Welfare effects of jointly managed non-targeted advertising in multi-sided media markets

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Abstract

We discuss the welfare effects of joint advertising by two media platforms that compete for viewers by choosing their subscription prices and also obtain revenue from selling to advertisers access to their subscriber base. Considering the case of non-targeted advertising, we show that welfare may increase with joint advertising and establish the conditions under which this is more likely to occur. We find that welfare may increase for intermediate values of the nuisance cost that advertising has on consumers.

JEL Classification: *L13; L41.*

Keywords: Joint advertising, Media; Product differentiation

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1 Introduction

The regulation of joint advertising management in multi-sided media markets has drawn increasing scrutiny. In December 2021, the Portuguese competition authority issued a Statement of Objections to the three largest pay-TV operators and a consulting firm for alleged anti-competitive practices, citing agreements on 30-second pre-recorded ads and standardized pricing for advertisers. Similarly, in 2010, the French competition authority imposed behavioral remedies on TF1 Group's acquisition of NT1 and TMC, requiring the independence of their advertising sales operations (Ivaldi & Zhang, 2022). These cases highlight concerns about the welfare effects of coordinated advertising strategies in two-sided markets. Motivated by such regulatory interventions, we analyze the implications of joint management of non-targeted advertising while preserving independent pricing decisions.

Our study builds on the literature examining competition between media platforms that generate revenue from both advertising and subscriptions. Anderson and Coate (2005) show that equilibrium ad levels may be inefficient, with subscriptions potentially reducing consumer surplus. Peitz and Valletti (2008) extend this by incorporating endogenous content provision, finding that free-to-air platforms provide more ads and less differentiated content when consumers dislike advertising. Other studies explore market structure and strategic behavior: Crampes et al. (2009) analyze entry and pricing, Maestre and Sanchez (2014) compare private versus public platforms, and Ambrus et al. (2016) and Anderson et al. (2019) examine multi-homing effects. Research on ad avoidance (e.g., Anderson & Gans, 2011; Stühmeier & Wenzel, 2011) and targeted advertising (e.g., Gong et al., 2019 and Johnston (2013)) further enriches the discussion.

We contribute by investigating how joint advertising affects welfare, showing that it can increase or decrease ad levels depending on viewers' marginal benefit.[1] Welfare is most likely to decline when

[1]Full mergers have been considered in the literature by Anderson and Coate (2005), Ambrus et al (2016) and Anderson et al (2018) for the case of non targeted advertising.

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ad nuisance is very high or low but improves at intermediate levels.

The paper is structured as follows: Section 2 presents the model and main results, Section 3 concludes, and proofs are in the online appendix.

2 The model

The model includes three types of agents: viewers (or consumers), platforms and advertisers (or producers). Viewers consider platforms to be horizontally differentiated and choose which platform to subscribe to based on their preferences for content, the subscription fee and the amount of advertising on the platform. Producers decide to advertise in one platform or in both by considering the advertising costs and the surplus they expect to obtain by reaching each platform's subscriber base, thus increasing viewer's demand for their products. Two platforms decide the price they charge for advertising and the viewers' subscription fees. Producers are not assumed to compete with each other in an explicit way.

In terms of timing: (1) platforms simultaneously choose advertising prices and subscription fees; (2) advertisers decide where to advertise; and finally (3) viewers choose their platform.

2.1 The relationship between viewers and advertisers

With respect to advertisers, we follow the model proposed by Peitz and Valletti (2008): all consumers have the same willingness to pay for each product, which depends on its quality, α, which also denotes the surplus the producer extracts from each sale. This quality α is uniformly distributed across producers in $[0, 1]$ and the mass of producers is normalized to 1. Contrary to Peitz and Valletti (2008), we allow consumers to retain some surplus, that is, the surplus generated by a product of quality α is $(1 + g)\alpha$, of which α will be captured by the advertiser and $g\alpha$ by the consumer, with

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g exogenously defined.[2]

We assume there are N viewers. Having been exposed to an ad on the platform that a viewer subscribed to, the probability of purchasing the advertised product is denoted by p. This is independent across consumers, quality levels and platforms.[3] With N_i denoting the number viewers on platform $i = 1, 2$, the expected sales of any product advertised in platform i is then $N_i p$.

