A Work Project, presented as part of the requirements for the Award of a Master's degree in Management from the Nova School of Business and Economics.

Balancing Patient Demand and Hospital Resources: A Newsvendor-Based Approach to Bed and Nurse Optimization

MARIANNA VICENTE TREMATERRA

Work project carried out under the supervision of:

Patrícia Xufre

17/12/2024

ABSTRACT

Using a two-period Newsvendor-based model, this study examines a data-driven strategy for optimizing nurse staffing and bed allocation in private hospitals. Leveraging a framework initially aimed at optimizing airline seat inventory, the model incorporates probabilistic demand estimation and overbooking strategies to balance patient demand with hospital resources. Despite limitations in assumptions, such as linear cost structures and parameter estimation, the analysis based on data from a cardiac unit in India reveals valuable insights for hospital management, demonstrating the model's potential to improve revenue and minimize costs associated with over and understaffing.

Keywords: newsvendor problem, cost optimization, nurse staffing, data-driven, hospital management, private healthcare

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INTRODUCTION

According to the World Health Organization (2016), nurses comprise half of the world's healthcare workforce, fulfilling an extremely important role within the healthcare and hospital systems globally. With responsibilities including administering and managing patients' medications, providing emotional support, and educating patients on how to manage disease and injuries, they frequently serve as the main point of contact for patients and their families. Numerous studies have highlighted the crucial role that nurses play, with strong evidence that higher nurse staffing levels result in better treatment for hospital patients and a lower risk of death (Griffiths et al. 2023). However, in an aging society where the demand for healthcare services continues to grow as the number of older adults increases (Grah et al. 2019), and where the nurses' salaries account for a large fraction of healthcare facilities' expenses, hospitals face increasing pressure from governments and stakeholders to lower operational expenses (Green, Savin, and Savva 2013). This pressure often leads to negative outcomes, such as reducing the number of nurses employed or overworking the existing workforce by admitting significantly more patients than can be adequately managed. The problem of nurse staffing optimization is therefore directly related to the number of patients admitted to hospitals, with emergency rooms, medical units and admission offices often overcrowded with patients waiting for rooms (Balaji and Brownlee 2018).

As such, optimizing bed allocation is essential for operational efficiency and quality care. This leads to the study's main research question: *Can a two-period model based on the Newsvendor problem be used to maximize nurse staffing levels and bed allocation in private hospitals?* By adapting a data-driven model originally developed by Albuquerque for optimizing airline seat inventory to the context of private hospitals, this study will try to address this question, with a focus on bed allocation and nurse staffing levels.

LITERATURE REVIEW

The Newsvendor problem (NVP), a widely known inventory management model, aims to optimize the inventory of perishable goods with stochastic demand over a single period (Albuquerque 2008). Originally, the problem is based on a newsvendor who must determine how many newspapers to buy at the start of a period in an attempt to maximize their profit or minimize their cost. In a similar context, hospital beds and nursing services can be considered perishable goods, as their availability is time-sensitive and can't be carried over to following periods, just as unsold newspapers in the original problem.

Early research on the Newsvendor problem assumed a known probability distribution for the demand in each period (Arrow, Harris, and Marschak 1951). However, given that real-world demand cannot be predicted, it is realistic to say that the demand distribution is uncertain (Benzion, Cohen, and Shavit 2010). Nevertheless, in the big-data age, there are datasets available which make it possible to develop data-driven models, allowing for a more accurate estimation of uncertainties and yielding better results to the Newsvendor problem (Huber et al. 2019; Bertsimas and Thiele, 2005). As such, this study leverages data-driven methods to enhance the application of the Newsvendor problem.

The traditional NVP considers a single period optimization, where all decisions concern a single discrete time frame, usually a single day. Some examples of this assumption can be found in Ban and Rudin (2019) or in Liu, Letchford and Svetunkov (2022). However, studies have shown that around 65% of nurses work 12 to 13 hour shifts (Stimpfel and Aiken 2013). As such, to achieve more realistic results, this work will follow Albuquerque's approach, developing an extension of the traditional problem with a two-period optimization model.

Relating to the objective function, Green, Savin, and Savva (2013), and Ban and Rudin (2019) opted to uncover the nurse staffing level that minimizes costs, while Liu, Letchford, and Svetunkov (2022) aimed to maximize estimated revenue. This distinction in objective function proves to be important only in the case of non-linear costs. Therefore, given that linear costs will be assumed throughout this work, we will opt to maximize profit, as to focus on achieving operational efficiency.

A further assumption in the NVP is the inclusion of underage (shortage) costs and overage (holding) costs. Shortage costs arise when inventory is insufficient, leading to unsatisfied demand; conversely, holding costs happen when excess inventory remains unsold and must be discarded (Oroojlooyjadid, Snyder, and Takáč 2020). For the purposes of this work, shortage costs will be assumed to occur when the number of patients exceeds predicted admissions, forcing the hospital to experience overcrowding. This situation can lead to adverse consequences, such as higher death and complication rates (Meredith et al. 2024), reputation damage and the financial burden of accommodating patients in non-standard care beds (Krochmal and Riley 1994). On the other hand, holding costs occur when there is surplus capacity, leading to inefficiencies in resource utilization, higher salary costs and unnecessary exposure to risk.

Building on these considerations, the process of solving the NVP is traditionally divided into two phases, first estimating demand for the given period, and later optimizing the order quantity based on the predicted demand (Liu, Letchford, and Svetunkov 2022). Similarly, the model employed in this study uses a disjoint approach. Based on the available historical data, it first models the cumulative density function and the probability density function for both periods to estimate demand. Then, it uses the derived demand to calculate the optimal quantities.

Thus, the purpose of this work project will be to build upon the work of Albuquerque, as to create a model able to optimize bed allocation and nurse staffing levels for a two-period time frame, aiming to provide actionable insights for hospital management.

MODEL FRAMEWORK

For the purpose of this work project, a private medical unit will be considered. This unit operates with two different shifts, the day shift (08h00 to 20h00) and the night shift (20h00 to 08h00). Admissions begin at 08h00 and are processed throughout the day, with no new admissions allowed after 20h00. Discharges are handled at the end of the day, coinciding with the closure of admissions at 20h00. As such, the number of patients is always higher during the day shift, since some patients are admitted for same-day treatment and do not stay overnight. Presuming hospital admission numbers reset at 20h00, the night shift will be regarded as the starting shift, or period 1. Consequently, the day shift for the following day will be considered period 2 of the total 24-hour interval.

After booking an admission at the hospital, patients are assigned to one of the N standard care beds existing in the medical unit. As a private hospital setting is being considered, each patient is required to pay an admission fee (r) at the time of scheduling the admission. This admission fee is considered as the revenue generated for each patient admitted to the hospital.

The demand for period i (D_i) represents the number of individuals seeking admission to the medical unit during that time period. Moreover, the term "admissions available" is defined as the maximum number of patients permitted to schedule an admission at the hospital for a certain time period. The hospital has a fixed limit on the number of admissions available for period 1 (A_1). Any "unsold" admissions from period 1 are carried over to period 2. Therefore, the total number of admissions available in 24 hours is $A = A_1 + A_2$, where A_2 represents the minimum number of admissions allocated for the day shift. It is important to note that A can exceed N, as in some situations patients do not show up for their scheduled appointments, leaving available capacity that can accommodate additional admissions. As such, the model considers overbooking to optimize bed utilization in anticipation of no-shows.

As in Albuquerque's airline model, this work will assume that the availability of night shift admissions (A_1) directly determines how much of the potential demand for night admissions is fulfilled. This study also assumes that a fixed fraction (u) of the unmet demand from period 1 will carry over to the demand in period 2, representing patients who, after being unable to obtain admission in period 1, will seek admission again in period 2. In practical terms, this translates into a certain proportion of patients who, after being unable to secure an admission during the night shift, will not abandon their need for care but will instead try again to gain admission during the day shift.

The hospital unit may face two types of shortage costs. The first type, denoted as s_{1i} , occurs when the demand for period i (D_i) exceeds the available admissions for that period (A_i). This situation forces patients to seek care at another healthcare facility, damaging the hospital's image and reputation. The second type of shortage cost, expressed as s_2, happens when the actual number of patients admitted exceeds N, and the patients must be placed on temporary beds, usually stretchers, leading to increased discomfort, limited mobility, and lower quality infrastructure for treatment and recovery, impacting patients both physically and mentally (Güzelbey Esengün and Alppay 2018). The medical unit can also incur in holding costs h, which represent the expenses incurred for each unoccupied bed among the N available beds at the end of the 24-hour period.

