

A Work Project, presented as part of the requirements for the Award of a Master's degree
in International Finance from the Nova School of Business and Economics.

TAIL RISK DYNAMICS IN EQUITY MARKETS: ESTIMATION, IMPLICATIONS,
AND PREDICTIVE POWER OF CONDITIONAL TAIL INDEXES

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16/01/2025

Abstract

This work project investigates the significance of tail risk in U.S. and European financial markets, focusing on stock-specific tail indexes and their implications for asset pricing and return predictability. Utilizing a flexible framework based on ordinary least squares (OLS), the work project estimates these tail indexes while incorporating various macroeconomic and financial covariates. By evaluating the risk premium and the predictive power of the tail indexes, this research contributes valuable insights into how financial risks affect market prices. The main finding of the analysis is the discovery of a significant tail risk premium for European stocks.

Keywords:

Tail Risk, Asset Pricing, Conditional Tail Index Estimation, Predictive Modeling, Extreme Market Events

Acknowledgement:

I would like to sincerely thank my supervisor, Professor Paulo M. M. Rodrigues, for his invaluable guidance and support throughout this work project.

This work used infrastructure and resources funded by Fundação para a Ciência e a Tecnologia (UID/ECO/00124/2013, UID/ECO/00124/2019 and Social Sciences DataLab, Project 22209), POR Lisboa (LISBOA-01-0145-FEDER-007722 and Social Sciences DataLab, Project 22209) and POR Norte (Social Sciences DataLab, Project 22209).

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1 Introduction

In recent years, the importance of understanding and managing tail risk in financial markets has become increasingly apparent. Traditional financial risk models, which often rely on assumptions of normal distribution, have proven inadequate in capturing the frequency and impact of extreme market events. This work project aims to contribute to the growing body of literature on tail risk by applying a novel approach to estimating stock-specific tail indexes and evaluating their significance in asset pricing and return predictability. Building upon the work of Nicolau and Rodrigues (2024), this study employs a flexible framework that allows for the estimation of conditional tail indexes using ordinary least squares (OLS). This method enables the assessment of the impact of various macroeconomic and financial covariates on the tail behavior of stock returns. The analysis encompasses a diverse set of stocks from both the U.S. and European markets, spanning multiple sectors and time periods.

The primary objectives of this study are threefold:

1. To estimate and analyze stock-specific tail indexes using a comprehensive set of covariates.
2. To evaluate the significance of these tail indexes in explaining stock returns beyond traditional factor models.
3. To assess the predictive power of tail indexes for future stock returns through a recursive forecasting exercise.

By addressing these objectives, this work project aims to provide valuable insights into the nature of tail risk in equity markets and its implications for asset pricing.

2 Literature Review

Tail risk has become a central focus in financial literature due to its potential to cause large price swings during extreme market events. Standard financial risk models, which do not account for the tails of the returns distribution, often fall short in the real world as highlighted by Du and Escanciano (2017). Therefore, understanding and managing tail risk is crucial for both market participants and regulators. Mandelbrot (1963) was one of the first to challenge traditional models of financial returns, such as the Gaussian model, by proposing power law distributions, which better capture the significant role of extreme events. Gabaix (2009) added to the literature regarding such distributions by demonstrating that power laws are pervasive in economics and finance, particularly in the distribution of stock market returns, firm sizes, and wealth. These works underscore that extreme events are more frequent than standard models predict, emphasizing the importance of tail risk in financial analysis.

Early contributions such as Hill (1975) laid the groundwork by introducing a method to estimate the tail of a distribution, relying on the Pareto distribution to approximate the behavior of extreme values. Davison and Smith (1990) build on the work of Hill (1975) by using the Generalized Pareto Distribution (GPD) to model exceedances over thresholds, along with a robust methodology for parameter estimation, threshold selection and diagnostics. The above-mentioned methods have proven to be very effective for tail index estimation and remain widely used (Gardes, Guillou, and Schorgen 2012; van Oordt and Zhou 2016; James et al. 2023).

Kelly and Jiang (2014) contributed to the field by developing a time-varying measure of

tail risk, identifying common fluctuations in the tail risks of individual firms and applying a dynamic power law structure. This measure tracks how extreme price movements across firms evolve over time, providing insight into the broader systemic risks that might emerge from common tail risk factors. Bollerslev, Todorov, and Xu (2015) offered a different perspective by isolating "jump tail risk" through the decomposition of the variance risk premium, using short-maturity option prices to capture the risk associated with sudden market shocks.

Another set of literature proposed using covariates to estimate the tail index with greater accuracy. Beirlant and Goegebeur (2003) model the tail index as a function of explanatory variables, which allows for capturing the conditional distribution's heaviness based on covariate information. They employ profile maximum likelihood estimation for parameter estimation. Wang and Tsai (2009) expand on this work by examining the asymptotic properties of these estimators, demonstrating their consistency and showing that, asymptotically, they follow a multivariate normal distribution with a non-zero mean.

Building on these advancements, Ma, Wei, and Huang (2020) developed a nonparametric estimator for the conditional tail index, employing a local maximum likelihood approach to link tail risk with predictive covariates. Nicolau, Rodrigues, and Stoykov (2023) extended the work by Wang and Tsai (2009) by adapting it to time series data, particularly by relaxing restrictive i.i.d. assumptions and demonstrating that, also in this context, estimates are asymptotically multivariate normal.

Recently, Nicolau and Rodrigues (2024) expanded on the work of Wang and Tsai (2009) and Beirlant and Goegebeur (2003) by proposing a novel framework that allows for the

application of estimation methods such as OLS to be used for the estimation of the tail index of heavy-tailed distributions. Their model is particularly flexible, allowing for the impact of covariates on the tails of return distributions to be assessed in a wide range of financial contexts. This paper forms the methodological basis for the tail risk estimation in this work project and demonstrates the power of econometric techniques in capturing conditional tail risk.

Another set of studies focuses on the impact that tail risk has on asset prices. Early works by Rietz (1988) and Barro (2006) establish that low-probability high-impact economic events can explain the historically high equity premium and other asset pricing puzzles. Using a more data-driven and empirical approach, Kelly and Jiang (2014) find that tail risk has strong predictive power for aggregate market returns, forecasting a 4.5% increase in excess market returns with a one standard deviation rise in tail risk. The finding that tail risk can act as a strong predictor of returns is further supported by Aboura and Arisoy (2019), who link size and idiosyncratic volatility anomalies to tail risk exposure, revealing that smaller stocks and those with higher volatility experience significant negative exposure during market stress. Suh, Yoo, and Yoon (2021) also demonstrate that tail risk indicators derived from S&P 500 options significantly predict future stock returns. Additionally, Gao, Lu, and Song (2019) highlight that assets with lower exposure to a global tail risk index tend to yield higher returns, emphasizing the global impact of extreme market events. Lastly, Andersen, Fusari, and Todorov (2020) note a disconnect between priced tail risk and realized volatility, suggesting that negative jump risk premiums are crucial drivers of variance risk premiums.