2.2 The relationship between platforms and advertisers

With r_i denoting the cost of placing an ad at platform i, a producer of quality α makes positive expected profit and decides to advertise in platform i if

$$\alpha p N_i - r_i > 0 \Leftrightarrow \alpha > \frac{r_i}{p N_i}.$$

Given the assumptions above, the demand for ads in platform i is $a_i = 1 - \frac{r_i}{pN_i}$ or $r_i = p\,(1 - a_i)\,N_i$. Each platform is a monopolist when selling access to its subscribers.[4] Hence, deciding over price r_i is equivalent to choosing the advertising level a_i. Platform i's advertising revenue per viewer is

$$z_i(a_i) := \frac{r_i a_i}{N_i} = \frac{p\,(1 - a_i)\,N_i a_i}{N_i} = p\,(1 - a_i)\,a_i$$

[2] Assume, for instance, that consumers have demand given $q = \beta - P$ for the product of a monopolist of type β, with zero costs. The monopolist will maximize its profit with price equal to $\beta/2$. The profit per consumer will then be $\beta^2/4$ and consumer surplus will be $\beta^2/8$. In this example, $\alpha = \beta^2/4$ and $g = \frac{1}{2}$.

[3] In Anderson and Coate (2005), a viewer has willingness to pay $\alpha > 0$ with probability p for a new good of type p and willingness to pay 0 with probability $1 - p$. Each new good is characterized by a different p, the probability of any consumer purchasing, that is, this probability increases with product quality while the willingness to pay is the same for all products. In Peitz and Valletti (2008) a product is produced at quality α and all consumers have willingness to pay α for this good, that is, $p = 1$. Each new good is characterized by a different willingness to pay.

[4] This corresponds to Armstrong (2006) "competitive bottlenecks" scenario, with multi-homing advertisers and single-homing viewers.

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Advertisers' aggregate expected profits are

$$\Pi_A = \pi_A - r_i a_i - r_j a_j = \int_{1-a_i}^{1} (\alpha p N_i - r_i)\, d\alpha + \int_{1-a_j}^{1} (\alpha p N_j - r_j)\, d\alpha$$
$$= \frac{1}{2} p \left(N_i a_i^2 + N_j a_j^2\right)$$

where π_A denotes all advertisers' expected profit from ad-induced sales.

2.3 The relationship between viewers and platforms

Viewers are assumed to select only one platform (i.e. they single-home). In terms of their preferences for platform characteristics (content, brand image, etc...), consumers are uniformly distributed along a Hotelling (1929) line, with platforms 1 and 2 located at 0 and 1. For a consumer located at $x \in [0, 1]$, the net utility of subscribing to platform i, located at l_i, is $v - t(x - l_i)^2 + X_i(a_i) - s_i$, where v is the willingness-to-pay for programming fully aligned with the viewer's preferences, which we assume to be sufficiently large to ensure that all viewers will subscribe to one platform; $-t(x - l_i)^2$ represents the disutility cost of platform i programming being misaligned with the viewer's preferences; $X_i(a_i)$ represents the impact on consumer utility of advertising on platform i and s_i denotes platform i's subscription fee.

$X_i(a_i)$ includes the following terms: On the one hand, we assume that consumers, when selecting their platform, anticipate their expected surplus from purchasing advertised products, which is $pg\alpha$ for a product of quality α. On the other hand, there is some disutility associated with each ad that is broadcast on platform i, measured by parameter δ. Hence,

$$X_i(a_i) := \int_{1-a_i}^{1} (pg\alpha)\, d\alpha - \delta a_i = pg \frac{a_i (2 - a_i)}{2} - \delta a_i$$

Some of the results below are framed in terms of the advertising average nuisance cost per product

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purchased, $\delta' = \frac{\delta}{p}$.

