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Public debt, iMPCs & fiscal policy transmission

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Abstract

In this paper, I examine the relationship between public debt and the effectiveness of fiscal policy, presenting evidence of an inverse relationship between government debt and fiscal multipliers. To explain the results, I develop and calibrate a HANK model tailored to the U.S. economy. The model reveals that higher public debt diminishes fiscal multipliers by making households less constrained; with more debt serving as a liquidity self-insurance tool, agents exhibit a weaker labor response to fiscal shocks. Theoretically, I show intertemporal marginal propensities to consume (iMPCs) are a sufficient statistics of public debt and consequently this influences fiscal multipliers. I then decompose the changes in iMPCs to those that come out of wealth distribution and policy function and I find that the primary factor driving variations in iMPCs is the change in interest rates due to the variation of government bonds. Although redistribution across households remains central to the transmission of fiscal policy, this paper is the first to show that other channels also influence discretionary fiscal policy's overall impact, especially in economies with higher debt.

Keywords: Fiscal multipliers; Public Debt; HANK; Government Spending; iMPCs.
JEL Classification: E21; E62; H31

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1 Introduction

After the financial crisis of 2008, fiscal policy became central in the debate for economists and policy-makers. The high increase in government expenditure across countries motivated the development of new empirical and quantitative models to study the impact on output growth. At the same time, the enormous fiscal stimulus in several countries was financed by an increase in public debt: the emission of government bonds to pay for expenditures has also increased during the COVID-19 pandemic. This persistent surge in debt levels raises important questions about the effectiveness of discretionary fiscal policy in such environments. Many advanced economies have seen their real debt balloon over the past few years. What does this imply for effectiveness of discretionary fiscal policy?

Motivated by empirical evidence that suggests a negative correlation between high levels of public debt and the effectiveness of fiscal policy, I develop and calibrate a Heterogeneous Agent New Keynesian (HANK) model for the U.S. economy. The model is designed to capture how fiscal policy interacts with varying levels of public debt, particularly by influencing the intertemporal marginal propensity to consume (iMPC) and labor supply decisions across heterogeneous households. The objective is to explore the specific role of domestic holdings of public debt in this relationship and to develop a theoretical model that captures the iMPCs of agents in response to varying levels of public indebtedness.

First, I estimate the state-dependent fiscal multiplier for the United States. I find that for higher debt, the state-dependent fiscal multiplier is smaller.

Second, to explain the mechanism, I develop a theoretical framework and calibrate a one-account HANK model to the US economy, featuring sticky wages and flexible prices, to examine how changes in the level of government debt held inside a country influence the responsiveness of the economy to fiscal policy shocks. The model is quantified to study the classic response of the economy to a fiscal policy shock, especially under conditions where agents are insured with varying levels of savings.

Theoretically, I study the mechanism through two channels: the insurance channel and the factor price channel. These two channels are used to explain the changes in MPCs due to higher government bonds. I decompose the two channels to a first order. Decomposing changes in the intertemporal marginal propensity to consume into components that stem from wealth distribution (insurance channel) and policy functions (factor price channel), I find that fiscal policy is less effective when agents hold more assets, as this increases the real interest rate and dampens the overall impact of fiscal interventions. Changes in real interest rates emerge as the dominant channel explaining the varying state-dependent fiscal multipliers. I corroborate these results using different fiscal rules and isolating the changes in taxes.

Literature. The concept of the fiscal multiplier, a summary statistic of the effectiveness of fiscal policy in stimulating economic activity, has been a subject of extensive research in macroeconomics. However, as highlighted by Ramey (2011) and Ramey (2019), there is no singular "fiscal multiplier." Instead, its magnitude can vary depending on several factors, including the type and size of policy change, economic conditions, and characteristics of the economy where the policy is implemented.

Woodford (1990) shows that public debt can act as a form of private liquidity. His idea challenges traditional views of government debt as a burden on future generations and instead suggests that it can serve as a valuable asset for private agents. He argues that government debt provides liquidity services to private agents by serving as a store of value and a means of payment. This liquidity function is crucial for facilitating transactions and smoothing consumption over time. He discusses the implications of his analysis for monetary policy. He suggests that central banks should consider the role of government debt in influencing liquidity conditions when formulating policy decisions. Aiyagari and McGrattan (1998) investigate the optimal quantity of government debt within a heterogeneous agent model framework, incorporating individual income risk, borrowing constraints, and precautionary savings. They challenge the traditional view that government debt should be minimized, demonstrating that a positive amount of government debt can enhance welfare by providing liquidity, thereby enabling better consumption smoothing for individuals facing income uncertainty. Their analysis highlights that the optimal debt level balances the benefits of liquidity against the costs associated with higher taxes needed to service the debt.

From the empirical side, recently Cho and Rhee (2023), using data from 24 OECD countries, find that fiscal policy is generally ineffective in high-debt economies but effective in low-debt economies, highlighting the importance of labor market stimulation for effective fiscal stimulus. Additionally, they show that aged economies experience negligible fiscal policy benefits regardless of debt levels, while non-aged economies benefit positively from fiscal policy in low-debt conditions but suffer negative effects in high-debt situations. Broner et al. (2022) explores the relationship between fiscal multipliers and the proportion of public debt held by foreign creditors. It posits that fiscal expansions can enhance domestic economic activity but may also cause crowding-out effects if domestic consumption and investment decline due to debt acquisition. These crowding-out effects are mitigated when governments sell debt to foreign investors, leading to larger fiscal multipliers. Auerbach and Gorodnichenko (2012), McKay and Reis (2016), Ramey and Zubairy (2018) and Berge et al. (2021) further explore how the effectiveness of fiscal policy can vary depending on the economic environment, including the stage of development, exchange rate regime, and openness of the economy.

Bayer et al. (2023) explore how expansionary fiscal policy influences the liquidity premium—the difference in returns between public debt and less liquid assets. Using an estimated HANK model, the authors show that increased public debt enhances private-sector liquidity, thereby stabilizing fixed-capital investment. They further quantify the long-term impact of higher public debt, finding minimal crowding out of capital but a significant reduction in the liquidity premium, which raises the fiscal cost of debt. The study also indicates that the optimal level of public debt, which maximizes revenue, has increased to 60% of US GDP since 2010.

Brinca et al. (2016) and Brinca et al. (2021) analyze the size and variability of fiscal multipliers depending on various characteristics of the country. They find multipliers depend on the fraction of the population facing binding credit constraints and the economy's average wealth level. The study also reveals significant cross-country differences in multiplier effects due to variations in economic structures and fiscal positions. Antunes and Ercolani (2020) find that the tightening of the household borrowing constraint over time can substantially magnify the government spending multiplier by strengthening the negative wealth effect on labor supply induced by the fiscal stimulus. Gorton and Ordonez (2022) find that the supply of government bonds discourages information acquisition about the heterogenous underlying qualities of private safe assets, improving their safety, crowding out the creation of private safe assets, but crowding in their safety. The optimal supply of government bonds should factor in the dual role of intra- and intertemporal smoothing and their impact on the quantity and safety of private assets. Moreover, the literature on state-dependent fiscal multipliers has emerged, aiming to elucidate how the effectiveness of fiscal policy varies under different economic conditions. Studies by Blanchard and Perotti (2002), Mountford and Uhlig (2009), Fernández-Villaverde et al. (2011), Ilzetzki et al. (2013), Woodford (2011) and Eggertsson (2011) have contributed valuable insights into the determinants and implications of state-dependent fiscal multipliers. These works highlight the importance of accounting for economic conditions, nominal rigidities, and the zero lower bound on nominal interest rates in assessing the efficacy of fiscal policy measures.

Other recent studies have investigated fiscal multipliers within the context of HANK models. Broer et al. (2021) compare the implications of different sources of nominal rigidity on fiscal multipliers in a HANK framework, while Auclert et al. (2024) introduce intertemporal marginal propensities to consume as sufficient statistics of fiscal multipliers.

Outline The remainder of the paper is organized as follows. In section 2 I briefly show the empirical association between the level of public debt and state-dependent fiscal multipliers. Section 3 describes the HANK model and section 4 the respective calibration. In section 5 I discuss the results of the model. Section 6 presents the main theoretical contribution. Section 7 concludes.

2 Empirical Evidence

In this section, I document the empirical relationship between fiscal multipliers and the level of public debt in the US. I start by describing the data and the empirical specification used, followed by a discussion of the results.

2.1 Data and Empirical Strategy

To empirically investigate the relationship between the level of debt and fiscal multipliers in the United States, I utilize data from the Jordà et al. (2017) Macrohistory Database and from Broner et al. (2022).

To investigate government spending multipliers based on the state of the economy, I follow the methodologies of Auerbach and Gorodnichenko (2012), Owyang et al. (2013) and Ramey and Zubairy (2018). I estimate state-dependent impulse responses to shocks in government purchases using Jordà (2005) local projections. This approach has become popular for estimating fiscal multipliers due to its advantages over vector autoregressions (VARs). It is more robust to misspecification because it does not impose implicit dynamic restrictions on the impulse responses' shape. This is true with observable structural shocks and for a fixed number of controls. Additionally, it allows for a more parsimonious specification since not all variables need to be included in every equation. It also easily accommodates state dependence and avoids potential biases when converting elasticities to multipliers.

The empirical baseline model specification is as follows:

$$ly_{t+h} = \alpha_{t+h} + \beta_{1h} lg_t + \beta_{2h}(lg_t * d_{t-1}) + controls + \epsilon_{t+h}$$

where:

- The horizons h of the local projection are 8
- l = log, y = real GDP, g = government expenditure, d = private debt[1]/GDP ratio

The set of controls is the following:

$$\sum_{k=1}^{2} \beta_{3h} y_{t-k} + \sum_{k=1}^{2} \beta_{4h} lg_{t-k} + \sum_{k=1}^{2} \beta_{5h}(lg_{t-k} * d_t) + \sum_{k=1}^{2} \beta_{6h}(lg_{t-k} * d_{t-1})$$

2.2 Results and discussion

The results of the first horizon of the local projection are reported in table 1.

The coefficients for β_1 and β_2 provide important insights into the relationship between government expenditure, debt-to-GDP ratio, and GDP. Specifically, the co-

[1] Private debt is intended to be the public debt held by domestic agents.

	Coefficient	Std. Err.	z	P> $|z|$	[95% Conf. Interval]	
β_1	0.52	0.09	5.34	0.000	0.33	0.72
β_2	-1.43	0.14	-9.94	0.000	-1.71	-1.14

Table 1: Local projection results for the first horizon.

efficient for β_2 is statistically significant and negative, indicating that the interaction between government expenditure and debt-to-GDP ratio has a significant impact on GDP. The level of debt-to-GDP is the level of public debt held domestically by agents in the United States. By analyzing the state-dependent fiscal multiplier impulse response, it is possible to explain how changes in government expenditure affect GDP differently depending on the level of debt-to-GDP ratio held domestically. The results show that β_2 is negative, indicating that higher levels of debt lead to smaller fiscal multipliers. This suggests that the effectiveness of fiscal policy in stimulating economic activity diminishes as the level of debt held by private investors increases inside the United States.

The empirical analysis includes several robustness checks to ensure the reliability of the results. I examine the sensitivity of previous findings to different time lags, alternative specifications, and control variables. Additionally, I find that the standard errors associated with the coefficients are small.

To address potential endogeneity concerns, I propose the same empirical framework by employing a shock-based approach using military expenditure news shocks, following the methodology of Ramey and Zubairy (2018). For this specification, for the United States, I use quarterly data, extending the dataset of Broner et al. (2022). By instrumenting government expenditure with exogenous shocks, I mitigate concerns about reverse causality and endogeneity in the regression analysis. The shock chosen is the narrative Ramey and Zubairy (2018) shock. The empirical model specification is as follows:

$$ly_{t+h} = \alpha_{t+h} + \beta_{1h} lg_t + \beta_{2h}(ls_t * d_{t-1}) + controls + \epsilon_{t+h}$$

where

- The horizons h of the local projection are 8
- g is instrumented by $s * d_{t-1}$

- l = log, y = rGDP, g = government expenditure, s = Ramey shock, d = private debt/GDP ratio

The set of controls is the following:

$$\sum_{k=1}^{2} \beta_{3h} y_{t-k} + \sum_{k=1}^{2} \beta_{4h} l g_{t-k} + \sum_{k=1}^{2} \beta_{5h} (l g_{t-k} * d_t) +$$

$$\sum_{k=1}^{2} \beta_{6h} (l g_{t-k} * d_{t-1}) + \sum_{k=1}^{2} \beta_{7h} s + \sum_{k=1}^{2} \beta_{8h} d_{t-1}$$

The results of the first horizon of the local projection with government expenditure instrumented by the Ramey shock are reported in table 2.

	Coefficient	Std. Err.	z	P> $\lvert z \rvert$	[95% Conf. Interval]	
β_1	0.89	0.82	1.09	0.277	-0.27	2.51
β_2	-0.32	0.12	-2.63	0.009	-0.56	-0.08

Table 2: Local projection instrumented by news shock results for the first horizon. The fiscal shock is government expenditure instrumented by the defense news shocks from Ramey and Zubairy (2018), normalized by potential GDP.

Figure 16 shows the impulse response function of the interaction term. As explained above, this shows how the US reacts differently to changes in government expenditure depending on the prevailing level of the debt-to-GDP ratio. Compared to the results provided in table 1, the values of β_1 and β_2 are capturing values closer to the literature. The value of β_1, the value of the response of output after a fiscal shock follows the estimates available in the literature. The value of β_2, statistically significant at the 95% level, shows that for a state where the level of debt is higher, the state-dependent fiscal multiplier is smaller. To explain the meaning of the negative response, it is possible to substitute standard values of the debt/GDP ratio inside the formula $\beta_{1h} + \beta_{2h} * d_{t-1}$. A level of debt/GDP of 150 % gives rise to a state-dependent fiscal multiplier of 0.41, while a lower level of debt/GDP of 50 % gives a multiplier of 0.73.