3 Data

To conduct an in-depth analysis of stock-specific tail indexes, daily covariate and equity price data was collected using a Bloomberg Terminal for the period from January 1st, 1999 to September 16th, 2024. Since tail events occur, by definition, only in the tails of a distribution, a large sample size is required to fully allow for accurate tail index estimation. Stock price data was collected for 66 stocks traded on the S&P 500 and for 91 stocks traded on major European exchanges. The stock selection for the American market was done based on data availability and with respect to a balanced distribution across the Global Industry Classification Standard (GICS) sectors. Stock selection for the European stocks was performed based on the current and historical constituents of the EURO STOXX 50 index. Including not only the current, but also the historical members of the index, allows for a more comprehensive analysis, as companies, which have been de-listed from the index, might have been strongly affected by various left-tail events. The sector classification for the European equities also follows the GICS sectors to allow for a consistent picture across different markets.

The choice of covariates used to estimate the tail indexes was influenced mainly by data availability (daily frequency required to match the equity price data) and by the need to cover as many different aspects of the macroeconomic and financial outlook as possible. Therefore, 10 distinct covariates were used in the estimation. To capture recent market moves, the daily log returns of the (i) S&P 500 and the (ii) EURO STOXX 50 were used. The inclusion of U.S. market data for the European tail indexes, and vice versa, is motivated by the high level of globalization in financial markets, where moves in one region can quickly reverberate in other areas of the world as well. Rapach, Strauss, and

Zhou (2013) have shown that lagged U.S. stock returns are a significant predictor of returns across other developed non-U.S. stock markets. Investor expectations about near-future volatility are characterized by the (iii) CBOE Volatility index, VIX, and its European counterpart the (iv) V2X. These volatility indexes are often used by investors as a “fear gauge” and high values of the indexes signal that investors may be expecting large moves (potential tail events) in the next 30 days. Complementing the volatility indexes, the (v) CBOE SKEW index focuses on downside tail risk specifically by looking at the skew in options prices with various strikes. Long-term macroeconomic expectations are often visible from the yield-curve, therefore (vi) U.S. 2-Year / 10-Year yield spread and the (vii) German 2-Year / 10-Year yield spread were included. The German yield curve was chosen to reflect the Eurozone macroeconomic outlook, as Germany is the largest economy in the region. To measure global currency risk, stemming particularly from less-developed countries that partake in international trade, (viii) USTWEME index was used. As a barometer of global economic health and inflationary pressures, (ix) WTI crude oil futures prices were used. Additionally, the price of crude oil futures has the potential to significantly impact the risks within Energy, Materials or Industrials sectors. Lastly, the (x) DXY index, which tracks the value of the U.S. dollar relative to a basket of major global currencies, reflects currency pressures from developed countries.

The final set of data that was used for performance evaluation of the tail index was data regarding the factor returns of the 5-factor model proposed by Fama and French (2015) as well as the momentum factor, also obtained from the authors’ website. The 6 used variables are available for both the U.S. and European markets, and both sets of geographical data were considered during evaluation. The 5-factor model includes: (i) *Market Risk Premium*, characterized by excess market returns, (ii) *Small Minus Big*,

calculated as the difference in returns between small-cap and large-cap stocks, (iii) *High Minus Low*, defined as the difference in returns of value and growth stocks, (iv) *Robust Minus Weak*, taken as the outperformance of stocks with high operating profitability, and (v) *Conservative Minus Aggressive*, which is computed as the difference in returns of firms that follow conservative investment policies and those that are more aggressive in their investments.

4 Methodology

4.1 Tail index estimation

As mentioned above in the literature review, the framework of the analysis done in this work project is based on the tail risk estimation approach proposed by Nicolau and Rodrigues (2024). The authors pioneered a flexible approach where the conditional tail index can be estimated by applying simple econometric methods, such as ordinary least squares (OLS). An important step in estimating the tail index is deciding which distribution the tail values (values exceeding a selected cut-off point) of the response variable adhere to. In literature, the prevailing approach is to assume either a Pareto or Pareto-type tail distribution, see Beirlant and Goegebeur (2003), James et al. (2023), Ma, Wei, and Huang (2020), Nicolau, Rodrigues, and Stoykov (2023).

To demonstrate the estimation process developed by Nicolau and Rodrigues (2024), consider the time series $(y_t, \mathbf{x}_t)$, $t = 1, \dots, n$, where $y_t \in \mathbb{R}$ is the response variable and $\mathbf{x}_t = (x_{1t}, \dots, x_{Kt})' \in \mathbb{R}^K$ is a K -dimensional vector of explanatory variables. Next, consider that y_t conditional on $\mathbf{x}_t$ is governed by the following Pareto distribution:

$$F(y_t | \mathbf{x}_t, w_n) = \begin{cases} 1 - (y_t / w_n)^{-\alpha(\mathbf{x}_t, \beta)} & \text{if } y_t > w_n, \\ 0 & \text{if } y_t < w_n \end{cases}, \quad t = 1, \dots, n, \quad (1)$$

where $w_n \in \mathbb{R}$ is the tail cut-off point, which is dependent on sample size n and is determined by using the discrepancy measure proposed by Wang and Tsai (2009). Importantly, $\alpha(\mathbf{x}_t, \beta) = \exp(\mathbf{x}_t' \beta)$ as in Beirlant and Goegebeur (2003), Wang and Tsai (2009) and Nicolau, Rodrigues, and Stoykov (2023). In this context, β represents the vector of parameters of all the explanatory variables. Next, focusing only on the subset of indices $S = \{t \in T : y_t > w_n\}$, where $T = \{1, 2, \dots, n\}$ and w_n is the tail cut-off point, we define τ as an index derived from S , representing the observations in this subset (the tail of the distribution). Logarithmic transformations are then applied to the Pareto distribution function leading to the following linear equation:

$$-\ln \left(\ln \left(\frac{y_\tau}{w_n} \right) \right) = \mathbf{x}_\tau' \beta + a_\tau, \quad (2)$$

where a_τ is an error term following a standard Gumbel distribution with mean γ , where $\gamma \approx 0.5777$ is Euler's constant. Lastly, by defining a new variable $z_\tau = -\ln \left(\ln \left(\frac{y_\tau}{w_n} \right) \right) - \gamma$, we arrive at the conditional tail index regression framework:

$$z_\tau = \mathbf{x}_\tau' \beta + \xi_\tau, \quad \tau = 1, \dots, n_0, \quad (3)$$

where ξ_τ is a zero-mean error term. In this form, the parameters associated with the explanatory variables, which characterize the right-tail index, can be straightforwardly computed using OLS. To obtain the parameters for the left-tail index, the response variable y_t is multiplied by -1. For the full proof and validity of the estimation process, see Nicolau and Rodrigues (2024).