When $pg > 0$ there is an expected marginal benefit of advertising for consumers which is decreasing with the number of ads and may be lower than the nuisance marginal cost. This happens because each additional ad comes from a lower quality producer that generates smaller surplus, but generates the same nuisance cost, δ. From the consumers' perspective the optimal level of advertising on platform i is $a_i^C = \max\left\{1 - \delta' \frac{1}{g}, 0\right\}$.

Assumption 1 ensures there is positive advertising in equilibrium.

Assumption 1: $p\,(g+1) - \delta > 0 \Leftrightarrow \delta' < 1 + g$.

A viewer located at $\widetilde{x} \in [0,1]$ is indifferent between the two platforms if

$$\begin{aligned} v - t(\widetilde{x})^2 + X_1(a_1) - s_1 = v - t(1-\widetilde{x})^2 + X_2(a_2) - s_2 \Leftrightarrow \\ \widetilde{x}(a_1, a_2, s_1, s_2) = \frac{X_1(a_1) - X_2(a_2) - s_1 + s_2}{2t} + \frac{1}{2} \end{aligned}$$

Hence $N_1 = \widetilde{x}N$ and $N_2 = (1-\widetilde{x})N$.

Assuming the costs per subscriber and per ad to be zero, platform i's profit is $\pi_i = (z_i(a_i) + s_i)\,N_i$. When platforms are independent, a_i and s_i are chosen to maximize each platform's profit whereas, under joint advertising, a_i is set to maximize joint profits while each platform chooses s_i to maximize its own profit.

Given these assumptions, consumer surplus is

$$\begin{aligned} CS &= \left(\int_0^{\widetilde{x}} (v - t(x)^2 + X_i - s_i)dx + \int_{\widetilde{x}}^1 (v - t(1-x)^2 + X_j - s_j)dx \right) N \\ &= vN - TN + CS_A - (s_i N_i + s_j N_j) \end{aligned}$$

where T denotes transportation cost and $CS_A = \int_{1-a_i}^1 (pg\alpha - \delta)\,d\alpha N_i + \int_{1-a_j}^1 (pg\alpha - \delta)\,d\alpha N_j$ de-

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notes the expected consumer surplus from advertised products, net of nuisance costs.

Social welfare is $W = vN - TN + CS_A + \pi_A$ where the assumptions of a covered market and platform symmetry make the first two terms invariant to how advertising is managed. The first-best level of advertising in platform i is the one that maximizes $\int_{1-a_i}^{1} (p(1+g)\alpha)\, d\alpha - \delta a_i$,

$$a_i^W = 1 - \frac{\delta'}{g+1}$$

As there is some nuisance cost associated with advertising, δ, and as transaction of low quality products ($\alpha \to 0$) do not generate any surplus, the socially optimal level of advertising is below 1: not all producers advertise.

2.4 Equilibria

The following Lemma presents the equilibrium, when platforms' decisions are independent.

Lemma 1: *In equilibrium with independent platforms, :*

$$\begin{aligned}
a_i^* &= \frac{p\,(g+1) - \delta}{p\,(g+2)} \\
s_i^* &= t - z_i^* = t - \frac{(p\,(g+1) - \delta)\,(\delta + p)}{p\,(g+2)^2} \\
\widetilde{x}^* &= \frac{1}{2} \\
r_i^* &= p\,(1 - a_i)\,N_i = \frac{\delta + p}{g+2}\frac{1}{2}N
\end{aligned}$$

The equilibrium level of advertising, a_i^*, increases with p and g and decreases with δ. Each platform is a monopolist when deciding how much advertising to accept. The cost of broadcasting more ads corresponds to the impact this has on the number of subscribers, which follows from

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$\frac{\partial X_i}{\partial a_i} = pg(1 - a_i) - \delta$. Increases in p and g make consumers more tolerant to advertising and thus lead to more ads in equilibrium. Likewise for a reduction in δ. Additionally, a higher p also increases advertisers' demand for ads which results in more ads in equilibrium.

