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Analysis of Quantitative Investment Strategies

Multifactor Earnings Surprise Strategy

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Abstract: This thesis explores the integration of four distinct investment strategies within a unified fund, employing the Markowitz framework for portfolio optimization. The study further incorporates an allocation scheme based on popular weighting principles. By synergizing diverse investment approaches and leveraging the Markowitz model, this research aims to enhance portfolio performance and risk management. The proposed allocation scheme seeks to capitalize on market trends and investor sentiment, offering a comprehensive strategy for constructing a well-balanced and resilient investment fund.

This paper implements and analyzes a multivariate momentum strategy based on three factors – earnings surprises, revenue surprises, and prior returns. The study explores market inefficiencies by examining the post-earnings announcement drift and price momentum anomalies, demonstrating the joint implications of integrating individual factors. The multifactor strategy significantly outperforms its univariate and bivariate counterparts, yielding an annualized excess return of 18.31%. Even after applying the CAPM and Fama-French pricing models, the portfolio generates a significant monthly excess return. The analysis also shows a stronger momentum effect in winner portfolios, evidenced by the statistical insignificance of loser portfolios.

Keywords: post-earnings announcement drift, earnings surprise, revenue surprise, momentum strategies

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1. Introduction

In this thesis, we conduct an in-depth analysis of four investment strategies, each characterized by its distinct approach and unique features. The initial sections are devoted to a thorough introduction of these strategies, outlining their key principles and the theoretical framework that underpins each of them. This is followed by a detailed comparative analysis where we examine and contrast the performance indicators associated with each strategy.

The core objective of this group part is to provide a comprehensive evaluation of these strategies both as standalone approaches and combined. We aim to investigate how strategic optimization and thoughtful allocation can potentially enhance their collective performance. By doing so, this study strives to not only assess the individual effectiveness of each strategy but also to explore the synergies that may arise when they are integrated. Through this approach, we seek to answer critical questions regarding the efficacy of these diverse investment strategies and to understand the potential advantages of their combination.

2. Individual Strategies

2.1 Multifactor Earnings Surprise Strategy (SUE)

2.1.1 Economic Motivation

The Efficient Market Hypothesis (EMH), introduced by Fama (1970), anchors modern investment theory with its assertion that stock prices in efficient markets fully reflect all available information. This hypothesis, however, has been challenged by various anomalies, indicating more intricate forces at work in financial markets. Notable among these are the post-earnings announcement drift (PEAD), originally identified by Ball and Brown (1968), and the concept of price momentum, as elaborated by Jegadeesh and Titman (1993), both later acknowledged by Fama (1998) as significant exceptions. PEAD, often labeled as the ‘granddaddy of underreaction events’, reveals that stock prices persist in responding to earnings news well beyond their initial announcement, hinting at a delayed market reaction. This phenomenon is especially pronounced in companies with ‘good news’, marked by high standardized unexpected earnings (SUE), which tend to outperform those with ‘bad news’, denoted by low SUE. Similarly, the price momentum anomaly, the inclination of stocks to sustain their prior performance trend, contests the EMH's premise of instantaneous price reaction to new information, leading to the expectation that historically successful stocks will likely continue to outperform their less successful counterparts in the short-term future.

2.1.2 Strategy

To bridge and exploit the anomalies mentioned above, I implement a multifactor earnings surprise investment strategy based on three key factors – earnings surprises, revenue surprises (SURGE), and prior returns – similar to Chen et al. (2014). By combining individual factors, the strategy aims to exploit the market's inefficiencies and delayed information processing.

The foundation of this strategy lies in the methodology for calculating these factors, which adopts the approach based on historical earnings as defined by Jegadeesh and Livnat (2006). This approach states that quarterly earnings typically follow a seasonal random walk with an underlying drift. Thus, earnings surprises are measured by the standardized unexpected earnings, defined as

$$SUE_{i,t} = \frac{Q_{i,t} - E(Q_{i,t})}{\sigma_{i,t}}, \quad (1)$$

where $Q_{i,t}$ is the quarterly earnings per share (EPS) for firm i in quarter t, $E(Q_{i,t})$ is the expected EPS for firm i in quarter t, and $\sigma_{i,t}$ is the standard deviation of earnings growth over the preceding eight quarters. The expected earnings $E(Q_{i,t})$ and standard deviation of earnings growth $\sigma_{i,t}$, respectively, are defined as

$$E(Q_{i,t}) = Q_{i,t-4} + \delta_{i,t} \quad (2)$$

and,

$$\sigma_{i,t} = \frac{1}{7}\sqrt{\sum_{j=1}^{8}\left(Q_{i,t-j} - E(Q_{i,t-j})\right)^2}, \tag{3}$$

where the drift term, $\delta_{i,t}$, is the average EPS growth over the past eight quarters, defined as

$$\delta_{i,t} = \frac{\sum_{j=1}^{8}(Q_{i,t-j} - Q_{i,t-j-4})}{8}. \tag{4}$$

Analogously, revenue surprises are measured by the standardized unexpected revenue surprise estimate (SURGE), defined as

$$SURGE_{i,t} = \frac{R_{i,t} - E(R_{i,t})}{\sigma_{i,t}}, \tag{5}$$

where $R_{i,t}$ is the quarterly revenue for firm i in quarter t, $E(R_{i,t})$ is the expected revenue for firm i in quarter t, and $\sigma_{i,t}$ is the standard deviation of revenue growth over the past eight quarters. For the price momentum factor, the three-month cumulative return is used as the trading signal.

The multifactor strategy is constructed by combining independent univariate strategies, each based on one factor. For each univariate strategy, five market-value weighted portfolios are formed monthly, based on the previous quarter's performance. In the context of revenue surprises, for example, the top-performing portfolio (R5) consists of firms with the highest positive revenue surprise from the preceding quarter, while the lowest-performing portfolio (R1) includes those with the most negative surprise. Building upon this, bivariate and multivariate strategies are developed by intersecting these univariate portfolios. A bivariate strategy might involve selecting firms that are top performers in both earnings and revenue surprises. Expanding on this concept, the multifactor portfolio P5xE5xR5 is created by combining the highest-performing univariate portfolios from all three factors.

2.1.3 Results

This section presents a structured summary of the results, beginning with a brief overview of the correlations among the three factors. Subsequently, it compares portfolios based on univariate, bivariate, and multivariate strategies, evaluating the profitability of combining factors. Thereafter, I will show the the significance and robustness of the multifactor strategy by applying common asset pricing models.

In testing the hypothesis that combining momentum factors enhances strategy returns, this paper first examines whether each factor contributes unique information to the strategy by calculating the correlations among the factors. Despite potential overlaps, the distinct insights offered by each factor could amplify the combined performance. The results show that SUE and SURGE are moderately correlated at 0.32, reflecting their relationship in the income statement. However, both show low correlations with prior returns (0.10 and 0.06), suggesting distinct information contribution to the multifactor approach.

Table 1 depicts the annualized excess portfolio returns for each momentum strategy. Panel A highlights distinct patterns in univariate strategies: top-performing quintiles P5, E5, and R5 show significant excess returns of 9.66%, 9.66%, and 9.54%, respectively. However, the lower quintiles lack statistical significance, implying that momentum effects are predominantly observed in winning portfolios. Additionally, the expected trend of higher quintiles yielding greater returns is not uniformly evident, contrasting prior research. Panel B shows portfolios combining two factors and reveals a typical momentum trend, with higher quintiles, such as P5xE5 and P5xR5, yielding higher returns (13.42% and 12.80%) than lower quintiles (again statistically insignificant). Notably, the zero-investment portfolio E5xR5 – E1xR1 yields a significant 7.70%, highlighting the effectiveness of integrating earnings and revenue surprises over price momentum. Panel C shows the multivariate strategy, combining all three factors, with the P5xE5xR5 portfolio delivering the highest excess return of 18.31%.

However, the corresponding zero-investment strategy's positive return isn't significant, implying that the benefits of the top-tier portfolio are potentially offset by the lower-tier portfolio to some degree, revealing complexities in the multivariate approach.

In conclusion, these results substantiate that, within my sample, momentum effects are predominantly present in top-quintile portfolios. Further, the results underscore the superiority of multifactor strategies over bivariate and univariate methods, reinforcing the joint implication of combining factors.

	Mean ER	t-value		Mean ER	t-value		Mean ER	t-value
Panel A: Univariate Strategies								
P1	0.0702	1.36	E1	0.0455	1.23	R1	0.0443	1.19
P2	0.0686	1.94	E2	0.0724	2.07	R2	0.0665	1.95
P3	0.0515	1.55	E3	0.0806	2.38	R3	0.0877	2.54
P4	0.0881	2.74	E4	0.0783	2.44	R4	0.0831	2.48
P5	0.0966	2.68	E5	0.0966	2.83	R5	0.0954	2.79
P5 – P1	-0.0034	-0.09	E5 – E1	0.0440	2.64	R5 – R1	0.0432	2.31
Panel B: Bivariate Strategies								
P1xE1	0.0281	0.50	P1xR1	0.0213	0.36	E1xR1	0.0247	0.59
P2xE2	0.0549	1.37	P2xR2	0.0765	1.98	E2xR2	0.0708	1.81
P3xE3	0.0502	1.35	P3xR3	0.0820	2.29	E3xR3	0.0915	2.42
P4xE4	0.1059	3.24	P4xR4	0.0949	2.88	E4xR4	0.0883	2.56
P5xE5	0.1342	3.35	P5xR5	0.1280	3.37	E5xR5	0.1151	3.00
P5xE5 – P1xE1	0.0594	1.20	P5xR5 – P1xR1	0.0567	1.16	E5xR5 – E1xR1	0.0770	2.82
Panel C: Multivariate Strategy								
			P1xE1xR1	-0.0088	-0.13			
			P2xE2xR2	0.0862	1.81			
			P3xE3xR3	0.0639	1.47			
			P4xE4xR4	0.0754	1.69			
			P5xE5xR5	0.1831	4.09			
			P5xE5xR5– P1xE1xR1	0.1170	1.85			

Table 1: Annualized Excess Returns of Momentum Strategies

Next, I apply the CAPM and Three-Factor Fama-French pricing models to capture the potential risk factors associated with the returns of P5xE5xR5. The regression results, depicted in Table 2, demonstrate that the multifactor strategy yields a significant monthly excess return of 0.95% under the CAPM, with a market beta of 0.92, indicating strong market exposure and volatility marginally lower than the broader market. In the Fama-French model, the monthly excess return is slightly lower at 0.77%. The market beta remains consistent at 0.92, while the positive SMB coefficient (0.32) suggests a small-cap preference, which aligns with previous research. The negative, though not significant, HML coefficient (-0.12) hints at a lesser focus on value stocks.

	α	β_M	R^2	α	β_M	β_{SMB}	β_{HML}	R^2
	0.0095	0.9172	0.468	0.0077	0.9170	0.3139	-0.1236	0.478
t-value	(3.44)	(14.44)	–	(2.61)	(14.50)	(2.07)	(-0.94)	–

Table 2: Regression Results (CAPM and FF3) of P5xE5xR5

In conclusion, the study confirms PEAD and price momentum in the S&P 500 and shows that multifactor strategies, particularly in winner portfolios, yield higher profitability than their univariate counterparts.

2.2 Quality Investing on the Oslo Stock Exchange (QMJ)

2.2.1. Introduction

The concept of quality investing has long been a fundamental strategy in the field of quantitative investment. A significant contribution to this was Asness et al. introduction of the Quality Minus Junk (QMJ) factor in 2013, and its further development in 2019. This method focuses on the returns from high-quality stocks, identified by factors such as profitability, growth, and safety, and challenges traditional investment models. The 2019 research by Asness et al. demonstrated the effectiveness of this approach in larger markets, but its performance in smaller markets, like Norway’s, has not been extensively studied.

Asness et al. (2019) integrated various research findings in quality investing into a single metric. Their results showed that stocks classified as high-quality by the QMJ factor tend to offer higher risk-adjusted returns, contradicting some traditional risk-return theories. The study explores reasons for this, including the possibility that markets might use more refined methods for evaluating quality, that quality stocks might represent unobserved risks, or that markets do not entirely reflect the quality aspect in stock prices. Their findings suggest potential inefficiencies in the market, indicating opportunities for investors.

