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Enhancing Multi-asset Portfolio Performances with Market Timing using the VIX

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Abstract

The ‘Fear gauge’, more commonly known as the VIX, or Market Volatility Index of the Chicago Board Options Exchange, is a popular market indicator which translates the degree of uncertainty in the market. This paper aims to exploit market timing using the VIX to enhance multi-asset portfolios performance. The strategy provides better performance than a simple risk-parity approach and works as a great diversifier of risk. Stocks/Bonds relationship and Style timing are both used effectively to enhance returns. However, gold fails to hedge against high volatility and other assets such as cash might be better options.

Keywords: Financial Markets, Asset Management, Trading Strategy, Volatility Index, Multi-Asset, Style Timing

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1. Introduction

John Maynard Keynes, in his seminal work "The General Theory of Employment, Interest, and Money" published in 1936, emphasized the significant impact of investors' "animal spirits" on market fluctuations. These fluctuations are quantified by a crucial financial metric known as volatility, which reflects the degree of risk associated with price changes and is calculated through the standard deviation of returns.

Various models, including ARCH (Autoregressive Conditional Heteroskedasticity) proposed by Engle (1982), have been developed to effectively forecast future volatility. Numerous studies, such as those conducted by Bollerslev, Chou, and Kroner (1992), Diebold and Lopez (1995), as well as Plam (1996), provide compelling evidence supporting the predictability of volatility. Many investors utilize these models and timing strategies to pursue returns in the financial markets. For example, Investment funds, particularly successful ones like Hedge Funds, strategically navigate markets with the aim of generating economic value. Remarkably, their proficiency in timing often remains robust even during bear markets, offering investors in these funds a layer of protection against unfavorable market conditions, as demonstrated by research conducted by Chen and Liang (2007). Moreover, additional studies, such as Fleming, Kirby, and Ostdiek (2001), provide further evidence of the economic value derived from effective volatility timing.

Investors express apprehension regarding future volatility, a concern often addressed through the estimation of past data. However, this approach challenges the weak form of market efficiency and may lack robustness. An alternative method for gauging future or implied volatility is the 'Fear gauge,' commonly referred to as the VIX or the Market Volatility Index of the Chicago Board Options Exchange (CBOE). The VIX (Figure 12) is a widely recognized market indicator that quantifies the anticipated level of uncertainty in the market. Serving as a potent tool for investors, CBOE's volatility index strongly signals

expectations of near-term market volatility.

Thomas and Maggie Copeland (1999) present findings that advocate for the implementation of Style and Size strategies using the VIX as a trading signal. They observe that adhering to the VIX timing signal results in positive excess returns, particularly as value and large-cap portfolios outperform when volatility increases. Conversely, growth and small-cap portfolios tend to outperform when volatility decreases. Copelands' strategy hinges on the premise that a decline in expected volatility signifies growing confidence in favor of growth stocks. They interpret this shift as a move towards a momentum strategy. In contrast, when volatility increases, indicating mean reversion, they define such shifts as opportunities for value strategies. This is because value stocks, characterized by low prices relative to intrinsic value (e.g., book value) and high dividends, are perceived as undervalued (as per French-Fama, 1992). Therefore, an increase in expected volatility suggests a potential performance boost for Value stocks. Additional studies, such as the one by Kao and Schumaker (1999), explore Equity Style timing in the short term. They find evidence supporting positive Style Spread (Value over Growth) over multiple periods for both Large and Small-Cap stocks. This aligns with Copeland's perspective, indicating that value tends to outperform during periods of increased expected volatility, a phenomenon they argue is more frequent in the market.

Conolly, Stivers, and Sun (2005) demonstrate that both stocks and bonds exhibit predictable behavior in response to market uncertainty. They observe that bond returns tend to be higher (lower) relative to stock returns on days when implied volatility experiences significant increases (decreases). The rationale behind this phenomenon is linked to the concept of cross-market hedging, where uncertainties in one market can influence prices in other non-affected markets, as outlined by Kodres and Pritsker (2002). In the context of Conolly, Stivers, and Sun's findings, heightened uncertainty in the stock market prompts

investors to reassess their portfolios. During periods of increased economic uncertainty, investors may redirect their focus to alternative assets such as bonds, which become more appealing as they seek returns in the face of elevated market unpredictability. This dynamic reflects a strategic response to market conditions, emphasizing the relation between various financial markets in the face of heightened uncertainty.

The strategy uses both Copeland (1999) and Conolly, Stivers & Sun (2005) findings to exploit a signal created on the VIX in a multi-asset portfolio. The trading signal, a percentage change on the rolling VIX, is analyzed historically to determine periods of high and low market uncertainties. The strategy allocates assets and then weights accordingly to the signal, leveraging Gold's safe heaven characteristic during periods of high volatility (Smirnova, 2016), Style timing and stock-bond relationship. The investment universe undergoes daily testing, wherein the VIX is applied at the market open, and the strategy's performance is assessed using closing prices. Despite the presence of intraday delta, this approach is deemed less biased compared to utilizing the VIX from the previous day's close and incurring a one-day lag that would encompass all overnight changes in the global market in addition to the intraday delta. Moreover, a risk-parity allocation scheme is used to allocate the weights between assets and diversify portfolio risk.

The remainder of this paper is organized as follows: Section 2 describes the data and the methodology used for the signal construction, Section 3 analyses the strategy historically with back-testing in and out-of-sample and discusses the strategy's implementation.

2. Data description

All data was retrieved from WRDS database. The strategy will be tested on the US equities and bonds markets, and Gold.

2.1 The portfolio's assets

The asset data utilized in this study comprises index tracking Exchange Traded Funds

(ETFs) extracted from the period around 2005 to 2023. The selection of these assets is based on their favorable attributes, including low transaction costs and a high volume of trading, rendering the strategy easily implementable. As suggested by Copeland, futures on indices, similar to those tracked by the chosen ETFs, can serve as substitutes.

All the equity ETFs are sourced from Vanguard (VTI, VV, VB, VUG, VTV, VBR, VBK), fixed-income ETFs are from iShares by BlackRock (TLT, IEF), and GLD is from SPDR (State Street). The rationale behind selecting these ETFs lies in their substantial trading volume compared to other ETFs and their cost-effectiveness, particularly for Vanguard, which claims to be 80% cheaper than similar ETFs. Additionally, these ETFs are index-tracking, meaning they follow an underlying index such as the CRSP US Total Market Index for VTI. Importantly, they are unlevered and uncapped, indicating no limitations on potential gains.

Longer-term securities ETFs were specifically chosen for bonds based on the insight from Conolly, Stivers & Sun (2005), which suggests that long-term bonds can function as closing maturity substitutes to stocks. The Equities ETFs were chosen to analyze and implement Copland's ideas and GLD was chosen because it is a popular gold ETF. These choices align with the overall strategy of navigating market uncertainties and optimizing asset allocations.

2.2 Signal construction

The VIX data used was extracted from 1990 until 2023 and consists of daily opening prices. As described by the CBOE Methodology (As of March 20, 2023), the VIX or Volatility Index was first introduced in 1993 by Chicago Board Options Exchange (CBOE) Global Markets and was originally designed to measure the market's expectation of 30-day volatility implied by at-the-money S&P 100 Index (OEX Index) option prices. In 2003, the index was upgraded with the collaboration of Goldman Sachs and now is based on the S&P 500 Index which is one of the most common US stock market proxies. This new VIX estimates market uncertainty by aggregating the weighted prices of the S&P 500 Index puts and calls (SPX Options) over a wide

range of strike prices. The generalized formula used in the VIX Index calculation is:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2 \quad (1)$$

Where the VIX index being $\sigma * 100$, T is Time to Expiration (in years), F is Option-implied forward price, K is the strike price, K_i is the strike price of the i^{th} out-of-money option (a call if $K_i > K_0$ or a put if $K_i < K_0$ or both if $K_i = K_0$), R is the risk-free interest rate to expiration and $Q(K_i)$ is the midpoint of the bid-ask spread of each option with strike K_i . The trading signal (see Figure 13) is a percentage change on the VIX with rolling t days observations:

$$\text{Signal}_t = \frac{X_t - \text{Rolling Mean}_t}{\text{Rolling Std Dev}_t} \quad (2)$$

$$\text{Rolling Mean}_t = \frac{1}{\text{window}} \sum_{j=t-\text{window}+1}^t X_j \quad (3)$$

Where Signal_t is the signal for the data point at time t, X_t is the VIX value at time t, window is the window size for the Rolling Mean_t . The signal is sensitive to changes in its window parameter because longer rolling window might lead to smoothing effects where the signal is less responsive, to lags where the signal takes time to adjust to the data or to persistence of trends where this signal amplifies trends because prolonged trends or period of high or low volatility are captured in the window. By contrast, shorter window size might lead to opposite effect. The effect of such parameters as the window size will be further discussed in the analysis.

3. Analysis

3.1 Strategy construction

Similarly to Copeland's signal, the signal is first created with a 75-day rolling window and then analysed on the 1990-2004 period in Figure 14. The different quantiles will be used to determine levels of high or low market uncertainty in our sample. A signal level of 0.29 or higher will be considered a high level of future market uncertainty, while a level of -0.21 or lower will be considered low. The median of -0.03 will be used as the bound to test Copeland (1999) and Conolly, Stivers & Sun (2005) in a first place. The signal skewness in this period is 1.31 which indicates more and higher extreme positive changes in the VIX.

Then, the various strategies from research papers were tested and regressed against the VIX in-sample between 2005 and 2014. Table 1 shows the contemporaneous relationship between changes in Volatility and the strategies. It is based on the following regression:

$$R_{asset\ 1,t} - R_{asset\ 2,t} = Coef + Slope * Signal_t + e_t \quad (4)$$

	Coef	t-Statistic	Slope	t-Statistic	R ²
Large/ Value – Growth	.011	2.36	1.74	1.76	.001
Small/ Value – Growth	.011	2.38	2.70	2.68	.003
Value/ Large – Small	.010	2.35	1.47	1.86	.001
Blend/ Large – Small	.011	2.36	1.77	2.51	.002
Growth/ Large – Small	.011	2.37	2.03	2.56	.003
Bond (mid-term) vs Stocks	.010	2.35	0.67	2.27	.002
Bond (long-term) vs Stocks	.010	2.34	0.64	2.65	.003
Gold vs Stocks	.010	2.32	0.05	0.21	.000

Table 1: Contemporaneous Relationship between Change in Volatility and the different strategies

All strategies do exhibit a positive slope indicating for example that value provides higher returns than growth for large stocks when future volatility (VIX signal) is high. Notice that the cross-asset relationships have lower slopes than the Style or Size Timing strategies and that the Gold/Stocks strategy is not statistically significant at 5%.

The strategies were then backtested with a trigger signal of -0.03 chosen earlier. Long and Long-Short portfolios were created. For Value vs Growth for example, the portfolio is either

long Value only or also short Growth when the signal is higher than the trigger and vice-versa when it is lower. The performances of the Style and Size Timing portfolios are in Table 2 and the performances of the cross-asset portfolios are in Table 3. Gold and bonds are tested against VTI to assess their relation to total stock market.

Portfolios	Style				Size					
	Large Cap		Small Cap		Value		Blend		Growth	
	LS	Long	LS	Long	LS	Long	LS	Long	LS	Long
Annual return (%)	6.05	11.07	4.83	13.11	5.70	10.86	4.99	11.88	4.40	12.92
Volatility (%)	7.44	20.79	7.30	25.14	9.31	23.73	8.92	22.74	9.30	22.36
Sharpe-Ratio	0.63	0.46	0.47	0.46	0.40	0.46	0.46	0.40	0.32	0.51
MDD (%)	-10.49	-46.41	-18.63	-51.94	-10.44	-59.50	-16.33	-56.71	-21.92	-52.23
Avg. DD (%)	-3.62	-6.88	-4.79	-7.97	-3.72	-9.80	-4.27	-8.90	-5.49	-7.80

Table 2: Size and Style timing Portfolio performances with a 75-day estimation window for the signal

The long portfolios have higher returns but also higher risk in comparison to the long-short portfolios. Overall, Copeland's strategies seems to produce positive returns in sample. For the cross-asset portfolios, the performance of long and long-short portfolios seems to be inverted to the Copeland strategies with long-short portfolios having better returns and higher risk (except for Stock-Gold). The stock-bond portfolios produces significant risk-adjusted returns.

Portfolios	Stock-Bond (7-10yrs)		Stock-Bond (20yrs+)		Stock-Gold	
	LS	Long	LS	Long	LS	Long
Annual return (%)	20.72	15.68	26.56	19.40	9.14	14.76
Volatility (%)	24.64	12.49	30.11	15.21	28.25	19.34
Sharpe-Ratio	0.78	1.18	0.83	1.14	0.27	0.69
MDD (%)	-34.65	-25.64	-27.16	-26.87	-68.02	-39.63
Avg. DD (%)	-6.01	-2.30	-7.87	-3.38	-30.37	-8.30

Table 3: Cross-Asset Portfolio performances with a 75-day estimation window for the signal

Copeland (1999) and Conolly, Stivers & Sun (2005) findings seem to work in our sample as the portfolios produce significant performances which aligns with the regression previously done.

The strategy combines both asset allocation and weight allocation based on the signal. The asset allocation is as follows:

$$\text{Signal} > \text{upper bound: 100\% in GLD} \quad (5)$$

$$\text{upper bound} > \text{Signal} > \text{middle bound: invest in Bonds and Gold} \quad (6)$$

$$\text{middle bound} > \text{Signal} > \text{lower bound: invest in Stocks and Gold} \quad (7)$$

$$\text{Signal} < \text{lower bound: 100\% in Growth stock ETF} \quad (8)$$

The upper bound is determined by the Q95 of the signal during the preceding period (1990-2004), while the lower bound corresponds to the Q5. These periods mark instances of unusually high or low volatility, prompting the portfolio to shift towards safe assets, such as gold, during high volatility periods, or assets that perform well in low volatility periods, such as growth stocks. The objective is to capitalize on volatility timing for a more effective allocation of future risk. It is crucial to recognize that these bounds may vary based on different time periods, and since they are derived from historical signal data, they serve as estimators for future volatility levels. Within the upper and lower bounds (6 & 7), the allocation of weights among the various assets follows a dynamic risk-parity weight allocation scheme with an estimation window of 75 days. For the stocks, the large value vs growth strategy from Copeland will be used in the portfolio because of VTV and VUG high daily volume which makes the strategy easier to implement for ‘high frequency’ rebalancing.