Below is a summary of the parameters used in this work, $i \in \{1, 2\}$:

N: number of physical standard beds in the medical unit

D_i: patient demand for period i

A_1, A: number of admissions made available in period 1, total number of admissions made available for both periods.

r_i: admission fee for period i, representing the hospital's revenue

u: fixed portion of the unsatisfied demand in period 1 that will join the demand in period 2.

h: holding costs, the costs of having empty beds.

s_{11}, s_{12}: type 1 shortage costs, reputation damage incurred when patients seek care elsewhere.

s_2: type 2 shortage costs, incurred when the actual number of admitted patients exceeds the number of beds, leading to reduced quality of care.

The number of admissions for period 1 (Q_1) is defined as the lesser of either the admissions made available for that period, or the actual demand for the night shift. This can be mathematically translated into:

$$Q_1 = \min(D_1, A_1)$$

Analogously, the number of admissions for period 2 (Q_2) will be the lesser of two values: the remaining admissions available after period 1, or the sum of the demand for period two and the portion of unmet demand from period 1 that carries over to period 2. This can be defined as:

$$Q_2 = \min(A - Q_1, D_2 + u(D_1 - Q_1))$$

The random variables representing incurred shortage and holding costs are described as:

$$H = \max(0, N - Q_1 - Q_2)$$

$$S_{11} = \max\big(0, (1 - u)(D_1 - Q_1)\big)$$

$$S_{12} = \max(0, D_2 + u(D_1 - Q_1) - (A - Q_1))$$

$$S_2 = \max(0, Q_2 + Q_1 - N)$$

As such, the profit function which will be maximized is defined as:

$$\pi = r_1 Q_1 + r_2 Q_2 - hH - s_{11} S_{11} - s_{12} S_{12} - s_2 S_2$$

To better understand how this model operates, a visual representation is provided in **Appendix A**.

Through detailed differentiation and application of first-order conditions, Albuquerque was able to arrive at two expressions for determining the optimal values for variables A_1 and A.

Condition 1

$$\big(r_1 - r_2 + (s_{11} - s_{12})(1-u)\big)[1 - F_1(A_1)] +$$

$$+(r_2 + s_{12} - s_2)(1-u)\int_0^{A-A_1} F_1\left(\frac{(A-(1-u)A_1 - D_2)}{u}\right) f_2(D_2)dD_2 -$$

$$-(r_2 + s_{12} - s_2)(1-u)F_1(A_1)F_2(A - A_1) - (h + s_2)(1-u)F_1(A_1)F_2(N - A_1) +$$

$$+(h + s_2)(1-u)\int_0^{N-A_1} F_1\left(\frac{(N-(1-u)A_1 - D_2)}{u}\right) f_2(D_2)dD_2 = 0$$

Condition 2

$$(r_2 + s_{12} - s_2)\Bigg[\int_0^{A_1}\big(1 - F_2(A - D_1)\big)f_1(D_1)dD_1 \; +$$

$$+\int_0^{A-A_1}\left(1 - F_1\left(\frac{A-(1-u)A_1 - D_2}{u}\right)\right) f_2(D_2)dD_2 +$$

$$+\big(1 - F_2(A - A_1)\big)\big(1 - F_1(A_1)\big)\Bigg] = 0$$

To ensure that the equations above lead to a maximum, the concavity of the profit function was guaranteed by demonstrating that its second derivative is non-positive. This assures that any stationary point derived from the first-order conditions is a global maximum. This condition holds as long as $s_2 \le r_2 + s_{12}$ and $F_2(N - A_1) \ge \frac{1}{1-u}\frac{r_2 - r_1}{r_2 + s_{12} + h}$. If these criteria stand, then $\pi(A_1, A)$ is concave. The condition $s_2 \le r_2 + s_{12}$ ensures that the cost of admitting patients beyond N does not go over the combined benefit of the revenue r_2 and the cost of avoiding unmet demand penalties s_{12}, since if it did, it would be more costly for the hospital to accommodate additional patients than to leave them unserved.

DATASET DESCRIPTION AND ANALYSIS

The dataset that will be used in this analysis was collected from Hero DMC Heart Institute, a tertiary care cardiac unit belonging to Dayanand Medical College and Hospital, in Punjab, India. To analyze and manipulate the dataset, Python, a versatile programming language, was employed.

The observations in the dataset cover a period of two years, from April 2017 to March 2019, during which the unit registered a total of 14.845 admissions. The dataset contains data on patients admitted to the hospital, specifically their admission date, discharge date and length of stay at the healthcare unit. This data was used to assess bed occupancy over the time period in question, assuming each patient occupied a single bed for the duration of their stay. Additionally, by assuming that the hospital only handled admissions and discharges during the daytime, estimation of the number of beds occupied in each daytime and nighttime shift was possible. As such, a patient admitted one day and discharged the next would count as occupying a bed during both daytime shifts (on the day they were admitted and on the day they were discharged) but only during the first nighttime shift, not the second.

After calculating bed occupancy for both shifts over this period, information before April 30, 2017 and after March 31, 2019 was excluded, to avoid inaccuracies. Since these dates are at the very start and end of the dataset, they could possibly lead to misleading occupancy counts. For example, there is data available on patients admitted before March 31, 2019 whose stay extended into April, but no information on new admissions from

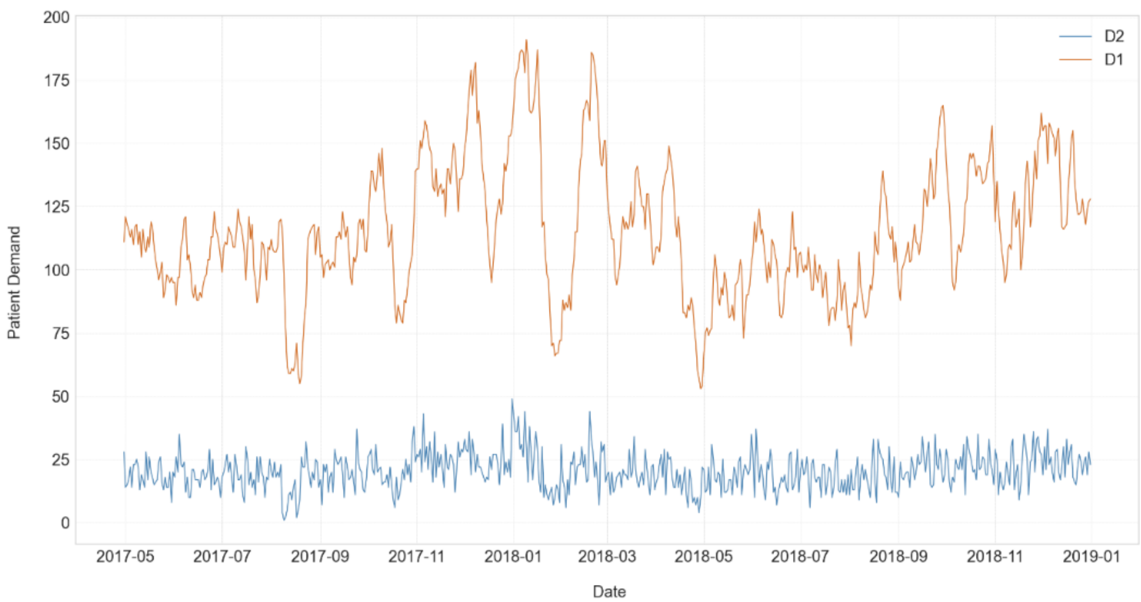
April 1 onward. On the other hand, the number of patients for the month of April 2017 can be misleading, as there are records for new patients being admitted from that day on, but no information on patients whose stay at the hospital started before that.

The number of beds occupied during each night shift is considered as the demand for period 1 (D_1), while the demand for period 2 (D_2) is calculated as the additional beds occupied during the following day shift beyond those already occupied in D_1.

After calculating the values for D_1 and D_2, splitting the data into a training set and a test set was necessary. This ensures that the model is not evaluated on the same data it learned from, preventing overfitting, and allowing us to assess the model's performance when presented with new, unseen data. As such, data from 2017 to 2018, containing 611 observations, or approximately 87% of the data, was used as the training set, while observations for 2019 were used to evaluate the model's performance, comprising 13% of the original dataset. The following analysis will only consider the training set.

