi_{t+1} < \phi^{*e} | \phi_t = \phi\right] \tag{2.3.5}$$

On the other hand, given the creditors' expectations, the government's problem is to determine the threshold ϕ^*, such that if the fundamentals are below the threshold, $\phi < \phi^*$, then the net benefit of repaying is negative and the government defaults, or above the threshold, where the net benefit is positive and the government repays:

$$\phi \mapsto \begin{cases} B\left(\phi, F(\phi, \phi^{*e})\right) < 0 & \text{if } \phi < \phi^* \\ B\left(\phi, F(\phi, \phi^{*e})\right) > 0 & \text{if } \phi > \phi^* \end{cases} \tag{2.3.6}$$

Because $B_1(\phi_t, \pi_t) > 0$, then there is a unique level of ϕ^* at which the net benefit is zero, that is, there is a unique equilibrium for each level of default expectations. The threshold level is denoted by $H(\phi^{*e})$ and Jeanne and Masson (2000) prove its existence. Moreover, since in a rational expectations equilibrium the agents beliefs must be true, then ϕ^* must be a fixed point of function $H(\cdot)$:

$$\phi^* = H(\phi^*) \tag{2.3.7}$$

This equation indicates that the level of fundamentals under which the creditors expect the government to default is the same as the threshold for which the government actually defaults.

The multiplicity of equilibria may arise because $H_1(\phi) > 0$, meaning that there is *strategic complementarity* between market expectations on the government's default rule and the government's actual decision rule. When creditors estimate a higher threshold, which translates into higher default expectations, then overall there is a decrease in the net benefit of not defaulting, which induces the government to revise upward its own threshold. In other words, the multiplicity of equilibria arises because for each state of default expectations there is level of fundamentals that triggers the default.

2.3.3 Sunspot equilibria

A sunspot equilibria is formalized by Jeanne and Masson (2000) as follows:

- There are n states $s = 1, 2, \cdots, n$ in the economy, each one corresponding to a different level of fundamentals that triggers a default, ϕ_s^*;
- At time t, if the state is s, then the government defaults if $\phi_t < \phi_s^*$.
- At time t, there are many critical thresholds as there are possible states. The thresholds may be ranked, $\phi_1^* < \phi_2^* < \cdots < \phi_n^*$, such that if the government decides to default at state s, so it will for any state higher than s.
- The transition across states follows a Markov process independent of the fundamentals and characterized by the matrix $\Theta = [\theta(i,j)]_{1 \leq i,j \leq n}$.

In a sunspot equilibrium, the default probability, previously defined in Eq. (2.3.5), depends not only on the current and future level of fundamentals, but also on the state. So, the probability of default is the sum of the default probabilities, $F(\phi_t, \phi_s^*)$, weighted by the probability to be in one of the n states, Θ, given the current state:

$$\pi_t = \sum_{s=1}^{n} \theta(s_t, s) \cdot F(\phi_t, \phi_s^*) \tag{2.3.8}$$

So, the government's net benefit does not only depends on the fundamentals' threshold, but also on the state and on the transition probability between states. Again, the optimal decision depends on whether the fundamentals are below the threshold, in which case the government defaults, or above the threshold, where the government repays. The government's problem in state s is to find the fundamentals level, ϕ_s^*, that make the net benefit zero:

$$\phi \mapsto \sum_{s'=1}^{n} B\left(\phi, \theta(s, s') \cdot F(\phi, \phi_{s'}^*)\right) = 0 \tag{2.3.9}$$

Analogously to the fundamental-based equilibria, a sunspot equilibrium is characterized by a vector of thresholds $(\phi_1^*, \phi_2^*, \cdots, \phi_n^*)'$ that satisfy the fixed points equations:

$$\forall s = 1, \cdots n, \phi_s^* = H_s(\phi_1^*, \phi_2^*, \cdots, \phi_n^*) = 0 \tag{2.3.10}$$

Jeanne and Masson (2000) prove the existence of these sunspot equilibria, for any number of states, if and only if there are multiple solutions to Eq. (2.3.7), that is, if there are multiple fundamental-based equilibria. Moreover, they also show that for a sunspot equilibria to exist, the probability of a decrease in the fundamental, $F(\phi, \phi)$, must be increasing with the fundamental, ϕ, at least over some range.

Note that the fundamental-base equilibria constitutes a degenerate case of the sunspot ones, where the transition matrix is the identity matrix, such that the economy never jumps between states and always remains in the initial state.

2.3.4 Multiple equilibria and Markov-switching regimes

In order to empirically test the model, Jeanne and Masson (2000) demonstrate that Markov-switching regimes model, where the regime switches correspond to jumps between multiple equilibria, can be interpreted as a linearized reduced form of the structural model with sunspots.

To bring the model to the data, the probability of default, Eq. (2.3.8), needs to be linearized. As a first step, the fundamental variable, ϕ is assumed to be a linear index aggregating the relevant macroeconomic variables plus a shock:

$$\phi_t = \alpha' \cdot x_t + \eta_t \tag{2.3.11}$$

where $\alpha = (\alpha_1, \cdots, \alpha_n)'$ is a vector of coefficients, $x_t = (x_{1t}, \cdots, x_{Kt})'$ is a vector of the relevant macroeconomic variables and η_t is an i.i.d. stochastic term which captures all other exogenous determinants of the government's decision.

As in Jeanne and Masson (2000), assuming that the fluctuations of the fundamentals are of limited magnitude at each state, then linearizing the default probability around the mean value yields:

$$\pi_t = \gamma_{s_t} + \beta' \cdot x_t + \nu_t \tag{2.3.12}$$

where γ_{s_t} is a constant that depends on the state, $\beta = (\beta_1, \cdots, \beta_K)'$ is a vector of coefficients and ν_t is an i.i.d. shock, all being written as functions of the structural parameters of the model.

Eq. (2.3.12) may be interpreted as a Markov-switching model with n regimes, where regime shifts affect the probability of default by changing the constant term, γ_{s_t} but do not affect the coefficients β. A jump to a state of higher expectations of default makes the default more likely and increases the constant term γ.

Hamilton (1994, p. 677-703) suggests that a Markov switching model is appropriate to model time series that have suffered a change in regime that may again occur in the future. Moreover, this framework is very flexible because it is consistent with several outcomes, even including a permanent change in regime or a short-lived change. In the permanent change case, even though different procedures could be used, the advantage of using a Markov switching model is that it allows to generate a meaningful forecast prior to the change.

To sum up, the Markov-switching regimes framework allows the theory of self-fulfilling expectations to be brought to the data with switches across regimes corresponding to jumps between different equilibria. As stated by Jeanne and Masson (2000), the model allowing for sunspots, performs better than a purely fundamental-based model, by improving the relationship between the economic fundamentals and expectations.

2.4 Empirical strategy

The empirical strategy is the following. Firstly, the dependent and possible independent variables are characterized based on existing studies. The period covered starts in January 2000, after the Euro implementation in Portugal (January 1999), and ends in December 2014, corresponding to the most recent available data for all variables, so it covers 180 months. Secondly, the most relevant variables are selected and their stationarity is analyzed. Finally, a multiple equilibria via self-fulfilling expectations model is estimated using a MSR, considering constant and regime coefficients, both with constant and time-varying transition probabilities.

2.4.1 The dependent variable

As previously mentioned, the dependent variable is the sovereign probability of default. The *proxy* for this variable is the sovereign bonds spreads, measured in basis points differences between German and Portuguese 10-year sovereign bond yield (Bruneau, Delatte and Fouquau, 2014; de Grauwe and Ji, 2013). In the currency crisis literature, the analogue to the default probability is the devaluation probability, usually approximated by the interest differential between two currency instruments, as in Jeanne and Masson (2000) seminal paper.

Fig. 2.2 plots German and Portuguese yields, as well as its difference, that is, the spread of Portuguese bonds.

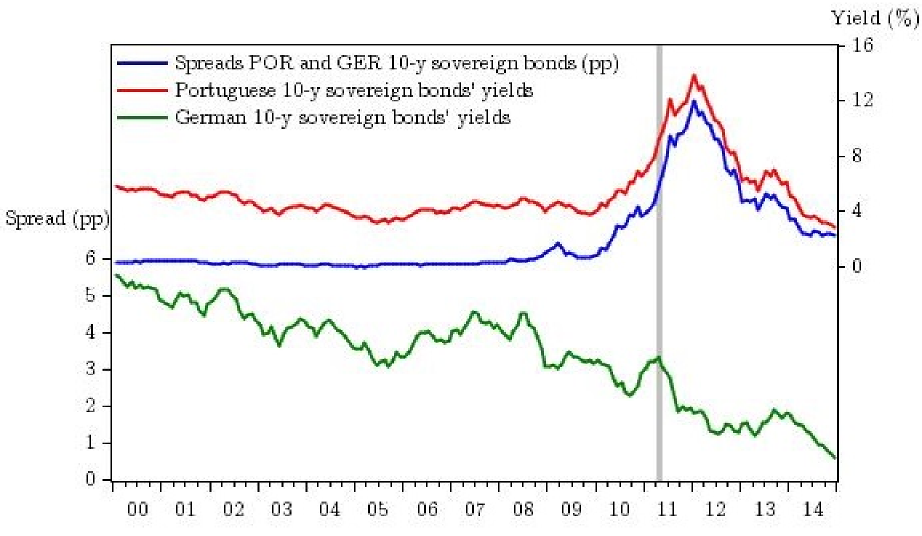
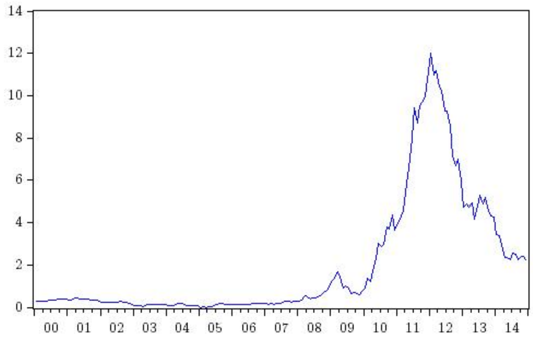
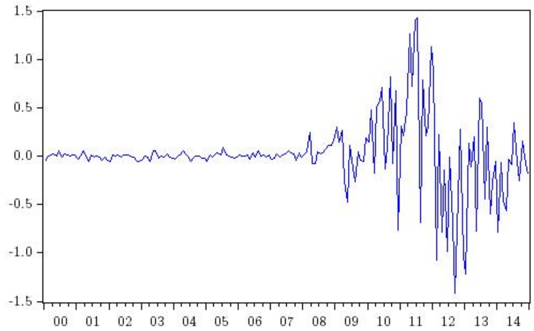


Figure 2.2: Spreads 10-year government bond yields
Source: Eurostat (2015*a*)

The large increase in the spreads (again plotted in Figure 2.3a) is accompanied by an increasing variability in its first differences during the crisis. This fact is visible in Figure 2.3b, that plots the absolute change in the spreads.



(a) Spreads



(b) Differenced spreads

Figure 2.3: Portuguese sovereign debt spreads and differenced spreads
Source: Eurostat (2015*a*) and author's calculations

As an alternative to the use of the yield differential as the dependent variable, several papers use the CDS spreads. As defined by the Deutch Research Bank (2015), CDS insures against losses stemming from a credit event. In the context of sovereign CDS, the contract protects against the default of the issuing sovereign. The premium (spread) which the protection buyer (e.g. a bank) pays to the protection seller (e.g. an insurance company) is determined by market forces and depends on the expected default risk of the respective country. In this context, CDS spreads act as an indicator of the market's current perception of sovereign risk, besides other factors such as market liquidity, counterparty risk and the global financial environment, in particular US interest rates and global risk appetite.

The relation between CDS spreads and the probability of default is operationalized by Deutch Research Bank (2015), which propose that the probability of default (π) can be estimated as a function of the CDS spread and the haircut fraction $1 - \theta$. In particular, for a 1-year CDS: $\pi = \frac{Spread}{1-\theta}$. However, Blundell-Wignall and Slovik (2011) emphasize the inconvenience of using a fixed rate of recovery, when this changes over time and over countries.

Badaoui, Cathcart and El-Jahel (2013) findings favor the use of bonds spreads, instead of CDS spreads. Their results reveal that the default risk represents on average 73% of the bond spreads, liquidity 26.86% and correlation risk 0.0014%. On the CDS side they find that, on average, default risk accounts only for 55.6% of the spreads, liquidity for 44.32% and correlation risk for 0.043%.

Moreover, the CDS Portuguese sovereign bonds market depth is low. Fontana and Scheicher (2010) notices that Portuguese net open CDS amounts to around 7% of its outstanding sovereign debt. Furthermore, although CDS market information indicates recent growing volumes and active trading, potentially variable liquidity is a major caveat in any price analysis.

For the reasons indicated, sovereign bond spreads potentially represent a better *proxy* for sovereign default probability.

2.4.2 Fundamental variables

The choice of the economic fundamentals constitutes a key aspect of the estimation and relies on the empirical literature on the determinants of spreads of government bonds. Since several studies find that different variables have different explanatory power for different countries and given the vast number of variables considered in the literature, the variables are grouped into the following six categories:

i) debt size sustainability

ii) external position

iii) competitiveness

iv) capacity to service the debt

v) international risk aversion

vi) liquidity risk.

Although recognizing that some of the variables (yet to be identified) could have been included in a different category, the option to categorize each variable mitigates colinearity issues, since at most one variable of each category is included, and allows for a sharper search of the most significant economic fundamentals.

A detailed description of the data set is provided in Appendix B.2, including the sources and procedures used to transform low to high frequency variables. Subsequently, these variables are described and their pertinence is discussed.

Debt sustainability

This group includes variables related to a country's ability to service all accumulated government debt at any point in time. As such, this group includes government debt as a share of GDP (Bruneau, Delatte and Fouquau, 2014; Fontana and Scheicher, 2010), debt to tax revenues ratio (de Grauwe and Ji, 2013; Aizenman, Hutchison and Jinjarak, 2013) or government deficit to GDP ratio (Maltritz, 2012).

A higher *debt to GDP* ratio corresponds to lower debt sustainability because there is a higher burden of debt service and expectations of a default are likely to be higher.

Figure 2.4 plots the evolution of this variable, showing the large increase after 2008. Additionally, this figure allows to draw two considerations. First, since 2010 the surge in the spread seems to be disconnected from the dynamics of the debt. Second, especially since 2008, the relationship between debt-to-GDP ratio and the sovereign spread shows a non-linear pattern.

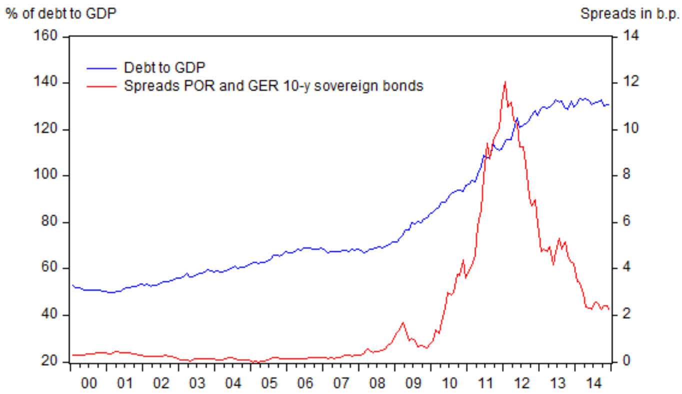


Figure 2.4: Debt as % of GDP
Source: Details in Appendix B.2.1

This non-linear relationship, usually implies that as the debt rises the impact on the spread of a one percentage point increase in the debt-to-GDP ratio also increases, is an empirical regularity, which generally holds for high debt countries. Indeed, as the public debt goes up the likelihood of a default grows too, thus leading investors in government bonds to demand a proportionally higher risk premium (Giordano, Linciano and Soccorso, 2012). As such, in the specification of the model, and as usual in the literature, debt to GDP is included in a non-linear form, implying that, as debt increases, investors realize that the default decision is closer, making them more sensitive to an increase in debt ratio (de Grauwe and Ji, 2013).

Notwithstanding, Figure 2.5 seems to indicate that, in the 2000-2014 period for the Portuguese case, this second order effect is negative, rather than positive.

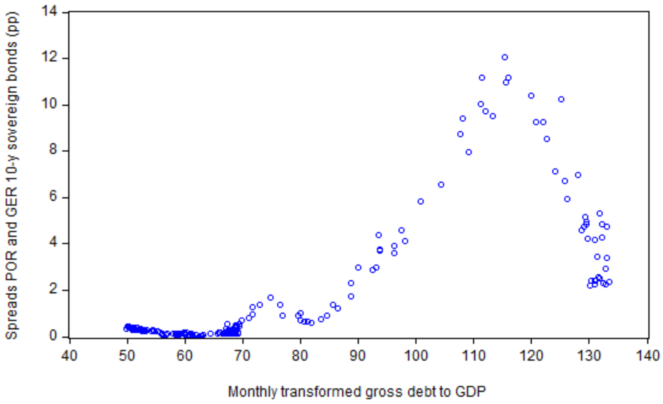


Figure 2.5: Scatter of spreads and debt as % of GDP
Source: Author's calculations

Concerning *debt-to-taxes*, designated as fiscal space, Aizenman, Hutchison and Jinjarak (2013) argue that this is a better measure of debt sustainability than the debt to GDP ratio, because a country may have a low debt to GDP ratio, yet find it difficult to service its debt because of a low capacity of raising taxes. Figure 2.6 plots this variable, showing that the ratio of government debt to tax revenues has been increasing, indicating that it would take an increasing number of years to generate the tax revenues necessary to service the debt (even considering that tax revenues were fully directed to debt repayment). The same Figure includes taxes as percentage of GDP showing that, on average, in periods where tax funding is decreasing, the debt ratio is increasing.

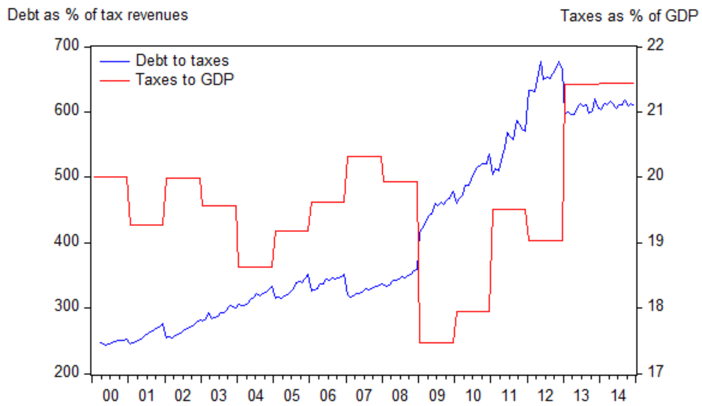


Figure 2.6: Monthly debt as % of tax revenues and yearly tax revenues as % of GDP
Source: Details in Appendix B.2.1

The same authors find that fiscal space (debt-to-taxes and deficit-to-taxes) is a statistically and economically important determinant of sovereign risk. Contrarily, de Grauwe and Ji (2013) found evidence that a large part of the surge in the spreads of the peripheral euro area countries during 2010-11 was disconnected from underlying increases in the debt to GDP ratios and fiscal space (except for Greece).

Regarding *deficit to GDP*, a higher government deficit implies a lower capacity to generate enough savings to repay the debt. In fact, Giavazzi and Pagano (1996) claim that reduction in the deficit can reduce interest rates either via a traditional crowding-in mechanism or by reducing the default premium on public debt if former policies were perceived as unsustainable and the fiscal correction restores government solvency.

The conventional arithmetic relation between deficit to GDP and a sustainable debt to GDP ratio, in the sense that this latter will not vary or will even reduce, emanates from the following condition (Pereira et al., 2012, p. 528):

$$\rho - g \geq (r - y) \cdot b \tag{2.4.1}$$

where ρ is the ratio of government revenues to GDP, g the ratio of government expenditure, excluding interest payments, to GDP; r corresponds to the real interest rate; y is the real GDP growth rate; and, b is the debt to GDP ratio. So, the ability of a government to honor its debt depends both on the size of the government's primary budget balance (surplus or deficit) before interest costs, and on the size of the debt burden relative to GDP, measured by the interest rate paid on that debt relative to the economy's growth rate. As such, Eq. 2.4.1 means that if the interest rate paid on public debt is higher than the economy's growth rate, the government debt will rise as a share of GDP unless governments run a primary budget surplus. The bigger the stock of debt, the bigger that surplus needs to be. This arithmetic suggests that countries with big primary deficits, big debt stocks and a big gap between interest rates and growth are the most vulnerable, thus possibly implying a higher probability of default.

Figure 2.7 plots both gross debt and general government deficit as percentage of GDP. The details regarding the series are provided in Appendix B.2.1. This figure shows that, on average, the variables move in opposite directions.

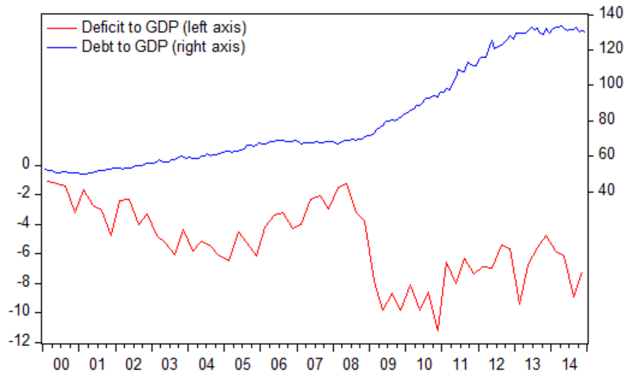


Figure 2.7: General government deficit as percentage of GDP
Source: Details in Appendix B.2.1

Finally, Table 2.1 reports the correlations between the variables included in this group, highlighting the fact that debt to GDP and debt to taxes evolve quit similarly and that debt and budget deficit move in opposite directions.

Table 2.1: Correlation between variables in *debt sustainability* group

Variable	Debt to GDP	Debt to taxes	Deficit to GDP
Debt to GDP	1.000		
	–		
Debt to taxes	0.969	1.000	
	(0.000)	–	
Deficit to GDP	−0.574	−0.613	1.000
	(0.000)	(0.000)	–

p-values in parentheses

External position

The external position is also included to explain the probability of default. The variables considered are the current account ratio to GDP (de Grauwe and Ji, 2013) and gross external debt to GDP (Aizenman, Hutchison and Jinjarak, 2013).