This thesis sets out to apply the QMJ framework to the Norwegian stock market with the aim of determining whether a quality investing strategy can be effective in this context. The focus will be on a practical evaluation of the QMJ strategy's performance in the Norwegian market, assessing its viability as a profitable investment approach.

2.2.2 Methodology

2.2.2.1 Data Description

This study's data is sourced from the Compustat databases via Wharton Research Data Services (WRDS), focusing on Norwegian firms. It comprises annual accounting data from June 1987 and daily stock price data from December 31, 1985, specifically targeting firms listed on the Oslo Stock Exchange.

Risk-free rates are proxied using the NIBOR 6-month and 1-month annualized rates, primarily from Norges Bank and, post-2012, Norske Finansielle Referanser AS (NoRe). Inflation trends are assessed using Consumer Price Index (CPI) data from Statistics Norway, and currency exchange rates are sourced from Sveriges Riksbank. Norwegian recession indicators are obtained from the Federal Reserve Economic Data (FRED) website.

The study confines its analysis to common ordinary shares, focusing on those with the highest market equity. Stock prices adjust for factors like stock splits and dividends. Accounting data is standardized to NOK, and stocks with less than six years of data are excluded due to metric requirements.

To ensure the relevance and avoidance of forward-looking biases, the study aligns fiscal year accounting data with a 6–12-month lag to the corresponding calendar year. This approach is in line with the methodology employed by Asness et al. (2019).

The analysis period is set from 2000 to 2023 to provide a comprehensive and representative sample, with the dataset averaging 174 unique stocks per year. This period is chosen based on the significant increase in stock data availability from around the year 2000.

2.2.2.2 Variable and Portfolio Construction

In analyzing quality investing in the Norwegian stock market, this study constructs variables and portfolios to capture quality across different dimensions. Quality is divided into profitability, growth, and safety, each measured through a series of financial metrics. Profitability includes gross profits over assets (GPOA), return on equity (ROE), return on assets (ROA), cash flow over assets (CFOA), gross margin (GMAR), and low accruals (ACC), reflecting the efficiency and effectiveness of income generation. Growth is measured over a five-year period, focusing on sustainable increases in profitability metrics such as ΔGPOA, ΔROE, ΔROA, ΔCFOA, and ΔGMAR. Safety is assessed through low beta (-β), low leverage (LEV), low earnings volatility (EVOL), and bankruptcy risk indicators like Altman's Z-score and Ohlson's O-score.

To standardize these diverse metrics for comparative analysis, z-scores are computed for each financial metric. This involves ranking the metrics within the stock universe and then standardizing these ranks to have a mean of zero and a standard deviation of one. This z-score transformation normalizes the various scales of financial metrics, enabling an aggregated assessment across different companies.

Portfolios are then constructed based on these standardized scores. Stocks are sorted into quintiles monthly, based on their overall quality scores, creating five portfolios that range from the lowest to the highest quality. This quintile method is preferred over the decile method for its suitability in the smaller Norwegian market, allowing for a more diversified portfolio.

The Quality-Minus-Junk (QMJ) factor is central to the portfolio analysis. Stocks are categorized by size and quality, dividing them into 'Big' or 'Small', and 'Quality' or 'Junk'. The QMJ factor is derived from the difference in returns between the high-quality and low-quality portfolios.

Additionally, the study constructs a market portfolio (MKT) as well as Small Minus Big (SMB) and High Minus Low (HML) factors, following the Fama and French model. These factors examine the return differences attributable to market risk, firm size, and book-to-market values, respectively, complementing the analysis of the QMJ factor.

2.2.3. Results

2.2.3.1 Quality Sorted Portfolios

The results of the study on quality investing in the Norwegian stock market, following the detailed construction of variables and portfolios, offer compelling insights. The analysis of quality-sorted portfolios reveals a clear trend: higher quality correlates with superior performance. This is evident from the progression of returns, where higher-quality portfolios consistently outperform their lower-quality counterparts. The 'P5-P1' differential, a comparison between the highest and lowest quality portfolios, demonstrates a significant monthly average return advantage of 1.11% for high-quality stocks. This finding is statistically significant, with a t-statistic of 2.42, effectively rejecting the null hypothesis of no difference in average returns.

Further analysis, adjusting for market and other factor risks, underscores the alpha of high-quality stocks. Their outperformance is statistically significant, attributed to their lower market and factor exposures. This aligns with their lower risk profile as indicated by both the Capital Asset Pricing Model (CAPM) and the three-factor model assessments. The strategy of long-high/short-low quality delivers abnormal returns ranging from 1.35% to 1.49% per month, supported by substantial t-statistics. The increase in Sharpe and information ratios across quality ranks further validates the effectiveness of a long-quality/short-junk strategy.

Table 1 Quality sorted portfolios

	P1	P2	P3	P4	P5	P5-P1
Excess Return	0.12%	0.13%	0.49%	0.52%	**1.23%**	**1.11%**
	(0.22)	(0.30)	(1.35)	(1.54)	(3.42)	(2.42)
CAPM alpha	**-0.74%**	**-0.59%**	-0.12%	-0.06%	**0.61%**	**1.35%**
	(-2.10)	(-2.30)	(-0.56)	(-0.34)	(3.14)	(3.04)
3-factor alpha	**-0.84%**	**-0.67%**	-0.18%	-0.04%	**0.66%**	**1.49%**
	(-2.65)	(-2.74)	(-0.86)	(-0.23)	(3.41)	(3.66)
CAPM beta	1.34	1.12	0.95	0.90	0.96	-0.38
Sharpe Ratio	0.05	0.06	0.28	0.32	0.70	0.50
Information Ratio	-0.55	-0.57	-0.18	-0.05	0.71	0.76
Adjusted R2	0.68	0.69	0.69	0.72	0.72	0.22

2.2.3.2 QMJ Portfolio

The performance of the QMJ factor portfolio exhibits significant excess returns and alpha relative to both the CAPM and the Fama-French three-factor model. The QMJ factor achieved monthly abnormal returns of 1.25% against CAPM and 1.34% against the Fama-French model, with robust t-values. The risk loadings of the QMJ factor indicate significant negative market, size, and value exposures, suggesting a strategic preference for low-beta, large stocks, and a short position in high beta, small stocks.

In terms of total returns, the QMJ strategy yielded approximately 1,350% cumulative returns over the sample period, significantly outperforming the market's 700%. This is complemented by a superior annualized Sharpe ratio of 0.79 and an information ratio of 1.19, indicative of a more favorable risk-adjusted return profile.

Table 2 QMJ

	Recession	Expansion	Severe bear market	Severe bull market	Low volatility	High volatility	Spike down in volatility	Spike up in volatility	All Periods
Excess Return	**1.53%**	**0.77%**	1.90%	0.38%	0.51%	**2.26%**	**1.19%**	**1.96%**	**1.04%**
	(3.63)	(2.16)	(1.58)	(0.94)	(1.24)	(4.40)	(2.27)	(4.01)	(3.88)
CAPM alpha	**1.47%**	**1.32%**	1.63%	0.75%	**1.72%**	**1.76%**	**2.53%**	**1.31%**	**1.25%**
	(3.69)	(3.93)	(1.32	(1.54)	(3.84)	(3.43)	(4.89)	(2.67)	(4.99)
3-factor alpha	**1.44%**	**1.52%**	**2.09%**	0.89%	**2.20%**	**1.71%**	**2.50%**	**1.28%**	**1.34%**
	(4.02)	(4.70)	(1.98)	(1.83)	(5.31)	(3.48)	(5.32)	(2.73)	(5.74)
CAPM beta	-0.26	-0.41	-0.12	-0.14	-0.52	-0.21	-0.53	-0.26	-0.32
Sharpe Ratio	1.14	0.61	1.26	0.38	0.46	1.65	0.85	1.50	0.79
Information Ratio	1.29	1.41	1.94	0.92	2.55	1.40	2.31	1.11	1.19
Adjusted R2	0.29	0.25	0.39	0.04	0.36	0.18	0.38	0.20	0.25

An examination of the QMJ strategy under varying market conditions reveals its robustness and adaptability. During both recessions and severe bear markets, the strategy consistently generates higher monthly excess returns, along with strong CAPM and three-factor alphas. This performance not only during economic expansions but also in times of market distress supports the 'flight to quality' phenomenon, where high-quality stocks become a haven for investors. The consistent delivery of positive alpha across different volatility scenarios further underscores the QMJ strategy’s resilience and its potential as a risk-conscious investment approach. These findings solidify the QMJ strategy’s position as a significant and effective investment approach in the Norwegian market, challenging traditional risk-return paradigms and offering a viable option for both defensive and growth-oriented portfolio components.

2.3 The Impact of Machine Learning on Stock Momentum Strategies (ML)

2.3.1 Economic Motivation

Stock market dynamics are complex, characterized by non-linear, chaotic, and volatile patterns, making investment inherently risky. The market's efficiency, as suggested by Malkiel und Fama (1970), indicates that price movements, reflecting all known information, are unpredictable and follow a random walk, aligning with new information's random nature. Advancements in computational intelligence and data mining technology have greatly expedited research in the field of quantitative investment (Yang, Lou and Bo 2017). The need for machine learning models capable of capturing and predicting the price fluctuations of single stocks or a collection of stocks in the market is on the rise (Yue u. a. 2021). Machine learning's pattern detection can refine trading strategies, particularly the widely used price momentum strategy. Its sporadic yet substantial gains make it a prime candidate for improvement via machine learning.

2.3.2 Methodology and Strategy

For the analysis, the Random Forest Classifier was employed to evaluate its effectiveness in forecasting stock movements. Subsequently, the Random Forest Regression model is used for predicting returns. Initially, a traditional momentum strategy is applied, ranking stocks based on their cumulative past returns from months t-12 to t-2, deliberately excluding the most recent month (t-1) to differentiate between intermediate-term momentum and short-term reversal trends. Stocks are then distributed into decile portfolios, typically weighted equally, and subjected to monthly rebalancing. The objective is to discern the efficacy of machine learning in enhancing the momentum strategy by monthly identification of winners and losers through predicted versus actual returns, and to subsequently measure the actual impact by utilizing realized returns during the holding period. To train the Random Forest Models, the following indicators were used:

- **Relative Strength Index (RSI)**

$$RSI = 100 - \frac{100}{1+RS} \quad (1)$$

$$RS = \frac{Average\ Gain\ over\ past\ 14\ days}{Average\ Loss\ over\ past\ 14\ days}$$

- **Stochastic Oscillator**

$$\%K = \left(\frac{C - L_{14}}{H_{14} - L_{14}}\right) * 100 \quad (2)$$

Where,

C = Current closing price

L_{14} = Lowest price over the past 14 days

H_{14} = highest price over the past 14 days

- **Williams Percent Range**

$$\%R = \left(\frac{H_{14} - C}{H_{14} - L_{14}}\right) * -100 \quad (3)$$

Where,

C = Current closing price

L_{14} = Lowest price over the past 14 days

H_{14} = highest price over the past 14 days

- **Price Rate of Change**

$$PROC_t = \frac{P_t - P_{t-n}}{P_{t-n}} \quad (4)$$

Where,

P(t) is the closing price at time t

P(t-n) is the closing price n periods before time t

- **On Balance Volume**

$$OBV(t) = OBV(t-1) + \begin{cases} Volume(t) & if\ P_t > P_{t-1} \\ 0 & if\ P_t = P_{t-1} \\ -Volume(t) & if\ P_t < P_{t-1} \end{cases} \quad (5)$$

Where, P(t) is the closing price at time t.

- **Moving Average Convergence Divergence**

$$MACD = EMA_{12} - EMA_{26} \quad (6)$$

$$SignalLine = EMA_9(MACD)$$

2.3.3 Random Forest Reliability

The Random Forest classifier achieved a predictive accuracy of approximately 70.77%, indicating its effectiveness in forecasting daily stock price movements. Precision rates for predicting downward and upward movements were 68.35% and 73.20% respectively, with recall rates of 71.99% for down days and 69.65% for up days. The F1-score corroborated these findings, indicating balanced performance. Feature importance analysis revealed that Williams R% and RSI were the most influential indicators, with MACD and Price Rate of Change also significant, while On Balance Volume had less impact.