The time series analysis of patient demand in **Figure 1** reveals that D_2 follows the trends observed in D_1 but on a significantly smaller scale, suggesting a relatively proportional relationship between the two variables. Additionally, there seems to exist some

Figure 1 *– Patient demand time series*

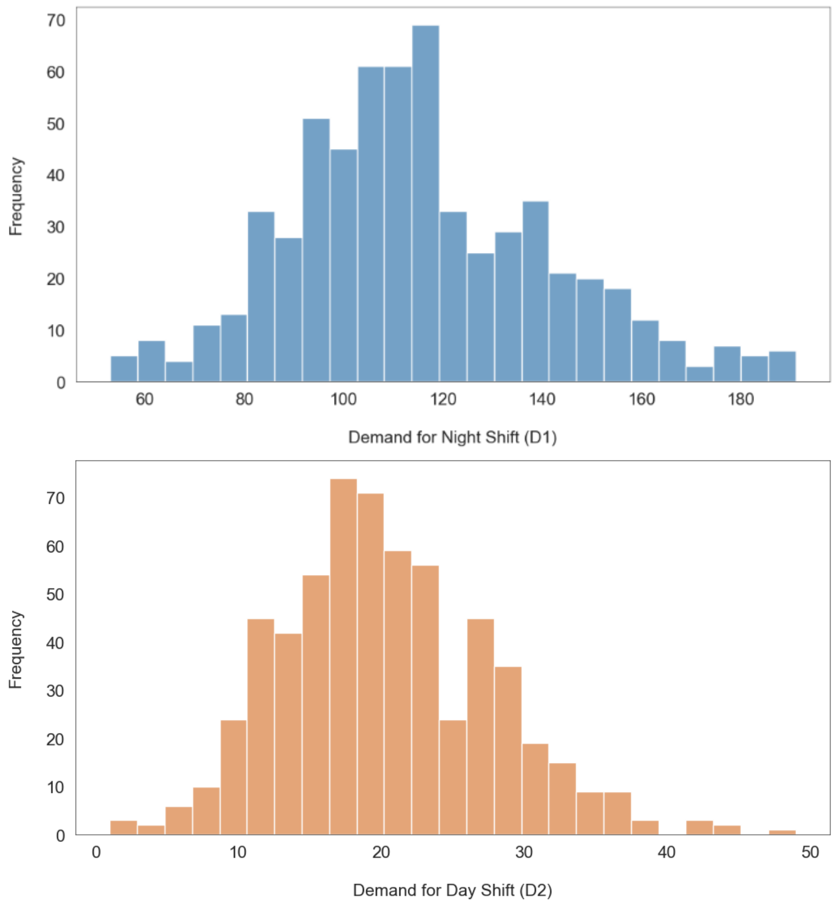


seasonality in the variation of the number of patients looking for admission, with a pronounced peak around early 2018 and later in the year, possibly reflecting higher patient needs due to winter illnesses.

The histograms in **Figure 2**, for both D_1 and D_2, were plotted using a total of 25 bins, allowing us to examine their frequency distribution. The histogram for the night shift is approximately symmetric with a slight right skew, while the histogram for the day shift is more prominently right skewed. While these histograms give a basic view of data distribution, they can be highly subjective structures, depending on the number of classes (or bins) used in dividing the sample.

As such, using the Kernel Density Estimate (KDE) might provide additional insights. KDE is a technique used for estimating the probability density function, providing a smoother and more detailed view of the distribution (Węglarczyk 2018). Unlike

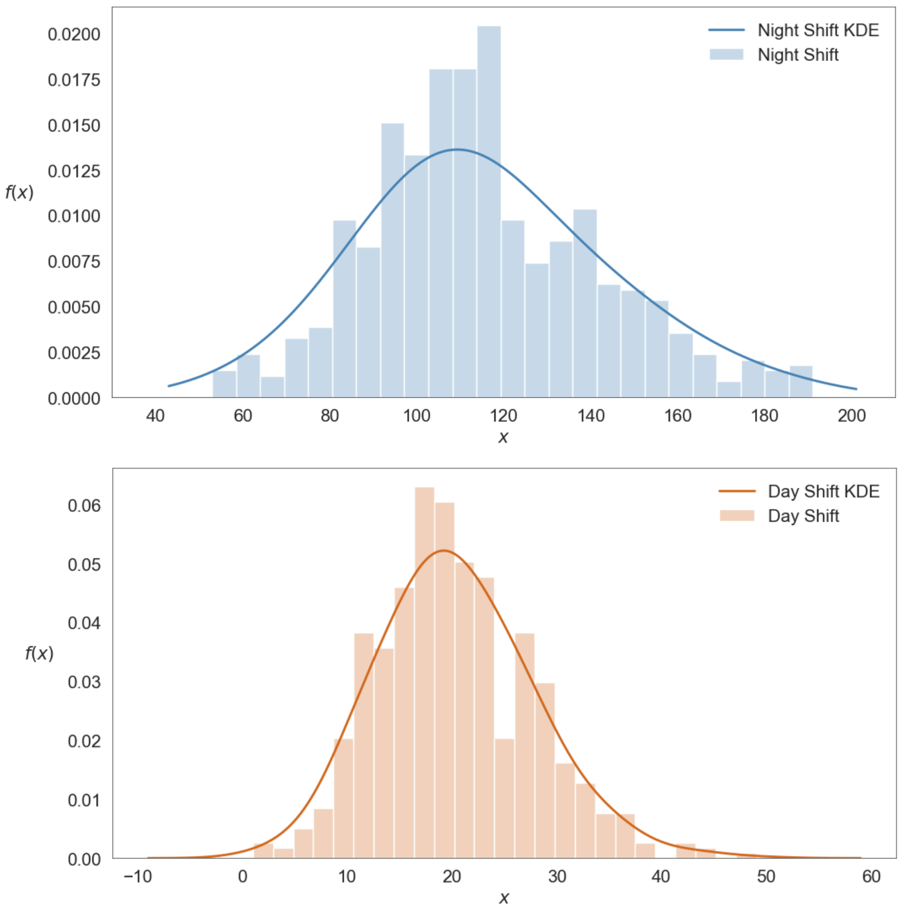
***Figure 2** – Night and day shift demand histograms*



histograms, KDE uses all sample points to create a continuous, smooth curve, making it easier to identify patterns in the data. This works by placing a smooth, bell-shaped kernel over each data point, and subsequently summing these kernels to produce a continuous estimate for the overall probability density function.

The performance of KDE is greatly influenced by two main factors: the selected kernel function, which is essentially a mathematical function used to estimate the probability density function; and the bandwidth, which controls the smoothness of the estimate, with higher values leading to a smoother curve (Soh et al. 2013). In this analysis, the Gaussian kernel will be used, due to its practical and effective nature, leading to smooth lines with flexibility as regards to bandwidth selection. As for the bandwidth value, `GridSearchCV` from the `sklearn.model_selection` module in Python was used to perform a grid search over a selection of possible bandwidth values. The grid search allows for