3.2 Results

Portfolios	In-sample (2005-2014)		Out-of-sample (2015-2023)	
	RP vix	RP	RP vix	RP
Annual returns (%)	12.56	6.50	13.53	2.84
Volatility (%)	14.94	6.57	11.29	7.08
Sharpe-Ratio	0.75	0.79	1.08	0.22
Skewness	.091	.220	.054	-.170
Kurtosis	18.36	3.85	6.96	6.22
MDD (%)	-22.13	-12.39	-24.05	-20.09
Avg. DD (%)	-5.42	-1.18	-3.39	-4.03

Table 4: Annual performances of the portfolios in and out of sample

Performance in-sample



Figure 1: Portfolio and components cumulative returns in-sample

Figure 1 depicts the cumulative returns of the strategy portfolio (RP vix) against a simple risk-parity allocation (RP) and the components of the portfolios. Table 4 provides the annual statistics of the two portfolios in the sample. To contextualize, the 2006-2014 period is primarily characterized by the 2008 financial crisis, succeeded by a phase of historically low interest rates that spurred economic recovery after the turmoil.

Overall, the strategy outperforms the simple risk-parity approach by leveraging the VIX signal to enhance returns while maintaining an attractive risk-adjusted return. The returns are significantly superior (12.56% vs 6.50%) but the risk also is higher (14.94% vs 6.57%) resulting in a marginally lower Sharpe ratio (0.75 vs 0.79) over the in-sample period.

The strategy is also compared to its individual components to assess its performance against

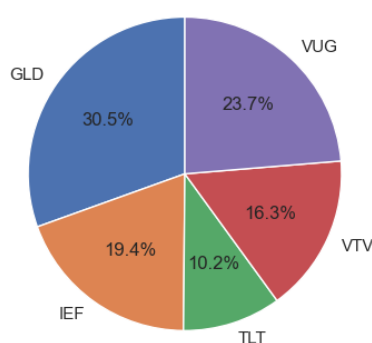


Figure 2: Average weights on each individual asset in the Strategy's portfolio (in-sample)

an investment in 100% in each of these components (see Table 11). The strategy's diversification proves more effective than that of the simple risk-parity approach as the cumulative returns over the sample are superior to any investment in a single asset. The sample of (2398 days) has 228 days with extreme high volatility (measured by the signal) and 180 days with extreme low volatility. These periods of extreme volatility

might have increase the average weight of GLD and VUG illustrated in Figure 2 because the

portfolio invested 100% during high (low) volatility periods.



Figure 3: Attribution of performance to each individual asset of the portfolio in-sample

The attribution of performance is plotted in Figure 3 and shows the cumulative performance of each sleeve (each asset class or individual asset in this case). It shows the contribution of each asset to the performance of the portfolio, and is explained by the weight allocation on each individual asset and their relative performance.

VUG's high cumulative returns and VTV's steady increase shows that Copeland's strategy is well implemented and works effectively. On the other hand, Gold's low cumulative returns is quite surprising because its important average weight in the portfolio and cumulative returns when 100% invested. This relatively low contribution of Gold could be explained by a failure from the strategy to exploit the asset with volatility timing. Indeed, important drawdowns can be observed at several points in Figure 15 and could explain the overall low performance of the Gold sleeve. In mid 2006, late 2008 and mid 2011, the gold sleeve dropped consequently due to spikes in the signal and significant negative returns. The consequent weight on GLD and the decrease in its performance can be associated with the major drawdowns of the strategy.

Deeper analysis was done during the financial crisis period (2007-2009) to further assess the strategy's performance during economic turmoils. Figure 16 depicts the cumulative performance of the strategy and its components during the crisis. The strategy barely survive the period with 2.51% annual returns and a volatility of 20.6% (see Table 12) which combine for surprising negative risk-adjusted returns that are however higher than the simple risk-parity

approach one's (Sharpe ratio of -0.016 vs -0.16). The weight on Equities considerably reduced while the weight on Bonds and Gold increased (see Figure 17) resulting from a period with relatively high signal. Similarly to the full in-sample, the strategy failed to exploit gold despite its high weight and individual cumulative performance (21% average annual return and Sharpe ratio of 0.69) relative to the other assets. The poor contribution of gold is illustrated in Figure 18 which emphasizes on the asset's high weight around October 2008 when the signal peaked at 1.5 while GLD's had a 30% drawdown (see Figure 19). This proves once more that gold was misexploited and might not be the perfect hedge against extreme high volatility.

Over the sample period, the strategy performed well as a diversifier and relative to any of the individual asset as it produced better annual returns et risk-adjusted returns. However, volatility timing and the weight allocation on GLD seem to fail as gold provokes most of the drawdowns in the portfolio during this period. The failure of timing on GLD is emphasized during the 2008 Financial crisis when Gold which is supposedly a safe heaven asset was misexploited for this purpose.

Performance out-of-sample



Figure 4: Portfolio and components cumulative returns out-of-sample

The out-of-sample performance depicted in Figure 4 above show the strategy absolute outperformance on the simple risk-parity approach in term of cumulative returns over the period. The strategy performed much better in this period than in the previous one (in-sample) with 13.53% average annual returns (vs 12.56%) and a Sharpe Ratio of 1.08 (vs 0.75). Also, the strategy out-of-sample relies much less on extreme returns with a Kurtosis of 6.96 (vs 18.36)

and a Skewness of 0.054 (vs 0.091). To contextualize, the period 2016-2024, is characterized by a relatively modest volatility with very few peaks of extreme high volatility. The major crisis was the Covid recession in 2020 due to the pandemic and the closure of international borders.

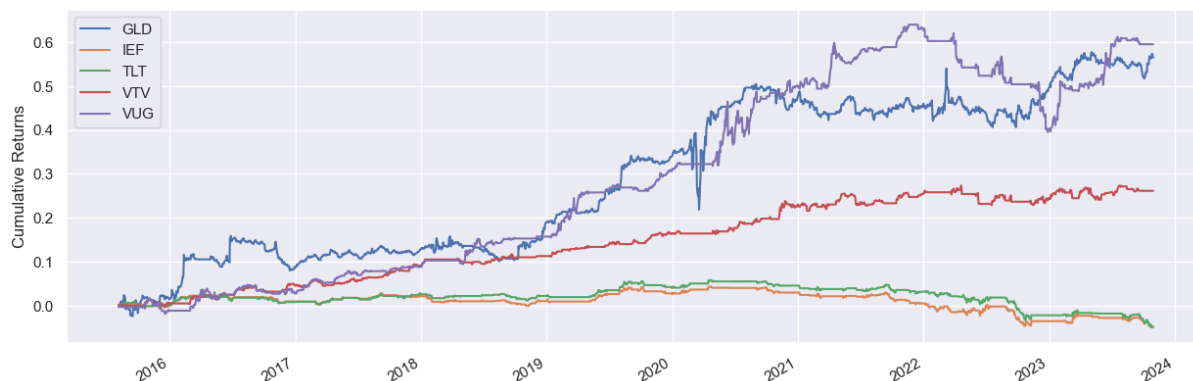


Figure 5: Attribution of performance to each individual asset of the portfolio in-sample

The attribution of performance in Figure 5 shows GLD and VUG's high contribution to the portfolio's performance. The average weights over the period (see Figure 6) are more or less

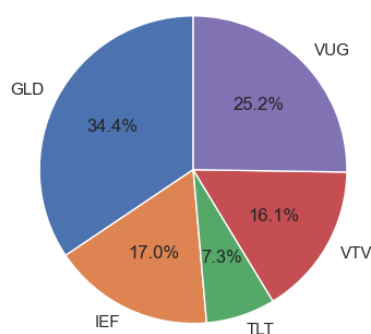


Figure 6: Average weights on each individual asset in the Strategy's portfolio (out-of-sample)

similar to the ones from the in-sample period with a slight increase on GLD and VUG for a decrease in the bonds ETFs. VUG's high average weight and strong performance (average returns of 12.75% with Sharpe ratio of 0.55) relative to the other assets (see Table 13) contributed heavily in the portfolio's performance over this period. VUG's returns were particularly strong after the Covid recession which triggered a period of

extremely low interest rates. Growth stocks may have outperformed thanks to the prevailing low interest rates, which enhance companies' borrowing capabilities to stimulate the economy. Additionally, since growth stocks derive a significant portion of their value from cash flows, their popularity likely increased due to their attractive income revenue. Furthermore, the substantial decline of VUG during the Covid recession and the Ukraine War had a significant impact on the portfolio, primarily because of its high weighting. The war had an inflationary impact, necessitating the Fed to raise interest rates. Consequently, this measure substantially

dampened economic output, specifically affecting growth stocks.

To expand on the analysis, a closer examination of the Covid recession, marked by the highest VIX peak in history, is undertaken to gain a deeper understanding of the portfolio's performance during this period. It is important to note that the Covid recession was a relatively brief downturn in comparison to the more prolonged Great Financial Crisis. The month of February 2020 was followed by an all-time high VIX and a strong decline in the financial markets, which was even more distinct on stocks as illustrates Figure 20. Nevertheless, the Federal Reserve's reduction of interest rates to nearly 0 had a stimulative effect on various assets, especially growth stocks, leading to a rapid recovery from the decline witnessed in March 2020. This period with the all time high VIX forced the portfolio to increase its weight on GLD with almost 50% average weight during these 6 months (see Figure 21). This high weight on GLD in the detriment of other assets is illustrated in Figure 22 which shows the attribution of performance. VTV, TLT and IEF remained flat for most of the recession due to extreme high and low weights and relatively poor performance. Also, a large drop can be observed for GLD, the peak in the signal forced the model to drastically increase its weight on the asset even though the latter performed poorly. The Covid recession is particularly unique as all the assets experienced a poor returns.

To conclude, the performance of the strategy out-of-sample is particularly strong compared to the in-sample period one. It is marked by a great leverage of the Growth stocks strategy which performed particularly well due to low interest rates. Also, the out-of-sample period experienced less large drawdowns (Figure 23) than the in-sample period which may have an important impact on the performance.

Implementation issues

Even though the strategy seems to perform well, some specificities related to its implementation in real life must be discussed to further assess the potential performance

implication it could produce.

The different costs related to managing and trading ETFs are one of the main implementability issue. Indeed, even though ETFs from Vanguard and iShares do not have transactions costs, they have an annual expense ratio which is calculated by dividing a fund's operational expenses by its average net assets. The expense ratio of the ETFs used in this strategy are relatively low (less than 15bps except GLD which is 40bps) and are therefore, not considered our analysis.

Also, a 75-day rolling window is used for both the signal and the dynamic risk-parity allocation in addition to a one day rebalancing period. These parameters were chosen because of historical literature. However, they must be tested to further assess the strategy's implementability and for optimization purposes. Figure 24 shows the out-of-sample strategy risk-adjusted variation given the window size and rebalancing frequency. It can be observed that only the one day rebalancing is consistent across the estimation window sizes because the signal is used as a daily change difference.

Conclusion

Copeland (1999) and Conolly, Stivers & Sun (2005) findings continue to hold relevance in the 21st century. Specifically, the stocks/bonds relationship is strong and proves efficient to diversify risk in high or low volatility periods. The Style timing works well as a standalone strategy but seems to be less efficient in a multi-asset portfolio. However, the relation between Growth stocks and interest rates is robust and could be exploited to further enhance the portfolio returns during periods of low volatility and interest rates. Overall, the extreme weights (up to 100%) on GLD and VUG due to high or low signal levels seem to largely influence the portfolio performance as they are attributable for the largest part of its returns. Also, the model seems to miss-exploit GLD as a hedge against extreme high volatility periods and other assets such as cash might be better options.

Group part.

1. Introduction

In this thesis, we conduct an in-depth analysis of four investment strategies, each characterized by its distinct approach and unique features. The initial sections are devoted to a thorough introduction of these strategies, outlining their key principles and the theoretical framework that underpins each of them. This is followed by a detailed comparative analysis where we examine and contrast the performance indicators associated with each strategy.

The core objective of this group part is to provide a comprehensive evaluation of these strategies both as standalone approaches and combined. We aim to investigate how strategic optimization and thoughtful allocation can potentially enhance their collective performance. By doing so, this study strives to not only assess the individual effectiveness of each strategy but also to explore the synergies that may arise when they are integrated. Through this approach, we seek to answer critical questions regarding the efficacy of these diverse investment strategies and to understand the potential advantages of their combination.

2. Individual Strategies

2.1 Multifactor Earnings Surprise Strategy (SUE)

2.1.1 Economic Motivation

The Efficient Market Hypothesis (EMH), introduced by Fama (1970), anchors modern investment theory with its assertion that stock prices in efficient markets fully reflect all available information. This hypothesis, however, has been challenged by various anomalies, indicating more intricate forces at work in financial markets. Notable among these are the post-earnings announcement drift (PEAD), originally identified by Ball and Brown (1968), and the concept of price momentum, as elaborated by Jegadeesh and Titman (1993), both later acknowledged by Fama (1998) as significant exceptions. PEAD, often labeled as the ‘granddaddy of underreaction

events', reveals that stock prices persist in responding to earnings news well beyond their initial announcement, hinting at a delayed market reaction. This phenomenon is especially pronounced in companies with 'good news', marked by high standardized unexpected earnings (SUE), which tend to outperform those with 'bad news', denoted by low SUE. Similarly, the price momentum anomaly, the inclination of stocks to sustain their prior performance trend, contests the EMH's premise of instantaneous price reaction to new information, leading to the expectation that historically successful stocks will likely continue to outperform their less successful counterparts in the short-term future.

2.1.2 Strategy

To bridge and exploit the anomalies mentioned above, I implement a multifactor earnings surprise investment strategy based on three key factors – earnings surprises, revenue surprises (SURGE), and prior returns – similar to Chen et al. (2014). By combining individual factors, the strategy aims to exploit the market's inefficiencies and delayed information processing.

