ABSTRACT

The Persistence of Wages

This paper provides comprehensive and detailed empirical regression analyses of the sources of wage persistence. Exploring a rich matched employer-employee data set and the estimation of a dynamic panel wage equation with high-dimensional fixed effects, our empirical results show that permanent unobserved heterogeneity plays a key role in driving wage dynamics. The decomposition of the omitted variable bias indicates that the most important source of bias is the persistence of worker characteristics, followed by the heterogeneity of firms' wage policy and last by the job-match quality. We highlight the importance of the incidental parameter problem, which induces a severe downward bias in the autoregressive parameter estimate, through both an in-depth Monte Carlo study and an empirical analysis. Using three alternative bias correction methods (the split-panel Jackknife (Dhaene and Jochmans, 2015), an analytical expression (Hahn and Kuersteiner, 2002), and a residual based bootstrap approach (Everaert and Pozzi, 2007, Gonçalves and Kaffo, 2015)), we observe that up to one-third of the reduction of the autoregressive parameter estimates induced by the control of permanent heterogeneity (high dimensional fixed effects) may not be justified.

JEL Classification: J31, J63, J65, E24

Keywords: wage persistence, high-dimensional fixed effects, match effects, incidental parameter problem

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1 Introduction

Relying on the idea that in an environment of search frictions and large heterogeneity in match quality there is a potential for wage growth over the working life via on-the-job search (Burdett and Mortensen, 1998), a growing body of research has emerged modelling earnings dynamics over the individual's life cycle (e.g., Postel-Vinay and Robin, 2002; Buchinsky et al., 2010; Low et al., 2010; Postel-Vinay and Turon, 2010; Altonji et al., 2013). To identify the mechanisms that drive earnings dynamics throughout an individual's career, this literature relies on the estimation of a structural model that takes individuals' mobility decisions over their working life and unemployment shocks into account. A key idea in these models is that a worker's future earnings and employment prospects will depend on his/her personal characteristics that are transferable across jobs, the job-match specific component of the current job, the job-to-job transitions, and the unemployment shocks. In this framework, wage persistence plays an important role in the sense that an individual's job search aspirations are largely determined by the job-specific component of the current job, which depends on previous wage offers, with job changes induced by offers of higher wages (Macleod and Malcomson, 1993; Postel-Vinay and Robin, 2002; Postel-Vinay and Turon, 2010; Altonji et al., 2013; and Bonhomme et al., 2019). Thus, earnings persistence is the combined effect of permanent observed and unobserved individual heterogeneity, permanent observed and unobserved job-match heterogeneity, and state dependence driven by cyclical innovations in the income process that may have persistent effects over time (Altonji et al., 2013; and Ejrnaes and Browning, 2014).[1]

The empirical analyses in this recent line of research provide some convincing conclusions. First, wages are highly persistent but do not exhibit random walk (unit root) type behavior (Alvarez and Arellano, 2004; Altonji et al., 2013; and Hospido, 2015). Second, time series dependence tends to decrease once permanent heterogeneity of the individual and the job is taken into account, suggesting that wage persistence stems largely from the time-invariant components of unobserved heterogeneity (Altonji et al., 2013; and Hospido, 2015). The relevance of the latter is also corroborated by an extensive empirical literature drawn from linked employer-employee data that has stressed the importance that observed and unobserved characteristics of workers, firms, and job specific match quality can have in explaining individual wages (see, among others, Abowd et al., 1999, Goux and Maurin, 1999, Woodcock, 2008, 2015, Torres et al., 2018, and Raposo et al., 2021). Third, job mobility choices play an important role in explaining wage fluctuations over the life cycle and they are driven by the value of the current match (Topel and Ward, 1992; Abowd et al., 2006; Low et al., 2010; Hospido, 2012b; Altonji et al., 2013; and Liu, 2019). Fourth, the degree of dependence of current wages on past wages seems to be lower across jobs than within jobs (Hospido, 2015; and Bonhomme et al., 2019).

[1] For an insightful discussion of the distinction between heterogeneity and structural state dependence see Heckman (1981), who illustrated these concepts developing a dynamic model of labor supply.

Framed in this influential literature, the aim of this paper is to show how much of the apparent persistence of wages is driven by worker, firm, and job-match quality heterogeneity, highlighting the role of omitted deterministics in driving wage persistence; and to illustrate the importance, both through Monte Carlo simulations and an empirical analysis, of the incidental parameter problem in the framework of dynamic wage models with high-dimensional fixed effects.

Exploring a rich matched employer-employee data set and an econometric model of wage growth over the career, we estimate a wage equation model that incorporates both state dependence (through a first-order autoregressive wage model) and job-match fixed effects. Then, building on Gelbach (2016), who uses the omitted variable bias formula to decompose the contribution of added covariates to changes in the estimates of the regression coefficients of interest, we compute the independent contribution of each unobserved fixed effect – worker, firm, and job-match quality - to the change in the autoregressive wage parameter estimates in order to provide a comprehensive decomposition of the sources of wage persistence.

The studies most closely related to ours in motivation and objectives are the recent contributions by Hospido (2012b, 2015). The originality of her analysis resides in the proposal of a model that takes into account individual and job unobserved heterogeneity and dynamics in the conditional variance of individual wages. Specifically, in Hospido (2012b), using data from the PSID 1968-1993, a nonlinear dynamic panel data model with worker (random) effects is estimated, concluding for the importance of permanent individual unobserved heterogeneity and state dependence effects in explaining the variance of wages. Along the same line of reasoning, Hospido extended her previous research by incorporating in the analysis the role of job-specific effects in the conditional variance of wages (Hospido, 2015). Estimation of a dynamic error components model showed that the autoregressive parameter estimate decreased considerably after netting out worker and job specific effects. Our approach overcomes two shortcomings in Hospido's studies: one is the lack of matched employer-employee data that allow us to uniquely identify workers and firms, and to track them over the years; and the other is the failure to decompose the job-match fixed effect into its components - worker, firm, and job-match quality. In our analysis, we favor an autoregressive fixed effects model to Hospido's autoregressive random effects approach for those two main reasons.

Our empirical analysis consists of the estimation of a dynamic panel wage equation with high-dimensional fixed effects using a balanced panel of full-time prime-age male (female) workers of 655,120 (383,456) observations and an unbalanced panel of full-time male (female) workers of 12,802,613 (9,800,784) observations over the 2002-2018 period. Specifically, the balanced panel comprises 40,945 (23,966) male (female) workers, 10,035 (4,680) firms, and 69,276 (38,795) job matches, and the unbalanced panel includes 2,014,995 (1,600,305) male (female) workers, 256,674 (225,318) firms, and 3,111,886 (2,372,438) job matches.

In estimating the model, special attention is given to the incidental parameter problem resulting from the estimation of a large number of fixed effects in short panels (Neyman and Scott, 1948; Chamberlain, 1980; Lancaster, 2000). Finally, an in-depth Monte Carlo exercise is performed to illustrate how the high-dimensional fixed effects may impact (bias) the autoregressive wage parameter estimate, conditional on worker, firm, and job-match quality fixed effects. Additionally, three alternative bias correction methods are used to address the incidental parameter bias: (i) the split-panel Jackknife estimator proposed in Dhaene and Jochmans (2015); (ii) the analytical solution of Hahn and Kuersteiner (2002), and (iii) a recursive-design residual-based wild bootstrap approach as suggested by Gonçalves and Kaffo (2015). Note that since all these methods have been introduced in the context of a conventional fixed effects model, the Monte Carlo results presented here will provide useful information on the performance of these approaches in the more complex framework of multiple high-dimensional fixed effects.

Results show that permanent observed and unobserved heterogeneity components (such as worker, firm, and job-match quality) play an important role in driving wage dynamics. Moreover, after accounting for these components and for the incidental parameter problem induced by the presence of high-dimensional fixed effects in the dynamic setting, there is still a positive (although smaller) relationship between current and past wages that is captured by the autoregressive parameter estimate. For instance, the empirical analysis of the unbalanced panel of male (female) workers shows that controlling for job-match fixed effects reduces the autoregressive wage parameter estimate from 0.949 (0.999) to 0.292 (0.255). The conditional decomposition of the autoregressive coefficient estimate shows that of the total wage persistence effects driven by worker, firm, and job-match quality permanent heterogeneity, 53.2% (58.1%) is due to worker, 31.8% (27.4%) to firm, and around 15% (14.5%) to the job-match quality fixed effects.

The contribution of this study to the wage dynamics literature is fourfold. First, we provide a detailed decomposition of the sources of wage persistence in light of the distinction between heterogeneity and state dependence (Heckman, 1981; and Bonhomme et al., 2019); second, we apply Gelbach's decomposition in the framework of a dynamic panel wage model with job-match fixed effects using bias corrections to overcome the incidental parameter problem, contributing in this way to illustrate the impact of these issues in the context of the estimation of high-dimensional fixed effects in short panels; third, we apply our empirical approach to a rich administrative matched employer-employee data set, using balanced and unbalanced panels of both male and female workers. Although the use of balanced panels is common in theoretical and applied work, the usefulness of the unbalanced samples is that they can be less restrictive in terms of requirements and more representative of all workers and job careers, and therefore, in our empirical analysis, for completeness we consider both frameworks; and fourth, we provide an in-depth Monte Carlo exercise that contributes to the scant literature on bias corrections in a high-dimensional fixed effects context (see, for instance, Hospido, 2012a and Charbonneau, 2017).

The rest of the paper is organized as follows. Section 2 discusses the decomposition approach in the context of models with worker and firm fixed effects, and job-match fixed effects; Section 3 provides an in-depth Monte Carlo analysis of the impact of omitted variables and the incidental parameter problem, as well as evidence on the performance of the three bias correction approaches; Section 4 provides an empirical analysis of wage persistence and the results of Gelbach's decomposition of the omitted variable bias due to the omission of the worker and firm, and job-match fixed effects, and, finally, Section 5 concludes the paper.

2 The Methodology

To better understand wage persistence we consider the dynamic wage equation,

$$w_{it} = \rho w_{i,t-1} + \alpha_i + \theta_{F(i,t)} + \phi_{iF(i,t)} + \gamma_t + \mathbf{X}_{it}\boldsymbol{\beta} + u_{it} \tag{1}$$

where w_{it} and $w_{i,t-1}$ are the wages of individual i in years t and $t-1$, respectively, $i = 1, ..., n$, $t = 1, ..., T$, $|\rho| < 1$ is the autoregressive parameter that characterizes wage persistence; α_i and $\theta_{F(i,t)}$ correspond to observed and unobserved worker and firm fixed heterogeneity, respectively; $\phi_{iF(i,t)}$ is a match quality fixed component that captures the wage earned by individual i while working in firm F relative to $\alpha_i + \theta_{F(i,t)}$; γ_t are time fixed effects, $\mathbf{X}_{it}$ is a vector of time-varying control variables, which includes tenure, the squares of age and tenure, schooling years, and firm size, and u_{it} is a zero mean idiosyncratic error term.

To measure the contribution of each fixed effect to wage persistence we use the decomposition proposed by Gelbach (2016). This decomposition is a computationally simple and econometrically meaningful procedure that takes advantage, in a surprisingly ingenious way, of the conventional OLS omitted variable bias formula. If the base specification is a parsimonious useful benchmark, which in our case corresponds to a conditional gross measure of wage persistence, the decomposition is also economically meaningful, providing an unambiguous measure of the contribution of each omitted variable to the change in the original coefficients of wage persistence.

It is important to highlight that for Gelbach's decomposition exercise, the full model needs to be well specified, i.e., the parameters of the full model have to be consistently estimated. Specifically, considering (1) as the full model, we assume that $E(u_{it}, u_{js}) = 0$, $i \neq j$ or $t \neq s$; $E(\alpha_i, u_{jt}) = 0$, $\forall i, j, t$; $E(\theta_{F(i,t)}, u_{js}) = 0$, $\forall i, j, s, t$, $E(\phi_{iF(i,t)}, u_{js}) = 0$, $\forall i, j, s, t$, $E(\gamma_t, u_{js}) = 0$, $\forall t, j, s$ and that $\mathbf{X}_{it}$ is strictly exogenous.