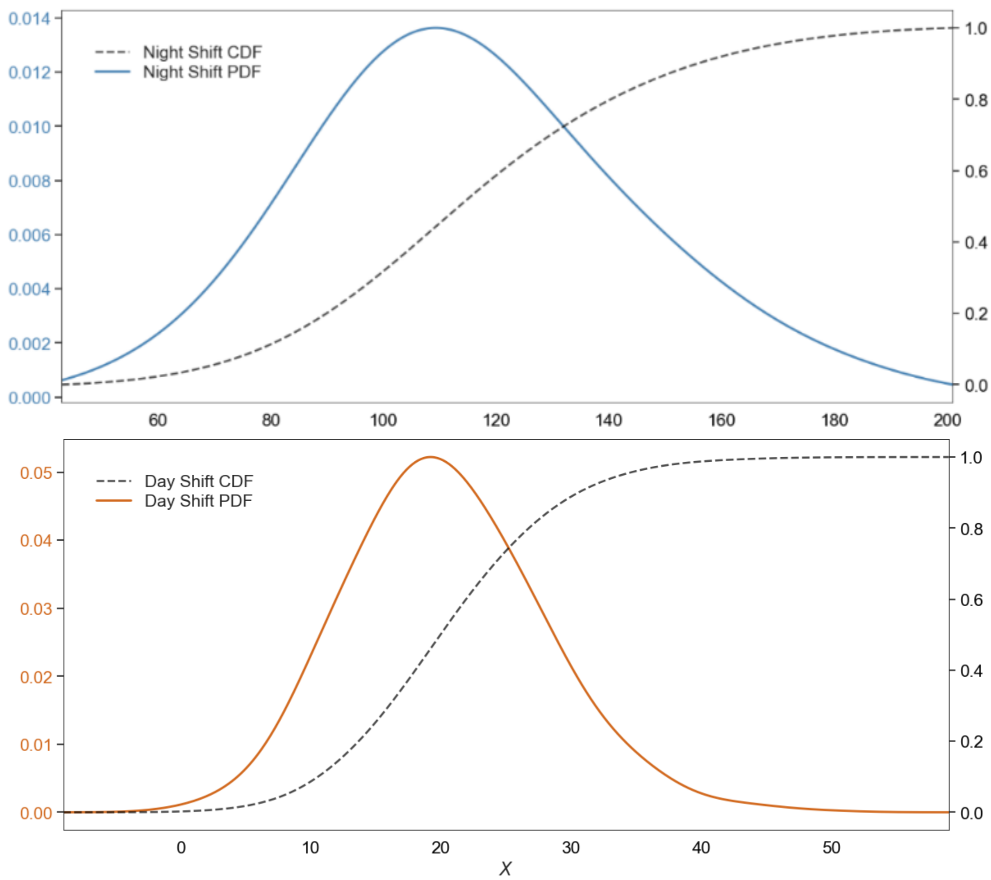
Figure 3 *– Night and day shift kernel density estimate (KDE)*



evaluation of each bandwidth, using cross-validation to select the optimal value that minimizes error, avoids overfitting, and provides the best fit for the KDE model. By iterating through a range of 100 evenly spaced numbers between 0,5 and 20, bandwidth values of approximately 16,06 for D_1, and 3,06 for D_2 were estimated. As such, the KDE curves in **Figure 3** were obtained.

The values for the probability density function (PDF), which represent the likelihood of observing data points at certain locations in the distribution, can be obtained from the KDE using the `score_samples` method, which returns the logarithmic density. These log-density values can then be converted to the regular density by applying exponentiation, the inverse of the logarithm. Thus, this process allows for the values of $f_1(x)$ and $f_2(x)$ to be computed. Based on the probability density function, the cumulative distribution function (CDF) can be computed as the integral of the PDF from the lower

Figure 4 – *Night and day shift probability and cumulative density functions*



bound up to a specific point, representing the total probability of observing a value less than or equal to that specific point. To do this, Simpson's Rule was employed by using the `simpson` function from the `scipy.integrate` module. This rule divides the area under the function $f(x)$ into vertical strips and approximates each section by fitting a parabola through every three consecutive points, providing a more precise integral approximation compared to other alternatives, such as the trapezoidal rule, which connects points with straight lines (Hahn and Valentine 2017). Once the cumulative probabilities were calculated, an interpolation function $F(x)$ for the CDF was created using the `interp1d` method, allowing for smooth evaluation of the CDF at any x-value within the specified range. Through this process, the values for $F_1(x)$ and $F_2(x)$ can be calculated. In **Figure 4**, the PDF and CDF for both demands are plotted. The next section will discuss the application of the model.

MODEL APPLICATION

To determine the optimal values for A_1 and A using conditions 1 and 2, defined in the previous section, the model parameters must first be defined. These values are essential for constructing the profit function $\pi(A_1, A)$ and validating its concavity, thus ensuring the optimality of A_1 and A. However, in the lack of real-world values for these parameters, assumptions will need to be made. These assumptions will be based in logical reasoning and supported by existent information whenever possible, ensuring that the model remains robust and the results meaningful.

To estimate N, data obtained from PORDATA was used. In 2022, Portugal had 36.209 hospital beds distributed across 243 hospitals, resulting in an average of approximately 149 beds per facility. Given that the dataset used in this analysis pertains to a smaller medical unit, it is reasonable to assume that the number of beds would be proportionally lower. As such, two-thirds of the referred average will be considered, resulting in an estimated capacity of 100 beds.

Regarding the admission fees r_1 and r_2, the analysis will consider values from Hospital da Luz Lisboa, a private hospital located in Lisbon and part of the Luz Saúde Group. As of 2024, the admission fee for a double room was 300€, which will be considered as the value for r_1. For the day shift revenue, the model will consider a value 33,3% higher in comparison to period 1, to account for higher fees from services available only during the daytime, such as outpatient treatments, specialist consultations or support healthcare activities. As such, r_2 will be equal to 400€.

Due to lack of available data, type 1 shortage costs (s_{11}, s_{12}) will be assigned a value of 100€, chosen as a reasonable estimate based on the potential impact of patients seeking care elsewhere and the associated reputational damage. Similarly, in the absence of precise data, holding costs h will be assigned the value of 100€ reflecting a reasonable estimate of the expenses sustained for maintaining empty beds, such as utilities and maintenance. Finally, type 2 shortage costs s_2 are set at 450€, reflecting not only the expense of accommodating patients on temporary beds but also the significant decline in care quality and comfort, potential reputational damage, and, in severe cases, the need to financially compensate patients for harm caused by inadequate bed availability.

With the model parameters defined and having derived the PDF and CDF for both demands in the previous sections, the model can be implemented.

The optimal admission levels can be found by minimizing the combined residuals from Condition 1 and Condition 2, which measure the deviation between the current values of A_1 and A and the expected equilibrium values predicted by the model. To do this, the `minimize` function from the `scipy.optimize` library was used, which iteratively adjusts the values of A_1 and A to minimize the combined residuals, in order to find the values

__Table 1__ – Optimal values for A and A_1

$r_1 = 300$ $r_2 = 400$ $s_{11} = 100$ $s_{12} = 100$ $s_2 = 450$ $h = 100$				
u	**0 < A1 < A**		**A1 = 0**	**A1 = A**
	A	A1	A	A
0,1	136	85	74	284
0,2	137	82	92	284
0,3	143	77	111	284
0,4	150	71	131	284
0,5	156	63	150	284
0,6	159	44	170	284
0,7	-	-	190	284
0,8	-	-	210	284
0,9	-	-	229	284
1	-	-	249	284

that lessen the discrepancies between the model's theoretical conditions and the practical solution. By minimizing both conditions, the optimal allocation of admissions was derived. The results from this implementation can be observed in **Table 1**.

Following the approach used by Albuquerque, the values for A_1 and A were computed for different values of parameter u, in order to assess its impact on the optimal solution. Since the increase in u does not guaranty the concavity of the profit function for $0 < A_1 < A$, two additional situations have to be assessed. The first case, where $A_1 = 0$, represents the first extreme case, in which no patients would be allowed in the hospital during the night shift, and it would only operate during the daytime. On the other hand, $A = A_1$ represents the instance where all admissions would be made available from the beginning, which would possibly lead to no available admissions for the day shift. To get the optimal admission levels for these extreme cases, the following formulas were used:

For $A_1 = 0$

$$(r_2 + s_{12} - s_2)\left[\int_0^A \left(1 - F_1\left(\frac{A - D_2}{u}\right)\right) f_2(D_2)dD_2 + \left(1 - F_2(A)\right)\right] = 0$$

For $A_1 = A$

$$(r_1 + s_{11} - s_2)(1 - F_1(A)) + (r_1 + s_{12} - s_2)\int_0^A (1 - F_2(A - D_1))f_1(D_1)d\,D_1 = 0$$

A summary of the formulas used to ascertain the functions' concavity and their respective values for the optimal levels used in this model can be found in **Appendix B, C** and **D**. From the values in **Table 1**, it can be concluded that, for the original condition of $0 < A_1 < A$, the value for total admissions (A) increases with u, while the admissions for period one (A_1) decrease. This outcome aligns with intuition: as the portion of unsatisfied demand in period 1 that is diverted to period 2 increases, it becomes more profitable for the hospital to decrease the number of patients admitted at 20h00, since night admissions represent a lower revenue than day ones. As the number of patients seeking admission at 08h00 increases, profits also increase.

A similar result is obtained for $A_1 = 0$. As a larger portion of the unmet demand for period 1 is diverted to period 2, it becomes increasingly profitable for the medical unit to increase the value of A. Regarding $A = A_1$, the value for u does not influence the optimal value of admissions. Since all admissions are allocated to period 1, this value is constant across all diversion levels. Any unmet demand from period 1 cannot be accommodated in period 2, as all admissions have been utilized in the first period.

