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The Italian Underground Economy:

Measurement, Dynamics, and Economic Impact on GDP and Tax Revenues

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Abstract

This work project aims to give valuable insights into the dimensions and dynamics of the Italian underground economy. We do so by first estimating its size by developing a Multiple Indicators Multiple Causes (MIMIC) model, a special case of Structural Equation Modeling (SEM). Afterward, we employ a VAR model to understand its dynamic relationships with GDP growth and tax revenues. We conclude that the underground economy has a significant size to GDP, has a strong negative effect on tax revenues, and an ambiguous relation with GDP growth in the short run.

Keywords

Underground Economy, Italy, Impulse Response, Tax Revenues, MIMIC Approach.

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I. Introduction

The underground economy (UE), often referred to as the shadow or informal economy, includes all the economic activities that are not captured in official statistics to avoid taxes, regulations, or compliance costs. This sector includes a wide range of activities, from illegal transactions involving goods and services, to unreported income from legal occupations. Estimating the underground economy, however, poses a considerable challenge due to its hidden nature. Understanding its size and dynamics is crucial for policymakers, as it has significant implications for fiscal policy, economic planning, and the accuracy of official statistics. The study of these activities should represent an essential element in the analysis of economic systems, due to their substantial impact on public finances and its distortionary effects on market dynamics and production, which propagates in the whole economic fabric. Additionally, it imposes significant social costs as the overall tax burden is disproportionately shared among a smaller number of compliant citizens, which increases economic inequality. While in certain situations the UE may offer immediate benefits, such as providing employment opportunities and meeting consumer demands in areas where the formal sector is lacking, these advantages are overshadowed by long-term disruptive effects. And even if it can provide income for those excluded from the formal labor market, such as migrants or individuals lacking necessary qualifications (Maloney 2004), these jobs often lack legal protections, benefits, and job security, perpetuating cycles of poverty and social exclusion.

In the case of Italy, controlling and understanding the UE dynamics and influence on economics performance, is even more important if considering that it is one of the largest among advanced economies. This pervasive shadow economy undermines fiscal revenues, exacerbates public debt, distorts market competition, and erodes social trust, all of which have significant consequences for the nation's economic stability and growth prospects. If not contrasted with efficient reforms, it will continue to be one of the main elements in limiting the Italian prospectives of growth, exasperating the complicated economic situation the country has been facing in the last decades.

One of the most immediate and detrimental effects of a sizable UE is the loss of tax revenues. Unreported income and untaxed transactions deprive the government of essential funds needed to finance public services and investments. According to the European Commission (2019), tax evasion in Italy amounts to approximately €110 billion annually. This significant shortfall contributes to Italy's high public debt levels, which almost led the country to bankruptcy in the 2011 Italian debt crisis. The fiscal strain limits the government's ability to engage in economically and socially beneficial investments in sectors such as education, healthcare, and in infrastructure, which causes a deterioration of the conditions and the accessibility of public services. A diminished tax base forces the state to make tough choices by cutting public spending, increasing borrowing, or raising taxes (Besley and Persson 2013). Raising taxes can be highly counterproductive, as it may further incentivize tax evasion and expand the UE, a concept known as the Laffer Curve effect (Laffer 2004). This is even more true if considering that the Italian tax system is already perceived as inefficient and burdensome. In fact, the World Bank's "Doing Business" report, which ranks countries based on the efficiency of their tax system, positions Italy 128th out of 190 countries (World Bank 2020). The complexity of the tax code, high tax rates, and bureaucratic inefficiencies make compliance both costly and time-consuming for businesses and individuals. This inefficiency not only discourages compliance but also diminishes Italy's attractiveness as a destination for domestic and foreign investment. High taxation and regulatory burdens can suppress economic activity, reduce competitiveness, and incentivize further participation in the UE. High taxes reduce the return on investment, making businesses less likely to invest in new projects or expand existing operations (Feld and Schneider 2010). Moreover, complex regulations and bureaucracy may raise the cost associated with operating in the formal sector, pushing entrepreneurs towards the informal sector where compliance costs are lower (Djankov et al. 2002). The disparity between businesses operating within the formal economy and those in the informal sector exacerbates these challenges. Firms that operate underground avoid taxes and regulatory compliance costs, enabling them to offer lower prices or achieve higher profit margins. This creates an uneven playing field, placing compliant businesses at a competitive disadvantage. As a result, formal sector businesses may respond by reducing their workforce, cutting investments, or, in some cases, relocating their operations to countries with more favorable economic conditions. In fact, Italy has witnessed a trend of manufacturing firms outsourcing production to countries with lower labor costs and less stringent regulatory environments, such as those in Eastern Europe and Asia (Federation of Italian Industries 2018). This outsourcing leads to job losses, a decline in domestic production, and a reduction in GDP, further weakening the national economy. A high tax burden, combined with perceived inefficiencies in public spending, can also lead to a decline in tax morale, which is the individual's motivation to pay taxes (Torgler and Schneider 2007). When taxpayers believe that tax evasion is widespread or that the government is not using tax revenues effectively, they may be more inclined to evade taxes themselves. This behavior creates a self-reinforcing cycle that expands the UE and further erodes the tax base. Moreover, the UE contributes to the erosion of social trust and compliance. When citizens perceive that tax evasion and regulatory non-compliance are widespread and go unpunished, their willingness to comply with laws diminishes (Alm, Martinez-Vazquez, and Schneider 2004). This decline in trust weakens the social contract between the state and its citizens, fostering a culture of non-compliance that can be difficult to reverse. The deterioration of public services, coupled with limited economic opportunities, may also prompt citizens to migrate to countries with better governance and more robust economies. Between 2008 and 2018, Italy experienced a net loss of approximately 420,000 citizens due to migration, many of whom were young and highly educated (OECD 2019). This "brain drain" not only depletes the country of skilled labor essential for innovation and economic growth but also signals a lack of confidence in Italy's prospects. The persistence of a large UE also has implications for economic policy and planning. Inaccurate economic data resulting from unreported activities undermines effective policymaking, since they rely on official statistics when establishing their reforms and strategies. When a large component of economic activity is unrecorded, policies may be misaligned with the actual needs of the economy, leading to suboptimal outcomes (Feige, 2016). Given these multifaceted challenges, there is a pressing need for comprehensive analysis of its size and consequences, and effective policy interventions to mitigate its negative impacts. This work project aims to provide insights into the dynamics and dimensions of the UE and its effect on economic growth and tax revenues. The findings serve to highlight that an excessive and uncontrolled dimension of the UE, like in the Italian case, propagate in the economy self-reinforcing negative trends, which will continue to harm the economic and social fabric if policy makers do not prioritize its reduction and the mitigation of its negative consequences. In the Italian economic and social context, addressing the inefficiencies within the tax system is crucial. Simplifying tax codes and reducing compliance costs can alleviate the burden on businesses and individuals, making formal participation more attractive (Feld and Schneider 2010). Strengthening tax enforcement and increasing the perceived risk of detection can also reduce the UE activities. Moreover, enhancing the quality and accessibility of public services can rebuild trust in government institutions, encouraging compliance and social cohesion (Torgler and Schneider 2007). The implications of reducing the UE extend beyond immediate fiscal benefits. By leveling the competitive playing field, Italy can retain businesses and discourage outsourcing, preserving jobs and bolstering GDP. Improving the economic environment may also stem the tide of emigration, not only retaining but also attracting talent and human capital essential for innovation and long-term growth.

II. Literature review

Various methodologies have been developed to estimate the size of the UE. Traditional approaches include the discrepancy methods that compare income and expenditure statistics, currency demand approach, and the physical input method (such as electricity consumption). While these methods offer valuable insights, they often rely on strong assumptions and may not capture the multifaceted nature of the UE. In response to these limitations, the Multiple Indicators Multiple Causes (MIMIC) model, grounded in structural equation modeling (SEM), has gained prominence. The MIMIC model, pioneered by Frey and Weck-Hannemann (1984), considers the UE as a latent variable influenced by observable causes and reflected in measurable indicators. This approach allows for a more comprehensive analysis by incorporating multiple factors that drive and signify underground activities. One of the seminal works in applying the MIMIC model to estimate the UE is Schneider (2005), who provided extensive estimates for various countries. Schneider and Enste (2000) highlighted the model's flexibility in capturing the relationship between the causes and effects of the UE. Recent studies have reinforced the understanding of the UE's impact on state functioning. For instance, Medina and Schneider (2019) provided updated estimates of the shadow economies worldwide, highlighting the persistent size of Italy's UE relative to other OECD countries. Their findings suggest that structural factors such as tax burden, unemployment rates, and regulatory quality significantly influence the size of the UE. Empirical evidence also indicates that efforts to reduce the UE can have positive effects on tax revenues and economic growth. A study by Elgin and Birinci (2016) showed that reforms aimed at decreasing tax evasion and improving regulatory efficiency can lead to a contraction of the UE and an expansion of the formal sector, thereby increasing the overall tax base. Roberto Dell'Anno's (2003) study applied the MIMIC model to the Italian context. Dell'Anno expanded on previous models by incorporating country-specific variables that influence the UE in Italy, such as taxation level, unemployment rates, and regulation intensity. His findings indicated that Italy has one of the largest underground economies among developed countries, underscoring the need for tailored policy responses. Subsequent studies have built upon Dell'Anno's work to refine the estimates and address methodological challenges. Ardizzi et al. (2014) utilized a cash-deposit ratio approach, arguing for its effectiveness in capturing local variations in UE within Italy. They emphasized the importance of regional analysis, given the economic disparities across Italian regions. Berger and Nitsch (2012) questioned the reliability of traditional estimation methods, advocating for a combination of approaches to cross-validate results. They suggested that reliance on a single method might lead to biased estimates due to the inherent assumptions and limitations of each approach. Research like the one conducted by Busato and Chiarini (2004) integrated macroeconomic models with UE estimates, incorporating it into a business cycle model for Italy, to test its significant impact on macroeconomic variables such as GDP and employment. The integration of UE estimates into macroeconomic models represents a growing area of research. Fratianni and Marchionne (2016) explored the relationship between UE and monetary policy in Italy using VAR models. They found that neglecting the UE could lead to misinterpretations of economic dynamics and policy effectiveness. Alm and Embaye (2013) studied the role of tax revenues in relation to the UE. They investigated the tax compliance behavior and its implications for the dimension of the UE, highlighting the feedback loop between tax policies and underground activities.

Building on this body of literature and given the complex interplay between the UE and various economic factors, there is a need for comprehensive analytical models that can capture these dynamics. Our study aims to help fill this gap. First, by estimating the size of the Italian UE using the MIMIC approach as formulated by Dell'Anno (2022). By updating the model with more recent data, this study seeks to provide an updated estimate of the UE in Italy. Second, by integrating our estimates into a VAR model, alongside GDP growth and tax revenues, we advance the literature by incorporating prior information and examining the dynamic interactions between these variables. The VAR model is particularly suited for this analysis because it treats all included variables as jointly endogenous. Capturing the complex interconnections over time among variables allows the data to inform us about these dynamics without imposing restrictive assumptions (Lütkepohl 2005). Moreover, by combining the VAR structure with identification strategies, we can impose meaningful structural shocks and trace their dynamic effects on the variables of interest. This flexibility is particularly appealing in a context where the functioning of the UE is not fully understood or where theoretical models may be silent on the precise dynamics between tax revenues, output growth, and hidden economic activities (Stock and Watson 2001; Canova 2007). The innovation of this paper stems from its methodological integration and the contemporary nature of its analysis. While previous studies have either focused on estimating the UE or analyzing its macroeconomic implications separately, this work project combines both aspects, providing a comprehensive framework that captures its dimension and the feedback mechanisms with economic growth, and tax revenues.