2.1 Wage equation with no job-match quality effect

Consider first the case in which the job-match quality fixed component, $\phi_{iF(i,t)}$, is not relevant, i.e., $\phi_{iF(i,t)} = 0$. In this situation the full model in (1) reduces to a dynamic wage equation with worker and firm fixed effects, such as,

$$w_{it} = \rho w_{i,t-1} + \alpha_i + \theta_{F(i,t)} + \gamma_t + \mathbf{X}_{it}\boldsymbol{\beta} + u_{it}. \tag{2}$$

As previously indicated, to determine the independent contribution of each fixed effect to the persistence of wages in model (2), we start by considering a simpler version of the full model, which we define as the base model, in which the worker and firm fixed effects are omitted, i.e.,

$$w_{it} = \rho w_{it-1} + \gamma_t + \mathbf{X}_{it}\boldsymbol{\beta} + u^0_{it}. \tag{3}$$

For convenience of presentation we write (3) in matrix notation,

$$\mathbf{W} = \rho \mathbf{W_{-1}} + \mathbf{Q}\boldsymbol{\vartheta} + \mathbf{U_0} \tag{4}$$

where $\mathbf{W}$ and $\mathbf{W}_{-1}$ are $n(T-1) \times 1$ vectors of wages and one-period lagged wages, respectively, $\mathbf{Q} := (\mathbf{X}, \boldsymbol{\mathcal{T}})$, $\mathbf{X}$ is the $n(T-1) \times k$ matrix of control variables, and $\boldsymbol{\mathcal{T}}$ is the $n(T-1) \times (T-1)$ matrix of time dummies.

To estimate ρ in (4), we use the Frisch-Waugh-Lovell theorem and express the least-squares estimate of ρ as the result of running a regression of $\mathbf{W}$ on $\mathbf{W}_{-1}$, after partialling out the effect of $\mathbf{Q}$, i.e.,

$$\hat{\rho}_0 = (\mathbf{W}'_{-1}\mathbf{P_Q}\mathbf{W}_{-1})^{-1}(\mathbf{W}'_{-1}\mathbf{P_Q}\mathbf{W}) \tag{5}$$

where $\mathbf{P_Q} := \mathbf{I} - \mathbf{Q}(\mathbf{Q}'\mathbf{Q})^{-1}\mathbf{Q}'$ projects onto the left null space of $\mathbf{Q}$, and is used to purge the effects of $\mathbf{Q}$ from $\mathbf{W}$ and $\mathbf{W}_{-1}$. The estimator in (5) can be written more compactly as,

$$\hat{\rho}_0 = \mathbf{A_Q}\mathbf{W} \tag{6}$$

where

$$\mathbf{A_Q} := (\mathbf{W}'_{-1}\mathbf{P_Q}\mathbf{W}_{-1})^{-1}\mathbf{W}'_{-1}\mathbf{P_Q}. \tag{7}$$

The representation in (6) will be instrumental for the analysis that follows, since pre-multiplying any variable by $\mathbf{A_Q}$ provides the corresponding coefficient estimate after controlling for the variables in $\mathbf{Q}$.

To identify the impact of worker and firm heterogeneity on wage persistence we also need to consider the full model in (2), which in matrix formulation is,

$$\mathbf{W} = \rho \mathbf{W_{-1}} + \mathbf{E}\boldsymbol{\alpha} + \mathbf{F}\boldsymbol{\theta} + \mathbf{Q}\boldsymbol{\vartheta} + \mathbf{U_1} \tag{8}$$

where $\mathbf{E}$ and $\mathbf{F}$ are the matrices of worker and firm dummies, and $\boldsymbol{\alpha}$ and $\boldsymbol{\theta}$ the corresponding unknown vectors of worker and firm fixed effects parameters. Given the high-dimensional fixed effects in (8), this linear regression model can be estimated using, for instance, the iterative procedure of Guimarães and Portugal (2010).[2]

Following Gelbach (2016) we resort to the OLS omitted variable bias expression, which essentially corresponds to the difference between the estimates of ρ computed from (8), which we define as $\hat{\rho}^{\mathrm{wf}}$, and $\hat{\rho}_0$ computed from (4), where the latter is biased due to the omission of the worker and firm fixed effects ($\mathbf{E}\boldsymbol{\alpha}$ and $\mathbf{F}\boldsymbol{\theta}$, respectively). Thus, to determine the contribution of these fixed effects to wage persistence, we estimate (8), $\mathbf{W} = \hat{\rho}^{\mathrm{wf}}\mathbf{W}_{-1} + \mathbf{E}\hat{\boldsymbol{\alpha}} + \mathbf{F}\hat{\boldsymbol{\theta}} + \mathbf{Q}\hat{\boldsymbol{\vartheta}} + \hat{\mathbf{U}}_1$, and multiply both sides by $\mathbf{A_Q}$, defined in (7). Since $\mathbf{A_Q}\mathbf{Q}\hat{\boldsymbol{\vartheta}} = 0$ and $\mathbf{A_Q}\hat{\mathbf{U}}_1 = 0$, it follows that the omitted variable bias is,

$$\hat{\rho}_0 - \hat{\rho}^{\mathrm{wf}} = \hat{\tau}_\alpha + \hat{\tau}_\theta \tag{9}$$

where $\hat{\tau}_\alpha = \mathbf{A_Q}\mathbf{E}\hat{\boldsymbol{\alpha}}$ and $\hat{\tau}_\theta = \mathbf{A_Q}\mathbf{F}\hat{\boldsymbol{\theta}}$ are the resulting biases of the omission in (4) of the worker and firm fixed effects, respectively. The difference between the estimates of ρ in (9) equals the sample analog of the omitted variable bias formula and Gelbach's algorithm allows us to decompose this difference into the separate effects deriving from each excluded fixed effect. In practice, all we need are the estimates of $\hat{\tau}_\alpha$ and $\hat{\tau}_\theta$, which are straightforwardly obtained from regressions of the estimated worker and firm fixed effects ($\mathbf{E}\hat{\boldsymbol{\alpha}}$ and $\mathbf{F}\hat{\boldsymbol{\theta}}$, respectively), on all the covariates included in the base model (4).

2.2 Wage equation with job-match quality effect

For estimation of (1) with $\phi_{iF(i,t)} \neq 0$, a feasible procedure that allows us to estimate the combination of the three sets of effects (worker, firm, and job-match quality fixed effects) is to replace them by a single set for each worker-firm pair, i.e., the job-match fixed effect, $\psi_{iF(i,t)}$, *viz.*,

$$w_{it} = \rho w_{i,t-1} + \psi_{iF(i,t)} + \gamma_t + \mathbf{X}_{it}\boldsymbol{\beta} + u_{it}. \tag{10}$$

Comparing (2) and (10) we observe that, on the one hand, the latter includes many more fixed effects (compare, for instance, the number of worker and firm fixed effects with that of the job-match fixed effects in the summary statistics of Table A.1 for the balanced and unbalanced panels considered in our empirical analysis). On the other hand, if job-match quality effects exist and are not accounted for, an omitted variable bias will emerge in (2), potentially making it unfeasible (this will be illustrated in detail in Section 3).

Thus, given our interest in understanding the impact of job-match heterogeneity on

[2]In Appendix B we briefly describe the iterative procedure of Guimarães and Portugal (2010) that is used in the estimation of a wage equation that incorporates two high-dimensional fixed effects.

wage persistence, we write (10) as,

$$\mathbf{W} = \rho \mathbf{W_{-1}} + \mathbf{M}\boldsymbol{\psi} + \mathbf{Q}\boldsymbol{\vartheta} + \mathbf{U_1} \tag{11}$$

where $\mathbf{M}$ corresponds to the matrix of job-match dummies and $\boldsymbol{\psi}$ is the corresponding unknown vector of parameters.

To determine the contribution of the job-match fixed effects to wage persistence we multiply both sides of $\mathbf{W} = \hat{\rho}^{\text{match}}\mathbf{W}_{-1} + \mathbf{M}\hat{\boldsymbol{\psi}} + \mathbf{Q}\hat{\boldsymbol{\vartheta}} + \hat{\mathbf{U}}_1$ by $\mathbf{A}_Q$, defined in (7). Noting that $\mathbf{A_Q}\mathbf{Q}\hat{\boldsymbol{\vartheta}} = 0$ and $\mathbf{A_Q}\hat{\mathbf{U}}_1 = 0$, the omitted variable bias resulting from the omission of the job-match fixed effects in (4) is,

$$\hat{\rho}_0 - \hat{\rho}^{\text{match}} = \mathbf{A_Q}\mathbf{M}\hat{\boldsymbol{\psi}} =: \hat{\tau}_{\psi}. \tag{12}$$

To separately identify the worker, firm, and job-match quality components, a workable assumption in this framework is to consider that the job-match quality effect is orthogonal to the worker and firm fixed effects (Woodcock, 2015 and Raposo et al., 2021). Hence, considering conditional orthogonality of these fixed effects essentially corresponds to specifying the regression model,

$$\mathbf{M}\hat{\boldsymbol{\psi}} = \tau_{\phi}\mathbf{W}_{-1} + \mathbf{E}\boldsymbol{\alpha}_m + \mathbf{F}\boldsymbol{\theta}_m + \mathbf{Q}\boldsymbol{\vartheta}_m + \boldsymbol{\varepsilon} \tag{13}$$

where $\mathbf{M}\hat{\boldsymbol{\psi}}$ is obtained from (11), $\mathbf{W}_{-1}$ and $\mathbf{Q}$ are as defined in (4), $\mathbf{E}$ and $\mathbf{F}$ are the matrices collecting the worker and firm dummies, respectively, $\boldsymbol{\alpha}_m$ and $\boldsymbol{\theta}_m$ are the corresponding vectors of unknown worker and firm parameters, and $\boldsymbol{\varepsilon}$ is an error term. τ_{ϕ} is the contribution of the job-match quality fixed effects to wage persistence. In line with the previous section, regression (13) will be estimated using the iterative procedure of Guimarães and Portugal (2010).

The decomposition of the job-match fixed effect, $\mathbf{M}\hat{\boldsymbol{\psi}}$, into three components, the worker (τ_{α}), firm (τ_{θ}), and job-match quality (τ_{ϕ}), allows us to identify their contribution to wage persistence. Hence, multiplying, as before, both sides of the estimated model in (13) by $\mathbf{A_Q}$, and noting that $\mathbf{A_Q}\mathbf{W}_{-1} = 1$, $\mathbf{A_Q}\mathbf{Q}\hat{\boldsymbol{\vartheta}}_{\boldsymbol{m}} = \mathbf{0}$ and $\mathbf{A_Q}\hat{\boldsymbol{\varepsilon}} = 0$, it follows that,

$$\hat{\tau}_{\psi} = \hat{\tau}_{\phi} + \hat{\tau}_{\alpha} + \hat{\tau}_{\theta}, \tag{14}$$

where $\hat{\tau}_{\alpha} = \mathbf{A_Q}\mathbf{E}\hat{\boldsymbol{\alpha}}_{\boldsymbol{m}}$ and $\hat{\tau}_{\theta} = \mathbf{A_Q}\mathbf{F}\hat{\boldsymbol{\theta}}_{\boldsymbol{m}}$.

Thus, (14) shows that in practice $\hat{\tau}_{\hat{\alpha}}$ and $\hat{\tau}_{\hat{\theta}}$ can be computed from regressions of the worker fixed effects estimates, $\mathbf{E}\hat{\boldsymbol{\alpha}}_m$, and the firm fixed effects estimates, $\mathbf{F}\hat{\boldsymbol{\theta}}_m$, respectively, on $\mathbf{W}_{-1}$ and $\mathbf{Q}$.