The interaction term helps understand how GDP responds to government consumption in states with low versus high debt-to-GDP ratios. The US reacts differently to changes in government expenditure depending on the prevailing level of the debt-to-GDP ratio. Specifically, states with higher debt-to-GDP ratios exhibit weaker GDP

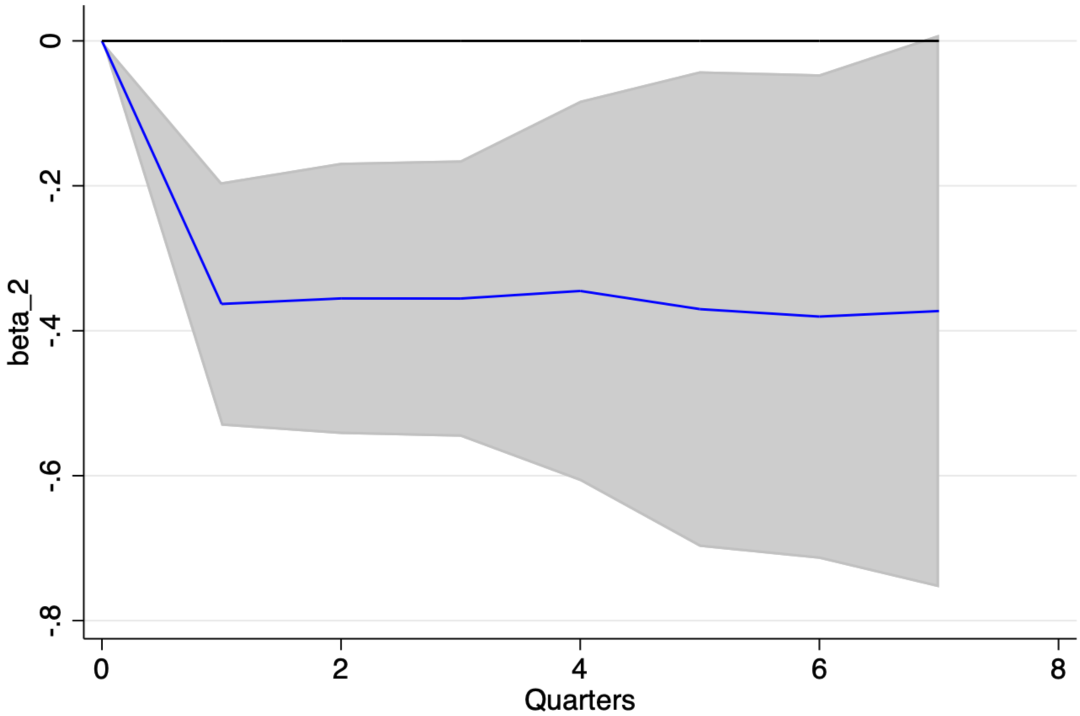

Figure 1: β_{2h} impulse response function over 6 quarters, with 68% confidence intervals.

growth responses than states with lower debt-to-GDP ratios. The response of the full state-dependent multiplier is in figure 17, for different levels of debt, showing the clear response in line with Ramey (2019).

To explain why the state-dependent fiscal multiplier is smaller for higher values of debt held inside a country, and how this is related to household decisions, in the next section I build a state-of-art HANK model to understand the mechanism.

3 Theoretical model

In this section, I describe the details of the quantitative framework used to study the economic response to a fiscal expansion shock for different levels of public debt. The model I propose to study this question is a Heterogeneous Agent New Keynesian (HANK) model, following Auclert et al. (2024) and McKay and Reis (2016). The model features sticky wages, flexible prices, a monetary authority that follows a standard Taylor rule, and a fiscal authority that can run a balanced budget, or financed itself with deficit.

3.1 Households

The economy is populated by a mass of heterogeneous agents that face idiosyncratic risk and aggregate uncertainty. At state s the household has a fixed transition matrix Π, and the mass of households in state s is equal to π_s, such that $\sum_s \pi_s e(s) = 1$. There exist n_e idiosyncratic states, and in any period t, agents transition between any two such states e and e' with exogenous probability $P(e, e')$. Each household decides how much to consume, and save given their state. The felicity function of an household at time t depends on consumption, c_t and work time, N_t and it is given by:

$$U(c_t, n_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} - \varphi \frac{n_t^{1+\eta}}{1+\eta}, \tag{1}$$

where σ is intertemporal elasticity of substitution, φ is a parameter that regulates the disutility of work, and η is the inverse of the Frisch labor elasticity.

Households work the same number of hours, N_t, which is determined by the labor union, as in Erceg et al. (2000). The labor union setting is presented in section 3.3.

Recursive formulation of the household problem At any given time, a household is characterized by the vector (e, a). A union chooses for the agents the hours worked n_{it}. They pay taxes proportionally on their income. The household's optimization problem over consumption and future asset holdings recursively is defined as follows:

$$
\begin{aligned}
V_t(e_{it}, a_{it-1}) = \max_{c_{it}, a_{it}} & \left\{ \frac{c_{it}^{1-\sigma}}{1-\sigma} - \phi \frac{n_{it}^{1+v}}{1+v} + \beta \mathbb{E}_t V_{t+1}(e_{it+1}, a_{it}) \right\} \\
\text{s.to} \quad & c_{it} + a_{it} = (1+r_t) a_{it-1} + \frac{W_t}{P_t} e_{it} N_t - \tau_t \\
& a_{it} \geq \underline{a},
\end{aligned} \tag{2}
$$

3.2 Firms

The firm setting is simple because the prices are flexible. There is a representative firm that follows an aggregate production function $Y_t = X_t N_t$ where X_t is the total factor productivity. This setting leads to flexible prices: $P_t = \frac{W_t}{X_t}$. The goods inflation is equal to the wage inflation minus TFP growth.

$$
\pi = \pi_w - (X_t - X_{t-1}) \tag{3}
$$

Discussion about the firm setting. The real wage in the economy is exogenous: it equals the marginal product of labor and the aggregate production function exhibits constant returns to scale. So there are no profits to distribute between agents in this economy. This is an advantage of the sticky wages compared to the sticky price setting. In fact, with sticky prices, after a positive demand shock, the price does not change but the firm needs to satisfy the demand, so it hires more workers and wage goes up. In a representative agent setting, wages become very procyclical, leading to countercyclical markups. This is not an issue in a standard new-Keynesian model, because the agents who earn wages and markup coincide, but in HANK, this is more problematic, because if an agent is poorer and gets a higher wage, the wage becomes procyclical and income becomes countercyclical. These redistribution effects across people can potentially have dangerous implications. That is the reason why I choose sticky wages, allowing for procyclical profits: for a positive demand shock, the wage and the markup do not adjust, and the price goes up increasing the profits, as in the data.

3.3 Labor Unions

Following Auclert et al. (2024), and as in a standard New Keynesian model with sticky wages, household labor hours, n_{it}, are determined by union labor demand. A continuum of unions exists, k, and a different labor union settles each labor type wage. Firms use labor in their production function, which is a CES bundle of type-specific labor inputs. This is each union, k, aggregates efficient units of work into a union-specific task $N_{kt} = \int e_{it} n_{ikt} di$. At a given time each union asks their members to supply hours according to, $n_{ikt} = N_{ikt}$, and setting wages to maximize the average utility of households, taking as given their consumption-savings decisions. Setting a nominal wage, W_{kt}, involves a quadratic adjustment cost similar to the price adjustment cost incurred by the firm:

$$\psi_t^w \left(W_{kt}, W_{kt-1}\right) = \left(\frac{\mu_w}{\mu_w - 1}\right)\left(\frac{1}{2\kappa_w}\right)\left[\log\left(W_{kt}/W_{kt-1}\right)\right]^2.$$

The union maximization problem leads to a Phillips curve[2] for wage inflation:

$$\pi_t^w = k_w\left(\phi N_t^v - \frac{\epsilon - 1}{\epsilon}\frac{W_t}{P_t}c_t^{-\sigma}\right) + \beta(\pi_{t+1}^w) \tag{4}$$

3.4 Monetary authority

The monetary authority follows a standard Taylor rule to set the nominal interest rate:

$$i_t = r_t^* + \phi_\pi \pi_t + \epsilon_t \tag{5}$$

where r_t^* is the optimal real interest rate, and ϕ_π and ϕ_y are the inflation Taylor rule coefficient and the Taylor rule coefficient on output, respectively.

3.5 Fiscal Authority

The government issues bonds, B^g, sets a proportional tax on labor income, $\tau_t w_t N_t$, and spends on goods and services, G_t, in order to balance its budget constraint period by

[2]Check Appendix C for the complete derivation.

period:

$$\tau_t = (1 + r_t)B_{t-1} + G_t - B_t \tag{6}$$

This holds in steady-state when the budget is balanced, assuming lump-sum transfers adjust to keep the real debt stock constant. In the case of deficit-financed changes in spending, I assume that the following fiscal rule for lump-sum transfers:

$$T_t = T_{ss} + \phi_T \left(B_{-1} - B_{ss}\right) + r_{ss} * B_{ss} + G_{ss} \tag{7}$$

3.6 Stationary Equilibrium

Definition (Competitive Equilibrium). Given a distribution of agents D, the competitive equilibrium can be summarized as follows:

1. The value function $V(e, a)$ and the policy functions $c(e, a)$, and $a'(e, a)$ solve the household problem, given in (2), taking factor prices and initial conditions as given.

2. Firms optimize their decisions.

3. Labor union chooses wages maximizing its objective function.

4. The monetary authority follows the Taylor rule, described by Equation (5).

5. The government budget is balanced. The fiscal authority spends G_t, issues one-period nominal bonds B, and adjusts the level of taxes τ_t to balance its budget period by period $\tau_t = (1 + r_t)B_{t-1} + G_t - B_t$.

6. Asset markets clear, that is, total saving by households equals government bonds:

$$B^g = \int adD$$

7. Goods market clears when the final good is used for private and public consumption:

$$Y = \int cdD + G$$

3.7 Balanced-Budget vs. Deficit-Financed Government Spending

In this section, I contrast two fiscal financing regimes for a one-time, unexpected increase in government consumption G. The overall structure of the economy—households, firms, and monetary policy—remains identical across the two experiments. The difference lies in how the fiscal authority finances this increase in G and how it subsequently manages the path of public debt.

3.7.1 Balanced-Budget Regime

Under the *balanced-budget* regime, the government chooses period-by-period taxes τ_t so that it does not issue any net *new* debt in response to higher spending.[3]

The government's per-period budget constraint in real terms is:

$$\tau_t \; = \; (1 + r_t)\, B_{t-1} \; + \; G_t \; - \; B_t, \tag{8}$$

with $B_t = B_{t-1}$ in a pure balanced-budget setting, so that τ_t adjusts each period to fully absorb spending changes.

Steady State. If government spending G_{ss} and debt B_{ss} are constant, the required tax burden simply covers the steady-state interest on B_{ss} plus G_{ss} itself.

After the Shock. Following a one-time shock that raises G_t by 1% relative to its steady-state level, taxes jump up immediately (since the government refuses to incur a new deficit). Consumption, therefore, tends to fall more sharply on impact (relative to the deficit-financed case), and private agents internalize the immediate increase in their tax burden. Debt shows little or no change from its steady-state path.

3.7.2 Deficit-Financed Regime

Under the *deficit-financed* regime, the government initially *permits* a rise in public debt to absorb most (or all) of the extra spending. Over time, taxes gradually adjust to bring the debt ratio back toward its steady-state target.

[3] Equivalently, one can say that the government "keeps debt at its steady-state level" if the economy already has some positive B_{ss} in the baseline.

A convenient way to model this is to posit a fiscal rule for lump-sum taxes:

$$T_t \;=\; T_{ss} \;+\; \phi_T \left(B_{t-1} - B_{ss}\right) \;+\; r_{ss}\, B_{ss} \;+\; G_{ss}, \tag{9}$$

with $\phi_T > 0$ determining the *speed* at which taxes respond to deviations of debt from its steady-state level B_{ss}. The law of motion for debt is then

$$B_t \;=\; (1 + r_t)\, B_{t-1} \;+\; G_t \;-\; T_t. \tag{10}$$

If ϕ_T is sufficiently small (or zero), the government initially finances most of the increase in G by issuing debt rather than raising current taxes. Over time, taxes rise above T_{ss} to service and gradually retire the extra debt.

3.8 Fiscal experiment and transition

The fiscal experiment I analyze in section 5 is a one-time increase in government consumption, G. I assume this increase in government consumption is an "MIT shock", i.e. an unpredictable and never-again-to-occur departure from the steady-state equilibrium. The analysis will be on the transition back to the steady-state along a perfect-foresight path, under the assumption that no shock will ever occur again.

For the main results of the paper, the financing rules consists of *deficit financed*: for the government consumption increase consists of covering the increase in G through deficit financing, meaning that the fiscal shock is absorbed by increasing public debt. Under this deficit-financed policy, the government commits to gradually restoring the debt level by adjusting taxes over time. Lump-sum transfers are assumed to follow a fiscal rule as in equation 7. Furthermore, taxes are chosen by the government such that public debt fully captures the government spending: $dB_t = \rho_B(dB_{t-1} + dG_t)$.[4] ρ_B is the degree determining the level of deficit financing: if $\rho_B = 0$, the policy keeps a balanced budget, while for greater ρ_B, the policy leads to a greater deficit.

[4] I use the same parameter or persistence for the government debt as for the government expenditure.

3.9 Computational strategy and definition of fiscal multiplier

For solving the model transition I use the approach firstly developed by Auclert et al. (2021) creating a rapid computation of Sequence-Space Jacobians, taking the derivatives of perfect-foresight equilibrium mappings between aggregate sequences around the steady state. I write the equilibrium conditions as a system of linear equations in the space of perfect-foresight sequences, i.e. the sequence space. These Jacobians summarize every aspect of the model that is relevant for the general equilibrium. The algorithm takes all relevant Sequence-Space Jacobians, and then composes and inverts these matrices to obtain the model's full set of impulse responses. I generate an MIT shock for government spending, and by using the Sequence-Space Jacobians and guesses for the sequences of prices along the transition, I get the respective impulse response functions of the aggregate variables.

Given the IRFs I can then compute the fiscal multipliers. I define the impact multiplier generated by the model as:

$$\text{impact mult} = \frac{\Delta Y_0}{\Delta G_0}, \tag{11}$$

where ΔY_0 is the change in output from period 0 to period 1 and ΔG_0 is the change in government spending in the same time interval. The cumulative multiplier follows the same definition, but it is computed over a period of four horizons, i.e. the four periods after the shock.