The estimation remains a critical but challenging task. The main challenges are related to the limited data available and to the difficulties in defining its causes and its range of operation. The MIMIC model offers a robust framework for capturing the latent nature of underground activities. By applying this model to Italy and integrating the findings into a VAR analysis, the current work project contributes to a more nuanced understanding of the UE's role within the broader economic system. It addresses gaps in the literature by updating estimates with recent data and exploring the dynamic relationships between key economic variables, offering valuable insights for policymakers and researchers alike.

III. Methodology

Underground economy estimates – MIMIC

This research aims to estimate the size of the Italian UE by employing the Multiple Indicators Multiple Causes (MIMIC) model, a special case of Structural Equation Modeling (SEM). In this section, I will refer to the UE as the Informal Economy (IE), in line with the Italian National Institute of Statistics' (ISTAT) denomination. The methodology closely follows the approach of Dell'Anno (2022), with some modifications tailored to the current research context and intents. The SEM is a statistical technique that allows for the analysis of complex relationships between latent and observed variables. The MIMIC model is particularly suitable for estimating unobservable variables like the IE, which in this model is a latent variable affected by observable variables, and which affects other observable indicators, capturing both its determinants and manifestations within a unified framework. The MIMIC model comprises two components, a Structural and a Measurement Model. The first model describes the relationship between the latent variable (η) and its six observed causes (x_i):

$$\eta_t = a_0 + \sum_{i=1}^{6} a_i x_{it} + \zeta_t \tag{1}$$

where a_i is the coefficient indicating the impact of each cause on the IE, and ζ, as ε in the measurement model below, are normally distributed error terms with zero mean and constant variance. The second model, links the latent variable to four observed indicators (y_j) dependent of the IE:

$$y_{1t} = b_0 + b_1\eta_t + \varepsilon_{1t}; \tag{2}$$

$$y_{2t} = b_0 + b_2\eta_t + \varepsilon_{2t}; \tag{3}$$

$$y_{3t} = b_0 + b_3\eta_t + \varepsilon_{3t}; \tag{4}$$

$$y_{4t} = b_0 + b_4\eta_t + \varepsilon_{4t}. \tag{5}$$

where b_j is the factor loading linking the IE to each indicator. By integrating these components, the MIMIC model accounts for both the factors influencing the IE and the outcomes resulting from it, specifying how the latent variables affect the observable indicators (y_j) and is affected by the exogenous indicators (x_i).

Data Sources and Variables Selection

The analysis utilizes annual data for Italy from 1995 to 2020. The primary data source is the ISTAT, which provides official statistics on national accounts, labor force surveys, sectoral value added and estimates of the non-observed economy (NOE). The Ministry of Economy and Finance provides data on tax revenues and the tax gap. The selection of the set of variables to be included in the model is based on economic theory and empirical evidence ensuring that the model captures the multifaceted nature of the IE.

The structural models incorporate several variables serving as proxies to capture economic factors influencing informality. First, we include three measures of the tax burden: the direct tax burden (x_{1a}), indirect tax burden (x_{1b}), and total tax burden (x_1), each expressed as a percentage of value added (VA). Theoretically, higher taxes increase the cost of compliances, incentivizing tax evasion and participation in the IE (Friedman et al. 2000). Second, we employ an index of labor market flexibility (x_2), calculated as the ratio of the income of workers with jobs of predetermined termination dates to the income of workers in working age. Greater flexibility is expected to reduce the cost of hiring and firing, decreasing the need for informal employment (Boeri and Garibaldi 2007). Third, an index of self-employment (x_3), computed as the ratio of full-time equivalent self-employed workers to total full-time equivalent employment (Dell'Anno 2022). Higher rates may be associated with a larger IE due to monitoring difficulties (Schneider and Enste 2000). Fourth, we use an index of labor cost (x_4), measured as the ratio of employers' social contributions to domestic compensation of employees (Dell'Anno 2022). Increased labor costs may push firms to hire workers informally to reduce expenses. Then, considering that investment fluctuations can predict future business activity, we include the ratio of gross fixed capital formation to VA (x_5) to account for the effect of the future business in IE. Lower investment may indicate capital constraints in the formal economy, pushing activities into the IE. We also employ the ratio of the VA by public administration, defense, and compulsory social security to total VA to reflect the regulatory burden and the presence of the public sector in the market (x_6). A larger public sector might reduce opportunities for informal activities (Dell'Anno 2022). Lastly, we incorporate the total number of hours worked (x_7) to account for the effect of the business cycle and overall economic environment.

Regarding the measurement models, we employ four potential indicators of the IE. The primary indicator is a proxy of the IE based on the National Accounts (NA) approach, specifically ISTAT's (2024) estimates of for the period 2011–2020 (y_1). Additionally, we include ISTAT's estimates of undeclared work (y_2), official estimates of the tax gap (y_3), defined as the difference between expected tax revenues and actual collections, and a proxy for informal employment in accommodation and food service activities (y_4) (Dell'Anno 2022).

Model Estimation

The estimation was conducted using the covariance-based SEM approach (CB-SEM) using maximum likelihood estimation. Six model specifications, three calibration methods, and two calibration periods were evaluated to identify the most appropriate structure. We have different model specifications to account for potential correlations and causal relationships not captured in the basic model, enhancing model accuracy and reflecting complex economic dynamics.

Model I (the base model) include the first six causes variables and all the indicators. Model II excludes certain variables and introduces covariances among exogenous variables where theoretically justified. Model III focuses on key causes such as self-employment rate (x_3), labor cost (x_4), and size of the public sector (x_6). Model IV introduces direct effects from certain causes to specific indicators, acknowledging potential relationships beyond the latent variable. Model V incorporates additional paths, including the effect of economic activity level (x_7) on the first indicator and direct effects from causes to multiple indicators. Model VI focuses on the most significant relationships, balancing model fit and parsimony and it includes four different cause variables (x_3, x_4, x_5, x_6). Before estimating the models, we test for multivariate normality, since in the CB-SEM the likelihood-ratio test assumes that observed variables are normally distributed. We initially addressed this issue performing the Henze-Zirkler and the Doornik-Hansen test, but both rejected the null hypothesis of multivariate normality. So, the Satorra-Bentler correction was applied to adjust standard errors and test statistics for non-normality, enhancing the robustness of estimates.

For the estimation, it was necessary to face the issue of indeterminacy, because of the need to convert the SEM estimates into actual values of the IE. We first set an identification constraint by fixing the first factor loading equal to one ($\lambda_1 = 1$) and setting the mean of the disturbance term (ζ) to zero. This standard practice in SEM allows the latent variable to be scaled appropriately. This first step is not enough to solve the indeterminacy issue, but it fixes a scale for the parameters, making it possible to estimate the SEM. Since the IE is unobservable, calibration was necessary to translate the latent variable estimates into meaningful and interpretable values. To enhance the robustness of the results, three calibration methods were employed: the first two with the intent of estimating the factor of scale to fix the coefficients estimated to predict the latent values, and the third used to fix inaccuracies, like inverted trend, in the first two model predictions. In addition to the identification and calibration we employed two different calibration periods (i.e., $t_1 = 2011 - 2018$ and $t_2 = 2014 - 2018$), which improve the reliability of our estimates by calibrating the model with exogenous values of IE. Based on the fit statistics and validation results, all the MIMIC models effectively represent the latent construct. Moreover, the coefficients of MIMIC III, IV, and VI exhibit considerable robustness to SEM estimators in terms of statistical significance (Dell'Anno 2022) (Table 1 in the appendix). Comparatively, Model VI shows the strongest statistical performance: the Satorra-Bentler chi-squared test indicates that is the only model where the hypothesis of a perfect fit between the model-implied covariance matrix and the sample variance matrix cannot be rejected at the 5% significance level (Dell'Anno 2022).

In fact, Model VI using Calibration Method 1 (detail below) for the period 2011–2018 is selected as the preferred model, as it balances model complexity with explanatory power. This model structure shows high correlation with ISTAT's IE estimates, reasonable magnitude and trends of the estimated IE, and theoretical coherence and statistical robustness. Calibration Method 2 and Method 3 sometimes yield negative scaling factors, which are theoretically inconsistent (Dell'Anno 2022). For this reason, in this section we explain in detail only calibration Method 1. The calibration Method 1 consists in adjusting the structural coefficients by latent scores. The first step consists in computing first-stage latent scores:

$$\widehat{IE}_t^{FS} = \sum_{i=1}^{6} \hat{\gamma}_i x_{i,t}. \tag{6}$$

In the second step, we utilize the exogenous estimate of the IE from ISTAT as an approximation for the latent η and we regress it on the first-stage latent scores. We implement ordinary least squares (OLS) regressions to obtain the inverse of the "true" scale coefficient, i.e. $\hat{\lambda}_1 = E(b_1)$, as well as the intercept of the structural equation governing the latent variable, $\hat{\gamma}_0 = E(a_0)$:

$$IE_t^{ISTAT} = \hat{\rho}_0 + \hat{\rho}_1 \widehat{IE}_t^{FS} + \varepsilon_t, \tag{7}$$

where the estimated scale coefficient is $\hat{\lambda}_1 = \frac{1}{\hat{\rho}_1}$. The third step consists in adjusting the estimated coefficients using the scaling factor to align with the external benchmark:

$$\hat{\gamma}_i^{Adj} = \frac{\hat{\gamma}_i}{\hat{\lambda}_1}. \tag{8}$$

The last step consists in calculating the time series of the IE in meaningful terms. We do so by combining the results obtained in the previous steps:

$$\widehat{IE}_t^{MIMIC} = \hat{\gamma}_0 + \frac{\widehat{IE}_t^{FS}}{\hat{\gamma}_i}. \tag{9}$$

VAR Model

Afterwards, we employ a Vector Autoregressive (VAR) model to examine the dynamic relationships among GDP growth, tax revenues, and the underground economy from 1995 to 2020. The VAR framework was chosen because it allows for a multivariate stochastic process that captures the dynamic interactions among the variables, treating each of them as endogenous and regressing its own lagged values and those of the other variables. The VAR framework is well-suited for our analysis as it permits an examination of how shocks to one variable propagate through the system, influencing other variables over time.

Data preparation involved collecting annual observations for the three variables of interest. GDP growth rates were calculated as year-over-year percentage changes, providing a measure of economic performance. Tax revenues were expressed as a percentage of GDP by dividing nominal tax revenues by nominal GDP and multiplying it by 100, facilitating comparison across time by adjusting for the size of the economy. The UE was estimated as a percentage of GDP, reflecting unrecorded economic activities that escapes official statistics and taxation.

Before estimating the VAR, we checked for the stationarity of the variables. We performed an Augmented Dickey-Fuller (ADF) test with constant and linear trend on each time series, which checks for the presence of a unit root in the series, with the null hypothesis being that the series is non-stationary. Incorporating a trend in the ADF regression allows for testing the null hypothesis of a unit root against a deterministic-trend-stationary alternative since GDP levels and fiscal variables often exhibit trends and are frequently integrated of order one. Stationarity is a critical assumption in time series analysis, as non-stationary data can lead to unreliable and spurious regression results, and it ensures that the asymptotic distribution theory for inference in VAR models is valid and guarantees standard inference. In this analysis, the ADF test indicated that while GDP growth and tax revenues as a percentage of GDP are stationary, the UE is non-stationary. We finally achieved stationarity for the UE by taking the first difference of it.