The evaluation of the Random Forest Regression model's predictive accuracy employed RMSE and MAE as key metrics. The RMSE, at 0.01763, indicated a high level of accuracy, suggesting that predicted values closely align with actual returns. The overall MAE, initially at 0.01141, further decreased to 0.01014 when considering predictions that matched the actual direction of stock movements. This notable reduction in MAE highlighted the model's proficiency in capturing market trend directionality.

Additionally, the model's predictive strength was further validated using the R^2 score. An overall R^2 score of 0.3035 suggested that the model accounted for approximately 30% of the variance in daily returns. This score improved significantly to 0.4852 for instances where the model's predictions and actual returns moved in the same direction, underscoring the model's enhanced explanatory power for consistent directional predictions.

In terms of feature importance within the Random Forest Regression analysis, RSI and Price Rate of Change emerged as key indicators in predicting future returns. The model assigned these features higher significance compared to On Balance Volume, which was found to have a lesser impact. This contrasted with the findings from the classifier model, highlighting the distinct roles that various features play in directional prediction versus value estimation in stock movements.

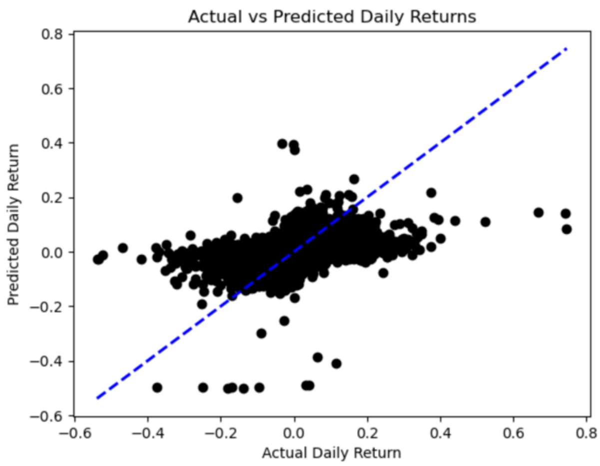

Figure 1: Regression Actual vs. Predicted

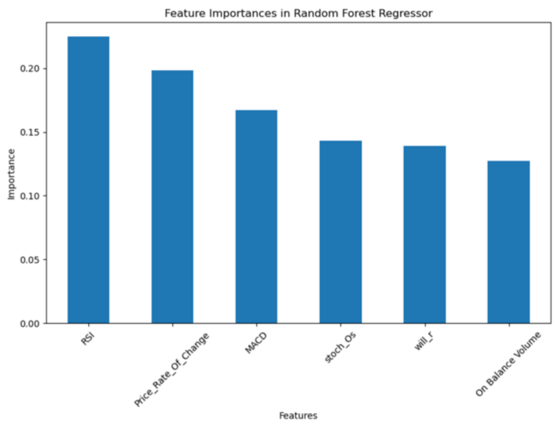

Figure 2: Future Importance Regression

2.3.4 Impact on Momentum and Stock Selection

Between 2017 and 2022, the machine learning-enhanced momentum strategy using predicted returns decreased by -2.16% in annualized returns, outperforming the actual returns strategy's -4.75% drop. In the volatile 2020-2022 post-COVID period, predicted returns limited losses to -3.26%, compared to -10.14% with actual returns.

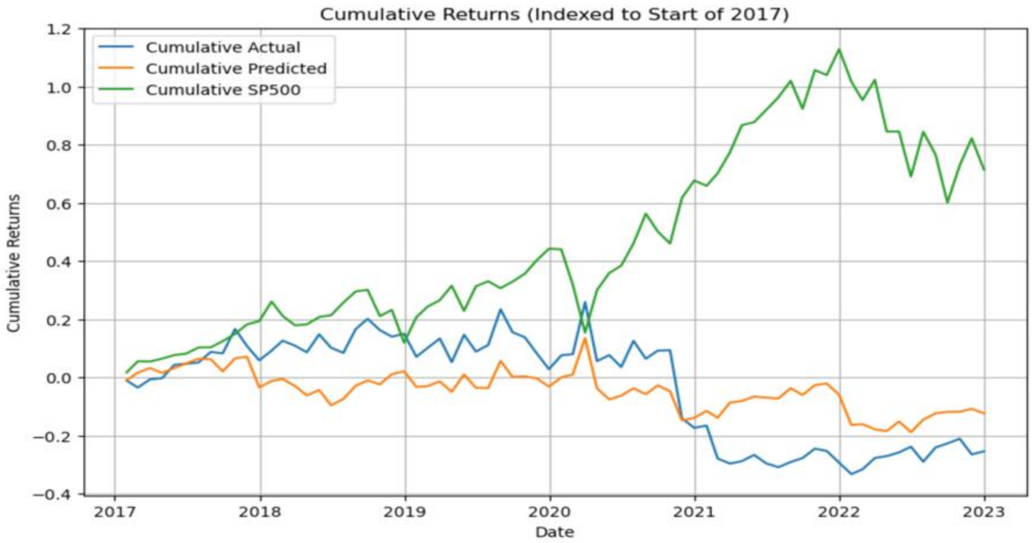

Figure 1: Cumulative Returns

Both strategies had negative Sharpe Ratios, indicating returns insufficient to offset their risk compared to the risk-free rate. The predicted strategy's Sharpe Ratio was -0.14, less negative than the actual strategy's -0.19. This was mirrored in the Sortino Ratios, where the predicted strategy had -0.18 compared to the actual's -0.25.

Period 2017-2022	**Strategy Actual**	**Strategy Predicted**	**Benchmark S&P500**
Volatility (%)	5,82%	4,35%	4,96%
Sharpe Ratio	-0,191	-0,14	0,547
Sortino Ratio	-0,247	-0,18	0,8
Average Drawdown	-0,184	-0,1267	-0,0434
Max Drawdown	-0,4695	-0,285	-0,2476

Table 1: Performance Overview

The predicted strategy also showed lower volatility (4.35%) and drawdowns (average -0.127, max -0.29) than the actual strategy (volatility 5.82%, average drawdown -0.18, max -0.467), indicating a lower risk profile.

2.4. Enhancing Multi-asset Portfolio Performances with Market Timing using the VIX (VT)

2.4.1. Economic motivation

Volatility has always been one of the main concerns for investors who not only seek better returns but also less risk. The ability to estimate future volatility enables investors for better timing for their strategies in terms of asset or weight allocation. Various models such as ARCH (Autoregressive Conditional Heteroskedasticity) proposed by Engle (1982), have been developed to effectively forecast future volatility. Also, historical literature provides evidence of the economic value derived from effective volatility timing (Fleming, Kirby and Ostdiek, 2001).

The VIX, a volatility index from the CBOE, is a popular indicator of future near-term volatility in the markets. It is a strong estimator and can also be used as the implied volatility in the Black-Scholes model.

2.4.2. Strategy

This strategy is built upon the framework of Thomas and Maggie Copeland (1999) and Conolly, Stivers and Sun (2005) who utilize VIX for asset allocation. It also extends their findings to a new time period to assess the reliability of their results before constructing the portfolio. This strategy aims to combine the findings in order to construct a portfolio enhancing performance for multi-asset strategies. Both, Style timing and cross-asset relationships are used to improve returns during low volatility periods and hedge against high volatility periods by optimally allocating assets and weights on the latter.

The signal created from the VIX is a rolling percentage change with an estimation window t:

$$\text{Signal}_t = \frac{X_t - Rolling\ Mean_t}{Rolling\ Std\ Dev_t} \tag{1}$$

$$Rolling\ Mean_t = \frac{1}{window} \sum_{j=t-window+1}^{t} X_j \tag{2}$$

The signal aims to capture the change in volatility relative to the past t days and follows Copeland (1999) construction. It is more robust than a simple percentage change on the VIX because it overcomes the noisiness in the index. To follow Copeland's findings, a 75-day (roughly three-month) estimation window is used for the analysis. The impact of the parameter on the strategy is studied later on.

The assets used in the strategy are ETFs in different asset classes (Equities, Bonds or Gold) and sub-classes (Growth and Stock or Long and mid-term bonds). Index tracking ETFs were chosen for this strategy to review Thomas and Maggie Copeland (1999) and Conolly, Stivers and Sun (2005) findings with indices. Also, the relative high volume and low costs of ETFs makes them more attractive for implementation. However, costs are not included in the strategy's performance analysis because these ETFs do not have transaction costs and their expense ratio is negligible (under 15 bps annually).

The strategy follows an asset allocations scheme such as:

Signal > upper bound: 100% in GLD (3)

upper bound > Signal > middle bound: invest in Bonds and Gold (4)

middle bound > Signal > lower bound: invest in Stocks and Gold (5)

Signal < lower bound: 100% in Growth stock ETF (6)

It also applies a weight allocation scheme when in between the upper and lower bound (3 & 4). The risk-parity allocation scheme is applied to evenly distribute risk across the selected assets which should perform better when the signal is in these bounds according to previous research findings.

2.4.3. Performance Overview

The performance of the portfolio (RP VIX) from June 2005 until December 2022 is plotted in Figure 1 alongside the portfolio individual components and a simple risk-parity portfolio (RP).

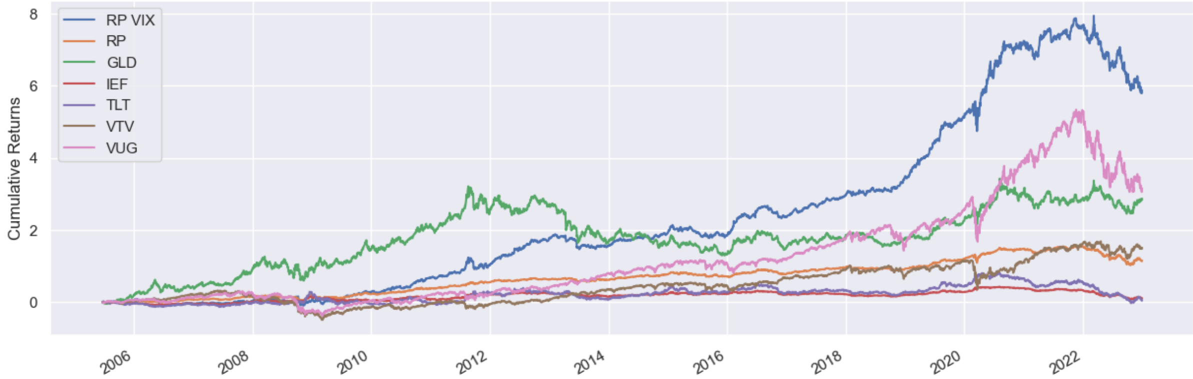

Figure 1: Cumulative returns of the portfolio and its components

The VIX trading portfolio performed much better than the simple risk-parity approach portfolio by yielding a total return of 5.87x vs 1.14x for the simple approach. Also, the strategy proves to be a great diversifier of risk with a higher Sharpe ratio than any of the individual assets. Table 1 presents the annual performances of the portfolios and the components.

	RP VIX	RP	GLD	TLT	IEF	VTV	VUG
Annual return (%)	11.88	4.40	9.26	0.94	1.58	7.10	10.1
Volatility (%)	13.22	6.79	17.93	6.57	14.67	19.94	20.53
Sharpe Ratio	0.89	0.64	0.51	0.13	0.10	0.35	0.49
Skewness	.071	.009	-.178	.096	.088	-.295	-.241
Kurtosis	17.74	5.05	6.24	3.17	3.97	11.84	10.13
Max. Drawdown (%)	-24.05	-20.95	-45.55	-24.48	-46.14	-61.31	-51.35
Avg. Drawdown (%)	-4.56	-2.99	-18.47	-5.38	-12.68	-11.81	-7.64

Table 1: Annual performance of the portfolio and its components

The strategy surpasses its individual components in terms of annual returns, achieving an impressive 11.88%, while maintaining a relatively modest volatility of 13.22% for its returns. Furthermore, the strategy's diversification renders it a safer investment when compared to its standalone components, exhibiting an average drawdown of -4.56% and a maximum drawdown of -24.05%. These figures highlight that the portfolio's losses are less severe than those incurred through a singular investment in TLT. Moreover, the portfolio's performance outshines a simple risk-parity approach in the realm of risk-adjusted returns, boasting a Sharpe ratio of 0.89 as opposed to 0.64. It is worth noting, however, that the portfolio's higher volatility is a result of its asset allocation's dependency on the VIX. Lastly, the Kurtosis value of 17.74 underscores the strategy's intentional reliance on extreme positive and negative returns, a deliberate aspect of the asset allocation model's design.