3 Monte Carlo Analysis

As indicated above, both specifications considered in (2) and (10) make use of a large number of fixed effects resulting in a significant incidental parameter problem. From the literature on panel data, we know that a critical situation arises when the dimension of one fixed effect (say n) increases without bound while T remains fixed. In this case, the number of individual parameters increases as n increases, raising the incidental parameter problem originally discussed by Neyman and Scott (1948) and recently reviewed by Lancaster (2000). In general, the estimators of the regression coefficients (the slopes) will be plagued by the incidental parameter problem and, for T fixed, will be inconsistent. In particular, when T is small, each individual fixed effect is very noisy, and this noise generally contaminates the estimates of ρ and the other parameters.[3]

In this Section, we perform an in-depth Monte Carlo analysis to illustrate the impact of this problem on the least-squares estimates of a panel autoregressive parameter ρ and to evaluate the possible suitability of three different bias correction approaches, that originally have been proposed for a conventional fixed effects model context. The main objective is to better understand the magnitude of the estimation bias (as a consequence of omitted variables and/or the incidental parameter problem) when different sets of fixed effects are considered in the estimation of the autoregressive slope parameter and to evaluate how well the different bias correction approaches perform.

3.1 The Data Generation Process

Our data generation process (DGP) is in line with (1), i.e.,

$$w_{it} = \rho w_{i,t-1} + \alpha_i + \theta_{F(i,t)} + \phi_{iF(i,t)} + u_{it} \tag{15}$$

where the error term $u_{it} \sim n.i.i.d.(0,1)$, $i = 1, ..., n$, $t = 1, ..., T$, $n = (1600, 3200)$, $T = (10, 20, 30)$ and $(\alpha_i, \theta_{F(i,t)}, \phi_{iF(i,t)})' \sim N(\mathbf{0}, \mathbf{\Omega})$, with $\mathbf{0}$ a 3×1 vector of zeros and $\mathbf{\Omega} := diag\{\sigma^2_\alpha, \sigma^2_\theta, \sigma^2_\phi\} = diag\{0.25, 0.25, 0.25\}$.[4]

For simplicity of presentation, but without loss of generality, we do not consider control variables in our DGP. For the number of firms we consider $J = (200, 400)$, which are indexed by $j = 1, ..., J$, with a random number of employees, N_j, drawn from a log-

[3]Similarly, if n is small each time fixed effect is very noisy.

[4]Note that, weak dependence, *i.e.* higher order AR dynamics or MA dependence (assuming that the latter is invertible), could be considered as long as this additional dynamics is accommodated in the model. In other words, as long as the residuals of the full model display close to white noise behavior. We conjecture that the use of the split-panel Jackknife and the Bootstrap should still be feasible with higher order models. For instance, in the context of higher order AR dynamics, keeping in mind that our focus is on the persistence of wages, one possibility would be to consider an autoregressive approximation as, $w_{it} = \mu_i^* + \rho w_{i,t-1} + \sum_{k=1}^{p} \alpha_k \Delta w_{i,t-k} + e_{it}$ where ρ corresponds to the measure of persistence used in our paper and p is a sufficiently large lag order which ensures that e_{it} displays close to white noise behavior.

normal distribution with mean $\mu_N = 8$. Each worker is given a realization of α_i and each firm is given a realization of θ_j. The first draws of α_i and θ_j, when $t = 1$, ensure that workers and firms with certain characteristics are matched.

We also allow for worker mobility between firms. For each worker we draw a potential new firm j from the list of currently existing firms. This new firm has its own set of characteristics θ_j. To ensure that a new match is drawn with a probability that is proportional to the firm size, the list of new firms is weighted by the size of the firm, and the movement from firm j to j' is determined by a random draw from a uniform distribution. We set the probability, p, of workers moving between firms to 10% and 25%. Changing p allows us to alter the number of workers that move each period. This will also change the number of job-match fixed effects that need to be used. The matching of workers and firms occurs once per period t. The number of periods, T, is varied to mimic real data. Typically, T is small because linked data are recorded annually and have become available only recently (for instance, in our empirical analysis below $T = 17$).

Once the identity of each firm has been established for every worker in all T periods, the dependent variable w_{it} is generated according to (15). The generated panel is balanced such that each worker is observed over T consecutive periods. However, it is not necessarily balanced in terms of firms, because small firms that experience worker exits may disappear.

In what follows, using these artificial panel data sets we investigate the bias of the autoregressive parameter estimates obtained from two dynamic panel data regression models, one that includes worker and firm fixed effects as in (2) and another that includes job-match fixed effects as in (10) (for clarity we will refer to the resulting autoregressive estimates obtained from the former model as $\hat{\rho}^{\mathrm{wf}}$ and from the latter as $\hat{\rho}^{\mathrm{match}}$). All results presented are based on 1000 Monte Carlo replications.

3.2 The Bias Correction Approaches

Bias-correction has been an important topic of research in the panel data models literature and has motivated the development of many bias reduction methods for (dynamic) panel data models with fixed effects, see e.g. Kiviet (1995), Hahn and Kuersteiner (2002), Hahn and Newey (2004), Bun and Carree (2005), Phillips and Sul (2007), Everaert and Pozzi (2007), Gourieroux et al. (2010) and Fernández-Val and Weidner (2016), among others.

For the purpose of our analysis we will focus on the analytical bias-correction approach proposed by Hahn and Kuersteiner (2002) (hereinafter HK), the split-panel Jackknife approach of Dhaene and Jochmans (2015), and a residual wild bootstrap approach as in Gonçalves and Kaffo (2015). The latter two approaches are interesting in the sense that no specific bias expression is needed to perform the correction, which in the present context of high-dimensional fixed effects, can prove advantageous. Note that all these procedures were originally designed for a conventional fixed effects context and it will therefore be useful to evaluate their performance in this high-dimensional fixed effects context. Typically, these bias correction approaches are designed to remove the first-order term of a large-T approximation of the bias of ρ. Below we provide a brief description of the approaches used in this work.

Hahn and Kuersteiner (2002) develop analytical expressions of the first-order bias of ρ that are very appealing from an empirical point of view due to their simplicity of application. This expression, evaluated at a maximum likelihood estimate of ρ with fixed effects, is subtracted from ρ to give a first-order bias-corrected estimate,[5] i.e.,

$$\hat{\rho}^{\mathrm{k}}_{HK} = \hat{\rho}^{\mathrm{k}} + \frac{1}{T}(1 + \hat{\rho}^{\mathrm{k}}), \text{ where k} = \text{wf or match.} \tag{16}$$

From an empirical point of view other bias correction approaches exist that do not require explicit bias formulas, such as the Jackknife approach, for example. In the context of the incidental parameter problem, Dhaene and Jochmans (2015) propose the split-panel Jackknife (see also Chudik et al., 2018).

To briefly describe the implementation of the split-panel Jackknife consider a balanced panel dataset of observations w_{it}, with $i = 1, \ldots, n$, $t = 1, \ldots, T$, and T even (see Dhaene and Jochmans, 2015, p. 999, for details for T odd). Hence, let $\hat{\rho}^{\mathrm{k}}$ denote the least-squares estimate of ρ computed from the full panel with either worker and firm fixed effects (k = wf) or with job-match fixed effects (k = match) and let ρ^{k}_1 and ρ^{k}_2 correspond to the estimates computed from the first half-panel, where $t = 1, \ldots, T/2$, and the second half-panel, where $t = T/2 + 1, \ldots, T$, respectively. The split-panel Jackknife estimator is,

$$\hat{\rho}^{\mathrm{k}}_{jk_{1/2}} = 2\hat{\rho}^{\mathrm{k}} - 1/2(\hat{\rho}^{\mathrm{k}}_1 + \hat{\rho}^{\mathrm{k}}_2). \tag{17}$$

For the unbalanced panel case, following Chudik et al. (2018), a simple way to implement the split-panel Jackknife bias correction is to assume that the T_i observations for individual i are even, and divide the unbalanced sample into two unbalanced sub-samples; the first sub-sample consisting of the first $T_i/2$ and the second sub-sample of the last $T_i/2$ observations (see Chudik et al., 2016 for further details).

Finally, the third approach that we use is a recursive-design residual-based wild bootstrap approach following Gonçalves and Kaffo (2015, Section 3.1). This approach resamples the residuals and recursively generates bootstrap observations for the dependent variable using the estimated autoregressive panel data model (see also Everaert and Pozzi, 2007). Specifically, the bootstrap bias correction is performed according to the algorithm described below.

Algorithm (Recursive-design residual-based wild bootstrap)

[5] Fernández-Val and Weidner (2018) provide the first-order bias formula for the case in which both individual effects and time fixed effects are present.

Step 1: *Estimate the first-order autoregressive panel data model with fixed effects (with worker and firm or job-match fixed effects) to obtain the estimated residuals* $\hat{u}_{it} = w_{it} - \hat{\lambda}^{\mathrm{k}}_{iF(i,t)} - \hat{\rho}^{\mathrm{k}} w_{i,t-1}$, $\mathrm{k} = \mathrm{wf}$ *or* match, *where* $\hat{\lambda}^{\mathrm{wf}}_{iF(i,t)} = \hat{\alpha}_i + \hat{\theta}_{F(i,t)}$ *and* $\hat{\lambda}^{\mathrm{match}}_{iF(i,t)} = \hat{\psi}_{iF(i,t)}$.

Step 2: *Generate bootstrap innovations* $u^*_{it} = \hat{u}_{it}\eta_{it}$, *where* $\eta_{it} \sim i.i.d.(0,1)$ *over* (i,t) *with* $\mathrm{E}(\eta^4_{it}) < \infty$, *which is independent of the sample data.*

Step 3: *Recursively generate a panel of pseudo observations* $\{w^*_{it}, i = 1, ..., n; t = 1, ..., T\}$ *from the panel AR(1) model,*

$$w^*_{it} = \hat{\lambda}^{\mathrm{k}}_{iF(i,t)} + \hat{\rho}^{\mathrm{k}} w^*_{i,t-1} + u^*_{it}, \quad i = 1, ..., n; t = 1, ..., T, \tag{18}$$

where $\mathrm{k} = \mathrm{wf}$ *or* match, $\hat{\lambda}^{\mathrm{k}}_{iF(i,t)}$ *are the fixed effects estimates and* $\hat{\rho}^{\mathrm{k}}$ *is the autoregressive least squares estimate computed from the original sample data in step 1. The initial condition is,* $w^*_{i0} = \hat{\lambda}^{\mathrm{k}}_{iF(i,1)}/(1-\hat{\rho}^{\mathrm{k}})$, $i = 1, ..., n$.

Step 4: *Using the bootstrap sample data,* $\left(w^*_t, w^*_{t-1},\right)'$, *in place of the original sample data,* $(w_t, w_{t-1})'$, *compute the bootstrap estimate of* ρ, $\hat{\rho}^{*,\mathrm{k}}_{b,rwb}$, $\mathrm{k} = \mathrm{wf}$ *or* match.

Step 5: *Repeat steps 2 to 4 B times and compute,*

$$\tilde{\rho}^{\mathrm{k}}_{rwb} = \frac{1}{B}\sum_{b=1}^{B} \hat{\rho}^{*,\mathrm{k}}_{b,rwb}.$$

B *is the number of bootstrap replications used. In the simulations and empirical analysis below we set* $B = 399$.

Step 6: *Taking the result of Step 5 we obtain the bias corrected estimate as,*

$$\hat{\rho}^{\mathrm{k}}_{rwb} = \hat{\rho}^{\mathrm{k}} + (\hat{\rho}^{\mathrm{k}} - \tilde{\rho}^{\mathrm{k}}_{rwb}). \tag{19}$$

As indicated by Gonçalves and Kaffo (2015) the residual wild bootstrap approach assumes cross-sectional independence as is common in the panel literature. However, time series dependence in the error term is allowed by assuming that u_{it} satisfies a martingale difference sequence assumption for each individual. Although this assumption rules out serial correlation, it is compatible with time series and cross sectional heteroskedasticity in u_{it}.

For interesting and detailed overviews of bias correction methods for dynamic panels see, inter alia, Dhaene and Jochmans (2015), Fernández-Val and Weidner (2018) and Arellano et al. (2017).