Model Estimation

Once we ensured the stationarity of the variables, we implemented the VAR (p) model with the following reduced form:

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + u_t \tag{10}$$

where y_t is a vector of endogenous variables at time t, c is a vector of constants, A_i are coefficient matrices, and u_t is a vector of error terms. The lag order p determines the number of past periods included in the model. It is crucial to identify its correct length for accurately capturing the temporal dependencies without overfitting. The lag order selection was performed using different information criteria: the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), the Final Prediction Error (FPE), and the Hannan-Quinn Information Criterion (HQIC). Models with lag orders ranging from one to four were estimated. Results showed that according to each test, the optimal lag length is 1. The lag ordering test ensures that the model is sufficiently flexible to capture the dynamics while avoiding unnecessary complexity that could reduce predictive accuracy, avoiding overfitting and loss of degrees of freedom.

Once the lag order is determined, we estimate the VAR using ordinary least squares (OLS) on each equation. After the estimation of the model, we check if the VAR is stable. To have a stable VAR we need to check if all the eigenvalues of the companion matrix have modulus (their absolute value) less than 1. Ensuring stability is crucial for meaningful dynamic analysis, as it implies that the system converges to equilibrium after a shock, allowing for meaningful inference on future trajectories and that the impulse response functions (IRFs) converge to zero over time, indicating that shocks to the system are temporary. After testing it we confirmed the stability of our model, allowing us to continue with our investigation of understanding and measuring the effect of a shock in one of our variables to the other variables.

Since the reduced-form VAR captures dynamic correlations and does not deliver economically interpretable shocks, it is central in our analysis to identify structural shocks that drive interpretable changes in the variables. Without structural identification, VAR-based impulse responses remain difficult to interpret, but by applying restrictions guided by economic theory, we extract structural shocks that lead to meaningful policy and economic interpretations. Unlike the reduced-form residuals u_t, structural shocks ε_t are assumed to be economically meaningful and mutually independent. To move from the reduced-form shocks u_t to economically interpretable structural shocks ε_t, we need identification conditions. A standard approach uses a matrix B such that:

$$u_t = B\varepsilon_t \tag{11}$$

where ε_t are orthogonal structural shocks with $E[\varepsilon_t \varepsilon'_t] = I$, and the matrix B is not a uniquely determined matrix without additional restrictions, which we decided to impose with sign restrictions. The sign restrictions approach, originally proposed by Uhlig (2005), exploits economic theory to impose conditions on the expected responses of variables to certain shocks.

For a GDP growth shock, which could arise from a productivity or demand shock, we impose a positive effect on tax revenues and a negative effect on the UE. As the formal economy expands, more economic agents are earning and spending through officially recorded channels. Even if tax rates remain unchanged, a higher volume of taxable transactions leads to greater tax collections. Moreover, with improved economic conditions the relative attractiveness of working in the informal economy diminishes as the official sector becomes more attractive due to higher incomes and improved employment opportunities in the official labor market. The UE tends to contract when the official economy is robust, reflecting that individuals and firms are less inclined to hide their activities when legitimate opportunities become more profitable.

Considering a shock on tax revenues, we impose a negative sign on GDP growth and a positive sign on the UE. When there is an unexpected increase in tax revenue, arising for example from heavier taxation or more intrusive tax policies, it can push some economic agents to reconsider their compliance. Facing new costs, they may seek refuge in the informal sector, inflating its size to avoid the added tax burden. At the same time, the increased taxation level on profits, labor, or consumption reduces disposable income and may distort economic decision-making. This combination can dampen aggregate demand, erode investment incentives, and reduce labor supply in the formal economy, leading to lower GDP growth.

Lastly, we impose a negative effect on both GDP growth and tax revenues arising from an unexpected surge in the UE, which might emerge from factors like weaker governance or more lucrative avenues for tax evasion. As more agents avoid the official radar, the government's ability to collect taxes diminishes, causing tax revenues to fall. Also, the impact on officially measured GDP is negative due to substitution between the formal and informal sectors, even if a part of the income generated in the informal will be spent in the formal economy. While total economic activity (formal plus informal) might remain constant in the short term, the part that is officially observed and included in GDP calculations could stagnate or decline.

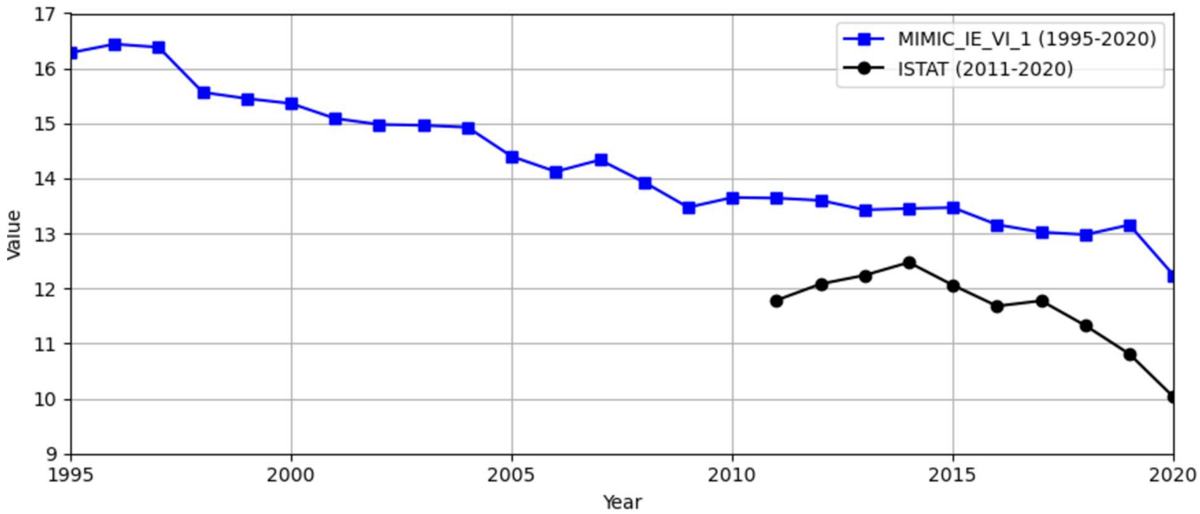
Once we defined the sign restrictions, we start the computational implementation by estimating the VAR and obtaining the reduced-form residual covariance matrix Σ_u. We compute a preliminary decomposition P, via a Cholesky factorization, such that $\Sigma_u = PP'$. Then we generate random orthonormal matrices Q to produce candidate impact matrices $B = PQ$. For each candidate B, we compute the resulting IRFs and check whether they satisfy the sign restrictions for each structural shock, accepting only those decompositions that match all the imposed restrictions. This approach draws repeatedly (thousands of times) from the set of orthonormal transformations. The accepted draws form a distribution of impulse responses that satisfy the imposed sign conditions. From this distribution, we compute median IRFs and confidence intervals. Once identified, sign-restricted IRFs trace out the dynamic effect of one structural shock on each endogenous variable over a 5-year horizon, capturing both immediate and long-term effects. The IRF associated with a shock in one variable provides insight on how it influences the path of the other two, showing the response over time following that particular structural innovation. Positive and negative shocks, with different standard deviations were considered to assess the sensitivity of the responses to the size and direction of the shocks. The estimated impulse responses were plotted along with 68% confidence intervals constructed using standard errors multiplied by the critical value from the standard normal distribution. These confidence intervals provide a measure of the statistical uncertainty associated with the estimated responses.

The last step is the ordering of the variables in the model. This choice is crucial when employing sign restrictions because it affects the identification of structural shocks. Although sign restrictions offer greater flexibility by not relying solely on a particular ordering, the initial ordering of variables still plays an important role. In macroeconomic applications, variables are often ordered from the most exogenous and slow-moving to the more endogenous and quickly adjusting variables. In our analysis, we adopted the following ordering: GDP growth, Tax Revenues, and UE. Justified by the fact that GDP growth is considered as a broad indicator of economic conditions that adjusts more slowly to shocks. Tax revenues, while influenced by current and past levels of GDP and policy decisions, may respond faster or reflect policymakers' reactions to the state of the economy. The UE, on the other hand, might be more reactive to both changes in tax policy and output conditions, which shapes the incentives and scale of underground activities, adjusting more immediately to avoid tax burdens or exploit new economic conditions. Having GDP first ensures that its innovations are interpreted before the feedback from other variables takes place. Thus, the ordering can interact with sign restrictions to deliver a set of economically interpretable structural shocks (Favero 2001; Canova and De Nicoló 2002).

IV. Results

The time series obtained from the CB-SEM MIMIC estimation provides a comprehensive view of the Italian underground economy's evolution over the 1995–2020 period. The results suggests that the size of the Italian UE as a share of official GDP gradually declined over time, reaching roughly 12.2% by 2020 (Figure 1).

Figure 1 - Underground Economy Estimates



Although the estimated shares are slightly higher than ISTAT's estimates, implemented using a National Accounts (NA) approach, the overall trajectory and temporal dynamics are in line with official figures and the existing literature (Dell'Anno 2022); working as an updated complementary source to understand and define UE dimension and path. In this study, the estimates are almost 2 percentage points higher than ISTAT's figures. Despite this difference, the qualitative direction and year-to-year changes are broadly aligned. Both series reflect a gradual contraction in the size of the UE over time, which may be attributable to several common underlying economic and policy factors. The slight upward bias in the MIMIC estimates can be rationalized through several channels. First, the SEM approach used in this study relies on theoretical constructs and latent variables that cannot be directly observed. Variables such as tax burden, labor market rigidity, and regulatory complexity are identified through a system of indicators and causal variables. The choice of indicators, their scaling, and the statistical properties inherent in the CB-SEM MIMIC methodology can introduce some upward shifts in the estimates. In contrast, ISTAT's NA approach, which by its nature may be more conservative, is tightly bound to officially recorded transactions and labor market information. In addition, higher estimates may reflect the MIMIC model's sensitivity to factors that extend beyond the scope of ISTAT's adjustments. For instance, the MIMIC approach can capture a broader spectrum of informal activity, including those that are not entirely captured by official labor force surveys or tax records. This might include micro-scale entrepreneurial activities, informal household production, and other small-scale, part-time, or seasonal activities that are partially or completely outside the reporting boundaries of national accounts. Moreover, the structural model's inclusion of indicators such as the index of self-employment, the index of labor cost, and an index of regulatory burden and the size of Public Administration, could accentuate even more the discrepancy with the NA estimates.

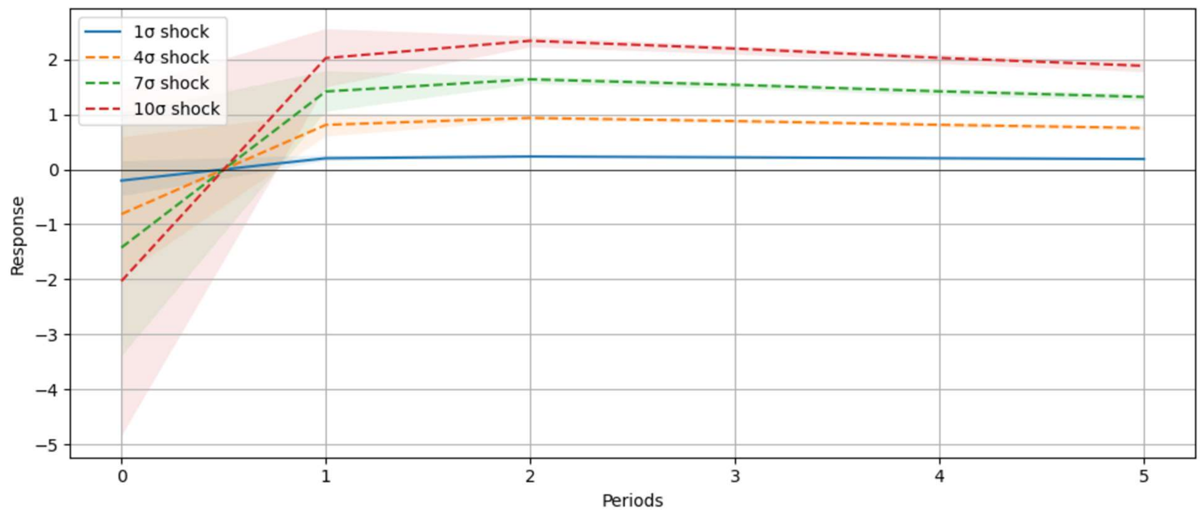
The differences observed here are consistent with the known difficulties of measuring the UE. No single method can claim to be the definitive measure of this unobservable phenomenon. While the NA provides a carefully audited, institutionally accepted benchmark, it can be somewhat conservative and may not fully incorporate the different manifestations of informal activities, that evade both administrative detection and survey-based corrections. On the other hand, MIMIC approaches strive to capture a latent construct through a combination of observable indicators, making it more expansive in terms of what is counted as "underground" activity. Moreover, as the Italian economy underwent different structural changes, such as the introduction of more stringent tax compliance measures, advances in digital payments reducing cash transactions, and the consolidation of certain informal employment arrangements, the MIMIC model could have responded to these shifts, showing a reduction of the UE overtime.