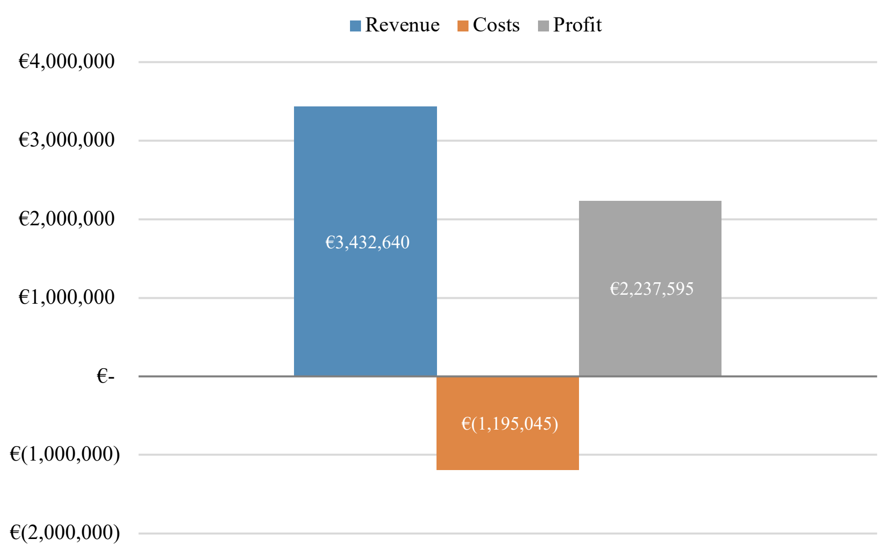
Since it is unrealistic to expect a large portion of the demand to be diverted between periods, aligning with the expectation that only a small fraction of unmet demand in period 1 will carry over to period 2, further analysis will be limited to the results for diversion $u = 0.1$, with optimal values of $A_1 = 85$ and $A = 136$.

MODEL PERFORMANCE ON UNSEEN DATA

Having derived the optimal admission levels based on data from 2017 and 2018, these values can now be applied to 2019. This allows for the estimation of the revenue and costs associated with the optimal admission levels, as well as to assess the required nursing staffing levels to support these admissions and see how they perform when compared with the observed demand in this time period.

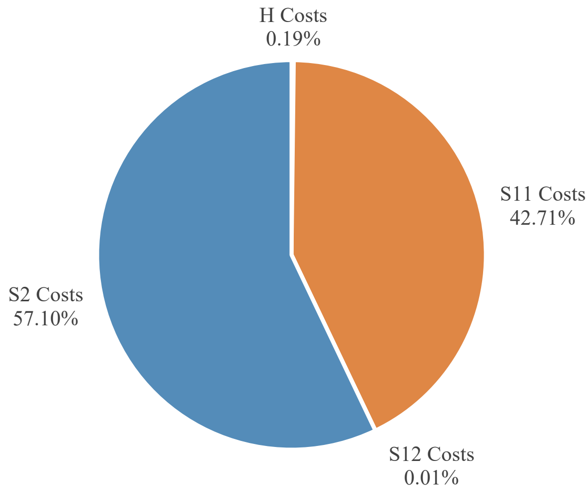
Appendix E presents data for the period from January 1, 2019, to March 31, 2019, reflecting the values following the implementation of the optimized admission levels. If the hospital had implemented the optimal admission levels derived from the previous two years, it would have achieved a total profit of 2.237.595€ for the first trimester of 2019, averaging a profit of 49.178€ per day. A visual comparison between the revenues, costs and profit incurred during this period can be observed in **Figure 5**.

Figure 5 *– Total revenue, total costs, and profit comparison*



With a total of 10.494 patients admitted during this period, the average profit per patient would have been 213,23€, which is lower than both admission fees. This reflects the high demand registered throughout the first trimester of this year, likely due to the seasonality of hospital demand. Specifically, the significant demand in period 1 exceeded the optimal levels predicted, resulting in substantial type 1 shortage costs for that period, as many patients seeking admission were turned away. Furthermore, type 2 shortage costs were considerable, as the consistently high demand made the hospital overbook admissions beyond its bed capacity, compromising care quality and increasing costs. Conversely, holding costs were very low, as demand for period 2 nearly always utilized all available beds, leaving virtually no beds unoccupied during the day. Shortage costs of type 1 for period 2 were almost nonexistent, as the hospital effectively met all demand during that period, even if it required overbooking to do so. For better visualization, a comparison between the different types of costs incurred can be observed in **Figure 6**.

***Figure 6** – Cost distribution per cost type*



Some insights can also be derived about nursing levels and how they might be related to the stochastic model. To estimate the necessary number of nurses, the nurse-to-patient ratio necessary for this healthcare unit must be assessed. The nurse staffing levels will be determined based on the number of admissions rather than bed occupancy, as all admitted patients, including those without an assigned bed, require nursing care.

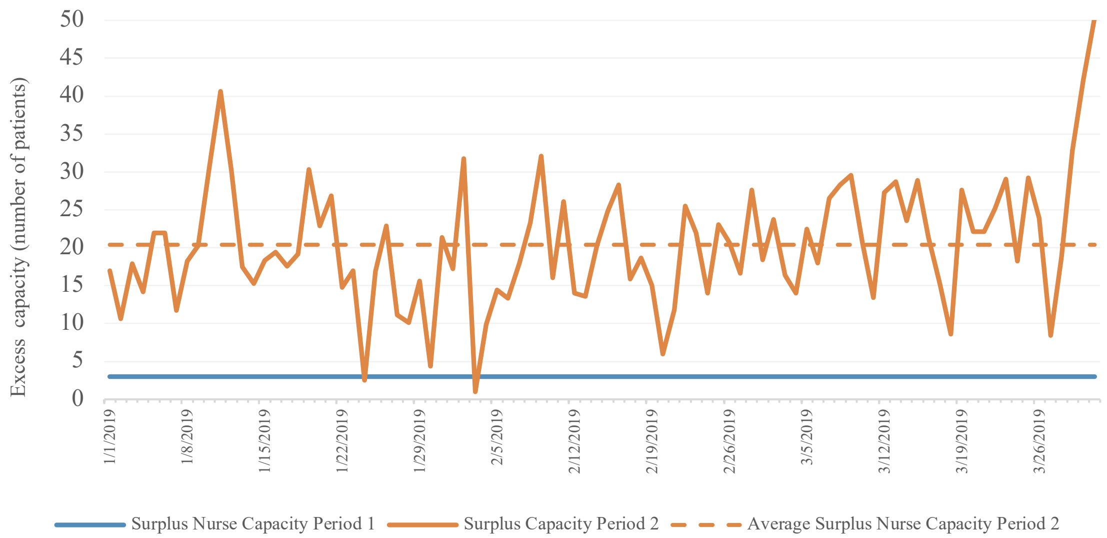
For this, a similar system to that used for bed management will be assumed. Nurse staffing for period 1 will be based on the optimal admission level A_1, with nurses hired for the night shift working the full 24 hours. For period 2, additional nurses will be employed to accommodate new patients and as such, allow for the total number of admissions, A.

According to Sharma and Rani (2020), who performed a critical study of existing nurse staffing norms in India, the benchmark for specialty wards, such as the cardiac unit used in this analysis, should be a 1:4 nurse-to-patient ratio. Thus, the number of nurses required for a given shift will be calculated by dividing the number of admissions by four and rounding up to the nearest whole number, as a fractional number of nurses is not feasible. Following this method, the optimal number of nurses for period 1 should be $\frac{85}{4} \approx 22$, and the additional number of nurses for period 2 approximately $\frac{51}{4} \approx 13$, coming to a total of 35 nurses for each 24-hour period.

HOSPITAL MANAGEMENT INSIGHTS

In **Figure 7**, the nurse capacity excess can be observed (representing the number of additional patients that could have been cared for using the surplus nursing staff available) for both shifts, during the first trimester of 2019.

***Figure** 7 – Daily excess nurse capacity*



It can be observed that, while nurse capacity excess is almost none for the night shift, too many nurses have been employed for the day shift, with some days reaching an excess nurse capacity of 40 (that is, with the surplus nurses, an additional 40 patients could have been taken care of). However, for days with higher demand, the number of employed nurses was adequate to meet the increased need. This once again underscores the inherent challenges faced by hospitals when it comes to nurse staffing, where uncertain demand and unpredictable patient volumes can lead to under or overstaffing, each case with its own set of problems.