3.3 Simulation Results

Tables 1 and 2 report the Monte Carlo results obtained for $n = 1600$ and $T = \{10, 20, 30\}$.[6] The artificial data generated allow for 10% and 25% worker mobility between firms.[7]

INSERT TABLES 1 and 2 ABOUT HERE

The results in Table 1 are based on data generated from (15) without a job-match quality effect, i.e., with $\phi_{iF(i,t)} = 0$. For the analysis of Table 1 both models (with worker and firm fixed effects, and with job-match fixed effects) can be considered as suitable approaches since $\phi_{iF(i,t)} = 0$. The first observation we can make from the uncorrected least-squares results in this table (see columns labeled wf and match) is that the bias reported is as we expect, i.e., the results are negative and decrease in absolute terms as T increases. Given that in our DGPs, $\rho \geq 0$, it is expected that the incidental parameter problem gives rise to an under estimation of the autoregressive parameter, and consequently that $E(\hat{\rho}^{\mathrm{k}} - \rho) < 0$, for k = wf or k = match. Specifically, we observe that the least-squares bias in the worker and firm fixed effects model falls from between $[-0.199, -0.132]$ for $T = 10$, to $[-0.063, -0.042]$ for $T = 30$, when 10% worker mobility is considered and that these magnitudes of the bias are essentially the same when 25% worker mobility is considered. When the model with job-match fixed effects is considered, the least-squares bias falls from between $[-0.253, -0.176]$ for $T = 10$ to $[-0.154, -0.108]$ for $T = 30$ when 10% worker mobility is considered, and from between $[-0.337, -0.244]$ for $T = 10$ to $[-0.249, -0.169]$ for $T = 30$ in the case of 25% worker mobility. The aggravation of the bias observed when worker mobility is increased to 25% is a consequence of the resulting increase in the number of job-match fixed effects (note that the number of worker and firm fixed effects remains constant). The incidental parameter problem is clearly illustrated in this table. Note, for instance, the increase in the absolute value of the least-squares bias from a worker and firm fixed effects model to that of job-match fixed effects models.

When the bias correction approaches are applied we observe that for the worker and firm fixed effects model, the split-panel jackknife ($jk_{1/2}$) provides the best bias correction. Specifically, we observe that for 10% worker mobility and $T = 10$ the bias of the split-panel Jackknife estimate is smallest for $\rho \leq 0.7$ (its bias is between $[-0.002, 0.013]$), and for $\rho = 0.9$ it is HK that preforms best (0.031). For $T = 30$ the split-panel Jackknife is still the best performing bias correction approach when $\rho \leq 0.7$ ($[-0.001, 0.006]$), and for $\rho = 0.9$ the best performing approach is now rwb (-0.010). For 25% worker mobility results are essentially the same.

From the bias correction results in the job-match fixed effects model we observe that for $T = 10$ and 10% worker mobility, the best performing approaches are rwb for $\rho \leq 0.5$

[6] Results for $n = 3200$ are qualitatively similar to those discussed in this section; see the Supplementary On-line Appendix for details.

[7] Note that in the empirical sample the fraction of movers is around 26%.

with bias between $[-0.077, -0.034]$, the split-panel Jackknife ($jk_{1/2}$) for $\rho = 0.7$ (-0.071), and HK for $\rho = 0.9$ (-0.012). For $T = 30$, rwb displays the best performance for all ρ ($[-0.031, -0.013]$). Considering 25% worker mobility and $T = 10$, rwb is best for $\rho \leq 0.7$ ($[-0.150, -0.076]$) and the split-panel Jackknife ($jk_{1/2}$) for $\rho = 0.9$ (-0.075). Note that as indicated above, the number of incidental parameters increases in this case which clearly aggravates the bias. For $T = 30$, the rwb is the best performing approach for all ρ with bias falling between $[-0.074, -0.039]$. Although the bias correction approaches used are not as effective in bias reduction as in the worker and firm fixed effects case, they still provide considerable reductions of the least-squares bias. In contrast to the conventional Nickel bias, which converges relatively quickly to zero as T increases (see, for instance, the results obtained from the model with worker and firm fixed effects - column wf), the bias resulting from the job-match fixed effects is more resilient the more worker mobility exists, as this will give rise to increased noise in the estimates of the fixed effects as a consequence of the growing number of smaller periods over which to estimate the job-match fixed effects.

Table 2 presents the Monte Carlo results obtained based on data generated from (15), but now allowing for job-match quality effects, i.e., $\phi_{iF(i,t)} \neq 0$. The incidental parameter problem is again quite visible in the column match of Table 2, corresponding to the case in which job-match fixed effects are used, and also from the column wf for the case of worker and firm fixed effects, but in the latter case some caution is required. Note that for $T > 10$ the results obtained from the latter model suggest a positive bias. This behavior is associated to the fact that worker and firm fixed effects will not capture the job-match quality effects present in the data, and therefore, the results provided do not only reflect the impact of the incidental parameter problem but also the resulting omitted variable bias. The latter seems to be positive and clearly emerges as T increases, since the incidental parameters problem is expected to diminish. The bias resulting from the omission of the job-match effects also seems to have a detrimental impact on the bias correction approaches used, as in the case of the parameter estimates computed from the model with worker and firm fixed effects the bias correction approaches seem to reduce the bias only when $T = 10$.

When the job-match fixed effects model is considered we observe that when 10% worker mobility is allowed the least-squares bias of the autoregressive parameter estimate for $T = 10$ is between $[-0.220, -0.143]$ (see column labeled match), which is quite large, and is aggravated as worker mobility is increased to 25% ($[-0.277, -0.195]$). Although the bias decreases in absolute value as the sample size increases, it is still sizable for $T = 30$ ($[-0.124, -0.073]$ and $[-0.182, -0.113]$, for the cases of 10% and 25% worker mobility, respectively).

The bias correction results for the match fixed effects model show that all three bias correction approaches (rwb, the split-panel Jackknife, and HK) provide significant bias reductions. The rwb is the approach that in general reveals the overall best performance in this case. Specifically, in the case of 10% worker mobility and $T = 10$, rwb achieves the best performance when $\rho \leq 0.5$ ($[-0.065, -0.028]$) and $jk_{1/2}$ when $\rho > 0.5$ ($[-0.045, -0.031]$). For $T = 30$, rwb achieves the best performance for all values of ρ considered ($[-0.006, -0.001]$). When 25% worker mobility is considered rwb performs best as $\rho \leq 0.7$ ($[-0.100, -0.055]$) and HK for $\rho = 0.9$, (-0.025). For $T = 30$ rwb is overall the best performing approach ($[-0.015, -0.040]$).

The results in Tables 1 and 2 emphasize the importance of the incidental parameter problem as the autoregressive parameter estimates are in general (sometimes substantially) smaller than the true ρ. Moreover, we also highlight that bias correction is clearly an important aspect to be considered in this framework and that the bias correction methods analyzed prove useful in this context. However, further work on bias reduction approaches suitable for short-time panels with high-dimensional fixed effects, such as bootstrap methods, is still needed.

4 Empirical Analysis

4.1 Data description

Our data come from a unique and rich longitudinal matched employer-employee dataset - *Quadros de Pessoal* (QP). QP is a mandatory annual employment survey collected by the Portuguese Ministry of Labor, Solidarity, and Social Security, which each firm with at least a single wage earner in the private sector is legally obliged to complete. QP has existed since 1985 and extends until 2018, which is the most recent available period.[8] Our study covers the 2002-2018 period. A shorter sample of the QP dataset (2002-2009) was recently used by Card et al. (2016).

QP contains information on the firm (location, industry, employment, sales, ownership, and legal setting, among other features), and on each of its workers (gender, age, education, skill, occupational category, tenure, wages, hours worked, and more). The information on earnings is very complete. It includes the monthly base wage (gross pay for normal hours of work), regular and non-regular benefits, and overtime pay, as well as the mechanism of wage bargaining. Information on normal and overtime hours of work is also available. From 1994 and thereafter data reported in QP refer to the month of October of each year.

Firms and workers entering the QP dataset are assigned a unique identification number that makes it possible to track firms and workers over time. Also, the worker files include the number of the firm to which each worker is affiliated in a given year, making it possible to match firms and their workers, and to identify each worker-firm pair. Currently, the

[8] Worker level data are not available for the years of 1990 and 2001.

dataset comprises information on around 300,000 firms and 3 million employees.

The possibility to match workers with their employers, the longitudinal nature of the data, and the long time span covered, makes QP an appropriate source to empirically evaluate wage persistence effects. Moreover, employer-reported wage information is known to be subject to less measurement error than worker-reported data.

Our data set includes the population of full-time male wage earners in the private nonfarm sector who worked at least 120 hours in the reference month, aged between 18 and 64, with a maximum of 50 years of tenure, and who earned at least 80 percent of the minimum wage.[9] We restrict our sample to the largest connected set, which represents more than 96% of the data.[10] The data include 655,120 (years $\times$ individuals) observations for a balanced panel of prime-age male workers, which corresponds to 40,945 workers, 10,035 firms, and 69,276 job matches. This sample includes continuously employed workers over the 2002-2018 period who may or may not have experienced a firm change. For comparison purposes, we also consider a balanced sample of 383,456 (years $\times$ individuals) observations of prime-age female workers, which corresponds to 23,966 workers, 4,680 firms, and 38,795 job matches.

Finally, in order to estimate the dynamic wage model in a more realistic setting, two alternative panels are considered: an unbalanced sample of male and an unbalanced sample of female workers. To be included in these samples workers must have been registered in the QP files for at least two years in the 2002-2018 period. In this case, the minimum number of spells per worker in QP data ranges from 2 to 17 years. Note that, workers who experienced an unemployment episode or who were employed in Public Administration or in self-employment are not covered by QP.[11]

The unbalanced panel of male workers includes 12,802,613 observations (2 million individuals, 250,000 firms, and 3.1 million job matches), while the unbalanced panel of female workers includes 9,800,784 observations (1.6 million individuals, 225,000 firms, and 2.4 million job matches).

Table A.1 reports the summary statistics for these four alternative panels. Real hourly wages are defined as the ratio between total regular (base wage and regular benefits) and irregular payroll (irregular benefits and overtime payments) in the reference month and the total number of normal and extra hours worked in the reference month (deflated using the Consumer Price Index: base-year 1986). The summary statistics indicate that female workers earn, on average, less than male workers are on average more educated, and employed in larger firms than their male counterparts. Regarding age and tenure, the balanced panels show that female workers are on average older (42.2 years old against

[9]Observations with missing data in the variables of interest were dropped. Multiple job-holder workers in a given year were also removed.

[10]A connected set is defined when at least one element of a worker-firm pair links the rest of the group (Abowd et al., 2002). This is done to warrant that the fixed effects are identified.

[11]Temporary exits from the data set may also occur if the survey form was not received by the Ministry of Labor before the date when the recording operations were closed.

40.3 years old for male workers) and have longer tenure (14.6 years of tenure against 13.1 years for male workers). In contrast, the unbalanced panels show that female workers are slightly younger (39.4 years old against 40.3 years old for male workers) and have lower tenure (9.3 years against 9.7 years for male workers).

4.2 Analysis of the Regression Results

In this section we seek to disentangle two sources of wage persistence: the persistence generated by the presence of unobserved heterogeneity and the persistence of earnings histories (Arellano et al., 2017). In the current exercise we explicitly incorporate worker, firm, and job-match quality heterogeneity in the wage model, enabling us to interpret the autoregressive coefficient as a measure of persistence of wage shocks over the worker's career history. Wage dependence may arise because firms base their wage offers to prospective workers on wages in their prior job (Altonji et al., 2013). In this set-up, the autoregressive coefficient may also reflect (or be interpreted as) the fraction of firms that counter-bid the offers of firms that engage in poaching their employees. Wage dependence is also engendered if better jobs improve the quality of the worker job search network. More broadly, job ladder models (Burdett and Mortensen, 1998; Huckfeldt, 2021) and risk insurance wage models (Guiso et al., 2005) generate wage persistence, by construction.

In Table 3 we present the regression results of the wage equation for both the base and the full model (with worker and firm fixed effects) specifications (discussed in Section 2), using a matched employer-employee balanced panel. In the absence of controls for heterogeneity, the estimates of the autoregressive parameter convey an indication of strong wage persistence for both male (0.89) and female (0.93) workers (see first column of Table 3). However, as hinted at earlier, the coefficient on lagged wages may capture the permanent effects of individual heterogeneity (who the worker is) and firm heterogeneity (where she works). The results provided in the second column of Table 3, in which the autoregressive coefficient estimate, $\hat{\rho}^{\mathrm{wf}}$, is reduced to 0.36 in the case of male and to 0.35 in the case of female workers, clearly show that permanent unobserved heterogeneity plays a key role in driving wage dynamics.