The foundation of this strategy lies in the methodology for calculating these factors, which adopts the approach based on historical earnings as defined by Jegadeesh and Livnat (2006). This approach states that quarterly earnings typically follow a seasonal random walk with an underlying drift. Thus, earnings surprises are measured by the standardized unexpected earnings, defined as

$$SUE_{i,t} = \frac{Q_{i,t} - E(Q_{i,t})}{\sigma_{i,t}}, \quad (1)$$

where $Q_{i,t}$ is the quarterly earnings per share (EPS) for firm i in quarter t , $E(Q_{i,t})$ is the expected EPS for firm i in quarter t , and $\sigma_{i,t}$ is the standard deviation of earnings growth over the preceding eight quarters. The expected earnings $E(Q_{i,t})$ and standard deviation of earnings growth $\sigma_{i,t}$, respectively, are defined as

$$E(Q_{i,t}) = Q_{i,t-4} + \delta_{i,t} \quad (2)$$

and,

$$\sigma_{i,t} = \frac{1}{7} \sqrt{\sum_{j=1}^8 (Q_{i,t-j} - E(Q_{i,t-j}))^2}, \quad (3)$$

where the drift term, $\delta_{i,t}$, is the average EPS growth over the past eight quarters, defined as

$$\delta_{i,t} = \frac{\sum_{j=1}^8 (Q_{i,t-j} - Q_{i,t-j-4})}{8}. \quad (4)$$

Analogously, revenue surprises are measured by the standardized unexpected revenue surprise estimate (SURGE), defined as

$$SURGE_{i,t} = \frac{R_{i,t} - E(R_{i,t})}{\sigma_{i,t}}, \quad (5)$$

where $R_{i,t}$ is the quarterly revenue for firm i in quarter t , $E(R_{i,t})$ is the expected revenue for firm i in quarter t , and $\sigma_{i,t}$ is the standard deviation of revenue growth over the past eight quarters. For the price momentum factor, the three-month cumulative return is used as the trading signal.

The multifactor strategy is constructed by combining independent univariate strategies, each based on one factor. For each univariate strategy, five market-value weighted portfolios are formed monthly, based on the previous quarter's performance. In the context of revenue surprises, for example, the top-performing portfolio (R5) consists of firms with the highest positive revenue surprise from the preceding quarter, while the lowest-performing portfolio (R1) includes those with the most negative surprise. Building upon this, bivariate and multivariate strategies are developed by intersecting these univariate portfolios. A bivariate strategy might involve selecting firms that are top performers in both earnings and revenue surprises. Expanding on this concept, the multifactor portfolio P5xE5xR5 is created by combining the highest-performing univariate portfolios from all three factors.

2.1.3 Results

This section presents a structured summary of the results, beginning with a brief overview of the correlations among the three factors. Subsequently, it compares portfolios based on univariate, bivariate, and multivariate strategies, evaluating the profitability of combining factors. Thereafter, I will show the the significance and robustness of the multifactor strategy by applying common asset pricing models.

In testing the hypothesis that combining momentum factors enhances strategy returns, this paper first examines whether each factor contributes unique information to the strategy by calculating the correlations among the factors. Despite potential overlaps, the distinct insights offered by each factor could amplify the combined performance. The results show that SUE and SURGE are moderately correlated at 0.32, reflecting their relationship in the income statement. However, both show low correlations with prior returns (0.10 and 0.06), suggesting distinct information contribution to the multifactor approach.

Table 1 depicts the annualized excess portfolio returns for each momentum strategy. Panel A highlights distinct patterns in univariate strategies: top-performing quintiles P5, E5, and R5 show significant excess returns of 9.66%, 9.66%, and 9.54%, respectively. However, the lower quintiles lack statistical significance, implying that momentum effects are predominantly observed in winning portfolios. Additionally, the expected trend of higher quintiles yielding greater returns is not uniformly evident, contrasting prior research. Panel B shows portfolios combining two factors and reveals a typical momentum trend, with higher quintiles, such as P5xE5 and P5xR5, yielding higher returns (13.42% and 12.80%) than lower quintiles (again statistically insignificant). Notably, the zero-investment portfolio E5xR5 – E1xR1 yields a significant 7.70%, highlighting the effectiveness of integrating earnings and revenue surprises over price momentum. Panel C shows the multivariate strategy, combining all three factors, with the P5xE5xR5 portfolio delivering the highest excess return of 18.31%.

However, the corresponding zero-investment strategy's positive return isn't significant, implying that the benefits of the top-tier portfolio are potentially offset by the lower-tier portfolio to some degree, revealing complexities in the multivariate approach.

In conclusion, these results substantiate that, within my sample, momentum effects are predominantly present in top-quintile portfolios. Further, the results underscore the superiority of multifactor strategies over bivariate and univariate methods, reinforcing the joint implication of combining factors.

	Mean ER	t-value		Mean ER	t-value		Mean ER	t-value
Panel A: Univariate Strategies								
P1	0.0702	1.36	E1	0.0455	1.23	R1	0.0443	1.19
P2	0.0686	1.94	E2	0.0724	2.07	R2	0.0665	1.95
P3	0.0515	1.55	E3	0.0806	2.38	R3	0.0877	2.54
P4	0.0881	2.74	E4	0.0783	2.44	R4	0.0831	2.48
P5	0.0966	2.68	E5	0.0966	2.83	R5	0.0954	2.79
P5 – P1	-0.0034	-0.09	E5 – E1	0.0440	2.64	R5 – R1	0.0432	2.31
Panel B: Bivariate Strategies								
P1xE1	0.0281	0.50	P1xR1	0.0213	0.36	E1xR1	0.0247	0.59
P2xE2	0.0549	1.37	P2xR2	0.0765	1.98	E2xR2	0.0708	1.81
P3xE3	0.0502	1.35	P3xR3	0.0820	2.29	E3xR3	0.0915	2.42
P4xE4	0.1059	3.24	P4xR4	0.0949	2.88	E4xR4	0.0883	2.56
P5xE5	0.1342	3.35	P5xR5	0.1280	3.37	E5xR5	0.1151	3.00
P5xE5 – P1xE1	0.0594	1.20	P5xR5 – P1xR1	0.0567	1.16	E5xR5 – E1xR1	0.0770	2.82
Panel C: Multivariate Strategy								
			P1xE1xR1	-0.0088	-0.13			
			P2xE2xR2	0.0862	1.81			
			P3xE3xR3	0.0639	1.47			
			P4xE4xR4	0.0754	1.69			
			P5xE5xR5	0.1831	4.09			
			P5xE5xR5– P1xE1xR1	0.1170	1.85			

Table 1: Annualized Excess Returns of Momentum Strategies

Next, I apply the CAPM and Three-Factor Fama-French pricing models to capture the potential risk factors associated with the returns of P5xE5xR5. The regression results, depicted in Table 2, demonstrate that the multifactor strategy yields a significant monthly excess return of 0.95% under the CAPM, with a market beta of 0.92, indicating strong market exposure and volatility marginally lower than the broader market. In the Fama-French model, the monthly

excess return is slightly lower at 0.77%. The market beta remains consistent at 0.92, while the positive SMB coefficient (0.32) suggests a small-cap preference, which aligns with previous research. The negative, though not significant, HML coefficient (-0.12) hints at a lesser focus on value stocks.

	α	β_M	R^2	α	β_M	β_{SMB}	β_{HML}	R^2
	0.0095	0.9172	0.468	0.0077	0.9170	0.3139	-0.1236	0.478
t-value	(3.44)	(14.44)	–	(2.61)	(14.50)	(2.07)	(-0.94)	–

Table 2: Regression Results (CAPM and FF3) of P5xE5xR5

In conclusion, the study confirms PEAD and price momentum in the S&P 500 and shows that multifactor strategies, particularly in winner portfolios, yield higher profitability than their univariate counterparts.

2.2 Quality Investing on the Oslo Stock Exchange (QMJ)

2.2.1. Introduction

The concept of quality investing has long been a fundamental strategy in the field of quantitative investment. A significant contribution to this was Asness et al. introduction of the Quality Minus Junk (QMJ) factor in 2013, and its further development in 2019. This method focuses on the returns from high-quality stocks, identified by factors such as profitability, growth, and safety, and challenges traditional investment models. The 2019 research by Asness et al. demonstrated the effectiveness of this approach in larger markets, but its performance in smaller markets, like Norway's, has not been extensively studied.

Asness et al. (2019) integrated various research findings in quality investing into a single metric. Their results showed that stocks classified as high-quality by the QMJ factor tend to offer higher risk-adjusted returns, contradicting some traditional risk-return theories. The study explores reasons for this, including the possibility that markets might use more refined methods for evaluating quality, that quality stocks might represent unobserved risks, or that markets do

not entirely reflect the quality aspect in stock prices. Their findings suggest potential inefficiencies in the market, indicating opportunities for investors.

This thesis sets out to apply the QMJ framework to the Norwegian stock market with the aim of determining whether a quality investing strategy can be effective in this context. The focus will be on a practical evaluation of the QMJ strategy's performance in the Norwegian market, assessing its viability as a profitable investment approach.

2.2.2 Methodology

2.2.2.1 Data Description

This study's data is sourced from the Compustat databases via Wharton Research Data Services (WRDS), focusing on Norwegian firms. It comprises annual accounting data from June 1987 and daily stock price data from December 31, 1985, specifically targeting firms listed on the Oslo Stock Exchange.

Risk-free rates are proxied using the NIBOR 6-month and 1-month annualized rates, primarily from Norges Bank and, post-2012, Norske Finansielle Referanser AS (NoRe). Inflation trends are assessed using Consumer Price Index (CPI) data from Statistics Norway, and currency exchange rates are sourced from Sveriges Riksbank. Norwegian recession indicators are obtained from the Federal Reserve Economic Data (FRED) website.

The study confines its analysis to common ordinary shares, focusing on those with the highest market equity. Stock prices adjust for factors like stock splits and dividends. Accounting data is standardized to NOK, and stocks with less than six years of data are excluded due to metric requirements.

To ensure the relevance and avoidance of forward-looking biases, the study aligns fiscal year accounting data with a 6–12-month lag to the corresponding calendar year. This approach is in line with the methodology employed by Asness et al. (2019).

The analysis period is set from 2000 to 2023 to provide a comprehensive and representative sample, with the dataset averaging 174 unique stocks per year. This period is chosen based on the significant increase in stock data availability from around the year 2000.

2.2.2.2 Variable and Portfolio Construction

In analyzing quality investing in the Norwegian stock market, this study constructs variables and portfolios to capture quality across different dimensions. Quality is divided into profitability, growth, and safety, each measured through a series of financial metrics. Profitability includes gross profits over assets (GPOA), return on equity (ROE), return on assets (ROA), cash flow over assets (CFOA), gross margin (GMAR), and low accruals (ACC), reflecting the efficiency and effectiveness of income generation. Growth is measured over a five-year period, focusing on sustainable increases in profitability metrics such as Δ GPOA, Δ ROE, Δ ROA, Δ CFOA, and Δ GMAR. Safety is assessed through low beta ($-\beta$), low leverage (LEV), low earnings volatility (EVOL), and bankruptcy risk indicators like Altman's Z-score and Ohlson's O-score.

To standardize these diverse metrics for comparative analysis, z-scores are computed for each financial metric. This involves ranking the metrics within the stock universe and then standardizing these ranks to have a mean of zero and a standard deviation of one. This z-score transformation normalizes the various scales of financial metrics, enabling an aggregated assessment across different companies.

Portfolios are then constructed based on these standardized scores. Stocks are sorted into quintiles monthly, based on their overall quality scores, creating five portfolios that range from the lowest to the highest quality. This quintile method is preferred over the decile method for its suitability in the smaller Norwegian market, allowing for a more diversified portfolio.

The Quality-Minus-Junk (QMJ) factor is central to the portfolio analysis. Stocks are

categorized by size and quality, dividing them into 'Big' or 'Small', and 'Quality' or 'Junk'. The QMJ factor is derived from the difference in returns between the high-quality and low-quality portfolios.

Additionally, the study constructs a market portfolio (MKT) as well as Small Minus Big (SMB) and High Minus Low (HML) factors, following the Fama and French model. These factors examine the return differences attributable to market risk, firm size, and book-to-market values, respectively, complementing the analysis of the QMJ factor.

2.2.3. Results

2.2.3.1 Quality Sorted Portfolios

The results of the study on quality investing in the Norwegian stock market, following the detailed construction of variables and portfolios, offer compelling insights. The analysis of quality-sorted portfolios reveals a clear trend: higher quality correlates with superior performance. This is evident from the progression of returns, where higher-quality portfolios consistently outperform their lower-quality counterparts. The 'P5-P1' differential, a comparison between the highest and lowest quality portfolios, demonstrates a significant monthly average return advantage of 1.11% for high-quality stocks. This finding is statistically significant, with a t-statistic of 2.42, effectively rejecting the null hypothesis of no difference in average returns.

Further analysis, adjusting for market and other factor risks, underscores the alpha of high-quality stocks. Their outperformance is statistically significant, attributed to their lower market and factor exposures. This aligns with their lower risk profile as indicated by both the Capital Asset Pricing Model (CAPM) and the three-factor model assessments. The strategy of long-high/short-low quality delivers abnormal returns ranging from 1.35% to 1.49% per month, supported by substantial t-statistics. The increase in Sharpe and information ratios across quality ranks further validates the effectiveness of a long-quality/short-junk strategy.

Table 1 Quality sorted portfolios

	P1	P2	P3	P4	P5	P5-P1
Excess Return	0.12%	0.13%	0.49%	0.52%	1.23%	1.11%
	(0.22)	(0.30)	(1.35)	(1.54)	(3.42)	(2.42)
CAPM alpha	-0.74%	-0.59%	-0.12%	-0.06%	0.61%	1.35%
	(-2.10)	(-2.30)	(-0.56)	(-0.34)	(3.14)	(3.04)
3-factor alpha	-0.84%	-0.67%	-0.18%	-0.04%	0.66%	1.49%
	(-2.65)	(-2.74)	(-0.86)	(-0.23)	(3.41)	(3.66)
CAPM beta	1.34	1.12	0.95	0.90	0.96	-0.38
Sharpe Ratio	0.05	0.06	0.28	0.32	0.70	0.50
Information Ratio	-0.55	-0.57	-0.18	-0.05	0.71	0.76
Adjusted R2	0.68	0.69	0.69	0.72	0.72	0.22

2.2.3.2 QMJ Portfolio

The performance of the QMJ factor portfolio exhibits significant excess returns and alpha relative to both the CAPM and the Fama-French three-factor model. The QMJ factor achieved monthly abnormal returns of 1.25% against CAPM and 1.34% against the Fama-French model, with robust t-values. The risk loadings of the QMJ factor indicate significant negative market, size, and value exposures, suggesting a strategic preference for low-beta, large stocks, and a short position in high beta, small stocks.