From a social perspective, equilibrium advertising on platform i is always insufficient:

$$a_i^w - a_i^* = 1 - \frac{\delta}{p(g+1)} - \frac{p(g+1) - \delta}{p(g+2)} > 0 \Leftrightarrow -(p(g+1) - \delta) < 0.$$

This follows from platforms being unable to capture all advertisers' profit (as non-discriminating monopolists) and also from being unable to capture consumer surplus from advertising (due to inter platform competition for subscribers).

The marginal impact of advertising on consumer net utility, in equilibrium, is

$$\left.\frac{\partial X_i}{\partial a_i}\right|_{a_i = a_i^*} = \frac{2p}{g+2}\left(\frac{g}{2} - \delta'\right) \gtrless 0$$

meaning that equilibrium advertising may be excessive or insufficient from the consumer's perspective.

The equilibrium subscription fee, s_i^*, increases with platform differentiation and decreases with z_i because a higher advertising revenue per subscriber makes the platform compete more aggressively for subscribers.

Platform equilibrium profits are

$$\pi_i = t\frac{N}{2}$$

which is the same as in a model with no advertising because there is a full pass-through of advertising revenues into lower subscription prices.[5]

[5] See Choi (2006) and Peitz and Valletti (2008).

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The equilibrium consumer surplus, advertiser profits and welfare are, respectively

$$CS^* = \left(v - \frac{1}{12}t - t + \frac{1}{2}\frac{\left(p\left(g+1\right)-\delta\right)^2}{p\left(g+2\right)}\right)N$$
$$\Pi_A^* = \frac{\left(p\left(g+1\right)-\delta\right)^2}{p\left(g+2\right)^2}\frac{N}{2}$$
$$W^* = \left(v - \frac{1}{12}t + \left(g+3\right)\frac{\left(p\left(g+1\right)-\delta\right)^2}{2p\left(g+2\right)^2}\right)N$$

When advertising is jointly managed, the advertising decisions are taken to maximize the platforms joint profits but the subscription fees are set to maximize each platform's individual profit. The equilibrium variables associated with this case are identified by a superscript J.

Lemma 2: *When advertising is jointly managed, in the symmetric equilibrium,*

$$a_i^{J*} = \frac{1}{2},\ s_i^{J*} = t - \ z_i^{J*} = t - \frac{p}{4},\ r_i^{J*} = \frac{p}{4}N$$

The corresponding equilibrium consumer surplus, platform profits, advertiser profits and welfare are

$$CS^{J*} = \left(v - \frac{1}{12}t - t + \frac{1}{8}\left(p\left(3g+2\right)-4\delta\right)\right)N$$
$$\pi_1^{J*} + \pi_2^{J*} = tN$$
$$\Pi_A^{J*} = \frac{p}{8}N$$
$$W^{J*} = \left(v - \frac{1}{12}t + \frac{1}{8}\left(3p\left(g+1\right)-4\delta\right)\right)N$$

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2.5 Welfare effects

The following Proposition presents the consequences of joint advertising.

Proposition 1: *When moving from independent to joint advertising management,*

i) Platform profits remain the same.

ii) Advertisers' profits increase if and only if $\delta' > \frac{g}{2}$.

iii) Consumer surplus decreases.

iv) Social welfare increases if and only if $\frac{g}{2} < \delta' < \frac{g+4}{2}\frac{g+1}{g+3}$.

Given that platforms are symmetric, the changes in advertising revenue per viewer are fully passed-on to subscription prices, as in Peitz and Valletti (2008). An increase in advertising revenue per viewer ($z_i^{J*} - z_i^* = \frac{p\left(\delta' - \frac{g}{2}\right)^2}{(g+2)^2} > 0$) is matched by a similar decrease in subscription prices and as each platform will have 50% of all viewers in both cases, the profit is the same under independent or joint advertising.[6]

The advertisers' aggregate profit increases with advertising:

$$\Pi_A = \frac{1}{2}p\left(N_i a_i^2 + N_j a_j^2\right) = \frac{1}{2}Npa_i^2$$

and advertising may either increase or decrease with joint advertising:

$$a_i^{J*} - a_i^* = \frac{1}{2} - \frac{p\,(g+1) - \delta}{p\,(g+2)} = \frac{\delta' - \frac{g}{2}}{g+2} \gtrless 0$$

If the marginal impact of advertising on consumer net utility is negative in the independent advertising management case, that is if $\left.\frac{\partial X_i}{\partial a_i}\right|_{a_i = a_i^*} < 0$, then, when a platform decides to increase advertising

[6] Joint advertising could increase platform profits in the presence of any fixed advertising costs that might not be duplicated under joint advertising. However, we have omitted such costs for simplicity.