4.2 Tail index evaluation

4.2.1 Covariate importance

Firstly, as the sample of stocks in the used dataset spans various regions and industries, a detailed analysis of the individual covariate significance is carried out on both the regional and industry level. Analyzing the covariates' significance helps uncover which factors influence both the left- and right-tail risks of different sectors in both the U.S. and Europe. To ensure comparability, the same set of 10 covariates is used for both U.S. and European stocks. This consistency allows differences in tail risk to reflect market or sector characteristics rather than variations in predictors.

4.2.2 Risk premium

Next, the risk premium associated with the tail index is tested by regressing the individual stock log returns on the Fama-French 5 factors, the momentum factor, and the tail indexes in the following setting:

$$\begin{aligned} ret_t^k = & \alpha + \beta_1 * left_{tail_t}^k + \beta_2 * right_{tail_t}^k + \beta_3 * (Mkt - Rf)_t + \beta_4 \\ & * SMB_t + \beta_5 * HML_t + \beta_6 * RMW_t + \beta_7 * CMA_t + \beta_8 \\ & * MOM_t + \varepsilon_t \end{aligned} \quad (4)$$

where ret_t^k are the log returns of stock k at time t and $left_{tail_t}^k$ and $right_{tail_t}^k$ are the left- and right-tail indexes of stock k at time t , estimated using data up to time $t-1$. This allows for a direct estimation of the tail index risk premium, which is not explained by the other, widely used, factors. Since the tail index is not represented in easily interpretable units, most of the attention is focused on whether the indexes are significant in the above specified regression.

4.2.3 Recursive predictive regressions

As the final step in the evaluation process, a recursive predictive regression procedure is implemented, which allows for a realistic forecasting exercise. The purpose of the forecasting exercise is to check whether the tail index holds any forecasting power when it comes to predicting individual stock returns.

Since tail events are inherently infrequent, for each stock the first 80% of available data is used for initial model training to allow for a sufficient number of observations and a robust model. Afterwards, the model uses information available at time t to make predictions for time $t+1$. Importantly, each day new information becomes available, and the model is re-trained at each step to mimic the behavior of a real investor using all the available data. The setup is thus the following:

At time t , the covariates' parameters are estimated from Equation 3 using lagged data available at time $t-1$. Next, the right- and left- tail index values at time t are predicted using the trained parameters and lagged data available at time t . To ensure a comprehensive analysis of the tail index forecasting power, 5 different predictive regression scenarios are considered with stock returns at time $t+1$ as the dependent variable:

$$ret_{t+1}^k = \alpha + \beta_1 * left_{tail_t}^k + \beta_2 * right_{tail_t}^k + \varepsilon_{1,t+1} \quad (5)$$

$$ret_{t+1}^k = \alpha + \beta_1 * factor_t^i + \varepsilon_{2,t+1}, i = 1 \dots 6 \quad (6)$$

$$ret_{t+1}^k = \alpha + \beta_1 * left_{tail_t}^k + \beta_2 * right_{tail_t}^k + \beta_3 * factor_t^i + \varepsilon_{3,t+1}, i = 1 \dots 6 \quad (7)$$

$$ret_{t+1}^k = \alpha + \beta_1 * factor_t^1 + \dots + \beta_6 * factor_t^6 + \varepsilon_{4,t+1} \quad (8)$$

$$ret_{t+1}^k = \alpha + \beta_1 * left_{tail_t}^k + \beta_2 * right_{tail_t}^k + \beta_3 * factor_t^1 + \dots + \beta_8 * factor_t^6 + \varepsilon_{5,t+1} \quad (9)$$

The scenarios above cover all the possible combinations of the tail indexes and the traditional factors highlighted in Equation 4. The forecasting power of the tail indexes is then evaluated based on the out-of-sample R^2 of Regression 5 and by comparing the out-of-sample R^2 of Regressions 6 & 7 and Regressions 8 & 9. The formula for the calculation of the OOS R^2 is the following:

$$R_{OOS}^2 = 1 - \frac{\sum_{i=1}^N (y_i - \hat{y}_i)^2}{\sum_{i=1}^N (y_i - \bar{y})^2}$$

where y_i is the actual observed value in the evaluation set, $\hat{y}_i$ is the predicted value in the evaluation set, and $\bar{y}$ is the historical mean value. Importantly, in the recursive predictive regression setting it is necessary to update $\bar{y}$ after each step as new data becomes available.

5 Results

5.1 Covariate importance

As right- and left-tail indexes were estimated individually for each stock in the dataset, covariate significance was recorded for each regression. Stock-specific results were later aggregated at the industry level to assess how each covariate affects the tail risks associated with each industry. Table 1 presents the results for the sample of U.S. equities. Table 2 displays the covariates significance for the European equities.

Table 1 Right- and Left-tail index regression covariate significance for U.S. equities

	Info Tech	Financial	Communi- cations	Healthcare	Consumer Disc.	Industrial	Consumer Staples	Energy	Utilities	Real Estate	Materials
	Right										
<i>Constant</i>	-0.261	0.978	1.397	0.680	0.030	1.543	0.934	3.083***	1.374	0.387	1.139
<i>SPX</i>	-0.353	-1.339	-0.867	0.288	-0.436	-0.749	-0.295	-0.181	-0.198	-0.451	-1.056
<i>SX5E</i>	0.046	1.115	1.137	0.114	0.067	0.285	0.811	0.735	0.021	1.094	0.570
<i>VIX</i>	-0.917	-3.251***	-2.123**	-1.048	-1.102	-1.919*	-1.196	-1.665*	-1.353	-3.778***	-2.500**
<i>V2X</i>	-0.752	0.929	-0.381	-0.892	-0.654	0.083	-0.737	-1.167	-1.906*	1.137	-0.105
<i>USTWEME</i>	-0.756	-0.542	0.721	0.152	0.057	0.393	0.415	0.892	0.838	0.967	0.312
<i>USYC2Y10</i>	1.555	0.273	1.648*	2.152**	1.061	1.366	2.240**	0.504	1.264	0.473	1.846*
<i>DEYC2Y10</i>	-0.083	0.019	-0.584	-0.647	0.021	-0.065	-0.851	1.591	0.811	-0.446	-0.727
<i>SKEW</i>	1.975**	1.565	1.772*	2.089**	1.696*	1.327	1.646*	-0.812	2.063**	2.191**	1.074
<i>CL1</i>	-0.509	-0.415	-0.308	0.054	-0.421	0.297	0.063	-0.343	0.027	0.559	0.571
<i>DXY</i>	-0.402	-1.490	-0.093	0.633	-0.032	-0.682	-0.394	-0.597	-0.171	-0.655	-0.323
	Left										
<i>Constant</i>	1.436	2.111**	1.196	0.822	1.142	2.025**	0.551	1.954*	1.305	1.392	1.665*
<i>SPX</i>	-0.872	-0.550	-0.031	0.459	-0.069	-0.413	0.501	0.028	-0.091	-0.378	-0.262
<i>SX5E</i>	0.464	-0.466	0.380	-0.337	-0.040	0.030	-0.385	0.937	-0.025	-0.627	-0.232
<i>VIX</i>	0.000	-2.191**	-1.186	-1.093	-1.232	-3.068***	-0.616	-2.547**	-1.253	-2.739***	-1.873*
<i>V2X</i>	-1.243	0.056	-0.622	-0.753	-0.385	0.526	-0.969	0.022	-1.257	0.343	-0.415
<i>USTWEME</i>	0.173	-0.852	0.087	-0.424	-0.003	-0.295	0.096	0.001	-0.844	-0.373	-0.546
<i>USYC2Y10</i>	0.994	-1.017	1.538	1.636	1.121	0.898	1.012	0.242	-0.413	-0.920	0.445
<i>DEYC2Y10</i>	0.624	1.070	0.317	-0.073	-0.096	0.026	-0.071	1.445	2.115**	0.576	0.232
<i>SKEW</i>	0.901	-0.022	0.303	1.826*	0.774	0.332	1.304	0.006	0.574	0.768	0.306
<i>CL1</i>	0.925	-0.375	0.488	-0.142	0.148	0.228	-0.111	0.695	-0.225	-0.590	0.526
<i>DXY</i>	0.433	0.925	0.267	-0.238	0.080	0.762	0.087	0.827	0.040	-0.234	-0.318