In terms of total returns, the QMJ strategy yielded approximately 1,350% cumulative returns over the sample period, significantly outperforming the market's 700%. This is complemented by a superior annualized Sharpe ratio of 0.79 and an information ratio of 1.19, indicative of a more favorable risk-adjusted return profile.

Table 2 QMJ

	Recession	Expansion	Severe bear market	Severe bull market	Low volatility	High volatility	Spike down in volatility	Spike up in volatility	All Periods
Excess Return	1.53% (3.63)	0.77% (2.16)	1.90% (1.58)	0.38% (0.94)	0.51% (1.24)	2.26% (4.40)	1.19% (2.27)	1.96% (4.01)	1.04% (3.88)
CAPM alpha	1.47% (3.69)	1.32% (3.93)	1.63% (1.32)	0.75% (1.54)	1.72% (3.84)	1.76% (3.43)	2.53% (4.89)	1.31% (2.67)	1.25% (4.99)
3-factor alpha	1.44% (4.02)	1.52% (4.70)	2.09% (1.98)	0.89% (1.83)	2.20% (5.31)	1.71% (3.48)	2.50% (5.32)	1.28% (2.73)	1.34% (5.74)
CAPM beta	-0.26	-0.41	-0.12	-0.14	-0.52	-0.21	-0.53	-0.26	-0.32
Sharpe Ratio	1.14	0.61	1.26	0.38	0.46	1.65	0.85	1.50	0.79
Information Ratio	1.29	1.41	1.94	0.92	2.55	1.40	2.31	1.11	1.19
Adjusted R2	0.29	0.25	0.39	0.04	0.36	0.18	0.38	0.20	0.25

An examination of the QMJ strategy under varying market conditions reveals its robustness and adaptability. During both recessions and severe bear markets, the strategy consistently generates higher monthly excess returns, along with strong CAPM and three-factor alphas. This performance not only during economic expansions but also in times of market distress supports the 'flight to quality' phenomenon, where high-quality stocks become a haven for investors. The consistent delivery of positive alpha across different volatility scenarios further underscores the QMJ strategy's resilience and its potential as a risk-conscious investment approach. These findings solidify the QMJ strategy's position as a significant and effective investment approach in the Norwegian market, challenging traditional risk-return paradigms and offering a viable option for both defensive and growth-oriented portfolio components.

2.3 The Impact of Machine Learning on Stock Momentum Strategies (ML)

2.3.1 Economic Motivation

Stock market dynamics are complex, characterized by non-linear, chaotic, and volatile patterns, making investment inherently risky. The market's efficiency, as suggested by Malkiel and Fama (1970), indicates that price movements, reflecting all known information, are unpredictable and follow a random walk, aligning with new information's random nature. Advancements in computational intelligence and data mining technology have greatly expedited research in the field of quantitative investment (Yang, Lou and Bo 2017). The need for machine learning models capable of capturing and predicting the price fluctuations of single stocks or a collection of stocks in the market is on the rise (Yue et al. 2021). Machine learning's pattern detection can refine trading strategies, particularly the widely used price momentum strategy. Its sporadic yet substantial gains make it a prime candidate for improvement via machine learning.

2.3.2 Methodology and Strategy

For the analysis, the Random Forest Classifier was employed to evaluate its effectiveness in forecasting stock movements. Subsequently, the Random Forest Regression model is used for predicting returns. Initially, a traditional momentum strategy is applied, ranking stocks based on their cumulative past returns from months $t-12$ to $t-2$, deliberately excluding the most recent month ($t-1$) to differentiate between intermediate-term momentum and short-term reversal trends. Stocks are then distributed into decile portfolios, typically weighted equally, and subjected to monthly rebalancing. The objective is to discern the efficacy of machine learning in enhancing the momentum strategy by monthly identification of winners and losers through predicted versus actual returns, and to subsequently measure the actual impact by utilizing realized returns during the holding period. To train the Random Forest Models, the following indicators were used:

- **Relative Strength Index (RSI)**

$$RSI = 100 - \frac{100}{1+RS} \quad (1)$$

$$RS = \frac{\text{Average Gain over past 14 days}}{\text{Average Loss over past 14 days}}$$

- **Stochastic Oscillator**

$$\%K = \left(\frac{C-L_{14}}{H_{14}-L_{14}} \right) * 100 \quad (2)$$

Where,

C = Current closing price

L₁₄ = Lowest price over the past 14 days

H₁₄ = highest price over the past 14 days

- **Williams Percent Range**

$$\%R = \left(\frac{H_{14}-C}{H_{14}-L_{14}} \right) * -100 \quad (3)$$

Where,

C = Current closing price

L₁₄ = Lowest price over the past 14 days

H₁₄ = highest price over the past 14 days

- **Price Rate of Change**

$$PROC_t = \frac{P_t - P_{t-n}}{P_{t-n}} \quad (4)$$

Where,

P(t) is the closing price at time t

P(t-n) is the closing price n periods before time t

- **On Balance Volume**

$$OBV(t) = OBV(t-1) + \begin{cases} Volume(t) & \text{if } P_t > P_{t-1} \\ 0 & \text{if } P_t = P_{t-1} \\ -Volume(t) & \text{if } P_t < P_{t-1} \end{cases} \quad (5)$$

Where, P(t) is the closing price at time t.

- **Moving Average Convergence Divergence**

$$MACD = EMA_{12} - EMA_{26} \quad (6)$$

$$SignalLine = EMA_9(MACD)$$

2.3.3 Random Forest Reliability

The Random Forest classifier achieved a predictive accuracy of approximately 70.77%, indicating its effectiveness in forecasting daily stock price movements. Precision rates for predicting downward and upward movements were 68.35% and 73.20% respectively, with recall rates of 71.99% for down days and 69.65% for up days. The F1-score corroborated these findings, indicating balanced performance. Feature importance analysis revealed that Williams R% and RSI were the most influential indicators, with MACD and Price Rate of Change also significant, while On Balance Volume had less impact.

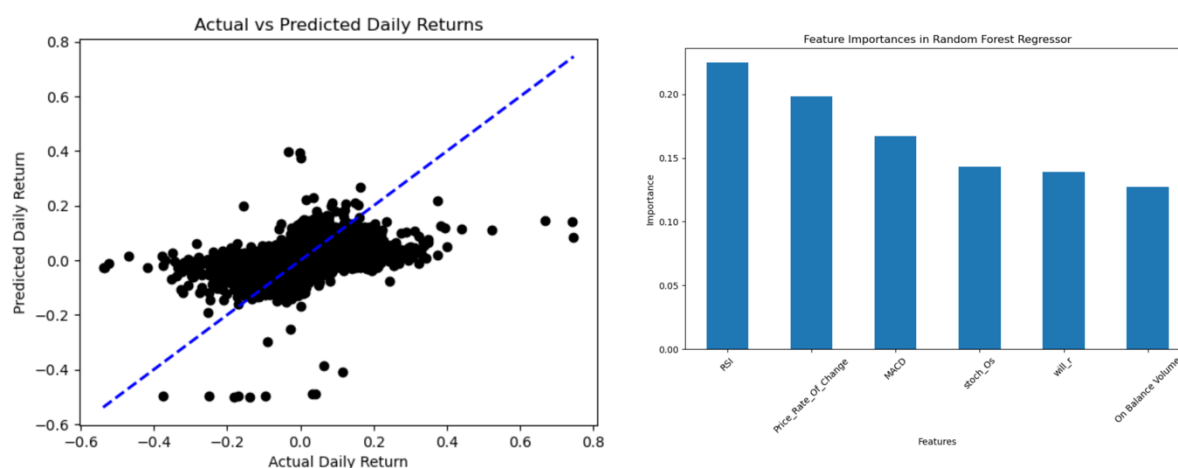
The evaluation of the Random Forest Regression model's predictive accuracy employed RMSE and MAE as key metrics. The RMSE, at 0.01763, indicated a high level of accuracy, suggesting that predicted values closely align with actual returns. The overall MAE, initially at 0.01141, further decreased to 0.01014 when considering predictions that matched the actual direction of stock movements. This notable reduction in MAE highlighted the model's proficiency in capturing market trend directionality.

Additionally, the model's predictive strength was further validated using the R^2 score. An overall R^2 score of 0.3035 suggested that the model accounted for approximately 30% of the variance in daily returns. This score improved significantly to 0.4852 for instances where the model's predictions and actual returns moved in the same direction, underscoring the model's enhanced explanatory power for consistent directional predictions.

In terms of feature importance within the Random Forest Regression analysis, RSI and Price Rate of Change emerged as key indicators in predicting future returns. The model assigned these features higher significance compared to On Balance Volume, which was found to have a lesser impact. This contrasted with the findings from the classifier model, highlighting the distinct roles that various features play in directional prediction versus value estimation in stock movements.

2.3.4 Impact on Momentum and Stock Selection

Between 2017 and 2022, the machine learning-enhanced momentum strategy using predicted returns decreased by -2.16% in annualized returns, outperforming the actual returns strategy's -



4.75% drop. In the volatile 2020-2022 post-COVID period, predicted returns limited losses to -3.26%, compared to -10.14% with actual returns.

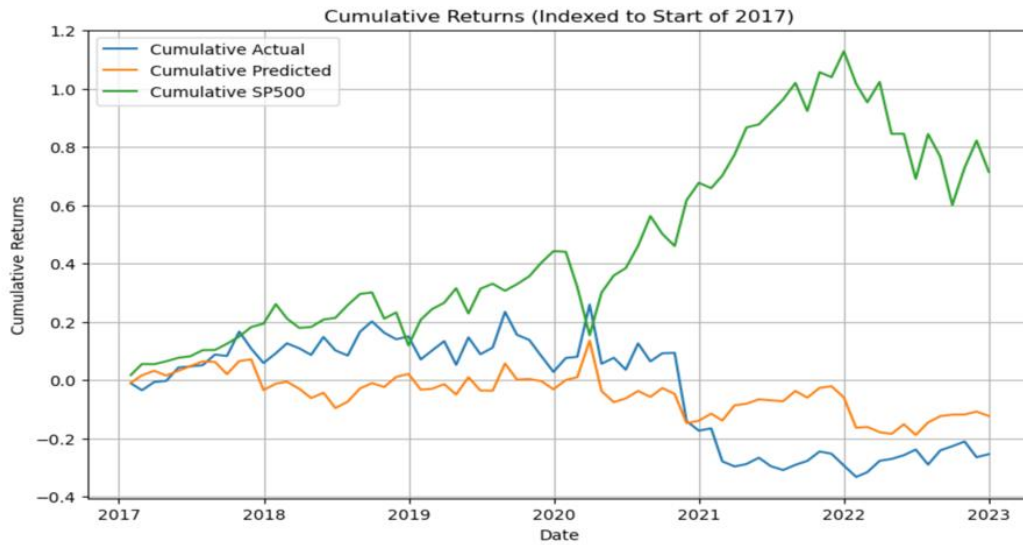


Figure 1: Cumulative Returns

Both strategies had negative Sharpe Ratios, indicating returns insufficient to offset their risk compared to the risk-free rate. The predicted strategy's Sharpe Ratio was -0.14, less negative than the actual strategy's -0.19. This was mirrored in the Sortino Ratios, where the predicted strategy had -0.18 compared to the actual's -0.25.

Period 2017-2022	Strategy Actual	Strategy Predicted	Benchmark S&P500
Volatility (%)	5,82%	4,35%	4,96%
Sharpe Ratio	-0,191	-0,14	0,547
Sortino Ratio	-0,247	-0,18	0,8
Average Drawdown	-0,184	-0,1267	-0,0434
Max Drawdown	-0,4695	-0,285	-0,2476

Table 1: Performance Overview

The predicted strategy also showed lower volatility (4.35%) and drawdowns (average -0.127, max -0.29) than the actual strategy (volatility 5.82%, average drawdown -0.18, max -0.467), indicating a lower risk profile.

2.4. Enhancing Multi-asset Portfolio Performances with Market Timing using the VIX (VT)

2.4.1. Economic motivation

Volatility has always been one of the main concerns for investors who not only seek better returns but also less risk. The ability to estimate future volatility enables investors for better timing for their strategies in terms of asset or weight allocation. Various models such as ARCH (Autoregressive Conditional Heteroskedasticity) proposed by Engle (1982), have been developed to effectively forecast future volatility. Also, historical literature provides evidence of the economic value derived from effective volatility timing (Fleming, Kirby and Ostdiek, 2001).

The VIX, a volatility index from the CBOE, is a popular indicator of future near-term volatility in the markets. It is a strong estimator and can also be used as the implied volatility in the Black-Scholes model.

2.4.2. Strategy

This strategy is built upon the framework of Thomas and Maggie Copeland (1999) and Conolly, Stivers and Sun (2005) who utilize VIX for asset allocation. It also extends their findings to a new time period to assess the reliability of their results before constructing the portfolio. This strategy aims to combine the findings in order to construct a portfolio enhancing performance for multi-asset strategies. Both, Style timing and cross-asset relationships are used to improve returns during low volatility periods and hedge against high volatility periods by optimally allocating assets and weights on the latter.

The signal created from the VIX is a rolling percentage change with an estimation window t :

$$\text{Signal}_t = \frac{X_t - \text{Rolling Mean}_t}{\text{Rolling Std Dev}_t} \quad (1)$$

$$\text{Rolling Mean}_t = \frac{1}{\text{window}} \sum_{j=t-\text{window}+1}^t X_j \quad (2)$$

The signal aims to capture the change in volatility relative to the past t days and follows Copeland (1999) construction. It is more robust than a simple percentage change on the VIX because it overcomes the noisiness in the index. To follow Copeland's findings, a 75-day (roughly three-month) estimation window is used for the analysis. The impact of the parameter on the strategy is studied later on.

The assets used in the strategy are ETFs in different asset classes (Equities, Bonds or Gold) and sub-classes (Growth and Stock or Long and mid-term bonds). Index tracking ETFs were chosen for this strategy to review Thomas and Maggie Copeland (1999) and Conolly, Stivers and Sun (2005) findings with indices. Also, the relative high volume and low costs of ETFs makes them more attractive for implementation. However, costs are not included in the strategy's performance analysis because these ETFs do not have transaction costs and their expense ratio is negligible (under 15 bps annually).