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it knows it will lose viewers. Under joint advertising, this effect is mitigated because the viewers lost by one platform will switch to the other. Therefore, advertising will increase when jointly managed, also increasing advertisers' aggregate profit. The opposite happens when if $\left.\frac{\partial X_i}{\partial a_i}\right|_{a_i=a_i^*} > 0$.

Thus, joint advertising always moves advertising in the "wrong" direction from the consumers' perspective, leading to a lower consumer surplus despite the fact that subscription fees always decrease:[7]

$$s_i^{J*} - s_i^* = -\frac{p\left(\frac{g}{2} - \delta'\right)^2}{(g+2)^2} < 0$$

With respect to welfare, recall only $CS_A + \pi_A$ change when moving from independent advertising to joint advertising. Figure 1 presents the expected social value that is generated by advertising-induced sales of each producer, that is $CS_A(\alpha) + \pi_A(\alpha)$, as a function of $\alpha \in [0, 1]$.

Advertising by any producer with quality larger than α_W increases social welfare and advertising by any producer with quality larger than α_C increases consumer surplus. The marginal advertiser with independent advertising management, $\alpha^* = 1 - a_i^*$ is such that $\alpha^* > \alpha_W$ but α^* may be larger or smaller than α_C. Hence, Figure 1 is divided in two panels: In panel 1a we have $\alpha^* > \alpha_C$ and in panel 1b we have $\alpha^* < \alpha_C$. The integral corresponds to expected social welfare derived from advertising-induced sales in both cases.

The arrows illustrate how the marginal advertiser changes when introducing joint advertising management. If this shifts to the left (right), advertising increases (decreases), which, as seen above can only happen when the marginal impact of advertising on consumers is negative (positive). In panel 1a welfare and consumer surplus can only decrease, whereas in panel 1b welfare can increase

[7] In our set up, viewers consider both the costs of advertising and the benefits associated with future purchases when selecting their platform. If viewers only took the nuisance cost into consideration (myopic viewers), joint advertising management would always lead to more advertising. This could lead to higher consumer surplus when the existing benefits of future additional purchases, disregarded by the myopic viewer when selecting the platform, more than outweigh the additional nuisance cost.

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or decrease, depending on how much advertising increases.

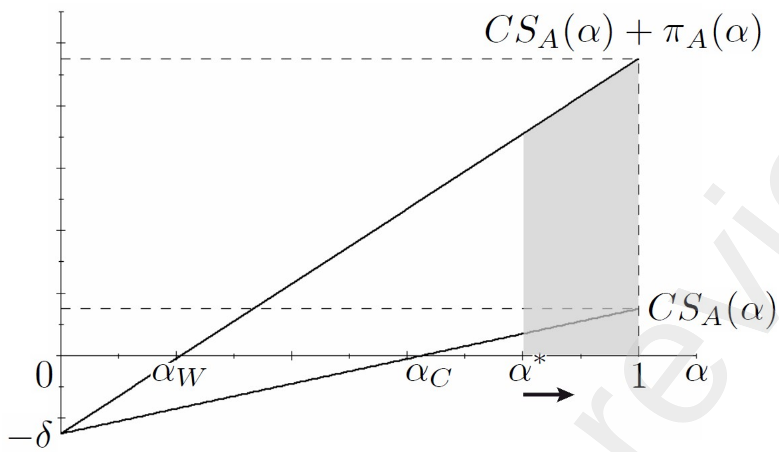

Figure 1a: Social welfare and consumer surplus as a function

of quality when $\alpha^* > \alpha_C$.