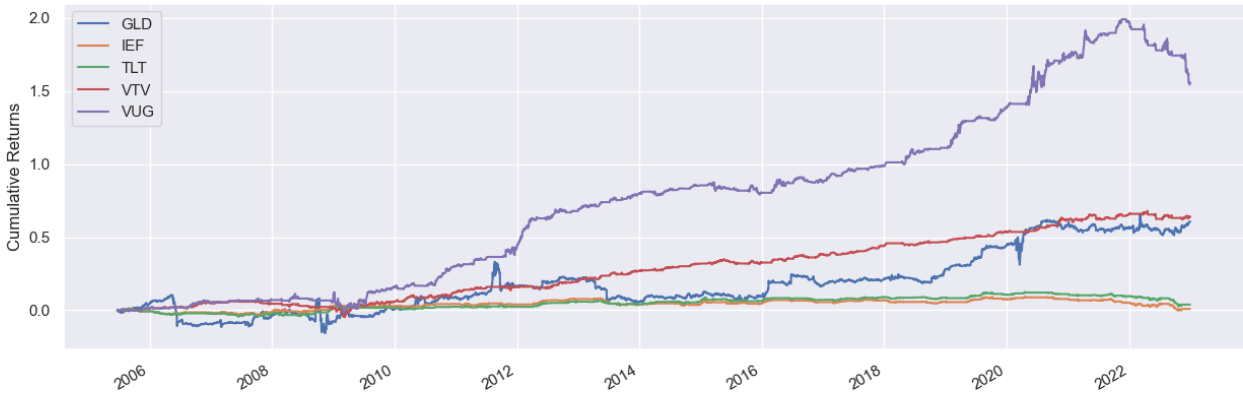

Figure 2: Attribution of performances to the individual components of the portfolio

Figure 2 presents the attribution of performance which highlights the individual contribution of each component to the portfolio. Growth stocks (VUG) heavily contributed to the portfolio’s total return because of their strong returns (10.1%) and weight in the portfolio (see Figure 3).

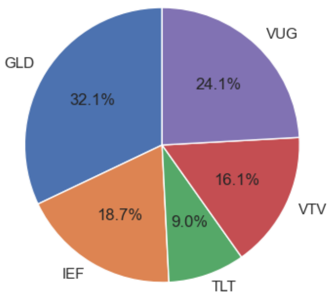

Figure 3: Average weight on each component of the portfolio

GLD and VUG consistently carry the highest weights on average in the portfolio, owing to their pronounced weights during both high and low volatility periods (refer to equations 4 and 5). Despite gold's commendable performance, yielding a 9.26% annual return and a substantial portfolio weight, its contribution to the overall portfolio remains modest. This could be attributed to a potential underutilization of gold with the VIX signal, suggesting that gold may not be the optimal asset for hedging against high volatility.

In conclusion, the strategy leverages the stock/bond relationship and employs style timing with the VIX signal to mitigate risk during periods of heightened volatility while enhancing returns during phases of low volatility.

3. Comparison of Individual Strategies

3.1 Cumulative Returns

Figure 1 presents the cumulative returns of our four investment strategies over two separate timeframes. The left panel illustrates the period from 2005 to 2016, where all three depicted strategies exhibit significant positive returns. During this phase, SUE achieves the highest cumulative return, although it exhibits pronounced volatility, particularly during the 2008 financial crisis. This contrasts with QMJ and VT, which show more consistent growth and lower volatility, suggesting greater resilience to market fluctuations. Notably, after the crisis, QMJ and SUE both demonstrate superior performance, diverging appreciably from VT starting in 2013. However, QMJ experienced a notable setback in 2016.

The right panel of *Figure 1* presents the performance from 2017 to 2022. Initially, QMJ, SUE, and VT exhibit comparable returns and experience similar levels of volatility. This trend persists until 2021, when SUE registers a significant increase in returns, outperforming QMJ and VT. In contrast, ML maintains a consistent but flat trend throughout the same period, characterized by low volatility, which is in line with previous research.

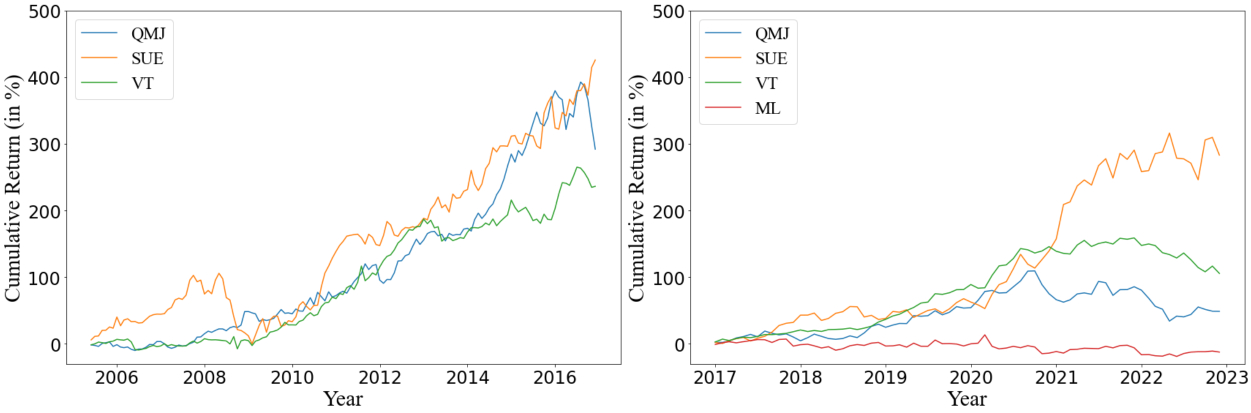

Figure 1: Cumulative Returns

3.2 Correlation of Individual Strategies

Figure 2 illustrates the correlation matrices for the returns of the four investment strategies from *Figure 1*, over two distinct periods. In both periods, QMJ and SUE exhibit a slight negative correlation, with coefficients of -0.15 and -0.17, respectively. This inverse relationship suggests that these strategies may offer diversification benefits when combined within a portfolio, as they tend to perform differently under the same market conditions.

The correlation between QMJ and VT is very weak in the first period at 0.03, rising modestly to 0.16 in the second. This trend indicates a slight increase in the similarity of their return movements over time. SUE and VT show the highest correlation in the datasets, with coefficients of 0.23 and 0.34, respectively. Although these correlations are the strongest within the matrices, they are still modest, suggesting only a moderate relationship between the strategies' returns.

ML, introduced in the second period, exhibits a notable correlation of 0.32 with QMJ, indicating a moderate positive relationship. However, its correlations with SUE and VT are -0.09 and -0.02, respectively, implying a very weak to negligible inverse relationship. This denotes that ML's performance does not closely follow that of Strategies SUE and VT.

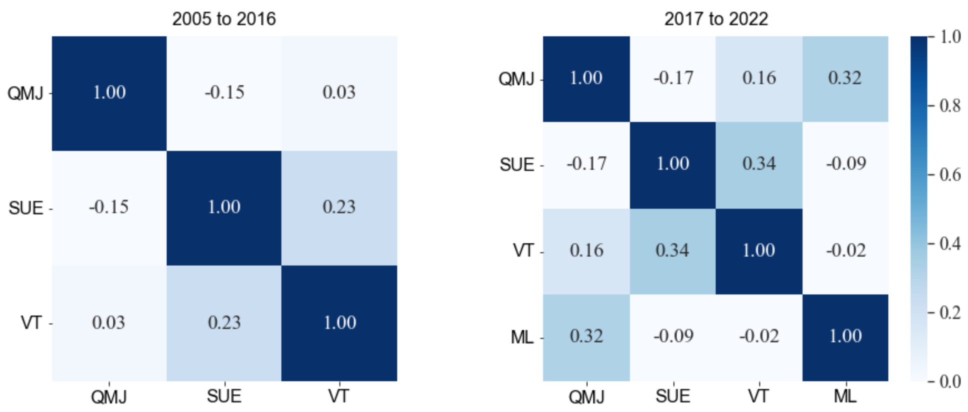

Figure 2: Correlation Matrices

3.3 Performance Analysis of Individual Strategies

Finally, we conduct an in-depth performance analysis, comparing the individual strategies. From June 2005 to December 2016, SUE yielded the highest total return of 426% and the greatest CAGR at 15.41%, but with the highest volatility, as seen by a standard deviation of 19.96%. Its risk-adjusted return, as reflected by a Sharpe ratio of 0.71, is less favorable when compared to QMJ and VT, which achieve Sharpe ratios of 0.90 and 0.82, respectively. This discrepancy can be attributed primarily to the notably lower volatility associated with QMJ and VT. Consistent with the trends observed in Figure 4, QMJ and VT exhibit more resilience during market downturns, as reflected by their substantially lower maximum drawdowns.

In the second period, VT improved its risk-adjusted performance with a Sharpe ratio of 1.22, on the back of higher returns and lower volatility. SUE's CAGR rose to 25.09%, boosting its Sharpe ratio to 1.16, despite increased risk. QMJ, however, significantly reduced annual returns while increasing volatility, resulting in a reduced Sharpe ratio of 0.42. ML, introduced in this period, experiences negative cumulative returns of -12%, translating to a negative Sharpe ratio of -0.14. This performance aligns with the challenges faced by price momentum strategies in recent years, as noted in contemporary research.

	06/2005 to 12/2016			01/2017 to 12/2022			
Strategy	QMJ	SUE	VT	QMJ	SUE	VT	ML
Total Return	292%	426%	236%	49%	283%	106%	-12%
CAGR	12.52%	15.41%	11.04%	6.86%	25.09%	12.78%	-2.16%
Standard Deviation	13.87%	19.96%	13.44%	16.25%	20.44%	10.46%	14.99%
Sharpe Ratio	0.90	0.71	0.82	0.42	1.16	1.22	-0.14
Excess Kurtosis	-1.78	-1.78	0.89	-3.00	-2.40	-2.64	-0.55
Skewness	-0.02	-0.17	-0.53	-0.23	0.31	0.22	-0.63
Best Month	15.81%	17.57%	13.10%	11.04%	20.06%	10.10%	12.39%
Worst Month	-10.66%	-14.88%	-16.60%	-11.73%	-9.86%	-5.17%	-15.21%
Positive Months	61.87%	62.59%	61.87%	55.56%	68.06%	65.28%	56.94%
Maximum Drawdown	-20.44%	-51.46%	-16.60%	-35.92%	-16.84%	-20.56%	-28.50%

Table 1: Descriptive statistics

4. Combined Strategy

4.1 Introduction

This section builds on the analysis of our individual investment strategies by exploring their combined use to potentially improve investor returns. Our goal is to create a unified investment strategy that capitalizes on the strengths of each approach. We apply the Markowitz Portfolio Optimization framework to integrate three strategies: Quality minus Junk (QMJ), Volatility Timing (VT), and Earnings Surprise (SUE). These strategies, based on long-term signals, contrast the Machine Learning-based strategy, which is computationally intensive. These strategies, oriented towards long-term signals, present a contrast to the Machine Learning-based strategy, which faces computational constraints, limiting its data scope.

We have decided to exclude the Machine Learning strategy from the combined approach. This decision is based on the extensive in-sample and out-of-sample analysis required, which the Machine Learning strategy's limited data set cannot fully support. It's important to note that this exclusion does not undermine the Machine Learning strategy’s potential effectiveness. Instead, our focus is on evaluating whether the integrated strategy, excluding Machine Learning, yields a higher Sharpe ratio compared to the performance of each strategy on its own.

4.2 Methodology

The centerpiece of our combined strategy is the Markowitz Portfolio Optimization framework, exemplified by the Tangency Portfolio. This approach combines the unique attributes of the QMJ, VT and SUE strategies into a single, optimized portfolio.

In our study, the Tangency Portfolio, constructed through the Markowitz Portfolio Optimization framework, is compared to four distinct investment models. Firstly, the 60/40 Portfolio, a traditional model, which allocates 60% to equities (S&P 500) and 40% to bonds (TLT ETF) and is renowned for its straightforwardness and conservative stance. This weighting scheme is popular among investors who aim for moderate risk. Secondly, the Equal-Weighted Portfolio which equally distributes weights across our three individual strategies, offering simple diversification without specifically targeting risk or return optimization. Thirdly, the Risk Parity Portfolio which equally balances risk across the three individual strategies based on each portfolio’s variance. This scheme is often referred to as the ‘All weather portfolio’ because its risk allocation seeks returns regardless of market conditions. Lastly, the Global Mean-Variance Portfolio seeks to minimize the portfolio's overall variance, standing in contrast to the Tangency Portfolio's focus on maximizing the Sharpe ratio.