The strategy follows an asset allocations scheme such as:

$$\text{Signal} > \text{upper bound: } 100\% \text{ in GLD} \quad (3)$$

$$\text{upper bound} > \text{Signal} > \text{middle bound: invest in Bonds and Gold} \quad (4)$$

$$\text{middle bound} > \text{Signal} > \text{lower bound: invest in Stocks and Gold} \quad (5)$$

$$\text{Signal} < \text{lower bound: } 100\% \text{ in Growth stock ETF} \quad (6)$$

It also applies a weight allocation scheme when in between the upper and lower bound (3 & 4). The risk-parity allocation scheme is applied to evenly distribute risk across the selected assets which should perform better when the signal is in these bounds according to previous research findings.

2.4.3. Performance Overview

The performance of the portfolio (RP VIX) from June 2005 until December 2022 is plotted in Figure 1 alongside the portfolio individual components and a simple risk-parity portfolio (RP).



Figure 1: Cumulative returns of the portfolio and its components

The VIX trading portfolio performed much better than the simple risk-parity approach portfolio by yielding a total return of 5.87x vs 1.14x for the simple approach. Also, the strategy proves to be a great diversifier of risk with a higher Sharpe ratio than any of the individual assets. Table 5 presents the annual performances of the portfolios and the components.

	RP VIX	RP	GLD	TLT	IEF	VTV	VUG
Annual return (%)	11.88	4.40	9.26	0.94	1.58	7.10	10.1
Volatility (%)	13.22	6.79	17.93	6.57	14.67	19.94	20.53
Sharpe Ratio	0.89	0.64	0.51	0.13	0.10	0.35	0.49
Skewness	.071	.009	-.178	.096	.088	-.295	-.241
Kurtosis	17.74	5.05	6.24	3.17	3.97	11.84	10.13
Max. Drawdown (%)	-24.05	-20.95	-45.55	-24.48	-46.14	-61.31	-51.35
Avg. Drawdown (%)	-4.56	-2.99	-18.47	-5.38	-12.68	-11.81	-7.64

Table 1: Annual performance of the portfolio and its components

The strategy surpasses its individual components in terms of annual returns, achieving an impressive 11.88%, while maintaining a relatively modest volatility of 13.22% for its returns. Furthermore, the strategy's diversification renders it a safer investment when compared

to its standalone components, exhibiting an average drawdown of -4.56% and a maximum drawdown of -24.05%. These figures highlight that the portfolio's losses are less severe than those incurred through a singular investment in TLT. Moreover, the portfolio's performance outshines a simple risk-parity approach in the realm of risk-adjusted returns, boasting a Sharpe ratio of 0.89 as opposed to 0.64. It is worth noting, however, that the portfolio's higher volatility is a result of its asset allocation's dependency on the VIX. Lastly, the Kurtosis value of 17.74 underscores the strategy's intentional reliance on extreme positive and negative returns, a deliberate aspect of the asset allocation model's design.

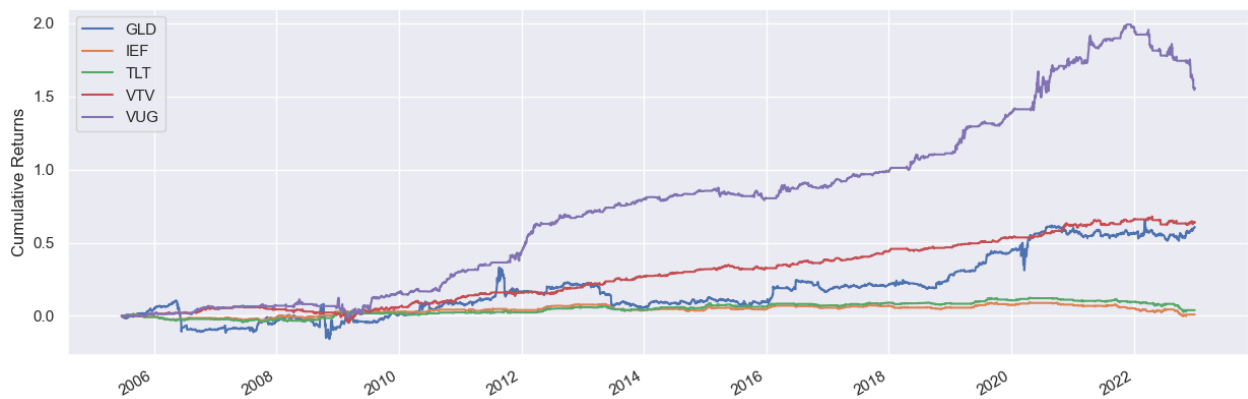


Figure 2: Attribution of performances to the individual components of the portfolio

Figure 2 presents the attribution of performance which highlights the individual contribution of each component to the portfolio. Growth stocks (VUG) heavily contributed to the portfolio's total return because of their strong returns (10.1%) and weight in the portfolio (see Figure 3).

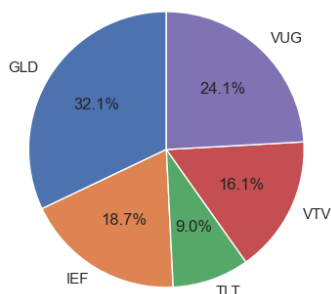


Figure 3: Average weight on each component of the portfolio

GLD and VUG consistently carry the highest weights on average in the portfolio, owing to their pronounced weights during both high and low volatility periods (refer to equations 4 and 5). Despite gold's commendable performance, yielding a 9.26% annual return and a substantial portfolio weight, its contribution to the overall portfolio remains modest. This could be attributed to a potential

underutilization of gold with the VIX signal, suggesting that gold may not be the optimal asset

for hedging against high volatility.

In conclusion, the strategy leverages the stock/bond relationship and employs style timing with the VIX signal to mitigate risk during periods of heightened volatility while enhancing returns during phases of low volatility.

3. Comparison of Individual Strategies

3.1 Cumulative Returns

Figure 7 presents the cumulative returns of our four investment strategies over two separate timeframes. The left panel illustrates the period from 2005 to 2016, where all three depicted strategies exhibit significant positive returns. During this phase, SUE achieves the highest cumulative return, although it exhibits pronounced volatility, particularly during the 2008 financial crisis. This contrasts with QMJ and VT, which show more consistent growth and lower volatility, suggesting greater resilience to market fluctuations. Notably, after the crisis, QMJ and SUE both demonstrate superior performance, diverging appreciably from VT starting in 2013. However, QMJ experienced a notable setback in 2016.

The right panel of Figure 7 presents the performance from 2017 to 2022. Initially, QMJ, SUE, and VT exhibit comparable returns and experience similar levels of volatility. This trend persists until 2021, when SUE registers a significant increase in returns, outperforming QMJ and VT. In contrast, ML maintains a consistent but flat trend throughout the same period, characterized by low volatility, which is in line with previous research.

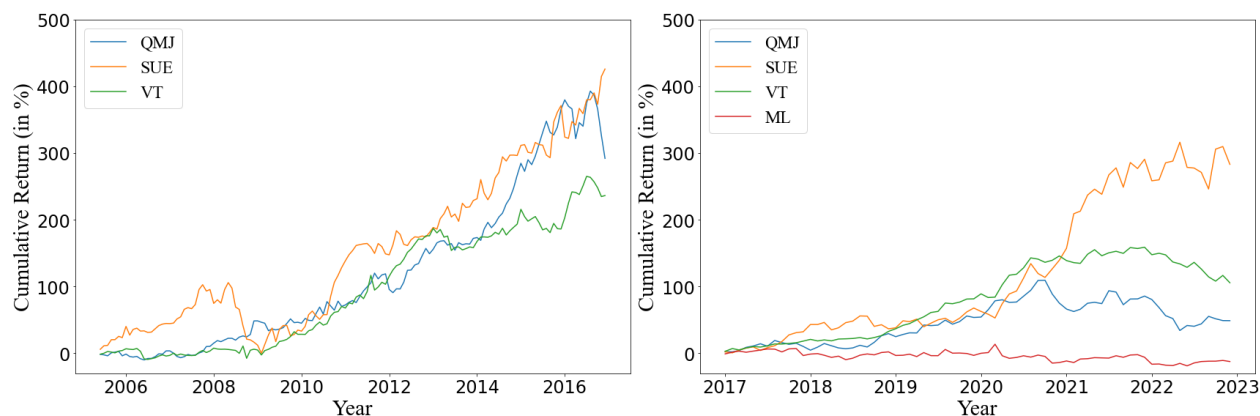


Figure 7: Cumulative Returns

3.2 Correlation of Individual Strategies

Figure 8 illustrates the correlation matrices for the returns of the four investment strategies from Figure 7, over two distinct periods. In both periods, QMJ and SUE exhibit a slight negative correlation, with coefficients of -0.15 and -0.17, respectively. This inverse relationship suggests that these strategies may offer diversification benefits when combined within a portfolio, as they tend to perform differently under the same market conditions.

The correlation between QMJ and VT is very weak in the first period at 0.03, rising modestly to 0.16 in the second. This trend indicates a slight increase in the similarity of their return movements over time. SUE and VT show the highest correlation in the datasets, with coefficients of 0.23 and 0.34, respectively. Although these correlations are the strongest within the matrices, they are still modest, suggesting only a moderate relationship between the strategies' returns.

ML, introduced in the second period, exhibits a notable correlation of 0.32 with QMJ, indicating a moderate positive relationship. However, its correlations with SUE and VT are -0.09 and -0.02, respectively, implying a very weak to negligible inverse relationship. This denotes that ML's performance does not closely follow that of Strategies SUE and VT.

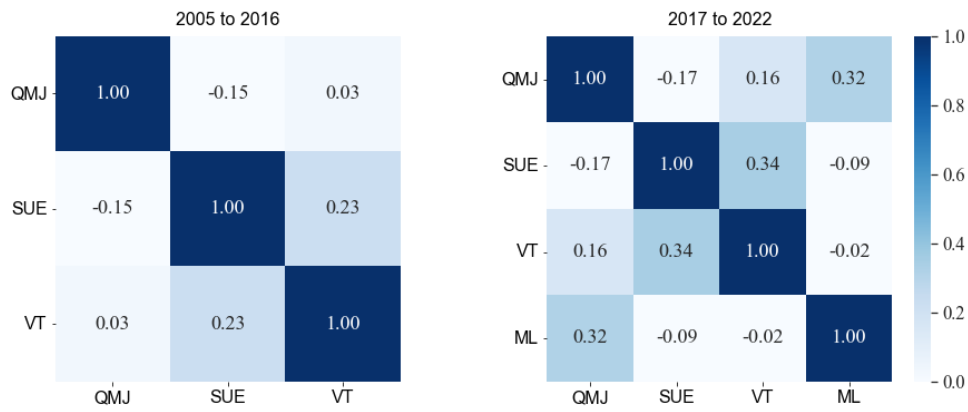


Figure 8: Correlation Matrices

3.3 Performance Analysis of Individual Strategies

Finally, we conduct an in-depth performance analysis, comparing the individual strategies. From June 2005 to December 2016, SUE yielded the highest total return of 426% and the greatest CAGR at 15.41%, but with the highest volatility, as seen by a standard deviation of 19.96%. Its risk-adjusted return, as reflected by a Sharpe ratio of 0.71, is less favorable when compared to QMJ and VT, which achieve Sharpe ratios of 0.90 and 0.82, respectively. This discrepancy can be attributed primarily to the notably lower volatility associated with QMJ and VT. Consistent with the trends observed in Figure 10, QMJ and VT exhibit more resilience during market downturns, as reflected by their substantially lower maximum drawdowns.

In the second period, VT improved its risk-adjusted performance with a Sharpe ratio of 1.22, on the back of higher returns and lower volatility. SUE's CAGR rose to 25.09%, boosting its Sharpe ratio to 1.16, despite increased risk. QMJ, however, significantly reduced annual returns while increasing volatility, resulting in a reduced Sharpe ratio of 0.42. ML, introduced in this period, experiences negative cumulative returns of -12%, translating to a negative Sharpe ratio of -0.14. This performance aligns with the challenges faced by price momentum strategies in recent years, as noted in contemporary research.

	06/2005 to 12/2016	01/2017 to 12/2022
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Strategy	QMJ	SUE	VT	QMJ	SUE	VT	ML
Total Return	292%	426%	236%	49%	283%	106%	-12%
CAGR	12.52%	15.41%	11.04%	6.86%	25.09%	12.78%	-2.16%
Standard Deviation	13.87%	19.96%	13.44%	16.25%	20.44%	10.46%	14.99%
Sharpe Ratio	0.90	0.71	0.82	0.42	1.16	1.22	-0.14
Excess Kurtosis	-1.78	-1.78	0.89	-3.00	-2.40	-2.64	-0.55
Skewness	-0.02	-0.17	-0.53	-0.23	0.31	0.22	-0.63
Best Month	15.81%	17.57%	13.10%	11.04%	20.06%	10.10%	12.39%
Worst Month	-10.66%	-14.88%	-16.60%	-11.73%	-9.86%	-5.17%	-15.21%
Positive Months	61.87%	62.59%	61.87%	55.56%	68.06%	65.28%	56.94%
Maximum Drawdown	-20.44%	-51.46%	-16.60%	-35.92%	-16.84%	-20.56%	-28.50%

Table 5: Descriptive statistics

4. Combined Strategy

4.1 Introduction

This section builds on the analysis of our individual investment strategies by exploring their combined use to potentially improve investor returns. Our goal is to create a unified investment strategy that capitalizes on the strengths of each approach. We apply the Markowitz Portfolio Optimization framework to integrate three strategies: Quality minus Junk (QMJ), Volatility Timing (VT), and Earnings Surprise (SUE). These strategies, based on long-term signals, contrast the Machine Learning-based strategy, which is computationally intensive. These strategies, oriented towards long-term signals, present a contrast to the Machine Learning-based strategy, which faces computational constraints, limiting its data scope.

We have decided to exclude the Machine Learning strategy from the combined approach. This decision is based on the extensive in-sample and out-of-sample analysis required, which the Machine Learning strategy's limited data set cannot fully support. It's important to note that this exclusion does not undermine the Machine Learning strategy's potential effectiveness. Instead, our focus is on evaluating whether the integrated strategy,

excluding Machine Learning, yields a higher Sharpe ratio compared to the performance of each strategy on its own.

