A Work Project, presented as part of the requirements for the Award of a Master's degree in Finance from the Nova School of Business and Economics.

Tail Risk and its Asset Pricing Implications on the Norwegian Stock Exchange

Michael Sollund

Work project carried out under the supervision of:

Paulo M. M. Rodrigues

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Abstract:

This work project reveals novel asset pricing implications of time-varying tail risk (TVTR) on the Norwegian stock market. On the aggregated market, TVTR holds significant positive in-sample predictive power for time horizons from one-year onwards. The newly published smooth time-varying tail risk holds significant positive predictive power even at short time horizons. When compared to the historical mean out-of-sample, TVTR performs significantly better. Cross-sectionally, constructed portfolio with high TVTR loading, compared to low TVTR loading, yields significantly higher average future returns, as well as positive CAPM-, Fama-French 3-factor- and Carhart 4-factor alphas, significant at the 1% and 5% level.

Keywords:

Tail Risk, Time-Varying Tail Risk, Hill Estimator, Forecast Evaluation, Return Prediction, Causality, Norway

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1 Introduction

Financial markets have always been volatile, with movements influenced by various global events. The Covid-19 pandemic has certainly had a significant impact on financial markets worldwide, with many experiencing unprecedented fluctuations. In addition to the pandemic, various other factors, such as geopolitical tensions, rising interest rates, energy crises, and the increasingly volatile cryptocurrency market, have added to the uncertainty and complexity of financial markets. However, despite these challenges, many equity indices have continued to reach new all-time highs, leaving investors with questions about stock returns and the potential for future market movements. As a result, the topic of equity return predictability has become increasingly topical, as investors search for insights and strategies to navigate the current financial market landscape.

The topic of equity return predictability has long been of high interest in finance, both in academia and in practice. Academically, equity return predictability is linked to deeper understanding of financial markets and price formation. In practice, accurate equity return prediction models are motivated by the prospect of driving profits through informed investing decisions. The ability to predict the direction and magnitude of future returns is essential for investors, asset managers, and financial institutions attempting to “beat the market”.

Equity return prediction models have been a subject of extensive research for decades. Many potential variables have been put forth as meaningful predictors, including historical returns, interest rates, financial ratios, and macroeconomic indicators. Examples include dividend-price ratio (Fama & French, 1988), earnings-price ratio (Campbell & Shiller 1988; Campbell & Shiller, 1998), book-to-market ratio (Kothari & Shanken, 1997; Pontiff & Schall, 1998), nominal interest rates (Campbell, 1987), inflation rate (Fama & Schwert, 1977; Nelson, 1976), term and default spreads (Fama & French, 1989), corporate issuing activity (Baker & Wurgler, 2000; Boudoukh et al., 2007), consumption-wealth ratio (Lettau & Ludvigson, 2001), and stock market volatility (French, Schwert, & Stambaugh, 1987).

Despite numerous models having been developed and tested, the "correct" model remains elusive to both researchers and practitioners. Empirical research findings within this topic are often not unanimous and can be specific to the sample used, leaving room for novel contributions to this area of research and broader understanding of financial markets.

This work project endeavors to examine the predictive capacity of properties of the lower extreme tails of the time dependent distribution of daily returns of Norwegian equities trading on the Norwegian stock exchange. The study has its basis in extreme value theory and draws its primary inspiration from findings of Kelly and Jiang (2014), who established that their proposed time-varying tail risk measure wields significant explanatory power for excess market returns in the United States.

There has been one similar study of tail risk on the Norwegian stock market as a masters' thesis (Kashif, 2016). However, this study produced insignificant results in most areas. Other than that, to the best of my knowledge, no previous study has applied this approach to the Norwegian equity market, making this investigation a novel contribution to the field.

The remainder of the work project is organized as follows. Section 2 presents a review of the current literature on the topic of equity return prediction models, particularly in light of extreme value theory and tail risk. Section 3 describes the used methodology and data collected. Section 4 presents the work project's findings. Section 5 discusses some of the work project's limitations. Lastly, section 6 concludes the work project.

2 Literary Review

Over the past decades, global financial markets have experienced major shocks, including the 2008 financial crisis, the 2010 European debt crisis and the 2019 Corona pandemic. These events presented challenges to dynamic asset pricing models on how they ought to accommodate the observed interdependencies between pricing, volatility, and tail risk. Extreme value theory methods have been successfully used in this context.

Extreme Value Theory. Extreme value theory (EVT) is an area of study that aims to model and measure events which occur with very small probability (Brodin & Kluppelberg, 2008). One particular aspect of EVT that has garnered interest is its application in tail risk analysis. Tail risk refers to the likelihood of rare and extreme events causing significant fluctuations in asset prices, which potentially can lead to substantial losses. Researchers have hypothesized that heavy-tailed shocks to economic fundamentals help explain certain asset pricing behavior that has proved otherwise difficult to reconcile with traditional macrofinance theory (Kelly & Jiang, 2014). EVT and tail risk analysis attempt to adjust asset pricing models to account for extreme downside risk. There are several challenges in implementing EVT risk modelling, including scarcity of extreme events, determining whether the series is "fat-tailed", choosing the tail threshold, and choosing the methods of estimating the parameters (Bensalah, 2000).