The *current account* of the balance of payments provides information on international trade in goods (traditionally the largest category), services, primary (factor) income and secondary income (transfer payments). A current account deficit shows that an economy is earning less from its international export transactions than spending abroad from import transactions with other economies, and is therefore a net debtor towards the rest of the world. As shown in Figure 2.8, the Portuguese economy as been exhibiting deficits through almost all the considered period. The net external borrowing to GDP (includes the current and capital) was also considered, but, as showed in the same Figure, its developments are not substantially different from the current account ones and thus was excluded.

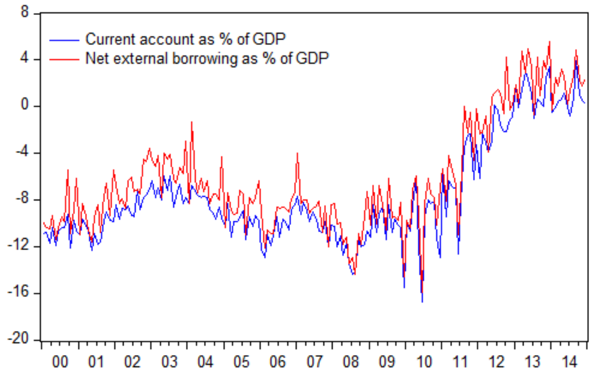


Figure 2.8: Current and capital account ratio to GDP
Source: Details in Appendix B.2.2

As mentioned by de Grauwe and Ji (2013), a larger current account deficit, which includes private and public residents, is likely to increase the government probability of default. In fact, if the imbalance stems from the private sector, it is likely to have a negative effect on economic activity, thus inducing a decline in government revenue and an increase in budget deficit. Alternatively, if the imbalance results from the public sector, an increase in foreign indebtedness leads to increases of the debt service and, as a consequence, of the default risk.

Portugal external imbalances have grown rapidly in the *pre*-crisis period, as seen by the fast current account deterioration since the adoption of the euro, while benefiting from exceptionally low interest rates and favorable external debt financing conditions.

Gross external debt corresponds to the outstanding liabilities side of a country's overall financial relations with foreign countries. A liability is considered to be a debt instrument when the holder has the unconditional right to receive in the future some payment(s) related to principal and/or interest (IMF, 2014). This variable is plotted in Figure 2.9.

Figure 2.9: Gross external debt ratio to GDP
Source: Details in Appendix B.2.2

Aizenman, Hutchison and Jinjarak (2013) find that during the crisis period (2008-10), an increase in the ratio of external debt to GDP had a positive impact on the probability of default. Similarly, Barrios et al. (2009) find that the impact of domestic factors on sovereign bond yield spreads increased significantly during the 2008 financial crisis, when international investors started to discriminate more between countries. In particular, the combination of high risk aversion and large current account deficits tend to magnify the incidence of deteriorated public finances on government bond yield spreads. Moreover, Mendoza and Yue (2012) present as an empirical regularity that defaulting countries on average have a high external debt as a share of GDP (and that this ratio gets higher when countries default).

In a cross section sample of 40 countries for 2008, Santos (2011) find that the *domestic savings rate* is lenders' main concern: economies with worse saving habits are penalized both in the CDS market and the sovereign bonds markets. Notwithstanding, for countries on the top quantiles of the implied default probabilities, public debt and external debt also play a significant role, increasing the likelihood of higher insurance premium in CDS market.

The correlations between the variables of this group are presented in Table 2.2 .

Table 2.2: Correlation between variables in *external position* group

Variable	Current account	Net external borrowing	Gross external debt
Current account	1.000		
	–		
Net external borrowing	0.963	1.000	
	(0.000)	–	
Gross external debt	0.527	0.489	1.000
	(0.000)	(0.000)	–

All variables are ratios to GDP; p-values in parentheses

Competitiveness

The real effective exchange rate (REER) and unit labour costs (ULC) are included as a *proxy* for competitiveness.

An increase in *REER* corresponds to a real appreciation of the currency and may induce a higher risk premium because a loss of international competitiveness may imply a deterioration of the country's future external position. Therefore an appreciation of the REER is likely to lead to an increase in the sovereign risk premium demanded by the investors (de Grauwe and Ji, 2013; Barrios et al., 2009).

Alternatively, Bruneau, Delatte and Fouquau (2014) consider that the *ULC* is a proxy for competitiveness, since it indicates the total labour cost per unit of output. Even though several critiques have been made to the appropriateness of ULC to measure competitiveness, e.g. Felipe and Kumar (2014), the discussion about the need to regain competitiveness in the euro area, in a context where nominal devaluation is not possible, fiscal rigidity has been imposed and the monetary policy is conducted by the ECB, suggests that the adjustment has to come through the labour market. Therefore, and even before the crisis, policy discussions have focused on analyses of ULC and the so-called internal devaluation or competitive disinflation: decrease in nominal wages or increase in productivity (Blanchard, 2007).

Figure 2.10 shows that these variables, on average, move in the same direction, confirmed by a significant correlation of 0.652.

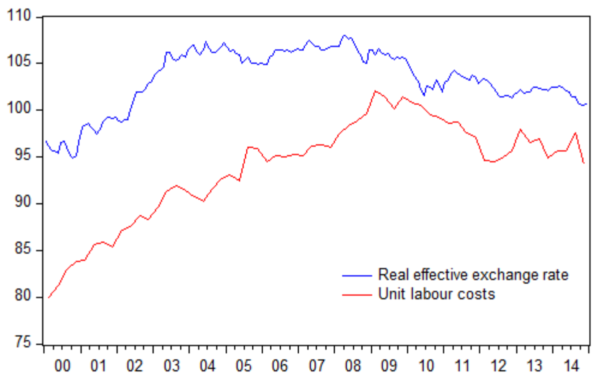


Figure 2.10: Real effective exchange rate and unit labour costs
Source: ECB (2015*a*), OECD (2015) and author's calculations

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Capacity to service the debt

The overall state of the economy is of crucial importance in determining the country's ability to meet its payment obligation. *Economic growth* is another fundamental variable that influences the probability of default by assessing the ability of the government to service the debt (de Grauwe and Ji, 2013). In fact, a higher economic growth implies a higher potential for tax collecting, which decreases the benefit of defaulting. By the same reasoning, an alternative specification uses *unemployment rate* as a proxy for the government's capacity to service debt (Bruneau, Delatte and Fouquau, 2014). Figure 2.11 plots both variables, which exhibit a correlation of -0.571.

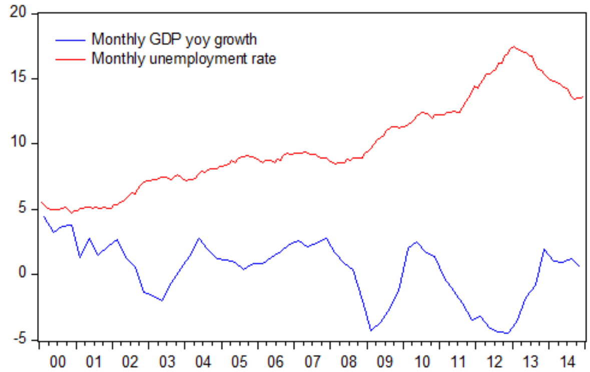
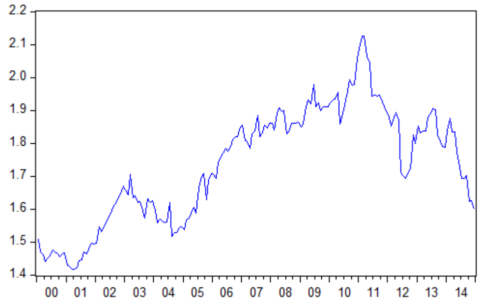


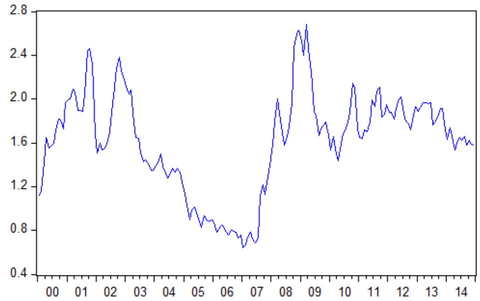
Figure 2.11: GDP year-on-year growth and unemployment rate
Source: Banco de Portugal (2015*g*,*b*), Eurostat (2015*b*) and author's calculations

Risk aversion and liquidity risk

In line with the literature, a *liquidity risk* proxy is included. This indicator measures the risk of having to sell or buy the asset in an illiquid market, at an unfair price and therefore bearing high transaction costs. The liquidity risk is usually measured either through bid-ask spreads or the size of the sovereign bond markets. In this paper, the latter measure is adopted. To capture the size of the government's bond market, we use the share of total outstanding Euro-denominated long-term government securities issued in the Euro-zone, as plotted in Figure 2.12a.



(a) Liquidity: share of total outstanding Euro-denominated long-term government securities issued in the Euro-zone
Source: ECB (2015*b*) and author's calculations



(b) International risk aversion: spread between sovereign and AAA corporate US long term bonds
Source: FRED (2015)

Figure 2.12: Liquidity and international risk aversion

The inclusion of a liquidity risk variable aims at capturing the possibility that the increase in the CDS spreads observed during the crisis period was mainly due to a surge in liquidity rather than to an increase in the default intensity, as suggested by Badaoui, Cathcart and El-Jahel (2013), and that liquidity effects are found to account for a sizable share of spreads' fluctuations, as found by Monfort and Renne (2014). In fact, during the crisis, several commentators expressed concern that manipulation of the CDS market by speculative investors was playing a crucial role in exacerbating the liquidity dry up in the market for Greek, Irish, Portuguese and Spanish sovereign debt (Calice, Chen and Williams, 2013).

As such, considering that a lack of liquidity increases the probability of default, then the expected sign for the coefficient of this variable is negative: higher liquidity implies lower default probability. Notwithstanding, the evidence brought by the empirical literature so far is controversial.

Finally, a *risk aversion* measure is also included. Following Bruneau, Delatte and Fouquau (2014) and Borgy et al. (2011), the international risk aversion is measured by the spread on sovereign and AAA corporate US long term bonds (Figure 2.12b). The inclusion of country risk measures in the specification may be seen as a problem, because the variables are unlikely to be exogenous in explaining the spread, however an international measure overcomes the problem. As pointed out by Borgy et al. (2011), principal component analysis regularly reveals that the first principal component (usually interpreted as time-varying risk aversion of international investors) accounts for more than 80% in the total variation of spreads. It is expected that a higher risk aversion, increases the risk premium.

Summary of fundamentals

Table 2.3 summarizes these variables, the expected signs for its coefficients and the effects they are capturing.

Table 2.3: Economic fundamentals variables

Variables	Description	Proxy
π	Difference between long-term sovereign bond yield and the risk-free rate - long-term German yield (p.p.)	Sovereign default probability
Debt (+)	Government gross debt ratio to GDP	Debt sustainability
Fiscal space (+)	Government gross debt ratio to tax revenues	
Deficit[a] (+)	General government deficit ratio to GDP	
Current account (-)	Current account balance ratio to GDP	External sustainability
Gross external debt (+)	Gross external debt ratio to GDP	
REER (+)	Real effective exchange rate	Competitiveness
ULC (+)	Unit Labor Cost % change over previous period	
GDP growth (-)	Economic growth	Capacity to service the debt
Unemployment (+)	Unemployment rate	
IRA (+)	Spread between US AAA corporate bonds and US 10-year sovereign bonds	International risk aversion
Liq (-)	Share of total outstanding Euro-denominated long-term government securities issued in the Euro zone	Liquidity

[a] In estimation results, the variable *deficit to GDP* corresponds to the symmetric of the government budget balance, thus a positive sign for its coefficient is expected.

Finally, the descriptive statistics in Table 2.4 show a significant heterogeneity across periods. In fact, volatility (as measured by the standard deviation) in the 2009-2014 period is significantly higher.

Table 2.4: Descriptive statistics

Variable	Mean	St. Dev.	Max.	Min.
Spread	1.983	2.952	12.030	0.000
2000-2008	0.234	0.160	0.950	0.000
2009-2014	4.607	3.210	12.030	0.580
Debt to GDP	80.179	28.021	133.539	49.849
2000-2008	60.440	6.970	71.666	49.849
2009-2014	109.787	20.606	133.539	71.602
Debt to taxes	405.953	137.866	675.501	243.471
2000-2008	302.431	36.265	359.673	243.471
2009-2014	561.237	71.988	675.506	416.835
Deficit to GDP	−5.234	2.463	−0.933	−11.200
2000-2008	−3.694	1.556	−0.933	−6.672
2009-2014	−7.544	1.618	−4.800	−11.200
Current account to GDP	−7.621	4.381	4.062	−16.803
2000-2008	−9.727	1.750	−5.946	−14.428
2009-2014	−4.463	5.188	4.062	−16.803
Gross external debt to GDP	189.821	37.697	240.465	103.928
2000-2008	164.481	26.550	204.375	103.928
2009-2014	227.830	8.872	240.465	204.005
REER	103.386	3.231	108.000	94.900
2000-2008	103.607	3.970	108.000	94.900
2009-2014	103.054	1.546	106.600	100.600
ULC	93.862	5.377	102.326	79.781
2000-2008	91.256	5.222	100.263	79.781
2009-2014	97.771	2.442	102.326	94.115
GDP growth	0.357	2.339	4.558	−4.672
2000-2008	1.383	1.579	4.558	−3.257
2009-2014	−1.183	2.454	2.819	−4.672
Unemployment	9.926	3.547	17.500	4.700
2000-2008	7.447	1.572	9.400	4.700
2009-2014	13.643	2.152	17.500	9.500
Liquidity	1.734	0.175	2.126	1.416
2000-2008	1.643	0.149	1.907	1.416
2009-2014	1.871	0.112	2.126	1.599
Risk aversion	1.613	0.462	2.680	0.640
2000-2008	1.464	0.515	2.630	0.640
2009-2014	1.836	0.235	2.680	1.440

The choice of the fundamental variables is a key aspect, as such the next section verifies the stationarity of the variables (including the independent variable) and presents results aiming at choosing the most significant variable of each group.

2.4.3 Preliminary analysis of the model

This subsection contains five parts. The first displays unit root tests, then a reduced set of variables is selected by OLS and cointegration tests are presented. Given the presence of a single cointegrating relationship, the fourth subsection shows the results of a Dynamic OLS and Fully Modified OLS. Finally, the last subsection includes comments on the results.

Stationarity analysis

To assess the stationarity of all variables, Table 2.5 presents the results of the usual ADF test (Dickey and Fuller, 1979), the Phillips and Perron (1988) Z_t test for a unit-root and the Kwiatkowski et al. (1992) test for stationarity.

Table 2.5: Results of unit-root and stationarity tests

Variable	I(d)	ADF[a]		PP[a]		KPSS[b]
		t-stat	*p-value*	Z_t*-stat*	*p-value*	*test-stat*
Spread	I(1)	−2.258	0.454	−1.783	0.709	0.158**
Δ Spread		−4.428***	0.003	−12.027***	0.000	0.099
Debt to GDP	I(1)	−1.725	0.736	−1.725	0.736	0.386***
Δ Debt to GDP		−14.891***	0.000	−14.829***	0.000	0.165
Debt to taxes	I(1)	−1.804	0.699	−1.847	0.678	0.340***
Δ Debt to taxes		−13.404***	0.000	−13.413***	0.000	0.110
Deficit to GDP	I(1)	−1.958	0.619	−2.408	0.374	0.081
Δ Deficit to GDP		−4.208	0.005***	−3.491**	0.043	0.048
CA to GDP	I(1)	−0.740	0.968	−3.316*	0.067	0.332***
Δ CA to GDP		−11.078***	0.000	−30.405***	0.000	0.093
External debt to GDP	I(1)	−1.519	0.820	−2.331	0.415	0.233***
Δ External debt to GDP		−5.403***	0.000	−2.518	0.319	0.045
REER	I(1)	−1.355	0.871	−1.356	0.870	0.381***
Δ REER		−10.865***	0.000	−10.651***	0.000	0.066
ULC	I(1)	−1.291	0.887	−1.048	0.934	0.391***
Δ ULC		−3.676**	0.027	−7.359***	0.000	0.042
GDP growth	I(0)	−3.163**	0.024	−2.641*	0.087	0.057
Unemployment	I(1)	−2.798	0.200	−1.966	0.616	0.144*
Δ Unemployment		−5.807**	0.000	−9.539***	0.000	0.102
Liquidity	I(1)	−0.708	0.97	−0.622	0.976	0.290***
Δ Liquidity		−14.226***	0.000	−14.252***	0.000	0.104
International risk aversion	I(1)	−2.726	0.227	−2.375	0.392	0.190**
Δ International risk aversion		−9.833***	0.000	−9.438***	0.000	0.062

Tests allow for a constant term and a trend regressor, except for GDP growth, that only includes constant.
[a] ADF and PP null hypothesis: Variable has a unit root.
[b] KPSS null hypothesis: Variable is stationary; critical values with constant: 0.739 / 0.463 / 0.347; critical values with constant and linear trend: 0.216 / 0.146 / 0.119 for 1, 5 and 10 percent.
***, ** and * denote significance at the 1, 5 and 10 percent level, respectively.

The results of both the ADF and PP tests point to the nonrejection of a unit-root in all of the variables, except GDP growth. For this variable, the ADF unit root with break test yields *t-stat*$= -6.394$, corresponding to a *p-value*< 0.01, which also rejects the null that growth has a unit root. These results are corroborated by KPSS test that leads to the rejection of stationarity for all series. Even though most of the variables are not stationary in levels, given the results of the tests applied to the differences, we take the evidence as pointing to the stationarity in first differences.

Cointegration analysis

After having established that unit root tests on sovereign spreads and their potential determinants cannot be rejected, we proceed to the verification of cointegration relationships. As pointed out by Engle and Granger (1987) a linear combination of two or more non-stationary series may be stationary. If such a stationary linear combination exists, the non-stationary time series are said to be cointegrated. The stationary linear combination is called the cointegrating equation and may be interpreted as a long-run equilibrium relationship among the variables.

Given the large set of fundamentals, and in order to select a reduced set of variables, linear equations are estimated using Ordinary Least Squares (OLS). All the combinations of independent variables in each group of fundamentals were successively tested and the final result is presented in Table 2.6 corresponding to the significant variables. The dependent variable is the spread.

Table 2.6: OLS estimates

	(1)		(2)	
Variables (expected sign)	Coefficients	Std. errors	Coefficients	Std. errors
Intercept	-26.680^{***}	(4.180)	-10.395^{***}	(3.715)
Debt to GDP (+)	1.174^{***}	(0.195)	0.436^{**}	(0.160)
Squared debt to GDP	-0.005^{***}	(0.001)	-0.002^{**}	(0.001)
Deficit to GDP (+)	-0.248^{***}	(0.090)		
Gross external debt to GDP (+)	-0.106^{***}	(0.021)		
GDP growth (−)	-0.288^{**}	(0.096)	-0.256^{**}	(0.135)
Liquidity (−)	-4.766^{**}	(2.076)	-5.973^{**}	(2.939)
Risk aversion (+)	1.267^{***}	(0.371)	1.018^{***}	(0.408)
BIC	3.376		4.027	
Log likelihood	-283.112		-346.842	
Durbin-Watson statistic	0.191		0.067	

Standard errors are corrected for serial correlation using Newey-West estimator.
***, ** and * denote significance at the 1, 5 and 10 percent level, respectively.

Table 2.6 shows that neither the external position nor competitiveness seem to have an impact on the probability of default. Moreover, the impact of *external debt* and *deficit* is not the expected (an issue further explored in the next section). For model selection, as suggested by Enders (2015, p. 149), we present the values of *Log likelihood* (a higher value is preferred) and Bayesian information criterion (BIC) (since it is a "loss of information" measure, a lower value is preferred). These statistics privilege Panel (1) over Panel (2). However, since in Panel (1) some of the signs are not theoretically understood and to keep a parsimonious model, we consider the model from Panel (2). In fact, the variables *debt to GDP*, *GDP growth*, *liquidity* and *risk aversion* are the ones that were consistently significant across the alternative specifications and whose coefficients more often present the expected sign.

At this point, there is a reduced set of four macroeconomic factors (debt to GDP, GDP growth, liquidity and international risk aversion) that may influence the probability of default. As such, also considering the independent variable, there are at most four cointegrating relationships. The cointegration tests follow the VAR-based methodology developed in Johansen (1995, 1991), allowing for linear trends in level data but only intercepts in the cointegrating equation. Table 2.7 contains the *Trace-test* and the *Eigenvalue-test* results. Both tests reject the hypothesis that there are no cointegrating relationships, but both tests fail to reject the hypothesis that there are at most one cointegrating relationship at the 5 percent level. As such, the conclusion is that there is one cointegrating relationship among the variables.

Table 2.7: Results of cointegration tests

Hypothesized No. of CE(s)	Eigenvalue	Rank Test (Trace)			Max-Eigenvalue Test		
		trace-stat	CV^a	*p-value*[b]	*max-eigen*	CV^a	*p-value*[b]
None***	0.217	89.352	69.819	0.001	43.346	33.877	0.003
At most 1	0.123	46.006	47.856	0.074	23.226	27.584	0.164

[a] 5 percent critical value
[b] MacKinnon-Haug-Michelis (1999) p-values
***, ** and * denote rejection at the 1, 5 and 10 percent level, respectively.

Estimates of fundamental-based model

Given the presence of one cointegration relation, the model is estimated in levels by the Dynamic OLS (DOLS) approach by Stock and Watson (1993) and also the Fully Modified OLS (FM-OLS) of Phillips and Hansen (1990). These methods are valid in the presence of one cointegrating relationship. In the context of finding the determinants of sovereign bonds spreads, these two procedures to deal with cointegration were also used by de Santis (2012) and Özmen and Yaşar (2015), respectively, applied to cross-sectional data.

As a robustness check, and given the possibility that the sample period is not very long, and therefore it might be difficult to reliably estimate the long-run relationships, the model is estimated in differences using OLS (where all explanatory variables are defined in differences, except for the GDP growth rate). Table 2.8 presents the results.