4.2 Methodology

The centerpiece of our combined strategy is the Markowitz Portfolio Optimization framework, exemplified by the Tangency Portfolio. This approach combines the unique attributes of the QMJ, VT and SUE strategies into a single, optimized portfolio.

In our study, the Tangency Portfolio, constructed through the Markowitz Portfolio Optimization framework, is compared to four distinct investment models. Firstly, the 60/40 Portfolio, a traditional model, which allocates 60% to equities (S&P 500) and 40% to bonds (TLT ETF) and is renowned for its straightforwardness and conservative stance. This weighting scheme is popular among investors who aim for moderate risk. Secondly, the Equal-Weighted Portfolio which equally distributes weights across our three individual strategies, offering simple diversification without specifically targeting risk or return optimization. Thirdly, the Risk Parity Portfolio which equally balances risk across the three individual strategies based on each portfolio's variance. This scheme is often referred to as the 'All weather portfolio' because its risk allocation seeks returns regardless of market conditions. Lastly, the Global Mean-Variance Portfolio seeks to minimize the portfolio's overall variance, standing in contrast to the Tangency Portfolio's focus on maximizing the Sharpe ratio.

The full sample is divided into two periods: an in-sample period (06/2005 to 12/2016) for optimizing strategy weights, and an out-of-sample period (12/2017 to 12/2022). The latter tests the optimized weights calculated in-sample to assess the portfolio's performance and robustness in a new period.

4.3 Performance Overview

4.3.1 In-Sample

4.3.1.1 Efficient Frontier

In our in-sample period, we sought the optimal asset allocation for our Tangency Portfolio. Through simulations, we constructed the efficient frontier and the feasible region to visualize (see *Figure 9*) where our different combined portfolios stand. The left side of the feasible region contains the best risk-adjusted portfolios among which are the various weight allocation schemes except for the 60/40 portfolio.

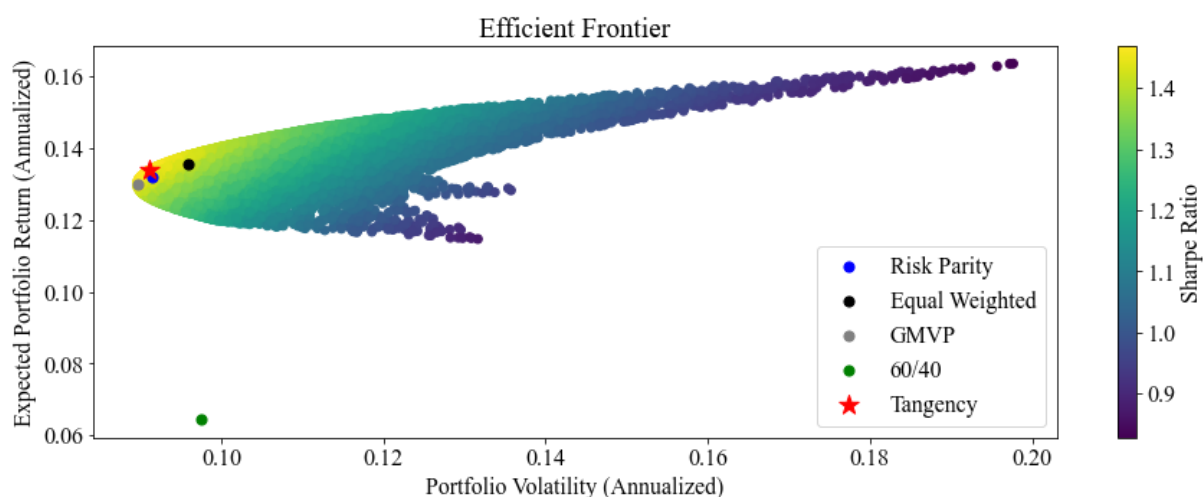


Figure 9: Efficient Frontier

As expected, the Tangency Portfolio achieves the highest Sharpe ratio, and hence, is the most desirable risk-adjusted portfolio, outperforming both the Risk Parity and Global Mean-Variance Portfolios.

	QMJ	SUE	VT
Sharpe Ratio	0.90	0.71	0.82
Weight	47.97%	25.81%	26.22%

Table 6: Tangency Portfolio Weights (06/2005 to 12/2016)

Strategy QMJ forms the backbone of the Tangency portfolio by contributing 48%, with SUE and VT also playing a substantial role. This weight distribution is not arbitrary, it echoes

the individual portfolios risk-adjusted performance across the in-sample period. Indeed, the tangency portfolio seeks for the best risk-adjusted returns possible and by definition allocates more weight to the components which have the highest Sharpe ratio.

	QMJ	SUE	VT
Global Min. Variance	44.77%	19.31%	35.92%
Risk Parity	36.68%	25.48%	37.84%
Equal Weighted	33.33%	33.33%	33.33%

Table 7: Portfolio Weights (06/2005 to 12/2016)

The Global Minimum Variance Portfolio's high allocation towards QMJ indicates a preference for less variance and covariance between the assets. The Risk Parity Portfolio presents a more even distribution, with SUE having the lowest weight because of its relatively higher risk. Meanwhile, the Equal Weighted Portfolio simplifies the approach by allocating a third to each strategy, serving as a straightforward benchmark against more complex weighting methods.

4.3.1.2 Cumulative Returns of Portfolios

Examining the in-sample cumulative returns, the Tangency, Risk Parity, Equal Weighted and Global Minimum Variance portfolios exhibit strikingly similar performances over the period. Each portfolio demonstrates robust growth, with their trajectories closely aligns, especially from 2008 onwards. The 60/40 portfolio, while starting the period at a similar level, diverges and shows a more subdued performance overall.

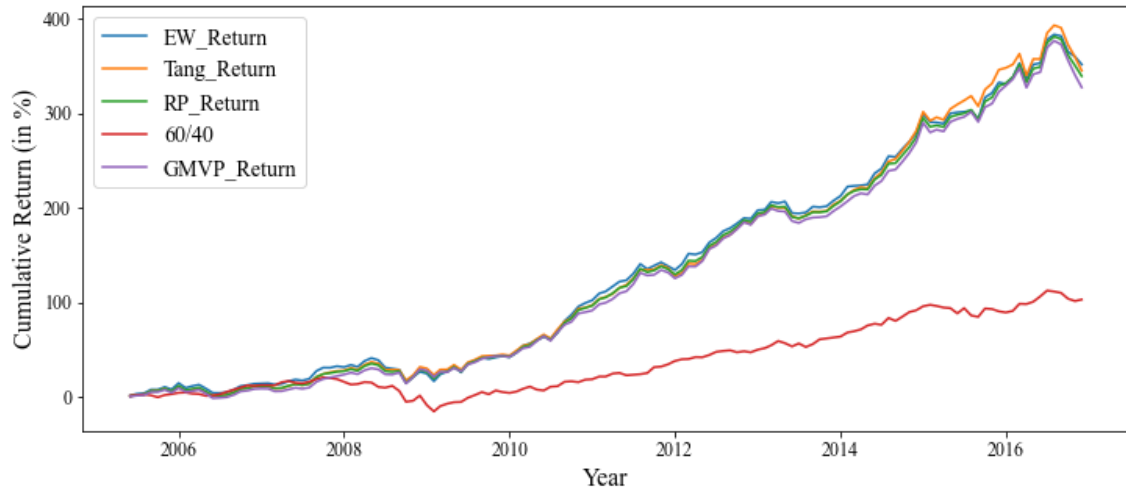


Figure 10: Cumulative Returns In-Sample

During the market turmoil of 2008, all portfolios experienced minor downturns, yet the recovery was swift and synchronized. The Tangency Portfolio, alongside the Risk Parity and Equal Weighted and Mean-Variance portfolios performed almost similarly, reflecting a shared sensitivity to the underlying market conditions, as seen in *Figure 10*. The convergence in performance suggests that, despite different strategic weightings, the portfolios were similarly capable of capitalizing on the market's performance.

4.3.1.3 Descriptive Statistics

Examining the performance statistics from the in-sample period, we observe a nuanced landscape of returns and risks. Table 8 shows that, as expected, the Tangency Portfolio stands out with a robust Sharpe ratio of 1.44, reflecting its superior risk-adjusted returns compared to the Equal Weighted and Risk Parity portfolios, which report Sharpe ratios of 1.36 and 1.39 respectively.

	Global Min. Variance	Risk Parity	Tangency	Equal Weighted	60 / 40
Total Return	328%	336%	345%	352%	103%
CAGR	13.37%	13.56%	13.76%	13.91%	6.30%
Standard Deviation	8.93%	9.11%	9.08%	9.55%	8.62%
Sharpe Ratio	1.41	1.39	1.44	1.36	0.73
Excess Kurtosis	-1.65	-0.95	-2.07	-0.93	1.95
Skewness	-0.63	-0.73	-0.57	-0.66	-1.34
Best Month	7.04%	6.71%	7.22%	7.63%	6.74%
Worst Month	-9.23%	-10.31%	-8.57%	-10.62%	-10.89%
Positive Months	73.38%	74.82%	73.38%	69.78%	63.31%
Maximum Drawdown	-12.48%	-15.52%	-14.24%	-18.45%	-29.94%

Table 8: In-Sample (2005-2016) Descriptive Statistics

In contrast, the 60/40 portfolio, while offering a respectable total return of 103%, does so at the cost of heightened risk relative to returns, as evidenced by its lower Sharpe ratio of 0.73 and a more pronounced maximum drawdown of nearly 30%. This underlines a trade-off between higher returns and increased risk exposure for the 60/40 portfolio. From a risk management perspective, the Tangency Portfolio presents an advantageous profile. Its maximum drawdown is limited to -14.24%, reflecting a well-managed risk level. Furthermore, its skewness and excess kurtosis suggest a relatively symmetrical distribution of returns with thinner tails than the Global Minimum Variance Portfolio, implying fewer extreme returns.

The Tangency portfolio proves to be a great diversifier of risk with substantially higher risk-adjusted returns than any of its individual components (Sharpe ratio of 1.44 vs 0.81 on average). Also, it shows higher total and annual returns than two of its three components which highlights once again its diversification effect where it improves returns by maintaining or lowering its risk.

4.3.2 Out-of-Sample

4.3.2.1 Cumulative Returns of Portfolios

In the out-of-sample period, the performance of the portfolios, when applying the optimal weights identified in the in-sample phase, presents a similar narrative where all the portfolios except the 60/40. The Tangency Portfolio, in particular, showcases its robustness. Its growth trajectory maintains proximity with the Risk Parity, Equal Weighted, and Global Minimum Variance portfolios, affirming the effectiveness of its strategic allocation.

Throughout this period, the Tangency Portfolio navigates market fluctuations with a resilience that echoes its in-sample performance. The 60/40 portfolio, again, underperforms relative to the other portfolios, which might be due to the difference in the composition of its components. The four other portfolios are composed majorly of stocks, resulting in this gap of performance. This contrast highlights the adaptive nature of the Tangency Portfolio, which, thanks to its foundation in the Markowitz optimization framework, could be more attuned to shifting market dynamics than the traditional 60/40 mix.

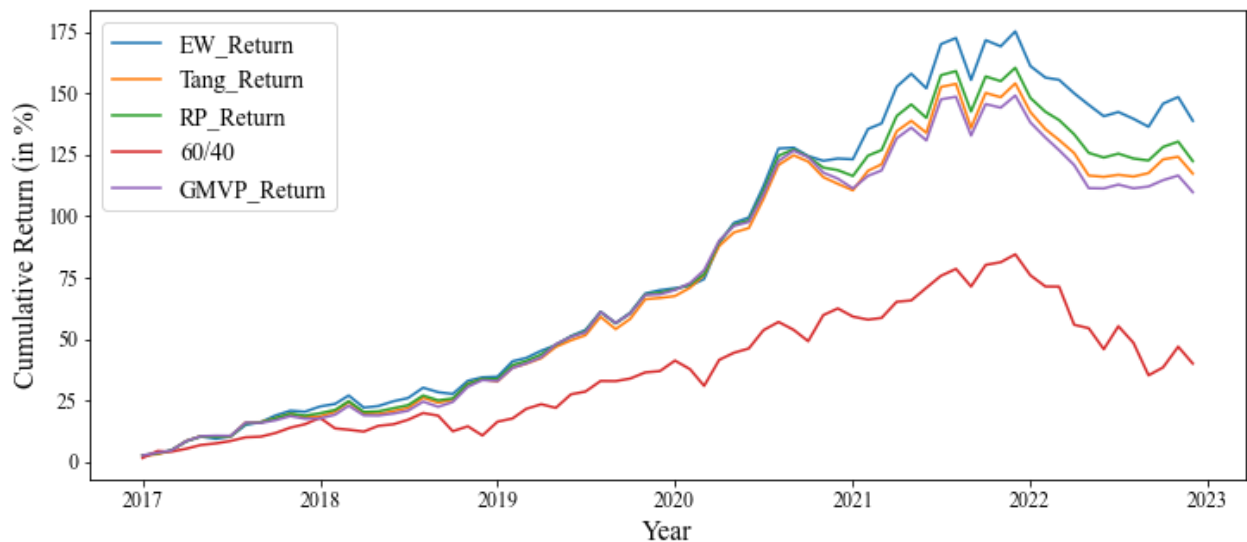


Figure 11: Cumulative Returns Out-of-Sample

As Figure 11 shows, after an initial period of parallel growth, the Tangency Portfolio, along with the other portfolios, faces a challenging period from 2020 onwards due to the Covid-

19 crisis. It's in this challenging climate that the portfolio's strategic weighting truly proves its mettle, avoiding the sharper declines seen in the 60/40 portfolio and demonstrating a disciplined return profile. This period validates the diversification and risk management principles that are integral to the Tangency Portfolio's design and affirms its potential as a resilient investment strategy in the face of market adversity.

4.3.2.2 Descriptive Statistics

Analyzing the Out-of-Sample performance (see Table 9), the Tangency Portfolio distinguishes itself with a commendable Sharpe ratio of 1.32. This figure not only surpasses the Global Minimum Variance Portfolio's 1.30 but also notably outperforms the traditional 60/40 portfolio's ratio of 0.50, underscoring its superior risk-adjusted returns. The Tangency's closest competitors, the Risk Parity and Equal Weighted portfolios, register Sharpe ratios of 1.44 and 1.49, respectively, suggesting that while the Tangency Portfolio does not lead in this measure, it holds its ground well within this competitive set.