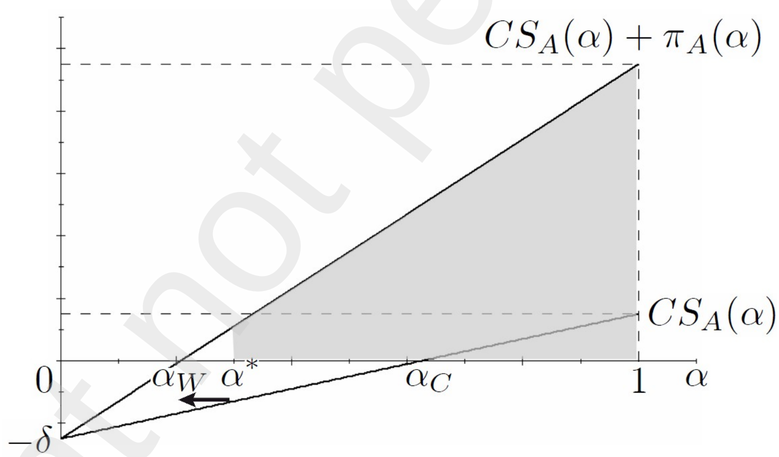

Figure 1b: Social welfare and consumer surplus as a function

of quality when $\alpha^* < \alpha_C$.

Figure 2 presents the relevant areas in the $(g, \frac{\delta}{p})$-space. If $\delta' < \frac{g}{2}$ the marginal impact of advertising on consumer utility, in the independent equilibrium, is positive and joint advertising leads to a reduction in advertising, to lower advertiser profits, consumer surplus and social welfare.

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If $\delta' > \frac{g}{2}$, joint advertising will lead to more ads and the increase is higher, the higher δ' because the effect that joint advertising internalizes is stronger:

$$a_i^{J*} - a_i^* = \frac{\delta' - \frac{g}{2}}{g+2}.$$

This may affect welfare positively or negatively. As the original number of ads is below the socially optimal level, a small increase in advertising will lead to higher welfare, with advertisers' profits increasing by more than the reduction in consumer surplus. This happens when δ' is not too high, that is, when $\delta' < \frac{g+4}{2}\frac{g+1}{g+3}$. When δ' is larger advertising significantly exceeds the socially optimal level and welfare decreases. As Figure 2 illustrates, welfare increases with joint advertising for "intermediate" levels of δ'.

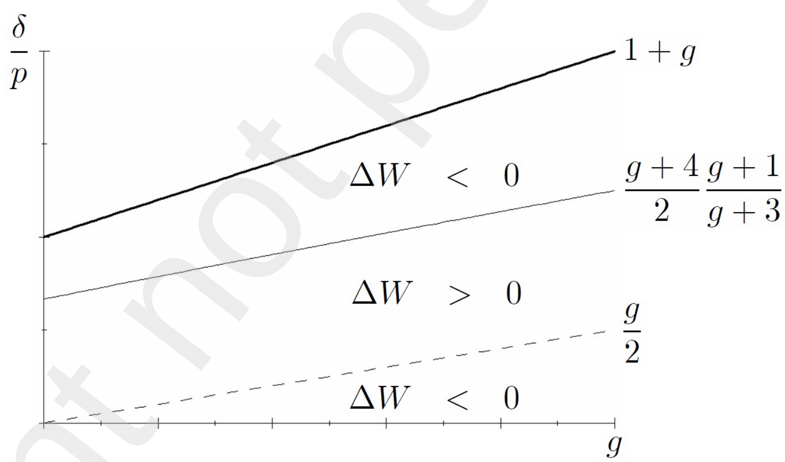

Figure 2: Welfare effects of joint advertising

3 Conclusions

This paper delves into the intricate dynamics of consumer surplus and welfare effects stemming from the collaborative management of advertising between competing platforms that facilitate advertisers

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and viewers interactions (while deciding independently on subscription prices).

Our analysis unveils that consumer surplus consistently diminishes with joint advertising, while the impact on welfare may sway unpredictably and depends on how much nuisance advertising imposes on viewers. For low or high values of this advertising "cost", social welfare decreases but when the parameter takes intermediate values, welfare is shown to increase.