4 Quantification

I calibrate the model to match the US economy with moments following the literature, in particular, Kaplan et al. (2018) and Auclert et al. (2024). I also report endogenously calibrated parameters, for the main calibrated benchmark economy. These remain fixed also when transitioning from one steady-state to another. All aggregate variables are in relation to GDP.

4.1 Preferences and Labor

I set the standard Frisch elasticity of labor supply to 1, similar to what is used in the literature. The disutility of work and the discount factor are among the parameters calibrated to match key moments in the data. The coefficient of risk aversion is set to be equal to 0.5 as in Bayer et al. (2019). As standard in the literature, the levels of β and ϕ are calibrated to hit a target for the level of government bonds in the economy. For the standard calibration, β is 0.972 and the disutility of labor is 1.69. These parameters are obtained to match the first level of debt/GDP ratio. When moving from one steady-state to another, I keep them fixed.

4.2 Government and Monetary Policy

I set government spending, G, to 16% of GDP and government bonds, as in Auclert et al. (2020). I use the value of ρ_G as in Nakamura and Steinsson (2014), *i.e.*, 0.9 at a quarterly frequency to calibrate the persistence of the government spending shocks. For monetary policy, I use the same parameters as in Auclert et al. (2020), that is, I set the response of the central bank to be equal to 1.5.

4.3 Other Parameters

The nominal rigidity of the New Keynesian Phillips Curve is set to be 0.1. Productivity and labor demand (when the economy is in full employment) are both set to 1. The factor prices are endogenously calibrated, for each calibration for the different levels of government debt. The markov chain points are 7, one for each income state. There are 500 points on the asset grids. The rest of the parameters are reported in appendix B.

5 Model results

In this section, I will first analyze the steady state of the baseline US-calibrated economy in a standard way. I will do this by making a comparative static analysis to compare the economy with low and high debt. Secondly, I will show the aggregate responses to an unexpected government consumption increase in the US-calibrated economy. In the second exercise, I let government consumption increase finance through a deficit-financed scheme. I then look at the relationship between the level of debt and the fiscal multiplier size, following the empirical evidence illustrated in section 2.

5.1 Baseline Steady-State and Comparative Statics

To understand the aggregate responses of the economy following a fiscal shock, it is essential first to examine how the economy behaves in each steady state. Each steady state represents a different equilibrium configuration of the economy, characterized by specific levels of government debt and associated macroeconomic variables. Suppose in steady-state the government follows a simple fiscal rule given by:

$$\tau = rB + G \tag{12}$$

In this framework, both B and G are treated as exogenous parameters. Government spending G is fixed and does not change across steady states, while the level of government debt B varies. In each steady state, the economy must satisfy the asset market clearing condition:

$$A(r) = B \tag{13}$$

where $A(r)$ represents the aggregate assets held by households, which depend on the real interest rate r. This condition ensures that the total assets supplied by households equal the government's demand for funds through debt issuance. As explained also in the parallel work of Campos et al. (2024), given market incompleteness, the stock of public debt determines how much households can self-insure against negative idiosyncratic shocks and, therefore, the interest rate at which the savings market clears. As B changes, the real interest rate r adjusts endogenously to maintain market equilibrium. Specifically, an increase in government debt B leads to a higher demand for funds, necessitating an adjustment in r to equilibrate the asset market.

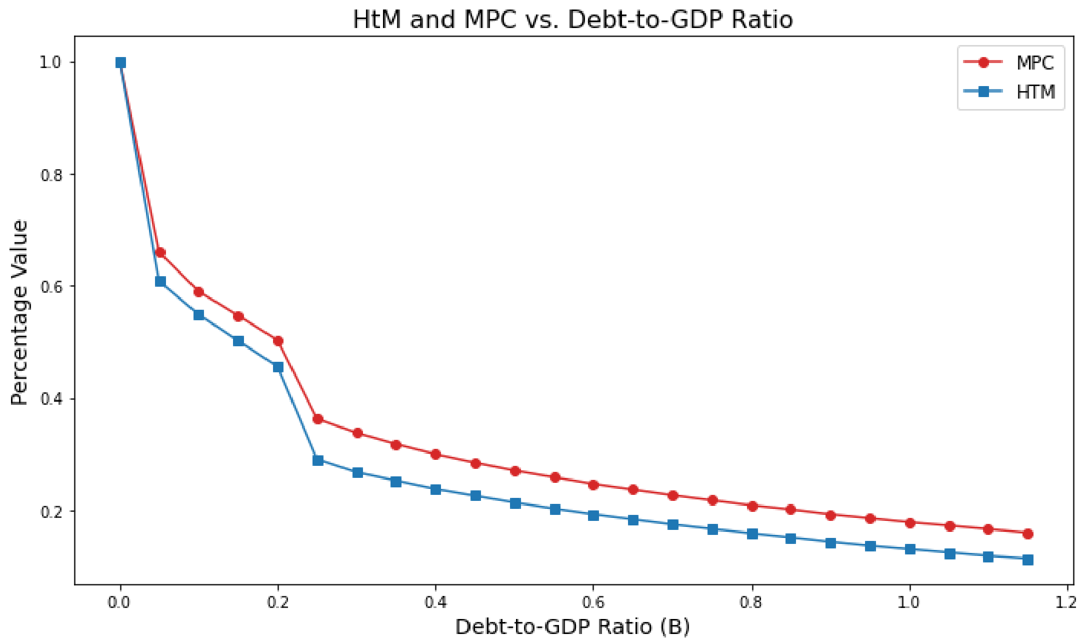

Figure 2: Changes of HtM households and aggregate MPC in the model.

5.2 Steady State Analysis

Figure 2 shows the steady states model behavior with different levels of government debt. With higher public debt, the aggregate marginal propensity consumption of households decreases. This is related to the fact that the share of constrained agents (hand-to-mouth) also declines. To reach different steady-state calibrated with different levels of public debt, I start from a benchmark level of government bond holdings held internally by households in the United States. To do this, as explained in section 4.1, I choose the discount factors to hit the target of debt/GDP. This is the benchmark level of the economy. From here, I increase the level of debt/GDP, without changing any other parameter: what will adjust will be only the interest rate. The preferences are kept fixed, across all the experiments reported.

Figures 3 and 4 illustrate how the level of aggregate marginal propensity of consumption goes down for a higher level of debt. More debt increases the level of assets in the economy, lowering the level of agents who are constrained. Figure 5 shows the different distribution of assets for the seven states of income in the economy. This shows the heterogeneous distribution of households, and how they differently behave in the states with low and high debt. These figures are relevant to understand

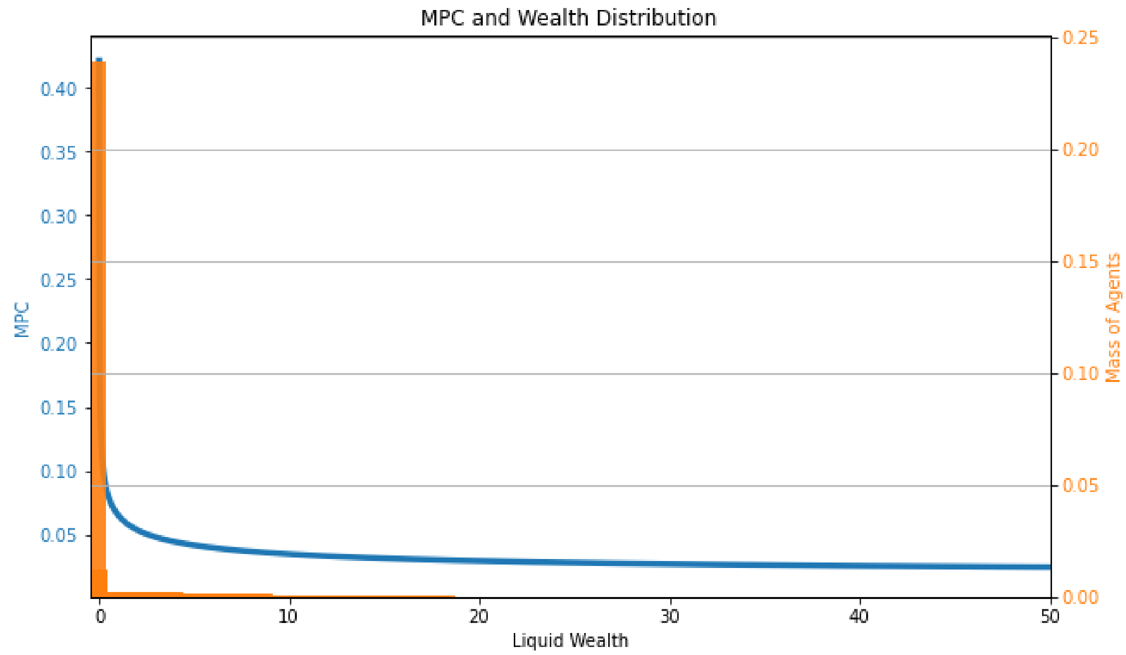

Figure 3: Distribution of assets and MPC for low debt SS.

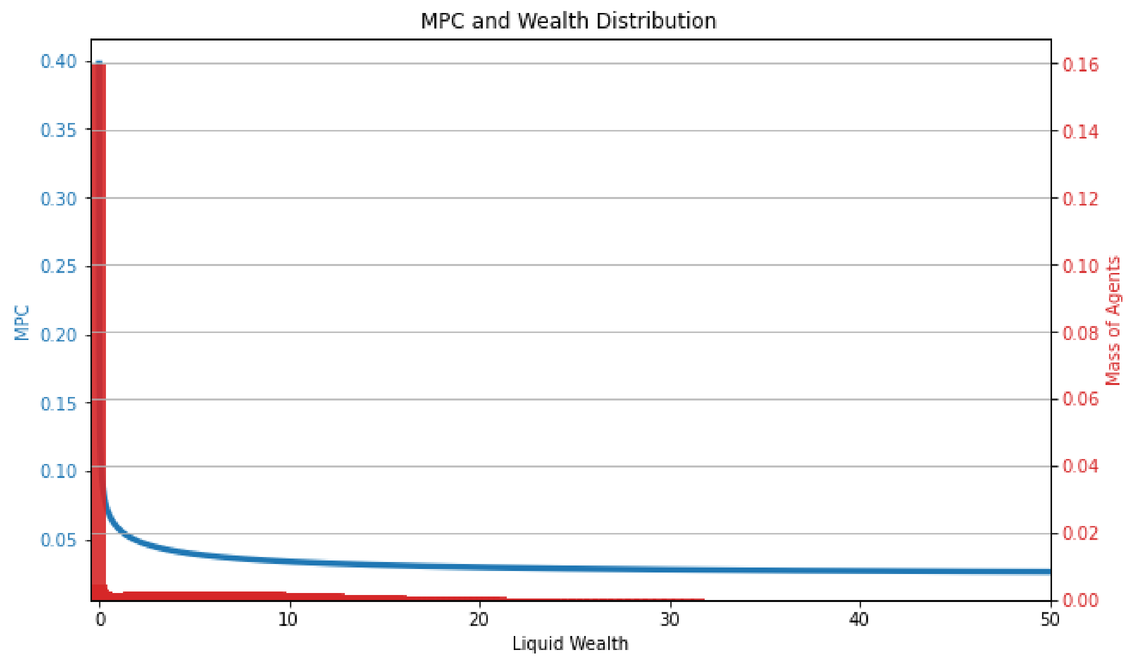

Figure 4: Distribution of assets and MPC for high debt SS.

the aggregate responses in the next section.[5]

[5]Note that no alternative calibration strategy can deliver high public debt and many constrained agents at the same time.

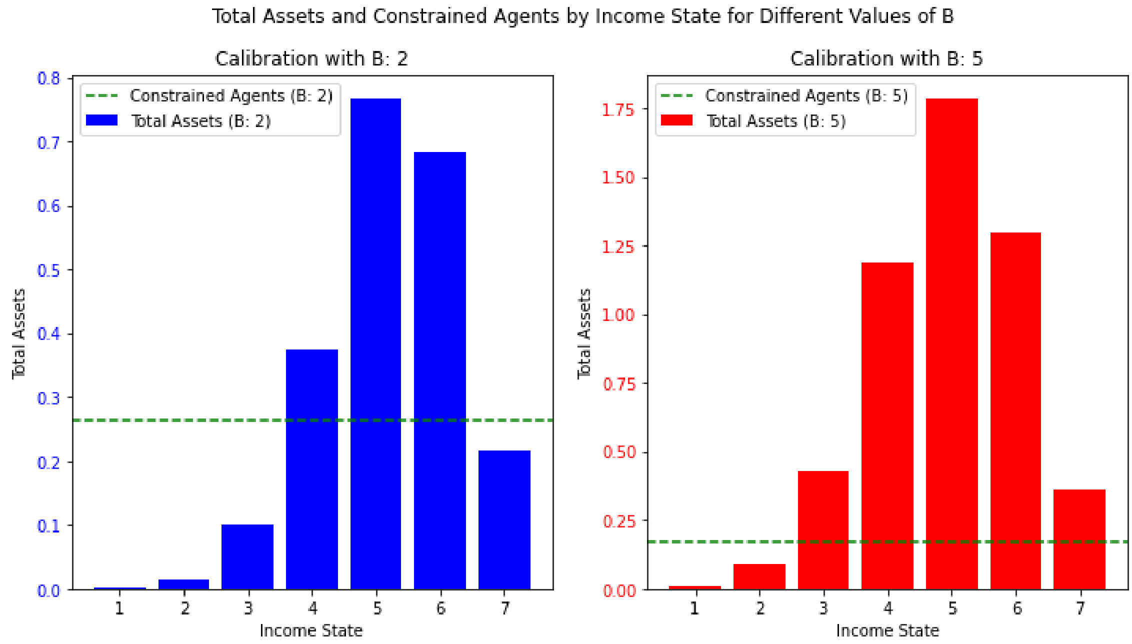

Figure 5: Income states and assets distribution of the 2 economies.