Empirical Findings of Return Distribution. For an extended period, it has been established that the distribution of unconditional returns deviates significantly from normality (Mandelbrot, 1963; Fama, 1963). Research has revealed that the return distribution for risky assets exhibits features such as fat tails, volatility clustering, asymmetry, mean reversion, and correlated volatility across different assets (Poon & Granger, 2003). These findings have significant statistical implications, including the insufficiency of using standard deviation and variance as measurement of market risk. This serves as one of the motivations driving the research of EVT to model a more accurate risk measure.

Volatility versus Tail Risk Return Compensation. Volatility captures the property of the entire return distribution. Intuitively, due to the risk-averse nature of investors, this seems inadequate. As a risk measure, more importance should be put on the lower left side of the return distribution as the upside does not explain financial risk. In line with this hypothesis and the aforementioned findings regarding the return distribution, volatility as a risk measure has been discovered not to be compensated for among both US and non-US stocks. Rather, there is compensation for tail risk with higher expected excess return (Xiong, Idzorek, & Ibbotson, 2014). A study from Anderson et al. (2020) further document these relationships, showing that tail risk premium is a potent predictor of future excess returns of American and European indices. Whereas, in contrast, the reward for pure diffusive variance risk is unrelated to future equity returns.

Value at risk. There exist several measures within the EVT area that attempt to capture extreme downside risk. Value at risk (VaR) is a very popular option and a key risk measure widely used by practitioners. VaR is a statistical measure that quantifies the possible financial losses of a firm, portfolio, asset, or position over a specific time frame with a given level of confidence. Part of its popularity comes from its conceptual simplicity and ability to reduce the market risk associated with any portfolio to just one monetary amount (Engle & Manganelli, 2004).

Value at Risk Criticism and Shortcomings. Despite VaR's popularity, it has been subject to various concerns and criticisms. Firstly, VaR has been criticized for not providing adequate information about the tail risk, as it focuses only on the threshold and thus neglects the magnitude of losses that might occur beyond the calculated risk (Krause, 2003). Secondly, VaR often makes assumptions about the distribution of returns that do not reflect the true distribution. Parametric VaR assumes some distribution, typically normal, which is limiting and often incorrect. Non-parametric VaR utilizes historical data and assumes that the distribution of historic returns reflects the distribution of future returns (Krause, 2003). VaR also fails to satisfy the sub-additivity property (Artzner et al., 1999), which entails that risk reduction benefits of diversifications are not captured.

Expected Shortfall. To address VaR's limitations of not capturing the effects of heavy tails, Expected Shortfall (ES) has emerged as an alternative risk measure. ES is defined as the expected loss of a financial portfolio given that the loss exceeds the VaR threshold, thus providing a more comprehensive view of the potential losses in extreme market scenarios (Rockafellar & Uryasev, 2002). Additionally, ES satisfies the sub-additivity property, which VaR does not (Embrechts & Wang, 2015).

ARCH & Variants. Empirical work suggests that the tail distribution varies over time (Quintos et al., 2001; Galbraith & Zernov, 2004; Werner & Upper, 2004; Wagner, 2003). To account for this time-varying nature of volatility and its effects on asset pricing, a different model is needed as VaR and ES do not capture this aspect. Engle (1982) introduced the Autoregressive Conditional Heteroskedasticity (ARCH) model to allow for dynamic volatility by modeling the conditional variance as a function of past errors. ARCH models have been widely adopted in financial research due to their ability to capture financial data's previously mentioned characteristics of volatility clustering and heteroscedasticity. The ARCH model has been developed and expanded to include variations, such as Generalized Autoregressive Conditional Heteroskedasticity (GARCH) (Bollerslev, 1986), among others.

Kelly & Jiang Approach. Kelly and Jiang (2014) contributed to the ongoing research on the importance of tail risk in asset pricing. Ideally, one would directly construct a measure of aggregate tail risk dynamics using time series data on market returns, similar to how dynamic volatility is estimated in a GARCH model. However, problematically, the nature of extreme events is that they occur infrequently. To circumvent this restriction, Kelly and Jiang developed a method of measuring tail risk by capturing extreme negative events of equity markets by pooling daily returns of equities and looking at the monthly cross-sectional return distribution to estimate the tail risk, which they hypothesized to be a valuable proxy for the aggregate market tail risk. Their time-varying tail risk measure was revealed to carry a significant risk premium and to be an important determinant of expected asset returns. The paper provided novel insights into the vast financial literature on risk-returns tradeoffs and has served as a foundation to further exploration on the topic of tail risk in asset pricing.

Empirical Background of Kelly & Jiang. Kelly & Jiang draw on several stands of empirical literature such as rare disaster hypothesis (Rietz, 1988; Barro, 2006), as in line with EVT, in a dynamic setting (Gabaix, 2012; Gourio, 2012; Wachter, 2013), as well as Bansal and Yaron's (2004) extension of long-run risk models that incorporate fat-tailed endowment shocks (Eraker & Shaliastovich, 2008; Bansal & Shaliastovich, 2010; 2011; Drechsler & Yaron, 2011).