To provide additional reassurance regarding the robustness of these findings, several alternative model specifications were considered during the estimation process (Figures 2, 3, 4, and 5 in the appendix), where different sets of causal variables and indicator variables were tested with the different calibration methods and periods. While variations in specification led to differences in level and timing, the overall pattern remained consistent: the UE in Italy has shown a decline over the last decades, generally mirroring improvements in institutional frameworks, greater integration with international markets, technological enhancements, and policy efforts aimed at reducing evasion and underreporting. A cross-comparison with other empirical studies and estimation techniques further supports these results. For instance, previous research by Dell'Anno (2022) and other scholars using MIMIC models like Medina and Schneider (2019), has also tended to produce estimates that are higher than those coming from official statistical agencies (Figure 6 in the appendix). These converging results, despite differences in methodologies and data sources, give robustness to the downward trend shown in this study.

For what concerns our VAR, the model output shows limited statistical significance between variables (Table 2 in the appendix). In fact, only the GDP growth lag for tax revenues is significant at 5% level. The low significance of the variables could be justified by the limited number of observations, which affects the precision of the estimated coefficients and standard errors. After differentiating the UE and applying a lag, we only have 24 observations in our sample. This limitation reduces the ability to detect statistically significant relationships, especially in time-series models like ours where the number of parameters increases with the number of lags and variables included. With 24 observations, the degrees of freedom are minimal, leading to less precise estimates and higher standard errors. The low significance of certain coefficients may also indicate weak short-term relationships between the variables, or that some relationships might require longer lags to capture their full impact. For instance, tax revenue adjustments or changes in informal activity often respond to GDP growth with delays, which may not be fully captured with a VAR (1) specification, which still, is the best compromise for our estimations considering the data limitations we had to face. Moreover, no shrinkage techniques combined with a small sample limits our ability to precisely identify meaningful relationships among the variables in the VAR, resulting in large standard errors and consequently small t-values. As a result, the confidence intervals are wide and the estimated coefficients are imprecise, making it statistically challenging to reject the null hypothesis that coefficients are zero.

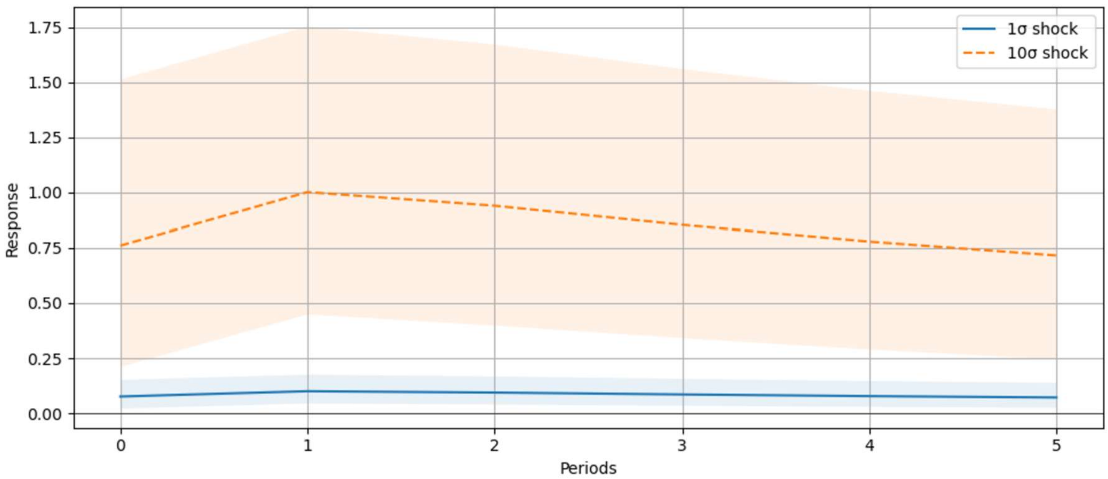
On the other hand, the analysis of the IRFs provides significant and economically meaningful results. We implement IRFs showing the impact that arises from a shock in each variable to the others. All the IRFs strictly follow the sign restrictions we imposed, except for the response of GDP growth to shock in the UE and vice versa. The IRF from a shock on the UE shows that after an initial non-significant decrease in GDP growth, it increases in the first and in the following years.

Figure 7 – Impulse Response of GDP growth to an Underground Economy 1,4,7,10 SD shocks



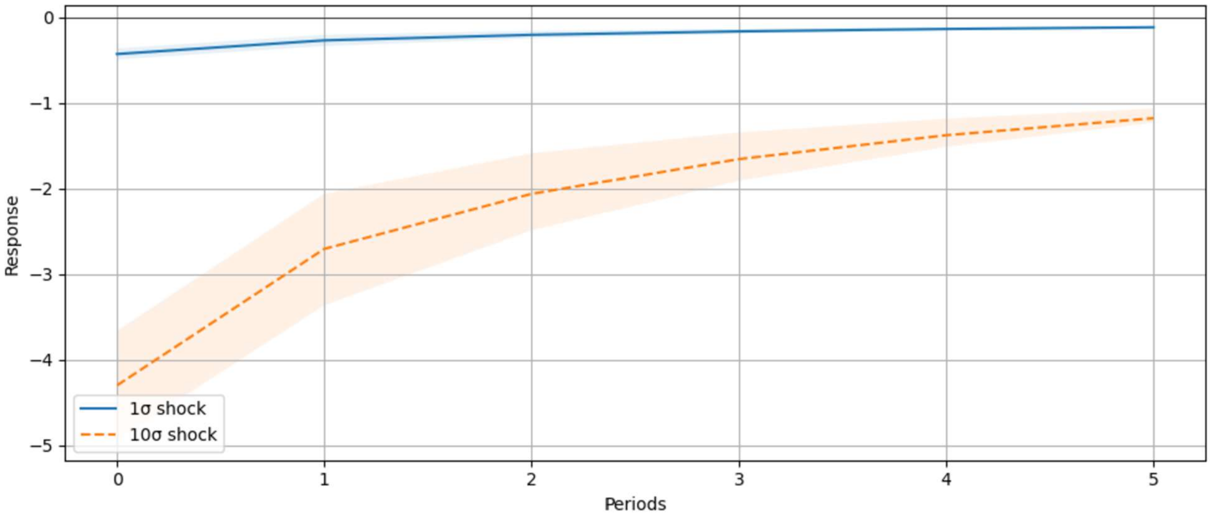
This increase can be justified by analyzing the ambiguous relationship between UE and GDP in the short run. In fact, even if a UE increase could arise from the transition of some activities from the formal to the informal sector, a significant portion of the revenues generated in the UE is afterward employed outside its boundaries, contributing to an increase in GDP. To really determine the sign of impact on GDP growth from a UE increase we need to consider if this increase is due to an enlargement of the already existing informal activities or from the abovementioned substitution effect in favor of informal activities and understand the dynamics of the total economy (formal plus informal). Table 3 in the appendix helps us understand which effect prevails, showing the percentage change resulting from different standard deviation (SD) shocks. We can see that a 10 SD positive shock in UE corresponds to a 2.4 % increase. When comparing the effect it has on GDP growth, we see that it generates an increase of only 2%. This 0.4% discrepancy between the two sustains the theory that, in our case, the short-term benefits arising from more "legal" transactions deriving from informal activities exceed the decrease in formal activities caused by the increase in the UE. But one thing is sure, this possible positive effect on GDP growth materializes only in the short-run, since in the long-run, the inefficiencies which the UE propagates in the economy prevail. As mentioned before, all the other IRFs follow the sign restriction, showing how important it is to comprehend the UE impact on the economy. Particularly relevant for our analysis is the understanding of the influence the UE and tax revenues have on each other. From figure 8 we see that an increase in tax revenues leads to an increase in the UE.

Figure 8 – Impulse Response of Underground Economy to a Tax Revenues 1 and 10 SD shocks



As Figure 9 shows, an increase in the UE leads to a contraction in tax revenues, with an even stronger impact and a larger statistical significance.

Figure 9 – Impulse Response of Tax Revenues to an Underground Economy 1 and 10 SD shocks



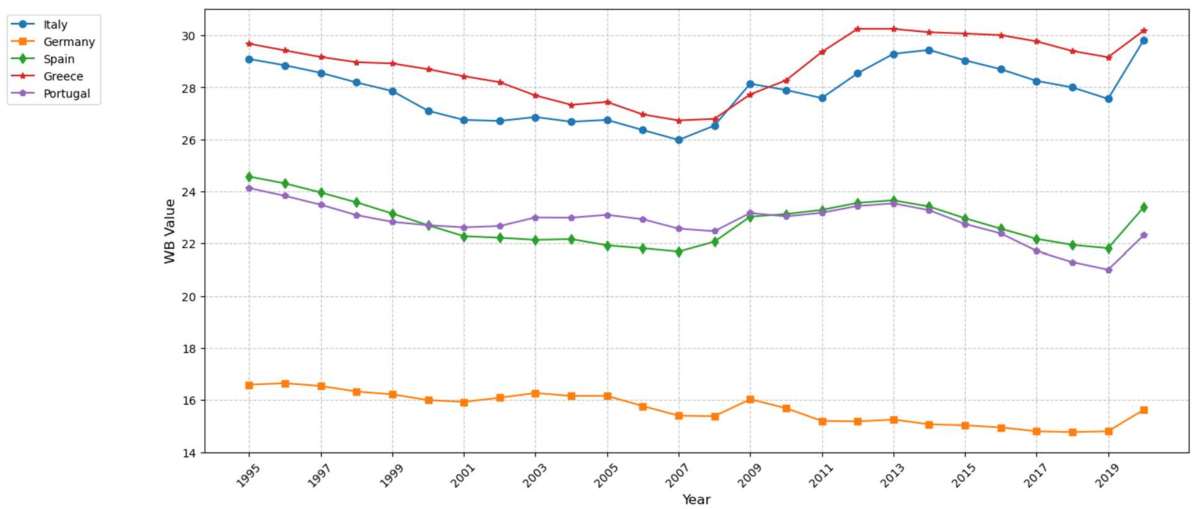
V. Conclusion and policy recommendations

To conclude, even if our estimates show a declining trend for the Italian UE, policymakers should prioritize policies and reform to reduce its size and consequences. Figure 6 in the appendix shows that the literature presents diverging results, for size and paths. Still, one thing is sure: the size of the Italian UE is massive in nominal terms and even more if compared with similar countries like France, and Germany. World Bank estimates show that the size of the Italian UE to GDP is twice that of Germany and France (Figure 10 in the appendix). This underlines how problematic the situation is. This not only leads to internal inefficiencies, limiting the country's ability to face recession, provide adequate services, and achieve economic growth and social stability, but also to a loss of importance and competitiveness at the European stage.