5.3 Aggregate responses

The aggregate responses to a 1% increase of the government consumption of the steady-state will allow to understand the reaction in the economy after a shock. The increase in public expenditure is fully financed by an increase in government debt in the same magnitude. This means that the public debt stock, B_t is allowed to increase during the transition. The figure 6 show how both the impact and cumulative multiplier decreases for higher level of bonds supply in the economy. An higher level of debt in the economy means, after an unexpected government shock, that the multipliers will be smaller. In terms of size, compared to the empirical evidence, the size of the multipliers fall is smaller. In particular there is a bigger decrease for a small amount of debt/GDP ratio, as the kink in the picture suggests.

Figure 7 illustrates the impulse response functions of different aggregate variables: public and private consumption, output, after-tax income, labor demand, and government debt response.

Under the fiscal experiment explained in section 3.8, the impact fiscal multiplier is around 0.81 when the level of public debt is 20 %, and it is 0.74 when the level of debt is 100 %. The cumulative multipliers in the first 4 quarters also change and are very close to the impact multipliers.

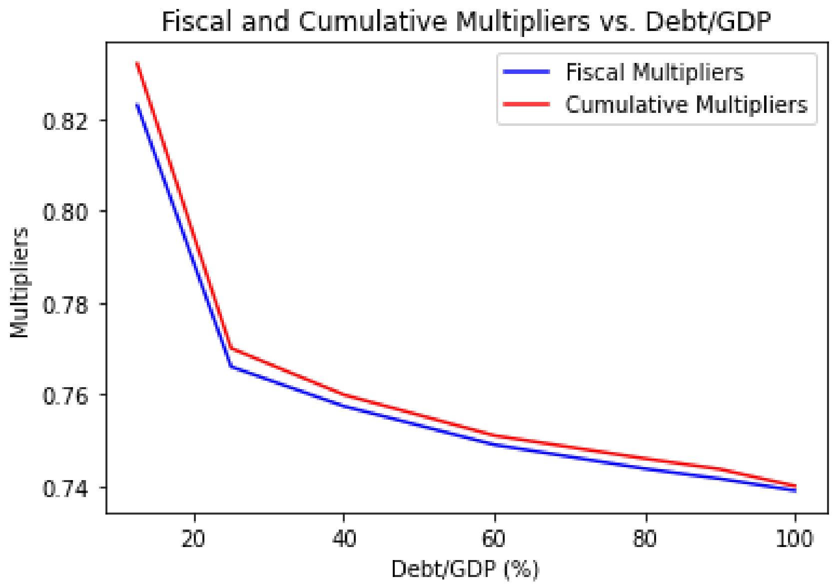

Figure 6: Plot of impact and fiscal multipliers for different levels of public debt in the HANK economy.

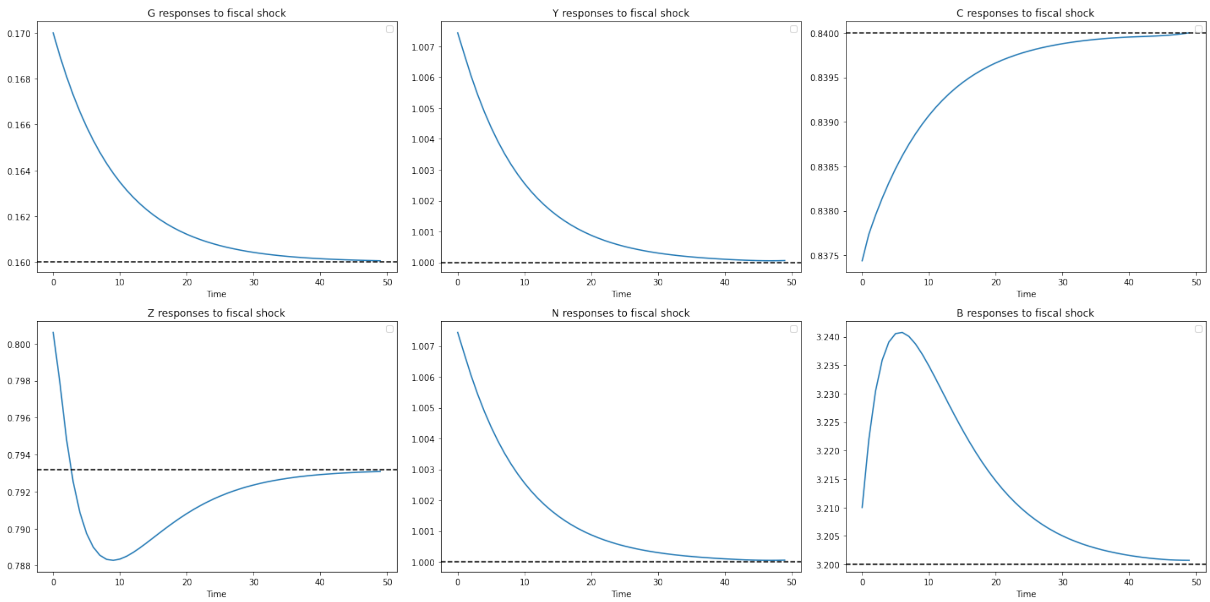

Figure 7: Impulse response functions for government expenditure, output, consumption, after-tax income, labor, and wage, for an economy with the 80% of debt/GDP ratio.

To understand the underlying mechanism it is crucial to focus on the marginal propensity to consume (MPC) of agents, as anticipated in the preceding sections. A higher level of debt translates to greater asset holdings and consequently, smaller MPCs among households. This diminished aggregate MPC leads to a lower aggregate demand response following the shock, resulting in a weaker employment response. The impulse response functions reveal that following an increase in government expenditure financed by public expenditure, output levels are higher in scenarios with lower debt burdens. After the shock, lower debt prompts agents to exhibit more pronounced adjustments in their labor response, thereby driving a more substantial change in aggregate labor demand. As a result, wage levels rise more sharply. While the interest rate and wage initially rise post-shock, the adjustment is more pronounced in contexts with lower debt burdens, showing a stronger negative impact. This dynamic ultimately influences consumption behavior, with agents exhibiting a greater propensity to consume in instances of lower debt, amplifying the output response. These results show why the impact and cumulative fiscal multiplier are diminished in scenarios characterized by higher debt. Conversely, agents exhibit less reactive behavior in situations of heightened debt, as they rely on government debt as a buffer against aggregate shocks. Given their enhanced liquidity and reduced constraints, their behavioral adjustments are comparatively subdued.

5.3.1 Impulse Responses in the Two Regimes - Comparison

Figure 8 compares the impulse responses under the two regimes. In the *deficit-financed* case, government debt B_t rises sharply on impact, then gradually returns. Taxes adjust *slowly*, leaving consumption higher initially but inducing a more persistent decline as taxes eventually rise. By contrast, in the *balanced-budget* scenario, there is no large change in B_t, and taxes τ_t jump immediately to fund G_t. Consumption thus drops more on impact, though the subsequent drag from increased taxes is shorter lived. *Quantitatively*, the deficit-financed shock produces a larger stimulus to aggregate demand and output in the short run, at the cost of higher future taxes and greater debt dynamics.

5.3.2 A Classic Mechanism

Figure 9 shows a classic explanation of the mechanism from the literature on fiscal multipliers. Impact multipliers are computed in the model for different levels of Frisch, the parameter that determines the elasticity of labor supply. When the labor supply is fully elastic, the multipliers decrease by more, and there is a more accentuated slope in the diminishing behavior of the multipliers. The red line shows the same impact multipliers computed in the main exercise, as in figure 6. When the

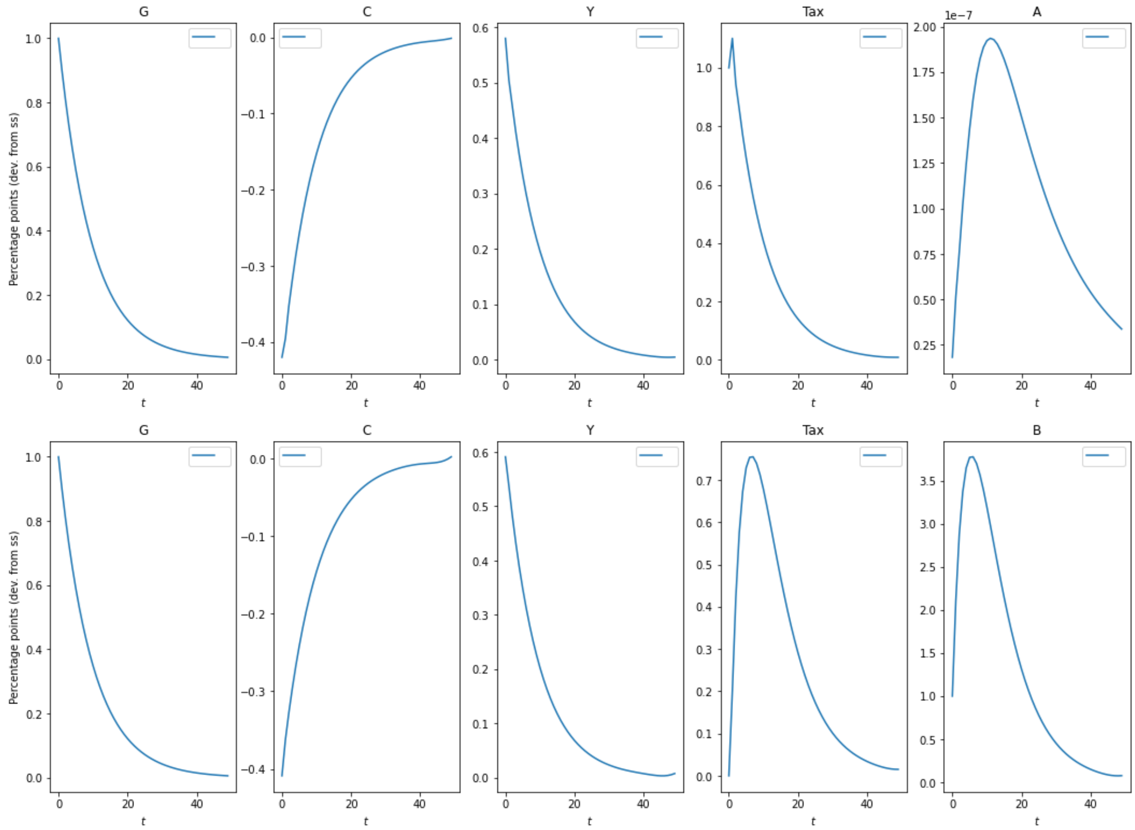

Figure 8: Impulse response functions for government expenditure, consumption, after-tax income, taxes, and assets/government bonds for an economy with the 40% of debt/GDP ratio.

labor supply is inelastic, the multipliers do not change.

This confirms a channel already present in the literature, showing how labor supply elasticity matters for fiscal multipliers. However, from this analysis, it is still not possible to distinguish the main channel driving the results. I will study this in more detail in the next section.

6 Theoretical Results

6.1 The IKC: Review and Applications

In the previous sections, I presented the results from the HANK quantitative model. In this section I will reinterpret the results in the spirit of Auclert et al. (2024). The authors show how relevant are iMPCs in deficit-financed fiscal policies. In the intertemporal Keynesian cross, the matrix **M** (iMPCs) is a sufficient statistic for the output response to fiscal policy.

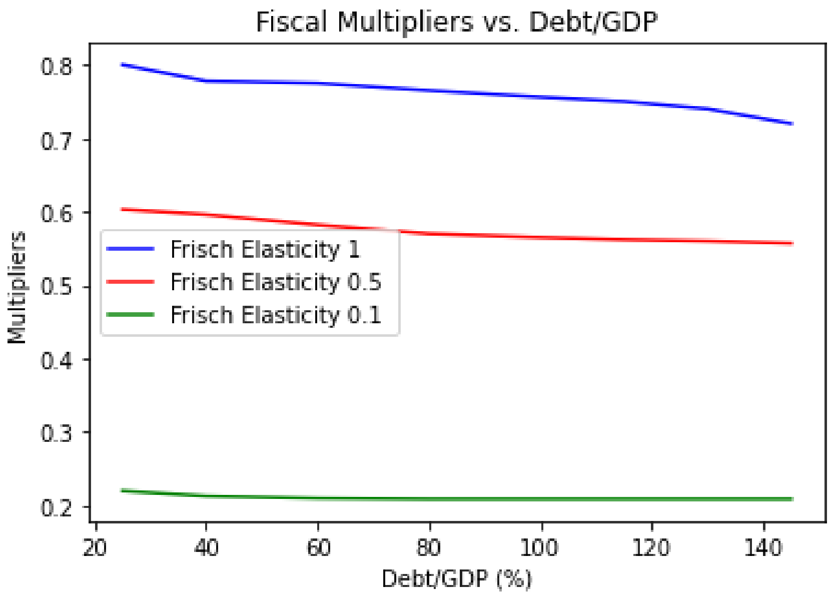

Figure 9: Sensitivity of the fiscal multipliers to the elasticity of the labor supply.

Proposition 1. [6] *(The Intertemporal Keyensian Cross). Let* $\boldsymbol{K} \equiv -\sum_{t=1}^{\infty}(1+r)^{-t}\boldsymbol{F}^{t}$*. The solution of the IKC problem is given by:*

$$d\boldsymbol{Y} = \mathcal{M} \cdot (d\boldsymbol{G} - \boldsymbol{M}d\boldsymbol{T}). \tag{14}$$

where the multiplier $\mathcal{M}$ *is the bounded linear operator defined by* $\mathcal{M} \equiv (\boldsymbol{K}(\boldsymbol{I}-\boldsymbol{M}))^{-1}\boldsymbol{K}$*.*

Proposition 1 highlights how the output response is driven by the interaction between iMPCs, determining $\mathcal{M}\cdot\mathbf{M}$ and primary deficits $d\mathbf{G} - d\mathbf{T}$.[7] This means that the assets agents hold in the economy affect the intertemporal Keynesian matrix, which becomes relevant theoretically in determining the results presented from the quantification of the model. There exists a solution if and only if $\mathbf{K}(\mathbf{I}-\mathbf{M})$ is invertible, and $\mathbf{F}$ is the lead operator that maps $x_0, x_1, ...$ to $x_0, x_1, ...$, corresponding to a matrix with ones directly above the diagonal.