Note: *, **, *** refer to statistical significance at the 10%, 5% and 1% significance levels, respectively. For inference purposes HAC standard errors are considered.

Table 2 Right- and Left-tail index regression covariate significance for European equities

	Info Tech	Financial	Communi- cations	Healthcare	Consumer Disc.	Industrial	Consumer Staples	Energy	Utilities	Real Estate	Materials
Right											
<i>Constant</i>	0.867	1.307	1.015	1.361	1.720*	1.173	0.849	2.479**	3.085***	1.337	0.536
<i>SPX</i>	-1.036	-0.686	-0.993	-0.668	-0.566	-0.951	-0.688	-2.309**	-1.345	-0.029	-1.081
<i>SX5E</i>	1.309	0.122	0.345	0.695	-0.021	0.595	0.413	1.541	0.428	-0.115	-0.078
<i>VIX</i>	-0.005	0.110	1.196	0.088	-0.744	-0.276	-0.244	-1.200	0.446	0.268	0.313
<i>V2X</i>	-1.433	-2.152**	-2.792***	-1.492	-1.041	-1.622	-1.918*	-0.962	-2.561**	-1.334	-2.235**
<i>USTWEME</i>	0.510	-0.015	-0.362	-0.600	-0.281	0.101	0.184	0.091	-0.449	-0.165	-0.289
<i>USYC2Y10</i>	0.251	0.291	1.784*	1.360	0.799	0.882	1.739*	0.651	1.078	1.145	0.753
<i>DEYC2Y10</i>	0.785	-0.274	-0.435	-0.146	0.154	-0.532	0.220	0.014	-0.743	1.286	-0.080
<i>SKEW</i>	0.974	0.652	1.304	0.558	0.160	1.089	1.345	-0.507	-0.649	0.372	0.944
<i>CL1</i>	0.206	0.323	-0.196	-0.681	0.380	0.486	0.601	-1.669*	-0.581	-0.670	0.517
<i>DXY</i>	-0.723	0.417	-0.241	-0.405	0.291	0.392	-0.606	-0.303	0.303	0.029	0.215
Left											
<i>Constant</i>	2.096**	1.783*	0.490	0.395	2.226**	2.145**	0.559	3.163***	1.944*	1.023	0.636
<i>SPX</i>	-0.792	0.649	0.554	0.473	0.032	0.264	0.099	2.165**	0.828	0.058	0.557
<i>SX5E</i>	-0.817	-0.689	-0.048	-0.082	-0.164	-0.073	-0.345	-1.801*	-0.641	0.759	-0.366
<i>VIX</i>	-0.373	-0.254	0.313	0.663	-0.589	-0.736	-1.038	-0.425	-0.046	-0.359	-0.443
<i>V2X</i>	-1.428	-2.095**	-2.299**	-2.080**	-1.434	-1.613	-1.009	-1.887*	-2.237**	-0.205	-1.268
<i>USTWEME</i>	-0.311	-0.142	0.195	-0.455	-0.161	0.487	-0.399	-0.151	-1.007	0.203	-0.699
<i>USYC2Y10</i>	0.406	-0.112	0.604	-0.394	0.455	0.116	0.388	0.211	-0.196	0.889	0.017
<i>DEYC2Y10</i>	1.036	0.477	0.869	1.150	0.419	0.331	0.557	0.412	0.383	1.422	0.518
<i>SKEW</i>	0.720	-0.042	1.530	1.188	-0.509	0.473	1.333	-0.682	0.112	-0.277	1.122
<i>CL1</i>	0.782	0.279	0.159	-1.166	0.385	0.229	0.335	0.877	0.171	0.796	0.129
<i>DXY</i>	0.866	-0.117	-0.005	0.180	0.716	0.400	0.493	1.253	0.790	-0.995	0.727

Note: *, **, *** refer to statistical significance at the 10%, 5% and 1% significance levels, respectively. For inference purposes HAC standard errors are considered.

Table 1 reveals that the most significant predictors of U.S. tail risk are *VIX* and *SKEW*, which makes intuitive sense as both represent forward-looking investor expectations about risk and volatility. Interestingly, *SKEW*, which is a left-tail risk measure, is a significant predictor of right-tail risk for 7 different industries (Info Tech, Communications, Healthcare, Consumer Discretionary, Consumer Staples, Utilities, and Real Estate), whereas when it comes to the left-tail it is only significant for the Healthcare sector. However, for the right-tail estimation, the coefficients of *SKEW* have a positive sign, which, given the fact that higher values of the tail index imply a lower probability of tail events, implies that when the *SKEW* index increases (i.e. investors are placing a higher premium on OTM put options, thus paying more for downside protection) the probability of right-tail events decreases. This can be explained by the fact that when investors are more worried about potentially impactful left-tail events, there is not a sufficient level of market optimism to fuel a right-tail event. The *VIX* index is a very significant predictor for 5 out of 11 sectors for the left-tail and for 6 out of 11 for the right-tail. The associated sign is always negative, which implies the logical interpretation that higher future expected volatility leads to a higher probability of tail events. The last variable that is significant for more than 1 sector is *USYC2Y10*, which represents the U.S. yield curve. Significance is observed with positive coefficients in the right-tail, meaning that the yield curve steepening decreases the probability of right-tail events. This could be due to either inflation concerns (increasing long-term yields) or economic slowdown fears (decreasing short-term yields).