The full sample is divided into two periods: an in-sample period (06/2005 to 12/2016) for optimizing strategy weights, and an out-of-sample period (12/2017 to 12/2022). The latter tests the optimized weights calculated in-sample to assess the portfolio's performance and robustness in a new period.

4.3 Performance Overview

4.3.1 In-Sample

4.3.1.1 Efficient Frontier

In our in-sample period, we sought the optimal asset allocation for our Tangency Portfolio. Through simulations, we constructed the efficient frontier and the feasible region to visualize (see *Figure 3*) where our different combined portfolios stand. The left side of the feasible region contains the best risk-adjusted portfolios among which are the various weight allocation schemes except for the 60/40 portfolio.

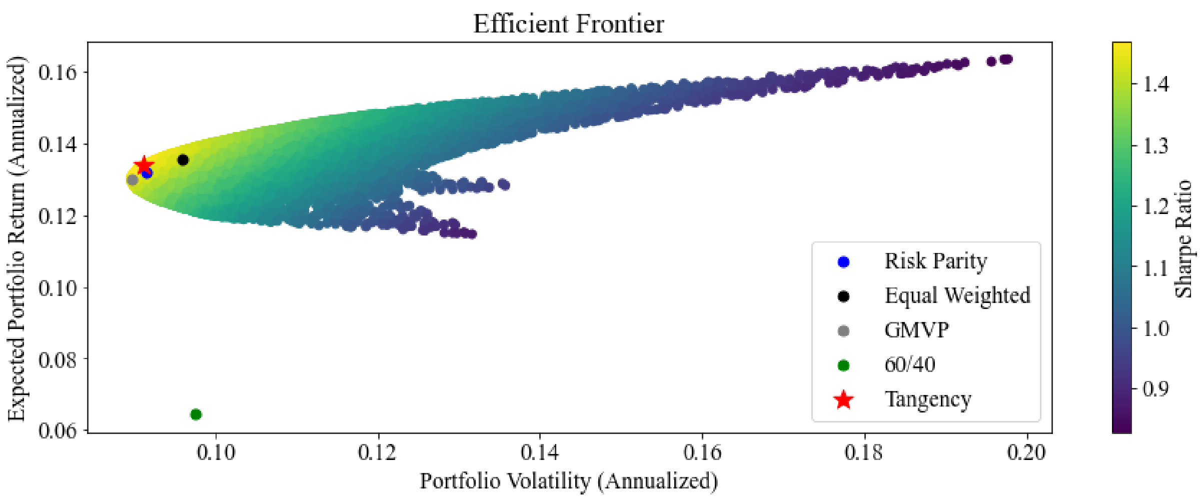

Figure 3: Efficient Frontier

As expected, the Tangency Portfolio achieves the highest Sharpe ratio, and hence, is the most desirable risk-adjusted portfolio, outperforming both the Risk Parity and Global Mean-Variance Portfolios.

	QMJ	SUE	VT
Sharpe Ratio	0.90	0.71	0.82
Weight	47.97%	25.81%	26.22%

Table 2: Tangency Portfolio Weights (06/2005 to 12/2016)

Strategy QMJ forms the backbone of the Tangency portfolio by contributing 48%, with SUE and VT also playing a substantial role. This weight distribution is not arbitrary, it echoes the individual portfolios risk-adjusted performance across the in-sample period. Indeed, the tangency portfolio seeks for the best risk-adjusted returns possible and by definition allocates more weight to the components which have the highest Sharpe ratio.

	QMJ	SUE	VT
Global Min. Variance	44.77%	19.31%	35.92%
Risk Parity	36.68%	25.48%	37.84%
Equal Weighted	33.33%	33.33%	33.33%

Table 3: Portfolio Weights (06/2005 to 12/2016)

The Global Minimum Variance Portfolio's high allocation towards QMJ indicates a preference for less variance and covariance between the assets. The Risk Parity Portfolio presents a more even distribution, with SUE having the lowest weight because of its relatively higher risk. Meanwhile, the Equal Weighted Portfolio simplifies the approach by allocating a third to each strategy, serving as a straightforward benchmark against more complex weighting methods.

4.3.1.2 Cumulative Returns of Portfolios

Examining the in-sample cumulative returns, the Tangency, Risk Parity, Equal Weighted and Global Minimum Variance portfolios exhibit strikingly similar performances over the period. Each portfolio demonstrates robust growth, with their trajectories closely aligns, especially from 2008 onwards. The 60/40 portfolio, while starting the period at a similar level, diverges and shows a more subdued performance overall.

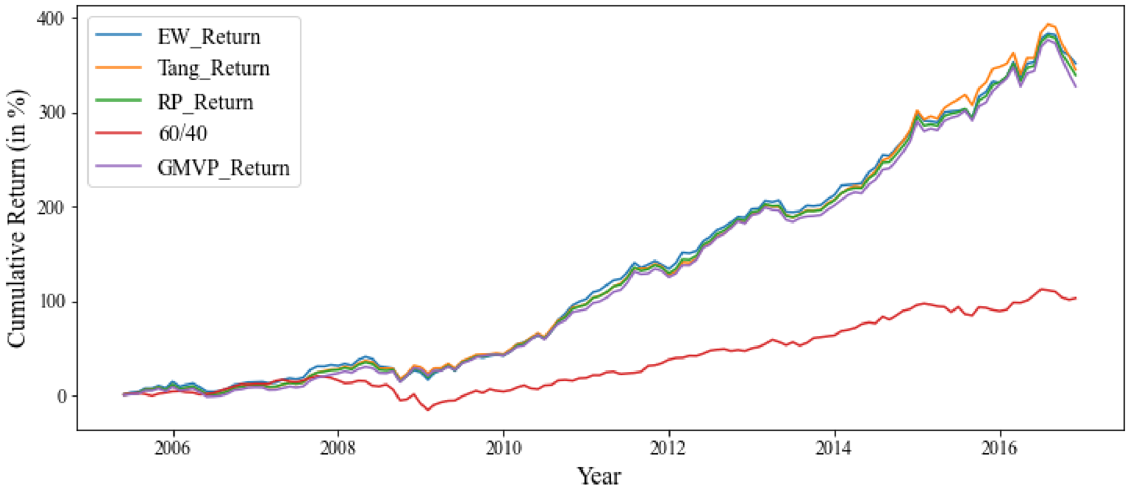

Figure 4: Cumulative Returns In-Sample

During the market turmoil of 2008, all portfolios experienced minor downturns, yet the recovery was swift and synchronized. The Tangency Portfolio, alongside the Risk Parity and Equal Weighted and Mean-Variance portfolios performed almost similarly, reflecting a shared sensitivity to the underlying market conditions, as seen in *Figure 4*. The convergence in performance suggests that, despite different strategic weightings, the portfolios were similarly capable of capitalizing on the market's performance.

4.3.1.3 Descriptive Statistics

Examining the performance statistics from the in-sample period, we observe a nuanced landscape of returns and risks. Table 4 shows that, as expected, the Tangency Portfolio stands out with a robust Sharpe ratio of 1.44, reflecting its superior risk-adjusted returns compared to the Equal Weighted and Risk Parity portfolios, which report Sharpe ratios of 1.36 and 1.39 respectively.

	Global Min. Variance	Risk Parity	Tangency	Equal Weighted	60 / 40
Total Return	328%	336%	345%	352%	103%
CAGR	13.37%	13.56%	13.76%	13.91%	6.30%
Standard Deviation	8.93%	9.11%	9.08%	9.55%	8.62%
Sharpe Ratio	1.41	1.39	1.44	1.36	0.73
Excess Kurtosis	-1.65	-0.95	-2.07	-0.93	1.95
Skewness	-0.63	-0.73	-0.57	-0.66	-1.34
Best Month	7.04%	6.71%	7.22%	7.63%	6.74%
Worst Month	-9.23%	-10.31%	-8.57%	-10.62%	-10.89%
Positive Months	73.38%	74.82%	73.38%	69.78%	63.31%
Maximum Drawdown	-12.48%	-15.52%	-14.24%	-18.45%	-29.94%

Table 4: In-Sample (2005-2016) Descriptive Statistics

In contrast, the 60/40 portfolio, while offering a respectable total return of 103%, does so at the cost of heightened risk relative to returns, as evidenced by its lower Sharpe ratio of 0.73 and a more pronounced maximum drawdown of nearly 30%. This underlines a trade-off between higher returns and increased risk exposure for the 60/40 portfolio. From a risk management perspective, the Tangency Portfolio presents an advantageous profile. Its maximum drawdown is limited to -14.24%, reflecting a well-managed risk level. Furthermore, its skewness and excess kurtosis suggest a relatively symmetrical distribution of returns with thinner tails than the Global Minimum Variance Portfolio, implying fewer extreme returns.

The Tangency portfolio proves to be a great diversifier of risk with substantially higher risk-adjusted returns than any of its individual components (Sharpe ratio of 1.44 vs 0.81 on average). Also, it shows higher total and annual returns than two of its three components which highlights once again its diversification effect where it improves returns by maintaining or lowering its risk.

4.3.2 Out-of-Sample

4.3.2.1 Cumulative Returns of Portfolios

In the out-of-sample period, the performance of the portfolios, when applying the optimal weights identified in the in-sample phase, presents a similar narrative where all the portfolios except the 60/40. The Tangency Portfolio, in particular, showcases its robustness. Its growth trajectory maintains proximity with the Risk Parity, Equal Weighted, and Global Minimum Variance portfolios, affirming the effectiveness of its strategic allocation.

Throughout this period, the Tangency Portfolio navigates market fluctuations with a resilience that echoes its in-sample performance. The 60/40 portfolio, again, underperforms relative to the other portfolios, which might be due to the difference in the composition of its components. The four other portfolios are composed majorly of stocks, resulting in this gap of performance. This contrast highlights the adaptive nature of the Tangency Portfolio, which, thanks to its foundation in the Markowitz optimization framework, could be more attuned to shifting market dynamics than the traditional 60/40 mix.

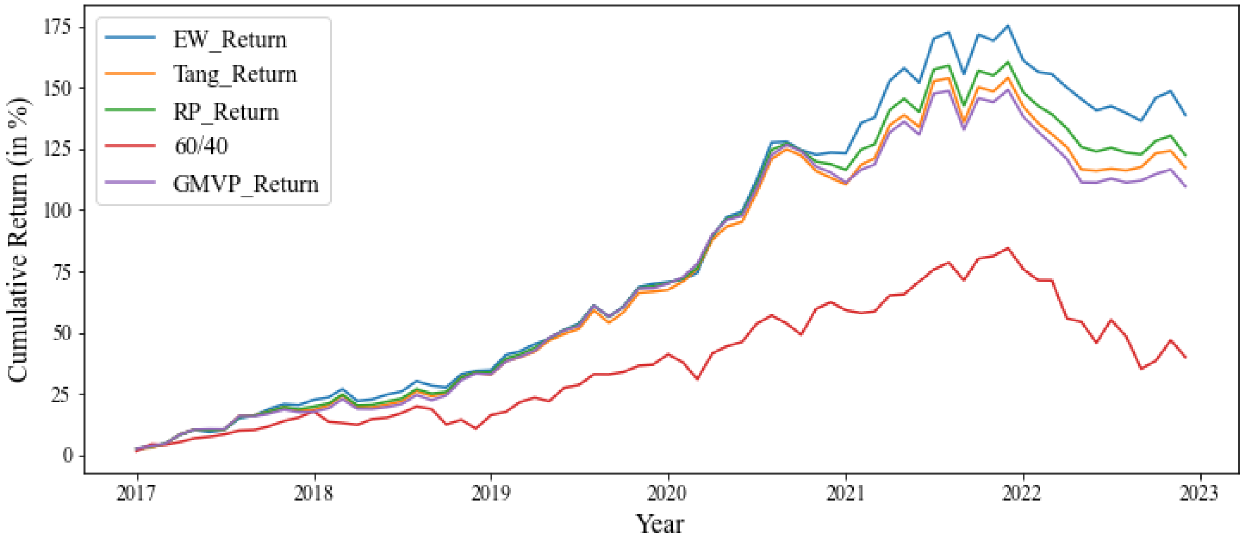

Figure 5: Cumulative Returns Out-of-Sample

As Figure 5 shows, after an initial period of parallel growth, the Tangency Portfolio, along with the other portfolios, faces a challenging period from 2020 onwards due to the Covid-

19 crisis. It's in this challenging climate that the portfolio's strategic weighting truly proves its mettle, avoiding the sharper declines seen in the 60/40 portfolio and demonstrating a disciplined return profile. This period validates the diversification and risk management principles that are integral to the Tangency Portfolio's design and affirms its potential as a resilient investment strategy in the face of market adversity.