	Global Min. Variance	Risk Parity	Tangency	Equal Weighted	60 / 40
Total Return	110%	124%	117%	139%	40%
CAGR	13.14%	14.41%	13.81%	15.61%	5.77%
Standard Deviation	9.60%	9.48%	9.97%	9.91%	11.55%
Sharpe Ratio	1.30	1.44	1.32	1.49	0.50
Excess Kurtosis	-2.93	-2.75	-2.74	-2.66	-2.21
Skewness	-0.02	0.08	-0.04	0.12	-0.55
Best Month	7.24%	7.74%	7.98%	8.36%	8.10%
Worst Month	-6.34%	-6.04%	-7.04%	-6.26%	-9.05%
Positive Months	68.06%	70.83%	69.44%	70.83%	65.28%
Maximum Drawdown	-15.79%	-14.69%	-14.97%	-14.10%	-26.70%

Table 9: Out-of-Sample (2017-2022) Descriptive Statistics

The Tangency Portfolio's out-of-sample Sharpe ratio reflects the performance shift of QMJ, which, despite a robust in-sample Sharpe ratio of 0.90, declined to 0.42 in the out-of-sample period. This significant reduction is attributed to its increased standard deviation, up from 13.87% to 16.25%, and a notable decline in annualized returns from 12.52% to 6.86%. The heavy weighting of 47.97% in QMJ within the Tangency Portfolio was based on its in-sample strength, yet the strategy's out-of-sample volatility and lower performance underlined the challenges of relying on past performance as a predictor of future returns.

Despite a lower total return compared to the Equal Weighted and Risk Parity portfolios, the Tangency Portfolio's Sharpe Ratio proves its balance of risk and reward, particularly when considering the 60/40 portfolio's trade-off between higher returns and risk exposure. The 60/40's higher volatility is reflected in its significantly larger maximum drawdown of -26.70% compared to the Tangency's -14.97%.

The Tangency Portfolio's skewness and kurtosis values point towards a more symmetric return distribution with fewer extreme deviations than its peers, indicating well-managed tail risk. Additionally, the Tangency Portfolio's consistency is evident with positive returns in 69.44% of the months, a testament to its strategic composition and the benefits of diversification, despite the challenging market conditions faced during the out-of-sample period.

4. Factor Analysis

In the subsequent analysis, we build upon the descriptive statistical insights from previous chapters to examine the risk factors influencing portfolio returns, utilizing the Capital Asset Pricing Model (CAPM) and the three-factor Fama-French (FF3) model. These models serve as a framework to explain the underlying risks that are hypothesized to influence portfolio performance. The CAPM is articulated through the following equation:

$$R_{p,t} - R_{f,t} = \alpha + \beta_1(R_{m,t} - R_{f,t}) + \varepsilon_t,$$

where $R_{p,t} - R_{f,t}$ represents the excess return of the portfolio over the risk-free rate, and $R_{m,t} - R_{f,t}$ represents the excess return of the market. The coefficient α captures the portfolio's abnormal returns, while β_1 quantifies the portfolio's sensitivity to market movements, with ε_t denoting the idiosyncratic, non-systemic component of return.

Expanding upon the CAPM, the Fama-French three-factor model integrates additional dimensions of risk associated with size and value:

$$R_{p,t} - R_{f,t} = \alpha + \beta_1(R_{m,t} - R_{f,t}) + \beta_2(SMB_t) + \beta_3(HML_t) + \varepsilon_t.$$

In this formula, *SMB* and *HML* introduce size and value factors, respectively. The coefficients β_2 and β_3 measure the portfolio's responsiveness to these additional risk factors. Collectively, these parameters estimate the degree to which market, size, and value factors systematically inform portfolio returns.

Table 10 presents the regression outputs for both models. Under the CAPM, the tangency portfolio consistently shows significant monthly excess returns of 0.95% across all periods analyzed. The portfolio's market beta ranges from 0.147 to 0.181, indicating lower volatility compared to the broader market and low exposure to market risk. However, the low R^2 values (0.068 to 0.086) suggest that the CAPM model has limited ability to explain the portfolio's variance, pointing towards the influence of factors beyond market risk. Considering the CAPM, the results are consistent throughout all periods.

	Full Sample (2005 – 2022)	In-Sample (2005 – 2016)	Out-of-Sample (2017 – 2022)
α	0.0095***	0.0095***	0.0095***
β_M	0.1668***	0.1814***	0.1474**
R^2	0.078	0.086	0.068

	Full Sample (2005 – 2022)	In-Sample (2005 – 2016)	Out-of-Sample (2017 – 2022)
α	0.0090***	0.0093***	0.0081***
β_M	0.2196***	0.2278***	0.1961***
β_{SMB}	-0.1389*	-0.0445	-0.2831**
β_{HML}	-0.2346***	-0.1990**	-0.2762***
R^2	0.167	0.123	0.281

Note: *, **, *** refer to the significance level of 90, 95, and 99 percent, respectively.

Table 10: Regression Results of Tangency Portfolio for CAPM & FF3 Model

In the Fama-French three-factor model analysis, the tangency portfolio consistently showcases a notable monthly excess return of 0.90% across the entire period, reaffirming its strong performance. The model further reveals the portfolio's relatively low exposure to broader market risk, as evidenced by its lower volatility compared to the market.

A significant shift is observed in the portfolio's sensitivity to company size. The negative coefficient for the *SMB* factor increases from an insignificant -0.045 in the In-Sample period to a significant -0.283 in the Out-of-Sample period. This transition highlights an increased inclination towards larger firms in the latter period. Concurrently, the negative coefficients for the *HML* factor, -0.199 and -0.276, underline a pronounced preference for growth stocks over value stocks. These coefficients collectively suggest a strategic shift in the portfolio towards larger, growth-oriented investments.

5. Limitations

This section delves into the diverse implementation issues that may arise when applying the combined strategy or its variants in real-life scenarios.

Firstly, a major limitation involves the various estimators used, particularly for the tangency portfolio, or the different weighting schemes employed for comparison. As discussed previously, the estimators for expected returns and volatility used in the weighting schemes are

based on past data. This issue is commonly referred to as 'parameter uncertainty' in literature, highlighting the restrictive assumptions inherent in models such as Markowitz's framework. The framework maximizes a single-period utility, assumes investors are myopic, markets are frictionless, and risk aversion is constant. These assumptions can lead to several potential errors if omitted or unstated. DeMiguel, Martin-Utrera, and Nogalas (2015) argue that, according to Garlenau and Pedersen's (2013) framework and in the absence of estimation error, the optimal portfolio is to trade toward the Mean-Variance. However, this optimal portfolio is sensitive to estimation error and trading costs. Numerous studies, such as Kan and Zhou (2007), have developed models to manage estimation errors.

Secondly, for the sake of simplicity and to focus on performance analysis, costs were initially excluded from the implementation of the combined strategy but will be addressed here. Costs, such as transaction or brokerage fees, are often overlooked in research where markets are assumed to be frictionless, yet they can significantly impact the execution of quantitative investment or trading strategies. In the combined strategy, there are no transaction costs as the weights are constant, and there is no rebalancing, but, in practice, transaction costs arise from the individual strategies constituting the portfolio. Additionally, volume is a crucial factor to consider in trading strategies, as some assets in the strategy may not be available for trade or might be available in limited quantities due to the respective illiquidity of their market.

6. Conclusion

In summary, the combined strategy validates the diversification of portfolio risk, outperforming individual strategies. Both the tangency portfolio and portfolios employing various weight allocation schemes demonstrate superior risk-adjusted returns compared to their individual components. This affirms that regardless of the weight allocation, diversification is a powerful strategy and a 'free lunch' as often described in literature.

Among the various combined portfolios, the tangency portfolio exhibits the best in-sample performance, yielding a total return of 345% and a Sharpe ratio of 1.44, aligning with expectations. According to the Markowitz framework, the tangency portfolio represents the optimal investment choice from a risk-adjusted perspective. Notably, QMJ significantly contributes to the tangency portfolio, because of having the highest Sharpe ratio among the three components. The remaining combined portfolios perform similarly, except for the simple 60/40 approach, which is markedly outperformed by the optimized portfolios.

However, the tangency portfolio falls short in out-of-sample performance, being surpassed by both the risk-parity and equal-weight approaches. This shift in comparative performance is attributed to the substantial weight assigned to QMJ (47.97%), whose out-of-sample performance notably declined (from a Sharpe ratio of 0.90 to 0.42). This downturn impeded the tangency portfolio during this subsequent period. The decline in the Sharpe ratio observed in the QMJ portfolio during the out-of-sample period can be attributed to the relatively improved performance of the Big Junk portfolio in this timeframe. Notably, the Big Junk portfolio experienced approximately 130% cumulative returns in the out-of-sample period. This significant gain adversely impacted the returns of the QMJ portfolio, which is structured to be short on the Big Junk portfolio.

To conclude, Markowitz's framework and popular weight-allocation scheme proves to be great diversifiers of risk. Also, the weights allocated by the tangency portfolio seems not to be working as well out-of-sample period than in-sample, which emphasizes overfitting as the strategy seems not robust.

Appendice

Individual assets	GLD	IEF	TLT	VTV	VUG
Annual returns (%)	12.17	2.42	4.02	6.64	9.43
Volatility (%)	20.77	7.11	14.49	21.62	19.92
Sharpe-Ratio	0.52	0.14	0.18	0.24	0.40
Skewness	-.205	.087	.062	-.165	-.026
Kurtosis	5.50	2.44	1.95	9.29	10.80
MDD (%)	-40.52	-11.86	-28.45	-61.31	-51.35
Avg. DD (%)	-12.17	-4.69	-12.22	-17.93	-9.30

Table 11: Annual performances of the individual asset in-sample

Individual assets	RP vix	RP	GLD	IEF	TLT	VTV	VUG
Annual returns (%)	2.51	1.58	21.24	6.31	7.68	-30.28	-19.31
Volatility (%)	20.65	7.58	26.54	8.36	14.95	32.74	29.39
Sharpe-Ratio	-0.01	-0.16	0.69	0.41	0.32	-1.01	-0.75
Skewness	.765	.267	.271	.125	.365	-.083	.172
Kurtosis	16.68	3.41	5.32	1.07	2.79	4.93	7.81
MDD (%)	-22.13	-12.39	-29.41	-7.12	-16.66	-57.40	-49.87
Avg. DD (%)	-4.65	-2.52	-7.56	-2.30	-3.87	-17.45	-12.60

Table 12: Annual performances of the portfolios and components during 2008

Individual assets	GLD	IEF	TLT	VTV	VUG
Annual returns (%)	6.58	-2.02	-3.97	6.82	12.75
Volatility (%)	13.96	6.67	15.32	17.65	21.35
Sharpe-Ratio	0.32	-0.45	-0.30	0.34	0.55
Skewness	-.046	.174	.123	-.547	-.422
Kurtosis	3.17	3.59	5.29	16.64	8.87
MDD (%)	-22.00	-27.71	-51.75	-36.78	-35.99
Avg. DD (%)	-7.94	-7.86	-15.96	-5.13	-7.36

Table 13: Annual performances of the individual assets out-of-sample

Individual assets	RP vix	RP	GLD	IEF	TLT	VTV	VUG
Annual returns (%)	39.05	18.50	34.04	20.70	42.41	-26.24	31.23
Volatility (%)	22.06	11.12	20.87	9.45	28.27	47.02	45.29
Sharpe-Ratio	1.73	1.59	1.59	2.11	1.47	-0.57	0.67
Skewness	-.435	-.876	-.001	.291	.199	-.248	-.585
Kurtosis	3.43	5.82	2.50	7.24	5.45	3.21	4.53
MDD (%)	-12.52	-9.89	-12.52	-4.67	-15.72	-36.78	-31.78
Avg. DD (%)	-1.8	-1.11	-1.82	-0.68	-3.03	-14.37	-8.57