When looking at the results for European stocks displayed in Table 2, *V2X* clearly stands out as the most significant covariate, attaining statistical significance in 5 out of 11 sectors for both the right- and left-tail. The associated sign with the European “fear gauge” is

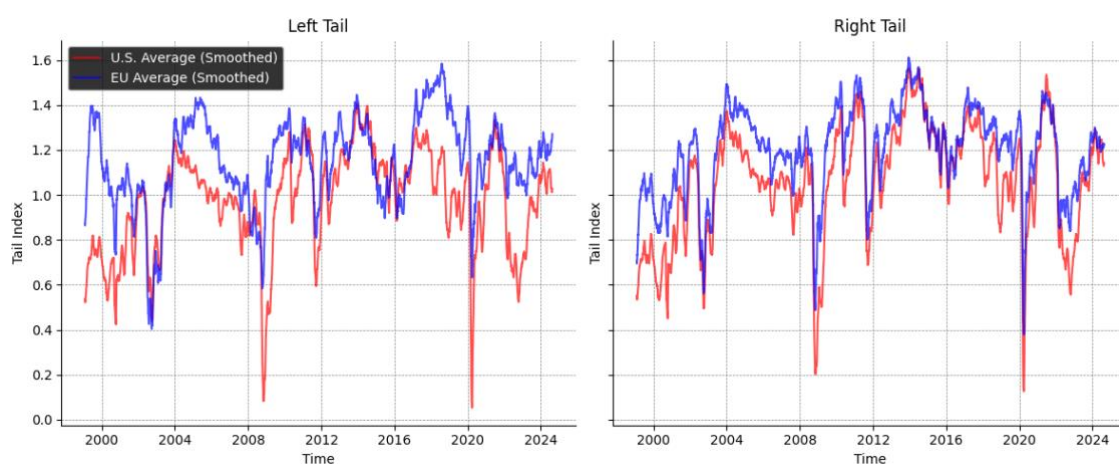
always negative, signaling that higher values of the index are linked to a higher probability of tail events. The only other covariate that is significant in more than 1 sector is, surprisingly, the U.S. yield curve, *USYC2Y10*, which has a significant positive coefficient for the Communications and Consumer Staples sectors in the right-tail. This matches the significance of the yield curve variable from Table 1 regarding U.S. equities, meaning that the right-tail risks of these sectors are influenced by the same factors across different geographies.

The most note-worthy results from Table 2 are found in the Energy sector, which has the highest number of significant covariates in both tails. For the right-tail, *SPX* (S&P500 returns) and *CL1* (price of crude oil futures) are significant with a negative sign, indicating that upwards movement across the U.S. stock market and rises in crude oil prices can significantly increase the probability of right-tail events for European energy companies. This inter-market effect can be explained by global dependence on a single commodity and by the leading position of U.S. energy producers. In the left-tail *SPX* has a positive sign, implying that when the S&P500 is rising, the likelihood of left-tail events in the European energy sector significantly decreases. Counterintuitively, *SX5E*, the variable representing returns of the EURO STOXX 50 index, has a significant negative coefficient, suggesting that increases across the European equity market increase the probability of left-tail events in the sector.

As an example, the aggregated and smoothed tail index series for the Healthcare sector are displayed below in Figure 1. The positioning of the line representing the U.S. tail index consistently below the European tail index line implies that Healthcare stocks on the U.S. market are more likely to experience tail events. This is particularly exemplified

during periods of high market stress, such as the period during the global financial crisis in 2008 and the onset of the COVID-19 pandemic in early 2020. Tail index plots for the other 10 sectors are included in the Appendix.

Figure 1 – Tail Index Averages for Healthcare sector



5.2 Tail Index Correlations

To gain insight into how tail indexes within industries and geographies are related, a correlation analysis was carried out. The results are presented in Table 3 and Table 4. As evident from the average values at the bottom of Table 3, the right-tail indexes are considerably more correlated than left-tail indexes for U.S. equities. This can possibly be explained by the fact that sectors often experience collective positive shocks due to macroeconomic factors, whereas negative shocks, such as lawsuits, fraud, or bankruptcies, are often idiosyncratic. For European equities the difference between correlations of left- and right-tail indexes is much smaller, which can be attributed to the fact that in Europe, despite the Eurozone, countries have distinct fiscal policies, regulatory environments, and economic dynamics, leading to more fragmented within-sector behavior.

Table 3 U.S. Tail index correlations

	Right-tail	Left-tail
Info Tech	0.77	0.60
Financial	0.66	0.54
Communications	0.72	0.57
Healthcare	0.81	0.74
Consumer Disc.	0.64	0.45
Industrial	0.73	0.65
Consumer Staples	0.62	0.57
Energy	0.76	0.76
Utilities	0.83	0.77
Real Estate	0.65	0.66
Materials	0.74	0.54
Average	0.72	0.62

Table 4 EU Tail index correlations

	Right-tail	Left-tail
Info Tech	0.37	0.40
Financial	0.51	0.51
Communications	0.52	0.62
Healthcare	0.54	0.41
Consumer Disc.	0.47	0.59
Industrial	0.65	0.53
Consumer Staples	0.72	0.59
Energy	0.54	0.58
Utilities	0.59	0.64
Real Estate	0.75	0.67
Materials	0.50	0.57
Average	0.56	0.55

5.3 Risk Premium

The risk premium associated with the right- and left-tail indexes is tested by estimating the regression specified in Equation 4 and then analyzing the significance of the tail index coefficients. The setting allows us to see if the tail indexes command a risk premium in excess of the Fama-French 5 factors and the momentum factor. The results of the risk premium analysis are displayed in Table 5 and Table 6 below.

<i>Table 5 U.S. Tail index significance</i>			<i>Table 6 EU Tail index significance</i>		
	Right-tail	Left-tail		Right-tail	Left-tail
Info Tech	-0.46	0.45	Info Tech	-1.88	-0.27
Financial	-1.45	1.85	Financial	-3.16	3.32
Communications	-0.77	0.81	Communications	-4.17	2.36
Healthcare	-0.31	0.38	Healthcare	-3.13	2.89
Consumer Disc.	-0.73	1.06	Consumer Disc.	-1.29	0.93
Industrial	-0.08	0.87	Industrial	-2.98	2.96
Consumer Staples	-0.34	0.58	Consumer Staples	-3.05	2.47
Energy	-0.88	0.85	Energy	-6.20	4.86
Utilities	0.08	0.39	Utilities	-3.25	2.96
Real Estate	-2.11	1.45	Real Estate	-0.27	1.97
Materials	-0.30	0.47	Materials	-2.98	3.21
Average	-0.67	0.83	Average	-2.94	2.52

As evident from Table 5, the U.S. tail indexes reach statistical significance at the 95% confidence level only in the right tail of the Real Estate industry, meaning that, in general, the tail indexes do not command a risk premium for U.S. stocks in excess of the established Fama-French 5 factors and the momentum factor. However, the consistent negative coefficient sign for the right-tail index and positive sign for the left-tail index are aligned with intuition as higher values of the index imply a lower probability of tail events. In this case, a negative coefficient of the right-tail index means that a higher value of the index (lower right-tail event probability) is associated with lower expected returns and vice versa for the left-tail index.