4.3.2.2 Descriptive Statistics

Analyzing the Out-of-Sample performance (see Table 5), the Tangency Portfolio distinguishes itself with a commendable Sharpe ratio of 1.32. This figure not only surpasses the Global Minimum Variance Portfolio's 1.30 but also notably outperforms the traditional 60/40 portfolio's ratio of 0.50, underscoring its superior risk-adjusted returns. The Tangency's closest competitors, the Risk Parity and Equal Weighted portfolios, register Sharpe ratios of 1.44 and 1.49, respectively, suggesting that while the Tangency Portfolio does not lead in this measure, it holds its ground well within this competitive set.

	Global Min. Variance	Risk Parity	Tangency	Equal Weighted	60 / 40
Total Return	110%	124%	117%	139%	40%
CAGR	13.14%	14.41%	13.81%	15.61%	5.77%
Standard Deviation	9.60%	9.48%	9.97%	9.91%	11.55%
Sharpe Ratio	1.30	1.44	1.32	1.49	0.50
Excess Kurtosis	-2.93	-2.75	-2.74	-2.66	-2.21
Skewness	-0.02	0.08	-0.04	0.12	-0.55
Best Month	7.24%	7.74%	7.98%	8.36%	8.10%
Worst Month	-6.34%	-6.04%	-7.04%	-6.26%	-9.05%
Positive Months	68.06%	70.83%	69.44%	70.83%	65.28%
Maximum Drawdown	-15.79%	-14.69%	-14.97%	-14.10%	-26.70%

Table 5: Out-of-Sample (2017-2022) Descriptive Statistics

The Tangency Portfolio's out-of-sample Sharpe ratio reflects the performance shift of QMJ, which, despite a robust in-sample Sharpe ratio of 0.90, declined to 0.42 in the out-of-sample period. This significant reduction is attributed to its increased standard deviation, up from 13.87% to 16.25%, and a notable decline in annualized returns from 12.52% to 6.86%. The heavy weighting of 47.97% in QMJ within the Tangency Portfolio was based on its in-sample strength, yet the strategy's out-of-sample volatility and lower performance underlined the challenges of relying on past performance as a predictor of future returns.

Despite a lower total return compared to the Equal Weighted and Risk Parity portfolios, the Tangency Portfolio's Sharpe Ratio proofs its balance of risk and reward, particularly when considering the 60/40 portfolio's trade-off between higher returns and risk exposure. The 60/40's higher volatility is reflected in its significantly larger maximum drawdown of -26.70% compared to the Tangency's -14.97%.

The Tangency Portfolio's skewness and kurtosis values point towards a more symmetric return distribution with fewer extreme deviations than its peers, indicating well-managed tail risk. Additionally, the Tangency Portfolio's consistency is evident with positive returns in 69.44% of the months, a testament to its strategic composition and the benefits of diversification, despite the challenging market conditions faced during the out-of-sample period.

5. Factor Analysis

In the subsequent analysis, we build upon the descriptive statistical insights from previous chapters to examine the risk factors influencing portfolio returns, utilizing the Capital Asset Pricing Model (CAPM) and the three-factor Fama-French (FF3) model. These models serve as a framework to explain the underlying risks that are hypothesized to influence portfolio performance. The CAPM is articulated through the following equation:

$$R_{p,t} - R_{f,t} = \alpha + \beta_1(R_{m,t} - R_{f,t}) + \varepsilon_t,$$

where $R_{p,t} - R_{f,t}$ represents the excess return of the portfolio over the risk-free rate, and $R_{m,t} - R_{f,t}$ represents the excess return of the market. The coefficient α captures the portfolio's abnormal returns, while β_1 quantifies the portfolio's sensitivity to market movements, with ε_t denoting the idiosyncratic, non-systemic component of return.

Expanding upon the CAPM, the Fama-French three-factor model integrates additional dimensions of risk associated with size and value:

$$R_{p,t} - R_{f,t} = \alpha + \beta_1(R_{m,t} - R_{f,t}) + \beta_2(SMB_t) + \beta_3(HML_t) + \varepsilon_t.$$

In this formula, *SMB* and *HML* introduce size and value factors, respectively. The coefficients β_2 and β_3 measure the portfolio's responsiveness to these additional risk factors. Collectively, these parameters estimate the degree to which market, size, and value factors systematically inform portfolio returns.

Table 6 presents the regression outputs for both models. Under the CAPM, the tangency portfolio consistently shows significant monthly excess returns of 0.95% across all periods analyzed. The portfolio's market beta ranges from 0.147 to 0.181, indicating lower volatility compared to the broader market and low exposure to market risk. However, the low R^2 values (0.068 to 0.086) suggest that the CAPM model has limited ability to explain the portfolio's variance, pointing towards the influence of factors beyond market risk. Considering the CAPM, the results are consistent throughout all periods.

	Full Sample (2005 – 2022)	In-Sample (2005 – 2016)	Out-of-Sample (2017 – 2022)
α	0.0095^{***}	0.0095^{***}	0.0095^{***}
β_M	0.1668^{***}	0.1814^{***}	0.1474^{**}
R^2	0.078	0.086	0.068

	Full Sample (2005 – 2022)	In-Sample (2005 – 2016)	Out-of-Sample (2017 – 2022)
α	0.0090***	0.0093***	0.0081***
β_M	0.2196***	0.2278***	0.1961***
β_{SMB}	-0.1389*	-0.0445	-0.2831**
β_{HML}	-0.2346***	-0.1990**	-0.2762***
R^2	0.167	0.123	0.281

Note: *, **, *** refer to the significance level of 90, 95, and 99 percent, respectively.

Table 6: Regression Results of Tangency Portfolio for CAPM & FF3 Model

In the Fama-French three-factor model analysis, the tangency portfolio consistently showcases a notable monthly excess return of 0.90% across the entire period, reaffirming its strong performance. The model further reveals the portfolio's relatively low exposure to broader market risk, as evidenced by its lower volatility compared to the market.

A significant shift is observed in the portfolio's sensitivity to company size. The negative coefficient for the *SMB* factor increases from an insignificant -0.045 in the In-Sample period to a significant -0.283 in the Out-of-Sample period. This transition highlights an increased inclination towards larger firms in the latter period. Concurrently, the negative coefficients for the *HML* factor, -0.199 and -0.276, underline a pronounced preference for growth stocks over value stocks. These coefficients collectively suggest a strategic shift in the portfolio towards larger, growth-oriented investments.

6. Limitations

This section delves into the diverse implementation issues that may arise when applying the combined strategy or its variants in real-life scenarios.

Firstly, a major limitation involves the various estimators used, particularly for the tangency portfolio, or the different weighting schemes employed for comparison. As discussed previously, the estimators for expected returns and volatility used in the weighting schemes are based on past data. This issue is commonly referred to as 'parameter uncertainty' in literature, highlighting the restrictive assumptions inherent in models such as Markowitz's framework. The framework maximizes a single-period utility, assumes investors are myopic, markets are frictionless, and risk aversion is constant. These assumptions can lead to several potential errors if omitted or unstated. DeMiguel, Martin-Utrera, and Nogalas (2015) argue that, according to Garlenau and Pedersen's (2013) framework and in the absence of estimation error, the optimal portfolio is to trade toward the Mean-Variance. However, this optimal portfolio is sensitive to estimation error and trading costs. Numerous studies, such as Kan and Zhou (2007), have developed models to manage estimation errors.

Secondly, for the sake of simplicity and to focus on performance analysis, costs were initially excluded from the implementation of the combined strategy but will be addressed here. Costs, such as transaction or brokerage fees, are often overlooked in research where markets are assumed to be frictionless, yet they can significantly impact the execution of quantitative investment or trading strategies. In the combined strategy, there are no transaction costs as the weights are constant, and there is no rebalancing, but, in practice, transaction costs arise from the individual strategies constituting the portfolio. Additionally, volume is a crucial factor to consider in trading strategies, as some assets in the strategy may not be available for trade or might be available in limited quantities due to the respective illiquidity of their market.

7. Conclusion

In summary, the combined strategy validates the diversification of portfolio risk, outperforming individual strategies. Both the tangency portfolio and portfolios employing various weight allocation schemes demonstrate superior risk-adjusted returns compared to their individual components. This affirms that regardless of the weight allocation, diversification is a powerful strategy and a ‘free lunch’ as often described in literature.

Among the various combined portfolios, the tangency portfolio exhibits the best in-sample performance, yielding a total return of 345% and a Sharpe ratio of 1.44, aligning with expectations. According to the Markowitz framework, the tangency portfolio represents the optimal investment choice from a risk-adjusted perspective. Notably, QMJ significantly contributes to the tangency portfolio, because of having the highest Sharpe ratio among the three components. The remaining combined portfolios perform similarly, except for the simple 60/40 approach, which is markedly outperformed by the optimized portfolios.

However, the tangency portfolio falls short in out-of-sample performance, being surpassed by both the risk-parity and equal-weight approaches. This shift in comparative performance is attributed to the substantial weight assigned to QMJ (47.97%), whose out-of-sample performance notably declined (from a Sharpe ratio of 0.90 to 0.42). This downturn impeded the tangency portfolio during this subsequent period. The decline in the Sharpe ratio observed in the QMJ portfolio during the out-of-sample period can be attributed to the relatively improved performance of the Big Junk portfolio in this timeframe. Notably, the Big Junk portfolio experienced approximately 130% cumulative returns in the out-of-sample period. This significant gain adversely impacted the returns of the QMJ portfolio, which is structured to be short on the Big Junk portfolio.

To conclude, Markowitz’s framework and popular weight-allocation scheme proves to be great diversifiers of risk. Also, the weights allocated by the tangency portfolio seems not to be working as well out-of-sample period than in-sample, which emphasizes overfitting as the strategy seems not robust.

1. Introduction

The Efficient Market Hypothesis (EMH), a cornerstone of modern investment theory proposed by Fama (1970), states that stock prices in efficient markets fully incorporate all available information. This hypothesis, while foundational, has been challenged by persistent anomalies that suggest subtler dynamics at play in financial markets. Among these, the post-earnings announcement drift (PEAD) identified by Ball and Brown (1968) and the concept of price momentum documented by Jegadeesh and Titman (1993), both of which were later acknowledged by Fama (1998), stand out as significant exceptions. PEAD, often termed the 'granddaddy of underreaction events', indicates that stock prices continue to react to earnings information well after their announcement, suggesting a delayed market response. This phenomenon is especially evident in firms experiencing 'good news' – characterized by high standardized unexpected earnings (SUE) – which typically outperform their counterparts facing 'bad news', indicated by low SUE. Similarly, price momentum, the tendency of stocks to continue their past performance trajectory, presents a challenge to the EMH's assertion of immediate information reflection in prices, with past winners expected to outperform past losers in the near future.

In this paper, I aim to bridge these anomalies with a multifactor earnings surprise investment strategy. By combining information from prior returns, earnings surprises, and revenue surprises, the strategy aims to exploit the market's inefficiencies and delayed information processing. This approach not only challenges the traditional understanding of market efficiency but also proposes a practical framework for leveraging these anomalies. This study aims to demonstrate that a multifactor strategy not only yields excess returns but also consistently outperforms its univariate and bivariate counterparts based on the same factors.

2. Literature Review

The exploration of market anomalies, especially post-earnings announcement drift and price momentum, has been a focal point in financial literature for over five decades. Initially identified by Ball and Brown (1968), PEAD has been continuously examined and validated by researchers, most convincingly by Bernard and Thomas (1989). In their paper, they showed that a zero-investment portfolio, capitalizing on PEAD, could generate annualized abnormal returns of 18% before transaction costs in the quarter following an earnings surprise. Building on this, Chan, Jegadeesh, and Lakonishok (1996) expanded the scope of analysis by illustrating that both past returns and past earnings surprises are significant predictors of future returns, underscoring the market's underreaction to various types of information. Decades later, the persistence of PEAD was reaffirmed by Jegadeesh and Livnat (2006). They discovered that the drift became more pronounced in cases where firms reported not only unexpected earnings but also unexpected revenue, suggesting joint implications.