This result is obtained under the assumption that the probability that a viewers' exposure to an ad leads to a purchase is not affected by joint management. Arguably, joint management may increase such probability if it leads to an improved ad targeting ability and in that case it would be more likely to increase welfare.

An interesting avenue for further research lies in explicitly introducing targeted advertising into the model. Though such a pursuit would undoubtedly entail a more complex model, hopefully the present analysis can be seen as a stepping stone in the direction of this more complete analysis.

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Online Appendix

Proof of Lemma 1: For platform 1, the profit function is

$$\pi_1 = (z_1(a_1) + s_1)\, N_1$$

with

$$\begin{aligned} N_1 &= \left(\frac{X_1(a_1) - X_2(a_2) - s_1 + s_2}{2t} + \frac{1}{2} \right) N \\ X_1(a_1) &= pg\frac{a_1\,(2 - a_1)}{2} - \delta a_1 \\ z_1(a_1) &= p\,(1 - a_1)\, a_1 \end{aligned}$$

So, the first-order conditions for profit maximization are:

$$\begin{aligned} \frac{\partial \pi_1}{\partial s_1} &= N_1 + (z_1(a_1) + s_1)\,\frac{-1}{2t} N \\ &= \left[\widetilde{x}(a_1, a_2, s_1, s_2) + (p\,(1 - a_1)\,a_1 + s_1)\,\frac{-1}{2t} \right] N = 0 \\ \frac{\partial \pi_1}{\partial a_1} &= \frac{\partial z_1}{\partial a_1} N_1 + (z_1(a_1) + s_1)\,\frac{1}{2t} N \frac{\partial X_1}{\partial a_1} \\ &= \left[p\,(1 - 2a_1)\,\widetilde{x}(a_1, a_2, s_1, s_2) + (p\,(1 - a_1)\,a_1 + s_1)\,\frac{1}{2t}(\frac{1}{2}pg\,(2 - 2a_1) - \delta) \right] N = 0 \end{aligned}$$

Replacing the first condition,

$$\widetilde{x}(a_1, a_2, s_1, s_2) = (z_1(a_1) + s_1)\,\frac{1}{2t} = (p\,(1 - a_1)\,a_1 + s_1)\,\frac{1}{2t}$$

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into the second one we obtain,

$$p\left(1-2a_1\right)\left(p\left(1-a_1\right)a_1+s_1\right)\frac{1}{2t}+\left(p\left(1-a_1\right)a_1+s_1\right)\frac{1}{2t}(\frac{1}{2}pg\left(2-2a_1\right)-\delta)=$$
$$\left(-p\left(g+1\right)+\delta+p\left(g+2\right)a_1\right)\frac{pa_1\left(1-a_1\right)+s_1}{2t}=0$$

which is equivalent to

$$a_1^*=\frac{p\left(g+1\right)-\delta}{p\left(g+2\right)}$$

if $(z_1(a_1)+s_1)>0$. The same holds for a_2, meaning that, for $i=1,2$

$$z_i^*=p\left(1-a_i^*\right)a_i^*=p\left(1-\frac{p\left(g+1\right)-\delta}{p\left(g+2\right)}\right)\frac{p\left(g+1\right)-\delta}{p\left(g+2\right)}=\left(\delta+p\right)\frac{\left(p\left(g+1\right)-\delta\right)}{p\left(g+2\right)^2}.$$
$$X_i^*=\left(p\left(g+1\right)-\delta\right)\frac{pg\left(g+3\right)-\delta\left(g+4\right)}{2p\left(g+2\right)^2}$$

The two conditions on subscription prices are

$$\frac{-s_1+s_2}{2t}+\frac{1}{2}+\left(p\left(1-\frac{p\left(g+1\right)-\delta}{p\left(g+2\right)}\right)\frac{p\left(g+1\right)-\delta}{p\left(g+2\right)}+s_1\right)\frac{-1}{2t}=0$$
$$(\frac{s_1-s_2}{2t}+\frac{1}{2})+\left(p\left(1-\frac{p\left(g+1\right)-\delta}{p\left(g+2\right)}\right)\frac{p\left(g+1\right)-\delta}{p\left(g+2\right)}+s_2\right)\frac{-1}{2t}=0$$