INSERT TABLE 3 ABOUT HERE

A key contribution of this study relies on the ability to unambiguously decompose the difference of the base and full models' estimates of ρ, in terms of the contribution of each component of unobserved heterogeneity. This is done taking advantage of the omitted variable bias formula (Gelbach, 2016), as discussed in Section 2. The (exact) decomposition exercise is also offered in the first line of Table 3. In the case of male workers it can be seen that the change in the estimates of ρ between the base and the full specification ($0.53 = 0.89 - 0.36$) can be attributed to the persistence of the individual characteristics (0.42) and to the persistence of firms' wage policies (0.11). The decomposition for female workers ($0.58 = 0.93 - 0.35$) places even more weight on the worker component (0.50), which translates into a smaller firm component (0.08). We tentatively conclude in favor of the dominance of worker heterogeneity over firm heterogeneity, a typical result in static wage models (Abowd et al., 1999; Torres et al., 2018). In summary, the notion that wages are persistent seems to arise primarily from perennial features of who the worker is and where she works and not as much from the permanence of economic shocks.

In the bottom panel of Table 3 we address the incidental parameter problem using the three alternative bias correction approaches of the least-squares estimate of ρ described in Section 3.2. First, we consider the results based on the split-panel Jackknife correction ($jk_{1/2}$). Then we give the analytical correction of Hahn and Kuersteiner (2002) (HK). The third correction alternative is the residual wild bootstrap (rwb). We provide bias corrected estimates for the base model's autoregressive parameter, ρ_0, because even in the absence of high-dimensional fixed effects finite sample bias is always present. We need the bias corrected estimate in order to be able to proceed with Gelbach's decomposition. Given our Monte Carlo simulations, we have the same preference for the split-panel Jackknife correction in the case of the worker and firm fixed effects model. The split-panel Jackknife correction attenuates visibly the reduction of the autoregressive parameter suggesting that the incidental parameter problem is driving the $\hat{\rho}^{\text{wf}}$ too low. The correction implies a revised ρ estimate of 0.55 for male and 0.54 for female workers, which means that around one third of the initial reduction of $\hat{\rho}$ is not justified. Furthermore, the split-panel Jackknife correction implies a substantial decrease in the worker component of wage persistence (toward 0.22 for male and 0.31 for female workers).

In Table 4 we account for worker-firm idiosyncratic match quality in the wage equation according to the definition of (10). By considering the job-match fixed effect (sometimes also called the job effect or job/period effect), $\hat{\rho}^{\text{match}}$ further falls to 0.33 in the case of male and 0.32 in the case of female workers, suggesting that match quality heterogeneity has a non-negligible impact on wage persistence. These results are broadly aligned with empirical studies that consider the inclusion of worker and job-match effects (Hospido, 2015). The match fixed effect subsumes, of course, the worker and firm fixed effects and a worker-firm idiosyncratic component that we called job-match quality. In general, without further hypotheses, it is not possible to disentangle the components of the job-match fixed effect. A convenient hypothesis is to assume that the match quality fixed effect is conditionally orthogonal to the worker and firm fixed effects (Raposo et al., 2021). Proceeding in this way, the contribution of the job-match fixed effect, $\hat{\rho}_0 - \hat{\rho}^{\text{match}}$, (0.56 for male and 0.61 for female workers) is decomposed into the corresponding contributions of the worker fixed effect (0.42 for male and 0.50 for female workers), the firm fixed effect (0.11 for males and 0.08 for females), and the job-match quality fixed effect (0.03 for both male and female workers).

INSERT TABLE 4 ABOUT HERE

In the case of the model with job-match fixed effects, our Monte Carlo simulations suggest that we should give preference to the residual wild bootstrap (rwb) bias correction. As before, the bias correction implies that part of the reduction in the estimate of ρ is undone and may not be justified, but the correction is smaller when compared to the one implied by the split-panel Jackknife. Furthermore, in this balanced panel, the rwb correction wipes out the contribution of the match quality.

Thus far we have been considering a balanced matched employer-employee panel. The requirements of a balanced panel make the sample unreasonably non-representative and severely biased toward workers with long and stable job careers. Moreover, applied researchers are typically confronted with unbalanced panels. In Tables 5 and 6 we no longer impose the restrictions of a balanced panel and consider a much larger and representative sample.

Table 5 reports the estimation results for the worker and firm fixed effects model, using the unbalanced matched employer-employee data panel. Not surprisingly, the ρ estimate is now smaller for both male and female workers in the uncorrected and bias corrected cases, but not by much. The main difference is the increased importance of the firm component of wage persistence, which more than doubled in the case of female workers (from 0.08 to 0.18 for the split-panel Jackknife bias correction) and increased by nearly 50% in the case of male workers (from 0.12 to 0.18). The bias correction implies, as before, that around one third of the change in the autoregressive coefficient estimates implied by the inclusion of worker and firm fixed effects is rooted in the incidental parameter problem.

INSERT TABLE 5 ABOUT HERE

Finally, in Table 6 we report the regression results and the corresponding decomposition exercise for the job-match specification, employing the unbalanced panel. Arguably, this is the specification that better accounts for unobserved heterogeneity. Interestingly, there is now a clear indication that job-match quality plays a relevant role in driving wage persistence. Taking the rwb as our preferred bias correction procedure, the job-match quality component equals a non-trivial 0.10 (0.11) for male (female) workers. Job-match quality heterogeneity also plays a key role in the empirical studies of Altonji et al. (2013), Raposo et al. (2021), and Woodcock (2020).

INSERT TABLE 6 ABOUT HERE

In summary, our empirical results highlight the importance of both the omitted variable bias and the incidental parameter bias. We show that neglecting the presence of worker, firm, and job-match quality heterogeneity severely biased the estimates of the autoregressive coefficient in an upward direction. Their inclusion, however, raises the incidental parameter problem. The split-panel Jackknife method implies the largest bias corrections - up to one third of the naive omitted variable bias correction - whereas the residual wild bootstrap generates the smallest. The analytical correction of Hahn and

Kuersteiner (2002) (HK) induces corrections contained between these two. The advantage of this procedure is its simplicity and the fact that it makes the trade-offs involved in the use of a large number of fixed effects transparent to the practitioner.

5 Conclusion

In this paper we provide a detailed analysis of wage persistence using high-dimensional fixed effects dynamic panel data regression models. We show through Monte Carlo simulations and a detailed empirical analysis that the estimation bias resulting from omitted variables and incidental parameters can be quite substantial.

Specifically, we draw five main findings from our results. First, we uncover what we believe is convincing evidence that time-series dependence of current wages on past wages is largely driven by the unobserved components of worker, firm, and job-match heterogeneity, contributing to the theoretical literature that seeks to model earnings dynamics over the life cycle. Neglecting heterogeneity severely biases upwards the autoregressive parameter estimate. Second, the decomposition of the omitted variable bias shows, in our favorite specification, that the most important source of bias is the persistence of worker characteristics (contributing to a reduction in the autoregressive coefficient of 0.35 for male and 0.43 for female workers) followed by heterogeneity of the wage policy of the firms (0.21 for male and 0.20 for female workers), and by job-match quality heterogeneity (0.10 for male and 0.11 for female workers). Third, we illustrate the importance, both through Monte Carlo simulations and in the empirical application, of the incidental parameter bias, which induced a downward bias in the autoregressive parameter estimate. Fourth, we also illustrate how the incidental parameter bias can be corrected using three alternative bias correction methods. We learned that up to one-third of the reductions of the autoregressive parameter estimates induced by the control of heterogeneity (high-dimensional fixed effects) may not be justified. Lastly, our simulation exercises showed that distinct bias corrections methods performed differently when we applied them to the model with worker and firm fixed effects or the model with job-match fixed effects, providing some guidance for future research on wage dynamics involving linked employer-employee data.

Acknowledgements

We are grateful to Editor Elie Tamer, Guest Editors David Card, Ian Schmutte, and Lars Vilhuber and three anonymous referees for their very useful comments and suggestions, which have contributed to improving the quality of the paper, as well as to Matei Demetrescu, Jonne Guyt and Geraldo Cerqueiro. This work was funded by Fundação para a Ciência e a Tecnologia through project number UID/Multi/00491/2019, PTDC/EGE-

ECO/28924/2017, UID/ECO/00124/2013, UID/ECO/00124/2019, UID/GES/00407/2013, UIDB/04105/2020 and Social Sciences DataLab (LISBOA-01-0145-FEDER-022209), POR Lisboa (LISBOA-01-0145-FEDER-007722, LISBOA-01-0145-FEDER-022209), and POR Norte (LISBOA-01-0145-FEDER-022209).

References

Abowd, J., Creecy, R., and Kramarz, F. (2002). Computing person and firm effects using linked longitudinal employer-employee data. Technical Papers 2002-06, Longitudinal Employer-Household Dynamics, Center for Economic Studies, U.S. Census Bureau.

Abowd, J., Kramarz, F., and Margolis, D. (1999). High wage workers and high wage firms. *Econometrica*, 67(2):251–334.

Abowd, J., Kramarz, F., and Roux, S. (2006). Wages, mobility and firm performance: advantages and insights from using matched worker-firm data. *Economic Journal*, 116:F245–F285.

Altonji, J., Smith Jr., A., and Vidangos, I. (2013). Modeling earnings dynamics. *Econometrica*, 81(4):1395–1454.

Alvarez, J. and Arellano, M. (2004). Robust likelihood estimation of dynamic panel data models. CEMFI Working Paper 0421.

Arellano, M., Blundell, R., and Bonhomme, S. (2017). Earnings and Consumption Dynamics: A Nonlinear Panel Data Framework. *Econometrica*, 85:693–734.

Bonhomme, S., Lamadon, T., and Manresa, E. (2019). A distributional framework for matched employer-employee data. *Econometrica*, 87(3):699–739.

Buchinsky, M., Fougère, D., Kramarz, F., and Tchernis, R. (2010). Interfirm mobility, wages and the returns to seniority and experience in the United States. *Review of Economic Studies*, 77:972–1001.

Bun, M. and Carree, M. (2005). Bias-corrected estimation in dynamic panel data models. *Journal of Business & Economic Statistics*, 23:200–210.

Burdett, K. and Mortensen, D. T. (1998). Wage differentials, employer size, and unemployment. *International Economic Review*, 39(2):257–273.

Card, D., Cardoso, A. R., and Kline, P. (2016). Bargaining, Sorting, and the Gender Wage Gap: Quantifying the Impact of Firms on the Relative Pay of Women. *The Quarterly Journal of Economics*, 131(2):633–686.

Chamberlain, G. (1980). Analysis of covariance with qualitative data. *The Review of Economic Studies*, 47(1):225–238.

Charbonneau, K. B. (2017). Multiple fixed effects in binary response panel data models. *The Econometrics Journal*, 20(3):S1–S13.

Chudik, A., Pesaran, M. H., and Yang, J.-C. (2016). Half-panel jackknife fixed-effects estimation of linear panels with weakly exogenous regressors. Working Papers 281, Federal Reserve Bank of Dallas.

Chudik, A., Pesaran, M. H., and Yang, J.-C. (2018). Half-panel jackknife fixed-effects estimation of linear panels with weakly exogenous regressors. *Journal of Applied Econometrics*, 33(6):816–836.

Correia, S. (2019). Reghdfe: Stata module to perform linear or instrumental-variable regression absorbing any number of high-dimensional fixed effects. Statistical Software Components S457874, Boston College Department of Economics.

Dhaene, G. and Jochmans, K. (2015). Split-panel Jackknife Estimation of Fixed-effect Models. *Review of Economic Studies*, 82(3):991–1030.

Ejrnaes, M. and Browning, M. (2014). The persistent-transitory representation for earnings process. *Quantitative Economics*, 5(3):555–581.

Everaert, G. and Pozzi, L. (2007). Bootstrap-based bias correction for dynamic panels. *Journal of Economic Dynamics and Control*, 31(4):1160–1184.