More recently, Chen et al. (2014) analyzed the profitability of revenue, earnings, and price momentum strategies in the U.S. market from 1974 to 2009. Their research revealed a persistent trend of market underreaction to each of these individual pieces of information. Significantly, their findings suggest that a multifactor strategy, which combines these three factors – especially when they are congruent – can yield monthly returns of 1.44%, highlighting the potential of multifactor momentum strategies. Building upon this, Sehgal and Jain (2015) applied a similar multivariate momentum strategy in the Indian market, corroborating Chen et al. (2014) findings. Their research demonstrated that a triple-sorted momentum strategy in India could achieve monthly returns of 2.28%.

This paper contributes to existing literature by exploring a multifactor momentum strategy on a U.S. index, similar to Chen et al. (2014), and extending the evaluation period to cover the past two decades.

3. Framework

3.1 Strategy

In this paper, I implement a multifactor momentum strategy based on three factors: earnings surprises, revenue surprises, and prior returns. Each factor forms the basis of a distinct univariate investment strategy and is independently calculated. These strategies are then synergistically combined to create the multifactor strategy.

For each univariate strategy, five market-value weighted portfolios are formed each month, based on the prior quarter's performance. For example, in the context of revenue surprises, the top-performing portfolio (R5) comprises firms with the most positive revenue surprise from the previous quarter, while the lowest-performing portfolio (R1) includes those with the most negative surprise.

The bivariate and multivariate strategies are then formulated by intersecting and uniting these univariate portfolios. A bivariate portfolio might, for instance, include firms that rank highest in both earnings and revenue surprises. Expanding on this concept, a multivariate portfolio, such as P5xE5xR5, is created by combining the highest-performing univariate portfolios across all three factors, capturing firms that demonstrate the most positive performance in prior returns, earnings surprise, and revenue surprise in the previous quarter.

The methodologies used to compute earnings and revenue surprises, as well as the calculation of price momentum will be detailed in Chapter 3.3.

3.2 Data

The data for this study was sourced from the Center for Research in Security Prices (CRSP) and Compustat databases. CRSP provided daily stock prices, returns, and shares outstanding, along with monthly data on 30-day U.S. Treasury bills, which served as the risk-free rate proxy. From Compustat, basic firm and quarterly accounting data was obtained, including earnings announcement dates, book values, revenues, and earnings per share, adjusted for splits, dividends, and acquisitions. The datasets were merged through the linking table from the CRSP/Compustat Merged Database, using unique identifiers for each database, namely PERMNO and GVKEY.

The dataset incorporates all firms in the S&P 500 index as of December 2022 and additionally includes prior constituents that exited the index after 2000, removing survivorship bias. The sample period is January 2000 until December 2022, however, to calculate revenue and earnings surprises, companies in the study must have 12 consecutive earnings announcements and three months of security returns before each formation date. As a result, the analysis of returns begins in 2003, providing a 20-year timeframe for assessing the efficacy of the trading strategy. The sample includes, on average, 733 firms annually, ranging from a high of 796 firms in the year 2000 to a low of 644 firms in 2022. The observed decline in firm numbers over the sample period can be attributed to limited survivorship due to a variety of factors such as bankruptcies, mergers, and acquisitions.

3.3 Methodology

The methodology section commences by establishing the signals utilized for each factor. In prior literature, two predominant methods are employed to estimate expected earnings and revenue: one based on historical financials and the other on analysts' forecasts. This study adopts the approach based on historical earnings as highlighted by Jegadeesh and Livnat (2006), which states that quarterly earnings typically follow a seasonal random walk with an underlying drift. To account for seasonality, the expected earnings for a given quarter are projected using the earnings from the same quarter in the preceding year, rather than the immediately preceding quarter. For this paper, earnings surprises are measured by the standardized unexpected earnings, defined as

$$SUE_{i,t} = \frac{Q_{i,t} - E(Q_{i,t})}{\sigma_{i,t}}, \qquad (1)$$

where $Q_{i,t}$ is the quarterly earnings per share (EPS) for firm i in quarter t, $E(Q_{i,t})$ is the expected EPS for firm i in quarter t, and $\sigma_{i,t}$ is the standard deviation of earnings growth over the preceding eight quarters. The expected earnings $E(Q_{i,t})$ and standard deviation of earnings growth $\sigma_{i,t}$, respectively, are defined as

$$E(Q_{i,t}) = Q_{i,t-4} + \delta_{i,t} \qquad (2)$$

and,

$$\sigma_{i,t} = \frac{1}{7}\sqrt{\sum_{j=1}^{8}\left(Q_{i,t-j} - E(Q_{i,t-j})\right)^2}, \qquad (3)$$

where the drift term, $\delta_{i,t}$, is the average EPS growth over the past eight quarters, defined as

$$\delta_{i,t} = \frac{\sum_{j=1}^{8}(Q_{i,t-j} - Q_{i,t-j-4})}{8}. \qquad (4)$$

Analogously, revenue surprises are measured by the standardized unexpected revenue surprise estimate (SURGE), defined as

$$SURGE_{i,t} = \frac{R_{i,t} - E(R_{i,t})}{\sigma_{i,t}}, \qquad (5)$$

where $R_{i,t}$ is the quarterly revenue for firm i in quarter t, $E(R_{i,t})$ is the expected revenue for firm i in quarter t, and $\sigma_{i,t}$ is the standard deviation of revenue growth over the past eight quarters.

For the price momentum factor, quintile portfolios are constructed using the cumulative return of the three months preceding the portfolio formation month. Consequently, the highest-performing portfolio (P5) in the univariate strategy encompasses past winners, while the lowest-performing portfolio (P1) includes past losers of the previous three months.

This study calculates portfolio returns monthly, sorting firms at each month's end into portfolios based on recent SUE, SURGE, and past returns. Portfolios are held until the next month's end for performance assessment. Subsequently, portfolios are rebalanced, and the cycle is repeated.

4. Results

4.1 Correlation of Momentum Factors

In testing the hypothesis that combining multiple momentum factors – revenue surprises, earnings surprises, and prior returns – can enhance strategy returns, this study first examines whether each factor contributes unique information to the strategy's profitability, before analyzing and comparing the constructed portfolios. The premise is that while these factors may overlap in the information they convey, each one also offers distinct insights that could lead to stronger performance when combined.

Therefore, the matrix in Figure 6 outlines the correlations between earnings surprises, revenue surprises, and prior returns. As expected, SUE and SURGE are positively correlated at 0.32, reflecting their intrinsic relationship within the income statement, where revenue serves as a primary driver of earnings. The correlations of prior returns with SUE and SURGE, however, are relatively low, at 0.10 and 0.06, respectively, indicating that market performance, as captured by prior returns, shares limited commonality with these accounting-based measures. These distinct correlation values support the notion that SUE, SURGE, and prior returns contribute unique information to the analysis, which is consistent with the findings reported by Jegadeesh and Livnat (2006) and Chen et al. (2014).

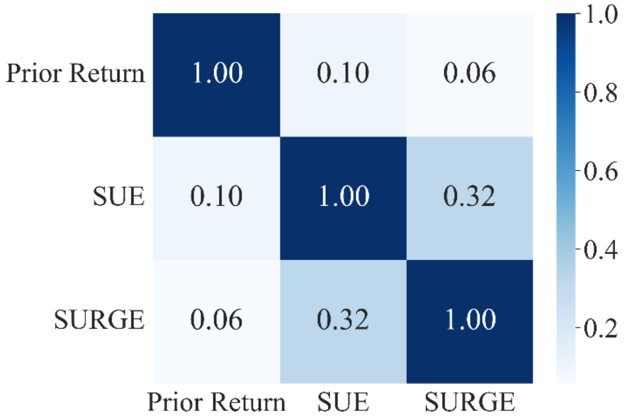

Figure 6: Correlation Matrix between Factors

4.2 Comparison of Momentum Portfolios

This section begins with a comparison of the three univariate momentum strategies. Aligned with Chordia and Shivakumar (2006), I assess the performance of the quintile portfolios for each factor and also create a zero-investment portfolio for each – going long in the highest quintile and short in the lowest – to effectively isolate the core momentum effects.

Table 7 summarizes the annualized mean excess returns (ER) for the three univariate momentum strategies. For price momentum, an irregular pattern emerges: P5 and P4 lead with returns of 9.66% and 8.81%, while P1 unexpectedly ranks third at 7.02%, breaking the anticipated trend. The earnings momentum strategy shows a clearer pattern, with E5 topping at 9.66% and E1 at the bottom at 4.55%, although E3 and E4 slightly deviate from a linear progression. For revenue momentum, R5 leads with 9.54% and R1 is the lowest at 4.43%, but the trend is again disrupted by R3 surpassing R4.

Notably, the returns of all three lowest quintile portfolios are not statistically significant, implying that the loser portfolios may not significantly differ from random market variations and that momentum effects might be predominantly present in the winner portfolios.

	Mean ER	t-value		Mean ER	t-value		Mean ER	t-value
P1	0.0702	1.36	E1	0.0455	1.23	R1	0.0443	1.19
P2	0.0686	1.94	E2	0.0724	2.07	R2	0.0665	1.95
P3	0.0515	1.55	E3	0.0806	2.38	R3	0.0877	2.54
P4	0.0881	2.74	E4	0.0783	2.44	R4	0.0831	2.48
P5	0.0966	2.68	E5	0.0966	2.83	R5	0.0954	2.79
P5 – P1	-0.0034	-0.09	E5 – E1	0.0440	2.64	R5 – R1	0.0432	2.31

Table 7: Univariate Strategies

The zero-investment portfolios for earnings and revenue surprises (E5 – E1 and R5 – R1) indicate positive and statistically significant mean returns of 4.40% and 4.32%, respectively, suggesting that these surprise factors provide valuable signals for creating profitable, selffinancing trading strategies. Conversely, the price momentum zero-investment portfolio (P5 – P1) demonstrates a negligible and statistically insignificant mean excess return, suggesting a lack of consistent price momentum effect across the assets within the period studied.

Transitioning from univariate to bivariate analysis, Table 8 explores the performance of portfolios combining two factors. It reveals a typical momentum trend, with higher quintiles such as P5xE5 and P5xR5 yielding the highest annual returns of 13.42% and 12.80%, respectively, both showing statistical significance. In contrast, the lowest quintiles, including P1xR1 and P1xE1, consistently show weaker performance with returns of 2.13% and 2.81%, though these are again not statistically significant, indicating possible random variations rather than a consistent negative trend. Notably, in the zero-investment portfolios, the significant return of 7.70% for E5xR5 – E1xR1 stands out. This underscores the efficacy of strategies that combine the power of earnings and revenue surprises, as opposed to those involving price momentum, which yield positive but statistically insignificant returns.

	Mean ER	t-value		Mean ER	t-value		Mean ER	t-value
P1xE1	0.0281	0.50	P1xR1	0.0213	0.36	E1xR1	0.0247	0.59
P2xE2	0.0549	1.37	P2xR2	0.0765	1.98	E2xR2	0.0708	1.81
P3xE3	0.0502	1.35	P3xR3	0.0820	2.29	E3xR3	0.0915	2.42
P4xE4	0.1059	3.24	P4xR4	0.0949	2.88	E4xR4	0.0883	2.56
P5xE5	0.1342	3.35	P5xR5	0.1280	3.37	E5xR5	0.1151	3.00
P5xE5 – P1xE1	0.0594	1.20	P5xR5 – P1xR1	0.0567	1.16	E5xR5 – E1xR1	0.0770	2.82

Table 8: Bivariate Strategies

Building upon the bivariate findings, Table 9 presents the performance of the multivariate strategy that combines all three momentum factors. Notably, the P5xE5xR5 portfolio emerges as the standout, achieving statistical significance with the highest annualized excess return of 18.31%, indicating the multifactor strategy's potential. However, when examining the zero-investment strategy (P5xE5xR5 – P1xE1xR1), the return, although positive, does not reach statistical significance. The lack of strong statistical backing for the zero-investment strategy implies that the benefits of the top-tier portfolio are potentially offset by the lower-tier portfolio to some degree, revealing complexities in the multivariate approach. It highlights that the multivariate approach, although theoretically sound, requires further empirical testing to validate its reliability in practice. Nevertheless, since my primary investment strategy involves going long the top-quintile portfolio P5xE5xR5, the statistical insignificance of the zero-investment portfolio does not affect the further analysis and significance of the main strategy.