The impulse response analysis reveals critical dynamics between the UE and tax revenues. A tax revenue shock triggers an increase in UE activity, suggesting a feedback loop where aggressive tax enforcement or tax hikes incentivize further non-compliance. While it raises tax revenue in the short-term, it might slow growth and push more activities underground, counteracting long-run fiscal objectives, canceling out or at least strongly decreasing the expected increase. Conversely, shocks to the UE show mixed effects on GDP growth, reflecting the complex interplay between formal and informal sectors. While short-term gains from informal activities may appear to boost GDP, long-term impacts are negative, amplifying inefficiencies and reducing fiscal resilience.

The UE is certainly a key contributor to the economic crisis the country has been facing in the last decades, compromising fiscal stability and the supply of public services. The Italian debt crisis is a striking example of how the UE perpetuates economic instability. By eroding the tax base, it reduces the government's capacity to service debt, which continues to accumulate at increasing interest rate levels, leading to austerity reforms. Figure 11 illustrates that a common factor among all the GIPS[1] countries is their high levels of UE activity, underscoring its detrimental impact on economic performance.

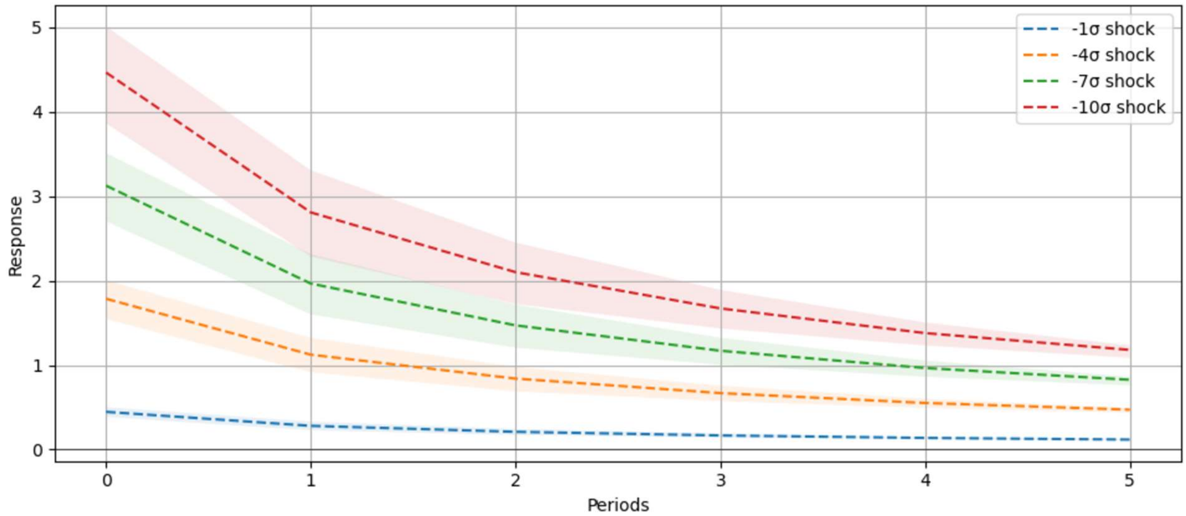
Figure 11 – World Bank MIMIC estimates for GIPS Countries (1995-2020)



[1] GIPS stands for Greece, Italy, Portugal and Spain. Those were the countries which suffered the most from the European Debt Crisis on last decade.

The pension system crisis and the worrying outlook for the public healthcare system are other examples of the degenerative consequences of the UE in the socio-economic sphere. Showing the need to establish strong policies aiming at the reduction of the underground economy. Thanks to our analysis we can see that, if a policy would be able to achieve a reduction of the UE by 2.4%, tax revenues would increase by 4.5% (Figure 12).

Figure 12 - Impulse Response of Tax Revenues to a negative shock to Underground Economy



This increase in government revenues would be crucial in facing the above-mentioned challenges, and in reverting the depressing economic environment the country has ahead.

Instead of raising taxes, which not only leads to an increase in the UE but also has counterproductive dynamics in the long-run, the Government should aim at reforms to incentivize individuals to leave the informal sector, contributing to the nation's economic revival. Policies that reduce compliance costs and enhance the efficiency of tax collection systems are crucial. Key measures include the introduction of tax incentives in sectors with high non-compliance, improving digital tax collection systems to monitor transactions and reduce evasion opportunities, and increasing transparency in public spending to ensure taxpayers see tangible benefits, rebuilding trust in governance. Strengthening penalties for large-scale evasion while focusing on supporting compliance for smaller entities would further boost revenues. Together, these measures can reduce incentives to operate informally, without compromising current productivity.

Addressing the UE is a long-term endeavor that requires a comprehensive and balanced approach. Further research would be fundamental in analyzing which are the main drivers of it. Understanding, predicting, and quantifying its behavior after specific internal and external shocks are fundamental in informing policymakers for predicting its dynamics and for better policymaking, achieving an optimal and predictable result, and avoiding an over-manipulation of the market, which could lead to other negative effects if not controlled accordingly. Once we have a better customized understanding of the economic and cultural causes of the Italian informality, policymakers could better shape reform to achieve fundamental and urgent benefits to revive the suffering Italian fiscal system and promote sustainable economic growth.

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Appendix

Table 1 – Potential Causes, Measurement Models, Model Assessment

Models →		I	II	III	IV	V	VI
Underground VA (UE=$\boldsymbol{\eta_1}$)		CB	CB	CB	CB	CB	CB
Direct Tax B. ($\boldsymbol{x_{1a}}$)	γ_{1a}	-0.02	--	--	--	--	--
Indirect Tax B. ($\boldsymbol{x_{1b}}$)	γ_{1b}	-0.021	--	--	--	--	--
Total Tax B. ($\boldsymbol{x_1}$)	γ_1	--	-0.025**	--	--	--	--
Flex. Lab.($\boldsymbol{x_2}$)	γ_2	0.01	0.01	--	--	--	--
Self-Empl.($\boldsymbol{x_3}$)	γ_3	0.104***	0.106***	0.091***	0.081***	0.078***	0.080***
Lab. Cost ($\boldsymbol{x_4}$)	γ_4	0.052**	0.052***	0.066***	0.068***	0.088***	0.069***
Investments ($\boldsymbol{x_5}$)	γ_5	-0.003	-0.005	--	0.016*	--	0.016*
Size P.A. ($\boldsymbol{x_6}$)	γ_6	-0.237***	-0.228***	-0.238***	-0.216***	-0.257***	-0.213***
Istat_Under. VA ($\boldsymbol{y_1}$)		--	--	--	--	--	--
UE=$\boldsymbol{\eta_1}$	λ_1	$1^{(constr.)}$	$1^{(constr.)}$	$1^{(constr.)}$	$1^{(constr.)}$	$1^{(constr.)}$	$1^{(constr.)}$
Off.W.hours ($\boldsymbol{x_7}$)	γ_7	--	--	--	--	0.033***	--
Constant		9.871***	9.980***	9.354***	9.115***	7.828***	9.073***
Undeclared Work ($\boldsymbol{y_2}$)		--	--	--	--	--	--
UE=$\boldsymbol{\eta_1}$	λ_2	1.62	1.642	1.804	6.240***	3.091***	--
Self-Empl.($\boldsymbol{x_3}$)	γ_8	--	--	--	-0.690***	-0.437***	--
Investments($\boldsymbol{x_5}$)	γ_9	--	--	--	-0.292***	--	--
Off.W.hours ($\boldsymbol{x_7}$)	γ_{10}	--	--	--	--	-0.304***	--
Constant		11.27***	11.39***	9.862**	22.54***	32.32***	--
Tax Gap %VA ($\boldsymbol{y_3}$)		--	--	--	--	--	--
UE=$\boldsymbol{\eta_1}$	λ_3	6.280***	6.299***	6.383***	7.022***	5.884***	6.991***
Investments($\boldsymbol{x_5}$)	γ_{11}	--	--	--	-0.184***	--	-0.182***
Constant		-10.117	-9.477*	-13.68***	-13.35***	-12.65***	-13.58***
Und.Work Acc.Food($\boldsymbol{y_4}$)		--	--	--	--	--	--
UE=$\boldsymbol{\eta_1}$	λ_4	17.34***	17.26***	17.548**	24.91***	18.413**	23.892**
Self-Empl..($\boldsymbol{x_3}$)	γ_{12}	--	--	--	2.018**	2.375***	2.129***
Lab. Cost($\boldsymbol{x_4}$)	γ_{13}	--	--	--	-3.151***	-2.955***	-3.101***
Investments($\boldsymbol{x_5}$)	γ_{14}	--	--	--	--	0.637***	--
Constant		32.925	35.001*	23.323	19.421	13.429	16.938
Satorra-Bentler χ^2 test	CB	134.993	129.353	68.321	21.218	46.89	8.778
Satorra-Ben. χ^2 p-value		0.000	0.000	0.000	0.017	0.001	0.344◊
Degrees of freedom◊◊		23	23	12	12	19	8

***, **, and * denote the significance levels corresponding to p-values below 1%, 5%, and 10%, respectively. ◊ indicates that we fail to reject the null hypothesis of the Satorra–Bentler scaled chi-squared test at the 5% significance level, signifying a good fit in CB-SEM. ◊◊ represents the degrees of freedom, calculated using the formula: $0.5(p+q)(p+q+1)+m-t$, where p is the number of indicators, q the number of causes, m the number of means and intercepts, and t the number of free parameters (Dell'Anno, 2022).

Figure 2 – MIMIC estimates with 1st Calibration Method and 1st Calibration Period

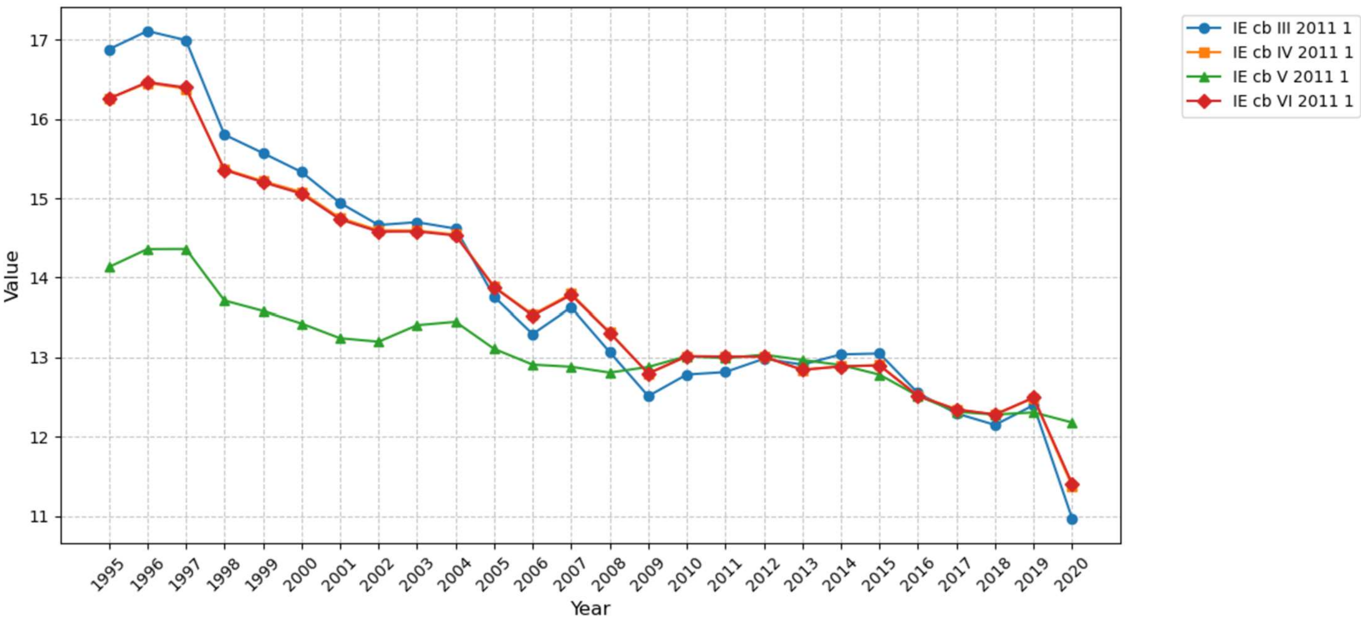

Figure 3 – MIMIC estimates with 2nd Calibration Method and 1st Calibration Period