Fernández-Val, I. and Weidner, M. (2018). Fixed effects estimation of large-t panel data models. *Annual Review of Economics*, 10(1):109–138.

Fernández-Val, I. and Weidner, M. (2016). Individual and time effects in nonlinear panel models with large N, T. *Journal of Econometrics*, 192(1):291–312.

Gelbach, J. (2016). When do covariates matter? And which ones, and how much? *Journal of Labor Economics*, 34(2):509–543.

Gonçalves, S. and Kaffo, M. (2015). Bootstrap inference for linear dynamic panel data models with individual fixed effects. *Journal of Econometrics*, 186(2):407–426.

Gourieroux, C., Phillips, P., and Yu, J. (2010). Indirect inference for dynamic panel models. *Journal of Econometrics*, 157(1):68–77.

Goux, D. and Maurin, E. (1999). Persistence of interindustry wage differentials: a reexamination using matched worker-firm panel data. *Journal of Labor Economics*, 17(3):492–533.

Guimarães, P. and Portugal, P. (2010). A simple feasible procedure to fit models with high-dimensional fixed effects. *Stata Journal*, 10(4):628–649.

Guiso, L., Pistaferri, L., and Schivardi, F. (2005). Insurance within the firm. *Journal of Political Economy*, 113(5):1054–1087.

Hahn, J. and Kuersteiner, G. (2002). Asymptotically unbiased inference for a dynamic panel model with fixed effects when both n and t are large. *Econometrica*, 70(4):1639–1657.

Hahn, J. and Newey, W. (2004). Jackknife and analytical bias reduction for nonlinear panel models. *Econometrica*, 72(4):1295–1319.

Heckman, J. (1981). Heterogeneity and state dependence. In *Studies in Labor Markets*, pages 91–140. National Bureau of Economic Research, Inc.

Hospido, L. (2012a). Estimating nonlinear models with multiple fixed effects: A computational note*. *Oxford Bulletin of Economics and Statistics*, 74(5):760–775.

Hospido, L. (2012b). Modelling heterogeneity and dynamics in the volatility of individual wages. *Journal of Applied Econometrics*, 27:386–414.

Hospido, L. (2015). Wage dynamics in the presence of unobserved individual and job heterogeneity. *Labour Economics*, 33:81–93.

Huckfeldt, C. (2021). Understanding the scarring effect of recessions. Technical report, Economics Department, Cornell University.

Kiviet, J. F. (1995). On bias, inconsistency, and efficiency of various estimators in dynamic panel data models. *Journal of Econometrics*, 68(1):53–78.

Lancaster, T. (2000). The incidental parameter problem since 1948. *Journal of Econometrics*, 95(2):391–413.

Liu, K. (2019). Wage risk and the value of job mobility in early employment careers. *Journal of Labor Economics*, 37(1):139–185.

Low, H., Meghir, C., and Pistaferri, L. (2010). Wage risk and employment risk over the life cycle. *American Economic Review*, 100(4):1432–67.

Macleod, W. B. and Malcomson, J. (1993). Investments, holdup, and the form of market contracts. *American Economic Review*, 83(4):811–37.

Neyman, J. and Scott, E. (1948). Consistent estimates based on partially consistent observations. *Econometrica*, 16(1):1–32.

Phillips, P. C. and Sul, D. (2007). Bias in dynamic panel estimation with fixed effects, incidental trends and cross section dependence. *Journal of Econometrics*, 137(1):162–188.

Postel-Vinay, F. and Robin, J.-M. (2002). Equilibrium wage dispersion with worker and employer heterogeneity. *Econometrica*, 70(6):2295–2350.

Postel-Vinay, F. and Turon, H. (2010). On-the-job search, productivity shocks, and the individual earnings process. *International Economic Review*, 51(3):599–629.

Raposo, P., Portugal, P., and Carneiro, A. (2021). The sources of wage losses of displaced workers: the role of the reallocation of workers into firms, matches, and job titles. *Journal of Human Resources*, 56(3):786–821.

Topel, R. H. and Ward, M. P. (1992). Job mobility and the careers of young men. *The Quarterly Journal of Economics*, 107(2):439–79.

Torres, S., Portugal, P., Addison, J. T., and Guimarães, P. (2018). The sources of wage variation and the direction of assortative matching: Evidence from a three-way high-dimensional fixed effects regression model. *Labour Economics*, 54(C):47–60.

Woodcock, S. D. (2008). Wage differentials in the presence of unobserved worker, firm, and match heterogeneity. *Labour Economics*, 15(4):771–793.

Woodcock, S. D. (2015). Match effects. *Research in Economics*, 69(1):100–121.

Woodcock, S. D. (2020). The Effect of the Hartz Labor Market Reforms on Post-unemployment Wages, Sorting, and Matching. IZA Discussion Papers 13300, Institute of Labor Economics (IZA).

Table 1: Bias comparison of alternative estimators in a panel AR(1) generated with worker and firm fixed effects only ($N = 1600, J = 200$)

		worker and firm fixed effects				*job-match fixed effects*			
		Bias				*Bias*			
T	ρ	wf	$jk_{1/2}$	HK	rwb	match	$jk_{1/2}$	HK	rwb
					10% Worker Mobility				
10	0.1	-0.132	-0.002	-0.035	-0.017	-0.176	-0.070	-0.084	-0.034
	0.3	-0.161	-0.000	-0.047	-0.031	-0.211	-0.078	-0.102	-0.053
	0.5	-0.187	0.001	-0.056	-0.052	-0.241	-0.084	-0.116	-0.077
	0.7	-0.199	0.013	-0.049	-0.077	-0.253	-0.071	-0.108	-0.101
	0.9	-0.145	0.072	0.031	-0.064	-0.183	0.015	-0.012	-0.079
20	0.1	-0.063	0.001	-0.012	-0.003	-0.125	-0.079	-0.077	-0.017
	0.3	-0.076	0.001	-0.015	-0.006	-0.147	-0.091	-0.090	-0.024
	0.5	-0.088	0.004	-0.017	-0.010	-0.167	-0.098	-0.100	-0.033
	0.7	-0.095	0.010	-0.015	-0.019	-0.177	-0.095	-0.101	-0.045
	0.9	-0.064	0.051	0.028	-0.018	-0.120	-0.015	-0.031	-0.033
30	0.1	-0.042	-0.001	-0.007	-0.001	-0.110	-0.080	-0.077	- 0.013
	0.3	-0.050	-0.000	-0.008	-0.003	-0.128	-0.092	-0.089	-0.018
	0.5	-0.057	0.002	-0.009	-0.004	-0.145	-0.101	-0.099	-0.024
	0.7	-0.063	0.006	-0.008	-0.008	-0.154	-0.103	-0.102	-0.031
	0.9	-0.042	0.034	0.020	-0.010	-0.108	-0.037	-0.048	-0.024
					25% Worker Mobility				
10	0.1	-0.130	-0.000	-0.033	-0.016	-0.244	-0.163	-0.159	-0.076
	0.3	-0.158	0.002	-0.043	-0.030	-0.288	-0.186	-0.187	-0.102
	0.5	-0.183	0.004	-0.052	-0.050	-0.325	-0.202	-0.208	-0.130
	0.7	-0.194	0.017	-0.044	-0.075	-0.337	-0.191	-0.201	-0.150
	0.9	-0.140	0.074	0.036	-0.062	-0.250	-0.075	-0.085	-0.109
20	0.1	-0.062	-0.000	-0.010	-0.003	-0.202	-0.164	-0.157	-0.053
	0.3	-0.074	-0.001	-0.013	-0.006	-0.235	-0.188	-0.182	-0.067
	0.5	-0.085	-0.005	-0.015	-0.010	-0.264	-0.207	-0.202	-0.082
	0.7	-0.092	-0.011	-0.012	-0.018	-0.275	-0.205	-0.204	-0.093
	0.9	-0.061	0.051	0.031	-0.018	-0.191	-0.082	-0.105	-0.054
30	0.1	-0.041	-0.000	-0.006	-0.001	-0.186	-0.157	-0.155	-0.046
	0.3	-0.049	0.000	-0.007	-0.003	-0.215	-0.180	-0.179	-0.056
	0.5	-0.055	0.003	-0.007	-0.004	-0.240	-0.196	-0.198	-0.067
	0.7	-0.060	0.006	-0.005	-0.008	-0.249	-0.195	-0.200	-0.074
	0.9	-0.041	0.034	0.021	-0.009	-0.169	-0.075	-0.111	-0.039

Notes: The column labeled ρ indicates the autoregressive parameter considered in the DGP in (15), and the columns labeled wf, match, $jk_{1/2}$, HK and rwb, present the results of the estimation bias $E(\hat{\rho}^k - \rho)$ and $E(\hat{\rho}^k_j - \rho)$, with $k = \text{wf}$ or $k = \text{match}$, and $j = jk_{1/2}, HK$ and rwb, computed as $1/R\sum_{s=1}^{R}(\hat{\rho}^k - \rho)$ and $1/R\sum_{s=1}^{R}(\hat{\rho}^k_j - \rho)$, respectively, where R is the number of Monte Carlo replications. $\hat{\rho}^k$ corresponds to the uncorrected least-squares estimate of ρ computed from a model with worker and firm fixed effects (when $k = \text{wf}$) and from a model with job-match fixed effects (when $k = \text{match}$). $\hat{\rho}^k_j$ corresponds to the bias corrected least-squares estimate of ρ computed with the split-panel Jackknife estimator ($j = jk_{1/2}$), the analytical correction proposed by Hahn and Kuersteiner (2002) ($j = HK$), and the residual wild bootstrap approach ($j = rwb$) described in Section 3.2.

Table 2: Bias comparison of alternative estimators for a panel AR(1) generated with worker, firms and job-match quality effects($N = 1600, J = 200$)

		worker and firm fixed effects *Bias*				*job-match fixed effects* *Bias*			
T	ρ	wf	$jk_{1/2}$	HK	rwb	$match$	$jk_{1/2}$	HK	rwb
				10% Worker Mobility					
10	0.1	-0.089	0.064	0.012	0.029	-0.169	-0.064	-0.076	-0.028
	0.3	-0.112	0.071	0.007	0.019	-0.198	-0.070	-0.088	-0.046
	0.5	-0.135	0.069	0.001	-0.001	-0.220	-0.069	-0.092	-0.065
	0.7	-0.150	0.061	0.005	-0.035	-0.216	-0.045	-0.068	-0.080
	0.9	-0.114	0.076	0.065	-0.042	-0.143	-0.031	0.033	-0.055
20	0.1	0.018	0.116	0.074	0.083	-0.118	-0.071	-0.069	-0.007
	0.3	0.014	0.128	0.080	0.089	-0.136	-0.079	-0.077	-0.011
	0.5	0.005	0.130	0.080	0.087	-0.148	-0.081	-0.080	-0.016
	0.7	-0.015	0.108	0.070	0.064	-0.144	-0.069	-0.067	-0.022
	0.9	-0.034	0.059	0.059	0.009	-0.087	0.004	0.003	-0.015
30	0.1	0.063	0.143	0.102	0.108	-0.100	-0.068	-0.067	-0.001
	0.3	0.065	0.154	0.110	0.117	-0.115	-0.077	-0.075	-0.003
	0.5	0.059	0.154	0.111	0.117	-0.124	-0.080	-0.078	-0.004
	0.7	0.039	0.130	0.097	0.097	-0.120	-0.071	-0.068	-0.006
	0.9	-0.005	0.054	0.058	0.028	-0.073	-0.021	-0.012	-0.004
				25% Worker Mobility					
10	0.1	-0.054	0.106	0.050	0.068	-0.221	-0.142	-0.134	-0.055
	0.3	-0.076	0.113	0.047	0.061	-0.255	-0.156	-0.150	-0.073
	0.5	-0.103	0.106	0.037	0.038	-0.277	-0.159	-0.155	-0.090
	0.7	-0.130	0.081	0.027	-0.007	-0.273	-0.137	-0.130	-0.100
	0.9	-0.116	0.074	0.063	-0.039	-0.195	-0.046	-0.025	-0.071
20	0.1	0.047	0.140	0.105	0.113	-0.176	-0.134	-0.130	-0.026
	0.3	0.042	0.146	0.109	0.117	-0.198	-0.148	-0.143	-0.031
	0.5	0.027	0.141	0.104	0.110	-0.211	-0.152	-0.147	-0.034
	0.7	-0.002	0.111	0.083	0.079	-0.205	-0.136	-0.130	-0.035
	0.9	-0.034	0.056	0.059	0.012	-0.138	-0.057	-0.050	-0.021
30	0.1	0.083	0.149	0.122	0.127	-0.154	-0.118	-0.123	-0.013
	0.3	0.081	0.154	0.127	0.132	-0.172	-0.130	-0.135	-0.015
	0.5	0.068	0.145	0.121	0.125	-0.182	-0.131	-0.138	-0.014
	0.7	0.039	0.114	0.098	0.098	-0.173	-0.116	-0.122	-0.011
	0.9	-0.009	0.047	0.054	0.025	-0.113	-0.048	-0.054	-0.004

See notes under Table 1.