Quantile Regression in Combination with Kelly & Jiang. As a result of Kelly and Jiang's (2014) model's significant results, further research on their time-varying tail risk measure has been conducted. Chevapatrakul et al. (2019) analyzed the significance of the time-varying tail risk on different points on the return distribution with the usage of quantile regression, taking Kelly and Jiang's research one step further. Their research presented evidence of the significance of strong predictive power of the time-varying tail risk of aggregate equity returns but revealed that the significance only applies to lower quantiles for forecast horizons up to one year. At higher quantiles, results showed no association between tail risk and excess market returns.

Averaging Kelly and Jiang. Faias (2023) modified Kelly & Jiang's monthly cross-sectional tail risk measure and achieved improved results from that of the original. Faias demonstrated that smooth cross-sectional tail risk yields positive and significant out-of-sample predictive power at both short- and long-term forecast horizons. Faias' version of monthly smooth cross-sectional tail risk uses the average of daily cross section tail risk within any given month, rather than the month's pooled daily returns. The concept of averaging and smoothing is nothing novel within the equity return prediction literature. Rapach et al. (2010) found that a combinatory equity return prediction model based on averaging achieved significant and significantly improved results. They empirically argued that combining forecasts incorporates information from numerous economic variables while sustainably reducing forecast volatility. The notion of reducing forecast volatility is the same principle applied by Faias' smoothening adaptation.

In-of-sample vs. Out-of-sample. Vast parts of the research on equity return predictors are based on in-sample prediction, as opposed to out-of-sample. Results that are significant in-sample but fail to deliver out-of-sample forecasts can be argued to be of little value. Welch and Goyal (2008) showed that an extensive list of predictors used in the literature were unable to deliver consistently superior forecasts in comparison to a simple constant model based on the historic average of return. Kelly and Jiang (2014) showcased statistically significant improvement of tail risk compared to the historic mean.

3 Methodology

3.1 Dataset Creation

Daily Norwegian stock returns are gathered from the TITLON database (Norges Arktiske Universitet, 2023). TITLON is a Norwegian database that provides access to financial time-series for all Norwegian universities. TITLON was chosen over Refinitiv due to Refinitiv's lack of dividend adjusted prices. Prices used are adjusted for both stock splits and dividends, which are of high importance. It is essential to adjust for dividends for accurately calculating the lower tail, as unadjusted stock prices can fall due to dividend payments or extreme drops resulting from certain acquisitions, which can cause daily returns to exceed the lower threshold in an inappropriate manner. TITLON also includes delisted stocks.

Monthly Norwegian risk-free rates, market return and pricing factors are gathered from Professor Bernt Arne Ødegård's resources for asset pricing (Ødegård, 2023). Risk-free rates are sourced from the Norwegian central bank. Market returns are the returns from two indices constructed from most stocks on the Norwegian Stock Exchange (the least liquid stocks are filtered out), one equally weighted index and one value weighted index. Pricing factors include Fama and French's factors HML and SMB and Carhart's momentum factor, UMD.

In November 2020, the old Norwegian Stock Exchange was acquired by Euronext. TITLON therefore provides two separate databases, 1985-2020:11 and 2020:12 to the present. For the analysis, I have merged the two. Euronext consists of more stocks than the prior exchange.

After merging, cleaning the dataset of unwanted errors and selecting the period 1990-2022, the dataset consists of 1,792,534 total daily return datapoints. I filtered out the stocks that have returns equal to 0 for more than 50% of the total trading days that the security has been active. After which, 1,352,830 daily return datapoints remain.

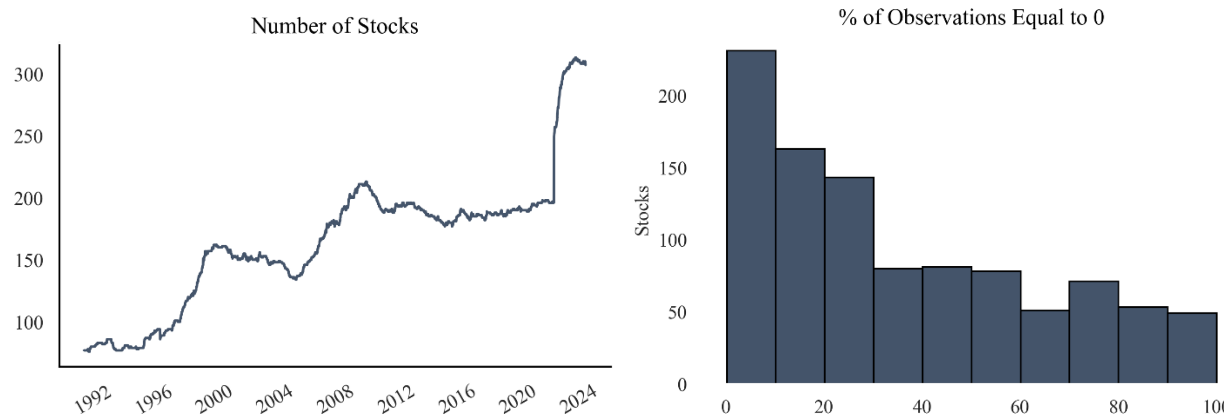

Figure 1 & 2: Figure 1 displays the numbers of stocks that are traded on the Norwegian stock exchange through time, before filtering illiquid stocks. The spike in late 2020 is due to the acquisition by Euronext. Figure 2 is a histogram of stocks' percentage of daily stock returns that are equal to zero. Those over 50% are removed.