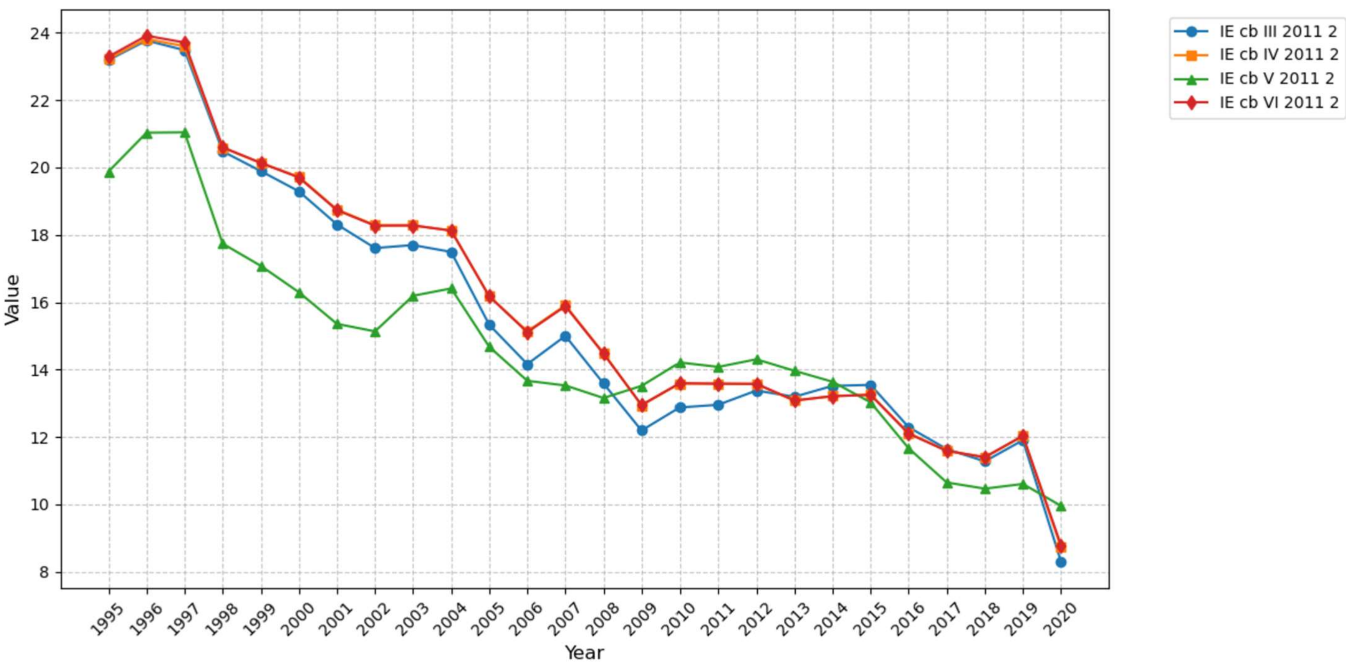

Figure 4 - MIMIC estimates with 1st Calibration Method and 2nd Calibration Period

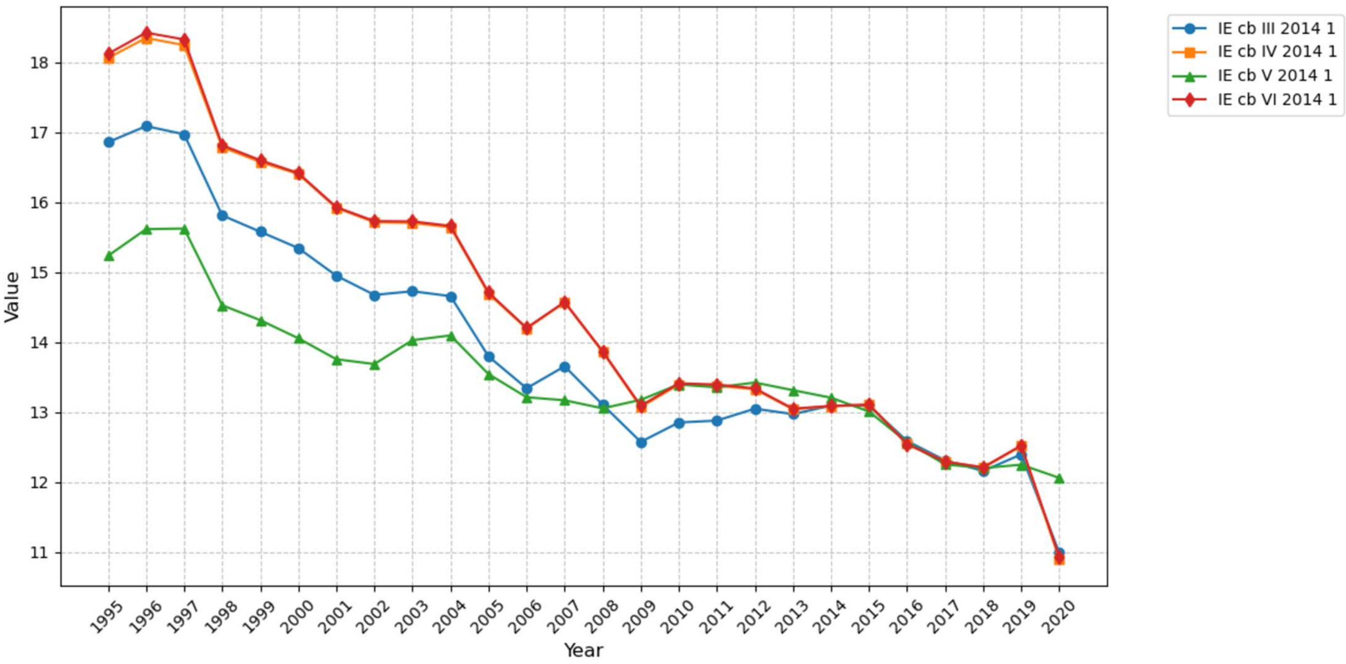

Figure 5 - MIMIC estimates with 2nd Calibration Method and 2nd Calibration Period