	Mean ER	t-value
P1xE1xR1	-0.0088	-0.13
P2xE2xR2	0.0862	1.81
P3xE3xR3	0.0639	1.47
P4xE4xR4	0.0754	1.69
P5xE5xR5	0.1831	4.09
P5xE5xR5 – P1xE1xR1	0.1170	1.85

Table 9: Multivariate Strategy

Concluding this chapter, it is evident that the strategies do not always follow the expected behavior nor are the results always statistically significant. However, the primary finding of this chapter is clear and confirms one of my research questions: the top portfolio in the multivariate strategy, P5xE5xR5, achieves significant excess returns and outperforms its univariate and bivariate counterparts, highlighting the joint implications from a multifactor approach.

4.3 Performance Analysis

The following chapter will delve into a detailed analysis of selected portfolios, specifically contrasting the top quintile portfolios from the univariate, bivariate, and multivariate strategies to further show the multivariate strategy's dominance. This comparison aims to substantiate the premise that factor integration can lead to superior investment strategies, a hypothesis not fully captured by the comparison of annualized returns alone, which omits an important factor, namely risk. Figure 7 illustrates the cumulative returns of all seven top-quintile portfolios, along with the market for benchmarking purposes. Consistent with expectations, the cumulative graph shows the multivariate strategy outpacing all other portfolios, including the market, over the entire sample period. Although the multivariate strategy's stark outperformance from 2021 onwards presents a challenge in visually scaling and comparing performances, it remains clear that all top-quintile portfolios have outperformed the market.

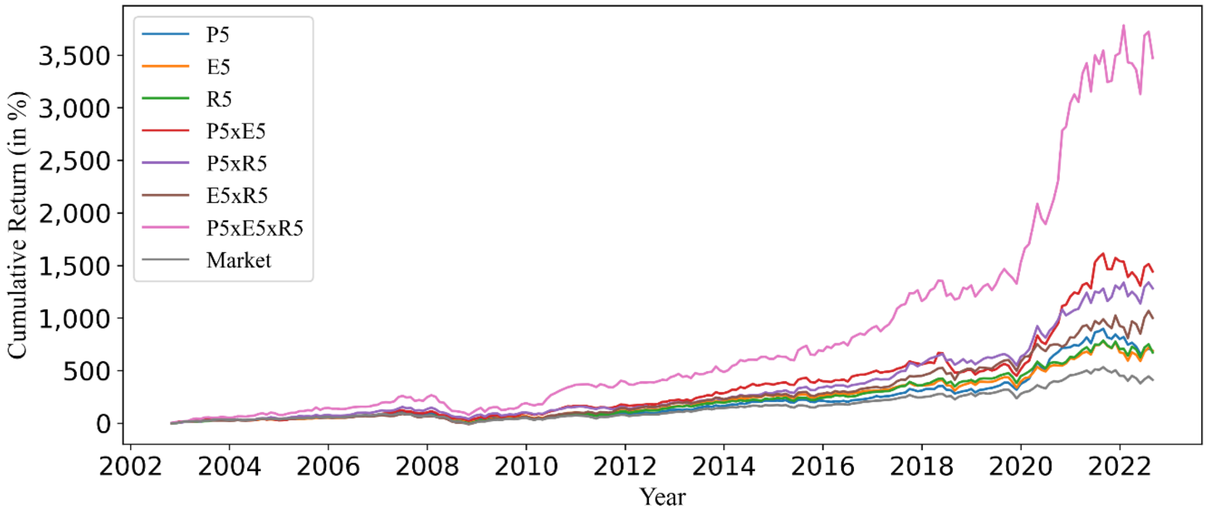

Figure 7: Cumulative Returns (2003 to 2022)

Diving into more detail, Table 10 contrasts relevant descriptive statistics of the primary strategy, P5xE5xR5, with other top-quintile univariate and bivariate portfolios, alongside the market as benchmarking. P5xE5xR5, despite its high annual volatility (19.86%), achieves the most favorable risk-adjusted performance with a Sharpe ratio of 0.95, confirming the superior returns detailed previously. The bivariate portfolios, P5xE5 and P5xR5, follow closely with Sharpe ratios of 0.77 each. All top quintile portfolios, including P5xE5xR5, outperform the market in risk-adjusted returns. Importantly, they do so without significantly increased exposure to extreme return fluctuations. This is evidenced by their similar maximum drawdown and distribution metrics such as excess kurtosis and skewness compared to the market.

In conclusion, P5xE5xR5 stands out for its superior risk-adjusted performance compared to other top-quintile portfolios and the market, further confirming the dominance of the multifactor strategy.

	P5	E5	R5	P5xE5	P5xR5	E5xR5	P5xE5xR5	Market
Total Return	711%	718%	750%	1,513%	1,338%	1,067%	3,722%	420%
CAGR	11.13%	11.18%	11.39%	15.05%	14.38%	13.19%	20.17%	8.67%
Standard Deviation	16.05%	15.18%	15.06%	17.77%	16.92%	17.03%	19.86%	14.85%
Sharpe Ratio	0.61	0.65	0.67	0.77	0.77	0.70	0.95	0.50
Excess Kurtosis	-1.82	-2.06	-2.05	-2.19	-2.57	-1.90	-2.04	-1.45
Skewness	-0.03	-0.28	-0.32	0.23	-0.03	0.07	0.01	-0.62
Best Month	16.31%	13.26%	13.58%	18.04%	15.38%	17.53%	20.06%	12.91%
Worst Month	-15.75%	-14.19%	-15.03%	-11.97%	-11.49%	-14.41%	-14.88%	-17.20%
Positive Months	62.18%	63.45%	63.87%	61.76%	64.71%	60.92%	64.71%	65.55%
Maximum Drawdown	-47.75%	-46.44%	-48.44%	-44.54%	-45.35%	-52.40%	-51.46%	-51.62%

Table 10: Descriptive Statistics Summary

4.4 Factor Analysis

Building upon the descriptive statistics analyzed in the preceding chapter, I now apply the Capital Asset Pricing Model (CAPM) by Black, Jensen, and Scholes (1972) and the three-factor model (FF3) by Fama and French (1993) to capture the potential risk factors associated with the portfolio returns. For this, I apply the following formulas:

$$R_{p,t} - R_{f,t} = \alpha + \beta_1 (R_{m,t} - R_{f,t}) + \varepsilon_t \quad (6)$$

$$R_{p,t} - R_{f,t} = \alpha + \beta_1 (R_{m,t} - R_{f,t}) + \beta_2 (SMB_t) + \beta_3 (HML_t) + \varepsilon_t. \quad (7)$$

Where $R_{p,t} - R_{f,t}$ is the excess portfolio return, $R_{m,t} - R_{f,t}$ is the excess market return, and SMB_t and HML_t are size and value factors. The coefficients in the equations – α for abnormal returns and β for market, size, and value sensitivities – reflect how these factors influence portfolio returns, with ε representing residual effects.

The market index used for calculating excess market returns is the S&P 500, adjusted for historical constituents, aligning with the market definition provided in Chapter 3.2. The size factor, SMB, is derived from the performance differentials between small and large companies, while the value factor, HML, reflects the divergence in returns between high and low book-to-market equities. The construction of the Fama-French risk factors follows the approach by Asness et al. (2018), utilizing the 80th percentile for market capitalization to categorize size and the 30th and 70th percentiles of the book-to-market ratio for the value breakpoint.

The regression results, summarized in Table 11, indicate that the Fama-French model explains a slightly larger proportion of the variance in portfolio returns (R^2 = 0.478) compared to the CAPM (R^2 = 0.468). Under the CAPM, the multifactor strategy is shown to generate monthly excess returns of 0.95%, with a market beta (β_M = 0.92) slightly below 1. This indicates a strong market exposure and volatility marginally lower than the broader market.

In the FF3 model, which controls for size and value, the portfolio still exhibits a statistically significant monthly excess return of 0.77%, at the 99% significance level. This outcome further strengthens the findings in Table 10, which highlights the portfolio's performance relative to the market. The estimated market beta (β_M = 0.92) under FF3 confirms the market exposure identified by the CAPM. The positive SMB coefficient (β_2 = 0.32) indicates a tilt towards smaller-cap stocks. This aligns with the findings of Bernard and Thomas (1989), who observed that small firms experienced significantly larger post-earnings announcement drifts compared to large firms (5.3% vs 2.8% abnormal returns over 60 days following earnings announcements). Conversely, the negative HML coefficient (β_3 = -0.12) suggests an inverse relationship with value stocks, although this result is not statistically significant.

	α	β_M	R^2	α	β_M	β_{SMB}	β_{HML}	R^2
	0.0095	0.9172	0.468	0.0077	0.9170	0.3139	-0.1236	0.478
t-value	(3.44)	(14.44)	–	(2.61)	(14.50)	(2.07)	(-0.94)	–

Table 11: Regression Results (CAPM and FF3) of P5xE5xR5

5. Limitations

In this chapter, I aim to present the limitations and issues identified in this study, specifically transaction costs and the choice of the sample market. Addressing these could further enhance the real-world meaningfulness of the implemented investment strategies.

A critical limitation of this study is the omission of transaction costs, specifically, commission fees and bid-ask spreads, in evaluating the profitability of the trading strategies employed. This is particularly significant considering the strategies' dependence on diversified portfolios and monthly rebalancing. The importance of this limitation is highlighted by Ng, Rusticus, and Verdi (2008), who found that transaction costs significantly reduce the profitability of PEAD strategies. Their research also indicates that firms with higher transaction costs often show higher abnormal returns, yet these returns, when adjusted for transaction costs, do not equate to increased profitability. This finding suggests that my analysis, by excluding transaction costs, may overestimate the net profitability of the strategies. Future research could, therefore, include a comprehensive analysis of transaction costs to assess the real-world effectiveness of these strategies more accurately.

The second limitation of this study arises from the sample market, specifically the S&P 500, in the context of constructing bivariate and multivariate portfolios. In this process, firms are initially independently sorted into quintiles for each factor, and these quintiles are then intersected to form multifactor portfolios. Despite the S&P 500 encompassing over 500 firms, this approach may lead to inadequate diversification. The limited overlap among quintiles, a consequence of this method, restricts the variety and number of firms in the resulting portfolios. To address this limitation in future research, two potential improvements could be implemented. Firstly, a broader and larger market, such as all U.S. stocks, could be selected. This approach, commonly used in previous studies, is likely to increase the potential for overlap, thereby enhancing diversification in the portfolios. Secondly, an alternative method for combining factors could be explored. For instance, employing a composite ranking system that considers all factors simultaneously, such as using the z-score to normalize each factor, could be effective. This method would not rely solely on sufficient overlap in portfolios, potentially offering a more robust approach to portfolio construction.

6. Conclusion

This study aimed to implement and analyze a multifactor investment strategy, incorporating three key momentum factors: prior returns, earnings surprises, and revenue surprises. The focus was to investigate the efficacy of trading on the post-earnings announcement drift within the S&P 500, spanning from 2000 to 2022. The primary objective was to determine if this approach generates significant excess returns, while also testing the joint implications of the individual factors. By doing so, this research aimed to unveil potential market inefficiencies and highlight the impact of delayed information processing in financial markets.

The findings of this study confirm my research questions and underscore the presence of the post-earnings announcement drift and price momentum anomalies in the S&P 500, in line with previous research. As explored in Chapter 4.2, integrating factors into multifactor strategies yields more profitable trading strategies. Notably, the results indicate a stronger momentum effect in winner portfolios, evidenced by the statistical insignificance of loser portfolios in my sample.

Consequently, my primary strategy, the multifactor portfolio P5xE5xR5, outperforms both its univariate and bivariate counterparts, delivering an annualized excess return of 18.31%. Even after applying the asset pricing models CAPM and FF3, the portfolio generates a monthly excess return of 0.95% and 0.77%, respectively, as indicated by significant positive alpha values.

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