3.2 Time-Varying Tail Risk

The basis of the analysis is the time-varying tail risk (TVTR), which falls under the extreme value theory. The calculation of TVTR is based on Hill's (1975) estimator which captures the nature of the lower tail of the return distribution. The underlying assumption of the Hill estimator is that the lower tail of asset returns behaves according to a power law. Equation (1) is a model of the conditional return tail and states that extreme return events obey a dynamic power law, i.e.,

$$P\left(R_{i,t+1} \middle| R_{i,t+1} < u_t \text{ and } \mathcal{F}_t\right) = \left(\frac{r}{u_t}\right)^{-\frac{a_i}{\lambda_t}} \tag{1}$$

The tail exponent a_i / λ_t is the key parameter which determines the shape of the tail. Equity returns have been documented to vary over time (Quintos et al., 2001), which in part is the background for the dynamic time-varying extension of the power law assumption championed by Kelly and Jiang (2014). The value of λ_t determines the heaviness of the tail and thus corresponds to high instances of extreme returns during the pooled period. The $1/\lambda_t$ term in the exponent may vary with the conditioning information set $\mathcal{F}_t$. The constant a_i determines the different level of tail risk among different assets. However, the dynamics are the same for all assets because they are all driven by the common process λ_t. Thus, λ_t is referred to as "tail risk" at time t.

The identifying assumption of the tail risk computation is that individual assets share similar dynamics. As a result, given a sufficiently large cross section of returns, enough stocks will experience individual tail events each period to provide accurate information about the overall market tail risk. This assumption avoids having to accumulate years of tail observations on the overall market in order to aggregate series to estimate the tail risk, and therefore avoid using observations that carry little information about current tail risk (Kelly & Jiang, 2014).

Using the cross-section of individual firms' returns allows the fluctuations of individual firms' tail through time to be captured. Applying Hill's (1975) tail index estimator to the time t cross-section provides an estimate for λ_t, i.e.,

$$\lambda_t^{Hill} = \frac{1}{K_t} \sum_{k=1}^{K_t} \ln\left(\frac{R_{k,t}}{u_t}\right) \tag{2}$$

where,

u_t is the extreme value threshold;
$R_{k,t}$ is the k-th logarithmic return that falls below u_t during month t;
K_t is the total number of exceedances within month t.

The threshold selection is an important part of extreme value theory. Opting for an inappropriately mild threshold will contaminate the tail exponent estimates because it will include data of the return distribution whose behavior can differ widely from that of the tail. On the other hand, a too limiting threshold can result in noisy estimates resulting in having too few datapoints. A set percentile threshold combats time-variation in volatility because the absolute threshold expands and contracts with volatility so that only a fixed set of observations is used to estimate the tail index estimate each month (Kelly & Jiang, 2014).

Kelly & Jiang follow Gabaix et al. (2006) who advocate a simple rule that fixes the u-exceedance probability at the 5th percentile for unconditional tail index estimation. In addition to the 5th

percentile, the succeeding analysis applies a discrepancy threshold selection model, as proposed by Wang and Tsai (2009). The purpose of the discrepancy threshold selector is to minimize the discrepancy between the empirical distribution and the uniform distribution.

A concern in calculating the tail index estimates is the influence that cross-sectional volatility heterogeneity may have. If volatility heterogeneity is present between different assets, e.g., industry, those particular assets with higher volatility will more often drive the lower tail of the return distribution and thus, the entire cross section of returns is not properly represented. One way to overcome this concern is to variance-standardize returns in advance of estimating the Hill estimator. However, this approach originates measurement related problems because the behavior of standardized returns can be highly sensitive to estimation error in the divisor (Kelly & Jiang, 2014). In addition, Kelly and Jiang (2014) showed through Monte Carlo simulation that variance heterogeneity has little effect on the performance of the Hill estimator. Therefore, this analysis employs raw returns for the tail index estimation rather than variance standardized returns.

3.3 Out-of-Sample Significance Testing

The TVTR is analyzed out-of-sample and is compared to the historical mean. To make informed inference conclusions, a measure of significance related to predictors' potential forecast improvements in relation to the historical mean must be evaluated and compared to appropriate critical values.