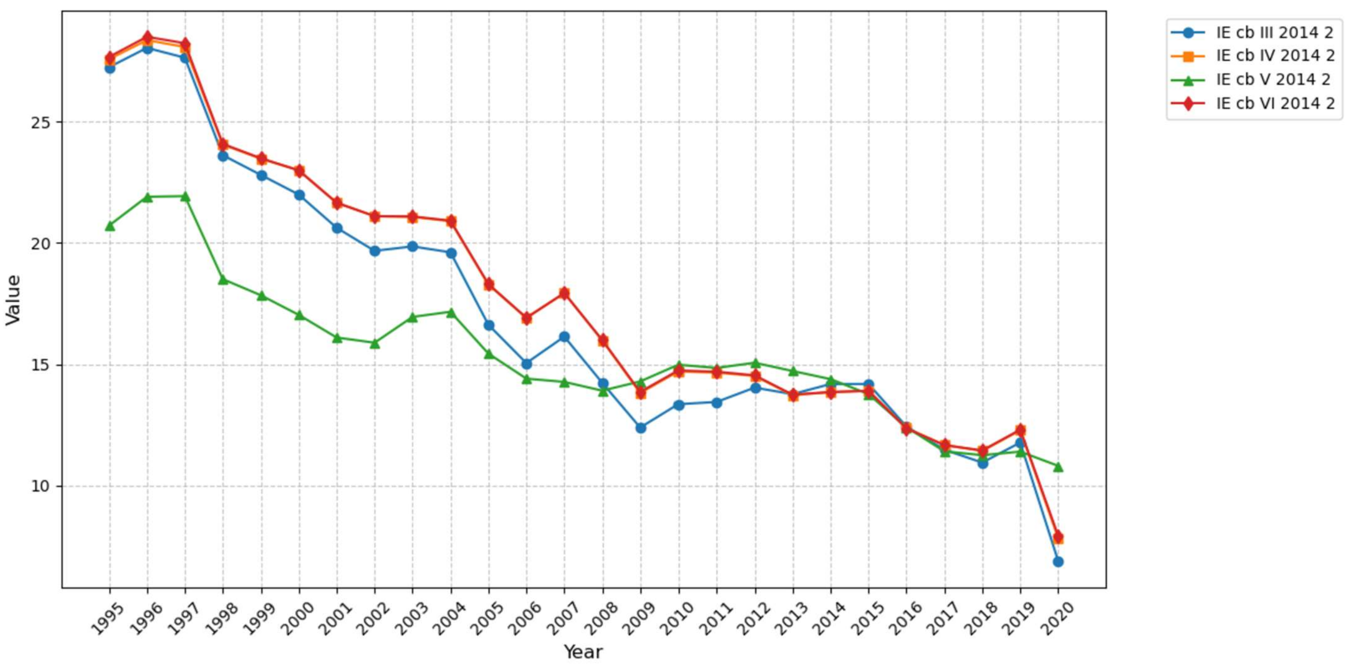

Figure 6 – Estimates Comparison

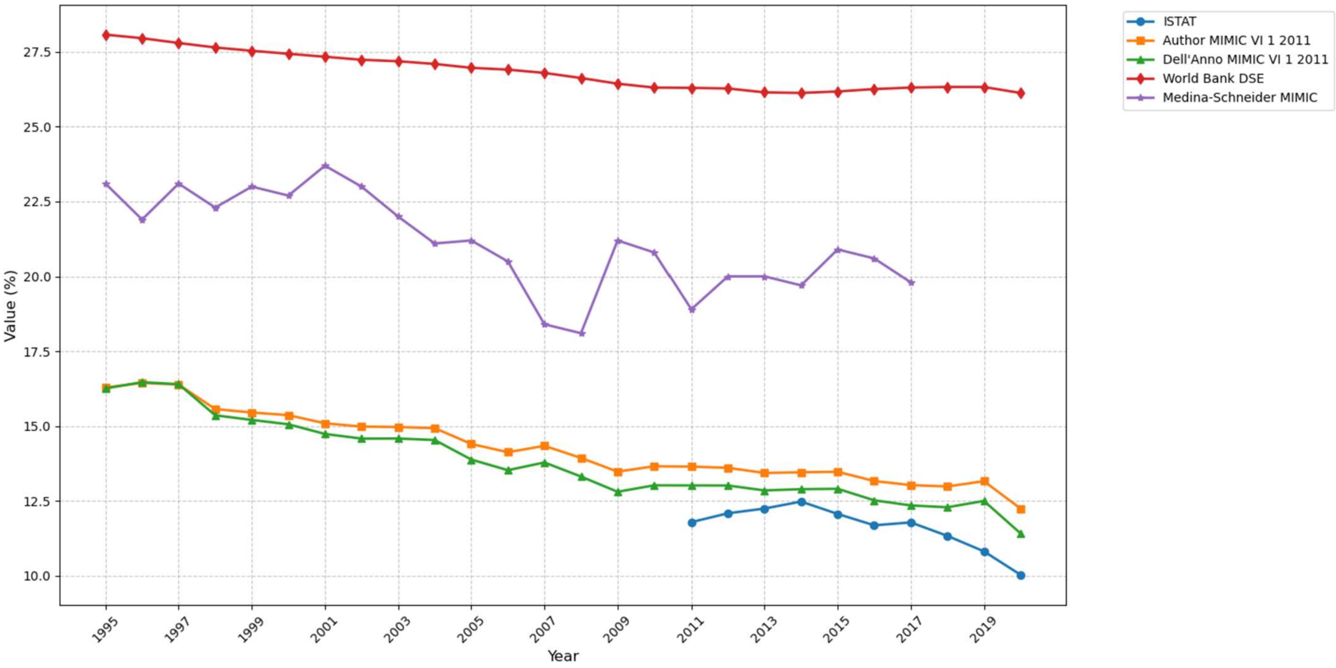

Figure 10 – World Bank MIMIC estimates comparison

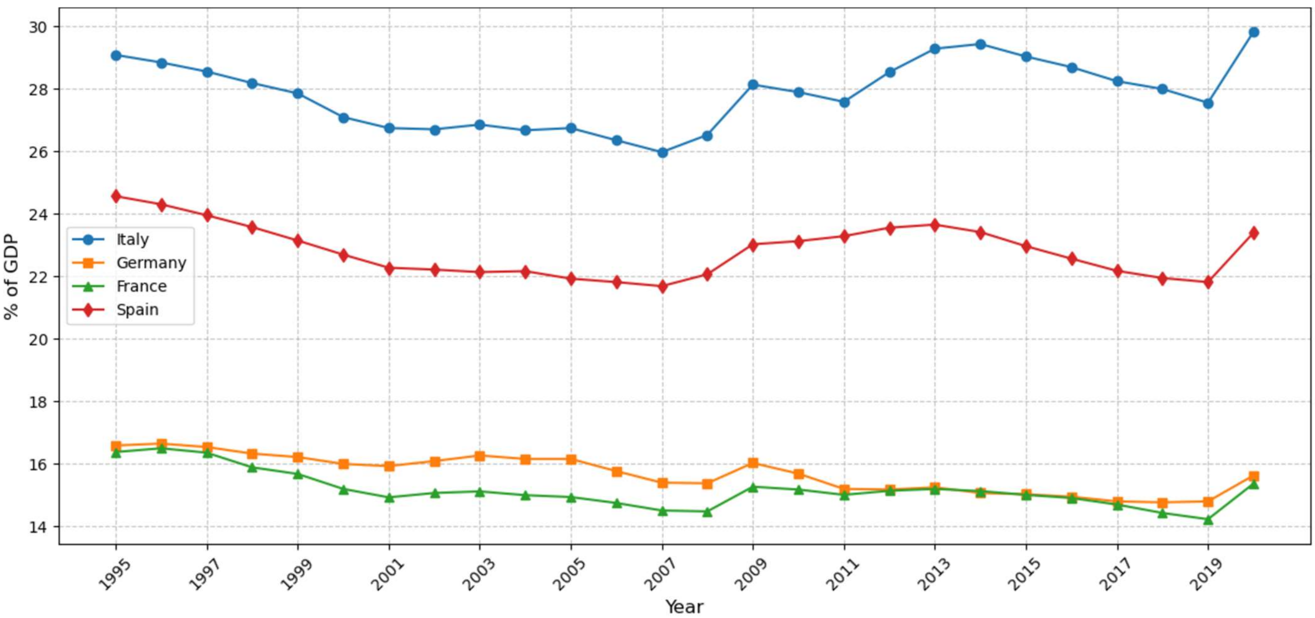

Table 2 – VAR (1) Model Output

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
GDPgrowth						
GDPgrowth L1.	.2449786	.1889108	1.30	0.195	-.1252798	.6152369
TaxrevenuesofGDP L1.	-.529868	.376379	-1.41	0.159	-1.267557	.2078212
dundergroundeconomy L1.	-1.159392	1.435205	-0.81	0.419	-3.972342	1.653558
_cons	15.43778	10.74286	1.44	0.151	-5.617828	36.4934
TaxrevenuesofGDP						
GDPgrowth L1.	-.1326335	.0610722	-2.17	0.030	-.2523328	-.0129342
TaxrevenuesofGDP L1.	.5627311	.121678	4.62	0.000	.3242466	.8012156
dundergroundeconomy L1.	-.1791269	.4639815	-0.39	0.699	-1.088514	.7302601
_cons	12.63087	3.473014	3.64	0.000	5.823887	19.43785
dundergroundeconomy						
GDPgrowth L1.	-.0269799	.0301735	-0.89	0.371	-.0861189	.0321591
TaxrevenuesofGDP L1.	.0162443	.0601166	0.27	0.787	-.1015821	.1340707
dundergroundeconomy L1.	-.2884873	.2292362	-1.26	0.208	-.7377819	.1608074
_cons	-.6538052	1.715888	-0.38	0.703	-4.016885	2.709274

Table 3 – Shocks Magnitudes

```
Own-Variable Shock Magnitudes:
==============================

GDP Growth Shock on gdp_growth:
1 SD shock: 0.9784 percentage points
4 SD shock: 3.9137 percentage points
7 SD shock: 6.8490 percentage points
10 SD shock: 9.7842 percentage points

Tax Revenue Shock on tax_revenue_pct_of_gdp:
1 SD shock: 0.1660 percentage points
4 SD shock: 0.6641 percentage points
7 SD shock: 1.1621 percentage points
10 SD shock: 1.6602 percentage points

Underground Economy Shock on underground_economy:
1 SD shock: 0.2392 percentage points
4 SD shock: 0.9569 percentage points
7 SD shock: 1.6746 percentage points
10 SD shock: 2.3924 percentage points
```

Table 4 – Augmented Dickey-Fuller Test

```
==========================
 Augmented Dickey-Fuller Test on "gdp_growth"
  -----------------------------------------------
 Null Hypothesis: Data has unit root (non-stationary).
 Significance Level = 0.05
 Test Statistic = -4.0722
 Number of Lags Used = 0
 Number of Observations Used = 25
 Critical Value 1% = -4.3750
 Critical Value 5% = -3.6035
 Critical Value 10% = -3.2382
 => P-Value = 0.0069. Reject the null hypothesis.
 => The series is stationary.


 Augmented Dickey-Fuller Test on "tax_revenue_pct_of_gdp"
  -----------------------------------------------
 Null Hypothesis: Data has unit root (non-stationary).
 Significance Level = 0.05
 Test Statistic = -3.8448
 Number of Lags Used = 1
 Number of Observations Used = 24
 Critical Value 1% = -4.3950
 Critical Value 5% = -3.6124
 Critical Value 10% = -3.2432
 => P-Value = 0.0144. Reject the null hypothesis.
 => The series is stationary.


 Augmented Dickey-Fuller Test on "underground_economy"
  -----------------------------------------------
 Null Hypothesis: Data has unit root (non-stationary).
 Significance Level = 0.05
 Test Statistic = -1.5741
 Number of Lags Used = 5
 Number of Observations Used = 20
 Critical Value 1% = -4.4993
 Critical Value 5% = -3.6583
 Critical Value 10% = -3.2689
 => P-Value = 0.8024. Cannot reject the null hypothesis.
 => The series is non-stationary.
```

Table 5 – Augmented Dickey-Fuller on the Underground Economy in First Difference

```
 Augmented Dickey-Fuller Test on "underground_economy_diff"
  -----------------------------------------------
 Null Hypothesis: Data has unit root (non-stationary).
 Significance Level = 0.05
 Test Statistic = -6.8854
 Number of Lags Used = 9
 Number of Observations Used = 15
 Critical Value 1% = -4.7284
 Critical Value 5% = -3.7568
 Critical Value 10% = -3.3235
 => P-Value = 0.0000. Reject the null hypothesis.
 => The series is stationary.
```

Table 4 – VAR order Selection

	AIC	BIC	FPE	HQIC
0	0.5004	0.6492	1.650	0.5355
1	-2.222*	-1.627*	0.1097*	-2.082*
2	-2.087	-1.046	0.1328	-1.842
3	-2.035	-0.5468	0.1621	-1.684
4	-1.626	0.3086	0.3340	-1.170

Table 5 – Stability Test

Eigenvalue stability condition

Eigenvalue	Modulus
.7183782	**.718378**
-.3458547	**.345855**
.146699	**.146699**

All the eigenvalues lie inside the unit circle.
VAR satisfies stability condition.