Proposition 2. *(Deficit-financed fiscal policies). Assume a unique equilibrium. For a deficit-financed policy, the output response to a fiscal policy shock* $\{d\boldsymbol{G}, d\boldsymbol{T}\}$ *is the sum of the govern-*

[6]Check appendix D for the numerical resolution.

[7]Note the household side is affected because of the structure of the fiscal rule: taxed are computed endogenously and the interest rate affects it directly. It would not be necessarily the same with a different fiscal rule in steady-state: $G_t = T - r_{t-1}B$, or $G_t = G - \phi_G(B_{t-1} - B)$.

ment spending policy dG and the effect on consumption dC,

$$d\boldsymbol{Y} = dG + \mathcal{M} \cdot \mathbf{M} \cdot (d\boldsymbol{G} - d\boldsymbol{T}). \tag{15}$$

Since the consumption response depends on the path of $\{dG, dT\}$, with more government bonds in the economy, the higher level of tax decreases deficits, leading to a lower response of output.

The empirical evidence on the impact of government bonds on fiscal policy transmission follows a deficit-financed policy. The model replicates the same type of policy. Theoretically, this means that the increase in government bonds positively affects the response to output, and the main channel from the quantification of the model is found in the lenses of the intertemporal Keynesian cross. This is also relevant for the following proposition 3. The intuition for this result is that iMPCs are an additional source of feedback from output back into consumption. For this reason, deficit-financed spending increases income without an immediate offset increase in taxes. Households spend this income both today and in the future, leading to an output boom in the future that triggers its intertemporal consumption feedback. The result from Auclert et al. (2024) shows a more persistent output effect, with additional amplification leading to a larger cumulative multiplier.

Proposition 3. [8] *(The role of taxes). Consider a bounded shock $\{d\boldsymbol{G}, d\boldsymbol{T}\}$ satisfying, $\sum_{t=0}^{\infty} \frac{dG_t}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{dT_t}{(1+r)^t}$. Then, any impulse response of output, $d\boldsymbol{Y}$, must satisfy:*

$$d\boldsymbol{Y} = dG - \mathbf{M} \cdot d\boldsymbol{T} + \mathbf{M} \cdot d\boldsymbol{Y}. \tag{16}$$

Higher taxes due to higher government bonds in the economy lead to lower output. The lower output level's size is determined by the iMPC matrix $\boldsymbol{M}$.

Proposition 3 shows the relevance of taxes for the household decision. Since the level of government bonds affects taxes, and taxes enter directly into the household budget constraint, it is possible to assess their impact on the output level, through iMPCs.

The propositions highlighted how relevant MPCs are for fiscal multipliers. Having iMPCs with heterogenous agent settings allows to study changes in wealth distribu-

[8] Check appendix **??** for the formalization.

tion. To study the relationship between government debt and state-dependent fiscal multipliers, the objective of the next section is to decompose the changes in MPCs to those that come out of changes in wealth distribution and changes in the policy function. If the former is not present in a representative agent model, it is present in a heterogeneous agent model.

Does the decrease in output after a fiscal shock depend on the wealth of agents, or on their consumption-savings decision?

6.2 Distribution or factor prices?

From a theoretical point of view, it is relevant to have a new-Keynesian setting featuring heterogeneous agents to study the implications of fiscal policy. Auclert et al. (2024) show how HANK matters for fiscal policy, and how this is different from RANK.[9] One primitive unsolved question is whether this difference in the fiscal multipliers depends on the distribution of assets or factor prices. Theoretically, these might be two ways to explain the heterogeneous state-dependence of fiscal multipliers, leading to the impulse response function response seen in the previous section. These two channels have the potential to explain the different state-dependent fiscal multipliers. Is it about the fact that literally people are holding a larger amount of assets? This is the insurance channel (distribution channel). Or is it the factor price channel (or bond price channel), which is the fact households are changing their behavior because real interest rates are different? To study which channel prevails to explain the mechanism, I propose a decomposition to first order: what would happen to the economy with low interest rates, but with the high-debt (steady-state) distribution of assets? To do this, I feed in different initial distributions of assets, to identify which channel is the most important. Moving from a stationary point to the other one, where there is more debt, there is more liquidity in a steady state. People prefer higher interest rates; the level of debt affects the level of the real interest rates and consequently affects household consumption policy function, but also the stationary distribution of households. Potentially there might be two different channels. One depends on the stationary distribution, which changes: the government issues more debt, and there

[9] This is not true for monetary policy. Kaplan et al. (2018) show how for monetary policy in the aggregate, a RANK captures the same aggregate effects of a HANK model.

are fewer hand-to-mouth agents in the economy because they are more insured by the additional liquidity they hold. The other one depends on the bond price channel: issuing more debt, interest rates increase, making consumption more costly. After a fiscal shock, the response will be fully captured by the labor supply response of agents that will be moving and generate different responses in output. In a RANK economy, the change is fully captured by changes in the policy functions, but in HANK, changes in the wealth distribution are also expected. So given this double potential channel, the question is what is generating this response in this HANK economy?

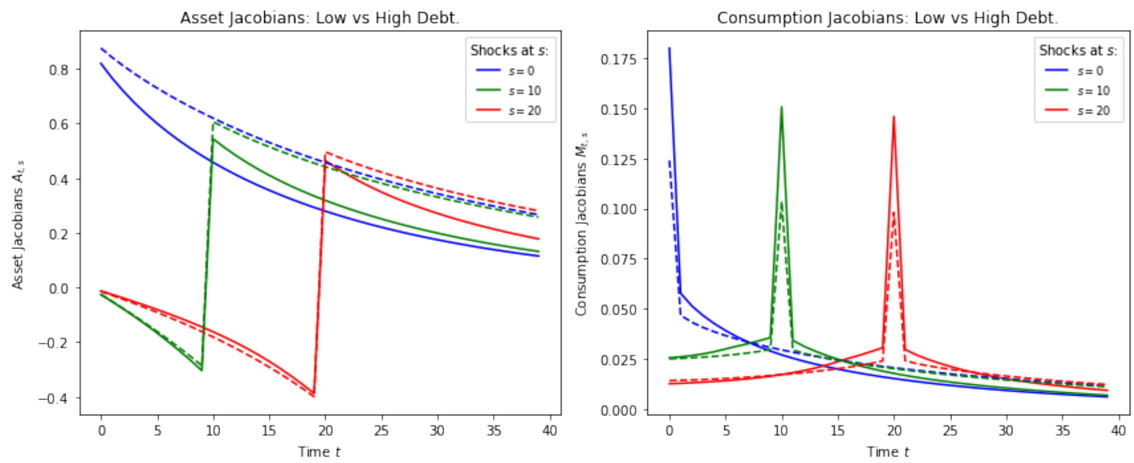

Figure 10: Assets and consumption Jacobians for income, of the two economies with low and high debt (solid line economy with low-debt, dashed line for the high-debt economy).

Figure 10 shows the different Jacobians of agents for consumption and asset holdings, for income. For given t and s[10], they capture the response of assets and consumption at date t to aggregate income shock at date s. The plot shows the income shock happening in periods 0, 10, and 20. The shape of the Jacobians is the standard of a one-account HANK model: agents dissave to anticipate the shock and accumulate afterward. From the plot, it is clear to see how the economy with high debt and low debt feature different Jacobians.

The decomposition consists of feeding different rates for different levels of public debt. To do this, after obtaining the policy functions and the distribution of each steady states I use them to decompose to first order the changes in MPCs. The results are that MPCs are mostly affected by changes in the real rate, rather than by changes in the wealth distribution. The decomposition shows that what matters for changes in

[10] The calibration is executed with quarterly data.

iMPCs are changes in the real interest rates: the changes in MPC come from changes in interest rates, and depend less on the changes on the level of liquidity in the economy.

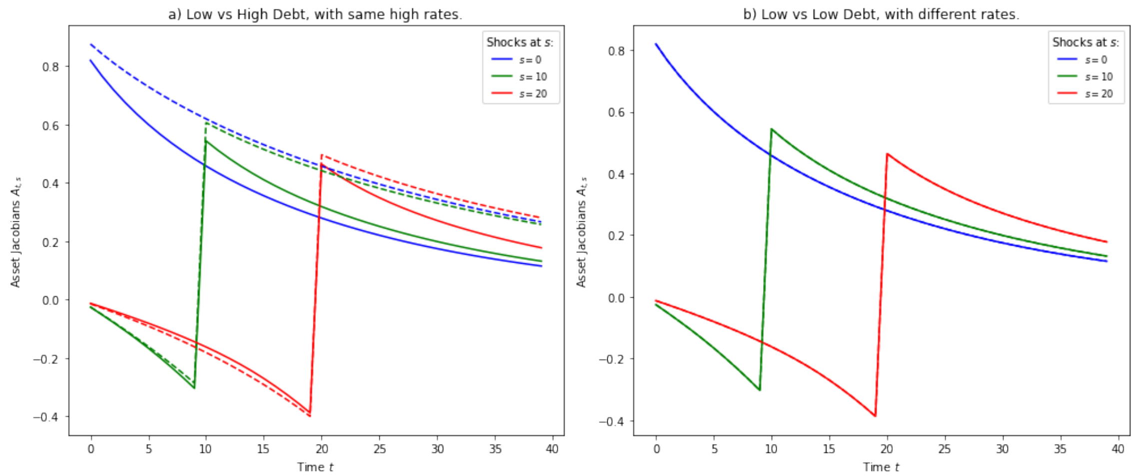

Figure 11: Asset Jacobians for income: on the left, two economies with low and high debt (solid line economy with low-debt, dashed line for the high-debt economy) and same rates. On the right, the same debt (benchmark level) for both economies, but different rates.

Proposition 4. *Changes in factor prices, affect individual MPC, leading to substantial changes in the fiscal multiplier, whereas changes in the wealth distribution* $\Psi(a,e)$ *have a limited impact on the aggregate MPC and fiscal multiplier.*

Considering the change in the level of government bonds B, the total variation of the aggregate MPC can be decomposed into two parts:

$$\frac{d}{dB}(\text{MPC}) = \underbrace{\int_{\mathcal{A}\times\mathcal{E}} \frac{\partial \text{MPC}(a,e)}{\partial B}\, d\Psi(a,e)}_{\text{Factor Price Channel}} + \underbrace{\int_{\mathcal{A}\times\mathcal{E}} \text{MPC}(a,e)\, \frac{\partial \Psi(a,e)}{\partial B}}_{\text{Insurance Channel}}$$

The changes in B affect individual MPCs through factor prices, while the wealth distribution effect at first order is negligible.

Proposition 1 shows how Jacobians and the iMPCs matrix are directly related to state-dependent fiscal multipliers. From proposition 4 it is possible to conclude that the factor price channel leads to lower state-dependent multipliers: households react to the change in the interest rates, rather than the change in wealth.

The consumption Jacobians reflect how households' consumption responds over time to an income impulse. By re-weighting the distribution or the policy function, I isolate how much of the consumption-income sensitivity is from bond-price changes contrary to the wealth distribution changes. Figure 12, shows the consumption–income Jacobians at three shock horizons (0, 10, 20 quarters). The left panel demonstrates that simply re-weighting the new (high-debt) distribution on the old (low-debt) policy function produces almost no shift in consumption responses relative to the baseline. By contrast, the right panel shows that changing the policy function-i.e., adopting the factor prices from the high-debt scenario–significantly alters consumption responses. Hence, I conclude that *factor-price changes*, not purely shifts in $\Psi(a, e)$, are what primarily drive the changes in the MPC and thus the fiscal multiplier.

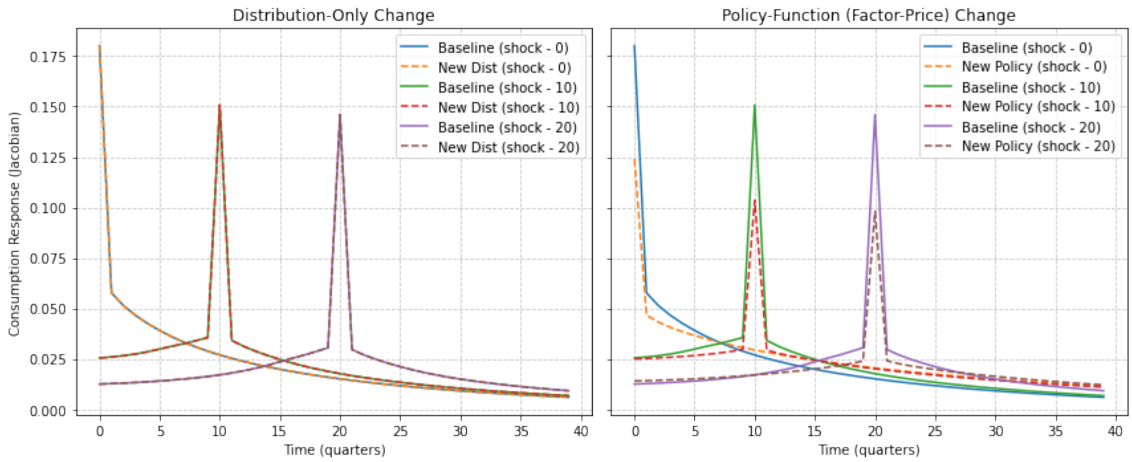

Figure 12: Consumption Jacobians for income: on the left, two economies with low and high debt (solid line economy with low-debt, dashed line for the high-debt economy) and same rates. On the right, the same debt (benchmark level) for both economies, but different rates.