Table 2.8: OLS, DOLS and FMOLS estimates

Variables (expected sign)	(1) OLS	(2) DOLS [a]	(3) FMOLS	(4) Δ OLS [b]
Intercept	-10.395^{***}	-7.102^{*}	-9.865^{***}	-0.029
	(3.715)	(4.222)	(3.792)	(0.038)
Debt to GDP (+)	0.436^{**}	0.470^{**}	0.456^{***}	0.073^{*}
	(0.160)	(0.183)	(0.160)	(0.038)
Squared debt to GDP	-0.002^{**}	-0.002^{**}	-0.002^{**}	
	(0.001)	(0.001)	(0.001)	
GDP growth (−)	-0.256^{**}	-0.288^{***}	-0.257^{**}	0.027
	(0.135)	(0.103)	(0.123)	(0.022)
Liquidity (−)	-5.973^{**}	-8.387^{***}	-6.473^{*}	-1.788
	(2.939)	(2.773)	(3.006)	(1.379)
Risk aversion (+)	1.018^{***}	1.037^{**}	1.021^{*}	0.277^{*}
	(0.408)	(0.431)	(0.558)	(0.142)
Bayesian information criterion (BIC)	4.027	3.103	4.090	0.944
Log likelihood	-346.842	-256.035	-350.520	-71.496
Durbin-Watson statistic	0.067	0.407	0.067	1.720

Standard errors in parentheses and for Panels (1)-(3) are corrected for serial correlation using Newey-West estimator.
[a] Automatic leads and lags specification: lead=4 and lag=0 based on SIC criterion.
[b] Dependent variable: spread in first differences; independent variables: all in first differences except GDP growth.
***, ** and * denote significance at the 1, 5 and 10 percent level, respectively.

Table 2.8 shows that the estimates from all three methods in levels are considerably similar, both in the sign and significance of the coefficients. However, the summary statistics and the inspection of residuals (Figure B.6 in Appendix) favors the DOLS estimates. Considering the specification in differences, debt to GDP and risk aversion are the only variables that determine the changes in spreads.

Engle and Granger (1987) demonstrated that once a number of variables are found to be cointegrated, there always exists a corresponding error-correction representation which implies that changes in the dependent variable are a function of the level of disequilibrium in the cointegrating relationship (captured by the error correction term) as well as changes in other explanatory variable(s).

The ECM model with one cointegration equation can be represented as follows:

$$\Delta S_t = \sum_{j=1}^{k} \phi_j \cdot \Delta S_{t-j} + \sum_{j=1}^{m} \mathbf{A'}_j \cdot \Delta \mathbf{X}_{t-j} + \gamma \cdot \left(S_{t-1} - \mathbf{B'X}_{t-1}\right) + u_t \tag{2.4.2}$$

For a lag selection of $k = m = 4$, Table 2.9 presents the results for the long run coefficients ($\mathbf{B}$) and the error correction term γ. Table B.2 in Appendix B.3 depicts the results for the short-term relations.

Table 2.9: VECM estimates of cointegrating relation

Cointegrating equation ($S_{t-1} - \mathbf{B}'\mathbf{X}_{t-1}$)	Coefficients	St. errors
Spread(-1)	1.000	
Debt GDP(-1)	-0.427^{**}	(0.183)
Squared debt GDP(-1)	0.002^{**}	(0.001)
GDP growth(-1)	0.212^{*}	(0.135)
Liquidity(-1)	7.708^{***}	(3.268)
Risk aversion(-1)	−0.924	(0.601)
C	5.750	
Error correction term γ of Δ Spread	-0.051^{**}	(0.021)

VECM dependent variable: Δ spread.
***, ** and * denote significance at the 1, 5 and 10 percent level, respectively.

When the variables are cointegrated, then in the short term, deviations from the long-term equilibrium ($S_{t-1} - \mathbf{B}'\mathbf{X}_{t-1}$) will feed back on the changes in the dependent variable (ΔS_t) in order to force the movement towards the long-term equilibrium. If the dependent variable is driven directly by this long-term equilibrium error, then it is responding to this feedback. If not, it is responding only to short-term shocks to the stochastic environment. The significance tests of the differenced explanatory variables ($\Delta\mathbf{X}_{t-1}$), provided in Table B.2 of Appendix B.3, give an indication of the "short-term" effects, whereas the "long-term" causal relationship is implied through the significance of the lagged error-correction term (γ), which contains the long-term information since it is derived from the long-term cointegrating relationship. The coefficient of the lagged error-correction term, however, is a short-term adjustment coefficient and represents the proportion by which the long-term disequilibrium (or imbalance) in the dependent variable is being corrected in each short period[1] (Masih and Masih, 1996). In this case, the value −0.051 indicates that in the case the spreads are off the long-run equilibrium, then the spreads decreases to its long-run equilibrium level with about 5.1 b.p. of the adjustment taking place within the first month.

Overall, the estimates of a long-run relationship between the spreads and the explanatory variables are consistent (in magnitude, sign and significance) across all estimation procedures. Even in OLS, the estimates are corrected for autocorrelation in the residuals, using the Newey-West standard errors (Newey and West, 1987) to account for serial correlation in the errors, as such the t-statistic has the usual, normal asymptotic distribution, confirming the following results.

[1]Non significance of the lagged error-correction term affects the implied long-term relationship and may be a violation of theory. The non-significance of any of the 'differenced' variables which reflects only a short-term relationship, however, does not involve such violations because, theory typically has nothing to say about short-term relationships (Masih and Masih, 1996).

Preliminary discussion of results

Regarding *debt to GDP*, the estimates for its coefficients are significant in all four equations and, as expected, an increase in this ratio leads to an increase in spreads. However, the impact (significance) of this variable tends to decrease when explaining the changes in spreads rather than levels. The negative coefficient in the squared ratio informs that additional increases in debt ratio have smaller impact on spreads. This may be considered as evidence that as debt is increasingly higher, investors to not take into account further increases.

GDP growth has a negative impact on the probability of default, indicating that investors are sensitive to business cycle. Likewise, unemployment also has the expected effect on spreads. However, the latter was dropped from the model because it has a lower explanatory power than growth.

In what concerns *liquidity*, the negative coefficients in Table 2.8 suggest, as expected, that the lack of liquidity increases the probability of default. However, this impact is no longer perceivable in differences. Contrarily, the *international risk aversion* variable shows a significant and positive effect on the probability of default, both in levels and differences, indicating that more averse investors are expected to demand a higher premium.

Regarding the factors not included in the model, the results suggest that competitiveness (either *proxied* by REER or ULC) is a factor that does not affect the perception of the markets regarding the probability of default.

With reference to *deficit to GDP* in Panel (1) of Table 2.6, it has a (not expected) negative impact on spreads, indicating that a reduction in government deficit leads to an increase in debt spreads, that is, an increase in the probability of default. This inconsistency may be explained by the fact that the widening in spreads starting from 2009 coincided with the rescue packages and fiscal tightening measures. As suggested by Bruneau, Delatte and Fouquau (2014), it may also show the potential counter-effective impact of fiscal austerity. This perspective is corroborated by Jorra (2012) who explores empirically how the adoption of IMF programs affects sovereign risk over the medium term. The author finds that those programs significantly increase the probability of subsequent sovereign defaults by approximately 1.5-2 percentage points. Furthermore, IMF programs turn out to be especially detrimental to fiscal solvency when the Fund distributes its resources to countries whose economic fundamentals are already weak. When the variable is considered in the differences model (not reported here), although the expected sign is obtained, changes in deficit are not significant to explain changes in spreads. Since the purpose of the analysis is to verify the appropriateness of a regime switching framework, the variable deficit to GDP is eliminated to avoid unexpected signaling of crises periods. As shown in Panel (2) of Table 2.6, the elimination of this variable (as well as external debt) does not change significantly the coefficients of other variables.

Moreover, these results support the findings of de Santis (2012) and the view by Aizenman, Hutchison and Jinjarak (2013) who argue that the euro area spreads fluctuations are difficult to reconcile with the fiscal stance in 2010. The authors also point out that, in their case, the least square estimator of the coefficients on the budget balance is strongly biased to zero due to measurement error, given that the budget balance data are available only every four months.

The coefficient of *external debt ratio*, although significant, has a negative impact on spreads. This result is contrary to the expected since it indicates that a reduction the country's indebtedness increases in the probability of default. Again, the widening of the spreads occurred simultaneously with the largest decrease in gross external debt ratio in the period of analysis (2000-2014). Also, the results in differences (not reported here) corroborate that statistically there is a negative relation between changes in external debt and changes in spreads. Again, the option is to exclude this variable, which does not change the value or significance of the other estimated coefficients.

The next section describes and presents the results regime switching models. First, we present the results for constant transition probabilities (CTP) between regimes. Afterwards, we re-estimate the model considering that one of the fundamental variables may be governing the transition probability, that is, we consider time-varying transition probabilities (TVTP). Finally, considering the extension to the original model by Bruneau, Delatte and Fouquau (2014), the model allows for regime-varying coefficients (RVC) both with CTP and TVTP.

2.5 Model specification and estimation results

2.5.1 Constant transition probabilities

The first model to be estimated is Eq. 2.3.12, which is reproduced here for the sake of clarity:

$$\pi_t = \gamma_{s_t} + \beta' \cdot x_t + \nu_t$$

where γ_{s_t} is a constant that depends on the state and x_t are the fundamentals. As previously mentioned, regime shifts affect the probability of default by changing the constant term, γ_{s_t}, but do not affect the coefficients β.

Considering two regimes or two states (s_t), then the process followed by a time series is in regime 1 if $s_t = 1$ and the process is in regime 2 if $s_t = 2$. As such, there is also the need of a process to model the discrete-valued random variable s_t. In a Markov-switching framework, s_t evolves according to a Markov chain and the change in regime is governed by transition probabilities p_{ij}, that denote the probability that the regime switches from regime i to regime j. These transition probabilities need to be estimated, along with the coefficients of the time series model.

Under a CTP framework, the probability of the system being in state $s_t = j$ only depends on the value of s_{t-1}, that is, $p_{ij} = P(S_t = j|S_{t-1} = i),\ t = 1, ..., T$.

The probabilities p_{ij} are *conditional* probabilities (Enders, 2015, p. 464-5), such that if the system is in regime 2, then p_{22} is the conditional probability that the system stays in regime 2 and $p_{21} = 1 - p_{22}$ is the conditional probability that the system switches to regime 1. The *unconditional* probabilities of the system being in regime 1 or in regime 2 are:

$$p_1 = \frac{1 - p_{22}}{2 - p_{11} - p_{22}} \qquad\qquad p_2 = \frac{1 - p_{11}}{2 - p_{11} - p_{22}} \tag{2.5.1}$$

The null hypothesis of a single regime is tested by comparing the constant parameter of the two models. The results of the two regime estimation are presented in Table 2.10, where Panel (1) corresponds to the model in levels and Panel (2) in differences.

Table 2.10: MSR-CTP estimates

	(1)[a]		(2)[b]	
Variables (expected sign)	R1: Tranquil	R2: Crisis	R1: Tranquil	R2: Crisis
Intercept	-16.453^{***}	-8.596^{***}	-0.001	0.019
	(0.418)	(0.444)	(0.004)	(0.067)
Debt to GDP (+)	0.748^{***}		0.006	
	(0.109)		(0.007)	
Squared debt to GDP	-0.004^{***}			
	(0.001)			
GDP growth (−)	0.119		0.001	
	(0.101)		(0.002)	
Liquidity (−)	-9.291^{***}		0.090^{***}	
	(1.718)		(0.017)	
Risk aversion (+)	1.221^{***}		0.054	
	(0.350)		(0.132)	
Regime standard error	1.007^{***}	2.826^{***}	0.028^{***}	0.567^{***}
	(0.107)	(0.499)	(0.002)	(0.044)
BIC	3.353		-1.297	
Log likelihood	-273.217		142.050	
Durbin-Watson statistic	0.334		1.580	

[a] Dependent variable: spread; independent variables in levels.
[b] Dependent variable: Δ spread; independent variables in first differences, except GDP growth.
***, ** and * denote significance at the 1, 5 and 10 percent level, respectively.
Standard errors in parentheses.

These estimates correspond to the case where the change in expectations exhibits sunspot dynamics, uncorrelated to any information about the fundamentals. When comparing the standard errors of regime 1 and 2, in both specifications, the results show that the crisis regime is more volatile than the no-crisis regime, as expected.

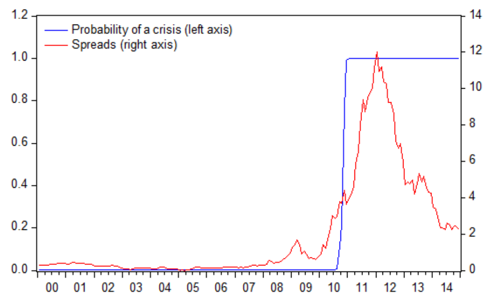
The estimate for the constant term in Panel (1), which captures the importance of agents' default expectations, is higher in the crisis regime and consistent with higher expectations of default in this regime. Moreover, the coefficients exhibit the expected sign, except for growth, which is not significant. Additionally, the estimates for the constant term indicate the presence of two regimes, apparently unrelated to the fundamentals. In the differences model, Panel (2), the constant term is not significant in either regime.

The estimated CTP are approximately the same in both specifications. Furthermore, the high estimated probability of staying in the same regime indicates that once the economy is in one state, it is very unlikely to change to the other state:

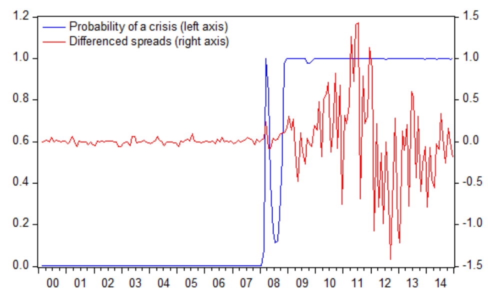
$$P^{(1)} = \begin{array}{r} \\ Tranquil \\ Crisis \end{array} \begin{array}{c} \begin{array}{cc} Tranquil & Crisis \end{array} \\ \begin{pmatrix} 0.997 & 0.003 \\ 0.001 & 0.999 \end{pmatrix} \end{array} \qquad P^{(2)} = \begin{array}{r} \\ Tranquil \\ Crisis \end{array} \begin{array}{c} \begin{array}{cc} Tranquil & Crisis \end{array} \\ \begin{pmatrix} 0.985 & 0.015 \\ 0.019 & 0.981 \end{pmatrix} \end{array}$$

The steady state probabilities of each regime are calculated using Eq.2.5.1. In the levels specifications, the steady state probability of being in the tranquil state is 25% and in the crisis regime is 75%. That is, over the long run, the fraction of time spent in the tranquil regime would equal 25% and the fraction of time spent in the crisis regime would equal 75%.

The smoothed probabilities estimates[2], indicating the months when the process was in the crisis regime, are shown in Figure 2.13.



(a) Model in levels: Panel (1)



(b) Model in differences: Panel (2)

Figure 2.13: Smoothed transition probabilities of being in a crisis regime: CTP

It can be seen that the specification in levels starts to switch from the tranquil to the crisis regime in August 2010. This regime change occurs after the creation of the European Financial Stability Facility in June 2010, a temporary crisis resolution mechanism created by the euro area Member States, that provided financial assistance to Ireland, Portugal and Greece. In November of the same year, EU and IMF provide 85 bn. rescue package for Ireland. The specification in differences signals the switch from tranquil to crisis in different periods, but, as previously stated, the model does not detect two significantly different regimes.

Both specifications indicate that by the end of 2014 there was a 100% probability of the economy being in the crisis regime, even though by this time the spreads of Portuguese bonds relatively to German bonds have reduced substantially. This result seems to indicate that by the end of 2014 expectations still dominate the fundamentals and the change in regime occurs sluggishly, as seen from the estimated probabilities of staying in the same regime.

According to Jeanne and Masson (2000) interpretation, the switch of the economy from low to high default expectations shows that the agents' increased expectations of a default had a significant contribution to the occurrence of a crisis, because for the same level of fundamentals a change in regime increases the probability of default.

Even though the summary statistics suggest an increase in the performance of this model when compared with single-regime estimates, in order to assess if the transitions probabilities are explained by the fundamentals, next the model includes time varying probabilities.

[2]The probability that the unobserved Markov chain for a Markov-switching model is in a particular regime in period t conditional on observing all sample information (Piger, 2014).

2.5.2 Time-varying transition probabilities

A MSR model with Time Varying Transition Probabilities (TVTP) involves the nonlinear parameterization of the transition probabilities in terms of a set of predetermined explanatory variables. In this specification the probability that $S_t = j$ depends not only on the value of S_{t-1} but also on a vector of exogenous variables G_t, where G_t may include elements of X_t. In this analysis, the probability of switching from regime i to regime j, denoted by $P_{ij}(t)$ evolves according to a logistic function of explanatory variables G_t (Diebold, Lee and Weinbach, 1994). So the probability of being in state 1 in period $t-1$ and staying in the same state in t is defined as:

$$p^{11}(t) \equiv P(S_t = 1|S_{t-1} = 1, G'_{t-1}; \alpha_1) \equiv \frac{exp(G'_{t-1} \cdot \alpha_1)}{1 + exp(G'_{t-1} \cdot \alpha_1)}. \tag{2.5.2}$$

Analogously, the probability of being in state 2 and in the next period staying in the same period is:

$$p^{22}(t) \equiv P(S_t = 2|S_{t-1} = 2, G'_{t-1}; \alpha_2) \equiv \frac{exp(G'_{t-1} \cdot \alpha_2)}{1 + exp(G'_{t-1} \cdot \alpha_2)}. \tag{2.5.3}$$

where $G_t = x_t$ is consistent with agents' expectations dependent on fundamentals and α_i are the parameters that govern the transition probabilities of staying in state i. Accordingly, the probability of being in state 1 and moving to state 2 in the next period is $P^{12}(t) = 1 - P^{11}(t)$ and similarly $P^{21}(t) = 1 - P^{22}(t)$.

The vector of economic variables, G_t, that may affect the transition probabilities consists of the same economic indicators which are the fundamental variables, x_t. As in Tamgac (2011), the model is firstly estimated such that the transition probabilities function includes all relevant explanatory variables previously identified (debt, GDP growth, liquidity and risk aversion). However, when these four variables are simultaneously included, neither of them is significant to explain the change in probabilities. Notwithstanding, individually the variables are significant. Moreover, Bruneau, Delatte and Fouquau (2014) suggest that to deal with endogeneity issues, the model should also be estimated introducing the lagged transition variables instead of the contemporaneous.

As such, to select the variable to include in the transition probability function, contemporaneous and lagged variables are included in the models in levels and in differences. Table B.3 and B.4 (in Appendix) report BIC, log-likelihood and DW-statistic of all the estimated models. As seen from those results, the summary statistics are very similar, whether the variable is included contemporaneously or lagged. Bruneau, Delatte and Fouquau (2014) take this as an evidence that the simultaneity bias does not influence the regime. Moreover, the estimates for the coefficients and standard errors in each model (not reported here) are also very similar across the various models.

This result indicates that no individual variable has a much higher explanatory power than the other variables to govern the probability of staying or changing the regime. Even so, the lowest BIC is obtained by including *international risk aversion* in the specification in levels and *deficit to GDP* in the model in differences. These results are reported in Table 2.11.

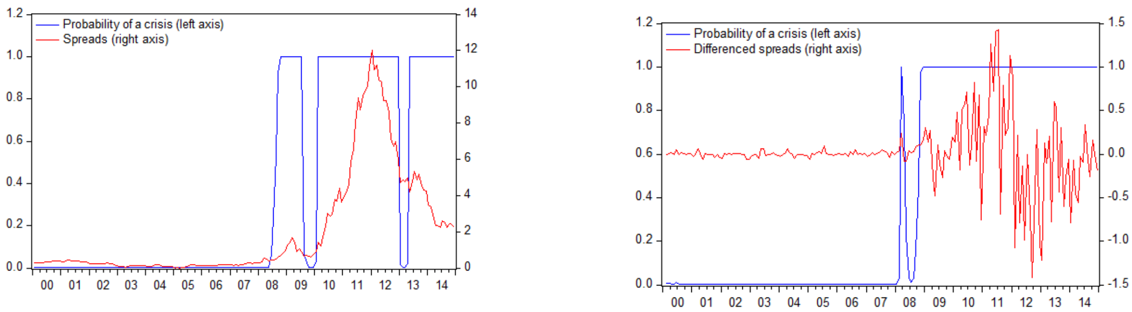
Table 2.11: MSR-TVTP estimates

	(1)[a]		(2)[b]	
Variables (expected sign)	R1: Tranquil	R2: Crisis	R1: Tranquil	R2: Crisis
Intercept	2.820***	4.527***	−.001	0.019
	(0.129)	(0.061)	(0.008)	(0.067)
Debt to GDP (+)	-0.115***		0.006	
	(0.006)		(0.006)	
Squared debt to GDP	0.001***			
	(0.000)			
GDP growth (−)	0.035***		0.001	
	(0.004)		(0.000)	
Liquidity (−)	0.284***		0.057	
	(0.097)		(0.129)	
Risk aversion (+)	0.191***		0.094***	
	(0.016)		(0.054)	
Regime standard error	0.058***	3.252***	0.028***	0.569***
	(0.003)	(0.391)	(0.004)	(0.053)
BIC	0.751		-1.341	
Log likelihood	-39.061		145.936	
Durbin-Watson statistic	0.050		1.579	

[a] Dependent variable: spread; independent variables in levels; transition probability function independent variable: international risk aversion.
[b] Dependent variable: Δ spread; independent variables in first differences, except GDP growth; transition probability function independent variable: deficit to GDP.
***, ** and * denote significance at the 1, 5 and 10 percent level, respectively.
Standard errors in parentheses.

Similarly to the CTP framework, the results suggest that in fact there are two different estimates for the constant term. However, in levels the only significant variable with the expected sign is risk aversion and in differences this is the only variable that has an impact on the changes in spreads. Notwithstanding, the Markov-switching framework with transition probabilities varying endogenously still identifies two different states in the economy: one with lower default expectations and one with higher default expectations.

Comparing the probabilities of being in a crisis from Figure 2.14 with the CTP framework, now the model identifies two switches from crisis to tranquil regime, with a return to crisis in May 2013, when Portugal regained access to international credit markets.