Table 3: Decomposition of the difference of the autoregressive parameter estimates of the base model in (3) and the worker and firm fixed effects model in (2) computed from balanced panels

	Male Workers					*Female Workers*				
				Decomposition:					Decomposition:	
	$\hat{\rho}_0$	$\hat{\rho}^{\text{wf}}$	$\hat{\rho}_0 - \hat{\rho}^{\text{wf}}$	*Worker*	*Firm*	$\hat{\rho}_0$	$\hat{\rho}^{\text{wf}}$	$\hat{\rho}_0 - \hat{\rho}^{\text{wf}}$	*Worker*	*Firm*
	0.8917	0.3642	0.5275	0.4147	0.1128	0.9255	0.3515	0.5740	0.4989	0.0751
	(0.0013)	(0.0035)		(0.0024)	(0.0022)	(0.0013)	(0.0048)		(0.0029)	(0.0027)
Bias correction:										
$jk_{1/2}$	0.8926	0.5517	0.3409	0.2184	0.1225	0.9263	0.5395	0.3868	0.3115	0.0753
HK	1.0030	0.4444	0.5586	-	-	1.0388	0.4310	0.6078	-	-
rwb	0.9455	0.4972	0.4483	0.3281	0.1202	0.9845	0.4778	0.5067	0.4494	0.0573

Notes: This Table presents the decomposition for male and female workers of the variation of the autoregressive coefficient estimate computed from the base model in (3) and the model with worker and firm fixed effects in (2). The results reported are obtained from balanced panels of 655,120 men-year observations and 383,456 women-year observations, respectively. The conditional decomposition of the variation of the autoregressive coefficient estimates ($\hat{\rho}_0 - \hat{\rho}^{\text{wf}}$) is based on Gelbach (2016). The contribution of each fixed effect is computed from an auxiliary regression in which the fixed effects are regressed on the covariates of the benchmark specification. The values in parentheses are clustered (at the level of the corresponding fixed effects) standard errors. The sum of the fixed effects' contributions equals ($\hat{\rho}_0 - \hat{\rho}^{\text{wf}}$). The rows labeled $jk_{1/2}$, HK and rwb correspond to bias corrected results obtained based on the split-panel Jackknife estimator, the analytical correction proposed by Hahn and Kuersteiner (2002), and the residual wild bootstrap approach, respectively, described in Section 3.2.

Table 4: Decomposition of the difference of the autoregressive parameter estimates of the base model in (3) and the job-match fixed effects model in (10) computed from balanced panels

	Male Workers						*Female Workers*					
				Decomposition:						Decomposition:		
	$\hat{\rho}_0$	$\hat{\rho}^{\text{match}}$	$\hat{\rho}_0 - \hat{\rho}^{\text{match}}$	*Worker*	*Firm*	*Match Quality*	$\hat{\rho}_0$	$\hat{\rho}^{\text{match}}$	$\hat{\rho}_0 - \hat{\rho}^{\text{match}}$	*Worker*	*Firm*	*Match Quality*
	0.8917	0.3329	0.5588	0.4145	0.1130	0.0313	0.9255	0.3198	0.6057	0.4991	0.0750	0.0316
	(0.0013)	(0.0036)		(0.0024)	(0.0022)	(0.0008)	(0.0013)	(0.0049)		(0.0029)	(0.0027)	(0.0014)
Bias correction:												
$jk_{1/2}$	0.8926	0.5065	0.3861	0.2200	0.1209	0.0452	0.9263	0.4905	0.4358	0.3093	0.0775	0.0490
HK	1.0030	0.4113	0.5917	-	-	-	1.0388	0.3974	0.6414	-	-	-
rwb	0.9455	0.4785	0.4670	0.3381	0.1289	0.0000	0.9845	0.4594	0.5251	0.4624	0.0621	0.0006

Notes: This Table presents the decomposition for male and female workers of the variation of the autoregressive coefficient estimate computed from the base model in (3) and the model with job-match fixed effects in (10). See notes under Table 3 for further details.

Table 5: Decomposition of the difference of the autoregressive parameter estimates of the base model in (3) and the worker and firm fixed effects model in (2) computed from unbalanced panels

	Male Workers					*Female Workers*				
				Decomposition into:					Decomposition into:	
	$\hat{\rho}_0$	$\hat{\rho}^{\text{wf}}$	$\hat{\rho}_0 - \hat{\rho}^{\text{wf}}$	Worker	Firm	$\hat{\rho}_0$	$\hat{\rho}^{\text{wf}}$	$\hat{\rho}_0 - \hat{\rho}^{\text{wf}}$	Worker	Firm
	0.8679	0.2939	0.5740	0.3823	0.1917	0.8749	0.2674	0.6075	0.4268	0.1807
	(0.0003)	(0.0007)		(0.0004)	(0.0003)	(0.0004)	(0.0010)		(0.0005)	(0.0004)
Bias correction:										
$jk_{1/2}$	0.8713	0.4774	0.3939	0.2119	0.1820	0.8779	0.4551	0.4228	0.2474	0.1754
HK	0.9778	0.3700	0.6078	-	-	0.9852	0.3420	0.6432	-	-
rwb	0.9492	0.3624	0.5868	0.3655	0.2213	0.9990	0.3256	0.6734	0.4569	0.2165

Notes: This Table presents the decomposition for male and female workers of the variation of the autoregressive coefficients estimate computed from the base model in (3) and the model with worker and firm fixed effects in (2). The results reported are obtained from unbalanced panels of 12,802,613 men-year observations and 9,800,784 women-year observations, respectively. The conditional decomposition of the variation of the autoregressive coefficient estimates $(\hat{\rho}_0 - \hat{\rho}^{\text{wf}})$ is based on Gelbach (2016). The contribution of each fixed effect is computed from an auxiliary regression in which the fixed effects are regressed on the covariates of the benchmark specification. The values in parentheses are clustered (at the level of the corresponding fixed effects) standard errors. The sum of the fixed effects' contributions equals $(\hat{\rho}_0 - \hat{\rho}^{\text{wf}})$. The rows labeled $jk_{1/2}$, HK and rwb correspond to bias corrected results obtained based on the split-panel Jackknife estimator, the analytical correction proposed by Hahn and Kuersteiner (2002), and the residual wild bootstrap approach, respectively, described in Section 3.2.

Table 6: Decomposition of the difference of the autoregressive parameter estimates of the base model in (3) and the job-match fixed effects model in (10) computed from unbalanced panels

	Male Workers						*Female Workers*					
				Decomposition:						Decomposition:		
	$\hat{\rho}_0$	$\hat{\rho}^{\text{match}}$	$\hat{\rho}_0 - \hat{\rho}^{\text{match}}$	*Worker*	*Firm*	*Match Quality*	$\hat{\rho}_0$	$\hat{\rho}^{\text{match}}$	$\hat{\rho}_0 - \hat{\rho}^{\text{match}}$	*Worker*	*Firm*	*Match Quality*
	0.8679	0.2228	0.6451	0.3823	0.1917	0.0711	0.8749	0.1976	0.6773	0.4267	0.1807	0.0699
	(0.0003)	(0.0008)		(0.0004)	(0.0003)	(0.0004)	(0.0004)	(0.0010)		(0.0005)	(0.0004)	(0.0004)
Bias correction:												
$jk_{1/2}$	0.8713	0.3832	0.4881	0.2118	0.1818	0.0945	0.8779	0.3572	0.5207	0.2474	0.1752	0.0980
HK	0.9778	0.2947	0.6831	-	-	-	0.9852	0.2680	0.7172	-	-	-
rwb	0.9492	0.2915	0.6577	0.3501	0.2092	0.0984	0.9990	0.2551	0.7439	0.4321	0.2043	0.1075

Notes: This Table presents the decomposition for male and female workers of the variation of the autoregressive coefficients estimate computed from the base model in (3) and the model with job-match fixed effects in (10). See notes under Table 5 for further details.

Appendix A - Summary Statistics

Table A.1: Summary Statistics

	Men		*Women*	
	Balanced	Unbalanced	Balanced	Unbalanced
Real hourly wages (in logs)	1.1597	0.6673	0.9814	0.4443
Age (in years)	40.3214	40.2751	42.2208	39.4291
Tenure (in years)	13.0845	9.6938	14.6281	9.2615
Firm size (in logs)	6.2642	4.4670	6.9368	4.5179
Schooling (in years)	11.4660	9.2638	11.9771	10.2398
N (number of observations)	655,120	12,802,613	383,456	9,800,784
Number of workers	40,945	2,014,995	23,966	1,600,305
Number of firms	10,035	256,674	4,680	225,318
Number of worker-firm matches	69,276	3,111,886	38,795	2,372,438

Note: This table reports the summary statistics from *Quadros de Pessoal* (2002-2018) for the four samples of workers used.

Appendix B - Estimating a two-way high-dimensional fixed effects regression model

In this Appendix we briefly describe the procedure of Guimarães and Portugal (2010) for estimating a wage equation that incorporates two high-dimensional fixed effects, the worker and firm fixed effects (in the case of job-match fixed effects the approach follows along similar lines). Specifically, the approach consists of a modified version of the methodology initially developed by Abowd et al. (1999) and Abowd et al. (2002) which was extended and simplified by Guimarães and Portugal (2010) to work with large datasets.[12]

For illustration, consider the matrix representation of the dynamic wage equation using

[12]The approach of Guimarães and Portugal (2010) is implemented in the *reghdfe* Stata procedure, see Correia (2019).

worker and firm fixed effects in (2) , i.e.,

$$\mathbf{W} = \mathbf{E}\boldsymbol{\alpha} + \mathbf{F}\boldsymbol{\theta} + \mathbf{Q}^{\star}\boldsymbol{\vartheta}^{\star} + \mathbf{V} \tag{B.1}$$

where $\mathbf{Q}^{\star} = (\mathbf{W}_{-1}, \mathbf{X})$ and $\boldsymbol{\vartheta}^{\star} = (\rho, \boldsymbol{\vartheta}')'$; $\mathbf{W}$ represents the vector of log wages, $\mathbf{W}_{-1}$ is the vector of one-period lagged log wages, $\mathbf{X}$ denotes the matrix of control variables (such as, time dummies, tenure, a quadratic in tenure, a quadratic in age, schooling years, and firm size), $\boldsymbol{\vartheta}^{\star}$ is a vector of regression coefficients that includes the wage persistence parameter ρ, $\mathbf{E}$ and $\mathbf{F}$, are matrices collecting the worker and firm dummies, respectively, and $\mathbf{V}$ stands for the error term.

As is well known, the least squares estimator of $\boldsymbol{\Phi} := (\boldsymbol{\vartheta}^{\star}, \boldsymbol{\alpha}, \boldsymbol{\theta})'$ solves the following equation:

$$\mathbf{Z}'\mathbf{Z}\boldsymbol{\Phi} = \mathbf{Z}'\mathbf{W} \tag{B.2}$$

where $\mathbf{Z} = (\mathbf{Q}^{\star}, \mathbf{E}, \mathbf{F})$. However, in the present context it is computationally difficult, or unfeasible, to invert $\mathbf{Z}'\mathbf{Z}$ due to the large number of worker and firm fixed effects.