6.3 Taxes or Government Bonds Prices?

The main result of this paper is that higher government debt tends to push up the real interest rate (r), which lowers the fiscal multiplier by diminishing households' marginal propensity to consume. Intuitively, when r is higher, households find saving more attractive than spending each additional unit of income, so their MPC declines. Lower MPC weakens the transmission of any given fiscal stimulus, translating into a smaller multiplier. In economies with relatively large public debt, interest rates exhibit stronger upward pressure in response to additional government borrowing, reducing households' willingness to consume. The result is that fiscal expansions in such economies produce smaller effects on output—hence, a lower multiplier. However in

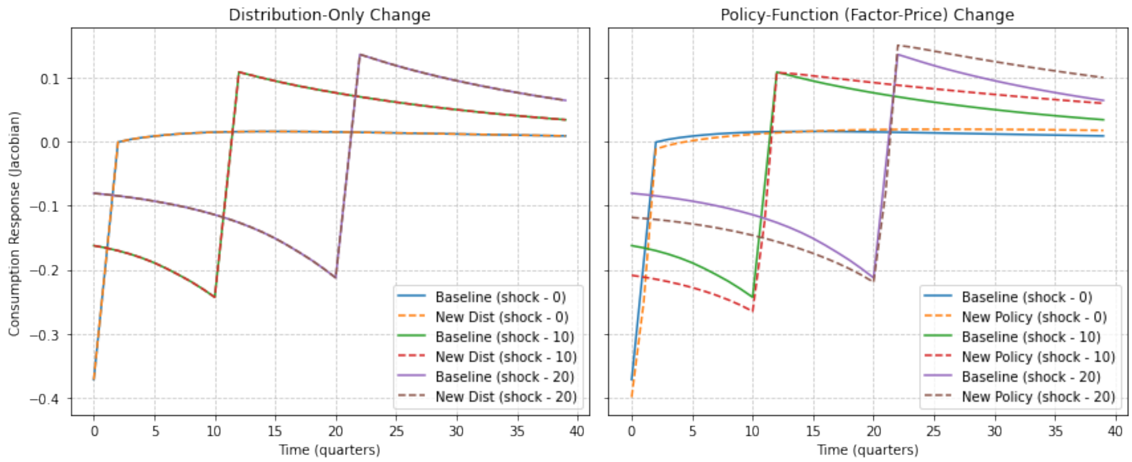

Figure 13: Consumption Jacobians for the interest rate: on the left, two economies with low and high debt (solid line economy with low-debt, dashed line for the high-debt economy) and same rates. On the right, the same debt (benchmark level) for both economies, but different rates.

the exercise above, in steady-state taxes also increase after the increase in government bonds. Someone might argue that an increase in taxes might lead to a decrease in the consumption choice of households. Starting the analysis from the poorest households (lowest level of the income grid) the increase in r raises returns on saving, shifting consumption for asset holders (especially at higher levels of assets a) as in figure 15. At the same time, the increase in T lowers disposable income, shifting consumption down for each asset level, but quantitatively this effect is minimal compared to the increase in r. If we also check households at a higher level of the income grid, the higher r yields to a more noticeable consumption shift at higher a, as these households significantly benefit from higher returns, while the higher T, lowers consumption for each level of asset, but marginally. Overall, at a higher income level, households hold more assets and thus the interest-rate shift strongly affects their consumption plan, i.e. the bond-price changes dominate over tax changes. Raising r significantly changes the consumption policy function, especially for asset-rich, higher-productivity households. On the other hand, raising T does affect consumption shifting it downward by reducing net labor income, but is often less impactful than the interest-rate channel in lowering MPCs. These findings parallel the same previous message: with higher B in general equilibrium, the rising real rate is the primary driver that reduces fiscal multipliers by lowering MPC.

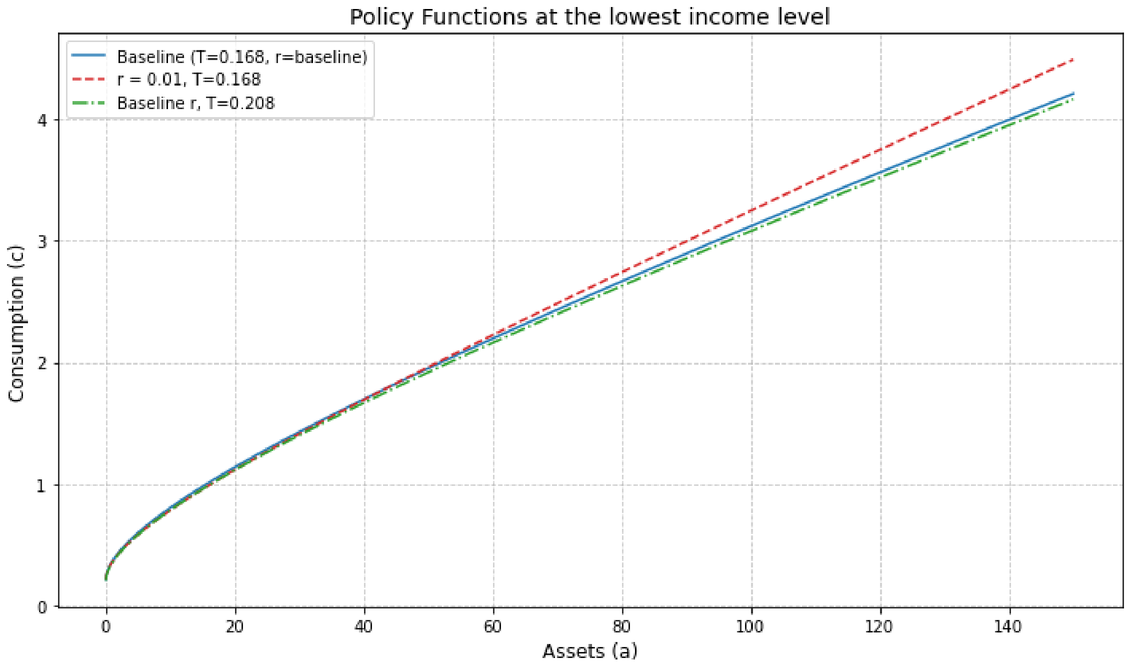

Figure 14: Policy functions for a change in r and T from the baseline level, for the agent with the lowest level of income

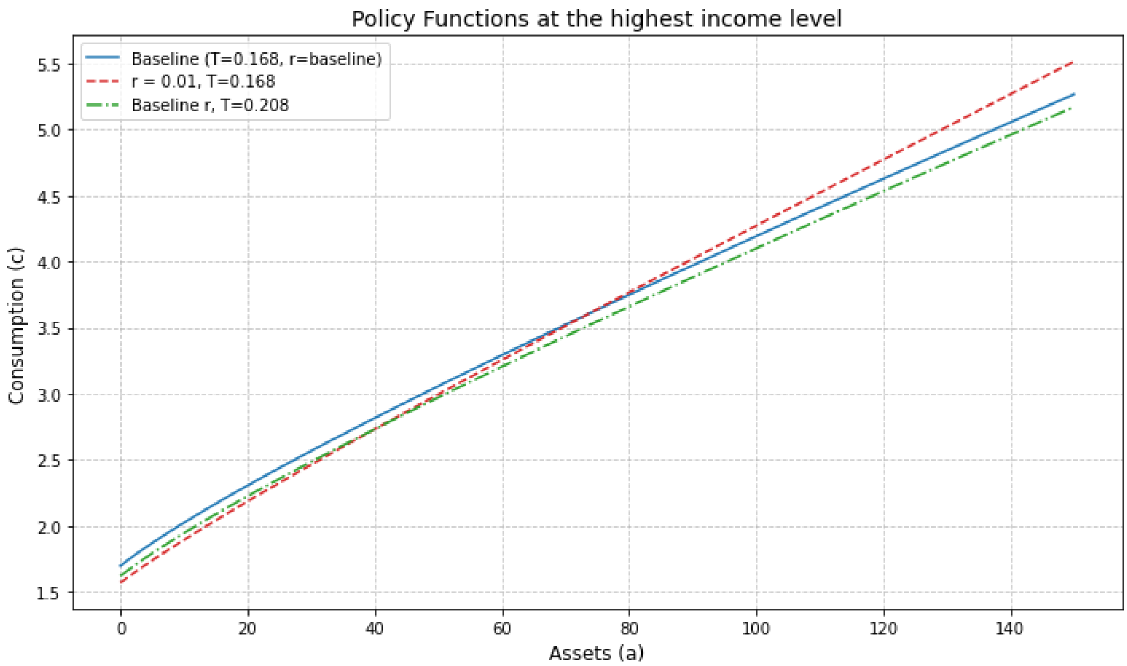

Figure 15: Policy functions for a change in r and T from the baseline level, for the agent with the highest level of income

6.4 Monetary - Fiscal Interaction: decreasing interest rates the central bank can help to maintain active the fiscal stimulus.

From the decomposition illustrated above, there is another straightforward result, that might need more (empirical) research in the future. The economy with higher debt suffers more from a potential increase in interest rates by the central bank. The decrease in multipliers is quantitatively stronger than the economy with lower debt. From this it is possible to conclude that higher rates are more recessive for fiscal policy for the economy with high debt. To study this, I allow the role of the central bank to become active. Let's suppose that in a state with high level of government debt, the central bank decreases the level of interest rates. Through this the central bank is able to satisfy two objectives at the same time: to help the government to sustain the high level of debt, and to improve the response of households to the fiscal stimulus. In fact, according to the previous main result, with a lower level of interest rate, for the same level of public debt, the response to the fiscal stimulus will be higher. Since what matters is the level of the real rate, when the rate decreases the response of households will go back to the one the one of the low - debt economy. This creates a classic monetary - fiscal policy interaction with its following implications: in times with high debt, decreasing the interest rate helps to maintain the fiscal stimulus active, while if the central bank needs to increase the rates, this leads to the total burden of high public debt, reducing the response of household to potential increase in income, or higher transfers. Higher rates are more recessive for fiscal policy for the economy with high debt.

6.5 Aggregate or individual MPC?

Proposition 1 shows there is a sufficient statistics that can express the fiscal multiplier as a function of iMPCs. Since the fiscal multiplier depends also on the level of debt, and this is interconnected with the iMPCs, one natural question is if this change is due to the fact the MPC of one individual is changing, or to the fact that the distribution is changing. The result is mainly driven by the change in individual MPC: with more debt, the aggregate distribution changes, leading to a different (lower) state-dependent fiscal multiplier.

Proposition 2 show how relevant are iMPCs in deficit-financed fiscal policies: there is a sufficient statistics which can express fiscal multipliers as its function. In the intertemporal Keynesian cross, the matrix iMPCs **M** is a sufficient statistic for the output response to fiscal policy. Proposition 2 and 3 show how iMPCs depend on the level of government debt, for a deficit-financed fiscal policy. The fact that iMPCs change, depends on the aggregate distribution. This leads to the following proposition:

Proposition 5. *(Aggregate Distribution). The response of output after a fiscal shock in government expenditure is a function of MPCs:*

$$d\boldsymbol{Y} = \hat{\mathcal{M}} \cdot (d\boldsymbol{G} - \boldsymbol{M} d\boldsymbol{T}) + G. \tag{17}$$

where the multiplier $\hat{\mathcal{M}}$ is the aggregate iMPCs across individuals.

This result follows directly from the decomposition. The factor price channel, due to the policy function change, dominates: the individual MPC of one agent is relevant in explaining the different responses in output. When we aggregate, the MPC of a single household stays the same, but the distribution over which the aggregation happens, changes. Higher government debt changes individuals' MPCs.

7 Conclusion

In conclusion, this paper provides a comprehensive analysis of the relationship between public debt, fiscal multipliers, and the channel relating the two, drawing on empirical evidence and insights from a theoretical model.

In this study, I present a Heterogeneous Agent New Keynesian (HANK) model with sticky wages and flexible prices, which I quantify to analyze the response of the economy to a fiscal policy shock for economies with different levels of government debt. The model allows for a decomposition of changes in the intertemporal marginal propensity to consume (iMPC) into effects driven by wealth distribution and policy functions.

The findings highlight that fiscal policy is less effective when governments issues more government bonds to households. Among the channels driving the state-dependent fiscal multiplier, changes in wealth distribution emerge as the dominant factor. Moreover, the HANK model underscores the importance of the bond price channel over the insurance channel, following intertemporal marginal propensity to consume.

The paper is firstly motivated by an empirical study where I document a negative correlation between higher government debt and the effectiveness of discretionary fiscal policy: in states with elevated debt, fiscal multipliers tend to be smaller. A properly calibrated HANK model, that can capture MPCs across agents, shows the underlying mechanisms driving the empirical patterns. By incorporating agents' marginal propensities to consume and matching a proper wealth distribution, I demonstrate how higher public debt levels lead to lower aggregate MPCs and diminished labor supply responses following fiscal shocks. This, in turn, dampens the overall impact of fiscal policy on output, contributing to the observed lower fiscal multipliers in highly indebted economies.

From a theoretical perspective, I show that intertemporal marginal propensities to consume act as a sufficient statistic for how public debt affects fiscal multipliers. Higher government debt makes people wealthier, leading to a different distribution of assets. However what really determines changes in MPCs is the individual change in MPC that affects the heterogeneous response of agents to the new aggregate distribution because of their change in wealth.

Additionally, the key factor in explaining the different output responses after a shock is that the MPC of an individual household changes while the distribution over which this aggregation occurs shifts. In particular, the factor price channel is the main driving the results. The individual household choice is relevant to understand how discretionary fiscal policy affects systematic fiscal policy.

These findings illuminate two broader policy questions. First, is the reduced potency of fiscal policy in high-debt economies necessarily undesirable, or does it reflect improved consumption-smoothing and a decline in overall constraints? Second, even when higher debt constrains policy efficacy, are there still vital macroeconomic roles for fiscal intervention in such contexts? Future research can explore how these mechanisms evolve in international contexts or under alternative policy regimes, and whether similar patterns hold for other asset classes beyond government bonds.