Figure 13 – Impulse Response of GDP growth to a GDP growth shock

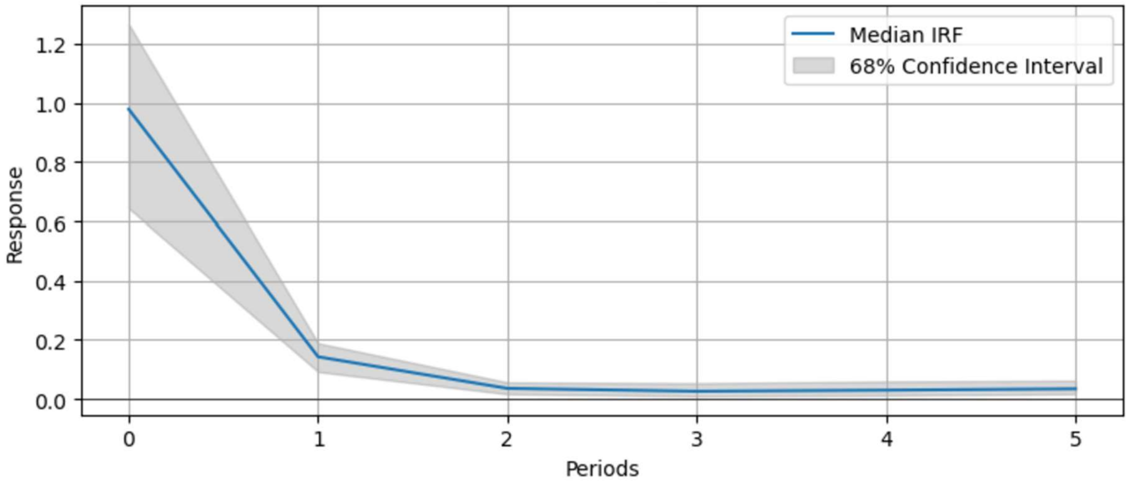

Figure 14 – Impulse Response of Tax Revenues to a GDP growth shock

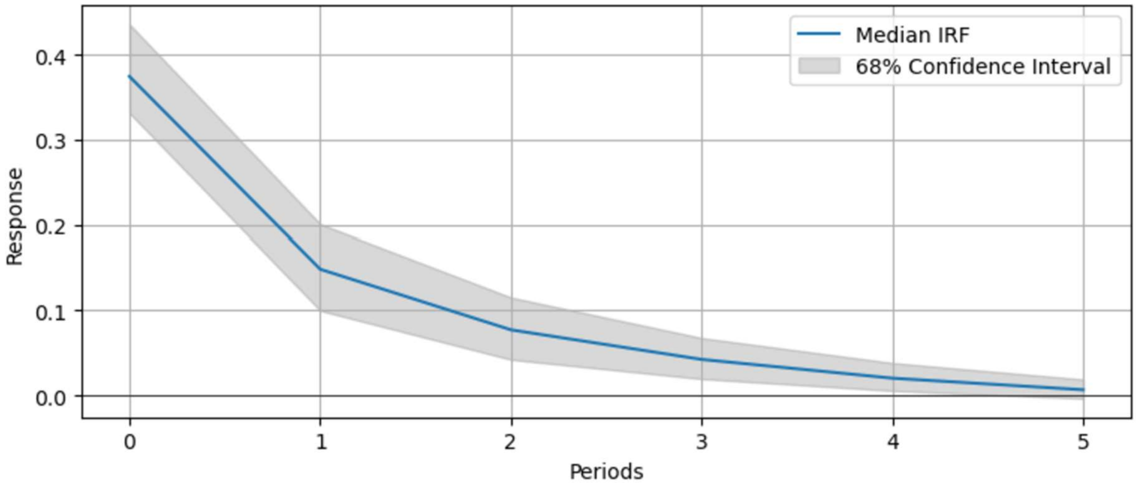

Figure 15 – Impulse Response of Underground Economy to a GDP growth shock

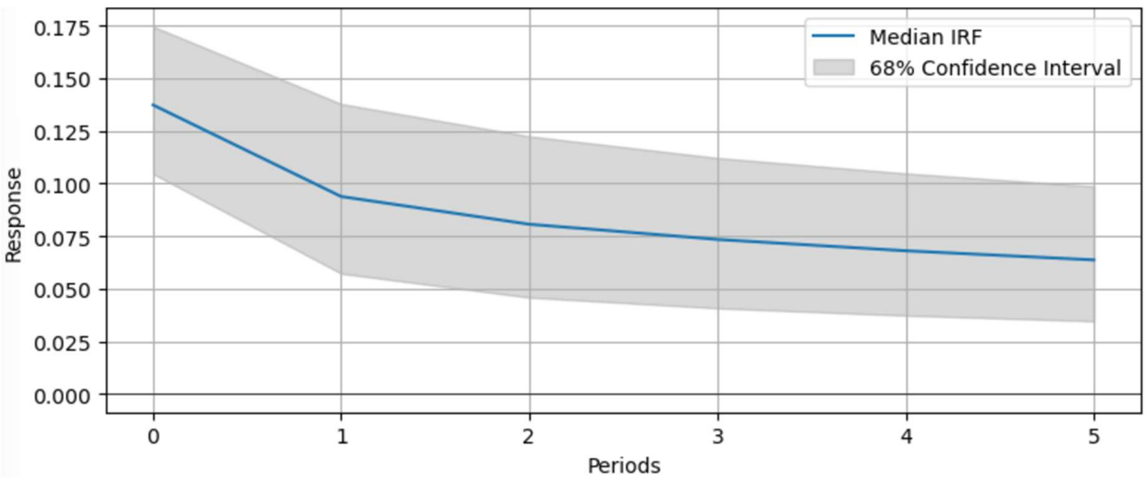

Figure 16 – Impulse Response of GDP growth to a Tax Revenues shock

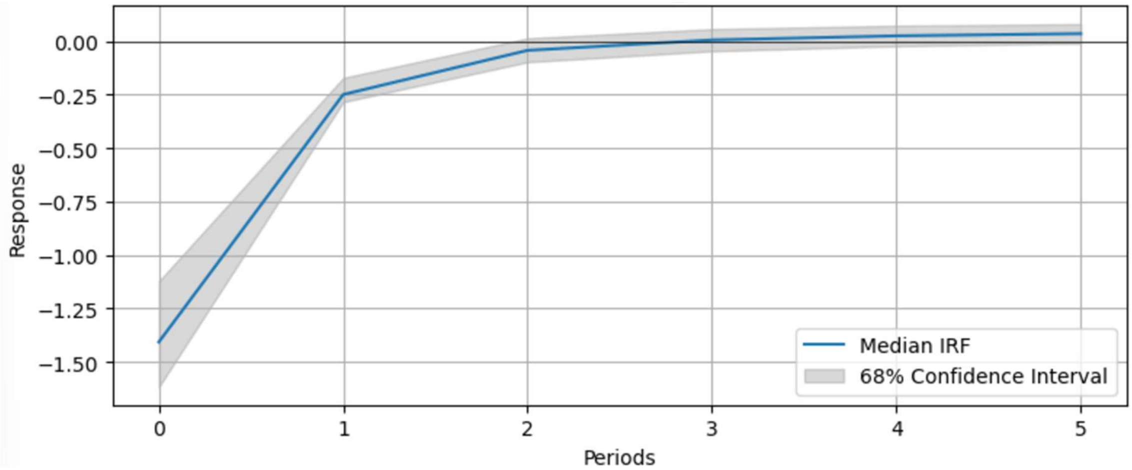

Figure 17 - Impulse Response of Tax Revenues to a Tax Revenues shock

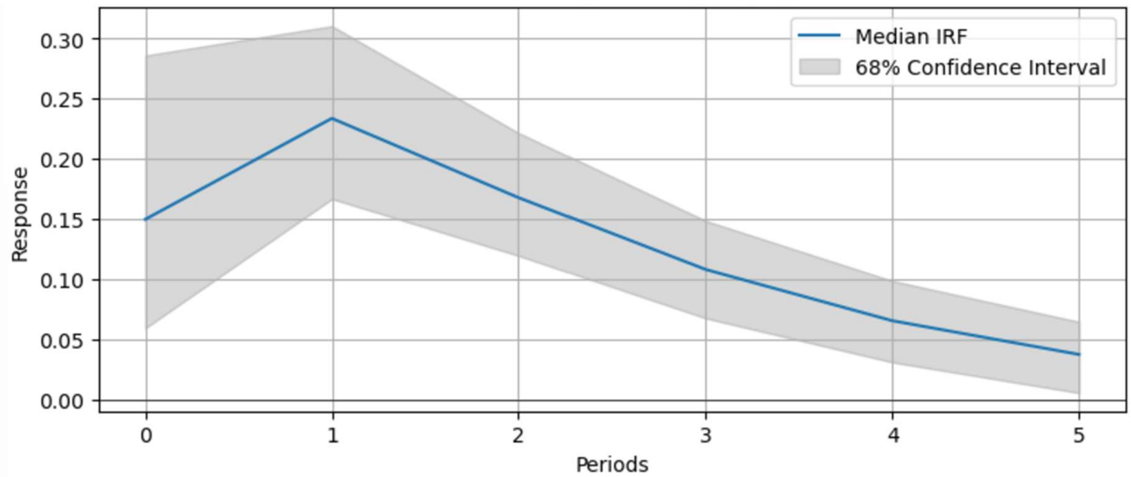

Figure 18 - Impulse Response of Underground Economy to a Tax Revenues shock

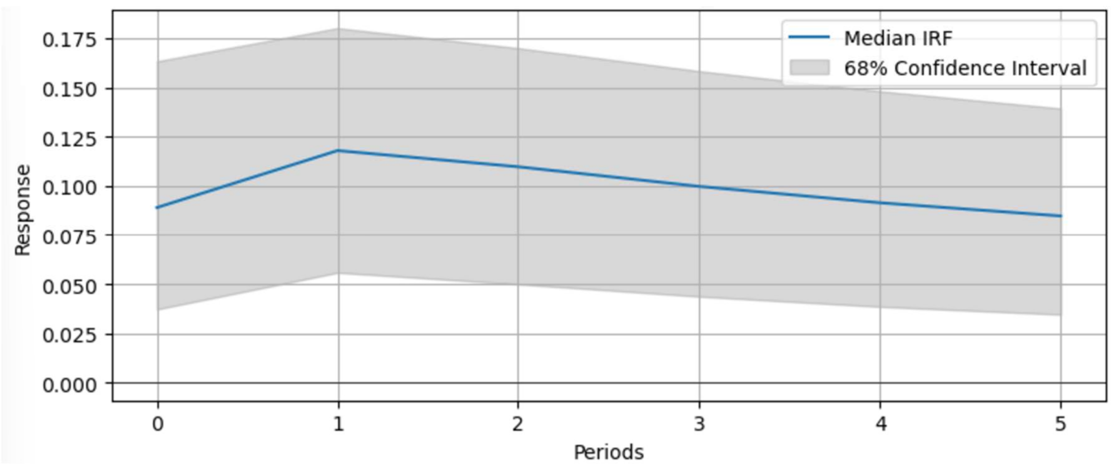

Figure 19 – Impulse Response of GDP growth to an Underground Economy shock

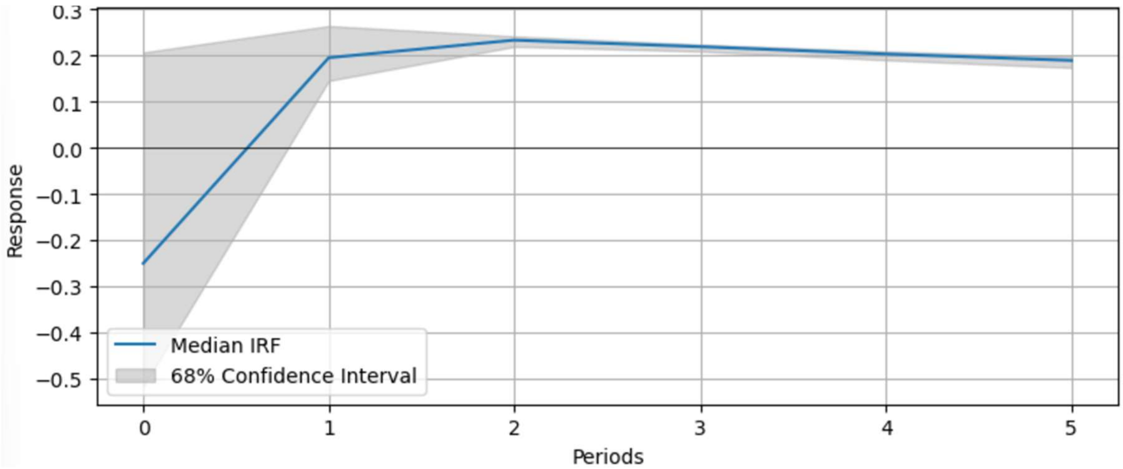

Figure 10 – Impulse Response of Tax Revenues to an Underground Economy shock

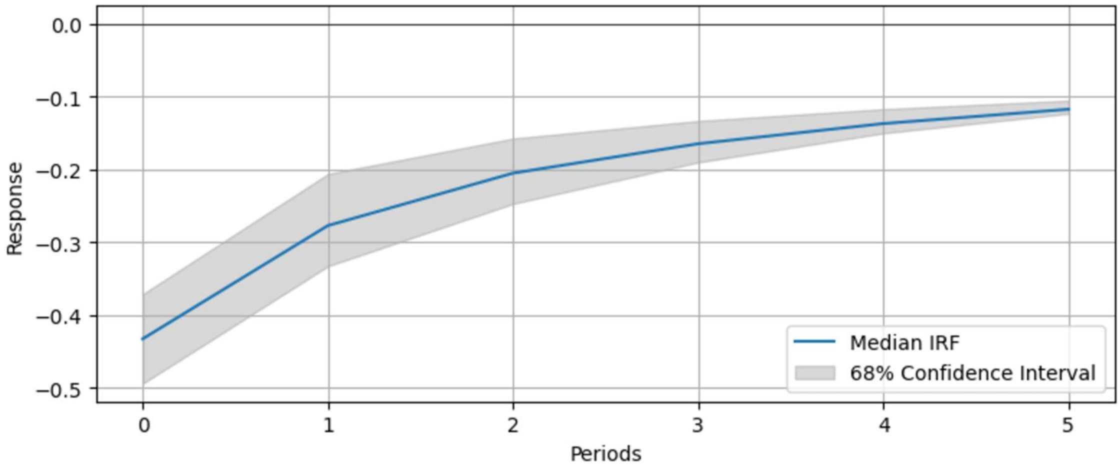

Figure 11 – Impulse Response of Undergound Economy to an Underground Economy shock