To test the out-of-sample predictive power I utilize Clark and McCracken's (2001) ENC-NEW significance test. This is the benchmark of out-of-sample predictive test in the forecasting literature (Kelly & Jiang, 2014). Clark and McCracken (2001) derived the limiting distributions of two standard tests and developed a new test of forecast encompassing applied to 1-step ahead predictions from nested linear models. Their developed test, ENC-NEW, and its critical values for significance levels of 10% and 5% for different ratios of initial window to forecast periods and number of predictors are numerically provided in their paper. This tests forecast encompassing between two models where the null hypothesis is that forecasts from model 1 encompass that from model 2 and that model 2 contains no more information that model 1. The ENC-NEW test-statistic is calculated as,

$$ENC - NEW = P\frac{\bar{c}}{MSE_2} = P\frac{P^{-1}\sum_t\left(\hat{u}_{1,t+1}^2 - \hat{u}_{1,t+1}\hat{u}_{2,t+1}\right)}{P^{-1}\sum_t \hat{u}_{2,t+1}^2} \tag{3}$$

where,

P is the number of out-of-sample forecasts;

$\hat{u}_{i,t+1}$ is the difference between the predicted value and the true value for model i.

4 Analysis

The hypothesis of the succeeding analysis is derived from Kelly and Jiang (2014), adjusted to application on the Norwegian market. The hypothesis is that, as traditional finance theory indicates, investors' marginal utility is increasing with tail risk and that tail risk itself is persistent and has asset pricing implications by being a valid measure of future market risk. As a result of investors being averse to risk, a positive shock in current tail risk would therefore imply an increased requirement of future excess returns in portfolios that are subject to this risk, including the market portfolio. Empirically, analysis of whether tail risk positively and significantly forecasts future excess market returns is conducted in- and out-of-sample. Additionally, a cross-sectional analysis is conducted where quintile portfolios are constructed based on their tail risk loading.

4.1 TVTR Series

Based the daily data of Norwegian stocks from the TITLON database from 1990 to 2022, I estimate the dynamic power law exponent from the monthly pooled logarithmic returns every month. Figure 3 plots the tail risk time-series computed with the Hill (05) estimator as well as the three-year future excess value-weighted market returns. Both time-series have been standardized to scale for comparative purposes. It is difficult to make out a definite pattern of the tail risk time-series in relation to the future aggregate returns. In broad strokes, they seem to move in similar directions on a large scale. Locally however, tail risk behaves both cyclical and counter-cyclical to the future return. The post 2000 dot com crash rally stands out with exceptionally high future returns and a tail risk that appears to evolve almost in parallel.

A large dataset is crucial to the accuracy of the extreme value estimate in order for it to be an informative estimator of market risk. The reason for this is that there is only a small portion of returns that fall under the threshold and hold all the power of the tail distribution, and assumed in the analysis is that TVTR contains information on the market as a whole. Certainly, a weakness of this analysis is the limited number of stocks on the Norwegian stock exchange. Compared to Kelly and Jiang's (2014) study, who use stocks on the NYSE, AMEX and NADSAQ, this analysis has far fewer stocks in the tail estimation.

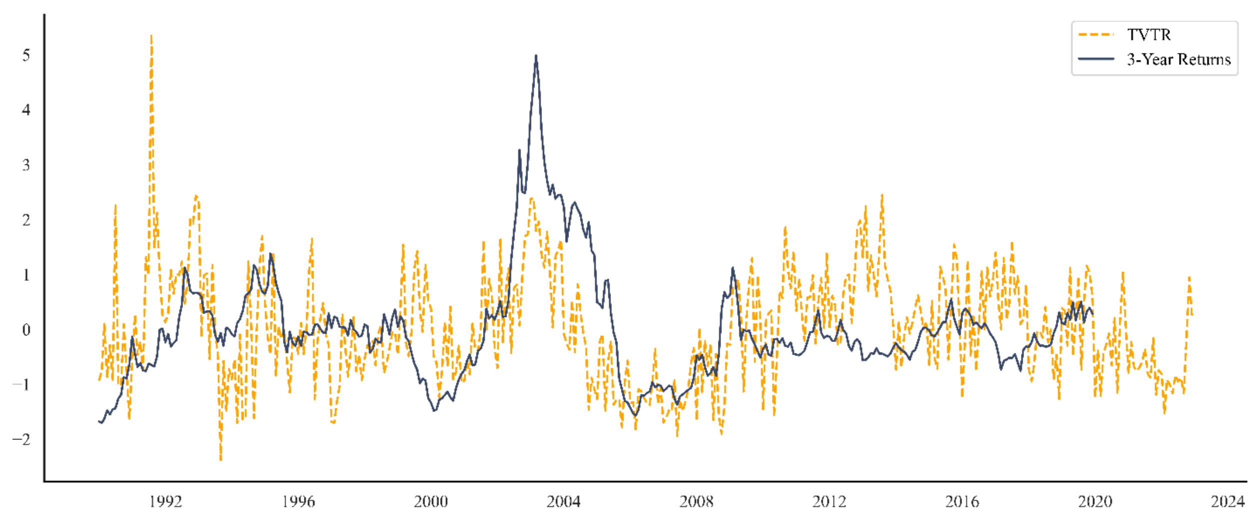

Figure 3: Plotted TVTR, as estimated with Hill 05, and the 3-year future excess market returns of the value-weighted market portfolio. Both time-series have been standardized.