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A Empirical Evidence

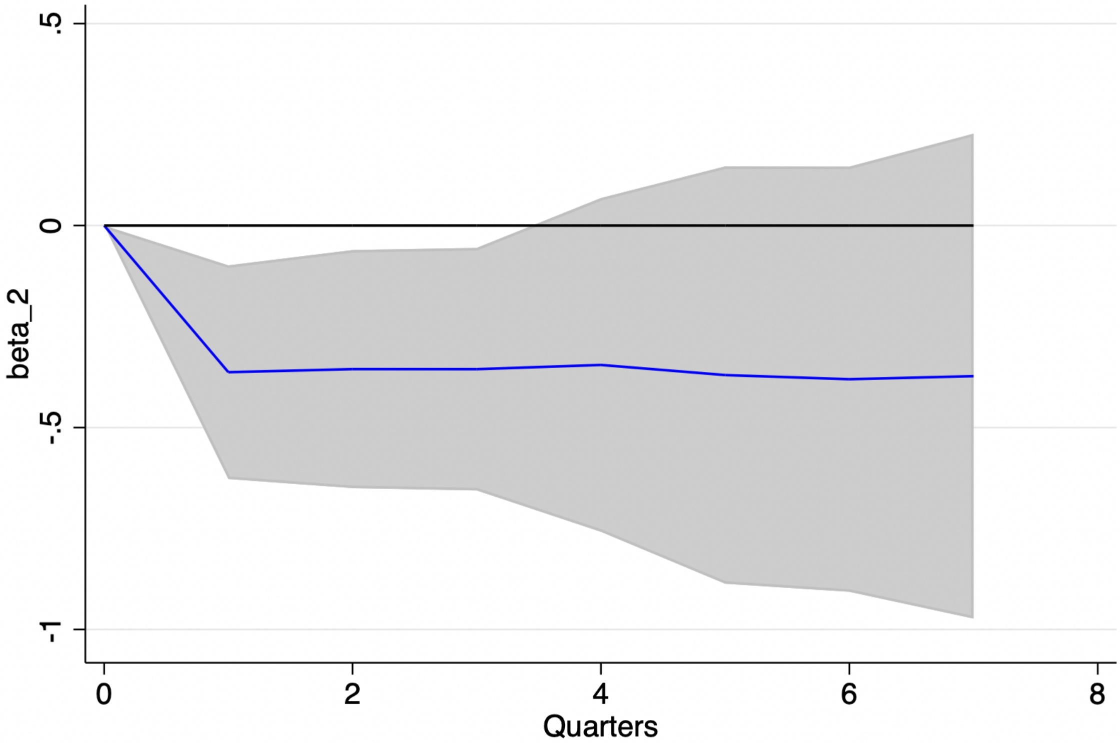

Figure 16: β_2 impulse response function over 8 horizons, with 90 % confidence intervals.

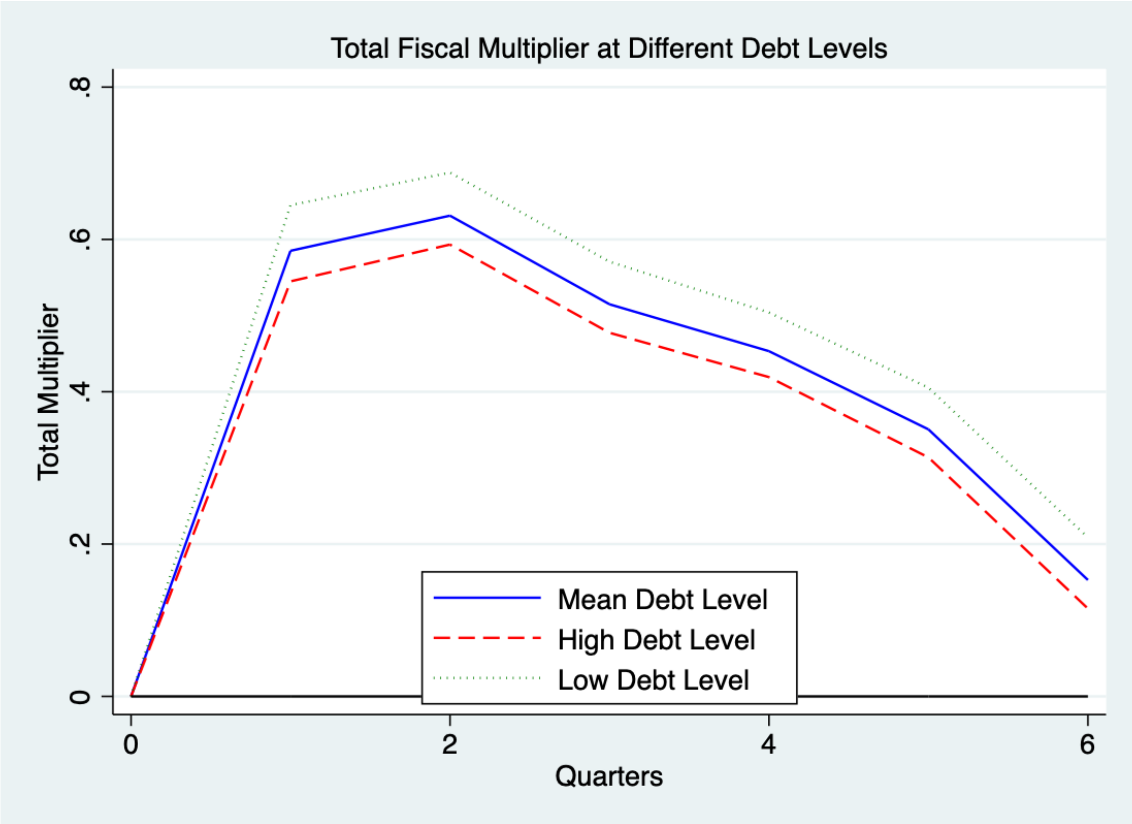

Figure 17: $\beta_1 + \beta_2$ impulse response function over 6 quarters.

B Full model parameters Calibration Description

B.1 Calibration Explanation

For the calibration exercises, to have two different steady-state with different levels of debt, I set the level of government bonds for the steady-state with low-debt level to 40% of GDP, and I set the one for the high-debt steady state to 80%. This value matches the value of internal public debt held by private investors in the US.

Parameter	Description	Value
Household		
β_1	Discount factor 1	0.972
ϕ	Disutility of labor	1.7
Inverse IES	0.5	
$\underline{b}$	Borrowing constraint	0.0
ρ_ε	Autocorrelation of earnings	0.95
σ_ε	Cross-sectional std of log earnings	0.50
Government		
θ_0	Income tax level	0.788
θ_1	Income tax progressivity	0.137
G	Government spending	0.16
B^g	Bond supply	1.6
ϕ_π	Taylor rule coefficient on inflation	1.25
Grid Parameters		
n_ε	Points in Markov chain for ε	7
n_A	Points on liquid asset grid	100
B	Liquid assets	1.6

C New-Keynesian Wage Phillips Curve derivation

The derivation of the wage new-Keynesian Phillips curve follows Auclert et al. (2024) which follows Erceg et al. (2000) but in a heterogeneous agents model. At any time t, the union k sets its wage W_{kt} to maximize, on behalf of all the workers it employs:

$$\max_{W_{kt}} \sum_{s=o}^{\infty} \left[\frac{c_t^{(1-\sigma)}}{1-\sigma} - \phi \frac{n^{(1+\eta)}}{1+\eta} - \frac{\psi}{2} \left(\frac{W_{kt+\tau}}{W_{kt+\tau-1}} \right)^2 \right] \text{s.to} \quad N_{kt} = \left(\frac{W_{kt}}{W_t}^{-\epsilon} \right) N_t \tag{18}$$

Unions take as given the initial distribution of households over idiosyncratic states. The price index for the aggregate employment services:

$$W_t = \left(W_{kt}^{1-\epsilon} dk \right)^{\frac{1}{1-\epsilon}} \tag{19}$$

From 18 it follows households real earnings are:

$$z_{it} = \int_0^1 \frac{W_k t}{P_t} N_{kt} dk = \frac{1}{P_t} \int_0^1 W_{kt} \left(\frac{W_{kt}}{W_t} \right)^{-\epsilon} N_t dk_t \tag{20}$$

The derivative of the household i total hours worked N are given by:

$$\frac{\delta n_{it}}{\delta W_{kt}} = -\epsilon \frac{N_{kt}}{W_{kt}} \tag{21}$$

Using the 20 and 21, the first order condition of the union with respect to W_{kt} is:

$$c_t^{-\sigma}(1-\epsilon)\frac{1}{P_t}N_{kt} + \epsilon\phi N_t^v \frac{N_{kt}}{W_{kt}} - \phi\frac{1}{W_{kt-1}}\left(\frac{W_{kt}}{W_{kt-1-1}}\right) + \beta\phi\frac{W_{kt+1}}{W_k t^2}\left(\frac{W_{kt+1}}{W_{kt}} - 1\right) = 0 \tag{22}$$

In equilibrium, all unions set the same wage, so $W_{kt} = W_t$ and $N_{kt} = N_t$. Moreover, wage inflation is defined as $\pi_t^w = \frac{W_t}{W_{t-1}} - 1$, so it follows:

$$c_t^{-\sigma}(1-\epsilon)\frac{1}{P_t}N_t + \epsilon\phi N_t^v \frac{N_t}{W_t} - \beta\phi(\pi_t^w + 1) + \beta\phi\pi_t(\pi_{t+1}^w) = 0 \tag{23}$$

Rearranging we get the wage New-Keynesian Phillips Curve:

$$\pi_t^w = k_w\left(\phi N_t^v - \frac{\epsilon - 1}{\epsilon}\frac{W_t}{P_t}c_t^{-\sigma}\right) + \beta(\pi_{t+1}^w) \tag{24}$$

D Proposition 1: Numerical Resolution of the IKC

For proposition 1, needed to get the Jacobians and to assess the impact of the matrix I follow the method of Auclert et al. (2024) and Auclert et al. (2021), I calculate Jacobians truncating to a horizon of T, so that $\mathbf{M}$ is a $T \times T$ matrix, and $\mathbf{dG}, \mathbf{dT}$ are $T \times 1$ vectors. Truncating $\mathbf{M}$ generally implies that $\mathbf{q}'(\mathbf{I} - \mathbf{M}) \neq 0$, since the "tents" corresponding to the final columns of $\mathbf{M}$ (see figure 3) are incomplete.

The approach I choose is to directly solve for the multiplier matrix $\mathbf{M}$, numerically computing $\mathbf{M} = \mathbf{A}^{-1}\mathbf{K}$, where $\mathbf{A}$ is the asset Jacobian, whose elements are given by $A_{ts} = \frac{\partial A_t}{\partial Z_s}$. After obtaining $\mathbf{A}$ either directly using the methods from Auclert et al. (2021), or indirectly from $\mathbf{M}$ using $\mathbf{A} = \mathbf{K}(\mathbf{I} - \mathbf{M})$. Then, given $\mathbf{M}$, form

$$\mathbf{dY} = \mathbf{M}(\mathbf{dG} - \mathbf{M}\mathbf{dT}) \tag{A.21}$$

E Formalization of the relationship between fiscal multipliers and the level of government debt.

To formally derive the relationship between fiscal multipliers and the level of public debt in the HANK model described in the paper, I start by considering how the household sector's consumption and savings decisions interact with government debt. As government debt increases, it introduces more bonds into the economy, which can alter households' consumption behavior via their marginal propensity to consume (MPC). The key mechanism at play is how higher levels of debt influence interest rates and asset holdings, which in turn affect fiscal multipliers.

E.1 Household Optimization Problem

Households maximize utility over consumption and labor. The household problem is given by:

$$V_t(e_{it}, a_{it-1}) = \max_{c_{it}, a_{it}} \left\{ \frac{c_{it}^{1-\sigma}}{1-\sigma} - \varphi \frac{n_{it}^{1+\eta}}{1+\eta} + \beta \mathbb{E}_t V_{t+1}(e_{it+1}, a_{it}) \right\}$$

subject to the budget constraint:

$$c_{it} + a_{it} = (1 + r_t)a_{it-1} + e_{it}\frac{W_t}{P_t}N_t - \tau_t$$

Given the recursive structure of the problem, the consumption function $c_t = C(a_{t-1}, e_t)$ can be derived as a function of past asset holdings a_{t-1}, labor income e_t, and interest rates r_t.

E.2 Intertemporal Marginal Propensity to Consume (iMPC)

The marginal propensity to consume (MPC) is defined as the derivative of consumption with respect to income:

$$iMPC_{it} = \frac{\partial c_{it}}{\partial y_{it}} \tag{25}$$

In the HANK framework, households face idiosyncratic risk and income heterogeneity, so the iMPC varies across households based on their asset holdings and income. Crucially, as public debt increases and more bonds are held by households, aggregate savings rise, and the distribution of wealth shifts. In general, wealthier households (those with more bonds) tend to have lower iMPCs due to the diminishing marginal utility of consumption: iMPC decreases as a_{it} increases.

Thus, when government debt increases, leading to more bonds (assets) in the economy, the aggregate MPC declines.

Consider a bounded shock $\{dG_t, dT_t\}$ to government spending G_t and taxes T_t. The impulse response of output $d\mathbf{Y}$ is governed by:

$$d\mathbf{Y} = dG - \mathbf{M} \cdot d\mathbf{T} + \mathbf{M} \cdot d\mathbf{Y}$$

where $\mathbf{M} \equiv [M_{ts}]$ is the infinite matrix of partial derivatives:

$$M_{ts} = \frac{\partial C_t}{\partial Y_s}$$

Solving for $d\mathbf{Y}$:

$$d\mathbf{Y} = (I - \mathbf{M})^{-1} (dG - \mathbf{M} \cdot d\mathbf{T})$$

E.3 Government Budget Constraint and Debt

The government's budget constraint is given by:

$$\tau_t = (1 + r_t)B_{t-1} + G_t - B_t.$$

In a deficit-financed fiscal expansion, government spending G_t increases, and this is matched by an increase in debt B_t. The fiscal multiplier measures the effect of this increase in government spending on output. Specifically, the **fiscal multiplier** is defined as the change in output (ΔY) in response to a change in government spending (ΔG):

$$\text{Impact Multiplier} = \frac{\Delta Y_0}{\Delta G_0}.$$

E.4 Interest Rates and Asset Demand

As government debt increases, the real interest rate r_t adjusts. In the model, the steady-state real interest rate r^* increases with higher debt levels because more bonds in the economy raise the demand for assets. The steady-state asset demand function is increasing in the real interest rate:

$$\frac{\partial A(r^*)}{\partial r^*} > 0,$$

where $A(r^*)$ is the aggregate demand for assets as a function of the real interest rate.