(a) Model in levels: Panel (1)

(b) Model in differences: Panel (2)

Figure 2.14: Smoothed transition probabilities of being in a crisis regime: TVTP

In contrast with the CTP estimates, the TVTP specification does not perform substantially better, specially taking into consideration that the coefficients present signs without theoretical support. This result may be interpreted as evidence of multiple equilibria, where shift in agents' expectations played role in the occurrence of the crisis making it self-fulfilling. In fact, if the transition probabilities between regimes cannot be predicted by economic fundamentals, then crisis episodes are driven by sunspots.

2.5.3 Regime-varying coefficients

This subsection considers that the relationship between government bond yields and economic fundamentals may be changing over time. In fact, a regime-varying coefficients model incorporates the possibility that the sensitivity of spreads to fundamentals may change when the regime changes. A possible modification in risk pricing over time period recommends the use of time-varying coefficient models, as confirmed by Bernoth and Erdogan (2012).

Bruneau, Delatte and Fouquau (2014) extend the original model introduced by Jeanne and Masson (2000) to obtain a linearized form of probability of default, where not only the constant but also the coefficients of the fundamentals are allowed to vary depending on observable variables. The specification of regime dependent coefficients is the following:

$$\pi_t = \gamma_{s_t} + \beta'_{s_t} \cdot x_t + \nu_t \tag{2.5.4}$$

where both γ_{s_t} and β'_{s_t} depend on the state. As such, regime shifts affect the probability of default by changing the constant term and the effect of the variables.

The regime switching may have CTP or TVTP. Table 2.12 and Table 2.13 present the results for each case, again considering the model in levels and in first differences.

Table 2.12: MSR with regime-varying coefficients - CTP estimates

	(1)[a]		(2)[b]	
Variables (expected sign)	R1: Tranquil	R2: Crisis	R1: Tranquil	R2: Crisis
Intercept	0.302	-89.215^{***}	-0.001	0.011
	(0.227)	(10.287)	(0.004)	(0.097)
Debt to GDP (+)	-0.036^{***}	1.806^{**}	0.005	0.076^{**}
	(0.009)	(0.157)	(0.008)	(0.034)
Squared debt to GDP	0.001^{***}	-0.008^{***}		
	(0.000)	(0.001)		
GDP growth (−)	0.052^{***}	-0.378^{***}	0.001	0.046
	(0.006)	(0.099)	(0.002)	(0.031)
Liquidity (−)	0.226	-5.291	0.078	-2.309
	(0.140)	(3.527)	(0.135)	(1.607)
Risk aversion (+)	0.254^{***}	4.070^{***}	0.088^{***}	0.554
	(0.024)	(1.041)	(0.017)	(0.633)
Regime standard errors	0.074^{***}	1.117^{***}	0.028^{***}	0.537^{***}
	(0.007)	(0.136)	(0.002)	(0.045)
BIC	0.180		-1.226	
Log likelihood	25.356		146.062	
Durbin-Watson statistic	0.644		1.781	

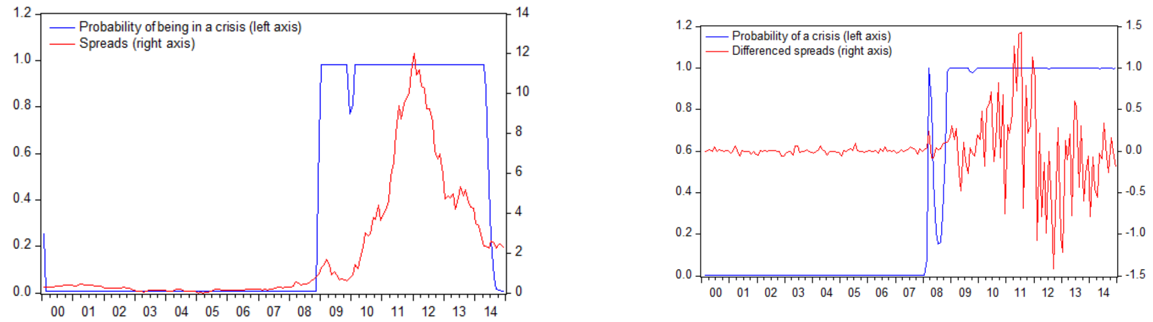
[a] Dependent variable: spread; independent variables in levels.
[b] Dependent variable: Δ spread; independent variables in first differences, except GDP growth.
***, ** and * denote significance at the 1, 5 and 10 percent level, respectively.
Standard errors in parentheses.

According to the estimates in Table 2.12, the variables have the expected and significant impact on spreads only in the crisis regime. In the tranquil regime, the results suggest that economic fundamentals do a poor job in explaining sovereign spreads, suggesting that the market pricing of sovereign risk may not have been fully reflecting fundamentals prior to the crisis, as found by Beirne and Fratzscher (2013). Moreover, liquidity does not seem to have any impact in no-crisis periods and risk aversion has the expected positive effect but with much lesser impact.

Similarly to the constant-coefficient framework with CTP, the regimes are stable as indicated by the high probability of staying in the same regime, shown below:

$$P^{(1)} = \begin{array}{c} \\ Tranquil \\ Crisis \end{array} \begin{array}{c} \begin{array}{cc} Tranquil & Crisis \end{array} \\ \begin{pmatrix} 0.993 & 0.007 \\ 0.019 & 0.981 \end{pmatrix} \end{array} \quad P^{(2)} = \begin{array}{c} \\ Tranquil \\ Crisis \end{array} \begin{array}{c} \begin{array}{cc} Tranquil & Crisis \end{array} \\ \begin{pmatrix} 0.983 & 0.017 \\ 0.014 & 0.987 \end{pmatrix} \end{array}$$

Moreover, as plotted in Figure 2.15a the model in levels signals a return to a tranquil regime starting in May 2014, the month when Portugal exited the bailout mechanism.



(a) Panel (1) (b) Panel (2)

Figure 2.15: Smoothed transition probabilities of being in a crisis regime: RVC-CTP

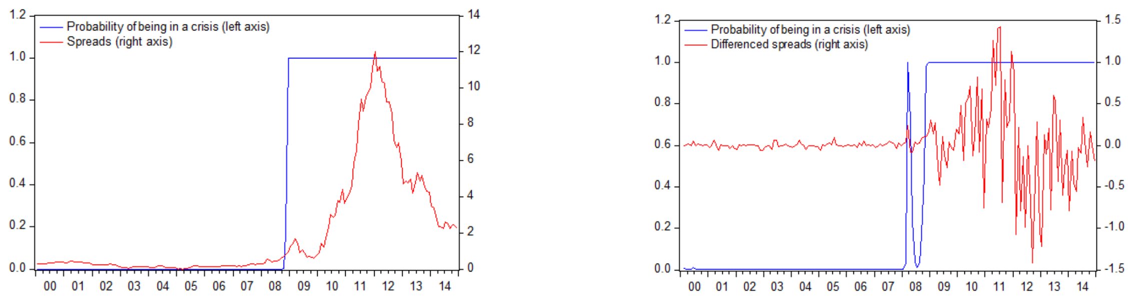
Finally, Table 2.13 shows the results of considering that the probability of switching regimes depends on the fundamentals. Using the same reasoning as in the time-invariant coefficients TVTP case, to select the variable to include in the transition function, each variable was considered contemporaneously and lagged. The results from Tables B.5 and B.6 (in Appendix) allow to opt for *debt to GDP* and *deficit to GDP* to model the transition probabilities in the levels and differences specifications, respectively.

Table 2.13: MSR with regime-varying coefficients - TVTP estimates

	(1)[a]		(2)[b]	
Variables (expected sign)	R1: Tranquil	R2: Crisis	R1: Tranquil	R2: Crisis
Intercept	7.866^{***}	-96.125^{***}	0.000	-0.067^{*}
	(0.429)	(7.236)	(0.006)	(0.037)
Debt to GDP (+)	-0.276^{***}	1.815^{***}	0.003	-0.596^{***}
	(0.015)	(0.143)	(0.008)	(0.184)
Squared debt to GDP	0.002^{***}	-0.008^{***}		
	(0.000)	(0.001)		
GDP growth (−)	0.023^{***}	-0.439^{***}	−0.001	-0.071^{***}
	(0.004)	(0.074)	(0.003)	(0.016)
Liquidity (−)	−0.059	−2.073	0.156	−1.025
	(0.102)	(2.103)	(0.109)	(0.875)
Risk aversion (+)	0.223^{***}	4.043^{***}	0.084^{***}	0.654^{***}
	(0.014)	(0.908)	(0.025)	(0.271)
Regime standard errors	0.052^{***}	1.098^{***}	0.028^{***}	0.541^{***}
	(0.001)	(0.128)	(0.002)	(0.047)
BIC	-0.051		-1.269	
Log likelihood	46.166		149.933	
Durbin-Watson statistic	0.633		1.786	

[a] Dependent variable: spread; independent variables in levels; transition probability function independent variable: debt to GDP.
[b] Dependent variable: Δ spread; independent variables in first differences, except GDP growth; transition probability function independent variable: deficit to GDP.
***, ** and * denote significance at the 1, 5 and 10 percent level, respectively.
Standard errors in parentheses.

These results are quit similar to the CTP ones, thus suggesting that economic fundamentals did not govern the probability of changes between regimes. The inclusion of debt as a determinant of the transition probabilities eliminates the change to tranquil regime in 2014 observed in the CTP model, as shown in Figure 2.16.



(a) Panel (1) (b) Panel (2)

Figure 2.16: Smoothed transition probabilities of being in a crisis regime: RVC-TVTP

To summarize, the results indicate no substantial improvement from CTP to TVTP, indicating that the transition probabilities are not governed by economic fundamentals.

2.5.4 Discussion of results

This section recaptures the results obtained and analyzed in the previous section and establishes a general discussion of the most significant elements.

Considering that the estimation of the probability of default using a *MSR framework* performs better than using a one-regime approach (OLS, DOLS, FMOLS, VECM) suggest that two regimes coexisted in Portugal during the 2000-2014 period. In fact, the results generated by all regime specifications show that it is possible to distinguish between two states: one with lower default expectations (tranquil regime) and another with higher default expectations (crisis regime). Moreover, the crisis regime is consistently more volatile in all models. Furthermore, the fact that a TVTP does not perform better than a CTP approach indicates that none of the economic fundamentals seems to particularly influence the probability of a change in regime, which can be interpreted as a sign of the importance of expectations in shaping the probability of default. These results suggest that economic indicators are insufficient to explain major shifts in the credit market spreads.

In particular, regarding the *constant-regime coefficients*, the estimate for the constant term, which captures the importance of the agents' devaluation expectations, is lower in the tranquil regime, indicating a lower probability of default in the tranquil regime. This shows that, besides the deteriorating fundamentals, agents' default expectations were in play in the Portuguese crisis episode. In fact, during observed crisis periods the model estimates the probability of being in the crisis regime as one, i.e. crises occurred when agents developed higher expectations for crisis occurring. Thus a crisis is influenced by agents' expectations making the crisis self-fulfilling.

The features of these two regimes are further investigated by the estimates of a *regime-varying coefficients* model. In fact, this model seems to perform better than the constant-coefficients, suggesting that the impact of the fundamentals on the probability of default is not the same when the economy is in tranquil or in crisis periods.

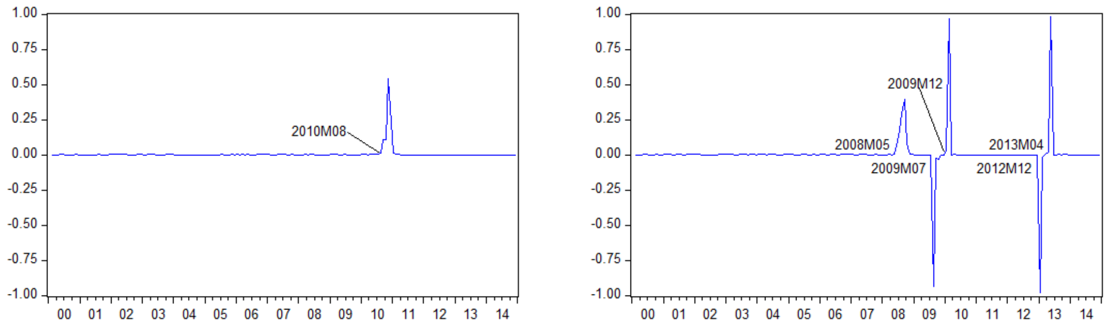
The results show that in crisis periods, all the variables have the expected impact. When the economy switches from the tranquil to the crisis regime, the impact of debt, liquidity and risk aversion unambiguously increase (notwithstanding the unexpected sign in the no-crisis regime for debt) and the coefficient of GDP growth becomes negative, as expected.

Moreover, the regime-varying results are consistent across both specifications (CTP and TVTP with alternative variables in the transition probabilities). The fact that in no-crisis periods the variables do not have the expected sign may indicate that economic fundamentals do a poor job in explaining sovereign spreads in tranquil periods. In fact, only risk aversion is theoretically justified. As stated by Beirne and Fratzscher (2013), empirical models with economic fundamentals generally are unsatisfactory in explaining sovereign risk in the pre-crisis period for European economies, suggesting that the market pricing of sovereign risk may not have been fully reflecting fundamentals prior to the crisis. These findings are corroborated by Giordano, Linciano and Soccorso (2012), who show that for 2007 and 2008, fundamentals have reduced the (fitted) spread; they also show that the 2008 increase in Italian spreads was mainly due to contagion effects. This is quite plausible given that at that time the overall state of economy remained still unaffected by the financial crisis. From 2009 onwards, as the general economic conditions deteriorated, worse fundamentals are estimated to have raised the spreads.

It is also worth mentioning that the fundamental variables in differences neither seem to explain the change in spreads, nor clearly distinguishes two regimes. This suggests that there is no short run relation between the variables. The only variable that is consistently significant in the models in differences is international risk aversion.

In order to compare the performance of each model on the identification of regime changes, Figure 2.17 and Figure 2.18 plot the first difference of the smoothed probabilities of being in the crisis regime obtained from the regime-constant coefficients and regime-varying coefficients, respectively. These plots give information about when a change of regime starts to occur: when the difference is positive, the model signals an increase probability of being in the crisis regime; contrarily, when the difference is negative, there is a decrease in the probability of being in the crisis regime; when the difference is 0, there is not change in regime. These figures also include the timing when the probability of being in a certain regime started to change.

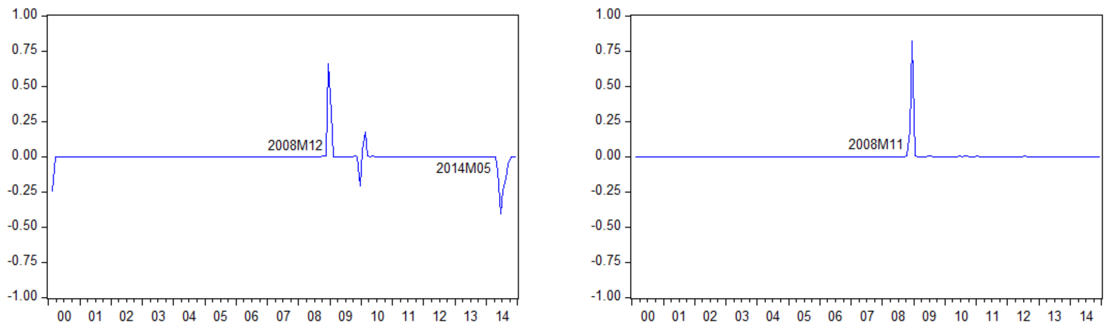
Figure 2.17a shows that starting in August 2010, the probability of being in the crisis regime started to increase and that until the end of 2014 the economy did not switch to a tranquil state. Alternatively, Figure 2.17b, which considers international risk aversion as determining the probability of regime switching, is much more erratic.



(a) CTP (b) TVTP

Figure 2.17: Timing of regime switch with constant-regime coefficients
Source: Author's calculations

Alternatively, when considering regime-varying coefficients, the beginning of the crisis is dated to the end-2008, with the CTP signaling a return to tranquil times by May 2014, when Portugal exited the bailout programme.



(a) CTP (b) TVTP

Figure 2.18: Timing of regime switch with regime-varying coefficients
Source: Author's calculations

Table 2.14 presents a chronicle of Portuguese sovereign debt crisis, including those dates in which different models signal a transition from crisis to tranquility (C-to-T) or from tranquility to crisis (T-to-C). The first column lists different dates following a chronological order. The time interval between these dates is different, since they relate to separate events, even though not necessarily independent.

Table 2.14: Timing of regime switch in each MSR specification

Date	Event	CTP	TVTP	RVC-CTP	RVC-TVTP
Jan 1999	Creation of euro area				
Mar 2005	EU finance ministers relax SGP[a]deficit rule				
May 2008			T-to-C		
Set 2008	Lehman Brothers filed for bankruptcy				
Nov 2008	EC announce 200 bn. stimulus plan				T-to-C
Dec 2008	Global financial crisis affects European banks and countries			T-to-C	
Jul 2009			C-to-T		
Nov 2009	Greek government announces deficits much higher than reported				
Dec 2009			T-to-C		
Mar 2010	Announcement of First update of Portuguese Stability and Growth Programme (PEC I)				
May 2010	EU and IMF provide 110 bn. rescue package for Greece				
	Eurosystem launches SMP[b] to buy Greek, Irish, Portuguese bonds over the next nine months				
	Announcement of Second update of Portuguese Stability and Growth Programme (PEC II)				
Jun 2010	Creation of EFSF[c]				
Aug 2010		T-to-C			
Set 2010	Announcement of Third update of Portuguese Stability and Growth Programme (PEC III)				
Nov 2010	EU and IMF provide 85 bn. rescue package for Ireland				
Mar 2011	Announcement of Fourth update of Portuguese Stability and Growth Programme (PEC IV) - not approved				
	Portuguese PM resigns				
May 2011	EU and IMF provide 78 bn. rescue package for Portugal				
Jun 2011	New Portuguese government				
Jul 2011	Moody's downgrades Portuguese public debt to junk				
	Plan to start second rescue package for Greece and to force 50% haircut on Greek bonds				
Aug 2011	ECB resumes SMP, buying Irish, Italian, Portuguese, Spanish bonds				
Dec 2011	ECB cuts repo rate[d]to 1% per year and eases collateral rules				
Jan 2012	Standard&Poors downgrades Portuguese public debt to junk				
Mar 2012	Second Greek rescue package of 130 bn. Enough bond holders agree to 53.5% face-value haircut				
Jul 2012	Draghi's "whatever it takes"				
Dec 2012			C-to-T		
Set 2012	SMP is replaced by the OMT [e]				
May 2013	Portugal regains complete access to lending markets		T-to-C		
May 2014	Portugal exits the bailout mechanism without additional need for support			C-to-T	

Source: Selected events associated with Eurozone debt crises adapted from Arellano, Conesa and Kehoe (2012) and Suh (2015)

[a] Stability and Growth Pact; [b]Securities Market Program; [c] European Financial Stability Facility - temporary crisis resolution mechanism created by the euro area Member States. The EFSF provided financial assistance to Ireland, Portugal and Greece; [d] Interest rate on repurchase agreements - loans to banks collateralized by bonds. [e] Outright Monetary Transactions.

Regarding the signal of crisis, the regime-constant coefficients CTP model captures the switch to a crisis in August 2010, that is in the most consensual date for the beginning of the euro area debt crisis (Arellano, Conesa and Kehoe, 2012) and the TPTV model is rather unstable. Taking into account that the latter model uses international risk aversion as the explanatory of the transition probabilities, the alternating regimes are due to shifts in risk perception.

Alternatively, when considering regime-varying coefficients, the crisis signal reports to end-2008. Even so, the CTP model indicates a return to the tranquil regime in May 2014, whereas the TPTV model indicates that at the end of 2014 the economy was still in the crisis regime. This result may be explained by the inclusion of debt as the variable that controls the transition probability, which in end-2014 was still in a a upward trend.

All things considered, the model that performs better is regime-varying coefficients model with CTP and confirms the following results.

During crisis periods, an increase in *debt to GDP* increases the probability of default and when the economy switches from the tranquil to the crisis regime, the impact of debt unambiguously increase (notwithstanding the unexpected sign in the no-crisis regime for debt). The variable *square debt to GDP* was included to capture potential non-linearities. The outcome of the model suggests that during tranquil time as the debt rises the impact on the spread of a one percentage point increase in the debt-to-GDP ratio also rises, which is taken as evidence that investors realize that the default decision is closer, making them more sensitive to an increase in debt ratio . However, during crisis periods (coincident with higher debt to GDP), investors seem to demand a proportionally lower risk premium, possibly valuing other macroeconomic factors (that become more significant in crisis periods).

Regarding *GDP growth* during tranquil periods, unexpectadly, increases the spreads; however, in crisis times growth reduces the spreads. So, when the economy switches from the tranquil to the crisis regime, the coefficient of GDP growth becomes negative, as expected. Bruneau, Delatte and Fouquau (2014) findings are quite similar to the ones reported here, but they use unemployment to measure the economic activity, instead of GDP growth. To this respect the empirical evidence is mixed; some studies conclude that the expected economic growth rate does not seem to matter (Attinasi, Checherita and Nickel, 2011), other studies confirm the relevance of the negative impact of economic growth on spreads (de Grauwe and Ji, 2013) or the positive relation between growth and spreads, as reported by Eichengreen and Mody (1998) for Latin America in the 1991-1995 period. In principle, a rising debt is not a problem as long as the economy grows at a faster pace than its public debt, however, after 2009 GDP growth exhibited a negative average rate, which indicates that a poor economic performance contributed to the widening of spreads.

Additionally, in tranquil periods *liquidity* does not seem to have any impact on the probability of default, suggesting that during tranquil times, investors do not refer to this indicator. On the contrary, liquidity has a negative and significant impact on crisis periods: lower liquidity leads to higher spreads. As reported by de Grauwe and Ji (2013), because members of a monetary union cannot issue their money, they are unable to guarantee debt repayment and as a result such countries are more vulnerable to negative market sentiments that in a self-fulfilling way can create a liquidity crisis. Dionne et al. (2011) also documents that illiquidity of the bond market is probably the main factor that explains the difference between default spread and credit spread and that this illiquidity effect is perhaps stronger in recession periods.

International risk aversion has the expected positive effect on both regimes, but with much higher impact in crisis periods, which may be taken as evidence that sovereign default risks and thus spreads are significantly triggered by global financial conditions. As already recalled in the previous section, sovereign bond spreads are driven not only by country specific factors but also by a time-varying international risk factor. A widening of this spread signals shifts in investors' preferences from the riskier sovereign bonds to the safer private sector assets. This result is corroborated by most recent studies on euro area debt crisis, as Giordano, Linciano and Soccorso (2012) or Bruneau, Delatte and Fouquau (2014) among others.