After solving with respect to s_1 and s_2 this yields

$$s_1^*=s_2^*=t-\left(p+\delta\right)\frac{p\left(g+1\right)-\delta}{p\left(g+2\right)^2}$$

Proof of Lemma 2:

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The first-order conditions for profit maximization are:

$$\frac{\partial\left(\left(z_1+s_1\right)N_1\right)}{\partial s_1}=N_1+\left(z_1+s_1\right)\frac{-1}{2t}N=0$$
$$\frac{\partial\left(\left(z_2+s_2\right)N_2\right)}{\partial s_2}=N_2+\left(z_2+s_2\right)\frac{-1}{2t}N=0$$
$$\frac{\partial\left(\left(z_1+s_1\right)N_1+\left(z_2+s_2\right)N_2\right)}{\partial a_1}=\frac{\partial z_1}{\partial a_1}N_1+\left(\left(z_1+s_1\right)-\left(z_2+s_2\right)\right)\frac{1}{2t}N\frac{\partial X_1}{\partial a_1}=0$$
$$\frac{\partial\left(\left(z_1+s_1\right)N_1+\left(z_2+s_2\right)N_2\right)}{\partial a_2}=\frac{\partial z_2}{\partial a_2}N_2+\left(\left(z_2+s_2\right)-\left(z_1+s_1\right)\right)\frac{1}{2t}N\frac{\partial X_2}{\partial a_2}=0$$

Plugging the first two conditions into the last two yields

$$\frac{\partial z_1}{\partial a_1}\left(z_1+s_1\right)+\left(\left(z_1+s_1\right)-\left(z_2+s_2\right)\right)\frac{\partial X_1}{\partial a_1}=0$$
$$\frac{\partial z_2}{\partial a_2}\left(z_2+s_2\right)+\left(\left(z_2+s_2\right)-\left(z_1+s_1\right)\right)\frac{\partial X_2}{\partial a_2}=0$$

from where the symmetric equilibrium is obtained.

Proof of Proposition 1:

i) See main text.

ii) Advertisers' profits increase if and only if

$$\frac{p}{8}N-\frac{\left(p\left(g+1\right)-\delta\right)^2}{p\left(g+2\right)^2}\frac{N}{2}>0\Leftrightarrow$$
$$\frac{1}{8}N\left(2\left(p\left(1+g\right)-\delta\right)+p\left(g+2\right)\right)\frac{2\delta-gp}{p\left(g+2\right)^2}>0\Leftrightarrow\frac{\delta}{p}>\frac{g}{2}$$

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iii) Consumers' surplus always decreases

$$\left(v - \frac{1}{12}t - t + \frac{1}{8}\left(2p - 4\delta + 3gp\right)\right)N - \left(v - \frac{1}{12}t - t + \frac{1}{2}\frac{\left(p\left(g+1\right) - \delta\right)^2}{p\left(g+2\right)}\right)N < 0 \Leftrightarrow$$

$$-N\frac{\left(2\delta - gp\right)^2}{8p\left(g+2\right)} < 0$$

which is always true.

iv) Social welfare increases if and only if

$$\left(v - \frac{1}{12}t + \frac{1}{8}\left(3p\left(g+1\right) - 4\delta\right)\right)N - \left(v - \frac{1}{12}t + \left(g+3\right)\frac{\left(p\left(g+1\right) - \delta\right)^2}{2p\left(g+2\right)^2}\right)N > 0 \Leftrightarrow$$

$$\left(\frac{\delta}{p}\right)^2\left(4g+12\right) - \frac{\delta}{p}4\left(4g + g^2 + 2\right) + g\left(g+4\right)\left(g+1\right) < 0 \Leftrightarrow$$

$$\frac{1}{2}g < \frac{\delta}{p} < \frac{1}{2}\left(g+4\right)\frac{g+1}{g+3}$$

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