E.5 Concavity of Consumption Function and Declining MPC

The consumption function $C(a_{t-1}, e_t)$ is concave in income due to diminishing marginal utility, which implies that the marginal propensity to consume (MPC) decreases as asset holdings increase. Mathematically, the MPC is given by:

$$\text{MPC}(e_t, a_{t-1}) = \frac{\partial C(a_{t-1}, e_t)}{\partial e_t}.$$

Given that $C(a_{t-1}, e_t)$ is concave in e_t, the second derivative of consumption with respect to income is negative:

$$\frac{\partial^2 C(a_{t-1}, e_t)}{\partial e_t^2} < 0.$$

This implies that as a_{t-1} (asset holdings) increases, the MPC decreases. In other words, wealthier households have a lower MPC because they consume a smaller fraction of additional income:

$$\text{MPC}(e_t, a_{t-1}) \quad \text{decreases as} \quad a_{t-1} \quad \text{increases.}$$

F Formalization of Proposition 4

This section formally demonstrate how an increase in government debt B affects the *aggregate* marginal propensity to consume via two key channels: (i) the *factor-price* (or policy-function) channel and (ii) the *wealth distribution* (or insurance) channel. I show that the real interest rate effect dominates, while shifts in the wealth distribution $\Psi(a, e)$ contribute at a higher order.

F.1 Setup and Definitions

Individual MPC. Each household has a consumption rule

$$c(a_t, e_t),$$

where $a_t \in \mathcal{A}$ denotes asset holdings and $e_t \in \mathcal{E}$ denotes productivity or income state. Let y_t be a small change in the household's (disposable) income; let define the *individual* marginal propensity to consume at state (a_t, e_t) by

$$MPC(a_t, e_t) \equiv \frac{\partial c(a_t, e_t)}{\partial y_t}. \tag{26}$$

Aggregate MPC. Let $\Psi(a, e)$ be the steady-state distribution of households over $\mathcal{A} \times \mathcal{E}$. Then aggregate consumption is

$$C = \int_{\mathcal{A}\times\mathcal{E}} c(a, e)\, \Psi(a, e),$$

and we define the *aggregate MPC* as

$$MPC_{\text{agg}} \;=\; \int_{\mathcal{A}\times\mathcal{E}} MPC(a,e)\; \Psi(a,e). \tag{27}$$

This measures the economy-wide sensitivity of consumption to a small, uniform change in households' income.

F.2 Decomposition of Changes in the Aggregate MPC

Debt, Factor Prices, and Distribution. Suppose the government raises its debt level B. Such an increase alters:

1. *Factor prices* (e.g. the real interest rate r), which can change households' consumption-savings policies and hence $MPC(a,e)$.
2. *Wealth distribution*, $\Psi(a,e)$, reflecting how households might reallocate assets or absorb new bonds differently.

I allow both MPC and Ψ to depend on B. Symbolically, write $MPC(a,e;B)$ for the individual MPC at debt level B, and $\Psi_B(a,e)$ for the corresponding distribution.

Then the *aggregate* MPC in an equilibrium with debt B is

$$MPC_{\text{agg}}(B) \;=\; \int_{\mathcal{A}\times\mathcal{E}} MPC\big(a,e;B\big)\, \Psi_B(a,e)\,.$$

My main goal is to differentiate $MPC_{\text{agg}}(B)$ with respect to B.

F.2.1 Total Derivative via the Chain Rule

Applying the product rule to the integral, we obtain:

$$\begin{aligned} \frac{d}{dB}\Big[MPC_{\text{agg}}(B)\Big] \;&=\; \frac{d}{dB}\Big[\int MPC(a,e;B)\,\Psi_B(a,e)\,da\,de\Big] \\ &= \underbrace{\int_{\mathcal{A}\times\mathcal{E}} \frac{\partial\, MPC(a,e;B)}{\partial B}\;\Psi_B(a,e)\,da\,de}_{\text{Factor-Price (Policy Function) Channel}} \;+\; \underbrace{\int_{\mathcal{A}\times\mathcal{E}} MPC(a,e;B)\;\frac{\partial\,\Psi_B(a,e)}{\partial B}\,da\,de}_{\text{Insurance Channel}} \end{aligned} \tag{28}$$

Interpretation.

- *Factor-Price (Policy Function) Channel.* The first term captures how each individual MPC changes with B, primarily via higher r or other factor prices. Integrating over the baseline distribution Ψ_B then aggregates these changes.

- *Distribution (Insurance) Channel.* The second term measures how reweighting the distribution from Ψ_B to $\Psi_{B+\Delta}$ affects the aggregate MPC, holding individual functions $MPC(a, e; B)$ fixed.

Dominance of the Factor-Price Channel In many plausible calibrations, $\partial\Psi_B/\partial B$ is negligible *at first order*, for example because an endogenous discount factor β adjusts so that new debt is absorbed with minimal reshuffling of the cross-sectional distribution. Formally:

$$\frac{\partial\,\Psi_B(a, e)}{\partial B} \;\approx\; 0 \quad \text{(at first order).}$$

Hence, the second (distribution) term in (28) vanishes, leaving only

$$\frac{d}{dB}\Big[MPC_{\text{agg}}(B)\Big] \;\approx\; \int_{\mathcal{A}\times\mathcal{E}} \frac{\partial\, MPC(a, e; B)}{\partial B}\, \Psi_B(a, e)\,.$$

Because $MPC(a, e; B)$ depends mainly on $r = r(B)$, the effect is typically dominated by

$$\left(\int \frac{\partial\; MPC(a, e; r)}{\partial r}\, d\Psi_B(a, e)\right) \frac{\partial r}{\partial B},$$

which is the *factor-price channel*. Intuitively, a rise in B that raises r will reduce household MPCs, and thus reduce the fiscal multiplier in a high-debt equilibrium. By contrast, a shift in Ψ_B alone contributes less at first order, under typical assumptions.

Therefore, the *real interest rate* (factor-price) effect explains most of the difference in MPC_{agg} across debt levels. This completes the argument that changes in B affect *aggregate* MPC—and hence the *fiscal multiplier*—primarily through the factor-price channel, rather than through a reshuffling of wealth distribution Ψ.

Formal Statement and Proof Let B be the level of government debt in a heterogeneous-agent model, potentially altering both

1. the factor price $r(B)$, which affects individual MPCs, and

2. the wealth distribution $\Psi_B(a, e)$.

Then, if the distribution does not shift significantly at first order ($\partial\Psi_B/\partial B \approx 0$), changes in B mostly change the aggregate MPC $_{\text{agg}}(B)$ *through* the factor-price channel, i.e. via $\partial r/\partial B$, rather than through reweighting Ψ.

Proof of Proposition 4. From (27), we have

$$MPC_{\text{agg}}(B) \;=\; \int MPC(a,e;B)\,\Psi_B(a,e).$$

Take a total derivative with respect to B, using the product rule:

$$\frac{d}{dB}MPC_{\text{agg}}(B) = \int \frac{\partial\, MPC(a,e;B)}{\partial B}\,\Psi_B(a,e) \;+\; \int MPC(a,e;B)\,\frac{\partial\,\Psi_B(a,e)}{\partial B}.$$

The first term is the *policy function (factor-price) channel*; the second is the *distribution (insurance) channel*. If $\partial\,\Psi_B(a,e)/\partial B$ is zero at first order, the second term vanishes and only the factor-price effect remains. Because raising B typically increases r, it thus directly lowers individual MPCs and the aggregate MPC, dominating the multiplier response to debt. □

Recall from the main text that the aggregate MPC in equilibrium with debt B is

$$MPC_{\text{agg}}(B) \;=\; \int_{\mathcal{A}\times\mathcal{E}} MPC(a,e;B)\;\Psi_B(a,e)\,da\,de\,.$$

When we differentiate with respect to B, we obtain the decomposition:

$$\frac{d}{dB}\Big[MPC_{\text{agg}}(B)\Big] \;=\; \underbrace{\int_{\mathcal{A}\times\mathcal{E}} \frac{\partial\,(a,e;B)}{\partial B}\,\Psi_B(a,e)\,da\,de}_{\text{Policy-Function / Factor-Price Channel}} \;+\; \underbrace{\int_{\mathcal{A}\times\mathcal{E}} MPC(a,e;B)\;\frac{\partial\,\Psi_B(a,e)}{\partial B}\,da\,de}_{\text{Distribution (Insurance) Channel}}\,. \tag{29}$$

I claim that, under certain conditions, the second integral (the "distribution channel") is negligible at first order (i.e. of smaller order than the factor-price term).

F.3 Formal Argument via an Envelope/Approximate Invariance Condition

To see why $\partial\Psi_B/\partial B$ can be zero (or very small) at first order, it is convenient to think of $\Psi_B(a,e)$ as the *stationary distribution* arising from households' optimal saving choices under equilibrium factor prices (interest rate $r(B)$, wages $w(B)$) and equilibrium tax policy (lump-sum taxes $\tau(B)$, etc.). I then show that as B changes *slightly*, the induced shift in the distribution must be of *second order* (or zero), provided the model setup satisfies certain smoothness and "approximate invariance" conditions.

Step 1: Households' Value Function & Policy Rules. Let $v(a, e; B)$ be the stationary (or steady-state) value function when the government debt is B. Households' individual policy function for assets next period, $\alpha(a, e; B)$, and the consumption function $c(a, e; B)$ solve the usual Bellman equation.

Because $r(B)$, $w(B)$, and $\tau(B)$ are *smooth* in B, small changes in B only induce *small* changes in the household's Bellman equation. As a result,

$$\alpha(a, e; B + \Delta B) \;=\; \alpha(a, e; B) \;+\; O(\Delta B), \quad c(a, e; B + \Delta B) \;=\; c(a, e; B) \;+\; O(\Delta B),$$

where $O(\Delta B)$ denotes terms of order ΔB or higher. Hence the *policy functions* (including $(a, e; B)$) change *at first order* in B.

Step 2: Stationary Distribution as a Fixed Point. The cross-sectional distribution $\Psi_B(a, e)$ solves a fixed-point or stationary condition (e.g. a Fokker-Planck equation in continuous time, or a law of motion under Markov transition in discrete time). Symbolically,

$$\Psi_B \;=\; \Gamma_B(\Psi_B),$$

where Γ_B is the operator that maps any trial distribution into the next period's (or next instant's) distribution given the policy rule $\alpha(\cdot, \cdot; B)$ and the Markov transitions for e. For small ΔB, we consider the new fixed point $\Psi_{B+\Delta B}$:

$$\Psi_{B+\Delta B} \;=\; \Gamma_{B+\Delta B}\Big(\Psi_{B+\Delta B}\Big).$$

Because $\Gamma_{B+\Delta B}$ is a *continuous* mapping in B and Ψ_B is already a fixed point of Γ_B, it follows (under typical Lipschitz or contraction conditions) that the *change* $\Psi_{B+\Delta B} - \Psi_B$ is of *order at most* ΔB. In practice, it may be *strictly smaller at first order* if there is an "approximate invariance" or "envelope" property, as I now detail.

Step 3: The Envelope/Approximate Invariance Argument. In many heterogeneous-agent models calibrated to match empirical wealth dispersion, two ingredients often imply *minimal shifts* in Ψ_B when B changes:

- *Lump-Sum Tax Adjustments:* Any additional interest cost from higher B is financed via a small, lump-sum tax $\Delta\tau$ on every individual. Lump-sum taxes do not fundamentally re-rank households in the cross-section; they shift everyone's budget constraint slightly.
- *Smooth Saving Motives:* Agents' saving rules $\alpha(a, e; B)$ adjust somewhat with r and τ, but these adjustments can *offset* each other in such a way that the overall wealth accumulation distribution changes little (at least to first order).

Under these conditions, one can show (technically via an "envelope theorem" argument at the distribution level) that the linear (first-order) term in $\partial\Psi_B/\partial B$ vanishes or is extremely small.[11]

Step 4: Conclusion of the Argument. Thus, for small ΔB, we have

$$\Psi_{B+\Delta B}(a, e) = \Psi_B(a, e) + O\big((\Delta B)^2\big),$$

and hence,

$$\frac{\partial \Psi_B(a, e)}{\partial B} = 0 + \text{(possible second-order terms)}.$$

This justifies the statement that at first order,

$$\int_{\mathcal{A}\times\mathcal{E}} MPC(a, e; B)\, \frac{\partial\, \Psi_B(a, e)}{\partial B}\, da\, de \approx 0.$$

Consequently, the second term in (29) —the "distribution channel"—is negligible at first order, so the effect of changes in B on MPC_{agg} is *dominated* by the first term, i.e. by changes in factor prices (the "policy-function channel").

Suppose:

1. Factor prices (r, w) and lump-sum taxes (τ) depend smoothly on B,

[11] In continuous-time models, one can write the stationary distribution as the solution to a differential equation (the Fokker-Planck equation). A small change in (r, τ) that keeps average saving rates roughly unchanged may shift Ψ_B only at second order. In discrete-time Markov chain settings, a similar argument applies by approximating transition kernels in a Taylor expansion around $\Delta B = 0$.

2. Households' policy functions $\alpha(a, e; B)$ and $MPC(a, e; B)$ are continuous and differentiable in B,

3. The stationary distribution Ψ_B is determined by a contraction mapping Γ_B (or continuous Markov transition) in the usual way,

4. Small changes in lump-sum tax $\tau(B)$ and factor prices $r(B)$ generate a *second-order* or negligible first-order change in Ψ_B, e.g. by an envelope argument.

Then, to first order in ΔB,

$$\Psi_{B+\Delta B}(a, e) \;=\; \Psi_B(a, e) \;+\; O\big((\Delta B)^2\big), \quad \text{so} \quad \frac{\partial \Psi_B(a, e)}{\partial B} \;=\; 0 \;+\; \text{(higher order terms)}.$$

Consequently, the distribution channel

$$\int_{\mathcal{A}\times\mathcal{E}} MPC(a, e; B)\, \frac{\partial \Psi_B(a, e)}{\partial B}\, da\, de$$

is negligible at first order, and the principal effect of B on the aggregate MPC comes through the *policy function (factor-price) channel*.

In particular, the key property is that the stationary distribution depends on B only at a higher order (second order or beyond). This is generically true under the standard assumptions on smoothness, lump-sum taxation, and the contraction property in stationary equilibrium, which limit the ability of a small ΔB to *re-rank or significantly shift* households across asset states. As a result, Ψ_B cannot change at first order, implying $\frac{\partial \Psi_B}{\partial B} = 0$ at $\Delta B = 0$.

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