4.2 TVTR Persistence

Time dependence in the tail risk series is a necessary condition for time-series effects. It is only when current or past shocks are informative about future levels that investors will dynamically adjust their discount rates based on the parameter in question. The tail risk series computed with the Hill (05) estimator has an AR(1) parameter of 0.55 significant at the 1 percent level, indicating that the TVTR may have predictive power for future extreme negative returns. Compared to Kelly and Jiang's (2014) tail risk series for the US, which has an AR(1) parameter of 0.93, it is quite low. However, it indicates some level of persistence, leading to the preliminary conclusion that TVTR has the potential to have asset pricing implication in the Norwegian market. Importantly, because returns of which TVTR is calculated from are unique cross sections, there is no mechanically caused persistence. Tail risk series computed with Hill discrepancy has an AR(1) parameter of 0.48.

From the autocorrelation function in Figure 4, it can be confidently concluded that TVTR displays time dependence. Shocks in TVTR extend to subsequent months' TVTR meaning that the TVTR of a given period is dependent on the former. The effects gradually fade away and escape the 5% significance level between lag 13 and 14, represented by the blue area.

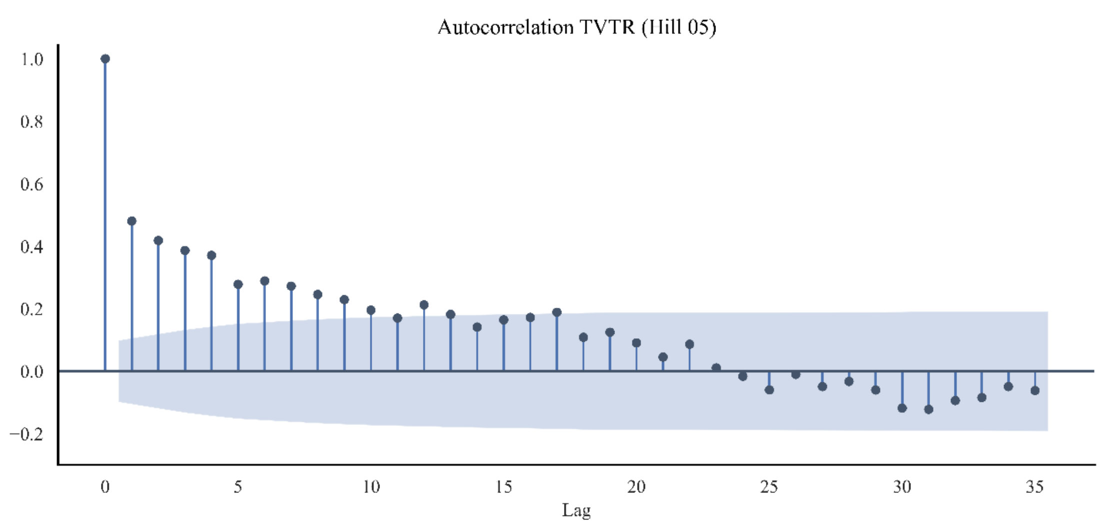

Figure 4: Autocorrelation function of TVTR (Hill 05 estimator). Blue area highlights the 5% significance level.

4.3 Volatility Heterogeneity Among Returns

A serious concern in the data is that of volatility heterogeneity among assets. Dynamic tail risk accounts for common-time volatility heterogeneity through the absolute threshold expanding and contracting with volatility because of the percentile threshold approach. However, Kelly and Jiang's (2014) tail risk approach does not correct for the possibility of volatility heterogeneity at the firm-level. In the case of extreme volatility heterogeneity, a small subset of stocks will be driving the extreme lower tail of the return distribution. The tail risk estimator is an attempt to measure the aggregated time-varying overall market risk and this hypothetical extreme volatile subset of stocks does not represent the overall market.

Kelly and Jiang (2014) concluded in their analysis that tail events at the firm-level are fairly evenly distributed across firms. Because the Norwegian market is different from the US, particularly in terms of amount of assets, this is analyzed. After data inspection, for this particular analysis I have opted to only include those firms that have more than 50 days of returns, because stocks with few returns are largely irrelevant for the overall analysis.

With a 5% threshold level and a large homogenous cross-section of returns, one would expect that each firm appears in the lower extreme tail 5% of the time. 99% of all the firms included in the analysis experience at least one tail event. 92% of firms appear in the tail more than 1% of the time. Only 3% of firms appear in the tail more than 15% of the time. On average, 51% of the actively traded firms in any given month appear in the cross-sectional lower tail on at least one day. With these insights, one can confidently conclude that the daily logarithmic return distribution among Norwegian firms is also fairly evenly distributed.

For refence, the metrics that lead Kelly and Jiang (2014) to the same are the following metrics: 99% of firms experience at least 1 tail event, 86% of firms appear in the tail more than 1% of the time, 8% of firms appear in the tail more than 15% of the time, and 45% of available firms did, on a monthly average, appear in the lower tail on at least one day. In comparison, the data in this work project contains less volatility heterogeneity among firms.