Herein, an iterative solution that alternates between estimation of $\widehat{\boldsymbol{\vartheta}}^{\star}$, $\widehat{\boldsymbol{\alpha}}$, and $\widehat{\boldsymbol{\theta}}$, can be used, i.e.,

$$\begin{bmatrix} \widehat{\boldsymbol{\vartheta}}^{\star,(r)} \\ \widehat{\boldsymbol{\alpha}}^{(r)} \\ \widehat{\boldsymbol{\theta}}^{(r)} \end{bmatrix} = \begin{bmatrix} (\mathbf{Q}^{\star\prime}\mathbf{Q}^{\star})^{-1}\mathbf{Q}^{\star\prime}(\mathbf{W} - \mathbf{E}\widehat{\alpha}^{(r-1)} - \mathbf{F}\widehat{\theta}^{(r-1)}) \\ (\mathbf{E}'\mathbf{E})^{-1}\mathbf{E}'(\mathbf{W} - \mathbf{F}\widehat{\theta}^{(r-1)} - \mathbf{Q}^{\star}\widehat{\boldsymbol{\vartheta}}^{\star,(r)}) \\ (\mathbf{F}'\mathbf{F})^{-1}\mathbf{F}'(\mathbf{W} - \mathbf{E}\widehat{\boldsymbol{\alpha}}^{(r)} - \mathbf{Q}^{\star}\widehat{\boldsymbol{\vartheta}}^{\star,(r)}) \end{bmatrix} \tag{B.3}$$

where $r = 1, \ldots,$ indicates the number of the "r^{th}" iteration. It is clear from (B.3) that at each iteration the estimates of the fixed effects are simply computed as averages of residuals. For instance, $(\mathbf{F}'\mathbf{F})^{-1}\mathbf{F}'$ is simply an average operator applied to the firm's residuals.

The iterative solution proceeds as follows. Through the recursive algorithm, the current value of $\widehat{\boldsymbol{\vartheta}}^{\star,(r)}$ is used to estimate the current value of $\widehat{\boldsymbol{\alpha}}^{(r)}$, and in the estimation of $\widehat{\boldsymbol{\theta}}^{(r)}$ the values of $\widehat{\boldsymbol{\alpha}}^{(r)}$ and $\widehat{\boldsymbol{\vartheta}}^{\star,(r)}$ are used. Then, the algorithm restarts and this will be repeated a sufficient number of times until the procedure converges.

Following Guimarães and Portugal (2010), to control for convergence of the algorithm, instead of transforming the variables, i.e., instead of using the Frish-Waugh-Lovell theorem to remove the influence of the two high-dimensional fixed effects from each individual variable for the estimation of $\widehat{\boldsymbol{\vartheta}}^{\star,(r)}$, alternatively a regression such as,

$$\mathbf{W} = \lambda_1\mathbf{E}\widehat{\boldsymbol{\alpha}}^{(r-1)} + \lambda_2\mathbf{F}\widehat{\boldsymbol{\theta}}^{(r-1)} + \mathbf{Q}^{\star}\boldsymbol{\vartheta}^{\star,(r)} + \mathbf{V} \tag{B.4}$$

is performed.

Note that here $\mathbf{E}\widehat{\boldsymbol{\alpha}}^{(r-1)}$ and $\mathbf{F}\widehat{\boldsymbol{\theta}}^{(r-1)}$ are estimated parameter vectors that are used as regressors to determine when convergence has been achieved. In other words, this regression will be used to compute the updated estimate of $\boldsymbol{\vartheta}$, $\widehat{\boldsymbol{\vartheta}}^{\star,(r)}$, and the usefulness of this model is that it allows us to determine when convergence has been achieved through the parameter estimates of λ_1 and λ_2. In particular, the algorithm will stop when $\widehat{\lambda}_1 = \widehat{\lambda}_2 = 1$.

Estimating a regression using the transformed variables with a correction for the degrees of freedom yields the exact least-squares solution for the coefficients and standard errors, for details see Guimarães and Portugal (2010) and Correia (2019).

On-Line Supplementary Appendix

to

"The persistence of wages"

by

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Summary of Contents

This supplement contains additional Monte Carlo results.

S.1 Additional Monte Carlo Results

Table S.1: Bias comparison of alternative estimators for a panel AR(1) generated with no job-match quality ($N = 3200$, $J = 400$)

		worker and firm fixed effects				*job-match fixed effects*			
		Bias				*Bias*			
T	ρ	wf	$jk_{1/2}$	HK	rwb	match	$jk_{1/2}$	HK	rwb
					10% Worker Mobility				
10	0.1	-0.133	0.025	-0.036	-0.017	-0.178	-0.048	-0.085	0.068
	0.3	-0.160	0.034	-0.046	-0.031	-0.212	-0.051	-0.103	0.065
	0.5	-0.187	0.041	-0.056	-0.052	-0.243	-0.051	-0.117	0.050
	0.7	-0.199	0.059	-0.049	-0.077	-0.254	-0.033	-0.109	0.026
	0.9	-0.144	0.126	0.032	-0.064	-0.184	0.061	-0.012	0.051
20	0.1	-0.064	0.006	-0.012	-0.003	-0.127	-0.077	-0.079	0.036
	0.3	-0.076	0.009	-0.014	-0.006	-0.149	-0.087	-0.092	0.038
	0.5	-0.088	0.013	-0.017	-0.010	-0.170	-0.094	-0.103	0.035
	0.7	-0.095	0.022	-0.015	-0.019	-0.179	-0.090	-0.103	0.025
	0.9	-0.063	0.067	0.029	-0.018	-0.122	-0.006	-0.033	0.024
30	0.1	-0.042	0.002	-0.007	-0.001	-0.112	-0.081	-0.079	0.022
	0.3	-0.049	0.004	-0.008	-0.002	-0.131	-0.093	-0.092	0.024
	0.5	-0.057	0.006	-0.009	-0.004	-0.148	-0.103	-0.103	0.022
	0.7	-0.062	0.011	-0.008	-0.008	-0.157	-0.104	-0.105	0.017
	0.9	-0.042	0.041	0.020	-0.009	-0.111	-0.038	-0.051	0.012
					25% Worker Mobility				
10	0.1	-0.130	0.027	-0.033	-0.017	-0.248	-0.150	-0.163	0.018
	0.3	-0.157	0.036	-0.043	-0.030	-0.292	-0.170	-0.191	0.007
	0.5	-0.183	0.044	-0.051	-0.051	-0.330	-0.183	-0.213	-0.013
	0.7	-0.193	0.063	-0.042	-0.074	-0.342	-0.168	-0.206	-0.031
	0.9	-0.139	0.128	0.037	-0.061	-0.254	-0.042	-0.089	0.004
20	0.1	-0.062	0.006	-0.010	-0.003	-0.209	-0.170	-0.164	-0.006
	0.3	-0.074	0.009	-0.012	-0.006	-0.243	-0.196	-0.191	-0.013
	0.5	-0.085	0.014	-0.014	-0.010	-0.273	-0.215	-0.212	-0.022
	0.7	-0.091	0.023	-0.011	-0.018	-0.286	-0.216	-0.215	-0.032
	0.9	-0.060	0.065	0.032	-0.017	-0.202	-0.092	-0.118	-0.011
30	0.1	-0.041	0.003	-0.005	-0.001	-0.195	-0.168	-0.164	-0.016
	0.3	-0.048	0.004	-0.006	-0.002	-0.226	-0.193	-0.190	-0.022
	0.5	-0.055	0.007	-0.007	-0.004	-0.252	-0.212	-0.211	-0.030
	0.7	-0.060	0.011	-0.005	-0.008	-0.264	-0.215	-0.216	-0.036
	0.9	-0.040	0.040	0.022	-0.009	-0.187	-0.099	-0.130	-0.014

Note: The column labeled ρ indicates the autoregressive parameter considered in the DGP in (15), and the columns labeled wf, match, $jk_{1/2}$, jk_{-1}, HK and rwb, present the results of the estimation bias $E(\hat{\rho}^k - \rho)$ and $E(\hat{\rho}^k_j - \rho)$, with $k = wf$ or $k =$ match and $j = jk_{1/2}, HK$ and rwb, computed as $1/R\sum_{s=1}^{R}(\hat{\rho}^k - \rho)$ and $1/R\sum_{s=1}^{R}(\hat{\rho}^k_j - \rho)$, respectively, where R is the number of Monte Carlo replications. $\hat{\rho}^k$ corresponds to the uncorrected least-squares estimate of ρ computed from a model with worker and firm fixed effects (when $k =$ wf) and from a model with job-match fixed effects (when $k =$ match). $\hat{\rho}^k_j$ corresponds to the bias corrected least-squares estimate of ρ computed with the split-panel Jackknife estimator ($j = jk_{1/2}$), the analytical correction of Hahn and Kuersteiner (2002) ($j = HK$), and the residual wild bootstrap approach ($j = rwb$).

Table S.2: Bias comparison of alternative estimators for a panel AR(1) generated with worker, firm and job-match quality effects ($N = 3200, J = 400$)

		worker and firm fixed effects				*job-match fixed effects*			
		Bias				*Bias*			
T	ρ	wf	$jk_{1/2}$	HK	rwb	match	$jk_{1/2}$	HK	rwb
		10% Worker Mobility							
10	0.1	-0.089	0.095	0.012	0.029	-0.170	-0.043	-0.077	0.072
	0.3	-0.111	0.108	0.008	0.020	-0.199	-0.043	-0.089	0.070
	0.5	-0.135	0.112	0.002	-0.001	-0.221	-0.038	-0.094	0.059
	0.7	-0.149	0.108	0.006	-0.034	-0.217	-0.009	-0.069	0.048
	0.9	-0.113	0.126	0.065	-0.042	-0.144	0.074	0.032	0.084
20	0.1	0.018	0.125	0.074	0.083	-0.121	-0.071	-0.072	0.044
	0.3	0.015	0.139	0.081	0.090	-0.139	-0.078	-0.081	0.048
	0.5	0.006	0.143	0.081	0.088	-0.151	-0.079	-0.084	0.049
	0.7	-0.014	0.123	0.070	0.065	-0.147	-0.065	-0.070	0.045
	0.9	-0.034	0.072	0.060	0.009	-0.089	0.003	0.002	0.046
30	0.1	0.064	0.149	0.103	0.109	-0.104	-0.073	-0.071	0.032
	0.3	0.066	0.160	0.111	0.118	-0.119	-0.081	-0.080	0.036
	0.5	0.060	0.160	0.112	0.117	-0.129	-0.085	-0.083	0.038
	0.7	0.039	0.136	0.097	0.097	-0.125	-0.075	-0.073	0.038
	0.9	-0.005	0.059	0.058	0.028	-0.076	-0.022	-0.015	0.034
		25% Worker Mobility							
10	0.1	-0.054	0.138	0.050	0.068	-0.226	-0.131	-0.138	0.039
	0.3	-0.075	0.151	0.047	0.061	-0.259	-0.142	-0.155	0.035
	0.5	-0.102	0.149	0.038	0.038	-0.282	-0.142	-0.160	0.028
	0.7	-0.129	0.127	0.028	-0.007	-0.277	-0.116	-0.135	0.022
	0.9	-0.115	0.122	0.063	-0.038	-0.198	-0.015	-0.028	0.052
20	0.1	0.047	0.148	0.105	0.113	-0.185	-0.145	-0.139	0.018
	0.3	0.042	0.156	0.109	0.117	-0.209	-0.161	-0.155	0.020
	0.5	0.027	0.151	0.103	0.109	-0.223	-0.165	-0.159	0.022
	0.7	-0.002	0.122	0.083	0.079	-0.217	-0.150	-0.143	0.024
	0.9	-0.034	0.067	0.059	0.012	-0.147	-0.064	-0.059	0.028
30	0.1	0.083	0.153	0.122	0.127	-0.168	-0.137	-0.137	0.012
	0.3	0.081	0.157	0.127	0.132	-0.188	-0.151	-0.151	0.014
	0.5	0.068	0.149	0.120	0.124	-0.199	-0.156	-0.156	0.018
	0.7	0.039	0.118	0.097	0.096	-0.191	-0.142	-0.141	0.023
	0.9	-0.009	0.052	0.054	0.025	-0.128	-0.067	-0.069	0.025

See notes under Table S.1