One potential explanation for preferring a CTP model for the switch from low to high probability of default *unrelated to economic fundamentals* is that markets are forward looking, not pricing entirely on current fundamentals but on expected further deterioration in future economic fundamentals, not fully reflected in current economic conditions. In fact, Aizenman, Hutchison and Jinjarak (2013) test differences in market pricing of risk in euro area peripheral countries and in countries outside Europe with similar fiscal conditions but with histories of debt restructuring. The authors suggests that market prices may not be based on current fundamentals, but rather on future fundamentals, expecting adjustment challenges in the euro are periphery to be more difficult than for the matched group of middle-income countries outside Europe, because of exchange rate and monetary constraints.

Another interpretation is that the economy switched to a "pessimistic" self-fulfilling expectations equilibrium. In fact, the results are consistent with multiple equilibrium with an abrupt switch from a "good" (tranquil) expectations equilibrium with low expected default rates when economic fundamentals are favourable to a "bad" (crisis) expectations equilibrium with high expected default rates when fundamental positions are unfavourable. Given turbulent conditions during the crisis period and if markets apparently did not focus on current fiscal fundamentals, Aizenman, Hutchison and Jinjarak (2013) suggest the markets simply overreacted and mispriced risk of default.

Within this perspective, sovereign risk was initially underpriced as contagious optimism, somewhat divorced from fundamentals, predominated market expectations in the euro area, especially at the time when the euro was perceived to be successful. However, in 2008 the tranquil expectations equilibrium switched to a crisis expectations equilibrium as markets overreacted to fiscal deterioration and generated extraordinarily high spreads. Accordingly, persistent pessimism, manifested in high risk permia, induces further aggravation of the economic fundamentals, leading a self-fulfilling prophecy crisis.

The presence of self-fulfilling beliefs is supported by de Grauwe and Ji (2013), because they find evidence that a significant part of the increase in the spreads of the peripheral euro area countries during 2010-11 was more related to negative investors' expectations, than to the deterioration of debt and fiscal positions, thus confirming the existence of a self-fulfilling crisis. Similarly, Aizenman, Hutchison and Jinjarak (2013) confirm that Portugal default risk is priced much higher than matched countries in 2010, even allowing for differences in fundamentals.

It is also worth mentioning that a third regime was also considered in order to verify whether assuming two regimes is a reasonable assumption. However, in an unreported analysis, the addition of another regime does not improve the fit of the model (in terms of BIC and log-likelihood). Therefore, the two-state regime switching model is an acceptable specification.

In conclusion, the model allowing for two regimes, performs better than a purely fundamental based model, by improving the relationship between the economic fundamentals and the default expectation.

2.5.5 Limitations of the results

Throughout this investigation, several limitations were found. When using short time series dimension, the statistical inference was found to be challenging, particularly because some macroeconomic determinants of sovereign bond yields are typically available only in low frequencies (quarterly). Yet another problem arises from the fact that information is available with different time lags. For example, monthly public debt data is available 50 days after the reference month, GDP flash estimates are available 50 days after the reference quarter and GDP estimates after 70 days. Finally, the errors autocorrelation was never found to be satisfactory. Notwithstanding, the consistency of the results (both in the coefficients and its significance) across the majority of the models, seems to validate the conclusions presented.

In the face of these limitations, there are some issues which were not address in this chapter, but that may be of interest for future research. First, the economic fundamentals variables correspond to the current period. The identification of early warning macroeconomic and financial indicators, in the line of Díez (2014), could help to find signals of likely crises in the near future. Second, having established that expectations also determine the spreads evolution, a question for future research is to find what is influencing these expectations. Possible clarifications can stem from the inclusion of ”news” to explain the transition probabilities, since several paper report large effects on spreads at the time of selected announcememts, both theoretically (Durdu, Nunes and Sapriza, 2013) and empirically (Pooter et al., 2015; Eser and Schwaab, 2013); the incorporation of contagion mechanisms (de Santis, 2012); or different financial variables than those considered here (Bruneau, Delatte and Fouquau, 2014). Third, to deal with autocorrelation issues within a MSR framework, it surely deserves further investigation the implementation of a Markov switching error correction model (MS-ECM), as proposed by Psaradakis and Spagnolo (2004) and applied by Frömmel and Karagyozova (2008).

2.6 Conclusion

Research has shown that market pessimistic expectations can contribute to the occurrence of a crisis that would not have taken place if such expectations could have been prevented at first place. In this context, it is pertinent to find whether the changes in the probability of default have been determined by economic fundamentals, by market sentiments or both.

To address this issue in the Portuguese case, following the existing empirical literature, the work estimates a model of the determinants of the 10-year Portuguese yield spreads relative to Germany. The econometric strategy involves three main steps. First, the literature is reviewed in order to select the candidate variables that should be included as determinants of sovereign debt spreads, the latter used as a *proxy* for the probability of default. Then, single-regime preliminary regressions were run, over the January 2000 - December 2014 to select the most significant variables. Second, it is shown that unit root tests on sovereign spreads and their potential determinants cannot be rejected during the period of analysis and estimate the model using cointegration methodologies to identify the plausible specification. The results of DOLS, FMOLS and VECM are quite similar to the OLS estimates, confirming the choice of the variables. Third, two regimes MSR models are estimated, both constant and time-varying regime coefficients, and with CTP and TVTP.

The regime-varying coefficient with CTP model was selected based on the adequacy of the expected impact of the variables, on the dates when regime switches were signaled and on the comparison of summary statistics with the alternative models.

This investigation reveals time-varying environment in the market for sovereign default risk that indicates that the macroeconomic fundamentals affect the spread differently, whether the economy is either experiencing a tranquil or a crisis state.

Specifically, there is empirical evidence of the key role of debt to GDP, GDP growth, liquidity and international risk aversion in pricing sovereign spreads. During crisis regimes, these macroeconomic fundamentals can be theoretically understood, as they exhibit the expected signs. During the tranquil state, only international risk aversion presents the expected sign (positive). This may suggest that during tranquil periods investors refer basically to this indicator, since the other signs find no theoretical support.

The link between the fundamentals and the probability of default is economically and statistically strong in crisis periods. However, the change from a tranquil to a crisis regime seemed potentially disconnected from fundamentals. One conceivable explanation for the switch from low to high default risk is that markets are forward looking, not pricing entirely on current fundamentals but on expected further deterioration in future fundamentals. Alternatively, the results are consistent with multiple equilibrium with an abrupt switch from a optimistic expectations equilibrium in Portugal with low expected default rates and low interest rates where fundamentals are favorable to a pessimistic expectations equilibrium with high expected default rates and high interest rates where fundamentals are unfavorable.

In face of the results and although the model identified a switch into a tranquil regime in May 2014 (the date of the bailout exit), it has to be recognized that, by then, the macroeconomic fundamentals were not significantly different from those that were identified as having prompted the crisis. This indicates that a close attention to public deficit and public debt remains of the utmost importance, notwithstanding those equally relevant concerns with growth and employment, without which consolidation of public finances is also put at risk.

These issues are especially relevant for a small open economy like Portugal, which has no control over liquidity and international risk aversion. These are the variables to which investors refer to during tranquil periods. But if a heightened risk perception hits the international market it may prompt a new switch into crisis, following which macroeconomic fundamentals will be scrutinized by investors. If this occurs, public finances play a starring role, either amplifying or leveling risk, and thus contributing to the duration of the crisis.

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Appendix A

Appendix of Chapter 1

A.1 Consumers problem

A.1.1 Crisis zone with no previous default

Here, if default has not occurred until period t, consumers assign a probability π to a default and a probability $1-\pi$ to no default. The representative consumer, given k_t, k^n_{t+2} and k^θ_{t+2} chooses c_t, k_{t+1} and c^n_{t+1}, c^θ_{t+1}, where the former correspond to the choices of consumption in periods $t+1$ contingent on the government not defaulting or defaulting, respectively. Thus, consumers' solve the variational problem:

$$\max_{c_t, k_{t+1}, c^n_{t+1}, c^\theta_{t+1}} c_t + \beta \cdot (1-\pi) \cdot c^n_{t+1} + \beta \cdot \pi \cdot c^\theta_{t+1}$$

subject to

$$\begin{aligned}
&c_t \leq (1-\tau) \cdot [y(k_t, 1) - \delta \cdot k_t] + k_t - k_{t+1} \\
&c^n_{t+1} \leq (1-\tau) \cdot [y(k_{t+1}, 1) - \delta \cdot k_{t+1}] + k_{t+1} - k^n_{t+2} \\
&c^\theta_{t+1} \leq (1-\tau) \cdot \left[y(k_{t+1}, \bar{\theta}) - \delta \cdot k_{t+1}\right] + k_{t+1} - k^\theta_{t+2} \\
&c_t, c^n_{t+1}, c^\theta_{t+1}, k_{t+1} \geq 0
\end{aligned}$$

Using the budget constraints to replace consumption in the utility function and taking k_t, k^n_{t+2} and k^θ_{t+2} as known, the first order condition for capital stock is:

$$1 = \beta \cdot (1-\pi) \cdot \left\{(1-\tau) \cdot \left[y'(k_{t+1}, 1) - \delta\right] + 1\right\} + \beta \cdot \pi \cdot \left\{(1-\tau) \cdot \left[y'(k_{t+1}, \bar{\theta}) - \delta\right] + 1\right\}$$

$$1 = \beta \cdot \left\{(1-\tau)\left[(1-\pi) \cdot y'(k_{t+1}, 1) + \pi \cdot y'(k_{t+1}, \bar{\theta}) - \delta\right] + 1\right\}$$

Since $y_t(k_t, \theta_t) = Z^{1-\theta_t} \cdot f(k_t)$, then the FOC becomes:

$$1 = \beta \cdot \left[(1-\tau)\left\{f'(k_{t+1})\left[(1-\pi)+\pi\cdot Z^{1-\bar{\theta}}\right]-\delta\right\}+1\right]$$

and there is a stationary capital stock at k^{π} obtained by:

$$1 = \beta \cdot \left[(1-\tau)\left\{f'(k^{\pi})\cdot\left[(1-\pi)+\pi\cdot Z^{1-\bar{\theta}}\right]-\delta\right\}+1\right].$$

Regarding consumption, if no default has occurred in t:

$$c^{\pi}(k) = (1-\tau)\cdot[f(k)-\delta\cdot k]+k-k^{\pi},$$

A.1.2 No default

In the case where no default is possible, we can derive the optimal level of capital accumulation k^n and the consumption rule $c^n(k_t)$ by setting $\pi = 0$:

$$1 = \beta \cdot \left\{(1-\tau)\left[f'(k^n)-\delta\right]+1\right\}$$

and

$$c^n(k) = (1-\tau)\cdot[f(k)-\delta\cdot k]+k_t-k^n$$

A.1.3 Default

Analogously, in the case where a default is the only possible outcome, the optimal level of capital accumulation can be derived by setting $\pi = 1$ and the optimal level of capital accumulation, k^{θ}, and consumption rule, $c^{\theta}(k)$, are derived from rules:

$$1 = \beta \cdot \left\{(1-\tau)\cdot\left[Z^{1-\theta}\cdot f'(k^{\theta})-\delta\right]+1\right\}$$

and

$$c^{\theta}(k) = (1-\tau)\cdot\left[Z^{1-\theta}\cdot f(k)-\delta\cdot k\right]+k-k^{\theta}$$

A.2 Government's problem

The derivation of the equilibrium is subdivided into two major steps. In a first step, the general problem of the government is solved, assuming that the possibility for a self-fulfilling crisis to occur is set to zero ($\pi = 0$). This enables on one hand the determination of the equilibriums in the "no-crisis zone" and the "default zone". Second, we proceed to the crisis zone ($\pi > 0$), in order to assure an appropriate derivation of this equilibrium.

A.2.1 Default Zone

In the default zone $q = \beta \cdot \theta$ and the government solves the variational problem (1.4.11), by choosing g_t, g_{t+1} and B_{t+1}, and taking as given B_t, B_{t+2}, k_t and k_{t+1}.

$$\max_{g_t, g_{t+1}, B_{t+1}} \beta^t \cdot v(g_t) + \beta^{t+1} \cdot v(g_{t+1})$$

subject to

$$g_t + B_t \leq \tau \cdot \left[Z^{1-\theta} \cdot f(k_t) - \delta \cdot k_t \right] + \beta \cdot \theta \cdot B_{t+1}$$

$$g_{t+1} + \theta \cdot B_{t+1} \leq \tau \cdot \left[Z^{1-\theta} \cdot f(k_{t+1}) - \delta \cdot k_{t+1} \right] + \beta \cdot \theta \cdot B_{t+2}$$

$$g_t, g_{t+1} \geq 0$$

$$B_{t+1} \geq \bar{B}(k_t)$$

$$+ \text{ transversality condition}$$

Forwarding the budget constraints and multiplying by θ, we find the present-value of the budget constraint:

$$\sum_{s=t}^{T} (\beta \cdot \theta)^{s-t} g_s + \theta \cdot B_t = \sum_{s=t}^{T} (\beta \cdot \theta)^{s-t} \cdot \tau \cdot \left[Z^{1-\theta} f(k_s) - \delta \cdot k_s \right] + (\beta \cdot \theta)^T \cdot B_{t+T}$$

yielding the transversality condition:

$$\lim_{T \to \infty} (\beta \cdot \theta)^T \cdot B_{t+T} = 0$$

The FOC is:

$$\frac{\partial}{\partial B_{t+1}} = \beta^t \cdot \beta \cdot \theta \cdot v'(g_t) - \beta^{t+1} \cdot \theta \cdot v'(g_{t+1}) = 0$$

which implies that $\theta \cdot v'(g_t) = v'(g_{t+1})$. So, the optimal government policy is to set $g_{t+1} = g_t$, and hence $B_{t+2} = B_{t+1}$.

In the general case where $k_0 \neq k^\theta$, the level of government consumption in period 0 is:

$$g^{\theta 0}(B_0, B_1, k_0) = \tau \cdot \left[Z^{1-\theta} \cdot f(k_0) - \delta \cdot k_0\right] + \beta \cdot \theta \cdot B_1 - B_0, \tag{A.2.1}$$

in period 1, the default takes place and $B_{t+1} = B_{t+2}$:

$$g^{\theta 1}(B_1, B_1, k^\theta) = \tau \cdot \left[Z^{1-\theta} \cdot f(k^\theta) - \delta \cdot k^\theta\right] - (\theta - \beta \cdot \theta) \cdot B_1$$

and thereafter:

$$g^{\theta 2}(B_1, B_1, k^\theta) = \tau \cdot \left[Z^{1-\theta} \cdot f(k^\theta) - \delta \cdot k^\theta\right] - (1 - \beta \cdot \theta) \cdot B_1$$

An average constant g^θ can be calculated by weighting the gs with the same weights as in the value function:

$$\frac{1}{1-\beta} \cdot g^\theta = g^{\theta 0}(B_0, B_1, k_0) + \beta \cdot g^{\theta 1}(B_1, B_1, k_0) + \frac{\beta^2}{1-\beta} \cdot g^{\theta 2}(B_1, B_1, k^\theta)$$

such that:

$$\begin{aligned} g^\theta =& \tau \cdot \left[Z^{1-\theta} \cdot f(k_0) - \delta \cdot k_0\right] + \beta \cdot \tau \cdot \left\{\left[Z^{1-\theta} \cdot f(k^\theta) - \delta \cdot k^\theta\right] - \left[Z^{1-\theta} \cdot f(k_0) - \delta \cdot k_0\right]\right\} \\ & - (1-\beta) \cdot B_0 - \beta^2 \cdot (1-\theta) \cdot B_1 \end{aligned}$$

To find the constant debt level, we make use of the stationary government budget constraint:

$$(1 - \beta \cdot \theta) \cdot B_1 = \tau \cdot \left[Z^{1-\theta} \cdot f(k^\theta) - \delta \cdot k^\theta\right] - g^\theta$$

and hence:

$$B^\theta(B_0, k_0) = \frac{1}{1 + \beta \cdot (1-\theta)} \cdot \left[B_0 + (1-\beta) \cdot \tau \cdot \left\{\left[Z^{1-\theta} \cdot f(k^\theta) - \delta \cdot k^\theta\right] - \left[Z^{1-\theta} \cdot f(k_0) - \delta \cdot k_0\right]\right\}\right].$$

In the particular case where $k_0 = k^\theta$, the constant debt level is:

$$B^\theta(B_0, k^\theta) = \frac{1}{1 + \beta \cdot (1-\theta)} \cdot B_0$$

and the government constant spending level simplifies to:

$$g^\theta = \tau \cdot \left[Z^{1-\theta} \cdot f(k^\theta) - \delta \cdot k^\theta\right] - \frac{1 - \beta \cdot \theta}{1 + \beta \cdot (1-\theta)} B_0$$

A.2.2 No default

In this zone, $q_t = \beta$, and the government solves the variational problem (1.4.16), by choosing g_t, g_{t+1} and B_{t+1}, and taking as given B_t, B_{t+2}, k_t and k_{t+1}.

$$\max_{g_t, g_{t+1}, B_{t+1}} \beta^t \cdot v(g_t) + \beta^{t+1} \cdot v(g_{t+1})$$

subject to

$$g_t + B_t \leq \tau \cdot [f(k_t) - \delta \cdot k_t] + \beta \cdot B_{t+1}$$

$$g_{t+1} + B_{t+1} \leq \tau \cdot [f(k_{t+1}) - \delta \cdot k_{t+1}] + \beta \cdot B_{t+2}$$

$$g_t, g_{t+1} \geq 0$$

$$B_{t+1} \leq \underline{b}(1, k_t)$$

$+$ transversality condition

The present-value of the budget constraint is:

$$\sum_{s=t}^{T} \beta^{s-t} g_s + B_t = \sum_{s=t}^{T} \beta^{s-t} \tau \cdot [f(k_s) - \delta \cdot k_s] + \beta^T \cdot B_{t+T}$$

yielding the transversality condition:

$$\lim_{T \to \infty} \beta^T \cdot B_{t+T} = 0$$

FOC:

$$\frac{\partial}{\partial B_{t+1}} = \beta^t \cdot \beta \cdot v'(g_t) - \beta^{t+1} \cdot \cdot v'(g_{t+1}) = 0$$

which implies that $v'(g_t) = v'(g_{t+1})$. So, the optimal government policy is to set $g_{t+1} = g_t$, and hence $B_{t+2} = B_{t+1}$.

In the general case where $k_0 \neq k^n$, the level of government consumption in period 0 is:

$$g^n(B_0, B_1, k_0) = \tau \cdot [f(k_0) - \delta \cdot k_0] + \beta \cdot B_1 - B_0$$

and thereafter:

$$g^n(B_1, B_1, k^n) = \tau \cdot [f(k^n) - \delta \cdot k^n] - (1 - \beta) \cdot B_1$$

An average constant g^n can be calculated by weighting $g^n(B_0, B_1, k)$ and $g^n(B_1, B_1, k^n)$ with the same weights as in the value function:

$$\frac{1}{1-\beta} \cdot g^n = g^n(B_0, B_1, k_0) + \frac{\beta}{1-\beta} \cdot g^n(B_1, B_1, k^n)$$

yielding the following result:

$$g^n(B_0, k_0) = \tau \cdot [f(k_0) - \delta \cdot k_0] + \beta \cdot \tau \cdot \{[f(k^n) - \delta \cdot k^n] - [f(k_0) - \delta \cdot k_0]\} - (1-\beta) \cdot B_0$$

To find the constant debt level, we make use of the government budget constraint in Eq. (1.3.4):

$$\beta \cdot B_1 = g^n + B_0 - \tau \cdot [f(k_0) - \delta \cdot k_0]$$

and hence:

$$B^n(B_0, k_0) = B_0 + \tau \cdot \{[f(k^n) - \delta \cdot k^n] - [f(k_0) - \delta \cdot k_0]\} .$$

In the case where, $k_0 = k^n$,

$$g^n(B_0, k^n) = \tau \cdot [f(k^n) - \delta \cdot k^n] - (1-\beta) \cdot B_0$$

and:

$$B^n(B_0, k^n) = B_0.$$

A.2.3 Crisis zone

Run down the debt in T periods The government plans to run debt down to $\underline{b}(k^n)$ in T periods and sets a constant level of spending at $g = g^T(B_0; \pi)$. In the crisis zone, where a default has not occurred but it is possible with probability π, we have $c = c^\pi(k^\pi)$, $k' = k^\pi$ and $q = \beta \cdot (1-\pi) + \beta \cdot \pi \cdot \theta = \hat{\beta}$. Applying this expressions to the governments' budget constraints, under the assumption that $B_t \neq B_{t-1}$, then:

$$\begin{aligned}
g^T + B_0 &= \tau \cdot [f(k^\pi) - \delta \cdot k^\pi] + \hat{\beta} \cdot B_1 \\
g^T + B_1 &= \tau \cdot [f(k^\pi) - \delta \cdot k^\pi] + \hat{\beta} \cdot B_2 \\
(...) &= (...) \\
g^T + B_{T-2} &= \tau \cdot [f(k^\pi) - \delta \cdot k^\pi] + \hat{\beta} \cdot B_{T-1} \\
g^T + B_{T-1} &= \tau \cdot [f(k^\pi) - \delta \cdot k^\pi] + \beta \cdot \underline{b}(k^n)
\end{aligned}$$

Multiplying each of these equations by $\hat{\beta}^t$ and adding them, we obtain:

$$\sum_{t=0}^{T-1} \hat{\beta}^t \cdot g^T + B_0 = \sum_{t=0}^{T-1} \hat{\beta}^t \cdot \tau \cdot [f(k^\pi) - \delta \cdot k^\pi] + \hat{\beta}^{T-1} \cdot \beta \cdot \underline{b}(k^n)$$

The constant government spending is:

$$\frac{1-\hat{\beta}^T}{1-\hat{\beta}} \cdot g^T + B_0 = \frac{1-\hat{\beta}^T}{1-\hat{\beta}} \cdot \tau \cdot [f(k^\pi) - \delta \cdot k^\pi] + \hat{\beta}^{T-1} \cdot \beta \cdot \underline{b}(k^n)$$

$$g^T = \tau \cdot [f(k^\pi) - \delta \cdot k^\pi] + \hat{\beta}^{T-1} \cdot \frac{1-\hat{\beta}}{1-\hat{\beta}^T} \cdot \beta \cdot \underline{b}(k^n) - \frac{1-\hat{\beta}}{1-\hat{\beta}^T} \cdot B_0 \tag{A.2.2}$$

Next we derive the expected payoff of running down debt to $\underline{b}(k^n)$ in T periods, when no default occurs during the debt reduction. To find $V^T(B_0, \pi)$, we computing the value functions $V_t^T(B_0; \pi)$ which denote the value of the policy for which there are still t periods left to run down debt. Each of these value functions has the following features:

- Since the economy is in the crisis zone, then $k = k^\pi$. Considering the possibility of not defaulting, consumers set $c = c^\pi(k^\pi)$ according to Eq. (1.4.3) and government sets $g = g^T$ as defined in Eq. (A.2.2); when considering the possibility of defaulting, agents set $c = c^\theta(k^\pi)$ as in Eq. (1.4.5) and $g = g^\theta(k^\pi)$ from Eq. (A.2.1).
- Afterwards, if no default had occur, the government will keep running down debt, or, if a default had occurred the payoff is $V^\theta = V^\theta(B_0, k^\theta)$ as in Eq. (1.4.14), where $k' = k^\theta$, government spending defined by (1.4.13) and keeping a constant debt level according to Eq. (1.4.12).