4.4 Market Return Predictability

I analyze the tail risk, measured by the Hill estimator, as a predictor of future aggregated excess market returns in the Norwegian stock exchange. To provide comprehensive insights, I have conducted separate investigations using equally weighted aggregated returns and value-weighted aggregated returns. My primary objective is to draw conclusions on the effectiveness of tail risk in predicting the directionality, magnitude, and significance of future market returns.

The following analysis is structured on a monthly frequency for the aggregated future one-, two-, three- and five-year excess returns. The dependent variable in this study is the Norwegian market excess return. The independent variables comprise the TVTR computed based on the Hill estimator at the 5% level and the Hill estimator formulated using a discrepancy threshold selector model. Both estimators are run in univariate regressions, allowing for a focused exploration of their individual predictive power in relation to market returns and potential differences.

To facilitate interpretation, I have scaled the Hill estimator variables to represent the effect of a one standard deviation increase on the dependent variable. Scaling allows for interpreting the magnitude of the influence of a standard deviation change in TVTR on market returns. This is also Kelly and Jiang's (2014) approach, which allows for direct comparisons. The dependent variables, the excess market returns, are not scaled. Instead, its coefficients are annualized and presented in percentage form.

Given the overlapping nature of the data, it is crucial to account for potential autocorrelation and heteroscedasticity in the standard errors. To address this, I use Hodrick's (1992) standard error correction method for overlapping data. The lag length for the correction is set equal to the periods of the respective excess future return. This has the purpose of ensuring robustness of the inferential conclusions drawn from the regressions. This approach is essential in providing a valid and reliable basis for inference and interpretation of the results. The regressions have the following form $r_{t+h} = \alpha + \beta TVTR_t + \varepsilon_{t+h}$.

Panel A: Equally Weighted Excess Returns						
	Hill 05			Hill Discrepancy		
	β	$t\ stat$	R^2	β	$t\ stat$	R^2
1 month	3.39	0.62	0.16	1.16	0.29	0.04
1 year	9.94*	1.66	5.78	7.74	1.29	3.52
2 years	8.94**	2.41	10.15	7.88**	2.23	7.86
3 years	9.50**	2.16	14.19	9.00**	2.23	12.71
5 years	8.69***	3.31	20.42	7.73***	3.71	15.85
Panel B: Value Weighted Excess Returns						
	Hill 05			Hill Discrepancy		
	β	$t\ stat$	R^2	β	$t\ stat$	R^2
1 month	-1.54	-0.40	0.05	-2.41	-0.61	0.12
1 year	3.93	1.14	2.12	2.61	0.75	0.94
2 years	5.28*	1.95	6.26	4.35*	1.65	4.24
3 years	5.97*	1.76	10.37	5.56*	1.70	9.00
5 years	5.85***	2.59	15.24	5.01**	2.58	11.03

Table 1: Regression output from in-sample predictive analysis of TVTR on future excess market returns over one-month, one-year, two-years, three-years and five-years. Panel A uses equally weighted market returns and Panel B uses value-weighted market returns. TVTR is measured with Hill 05 and Hill Discrepancy and is standardized to facilitate interpretation. Market returns are in percentage and the coefficients are annualized. R^2 is in percentage. Hodrick's corrected standard errors are used to calculate t-statistics. Asterisk (***), (**) and (*) indicate statistical significance at 1%, 5% and 10%, respectively.

The results from Table 1 reveal interesting insights into the relationship between the tail risk and the future excess market returns on the Norwegian market.

In the case of the equally weighted future aggregated excess market returns, the regression results indicate a positive relationship with both the Hill 05 and Hill Discrepancy variables for all the time intervals. This suggests that an increase in tail risk, as measured by the Hill estimator, predicts an increase in future market returns, which argues in favor of the initial hypothesis that investors require higher returns as market risk increases.

The one-month regression does not show a statistically significant relationship for either of the TVTR measures, as evidenced by the t-statistics of 0.63 and 0.29 for Hill 05 and Hill Discrepancy, respectively. However, as we progress to longer time horizons, the predictive power of the TVTR becomes more apparent. For instance, the Hill 05 estimator at the one two-year, three-year, and five-year horizons yield statistically significant coefficients at the 5% and 1% level and indicate an increase in future annualized returns of 8.9%, 9.5% and 8.7%, respectively, as a result of a one standard deviation increase in TVTR.

A similar pattern is observed with the Hill Discrepancy estimator, with positive and statistically significant coefficients at the two-year, three-year, and five-year horizons at the 5% and 1% significance levels. The explanatory power, as indicated by the R-squared values, increases with the time horizon, reaching up to 15.9% for the five-year interval.

When considering the value-weighted future excess market returns, the relationship with the Hill estimators is less consistent. For the one-month horizon, both the Hill 05 and Hill Discrepancy estimators indicate a negative relationship with the market returns, although these relationships are statistically insignificant at a meaningful level.