Nurse overstaffing can be a problem from a hospital management perspective, increasing operational costs and directly impacting the hospital's financial performance. These operational inefficiencies can potentially lead to resource underutilization, in case nurses have insufficient tasks, or employee dissatisfaction. However, recent studies have shown that, not only is overstaffing extremely rare in current hospitals, but also that it's effect can sometimes be positive. As mentioned in Griffiths et al. (2020), overstaffing can be associated with salvage value, as excess staff might still add value, either by being deployed to another medical unit, or adding value to their own unit. In some cases, overstaffing might not even be inefficient, as the value added to patient care by surplus nurses may overcome the financial detriment caused by additional staff.

On the other hand, although understaffing may reduce operational costs to some extent, it can present adverse outcomes to hospital service quality. Lower staffing levels have been associated with higher patient mortality rates, higher rates of medical errors of omission (an improper increased risk of health-related adverse events resulting from a medical error, typically from receiving too little treatment, such as delays in diagnosis or failure to provide treatments (Hayward et al. 2005)), fatigue, and nurse burnout (Garrett 2008). These high levels of stress and burnout in nurses have been associated with a mental health crisis amongst healthcare professionals, leading to increased nurse turnover in recent years (Kelly, Gee, and Butler 2021).

This high turnover and uncertainty also have a negative impact on managers' ability to plan, organize and coordinate medical units. As such, a static staffing model as the one employed in this work might not be optimal. Instead, dynamic staffing models might be useful, adjusting staffing levels according to a multitude of factors, such as historical admission patterns, seasonal trends, and real-time demand fluctuations. Additionally, shift-specific staffing strategies can be used to ensure a good alignment with demand. Furthermore, data-driven decision-making is crucial to better understand trends and fluctuations in demand, as well as to anticipate future staffing needs and optimize resource allocation. By moving beyond a perspective based solely on financial insights and incorporating patient contentment and quality care metrics into their decision process, healthcare units can ensure cost efficiency while achieving a more holistic approach, aligning operational goals with patient experience.

CONCLUSIONS

This work project presented a data-driven approach to the Newsvendor Problem to optimize bed allocation and nurse staffing in private healthcare units, leveraging a model initially developed by Albuquerque (2008) for airline seat optimization. Through the incorporation of probabilistic demand estimation, overbooking strategies and multi-period cost considerations, this study has provided valuable insights into hospital operations and resource allocation.

The results emphasize the noteworthy impact of stochastic demand on resource allocation decisions. Modeling patient demand and introducing overbooking strategies can aid in the optimal utilization of resources while accounting for no-shows and unpredictable patient behavior. Additionally, it can also be concluded that, under realistic assumptions, balancing admissions between day and night shifts can improve revenue and minimize holding and shortage costs.

This work project also provides actionable insight from a hospital management perspective. Predictive scheduling of bed occupation and hospital staffing is crucial for effective healthcare management, as adjustment of nurse staffing levels and admission availability can help achieve a better alignment between demand and resources, allowing for a reduction in inefficiencies and costs associated with over and understaffing.

Despite the promising results, the study has several limitations that must be acknowledged. The benefits of this approach are greatly restricted by the accuracy of demand forecasting, with the assumptions supporting the cost parameters playing a substantial role in the results obtained as well. As such, the model's performance can be limited when faced with real-world scenarios, such as the variability of patient demand and the complexity of hospital operations. The reliance on hypothetical values for the model's parameters, although supported by empirical evidence, also represents a considerable limitation of this study. Additionally, assuming a linear cost structure may be an oversimplification of the real-life hospital operations. Furthermore, the use of a disjoint approach in this model does not account for the interdependencies between demand prediction and optimization, which could lead to suboptimal solutions when demand estimation is not accurate. Ethical implications must also be considered. Overbooking can be a helpful strategy to manage inefficiencies related to patient no-shows, but if demand unexpectedly exceeds capacity, it may cause hospitals to sacrifice the quality of service. As such, maintaining an equilibrium between financial optimization and patient-centered care is necessary. Moreover, factors such as staff burnout and patient acuity were not considered, further limiting its scope.

Several paths for future development and research remain available, such as expanding the model to consider non-linear cost structures, multi-period frameworks or variable nurse-to-patient ratios. Additionally, the development of a dynamic model might help mitigate limitations associated with seasonal trends and unpredictable fluctuations in patient demand, such as epidemics or unforeseen events. Finally, a one-step approach might better capture the interdependencies between demand prediction and optimization.

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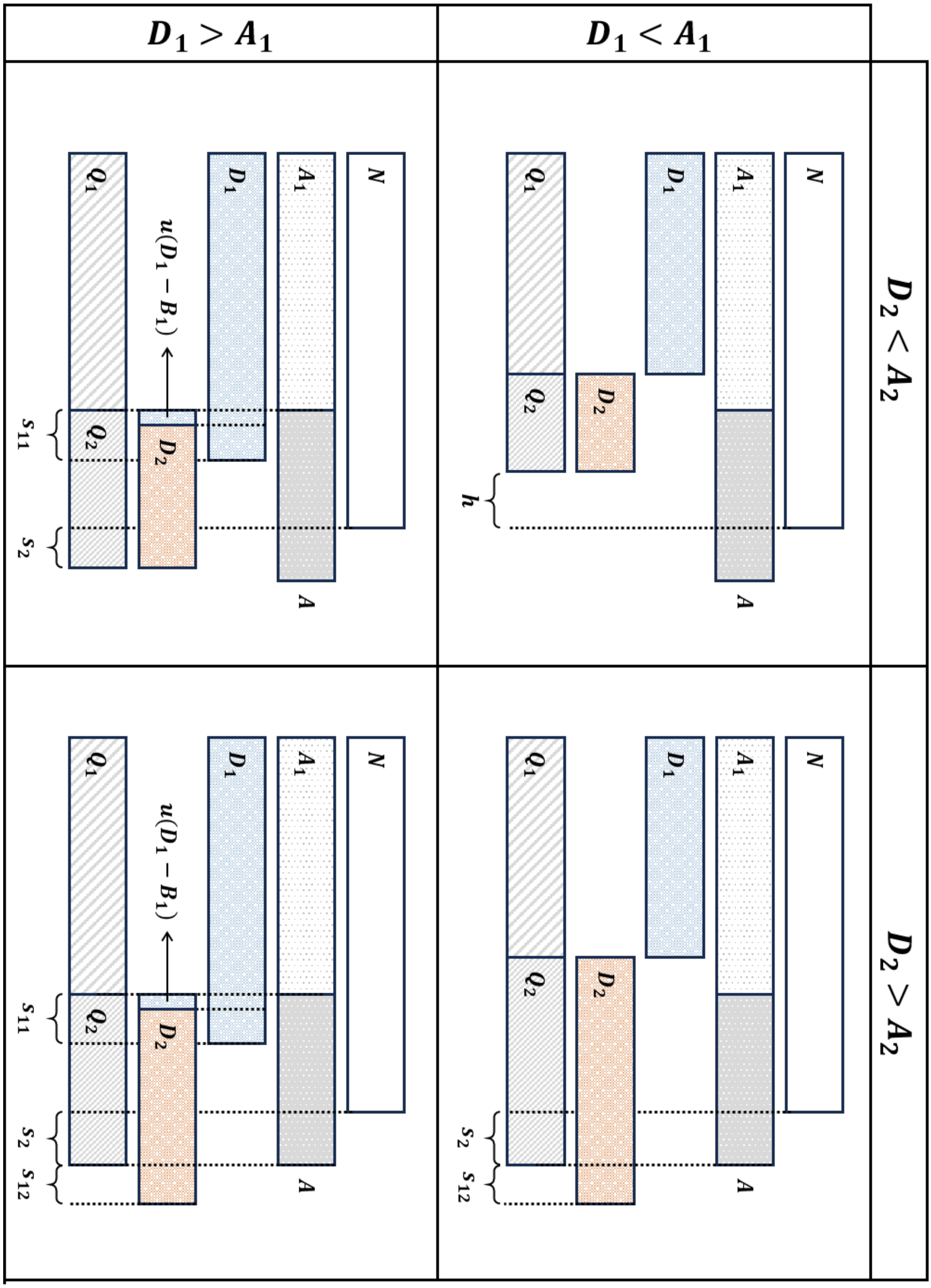
APPENDIX

Appendix A – Model Visualization



Appendix B – Conditions for function concavity for $0 < A_1 < A$

Concavity condition 1

$s_2 \le r_2 + s_{12}$

Concavity condition 2

$F_2(N - A_1) \ge \frac{1}{1-u}\frac{r_2 - r_1}{r_2 + s_{12} + h}$.