The value functions are:

$$\begin{aligned}
V_T^T(B_0) &= (1-\pi)\cdot[c^\pi(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi))]+\pi\cdot\beta\cdot V^\theta(B_0)+(1-\pi)\cdot\beta\cdot V_{T-1}^T(B_0)\\
V_{T-1}^T(B_0) &= (1-\pi)\cdot[c^\pi(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi))]+\pi\cdot\beta\cdot V^\theta(B_0)+(1-\pi)\cdot\beta\cdot V_{T-2}^T(B_0)\\
(...) &= (...)\\
V_3^T(B_0) &= (1-\pi)\cdot[c^\pi(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi))]+\pi\cdot\beta\cdot V^\theta(B_0)+(1-\pi)\cdot\beta\cdot V_2^T(B_0)\\
V_2^T(B_0) &= (1-\pi)\cdot[c^\pi(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi))]+\pi\cdot\beta\cdot V^\theta(B_0)+(1-\pi)\cdot\beta\cdot V_1^T(B_0)\\
V_1^T(B_0) &= (1-\pi)\cdot[c^n(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi))]+\pi\cdot\beta\cdot V^\theta(B_0)+(1-\pi)\cdot\beta\cdot V^n[\underline{b}(k^n)]
\end{aligned}$$

Notice that in period T, government spending changes from g^T to $g^n(k^n)$ and $B=\underline{b}(k^n)$, thus using 1.4.18

$$V_0^T = V^n\left[\underline{b}(k^n)\right] = \frac{1}{1-\beta}\cdot\{c^n(k^n)+v\left[g^n(\underline{b}(k^n),\underline{b}(k^n),k^n)\right]\},$$

with

$$g^n(\underline{b}(k^n),\underline{b}(k^n),k^n) = \tau\cdot[f(k^n)-\delta\cdot k^n]-(1-\beta)\cdot\underline{b}(k^n)$$

To calculate the value function $V^T(B_0;\pi)$, we apply backward induction (to simplify notation we redefine $V^n[\underline{b}(k^n)] = V^n$):

$$\begin{aligned}
V_2^T &= (1-\pi)\cdot[c^\pi(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi))]+\pi\cdot\beta\cdot V^\theta+\\
&\quad+(1-\pi)\cdot\beta\cdot\{(1-\pi)\cdot[c^n(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi))]+\pi\cdot\beta\cdot V^d+(1-\pi)\cdot\beta\cdot V^n\}\\
&= (1-\pi)\cdot[c^\pi(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi))]+\pi\cdot\beta\cdot[1+(1-\pi)\cdot\beta]\cdot V^\theta+\\
&\quad+(1-\pi)\cdot\beta\cdot\{(1-\pi)\cdot[c^n(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi))]\}+[(1-\pi)\cdot\beta]^2\cdot V^n
\end{aligned}$$

$$\begin{aligned}
V_3^T &= [1+(1-\pi)\cdot\beta]\cdot\{(1-\pi)\cdot[c^\pi(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi))]\}+\\
&\quad+\pi\cdot\beta\cdot[1+(1-\pi)\cdot\beta+((1-\pi)\cdot\beta)^2]\cdot V^\theta+\\
&\quad+((1-\pi)\cdot\beta)^2\cdot\{(1-\pi)\cdot[c^n(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi))]\}+\\
&\quad+((1-\pi)\cdot\beta)^3\cdot V^n\\
&\ \vdots\\
V_T^T &= [1+(1-\pi)\cdot\beta+\cdots((1-\pi)\cdot\beta)^{T-2}]\cdot\{(1-\pi)\cdot[c^\pi(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi))]\}+\\
&\quad+\pi\cdot\beta\cdot[1+(1-\pi)\cdot\beta+\cdots+[(1-\pi)\cdot\beta]^{T-1}]\cdot V^\theta+\\
&\quad+[(1-\pi)\cdot\beta]^{T-1}\cdot\{\pi^n\cdot[c^n(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi))]\}+\\
&\quad+[(1-\pi)\cdot\beta]^T\cdot V^n
\end{aligned}$$

As per definition $V_T^T(B_0;\pi) = V^T(B_0;\pi)$, then the expected payoff of running down the debt in T periods while a default does not occur is:

$$V^T(B_0) = \frac{1-[(1-\pi)\cdot\beta]^{T-1}}{1-(1-\pi)\cdot\beta}\cdot\{(1-\pi)\cdot[c^\pi(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)]\}+$$
$$+[(1-\pi)\cdot\beta]^{T-1}\cdot\{(1-\pi)\cdot[c^n(k^\pi)+v(g^T)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)]\}+$$
$$+\pi\cdot\beta\cdot\frac{1-[(1-\pi)\cdot\beta]^T}{1-(1-\pi)\cdot\beta}\cdot V^\theta(B_0,k^\theta)+[(1-\pi)\cdot\beta]^T\cdot V^n[\underline{b}(k^n)]$$

Run up debt in W periods The government plans to run up debt to $\bar{B}(k^\theta)$. In period W the debt level falls, and afterwards it follows a stationary level. During the process, while a default does not occur, the government sets a constant level of spending at $g = g^W(B_0;\pi)$, and we have $c = c^\pi(k^\pi)$, $k' = k^\pi$ and $q = \beta\cdot(1-\pi)+\beta\cdot\pi\cdot\theta = \hat{\beta}$. Applying this expressions to the governments' budget constraints, under the assumption that $B_t \neq B_{t-1}$, then:

$$\begin{aligned}
g^W + B_0 &= \tau\cdot[f(k^\pi)-\delta\cdot k^\pi]+\hat{\beta}\cdot B_1\\
g^W + B_1 &= \tau\cdot[f(k^\pi)-\delta\cdot k^\pi]+\hat{\beta}\cdot B_2\\
(...) &= (...)\\
g^W + B_{W-2} &= \tau\cdot[f(k^\pi)-\delta\cdot k^\pi]+\hat{\beta}\cdot B_{W-1}\\
g^W + B_{W-1} &= \tau\cdot[f(k^\pi)-\delta\cdot k^\pi]+\beta\cdot\theta\cdot\bar{B}(k^\theta)
\end{aligned}$$

Multiplying each of these equations by $\hat{\beta}^w$ and adding them, we obtain:

$$\sum_{w=0}^{W-1}\hat{\beta}^w\cdot g^W + B_0 = \sum_{w=0}^{W-1}\hat{\beta}^w\cdot\tau\cdot[f(k^\pi)-\delta\cdot k^\pi]+\hat{\beta}^{W-1}\cdot\beta\cdot\theta\cdot\bar{B}(k^\theta)$$

The constant government spending is:

$$\frac{1-\hat{\beta}^W}{1-\hat{\beta}}\cdot g^W + B_0 = \frac{1-\hat{\beta}^W}{1-\hat{\beta}}\cdot\tau\cdot[f(k^\pi)-\delta\cdot k^\pi]+\hat{\beta}^{W-1}\cdot\beta\cdot\theta\cdot\bar{B}(k^\theta)$$

$$g^W = \tau\cdot[f(k^\pi)-\delta\cdot k^\pi]+\hat{\beta}^{W-1}\cdot\frac{1-\hat{\beta}}{1-\hat{\beta}^W}\cdot\beta\cdot\theta\cdot\bar{B}(k^\theta)-\frac{1-\hat{\beta}}{1-\hat{\beta}^W}\cdot B_0 \tag{A.2.3}$$

We derive the expected payoff of running up the debt to $\bar{B}(k^\theta)$ in W periods, $V^W(B_0,\pi)$, by computing the value functions $V_w^W(B_0;\pi)$ which denote the value of the policy for which there are still w periods left to run up debt. These value functions are:

$$V_W^W(B_0) = (1-\pi)\cdot[c^\pi(k^\pi)+v(g^W)] + \pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)] + \pi\cdot\beta\cdot V^\theta(B_0) + (1-\pi)\cdot\beta\cdot V_{W-1}^W(B_0)$$
$$V_{W-1}^W(B_0) = (1-\pi)\cdot[c^\pi(k^\pi)+v(g^W)] + \pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)] + \pi\cdot\beta\cdot V^\theta(B_0) + (1-\pi)\cdot\beta\cdot V_{W-2}^W(B_0)$$
$$\vdots$$
$$V_3^W(B_0) = (1-\pi)\cdot[c^\pi(k^\pi)+v(g^W)] + \pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)] + \pi\cdot\beta\cdot V^\theta(B_0) + (1-\pi)\cdot\beta\cdot V_2^W$$
$$V_2^W(B_0) = (1-\pi)\cdot[c^\pi(k^\pi)+v(g^W)] + \pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)] + \pi\cdot\beta\cdot V^\theta(B_0) + (1-\pi)\cdot\beta\cdot V_1^W$$
$$V_1^W(B_0) = (1-\pi)\cdot[c^n(k^\pi)+v(g^W)] + \pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)] + \pi\cdot\beta\cdot V^\theta(B_0) + (1-\pi)\cdot\beta\cdot V^\theta[\bar{B}(k^\theta)]$$

When there is 1 period left to run up the debt (in period $W-1$), if a default does not happen, then in the next period (W), government spending changes from g^W to $g^\theta(k^\theta)$ and the government defaults on $B = \bar{B}(k^\theta)$.

Making use of the constant debt level from Eq.(1.4.12), then

$$B^\theta(\bar{B}(k^\theta), k^\theta) = \frac{1}{1+\beta\cdot(1-\theta)}\cdot\bar{B}(k^\theta). \tag{A.2.4}$$

The payoff of defaulting from period W onwards is Eq. (1.4.14):

$$V_0^W = V^\theta(\bar{B}(k^\theta), k^\theta) = c^\theta(k^\theta) + v\left[g^\theta(\bar{B}(k^\theta), B^\theta, k^\theta)\right] + \beta\cdot\frac{1}{1-\beta}\cdot\left\{c^\theta(k^\theta) + v\left[g^\theta(B^\theta, B^\theta, k^\theta)\right]\right\} \tag{A.2.5}$$

with

$$g^\theta(\bar{B}(k^\theta), B^\theta(k^\theta), k^\theta) = \tau\cdot[Z^{1-\theta}\cdot f(k^\theta) - \delta\cdot k^\theta] - \theta\cdot\bar{B}(k^\theta) + \beta\cdot\theta\cdot B^\theta(\bar{B}(k^\theta), k^\theta)$$

$$g^\theta(B^\theta(k^\theta), B^\theta(k^\theta), k^\theta) = \tau\cdot[Z^{1-\theta}\cdot f(k^\theta) - \delta\cdot k^\theta] - (1-\beta\cdot\theta)\cdot B^\theta(k^\theta).$$

To calculate the value function $V^W(B_0;\pi)$, we apply backward induction. To simplify notation we redefine $V^\theta(B_0) = V^d$ and $V^\theta[\bar{B}(k^\theta)] = V^\theta$, to obtain:

$$\begin{aligned}
V_2^W &= (1-\pi)\cdot[c^\pi(k^\pi)+v(g^W)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)]+\pi\cdot\beta\cdot V^d+\\
&+(1-\pi)\cdot\beta\cdot\{(1-\pi)\cdot[c^n(k^\pi)+v(g^W)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)]+\pi\cdot\beta\cdot V^d+(1-\pi)\cdot\beta\cdot V^\theta\}\\
&=(1-\pi)\cdot[c^\pi(k^\pi)+v(g^W)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)]+\pi\cdot\beta\cdot[1+(1-\pi)\cdot\beta]\cdot V^d+\\
&+(1-\pi)\cdot\beta\cdot\{(1-\pi)\cdot[c^n(k^\pi)+v(g^W)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)]\}+[(1-\pi)\cdot\beta]^2\cdot V^\theta
\end{aligned}$$

$$\begin{aligned}
V_3^W &= [1+(1-\pi)\cdot\beta]\cdot\{(1-\pi)\cdot[c^\pi(k^\pi)+v(g^W)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)]\}+\\
&+\pi\cdot\beta\cdot[1+(1-\pi)\cdot\beta+((1-\pi)\cdot\beta)^2]\cdot V^d+\\
&+((1-\pi)\cdot\beta)^2\cdot\{(1-\pi)\cdot[c^n(k^\pi)+v(g^W)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)]\}+\\
&+((1-\pi)\cdot\beta)^3\cdot V^\theta\\
&\vdots\\
V_W^W &= [1+(1-\pi)\cdot\beta+\cdots((1-\pi)\cdot\beta)^{W-2}]\cdot\{(1-\pi)\cdot[c^\pi(k^\pi)+v(g^W)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)]\}+\\
&+\pi\cdot\beta\cdot[1+(1-\pi)\cdot\beta+\cdots+[(1-\pi)\cdot\beta]^{W-1}]\cdot V^d+\\
&+[(1-\pi)\cdot\beta]^{W-1}\cdot\{\pi^n\cdot[c^n(k^\pi)+v(g^W)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)]\}+\\
&+[(1-\pi)\cdot\beta]^W\cdot V^\theta
\end{aligned}$$

As per definition $V_W^W(B_0;\pi)=V^W(B_0;\pi)$, then the expected payoff of running up the debt in W periods while a default does not occur is:

$$\begin{aligned}
V^W(B_0) = \frac{1-[(1-\pi)\cdot\beta]^{W-1}}{1-(1-\pi)\cdot\beta}&\cdot\{(1-\pi)\cdot[c^\pi(k^\pi)+v(g^W)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)]\}+\\
&+[(1-\pi)\cdot\beta]^{W-1}\cdot\{(1-\pi)\cdot[c^n(k^\pi)+v(g^W)]+\pi\cdot[c^\theta(k^\pi)+v(g^\theta(k^\pi)]\}+\\
&+\pi\cdot\beta\cdot\frac{1-[(1-\pi)\cdot\beta]^W}{1-(1-\pi)\cdot\beta}\cdot V^\theta(B_0)+[(1-\pi)\cdot\beta]^W\cdot V^\theta[\bar{B}(k^\theta)]
\end{aligned}$$

A.2.4 Participation constraint

In the crisis zone, where a default has not occurred but it is possible with probability π, we have $c = c^{\pi}(k^{\pi})$, $k' = k^{\pi}$ and $q = \beta \cdot (1-\pi) + \beta \cdot \pi \cdot \theta = \hat{\beta}$. To define the participation constraint, we are assuming a stationary debt level B. Then g^{π} is stationary at:

$$g^{\pi} = \tau \cdot [f(k^{\pi}) - \delta \cdot k^{\pi}] - (1-\hat{\beta}) \cdot B$$

If the country defaults in $t+1$, but agents have not anticipated it, then there is a drop in productivity and the price of bonds increases to $q = \beta \cdot \theta$:

$$\begin{aligned} g^{\theta 1} &= \tau \cdot [Z^{1-\theta} \cdot f(k^{\pi}) - \delta \cdot k^{\pi}] - (\theta - \beta \cdot \theta) \cdot B \\ U^{\theta}(k^{\pi}) &= c^{\theta}(k^{\pi}) + v(g^{\theta 1}) \end{aligned}$$

Afterwards:

$$\begin{aligned} g^{\theta 2} &= \tau \cdot [Z^{1-\theta} \cdot f(k^{\theta}) - \delta \cdot k^{\theta}] - (1 - \beta \cdot \theta) \cdot B \\ U^{\theta}(k^{\theta}) &= c^{\theta}(k^{\theta}) + v(g^{\theta 2}) \end{aligned}$$

$$V^{\theta}(k^{\pi}) = c^{\theta}(k^{\pi}) + v(g^{\theta 1}) + \frac{\beta}{1-\beta} \cdot [c^{\theta}(k^{\theta}) + v(g^{\theta 2})]$$

The payoff of not defaulting in t, when there is a probability π of defaulting in posterior periods is:

$$V_t^n(\bar{B}, k^{\pi}) = [c^{\pi}(k^{\pi}) + v(g^{\pi})] + (1-\pi) \cdot \beta \cdot V_{t+1}^n(\bar{B}) + \pi \cdot \beta \cdot V^{\theta}(k^{\pi})$$

$$\begin{aligned} V_t^n(\bar{B}, k^{\pi}) &= [c^{\pi}(k^{\pi}) + v(g^{\pi})] + \\ &+ (1-\pi) \cdot \beta \cdot \left\{[c^{\pi}(k^{\pi}) + v(g^{\pi})] + (1-\pi) \cdot \beta \cdot V_{t+2}^{\theta}(\bar{B}) + \pi \cdot \beta \cdot V^{d}(k^{\pi})\right\} + \pi \cdot \beta \cdot V^{\theta}(k^{\pi}) \end{aligned}$$

$$V_t^n(\bar{B}, k^{\pi}) = \frac{1}{1-(1-\pi) \cdot \beta} \cdot [c^{\pi}(k^{\pi}) + v(g^{\pi})] + \pi \cdot \beta \cdot \frac{1}{1-(1-\pi) \cdot \beta} \cdot V^{\theta}(k^{\pi})$$

$$\begin{aligned} V_t^n(\bar{B}, k^{\pi}) &= \frac{1}{1-(1-\pi) \cdot \beta} \cdot [c^{\pi}(k^{\pi}) + v(g^{\pi})] + \pi \cdot \beta \cdot \frac{1}{1-(1-\pi) \cdot \beta} \cdot [c^{\theta}(k^{\pi}) + v(g^{\theta 1})] + \\ &+ \pi \cdot \beta \cdot \frac{1}{1-(1-\pi) \cdot \beta} \cdot \frac{\beta}{1-\beta} \cdot [c^{\theta}(k^{\theta}) + v(g^{\theta 2})]] \end{aligned}$$

A.3 MATLAB Code

A.3.1 Steady state

$$y_t(k_t) = Z^{1-\theta} \cdot A \cdot k_t^{\alpha}$$
$$y_t'(k_t) = Z^{1-\theta} \cdot A \cdot \alpha \cdot k_t^{\alpha-1}$$

No default $\theta = 1$

$$1 = \beta \cdot \left\{(1-\tau) \cdot \left[y'(k^n) - \delta\right] + 1\right\}$$
$$k^n = \left\{\left[\frac{1-\beta}{\beta \cdot (1-\tau)} + \delta\right] \cdot \frac{1}{A \cdot \alpha}\right\}^{\frac{1}{\alpha-1}}$$

Default

$$1 = \beta \cdot \left\{(1-\tau) \cdot \left[Z^{1-\theta} \cdot y'(k^{\theta}) - \delta\right] + 1\right\}$$
$$k^{\theta} = \left\{\left[\frac{1-\beta}{\beta \cdot (1-\tau)} + \delta\right] \cdot \frac{1}{Z^{1-\theta} \cdot A \cdot \alpha}\right\}^{\frac{1}{\alpha-1}}$$

Crisis

$$1 = \beta \cdot \left\{(1-\tau) \cdot \left[y'(k^{\theta}) \cdot [(1-\pi) + \pi \cdot Z^{1-\theta}] - \delta\right] + 1\right\}$$
$$k^{\theta} = \left\{\left[\frac{1-\beta}{\beta \cdot (1-\tau)} + \delta\right] \cdot \frac{1}{[(1-\pi) + \pi \cdot Z^{1-\theta}] \cdot A \cdot \alpha}\right\}^{\frac{1}{\alpha-1}}$$