Conversely, Table 6 shows much more promising results in terms of risk premium for

European stocks. The right-tail index is a significant predictor at the 95% confidence level for 8 out of the 11 sectors and the left-tail index for 9 out of the 11 sectors, meaning that the tail indexes command a very significant risk premium not explained by the other factors. The highest significance was observed for the Energy sector, which can largely be attributed to recent global events that have heightened tail risks specific to this industry. The Energy sector has faced substantial volatility due to geopolitical disruptions, including Russia's invasion of Ukraine and ongoing tensions in the Middle East, which have disrupted global oil and gas supplies. These events have increased the likelihood of extreme negative shocks, reflecting heightened sensitivity to downside risks. Likewise, sudden surges in oil prices, driven by breaking news or market developments, frequently trigger significant right-tail moves in the stock prices of energy companies. Other European sectors which display a significant risk premium related to the tail indexes include the Financial, Communications, Healthcare, Industrial, Consumer Staples, Utilities, and Materials sectors.

5.4 Recursive predictive regressions

The realistic forecasting exercise explained in Section 4.2.3 allows for a detailed analysis of the forecasting power associated with the tail indexes. First, the forecasts are made only using the estimated right-tail and left-tail indexes in order to establish their lone forecasting power. Next, each of the traditional factors is used individually to create baseline factor models, to which the tail indexes are later added and the changes in OOS R^2 are observed. Lastly, a baseline forecasting model using all the 6 traditional factors is estimated and later augmented with the tail indexes to measure their complementary forecasting power. Summarized results from the extensive forecasting exercise are

displayed in Table 7 below.

Table 7 Average OOS R^2 for all forecasting scenarios

	Average OOS R^2 (in % form)	
	U.S.	EU
Tails	-0.25	-0.56
1 Factor	-0.07	-0.33
1 Factor + Tails	-0.26	-0.86
6 Factors	0.02	-1.65
6 Factors + Tails	0.06	-1.80
Average	-0.10	-1.04

The only setting where an improvement resulting from adding the tail indexes was found was the 6-factor model for U.S. stocks, where the OOS R^2 increased from 0.02% to 0.06%. For European stocks no aggregate positive OOS R^2 values were observed. The results above suggest that the daily tail indexes may have limited predictive power for one-period-ahead returns. One possible explanation is that the tail indexes primarily capture more long-horizon risks or rare events that do not manifest in one-period returns.

6 Conclusions

This work project has explored the estimation and evaluation of stock-specific tail indexes using a novel OLS-based approach. The findings contribute to the existing literature on tail risk in several ways. First, the study has demonstrated the effectiveness of using a diverse set of macroeconomic and financial covariates in estimating conditional tail indexes for individual stocks. This approach allows for a more nuanced understanding of the factors that influence extreme market events. Second, the analysis of covariate importance across different regions and industries revealed valuable insights into the varying nature of tail risk across different market segments. Most notably, the analysis

uncovered that the *VIX* index, and its European counterpart *V2X*, were consistently significant predictors of tail risk across most sectors in their respective markets. Two other variables that stood out were the *SKEW* index and the U.S. yield spread, which had an effect on both U.S. and European stocks. This information can be particularly useful for risk managers and policymakers in developing targeted strategies for mitigating tail risk. Third, by examining the risk premium associated with tail indexes, the work project has shed light on the role of tail risk in asset pricing. The results suggest that tail risk carries a significant premium on the European equity market that is not fully captured by traditional factor models, underscoring the importance of incorporating tail risk measures into investment decision-making processes. Lastly, the recursive forecasting exercise from Section 5.4 showed that the daily right- and left-tail indexes do not possess much forecasting power. This finding, in light of existing literature (Kelly and Jiang, 2014), suggests that using the tail risk index for forecasting requires a longer time frequency, such as monthly data.

While this study provides important contributions to the field, it also has certain limitations. The analysis was confined to a specific set of stocks and time periods, which may limit the generalizability of the results. Additionally, the choice of covariates, while comprehensive, may not capture all relevant factors influencing tail risk. These limitations open up several avenues for future research. These may include:

1. Exploring alternative sets of covariates, including firm-specific characteristics, to further refine tail risk estimation.
2. Extending the analysis to other asset classes, such as bonds or currencies, to provide a more comprehensive view of tail risk across financial markets.

3. Investigating the implications of tail risk for portfolio construction and risk management strategies.

In conclusion, this work project highlights the critical role of tail risk in financial markets and demonstrates the potential of advanced econometric techniques in capturing and analyzing this risk. The findings underscore the importance of considering tail risk in financial modeling and decision-making processes. As financial markets continue to evolve and face new challenges, a deeper understanding of tail risk will remain crucial for investors, risk managers, and regulators alike.

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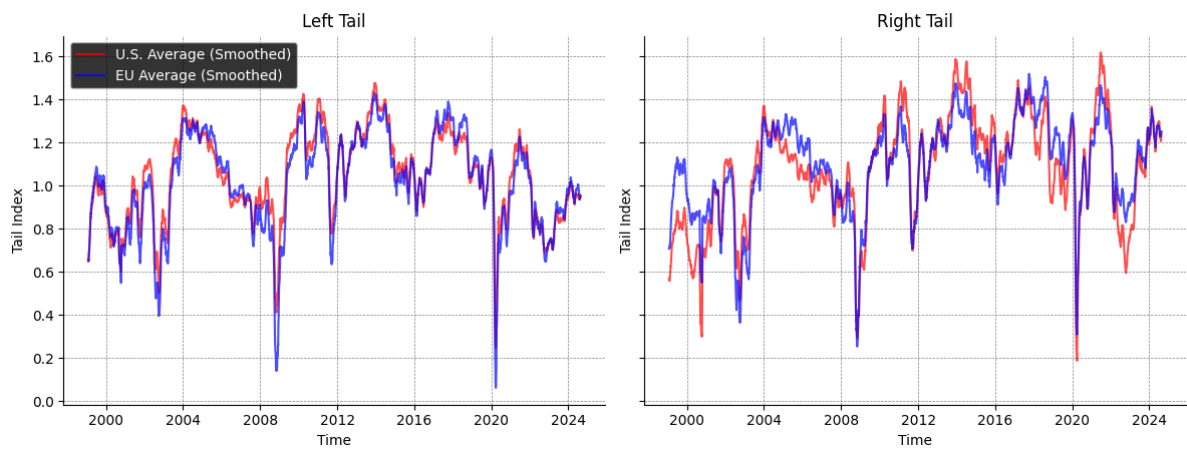
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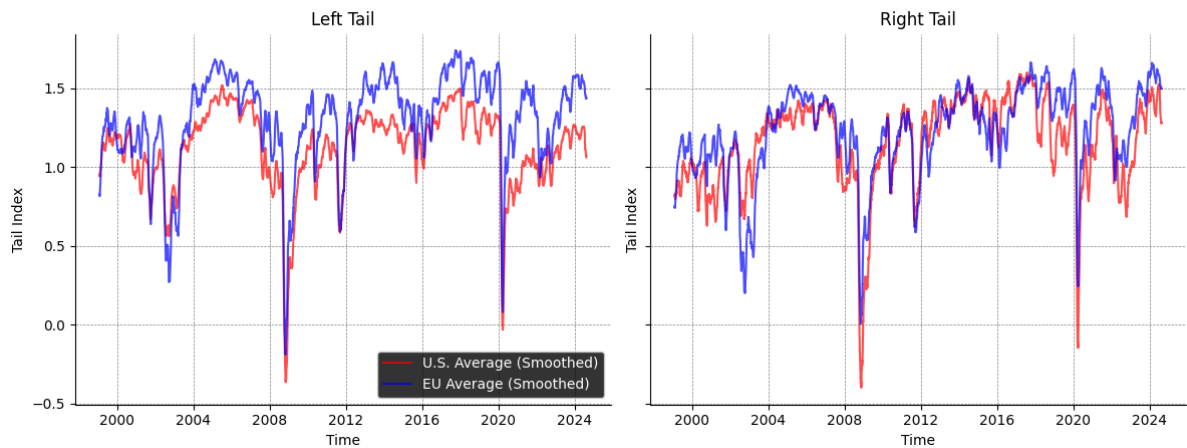
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Appendix