Values for the model

u	A	A_1	s_2	$r_2 + s_{12}$	$F_2(N - A_1)$	$\frac{1}{1-u}\frac{r_2 - r_1}{r_2 + s_{12} + h}$	Concavity
0,1	136,008036	85,593497	450	500	0,228962	0,185185	Yes
0,2	137,251383	81,655184	450	500	0,416029	0,208333	Yes
0,3	143,204458	77,349499	450	500	0,634235	0,238095	Yes
0,4	150,353895	71,80037	450	500	0,844274	0,277778	Yes
0,5	156,456516	63,013037	450	500	0,975111	0,333333	Yes
0,6	159,902354	44,818413	450	500	0,999965	0,416667	Yes
0,7	201,109694	0	450	500	1	0,555556	Yes
0,8	259,280829	259,280829	450	500	0	0,833333	No
0,9	257,060647	235,911744	450	500	0	1,666667	No
1	255,850772	222,903293	450	500	0	inf	No

Appendix C – Conditions for function concavity for $A_1 = 0$

Concavity condition

$$-\frac{1}{u}(r_2 + s_{12} - s_2)\int_0^A f_1\left(\frac{A - D_2}{u}\right) f_2(D_2)dD_2 < 0$$

Values for the model

u	A	Condition	Concavity
0,1	73,72973	-0,000097	Yes
0,2	92,286286	-0,000118	Yes
0,3	111,441441	-0,000156	Yes
0,4	130,895896	-0,000211	Yes
0,5	150,35035	-0,000309	Yes
0,6	170,104104	-0,000437	Yes
0,7	189,857858	-0,000619	Yes
0,8	209,611612	-0,000852	Yes
0,9	229,365365	-0,001123	Yes
1	249,119119	-0,001418	Yes

Appendix D – Conditions for function concavity for $A_1 = A$

Concavity condition

$$-(s_{11} - s_{12})f_1(A) - (r_1 + s_{12} - s_2)\int_0^A f_2(A - D_1)f_1(D_1)dD_1 < 0$$

Values for the model

u	A	Condition	Concavity
0,1	284,263158	-0,0480841	Yes
0,2	284,263158	-0,0480841	Yes
0,3	284,263158	-0,0480841	Yes
0,4	284,263158	-0,0480841	Yes
0,5	284,263158	-0,0480841	Yes
0,6	284,263158	-0,0480841	Yes
0,7	284,263158	-0,0480841	Yes
0,8	284,263158	-0,0480841	Yes
0,9	284,263158	-0,0480841	Yes
1	284,263158	-0,0480841	Yes

Appendix E – Financial results with optimal values of A and A_1

Date	Q_1 Revenue	Q_2 Revenue	Total Revenue	H Costs	S_{11} Costs	S_{12} Costs	S_2 Costs	Total Costs	Profit
01/01/2019	25.500 €	14.000 €	39.500 €	0 €	-3.600 €	0 €	-9.000 €	-12.600 €	26.900 €
02/01/2019	25.500 €	16.560 €	42.060 €	0 €	-3.960 €	0 €	-11.880 €	-15.840 €	26.220 €
03/01/2019	25.500 €	13.640 €	39.140 €	0 €	-5.490 €	0 €	-8.595 €	-14.085 €	25.055 €
04/01/2019	25.500 €	15.120 €	40.620 €	0 €	-6.120 €	0 €	-10.260 €	-16.380 €	24.240 €
05/01/2019	25.500 €	12.000 €	37.500 €	0 €	-6.300 €	0 €	-6.750 €	-13.050 €	24.450 €
06/01/2019	25.500 €	12.000 €	37.500 €	0 €	-6.300 €	0 €	-6.750 €	-13.050 €	24.450 €
07/01/2019	25.500 €	16.120 €	41.620 €	0 €	-5.670 €	0 €	-11.385 €	-17.055 €	24.565 €
08/01/2019	25.500 €	13.520 €	39.020 €	0 €	-7.020 €	0 €	-8.460 €	-15.480 €	23.540 €
09/01/2019	25.500 €	12.680 €	38.180 €	0 €	-5.130 €	0 €	-7.515 €	-12.645 €	25.535 €
10/01/2019	25.500 €	8.440 €	33.940 €	0 €	-5.490 €	0 €	-2.745 €	-8.235 €	25.705 €
11/01/2019	25.500 €	4.560 €	30.060 €	-360 €	-4.860 €	0 €	0 €	-5.220 €	24.840 €
12/01/2019	25.500 €	8.720 €	34.220 €	0 €	-3.420 €	0 €	-3.060 €	-6.480 €	27.740 €
13/01/2019	25.500 €	13.800 €	39.300 €	0 €	-3.150 €	0 €	-8.775 €	-11.925 €	27.375 €
14/01/2019	25.500 €	14.680 €	40.180 €	0 €	-4.230 €	0 €	-9.765 €	-13.995 €	26.185 €
15/01/2019	25.500 €	13.480 €	38.980 €	0 €	-5.130 €	0 €	-8.415 €	-13.545 €	25.435 €
16/01/2019	25.500 €	13.040 €	38.540 €	0 €	-5.040 €	0 €	-7.920 €	-12.960 €	25.580 €
17/01/2019	25.500 €	13.760 €	39.260 €	0 €	-4.860 €	0 €	-8.730 €	-13.590 €	25.670 €
18/01/2019	25.500 €	13.120 €	38.620 €	0 €	-5.220 €	0 €	-8.010 €	-13.230 €	25.390 €
19/01/2019	25.500 €	8.680 €	34.180 €	0 €	-6.030 €	0 €	-3.015 €	-9.045 €	25.135 €
20/01/2019	25.500 €	11.640 €	37.140 €	0 €	-5.490 €	0 €	-6.345 €	-11.835 €	25.305 €
21/01/2019	25.500 €	10.040 €	35.540 €	0 €	-4.590 €	0 €	-4.545 €	-9.135 €	26.405 €
22/01/2019	25.500 €	14.880 €	40.380 €	0 €	-4.680 €	0 €	-9.990 €	-14.670 €	25.710 €
23/01/2019	25.500 €	14.000 €	39.500 €	0 €	-5.400 €	0 €	-9.000 €	-14.400 €	25.100 €
24/01/2019	25.500 €	19.800 €	45.300 €	0 €	-6.750 €	0 €	-15.525 €	-22.275 €	23.025 €
25/01/2019	25.500 €	14.040 €	39.540 €	0 €	-8.190 €	0 €	-9.045 €	-17.235 €	22.305 €
26/01/2019	25.500 €	11.640 €	37.140 €	0 €	-7.290 €	0 €	-6.345 €	-13.635 €	23.505 €
27/01/2019	25.500 €	16.360 €	41.860 €	0 €	-7.110 €	0 €	-11.655 €	-18.765 €	23.095 €
28/01/2019	25.500 €	16.760 €	42.260 €	0 €	-7.110 €	0 €	-12.105 €	-19.215 €	23.045 €
29/01/2019	25.500 €	14.560 €	40.060 €	0 €	-8.460 €	0 €	-9.630 €	-18.090 €	21.970 €
30/01/2019	25.500 €	19.040 €	44.540 €	0 €	-7.740 €	0 €	-14.670 €	-22.410 €	22.130 €
31/01/2019	25.500 €	12.240 €	37.740 €	0 €	-8.640 €	0 €	-7.020 €	-15.660 €	22.080 €
01/02/2019	25.500 €	13.920 €	39.420 €	0 €	-8.820 €	0 €	-8.910 €	-17.730 €	21.690 €
02/02/2019	25.500 €	8.080 €	33.580 €	0 €	-8.280 €	0 €	-2.340 €	-10.620 €	22.960 €
03/02/2019	25.500 €	20.400 €	45.900 €	0 €	-7.200 €	-100 €	-16.200 €	-23.500 €	22.400 €
04/02/2019	25.500 €	16.840 €	42.340 €	0 €	-8.190 €	0 €	-12.195 €	-20.385 €	21.955 €
05/02/2019	25.500 €	15.040 €	40.540 €	0 €	-7.740 €	0 €	-10.170 €	-17.910 €	22.630 €
06/02/2019	25.500 €	15.480 €	40.980 €	0 €	-7.830 €	0 €	-10.665 €	-18.495 €	22.485 €
07/02/2019	25.500 €	13.600 €	39.100 €	0 €	-7.200 €	0 €	-8.550 €	-15.750 €	23.350 €
08/02/2019	25.500 €	11.480 €	36.980 €	0 €	-6.930 €	0 €	-6.165 €	-13.095 €	23.885 €
09/02/2019	25.500 €	7.960 €	33.460 €	0 €	-6.210 €	0 €	-2.205 €	-8.415 €	25.045 €
10/02/2019	25.500 €	14.400 €	39.900 €	0 €	-5.400 €	0 €	-9.450 €	-14.850 €	25.050 €