A.3.2 Code

```
%MODEL WITH CONSTANT CAPITAL
%1A) PARAMETERS
clear
close all
tic
ngrid=201; % Size of debt grid
% Technology and production parameters
    alfa=0.4;   % Share of capital
    delta=0.05; % Depreciation rate
    A=2;        % Technology
    Z=0.95;     %Default penalty proposed by CK(1996)
    tau=0.2;    % Tax rate
% Preference Parameters
    beta=0.97;  %0.832972=0.97**6 % Individual discount factor
       (in year terms)
%Crisis parameters
    pi=0.02;    %Probability of a panic
    theta=0.90;    % Recovery rate

ZZ=[1 (1-pi)+pi*Z^(1-theta) Z^(1-theta)];
q=[beta beta*(1-pi)+beta*pi*theta beta*theta];

% STEP 1C) FUNCTIONS
prod=@(kk) A*kk^alfa; % Production function
util=@(cc,gg) cc + log(gg);    % Utility from consumption

% STEP 2A) STEADY STATE
k_n=(((1-beta)/(beta*(1-tau))+delta)*(1/(A*alfa*ZZ(1))))^(1/(
   alfa-1));
k_pi=(((1-beta)/(beta*(1-tau))+delta)*(1/(A*alfa*ZZ(2))))^(1/(
   alfa-1));
k_d=(((1-beta)/(beta*(1-tau))+delta)*(1/(A*alfa*ZZ(3))))^(1/(
   alfa-1));

y_n=prod(k_n);
y_pi=prod(k_pi);
y_d=Z^(1-theta)*prod(k_d);

c_n=(1-tau)*(y_n-delta*k_n);
c_pi=(1-tau)*(y_pi-delta*k_pi);
c_d=(1-tau)*(y_d-delta*k_d);

% STEP 1B) DEBT GRID AND THRESHOLDS
bmin=0;     %Minimum level of debt in the grid
Max_debt_gdp=1.5;
bmax=Max_debt_gdp*(y_pi*3/2); %Maximum level of debt in the
   grid
B=linspace(bmin,bmax,ngrid)';

BLowNew=0;
BHighNew=0;
BLow=3;
```

```
BHigh=16;
Bopt=ones(ngrid,1);   %Policy function
posBopt=zeros(ngrid,1);    %Position in the grid

% STEP 2 CONSTRUCT PRIVATE CAPITAL GRID
k_grid=zeros(ngrid,1);
y_grid=zeros(ngrid,1);
q_grid=zeros(ngrid,1);
cc=ones(ngrid);
gg=ones(ngrid);

% STEP 3A Define the initial value function and compute first-
   period value
VFunAux=zeros(ngrid,1);
VDefAux=zeros(ngrid,1);
VFun=zeros(ngrid,1);
VDef=zeros(ngrid,1);

% counters
iter_thres=1;
tol=1.0e-3;
distance_thres=1;

% STEP 4 Iteration on the Bellman's equation
%hold all
while distance_thres>tol
    % CONSTRUCT PRIVATE CAPITAL GRID AND BONDS' PRICE GRID
    for cdebt=1:ngrid   %period t
        if B(cdebt)<=BLow
            k_grid(cdebt)=k_n;
            y_grid(cdebt)=y_n;
            q_grid(cdebt)=beta;
        elseif B(cdebt)<BHigh
            k_grid(cdebt)=k_pi;
            y_grid(cdebt)=y_pi;
            q_grid(cdebt)=beta*(1-pi)+beta*pi*theta;
        else
            k_grid(cdebt)=k_d;
            y_grid(cdebt)=y_d;
            q_grid(cdebt)=beta*theta;

        end
    end

    % CONSTRUCT PRIVATE AND PUBLIC CONSUMPTION GRID IN PERIODS
       WHERE A
    % DEFAULT DOES NOT OCCUR
    % Private Consumption c_t=(1-tau)*[y(k_t)-delta*k_t] + k_t-
       k_(t+1)
    % Public Consumption g_t= tau*[y(k_t)-delta*k_t] + q*b_t -
       theta*b_(t+1)
    for cdebt=1:ngrid   %period t
        for cdebt2=1:ngrid   %period t+1
```

```
            if B(cdebt2)<=BLow
                gg(cdebt,cdebt2)= tau*(y_grid(cdebt)-delta*
                   k_grid(cdebt))+q_grid(cdebt2)*B(cdebt2)-B(
                   cdebt);
                cc(cdebt,cdebt2)= (1-tau)*(y_grid(cdebt)-delta*
                   k_grid(cdebt))+k_grid(cdebt)-k_grid(cdebt2);
            elseif B(cdebt2)<BHigh
                gg(cdebt,cdebt2)= tau*(y_grid(cdebt)-delta*
                   k_grid(cdebt))+q_grid(cdebt2)*B(cdebt2)-B(
                   cdebt);
                cc(cdebt,cdebt2)= (1-tau)*(y_grid(cdebt)-delta*
                   k_grid(cdebt))+k_grid(cdebt)-k_grid(cdebt2);
            else
                gg(cdebt,cdebt2)= tau*(y_grid(cdebt)-delta*
                   k_grid(cdebt))+q_grid(cdebt2)*B(cdebt2)-B(
                   cdebt);
                cc(cdebt,cdebt2)= (1-tau)*(y_grid(cdebt)-delta*
                   k_grid(cdebt))+k_grid(cdebt)-k_grid(cdebt2);
            end
        end
    end

Udebt=util(cc,gg);
Udebt(gg<=0)=-9999999;
Udebt(cc<=0)=-9999999;

% CONSTRUCT PRIVATE AND PUBLIC CONSUMPTION GRID IN PERIOD WHERE
    A
    % DEFAULT OCCURS
    % Private Consumption c_t=(1-tau)*[y(k_t)-delta*k_t] + k_t-
       k_(t+1)
    % Public Consumption g_t= tau*[y(k_t)-delta*k_t] + q*b_t -
       theta*b_(t+1)
    for cdebt=1:ngrid   %period t
        for cdebt2=1:ngrid   %period t+1
            if B(cdebt2)<=BLow
                gg(cdebt,cdebt2)= tau*(y_grid(cdebt)-delta*
                   k_grid(cdebt))+q_grid(cdebt2)*B(cdebt2)-B(
                   cdebt);
                cc(cdebt,cdebt2)= (1-tau)*(y_grid(cdebt)-delta*
                   k_grid(cdebt))+k_grid(cdebt)-k_grid(cdebt2);
            elseif B(cdebt2)<BHigh
                gg(cdebt,cdebt2)= tau*(y_grid(cdebt)-delta*
                   k_grid(cdebt))+q_grid(cdebt2)*B(cdebt2)-B(
                   cdebt);
                cc(cdebt,cdebt2)= (1-tau)*(y_grid(cdebt)-delta*
                   k_grid(cdebt))+k_grid(cdebt)-k_grid(cdebt2);
            else
                gg(cdebt,cdebt2)= tau*(y_grid(cdebt)-delta*
                   k_grid(cdebt))+q_grid(cdebt2)*B(cdebt2)-theta
                   *B(cdebt);
                cc(cdebt,cdebt2)= (1-tau)*(y_grid(cdebt)-delta*
                   k_grid(cdebt))+k_grid(cdebt)-k_grid(cdebt2);
```

```
            end
        end
    end

UDef_t=util(cc,gg);
UDef_t(gg<=0)=-9999999;
UDef_t(cc<=0)=-9999999;

iter=1;
distance_fun=1;
distance_def=1;
distance=2;
   while distance>tol
        for cdebt=1:ngrid
            VFun(cdebt)=-1.0e5;
            VDef(cdebt)=-1.0e5;
            for cdebt2=1:ngrid
                %Zone 1: Choose debt tomorrow in no crisis zone
                if B(cdebt2)<=BLow
                    val1=Udebt(cdebt,cdebt2)+beta*VFunAux(
                       cdebt2);
                    if val1>=VFun(cdebt)
                        VFun(cdebt)=val1;
                        Bopt(cdebt)=B(cdebt2);
                    elseif val1<VFun(cdebt)
                        %VDef(cdebt)=val1;
                    end
                %Zone 3: Choose debt tomorrow in default zone
                elseif B(cdebt2)>=BHigh
                    val3=UDef_t(cdebt,cdebt2)+beta*VDefAux(
                       cdebt2);
                    %val=Vdef(cdebt2);
                    if val3>=VDef(cdebt)
                        VDef(cdebt)=val3;
                        VFun(cdebt)=val3;
                        Bopt(cdebt)=B(cdebt2);
                    end
                %Zone 2: Choose debt tomorrow in crisis zone
                elseif B(cdebt2)<BHigh
                    val2=Udebt(cdebt,cdebt2)+beta*(1-pi)*
                       VFunAux(cdebt2)+beta*pi*UDef_t(cdebt,
                       cdebt2)+beta^2*pi*VDefAux(cdebt2);
                    if val2>=VFun(cdebt)
                        VFun(cdebt)=val2;
                        Bopt(cdebt)=B(cdebt2);
                    elseif val2<VFun(cdebt)
                        VDef(cdebt)=val2;

                    end
                end
            end
        end
        distance_fun=norm(abs(VFun-VFunAux));
```

```
        distance_def=norm(abs(VDef-VDefAux));
        distance=distance_fun+distance_def;
        iter=iter+1;
        if iter/100==round(iter/100)
            iter
            distance_fun
            distance_def
        end
        VFunAux=VFun;
        VDefAux=VDef;
    end
    iter
    %legendInfo{iter_thres} = ['X = ' num2str(iter_thres)];
    %plot(B,VFun)
    %xlabel('B(t)');
    %ylabel('V');
    %legend(legendInfo);
    %drawnow
    %plot(B,B,'--',B,Bopt)
    for thres=1:ngrid
        %% UPDATE LOW DEBT THRESHOLD
        % BLow is the max value of not defaulting even if the
           price is higher

        % Utility of NO default with higher price
        gg_1=tau*(y_n-delta*k_n)-B(thres)+q(3)*B(thres);
        cc_1= (1-tau)*(y_n-delta*k_n);
        UU_1=util(cc_1,gg_1);
        UU_1(gg_1<=0)=-9999999;

        % Utility of default in period T;
        gg_2=tau*(y_d-delta*k_d)-theta*B(thres)+q(3)*Bopt(thres)
           ;
        cc_2= (1-tau)*(y_d-delta*k_d);
        UU_2=util(cc_2,gg_2);
        UU_2(gg_2<=0)=-9999999;

        if 1/(1-beta)*UU_1>VDef(thres)
            BLowNew=B(thres);
        else
        end
        %% UPDATE HIGH DEBT THRESHOLD
        % BHigh is the maximum value for which it is optimal not
            to default when bankers lend at price q2

        % Utility of default in period T in crisis zone;
        gg_3=tau*(y_d-delta*k_d)+q_grid(thres)*Bopt(thres)-theta
           *B(thres);
        cc_3= (1-tau)*(y_d-delta*k_d);
        UU_3=util(cc_3,gg_3);
        UU_3(gg_3<=0)=-9999999;

        % Utility of default after period T;
```

```
        gg_4=tau*(y_d-delta*k_d)+q(3)*Bopt(thres)-Bopt(thres);
        cc_4= (1-tau)*(y_d-delta*k_d);
        UU_4=util(cc_4,gg_4);
        UU_4(gg_4<=0)=-9999999;

        % Utility of NOT default in crisis zone at price q2==
           VFun
        gg_5=tau*(y_pi-delta*k_pi)+q(2)*B(thres)-B(thres);
        cc_5= (1-tau)*(y_pi-delta*k_pi);
        UU_5=util(cc_5,gg_5);
        UU_5(gg_5<=0)=-9999999;

        if VFun(thres)>UU_3+beta/(1-beta)*UU_4
            BHighNew=B(thres);
        else
            VFun(thres)=VDef(thres);
        end
    end
    % Check convergence in Thresholds
    distance_thres=abs(BLowNew-BLow)+abs(BHighNew-BHigh)
    iter_thres=iter_thres+1
    BLow=BLowNew
    BHigh=BHighNew
end
%hold off

B_y=B/(y_grid*3/2);
Bopt_y=Bopt/(y_grid*3/2);

BLow_y=BLow/(y_pi*3/2)
BHigh_y=BHigh/(y_pi*3/2)
bmax_y=bmax/(y_pi*3/2);

figure
plot(B_y,Bopt_y,B_y,B_y,'--')
axis([0 Max_debt_gdp 0 Max_debt_gdp]) %sets the limits for the
   x- and y-axis of the current axes.
line('XData', [BLow_y BLow_y], 'YData', [0 Max_debt_gdp], '
   LineStyle', ':', 'Color','m');
line('XData', [BHigh_y BHigh_y], 'YData', [0 Max_debt_gdp], '
   LineStyle', ':', 'Color','m');
xlabel('Initial Debt-to-gdp')
ylabel('Future Debt-to-gdp')
set(0,'DefaultTextInterpreter','LaTex')
set(gca,'XTick',0:0.1:Max_debt_gdp)
set(gca,'YTick',0:0.1:Max_debt_gdp)
set(gca,'fontsize',8)
text(BLow_y, 0.95, '$\underline{b}(k)$', 'Color', 'k')
text(BHigh_y, 0.95, '$\bar{B}(k)$', 'Color', 'k')
text(0.5, 0.4, 'B''(B)', 'Color', 'k')
%title({'Policy Function for debt'})

toc
```

```
Gopt=ones(ngrid);
for i=1:ngrid
        if B(i)<=BLow
            Gopt(i)= tau*(y_n-delta*k_n)+q(1)*Bopt(i)-B(i);
            Gopt_y(i)=Gopt(i)/y_n;
        elseif B(i)<=BHigh
            Gopt(i)= tau*(y_pi-delta*k_pi)+q(2)*Bopt(i)-B(i);
            Gopt_y(i)=Gopt(i)/y_pi;
        else
            Gopt(i)= tau*(y_d-delta*k_d)+q(3)*Bopt(i)-theta*B(i
               );
            Gopt_y(i)=Gopt(i)/y_d;
        end
end

%figure
%Max_g_gdp=0.5;
%plot(B_y,Gopt_y)
%axis([0 Max_debt_gdp 0 Max_g_gdp]) %sets the limits for the x-
    and y-axis of the current axes.
%line('XData', [BLow_y BLow_y], 'YData', [0 Max_g_gdp], '
   LineStyle', ':', 'Color','m');
%line('XData', [BHigh_y BHigh_y], 'YData', [0 Max_g_gdp], '
   LineStyle', ':', 'Color','m');
%xlabel('Initial Debt-to-gdp')
%ylabel('Government spending-to-gdp')
%set(0,'DefaultTextInterpreter','LaTex')
%set(gca,'XTick',0:0.1:Max_debt_gdp)
%set(gca,'YTick',0:0.1:Max_g_gdp)
%set(gca,'fontsize',8)
%text(BLow_y, 0.95, '$\underline{b}(k)$', 'Color', 'k')
%text(BHigh_y, 0.95, '$\bar{B}(k)$', 'Color', 'k')
%text(0.6, 1, 'G(B)', 'Color', 'k')
```

Appendix B

Appendix of Chapter 2

B.1 Summary of empirical literature

Table B.1: Summary of empirical literature on the determinants of sovereign bond yields in the euro area

Reference	Sample	Variables and econometric models
Gerlach, Schulz and Wolff (2010)	12 EMU countries; 01 Jan 1999-28 Feb 2009	**Dependent**: yield to maturity spread **Explanatory**: lag of spread, time to maturity, debt to GDP, three-year-ahead deficit forecasts, banking sector total assets to GDP and the equity over total assets ratio, aggregate risk factor, liquidity, dummies for crises **Model**: panel fixed effect model, random effect model, dynamic panel
Attinasi, Checherita and Nickel (2011)	10 EMU countries; June 2007-March 2009	**Dependent**: monthly and quarterly averages of the yield spread **Explanatory**: public debt to GDP ratio, GDP growth, private debt to GDP, current account to GDP, VIX index, financial factors (volatility of the sovereign spread, volatility of bank stocks, spread on corporate bonds having the same rating), systemic risk indicators **Model**: Panel model
Von Hagen, Schuknecht and Wolswijk (2011)	15 EU countries, 1991 and mid-May 2000	**Dependent**: US$ and DM euro-denominated government bond spreads relative to US and German benchmark bonds **Explanatory**: public debt and deficit relative to GDP as differences relative to the benchmark country; size of debt issue (proxy for liquidity); general investors' risk aversion (yield spread between low grade US corporate bonds (BBB) and benchmark US government bonds); short-term interest rate (proxy for investors' risk aversion); time to maturity of the bonds at the time of issue (additional control related to the investors' risk premium); fiscal variables interacted with EMU dummy and Lehman default dummy **Model**: Panel analysis with time fixed effects
Borgy et al. (2011)	8 EMU countries, January 1999-June 2011	**Dependent**: spreads at 1, 5 and 10 years maturities **Explanatory**: 1-month risk free short term rate, global volatility (VIX), position in the euro-area business cycle (business confidence indicator), a national fiscal balances (the change expected in the debt/GDP ratio over the next 12 months) **Model**: Affine models
Alessandrini et al. (2012)	10 EMU countries; 2000:Q1-2011:Q2.	**Dependent**: spread computed for yields to maturity of 10-y government bonds

Table B.1 Continued: Summary of empirical literature on the determinants of sovereign bond yields in the euro area

Reference	Sample	Variables and econometric models
		Explanatory: primary balance to GDP; public debt to GDP; bid-ask; general risk aversion, as first component of 4 measures of risk; real GDP growth; inflation; labor productivity growth; trade balance to GDP; liabilities to German banks; political risk rating; dummies for crises (where appropriate, variables are measured relative to German values) **Model**: panel corrected standard error (PCSE)
Bernoth and Erdogan (2012)	10 EMU countries; 1999:Q1-2010:Q2.	**Dependent**: yield spreads of individual countries calculated as the end-quarter yield differential of their 10-year benchmark bonds relative to the 10-year German Bund **Explanatory**: debt to GDP ratio and the projected (12-months ahead) deficit to GDP ratio in level and quadratic; bid-ask spread; general risk aversion measured as the spread between the yield of US BBB corporate bonds and the yield of US Treasuries (for Europe data available only from 2002) **Model**: Semiparametric time-varying coefficient model
di Cesare et al. (2012)	10 EMU countries	**Dependent**: monthly and quarterly averages of the yield spread **Explanatory**: public debt to GDP ratio, GDP growth, private debt to GDP, current account to GDP, the VIX index, financial factors (volatility of the sovereign spread, volatility of bank stocks, spread on corporate bonds having the same rating), systemic risk indicators **Model**: Panel model
Favero and Missale (2012)	10 EMU countries; weekly data from June 2006 August 2011	**Dependent**: yield spread. **Explanatory**: expected deficit and debt to GDP (EC forecast) differenced to Germany; global risk factor (US corporate Baa-Aaa spread), "global spread" (at t and t-1) **Model**: VAR; Multivariate GARCH model for Italy and Spain
Giordano, Linciano and Soccorso (2012)	10 EMU countries; January 2002-May 2012; Monthly and quarterly transformed	**Dependent**: Difference in yields to maturity of 10-year government bonds relative to Germany's

Table B.1 Continued: Summary of empirical literature on the determinants of sovereign bond yields in the euro area

Reference	Sample	Variables and econometric models
		Explanatory: fiscal position: gross government debt over GDP, primary balance over GDP, government budget deficit/surplus over GDP, gross government debt over total tax revenues; economic activity: GDP growth, industrial production; external sector: current account balance over GDP, REER; global risk aversion indicator: spread between the yield of US AAA corporate bonds and the yield of US BBB corporate bonds; debt share **Model**: Feasible generalized least square estimator (FGLS) accounting for the presence of AR(1) autocorrelation within panels
Maltritz (2012)	10 EMU countries; 1999-2009	**Dependent**: yield spread **Explanatory**: deficit to GDP, debt to GDP, average interest rate paid on debt, GDP growth, Trade balance to GDP, openness(import+export) to GDP, change in terms of trade index, inflation and its variation, capital formation to GDP, liquidity(proxied by: total amount of outstanding bonds and country's debt in relation to the overall debt of all EMU countries), US riskless interest rate (bond yield from US treasury yield curve for one-year maturity), spread between the yield of BBB US corporate bonds and the yield of US Treasuries **Model**: Bayesian Model Averaging.
Aizenman, Hutchison and Jinjarak (2013)	60 countries (advanced and emerging)from 2005 to 2010	**Dependent**: 5-year sovereign CDSs **Explanatory**: ratio of government debt to tax revenue, ratio of the fiscal deficit to tax revenue, US interest rates, trade openness (import plus export/GDP), GDP growth, per capita GDP. **Model**: Arellano-Bond dynamic panel estimator
de Grauwe and Ji (2013)	EMU countries and 14 stand-alone developed countries from 2000 to 2011	**Dependent**: yield spread with respect to the German Bund **Explanatory**: public debt to GDP ratio, fiscal space, current account, real effective exchange rate, GDP growth, time dummies **Model**: Fixed effect model
Afonso, Arghyrou and Kontonikas (2014)	10 euro area countries; January 1999-November 2010	**Dependent**: 10 year government bond yield (differential versus Germany)

Table B.1 Continued: Summary of empirical literature on the determinants of sovereign bond yields in the euro area

Reference	Sample	Variables and econometric models
		Explanatory: (Log of) S&P 500 implied stock market volatility index (VIX); (Minus) Second principal component of spread; 10 year government bond bid-ask spread; (Log of) CPI based real effective exchange rate; Expected budget balance/GDP (differential versus Germany); Expected debt/GDP (differential versus Germany); Industrial production annual growth (differential versus Germany); Long-term/Total general government debt; Dummy variable: 1 from 2007.08 onwards, zero otherwise; Dummy variable: 1 from 2009.03 onwards, zero otherwise **Model**: Panel analysis; two stage least squares with cross section weights accounting for cross-sectional heteroskedasticity
Bruneau, Delatte and Fouquau (2014)	5 European peripheral countries; January 2006-September 2011	**Dependent**: sovereign bond spread **Explanatory**: debt-to-GDP ratio, unemployment, unit labor cost, risk, liquidity **Model**: Time varying panel smoothing threshold regression
Poghosyan (2014)	22 advanced economies from 1980 to 2010	**Dependent**: Change in real bond yields **Explanatory**: Long-run determinants: potential growth rate and debt-to-GDP ratio; Dummy to account the introduction of the euro; short-run determinants: changes in debt-to-GDP ratio, changes in real money market rate (monetary policy effect), changes in inflation (nominal shocks), changes in the primary balance ratio (short-term fiscal policy), changes in the growth rate (cyclical fluctuations) **Model**: Pooled mean group - panel data version of the error-correction model
Schumacher (2014)	26 European countries from 1996 to 2013	**Dependent**: change in the averaged ratings over the three rating agencies **Explanatory**: consumer sentiment, growth of GDP per capita and Blanchard's primary fiscal gap indicator **Model**: Panel VAR
Afonso and Nunes (2015)	15 EU countries; January 1999-January 2012	**Dependent**: 10-year government bond yields **Explanatory**: corrections made to the EC macro and fiscal forecasts (GDP growth rate, inflation, budget balance, debt ratio, current account, real effective exchange rate); short-term interest rates, fiscal rule index and VIX

Table B.1 Continued: Summary of empirical literature on the determinants of sovereign bond yields in the euro area

Reference	Sample	Variables and econometric models
		Model: panel and individual Seemingly Unrelated Regressions (SUR)
Afonso and Rault (2015)	17 OECD countries; 1973-2008	**Dependent**: long-term sovereign bond yields **Explanatory**: inflation, budgetary and current account imbalances, real effective exchange rates, and liquidity **Model**: Panel cointegration
Afonso et al. (2015)	10 Euro area countries, January 1999-August 2011	**Dependent**: euro area sovereign bond yield spreads against Germany, lagged value **Explanatory**: (Log) S&P 500 implied stock market volatility index (VIX); bid-ask spread of 10-year government bonds; expected (one-year ahead) government budget balance-to-GDP ratio and government debt-to-GDP ratio, measured as differentials versus Germany; annual growth rate of industrial production; (log) real effective exchange rate **Model**: dynamic general-to-specific (GETS) model selection methodology
Suh (2015)	10 EMU countries; January 1999 to June 2007	**Dependent**: Difference between 10-year sovereign debt yield and model-implied long-term yields (reflect the dynamics of the policy rate) **Explanatory**: GDP growth rate, inflation, and government debt to GDP ratio **Model**: propose new measures of financial contagion
Özmen and Yaşar (2015)	23 developing countries; daily data 1998-2012	**Dependent**: sovereign bond spreads **Explanatory**: Country credit ratings as a proxy for domestic fundamentals; volatility implicit in US stock options as a measure of international risk appetite of international ; 3-month USD libor rate to proxy global liquidity conditions **Model**: Conventional panel data procedures; common correlated effects pooled (CCEP) method; FM-OLS; panel autoregressive distribute lag (PARDL) procedure

Source: Giordano, Linciano and Soccorso (2012), updated and extended to 2015

B.2 Data set description

Spreads 10-years government bond yields for Portugal and Germany were retrieved from Eurostat (2015*a*).