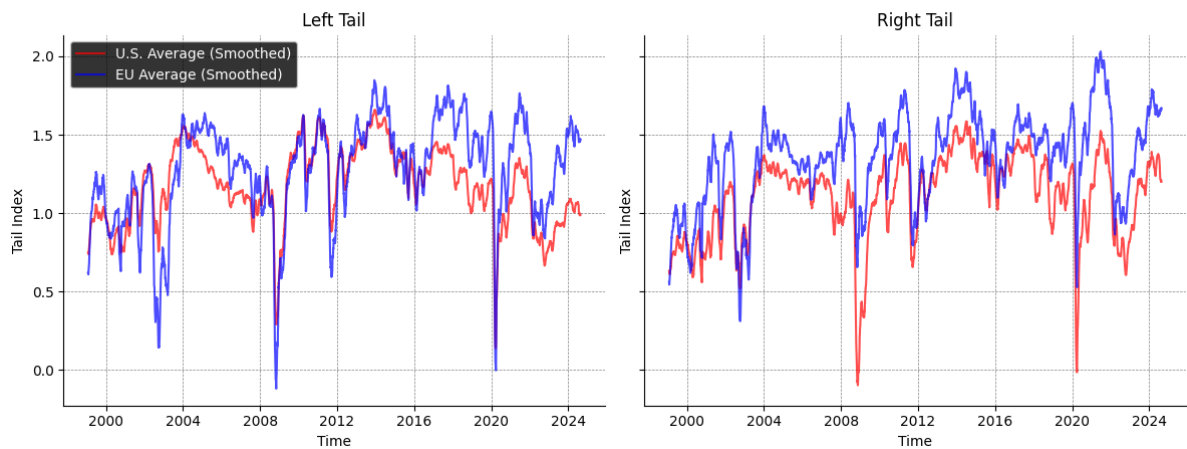
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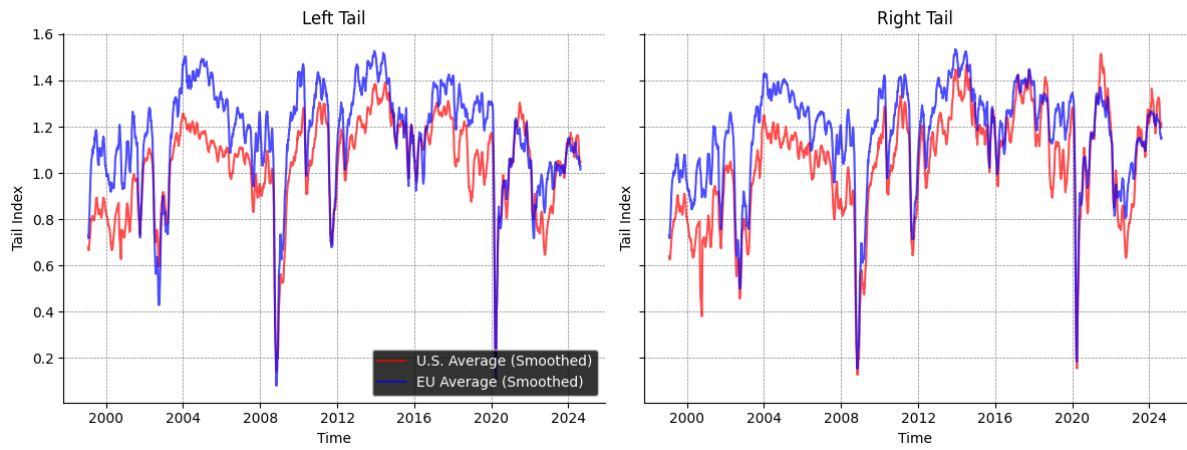
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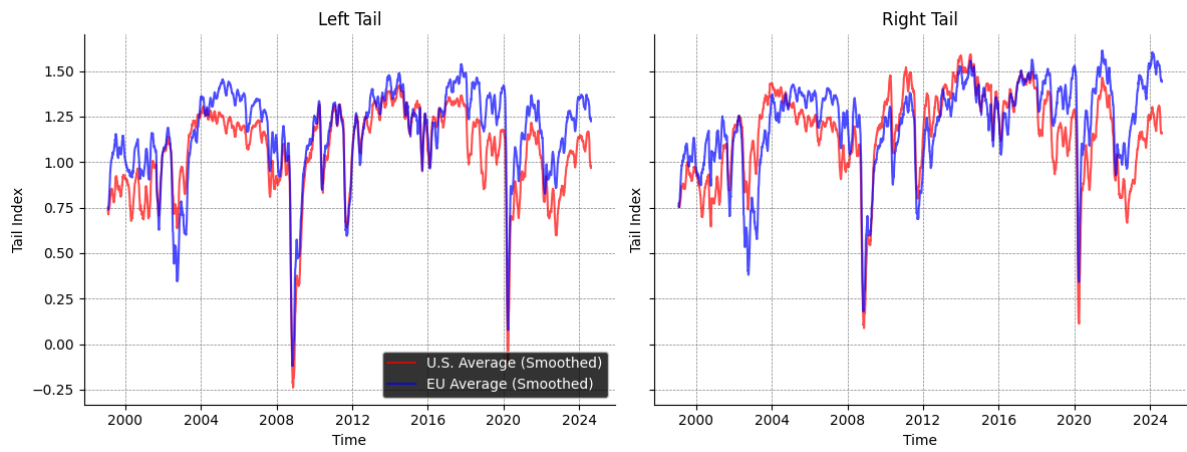
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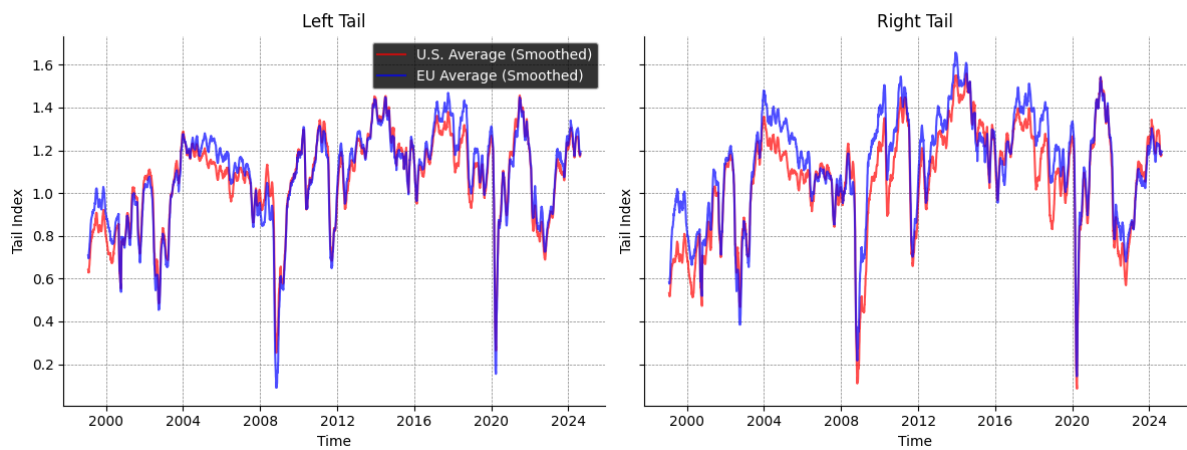
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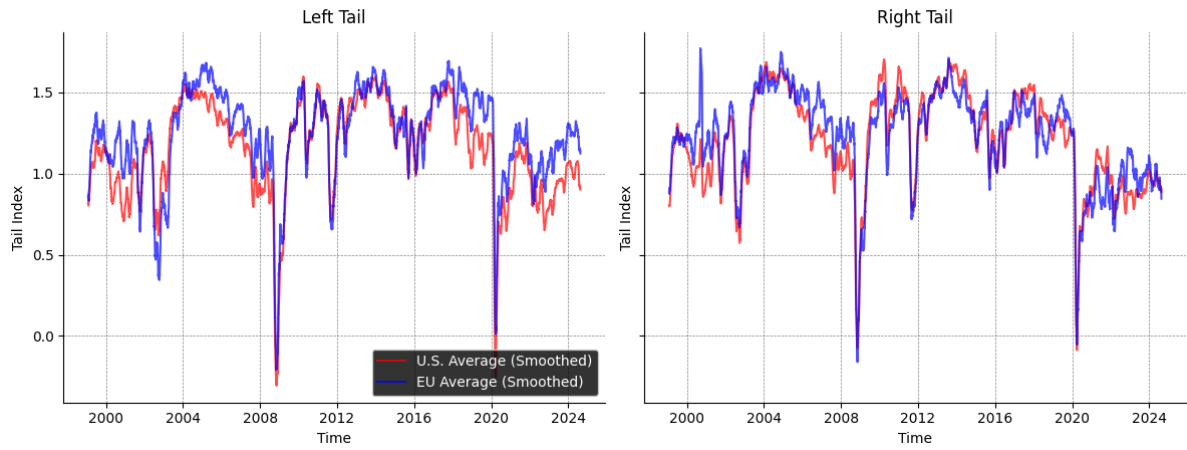