Table 14: Annual performances of the portfolios and components during 2020

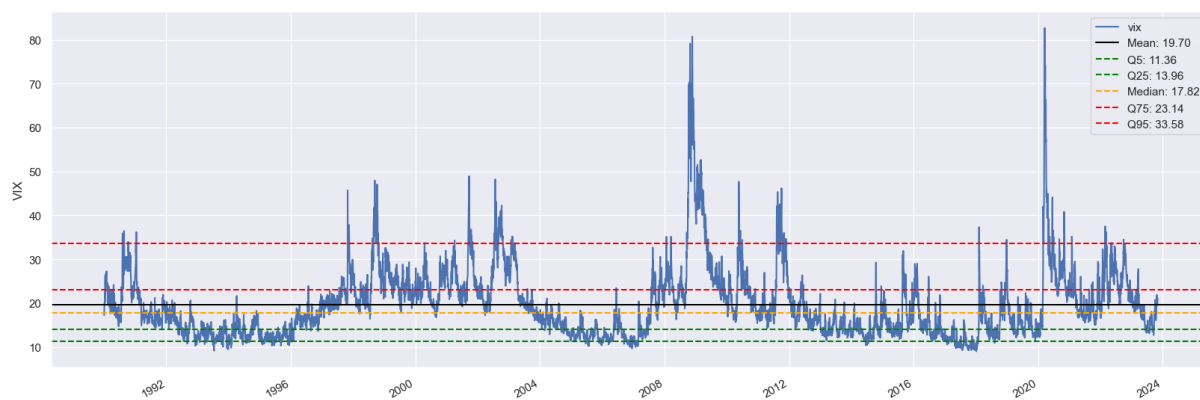


Figure 12: VIX level from 1990 to 2023

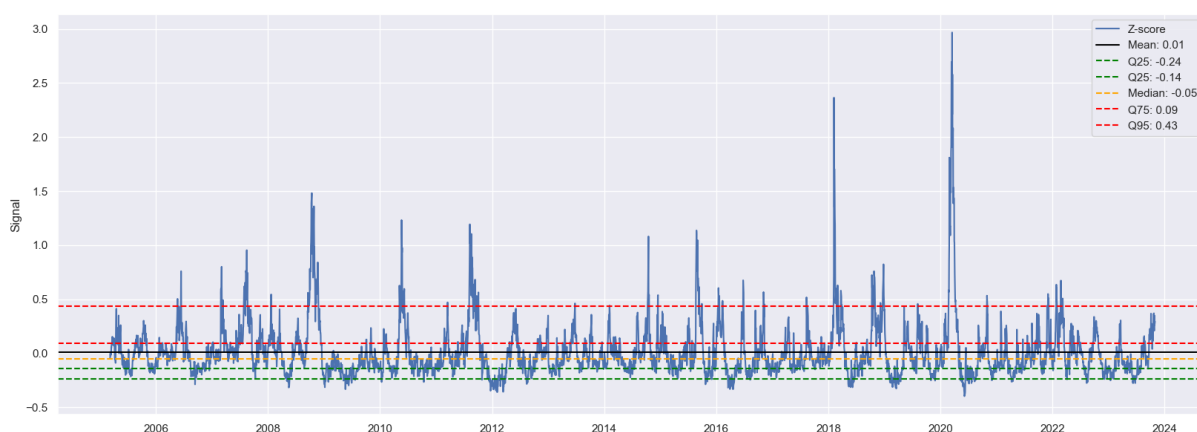


Figure 13: Signal (in and out-of-sample) with an estimation window of 75-days

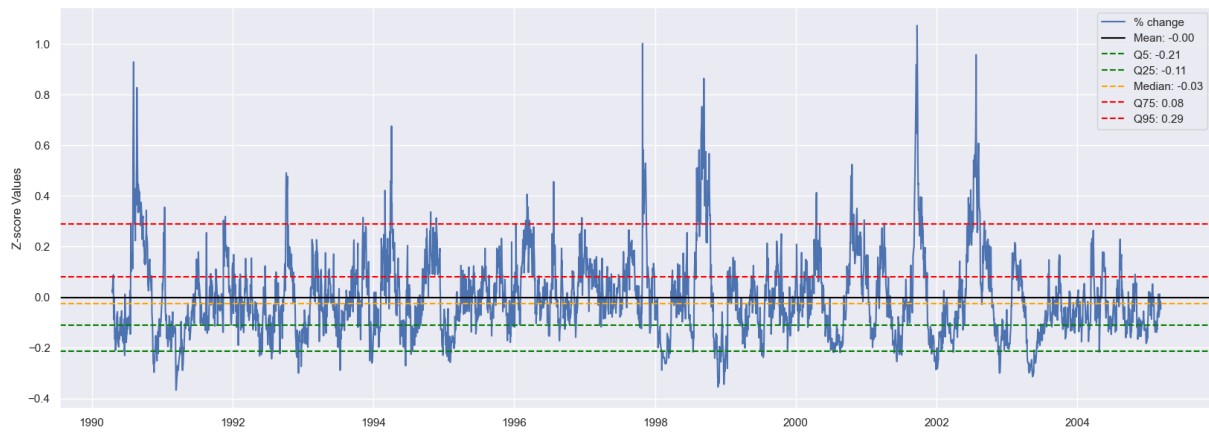


Figure 14: Signal (before sample) with a 75-day estimation window

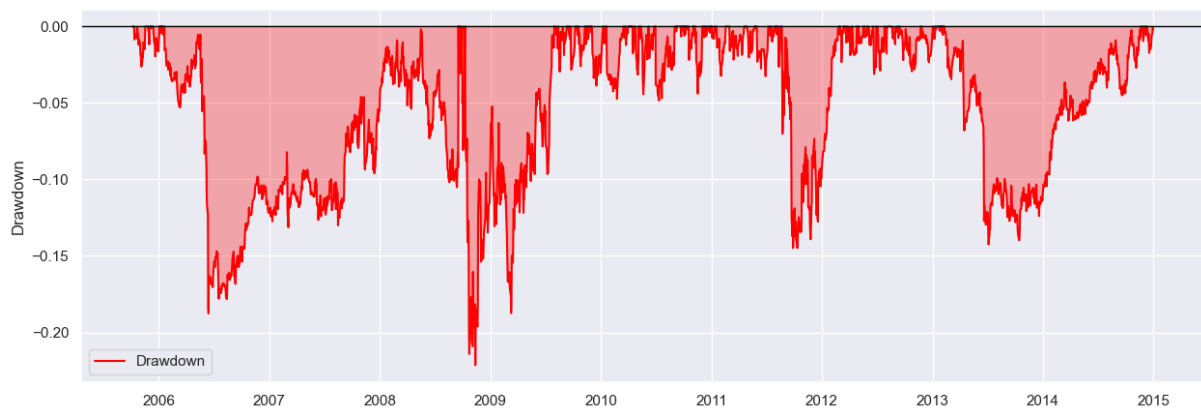


Figure 15: Portfolio drawdowns in-sample

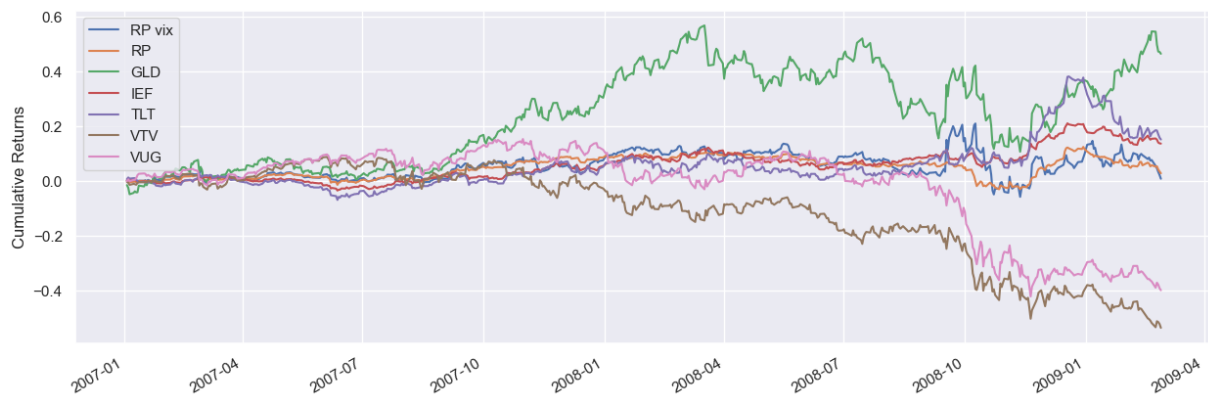


Figure 16: Portfolio and components cumulative returns during the Great financial crisis

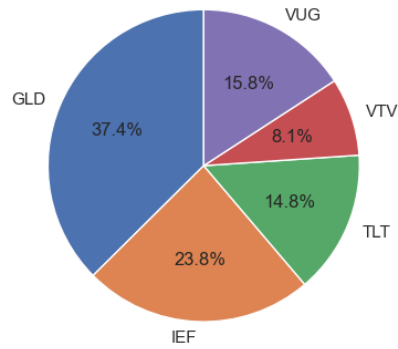


Figure 17: Average weights on each individual asset in the Strategy's portfolio (out-of-sample)

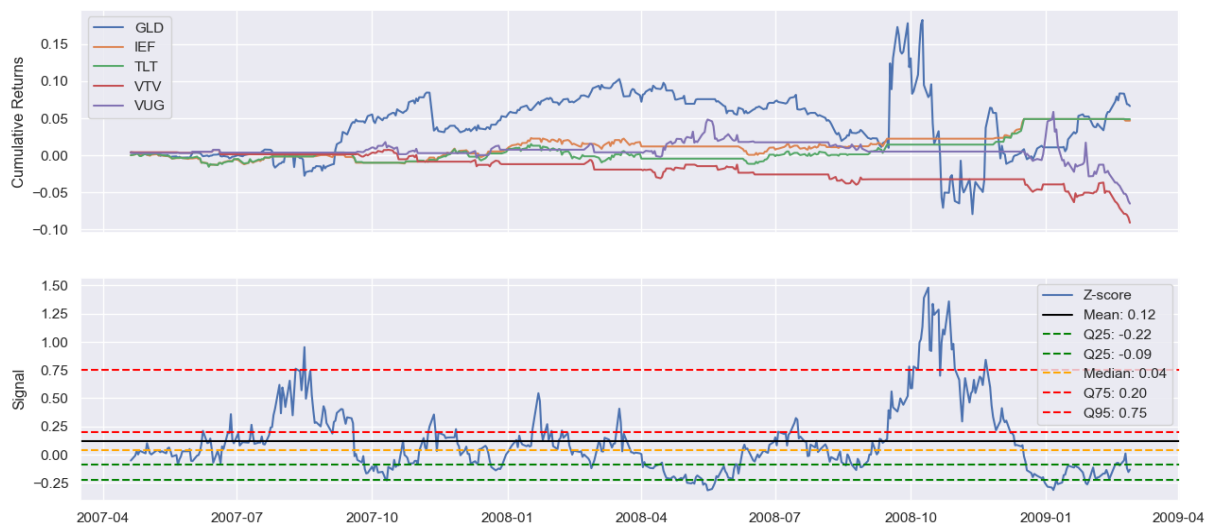
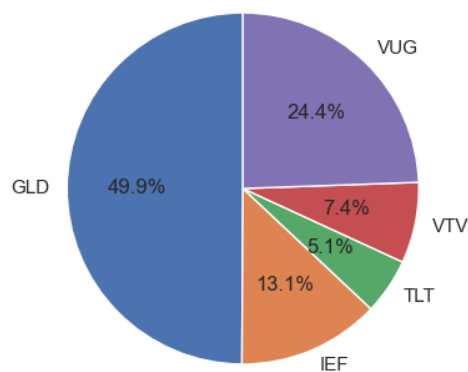
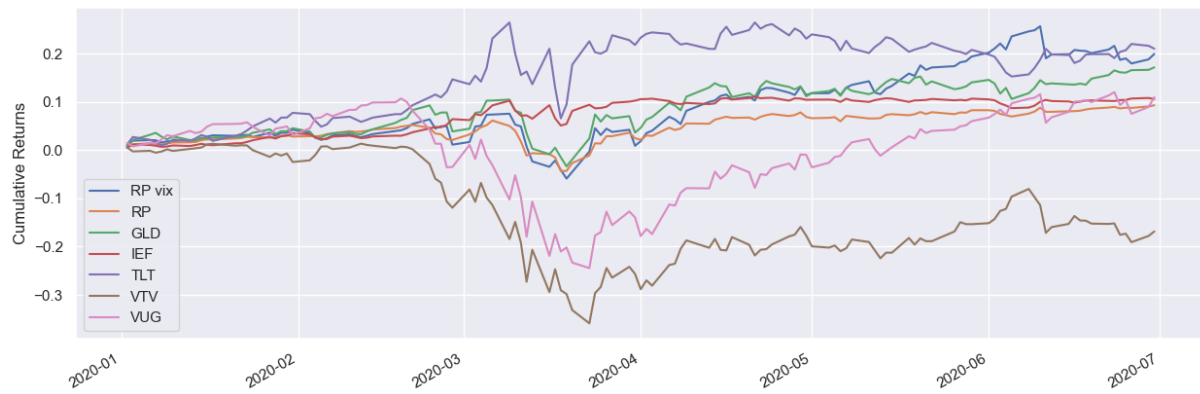


Figure 18: Attribution of performance of the individual assets and signal fluctuation during the Great financial crisis



Figure 19: GLD drawdowns in-sample



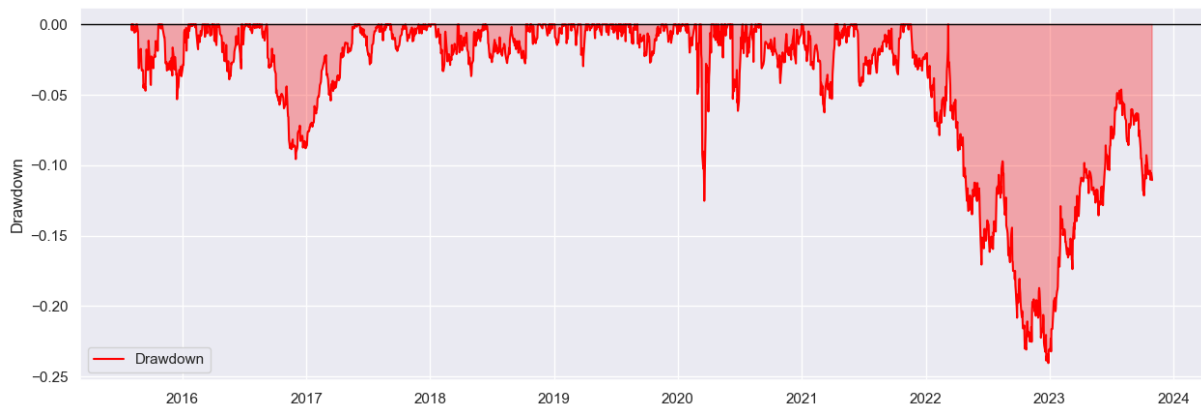


Figure 23: Portfolio drawdowns out-of-sample

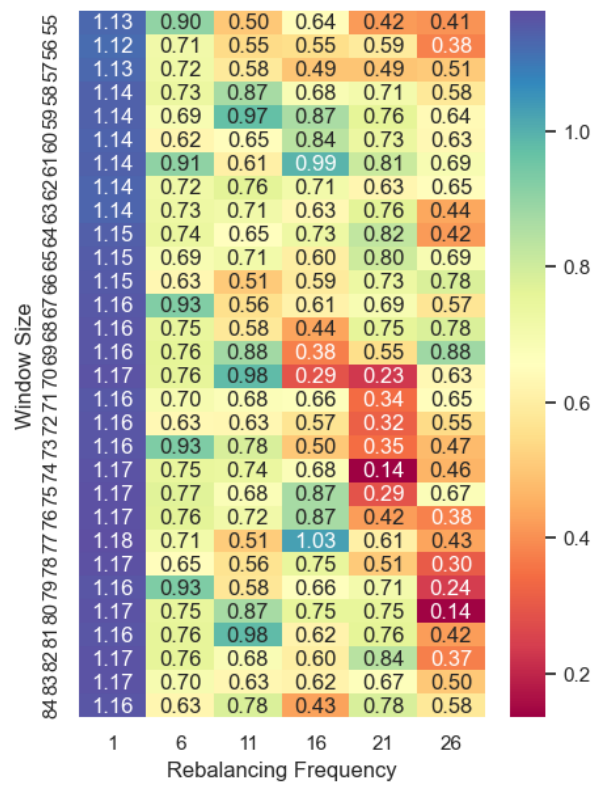


Figure 24: Heatmap of the Sharpe ratio of the portfolio out-of-sample

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