11/02/2019	25.500 €	10.360 €	35.860 €	0 €	-5.310 €	0 €	-4.905 €	-10.215 €	25.645 €
12/02/2019	25.500 €	15.200 €	40.700 €	0 €	-5.400 €	0 €	-10.350 €	-15.750 €	24.950 €
13/02/2019	25.500 €	15.360 €	40.860 €	0 €	-4.860 €	0 €	-10.530 €	-15.390 €	25.470 €
14/02/2019	25.500 €	12.680 €	38.180 €	0 €	-5.130 €	0 €	-7.515 €	-12.645 €	25.535 €
15/02/2019	25.500 €	10.880 €	36.380 €	0 €	-4.680 €	0 €	-5.490 €	-10.170 €	26.210 €
16/02/2019	25.500 €	9.480 €	34.980 €	0 €	-4.230 €	0 €	-3.915 €	-8.145 €	26.835 €
17/02/2019	25.500 €	14.440 €	39.940 €	0 €	-4.590 €	0 €	-9.495 €	-14.085 €	25.855 €
18/02/2019	25.500 €	13.320 €	38.820 €	0 €	-5.670 €	0 €	-8.235 €	-13.905 €	24.915 €
19/02/2019	25.500 €	14.800 €	40.300 €	0 €	-6.300 €	0 €	-9.900 €	-16.200 €	24.100 €
20/02/2019	25.500 €	18.400 €	43.900 €	0 €	-6.300 €	0 €	-13.950 €	-20.250 €	23.650 €
21/02/2019	25.500 €	16.080 €	41.580 €	0 €	-7.380 €	0 €	-11.340 €	-18.720 €	22.860 €
22/02/2019	25.500 €	10.600 €	36.100 €	0 €	-7.650 €	0 €	-5.175 €	-12.825 €	23.275 €
23/02/2019	25.500 €	12.000 €	37.500 €	0 €	-6.300 €	0 €	-6.750 €	-13.050 €	24.450 €
24/02/2019	25.500 €	15.200 €	40.700 €	0 €	-5.400 €	0 €	-10.350 €	-15.750 €	24.950 €
25/02/2019	25.500 €	11.560 €	37.060 €	0 €	-5.310 €	0 €	-6.255 €	-11.565 €	25.495 €
26/02/2019	25.500 €	12.520 €	38.020 €	0 €	-5.670 €	0 €	-7.335 €	-13.005 €	25.015 €
27/02/2019	25.500 €	14.160 €	39.660 €	0 €	-5.760 €	0 €	-9.180 €	-14.940 €	24.720 €
28/02/2019	25.500 €	9.760 €	35.260 €	0 €	-5.760 €	0 €	-4.230 €	-9.990 €	25.270 €
01/03/2019	25.500 €	13.440 €	38.940 €	0 €	-5.040 €	0 €	-8.370 €	-13.410 €	25.530 €
02/03/2019	25.500 €	11.320 €	36.820 €	0 €	-5.670 €	0 €	-5.985 €	-11.655 €	25.165 €
03/03/2019	25.500 €	14.240 €	39.740 €	0 €	-5.940 €	0 €	-9.270 €	-15.210 €	24.530 €
04/03/2019	25.500 €	15.200 €	40.700 €	0 €	-6.300 €	0 €	-10.350 €	-16.650 €	24.050 €
05/03/2019	25.500 €	11.800 €	37.300 €	0 €	-7.650 €	0 €	-6.525 €	-14.175 €	23.125 €
06/03/2019	25.500 €	13.600 €	39.100 €	0 €	-6.300 €	0 €	-8.550 €	-14.850 €	24.250 €
07/03/2019	25.500 €	10.200 €	35.700 €	0 €	-6.750 €	0 €	-4.725 €	-11.475 €	24.225 €
08/03/2019	25.500 €	9.480 €	34.980 €	0 €	-6.030 €	0 €	-3.915 €	-9.945 €	25.035 €
09/03/2019	25.500 €	8.960 €	34.460 €	0 €	-4.860 €	0 €	-3.330 €	-8.190 €	26.270 €
10/03/2019	25.500 €	12.560 €	38.060 €	0 €	-4.860 €	0 €	-7.380 €	-12.240 €	25.820 €
11/03/2019	25.500 €	15.440 €	40.940 €	0 €	-5.040 €	0 €	-10.620 €	-15.660 €	25.280 €
12/03/2019	25.500 €	9.880 €	35.380 €	0 €	-4.230 €	0 €	-4.365 €	-8.595 €	26.785 €
13/03/2019	25.500 €	9.320 €	34.820 €	0 €	-3.870 €	0 €	-3.735 €	-7.605 €	27.215 €
14/03/2019	25.500 €	11.360 €	36.860 €	0 €	-3.960 €	0 €	-6.030 €	-9.990 €	26.870 €
15/03/2019	25.500 €	9.240 €	34.740 €	0 €	-3.690 €	0 €	-3.645 €	-7.335 €	27.405 €
16/03/2019	25.500 €	12.240 €	37.740 €	0 €	-4.140 €	0 €	-7.020 €	-11.160 €	26.580 €
17/03/2019	25.500 €	14.720 €	40.220 €	0 €	-4.320 €	0 €	-9.810 €	-14.130 €	26.090 €
18/03/2019	25.500 €	17.360 €	42.860 €	0 €	-5.760 €	0 €	-12.780 €	-18.540 €	24.320 €
19/03/2019	25.500 €	9.760 €	35.260 €	0 €	-6.660 €	0 €	-4.230 €	-10.890 €	24.370 €
20/03/2019	25.500 €	11.960 €	37.460 €	0 €	-5.310 €	0 €	-6.705 €	-12.015 €	25.445 €
21/03/2019	25.500 €	11.960 €	37.460 €	0 €	-6.210 €	0 €	-6.705 €	-12.915 €	24.545 €
22/03/2019	25.500 €	10.720 €	36.220 €	0 €	-5.220 €	0 €	-5.310 €	-10.530 €	25.690 €
23/03/2019	25.500 €	9.160 €	34.660 €	0 €	-4.410 €	0 €	-3.555 €	-7.965 €	26.695 €
24/03/2019	25.500 €	13.520 €	39.020 €	0 €	-5.220 €	0 €	-8.460 €	-13.680 €	25.340 €
25/03/2019	25.500 €	9.120 €	34.620 €	0 €	-4.320 €	0 €	-3.510 €	-7.830 €	26.790 €
26/03/2019	25.500 €	11.240 €	36.740 €	0 €	-4.590 €	0 €	-5.895 €	-10.485 €	26.255 €
27/03/2019	25.500 €	17.440 €	42.940 €	0 €	-4.140 €	0 €	-12.870 €	-17.010 €	25.930 €
28/03/2019	25.500 €	13.280 €	38.780 €	0 €	-5.580 €	0 €	-8.190 €	-13.770 €	25.010 €
29/03/2019	25.500 €	7.640 €	33.140 €	0 €	-5.490 €	0 €	-1.845 €	-7.335 €	25.805 €
30/03/2019	25.500 €	3.920 €	29.420 €	-520 €	-3.420 €	0 €	0 €	-3.940 €	25.480 €
31/03/2019	25.500 €	640 €	26.140 €	-1.340 €	-1.440 €	0 €	0 €	-2.780 €	23.360 €
TOTAL	**2.295.000 €**	**1.137.640€**	**3.432.640€**	**-2.220€**	**-510.390€**	**-100€**	**-682.335€**	**-1.195.045€**	**2.237.595€**