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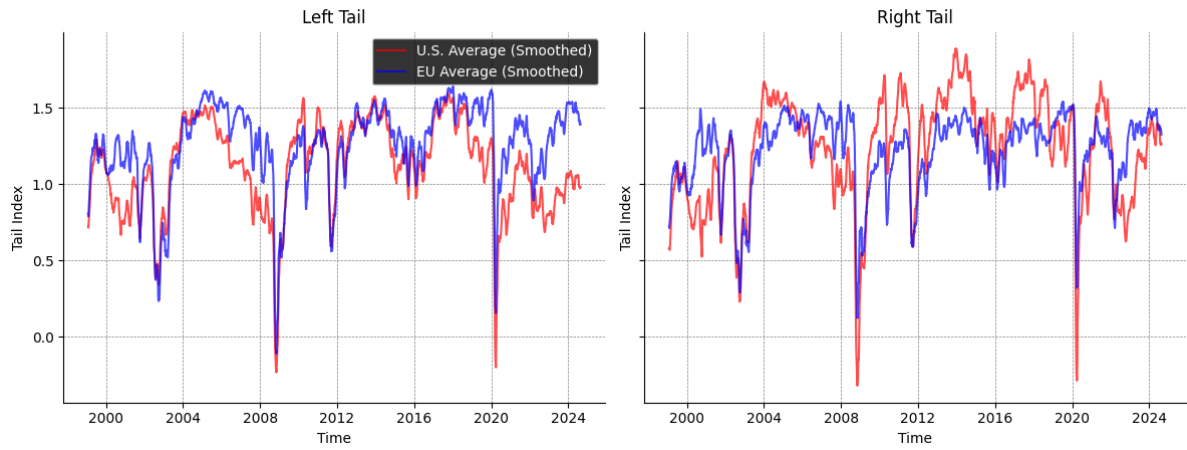
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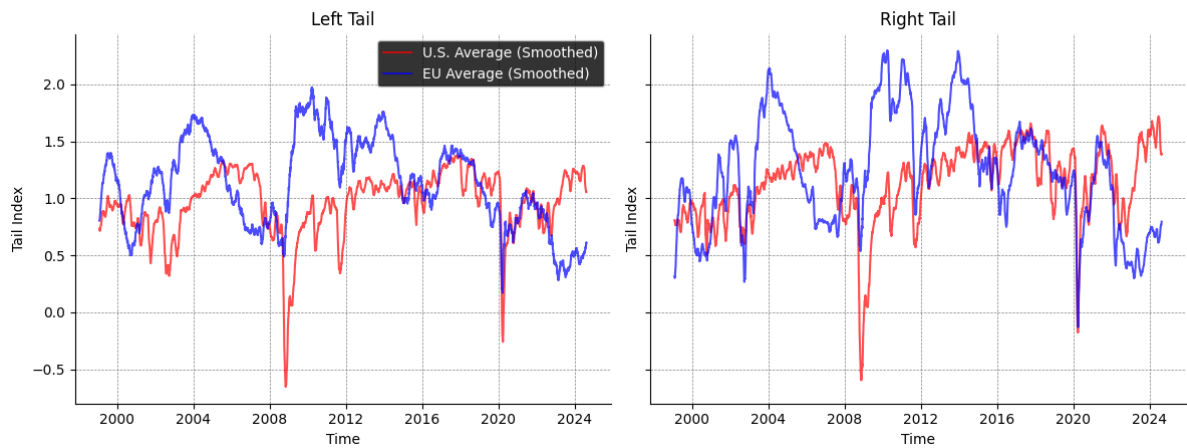
Tail Index Averages for Sector: Energy



Tail Index Averages for Sector: Utilities



Tail Index Averages for Sector: Real_Estate



Tail Index Averages for Sector: Materials