GDP Quarterly seasonally and calendar adjusted data was obtained from Banco de Portugal (2015*f*) and Banco de Portugal (2015*e*), respectively for chain linked volumes and current prices. Eviews 9 software was used to transform the data into monthly frequencies (quadratic interpolation, such that the sum of monthly GDP matched quarterly data).

B.2.1 Debt sustainability

Debt-to-gdp The monthly data for the gross debt of general government (Excessive Deficit Procedure) is available starting from June 2011 at Banco de Portugal (2015*c*). Before that period, quarterly data is available at Banco de Portugal (2015*d*). The transformation into monthly data was made through Denton's procedure (Eviews 9), which finds an interpolated series x by relating a higher-frequency indicator series z to a lower-frequency benchmark series y. The benchmark series used was the monthly gross government debt (not according to Maastrich criteria) extracted from IGCP (2015). To obtain debt to GDP ratio, the gross debt was divided by annualized GDP at market prices, that is the moving sum of the current month and previous eleven months.

To check the appropriateness of this monthly transformation, Figure B.1 plots the quarterly debt to GDP directly obtained from the same data source (Banco de Portugal, 2015*d*) and the monthly transformed data.

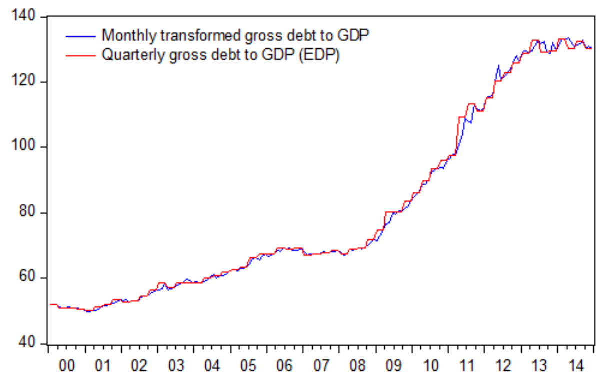
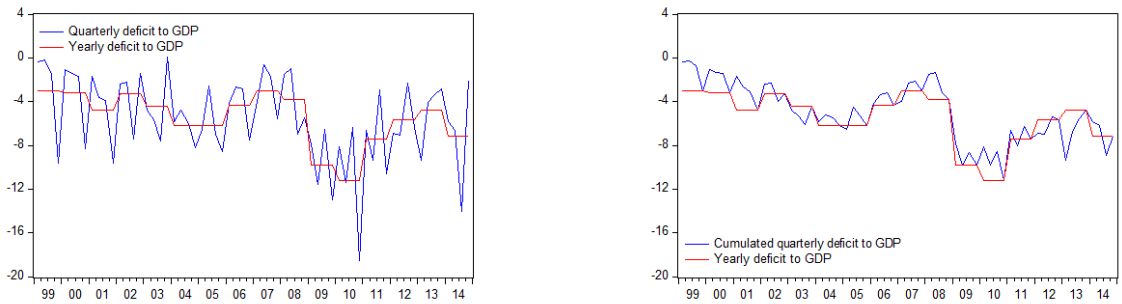


Figure B.1: Comparison of monthly transformed and quarterly original debt as % of GDP
Source: Banco de Portugal (2015*d*) and author's calculations

Debt-to-tax revenues Tax revenue was obtained from DGO (2015*b*) and include direct and indirect taxes. To calculate debt-to-tax ratio, end-of-month debt positions were divided by annualized tax revenues.

Deficit-to-gdp Data on quarterly and yearly government deficit to GDP was extracted from DGO (2015*a*). Figure B.2a plots these series. Given the seasonal pattern of the quarterly data, the quarterly series considered was calculated as the cumulative flows since beginning of the year. This time series is plotted in Figure B.2b.



(a) Yearly and quarterly
Source: DGO (2015*a*)

(b) Yearly and cumulative quarterly
Source: DGO (2015*a*) and author's calculations

Figure B.2: Deficit as percentage of GDP

Finally, to obtain monthly transformed data, a cubic interpolation (Eviews 9) was used with monthly data matching the last quarter value. Figure B.3 plots the quarterly data and the monthly transformed data.

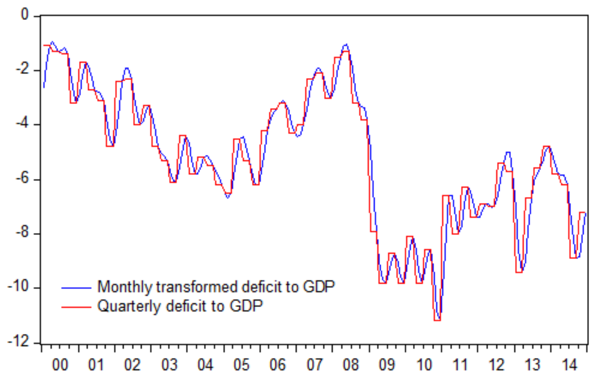


Figure B.3: Monthly transformed and quarterly deficit as percentage of GDP
Source: DGO (2015*a*) and author's calculations

B.2.2 External position

Current account to GDP and net borrowing to GDP Data on the current account was obtained from Banco de Portugal (2015*a*) and correspond to monthly desseasonalized cumulative flows since the beginning of year. To find current account to GDP ratio, those values are divided by the cumulative current prices GDP since beginning year. Net borrowing corresponds to the current and capital account balance. This data set has the same source and analogous procedures as the current account to GDP.

Gross external debt to GDP Corresponds to IMF's financial account's liabilities *vis-a-vis* the rest of the world and focuses on all resident institutional sectors. Quarterly data were obtained from Banco de Portugal (2015*h*) and transformed into monthly data by using a quadratic transformation matching the end of period value. Figure B.4 plots the original quarterly data and the transformed monthly data.

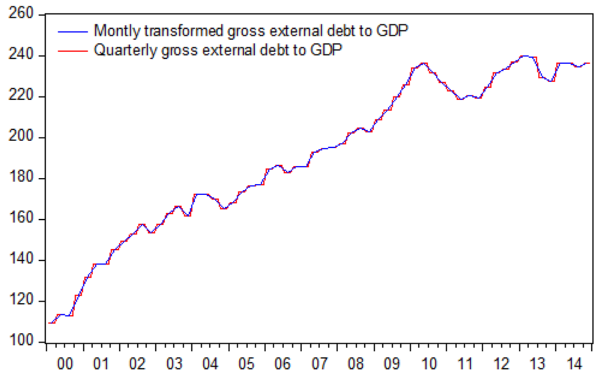


Figure B.4: Quarterly and monthly transformed external debt to GDP
Source: Banco de Portugal (2015*h*) and author's calculations

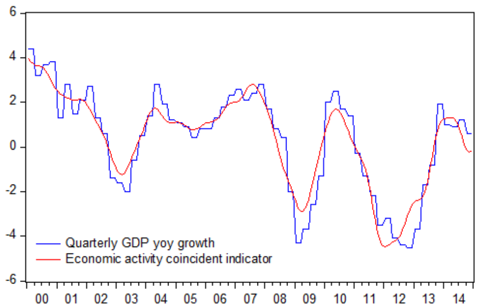
B.2.3 Competitiveness

Real effective exchange rate Monthly REER indices are obtained from ECB (2015*a*), and correspond to ECB Real harmonized competitiveness indicator CPI deflated and calculated on the basis of weighted averages of bilateral exchange rates vis-á-vis the currencies of trading partners: EER-38 group of currencies and Euro area (latest composition) currencies.

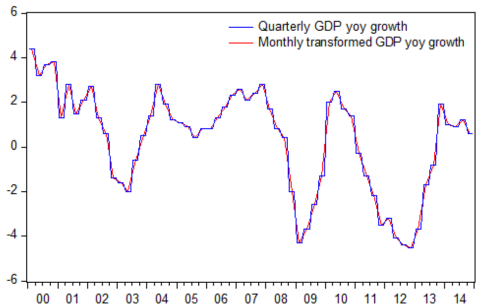
Unit labor costs ULC correspond to the average cost of labour per unit of output and are calculated as the ratio of total labour costs to real output. Quarterly data were collected from OECD (2015) and transformed into monthly frequencies, using a quadratic interpolation such that the ULC average of monthly data matched to the source quarterly data.

B.2.4 Capacity to service the debt

GDP growth Quarterly data for GDP year-on-year growth rate was obtained from Banco de Portugal (2015*g*). The transformation into monthly data was made through Denton's procedure (Eviews 9), using as indicator the economic activity coincident indicator provided by Banco de Portugal (2015*b*).



(a) Quarterly GDP growth and Economic activity indicator
Source: Banco de Portugal (2015*g*,*b*)



(b) Quarterly and monthly transformed GDP growth
Source: Banco de Portugal (2015*g*,*b*) and author's calculations

Figure B.5: GDP year on year growth rate

Unemployment Monthly data of seasonally adjusted unemployment rates were retrieved from Eurostat (2015*b*).

B.2.5 Risk and liquidity

International risk aversion Measured by the spread between US AAA corporate 10-Y yields and US 10-Y sovereign bonds' yields available at FRED (2015).

Liquidity risk Share of Portuguese long-term government securities in total outstanding Euro-denominated long-term government securities issued in the Euro zone (ECB, 2015*b*). General government includes central government, state government, local government and social security funds.

B.3 Additional results

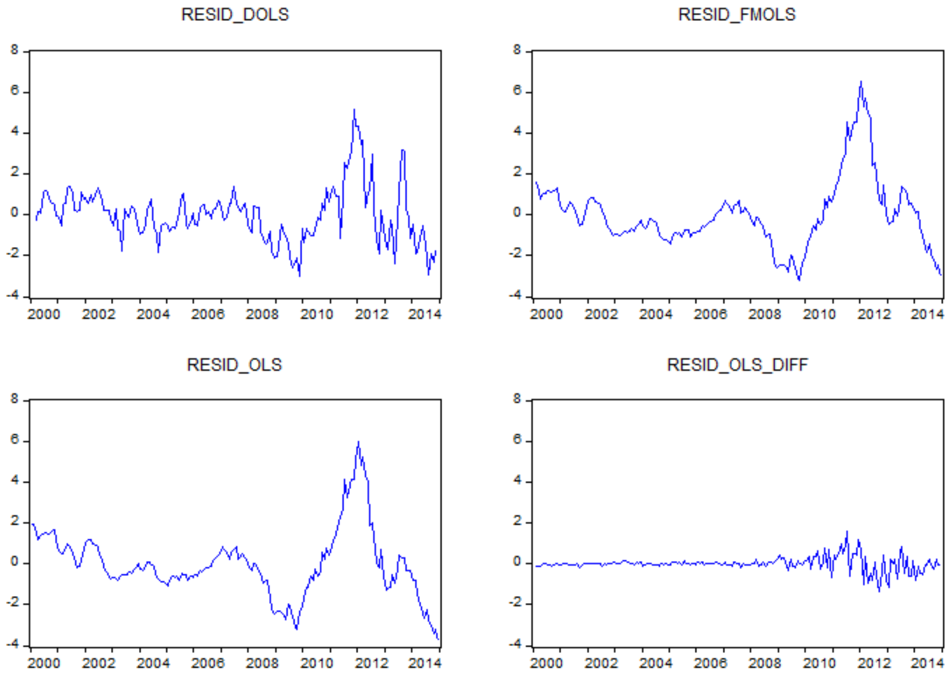


Figure B.6: Residuals of the estimates of Table 2.8
Source: Author's calculations [EViews 9]

Table B.2: VECM short run estimates

	D(spread)	D(debt)	D(sq. debt)	D(growth)	D(liquid)	D(risk)
ECT	−0.051**	0.160**	50.031***	−0.050*	−0.004*	0.004
	(0.021)	(0.078)	(16.679)	(0.031)	(0.002)	(0.008)
D(spread(-1))	0.070	−0.374	−111.802	−0.092	−0.003	0.022
	(0.080)	(0.290)	(62.411)	(0.116)	(0.008)	(0.031)
D(spread(-2))	0.244	0.369	73.539	−0.021	−0.004	0.036
	(0.080)	(0.289)	(62.034)	(0.115)	(0.008)	(0.031)
D(spread(-3))	0.236	−0.337	−111.677	−0.050	0.002	−0.052
	(0.080)	(0.290)	(62.308)	(0.116)	(0.008)	(0.031)
D(spread(-4))	0.218	0.365	95.340	0.162	0.014	0.030
	(0.086)	(0.311)	(66.868)	(0.124)	(0.009)	(0.033)
D(debt(-1))	−0.045	0.605	180.154	0.072	−0.015	−0.041
	(0.116)	(0.422)	(90.600)	(0.168)	(0.012)	(0.045)
D(debt(-2))	−0.037	0.374	96.864	−0.161	−0.009	−0.023
	(0.116)	(0.420)	(90.257)	(0.168)	(0.012)	(0.044)
D(debt(-3))	−0.161	0.246	61.248	−0.102	0.002	0.007
	(0.114)	(0.413)	(88.784)	(0.165)	(0.012)	(0.044)
D(debt(-4))	−0.039	1.068	218.135	−0.279	0.005	−0.029
	(0.109)	(0.393)	(84.459)	(0.157)	(0.011)	(0.042)
D(sq.debt(-1)	0.000	−0.003	−1.016	−0.000	0.000	0.000
	(0.001)	(0.002)	(0.418)	(0.001)	(0.000)	(0.000)
D(sq.debt(-2)	0.000	−0.003	−0.675	0.001	0.000	0.000
	(0.001)	(0.002)	(0.418)	(0.001)	(0.000)	(0.000)
D(sq.debt(-3)	0.001	−0.001	−0.198	0.001	0.000	0.000
	(0.001)	(0.002)	(0.414)	(0.001)	(0.000)	(0.000)
D(sq.debt(-4)ˆ2)	0.000	−0.004	−0.781	0.001	−0.000	0.000
	(0.000)	(0.002)	(0.385)	(0.001)	(0.000)	(0.000)
D(growth(-1))	0.008	−0.218	−60.253	0.753	−0.005	−0.015
	(0.055)	(0.201)	(43.093)	(0.080)	(0.006)	(0.021)
D(growth(-2))	−0.054	0.279	73.256	−0.387	0.013	−0.030
	(0.068)	(0.247)	(53.000)	(0.098)	(0.007)	(0.026)
D(growth(-3))	0.090	−0.113	−25.688	−0.104	−0.006	0.021
	(0.070)	(0.252)	(54.233)	(0.101)	(0.007)	(0.027)
D(growth(-4))	−0.060	−0.116	−9.606	0.209	0.001	−0.011
	(0.057)	(0.205)	(44.019)	(0.082)	(0.006)	(0.022)

Table B.2 Continued: VECM short run estimates

	D(spread)	D(debt)	D(sq. debt)	D(growth)	D(liquid)	D(risk)
D(LR(-1))	−1.905	−2.099	−341.108	−0.071	0.007	0.406
	(0.883)	(3.198)	(687.139)	(1.275)	(0.090)	(0.338)
D(LR(-2))	0.658	1.952	555.611	−0.910	0.128	−0.091
	(0.896)	(3.245)	(697.316)	(1.294)	(0.091)	(0.343)
D(LR(-3))	2.110	−0.283	−272.339	1.054	−0.149	−0.097
	(0.895)	(3.242)	(696.686)	(1.293)	(0.091)	(0.343)
D(LR(-4))	0.998	−6.049	−1359.753	0.395	−0.050	0.200
	(0.911)	(3.297)	(708.573)	(1.315)	(0.092)	(0.348)
D(IRA(-1))	0.422	0.623	153.020	−0.466	−0.009	0.312
	(0.213)	(0.772)	(166.003)	(0.308)	(0.022)	(0.082)
D(IRA(-2))	−0.649	−0.619	−100.049	−0.013	0.012	−0.136
	(0.222)	(0.804)	(172.838)	(0.321)	(0.023)	(0.085)
D(IRA(-3))	0.217	1.427	269.947	−0.413	0.022	−0.046
	(0.223)	(0.809)	(173.870)	(0.323)	(0.023)	(0.085)
D(IRA(-4))	0.023	−1.049	−243.511	0.094	0.006	−0.064
	(0.213)	(0.773)	(166.040)	(0.308)	(0.022)	(0.082)
C	−0.013	0.301	53.573	−0.019	0.004	0.010
	(0.034)	(0.124)	(26.692)	(0.050)	(0.003)	(0.013)
Log likelihood	−44.008	−269.193	−1208.969	−108.342	356.441	124.119
BIC	1.270	3.844	14.584	2.006	−3.306	−0.651
F-statistic	2.935	1.992	2.517	5.717	1.111	1.439

Standard errors in parenthesis

Table B.3: Selection of variables to include in TVTP in the specification in levels

Variables in transition probabilities	Contemporaneous transition variables			Lagged transition variables		
	BIC	LL	DW	BIC	LL	DW
All relevant[a]	3.668	−285.965	0.126	1.490	−89.956	0.050
Debt	0.791	−42.608	0.106	0.796	−43.121	0.107
Fiscal space	0.774	−41.109	0.107	0.780	−41.662	0.108
Deficit	0.769	−40.700	0.061	0.786	−42.150	0.036
Current account	1.039	−64.968	0.080	1.035	−64.600	0.095
Gross external debt	0.795	−42.989	0.107	1.718	−126.117	0.072
REER	0.767	−40.475	0.107	0.773	−41.033	0.078
ULC	0.774	−41.056	0.107	0.780	−41.650	0.107
GDP growth	1.374	−95.118	0.111	1.308	−89.136	0.089
Unemployment	0.776	−41.288	0.081	0.806	−43.951	0.108
IRA	0.751	−39.061	0.050	0.826	−45.808	0.188
Liquidity	0.778	−41.457	0.107	0.784	−42.023	0.108

[a] Debt to GDP, GDP growth, International risk aversion and Liquidity.

Table B.4: Selection of variables to include in TVTP in the specification in differences

Variables in transition probabilities	Contemporaneous transition variables			Lagged transition variables		
	BIC	LL	DW	BIC	LL	DW
Relevant variables[a]	−0.758	109.365	1.586	−1.188	147.851	1.578
Debt	−1.301	142.362	1.579	−1.301	142.384	1.580
Fiscal space	−1.320	144.126	1.580	−1.321	144.159	1.580
Deficit	−1.341	145.936	1.580	−1.334	145.357	1.580
Current account	−1.141	128.026	1.599	−1.153	129.159	1.597
Gross external debt	−1.285	140.988	1.581	−1.285	140.967	1.581
REER	−1.292	141.639	1.581	−1.293	141.658	1.581
ULC	−1.292	141.528	1.580	−1.292	141.551	1.581
GDP growth	−0.840	101.121	1.582	−0.829	100.107	1.579
Unemployment	−1.292	141.537	1.580	−1.291	141.530	1.580
IRA	−1.262	138.881	1.582	−1.270	139.628	1.582
Liquidity	−1.288	141.184	1.582	1.289	141.313	1.512

[a] Debt to GDP, GDP growth, International risk aversion and Liquidity.

Table B.5: Selection of variables to include in RVC-TVTP in the specification in levels

Variables in transition probabilities	Contemporaneous transition variables			Lagged transition variables		
	BIC	LL	DW	BIC	LL	DW
Relevant variables[a]	1.960	−119.295	0.287	4.464	−344.697	0.069
Debt	−0.051	46.166	0.633	0.219	21.854	0.533
Fiscal space	0.185	24.901	0.546	1.360	−80.846	0.431
Deficit	0.027	39.065	0.383	0.211	22.598	0.377
Current account	0.436	2.313	0.383	0.314	13.325	0.210
Gross external debt	0.196	23.849	0.527	0.213	22.341	0.541
REER	0.181	25.282	0.649	0.197	23.781	0.622
ULC	0.184	25.012	0.631	0.073	35.017	0.605
GDP growth	1.077	−55.394	0.137	0.903	−39.696	0.054
Unemployment	0.200	23.528	0.475	0.218	21.953	0.482
IRA	0.736	−24.680	0.638	0.244	−19.531	0.540
Liquidity	0.185	24.852	0.602	0.202	23.356	.620

[a] Debt to GDP, GDP growth, International risk aversion and Liquidity.

Table B.6: Selection of variables to include in RVC-TVTP in the specification in differences

Variables in transition probabilities	Contemporaneous transition variables			Lagged transition variables		
	BIC	LL	DW	BIC	LL	DW
Relevant variables[a]	−0.686	113.246	1.763	−1.059	162.200	1.769
Debt	−1.230	146.365	1.786	−1.230	146.389	1.786
Fiscal space	−1.249	148.124	1.787	−1.250	148.159	1.787
Deficit	−1.269	149.933	1.786	−1.263	149.363	1.786
Current account	−1.067	131.813	1.778	−1.081	133.031	1.766
Gross external debt	−1.216	145.104	1.779	−1.215	145.083	1.779
REER	−1.222	145.706	1.777	−1.223	145.746	1.777
ULC	−1.222	145.645	1.778	−1.222	145.658	1.778
GDP growth	−0.767	104.957	1.702	−0.757	104.069	1.701
Unemployment	−1.220	145.532	1.786	−1.220	145.534	1.786
IRA	−1.192	143.000	1.779	−1.200	143.748	1.780
Liquidity	−1.218	145.298	1.778	−1.219	145.427	1.778

[a] Debt to GDP, GDP growth, International risk aversion and Liquidity.