From the one-year horizon onwards, the relationship becomes positive for both the Hill 05 and Hill Discrepancy estimators. The Hill 05 estimator shows statistically significant coefficients at the 10% level for the two-year and three-year horizon, and at the 1% significance level for the five-year horizon. The Hill Discrepancy estimator shows statistically significant coefficients at the 10% level for the two-year and three-year intervals, and at the 1% significance level for the five-year interval. Similar to the equally weighted returns, the explanatory power of the Hill estimators increases with the time horizon in the case of value-weighted returns.

When using the value weighted returns, coefficients are less statistically significant when compared to the equally weighted returns for all time intervals and both Hill estimators. Intuitively, this is very logical. Because the tail risk is calculated from the pooled cross-sectional returns of all stocks traded in the Norwegian market where all stocks are treated equally, it makes sense that the equally weighted returns better correspond to the tail risk measure compared to that where bigger stocks are overrepresented.

In conclusion, the results suggest that the tail risk, as measured by the Hill estimators, is a powerful predictor of future market returns in the Norwegian stock exchange, particularly over longer time horizons. However, the strength and directionality of this predictive relationship seem to vary depending on whether the returns are equally weighted or value weighted. The difference between TVTR estimated on the 5^{th} percentile and using the discrepancy model is largely insignificant.

4.5 TVTR Out of Sample Forecasting Analysis

After concluding that the tail risk carries significant in-sample predictive power of aggregate market return of the Norwegian markets, I investigate the out-of-sample predictive ability. I compare the predictive power to the historical mean return, the standard benchmark.

Using data through month t, beginning at $t = 120$ to accommodate a sufficiently large initial estimation period, I run a recursive univariate predictive regression of Norwegian excess market returns on tail risk, $r_{t+1} = \alpha + \beta_t TVTR_t$,where $t = 120, \dots, T$. The coefficient is used to forecast the excess market return for the next period, $t + 1$. The estimation window is increased by one, a new coefficient is obtained, and a new out-of-sample forecast of the following month's return is computed. This procedure is repeated until the entire sample has been exhausted. This procedure mimics that of an investor in real time. By using the forecast errors from this analysis, I calculate the out-of-sample R^2, which measures improvement of fit of TVTR compared to the historical mean. A negative R^2 implies that the TVTR prediction performs worse than a prediction based on the historical mean return, whereas a positive R^2 implies that it performs better.

	Equally Weighted Excess Returns		Value Weighed Excess Returns	
	Hill 05	Hill Discrepancy	Hill 05	Hill Discrepancy
1 month	2.89*	1.04*	5.32*	3.65*
1 year	4.37*	3.89*	3.00*	1.88*
2 years	11.00*	8.69*	9.14*	6.29*
3 years	16.11*	14.46*	12.97*	10.98*
5 years	24.25*	19.23*	20.52*	15.22*

Table 2: Out-of-sample R^2 in % for market returns over one-month, one-year, two-years, three-years and five-years. R^2 represents improvement of TVTR, here estimated by Hill 05 and Hill Discrepancy, compared to the historical mean. $R^2 = 1 - (\sum_t (r_{m,t+1} - \hat{r}_{m,t+1|t})^2)/(\sum_t (r_{m,t+1} - \bar{r}_{m,t})^2)$, where $r_{m,t+1}$ is the actual return in the period $t + 1$, $\hat{r}_{m,t+1|t}$ is the out-of-sample forecast of the $t + 1$ return based on TVTR and only data through t, and $\bar{r}_{m,t}$ is the historical mean return through t. Asterisk (*) represents significance of 5%, based on Clark and McCraken (2001) ENC-NEW test for out-of-sample predictability.

The results from Table 2 indicate that tail risk forecasts demonstrate similar predictive success out-of-sample. Both Hill estimates outperform the historical return mean for all time intervals. For the equally weighed index and the Hill 05 predictor, the out-of-sample at the one-month, one-year, two-years, three-years and five-year horizons showcase an R^2 of 2.9%, 4.4%, 11.0%, 16.1% and 24.3%, respectively, all significant at the 5% level in accordance with Clark and McCracken (2001) ENC-NEW test. These results are very similar to those of Kelly & Jiang (2014).

4.6 Smooth Cross Sectional Tail Risk

Faias (2023) developed a modified version of Kelly and Jiang's (2014) time-varying tail risk measure. Instead of using the monthly pooled daily returns, Faias (2023) advocated for the method of calculating the Hill estimator daily and averaging the daily Hill estimator per month. I perform the similar analysis as before with this modified version to test whether Faias' (2023) improved results are applicable to the Norwegian market as well. The smooth time-varying risk (SCSTR) is calculated from the 5th percentile.

The smooth time-varying tail risk does differ from the TVTR although the correlation between the two time-series is high at 80%. The volatility of the SCSTR is lower compared to TVTR with 6.1% and 7.0%, respectively, as one would expect the effect from averaging to be (Figure 5). The AR(1) parameter equals 0.56, slightly higher than TVTR, but the number of lags of persistence with 5% significance is the same, disappearing between lag 13 and 14.

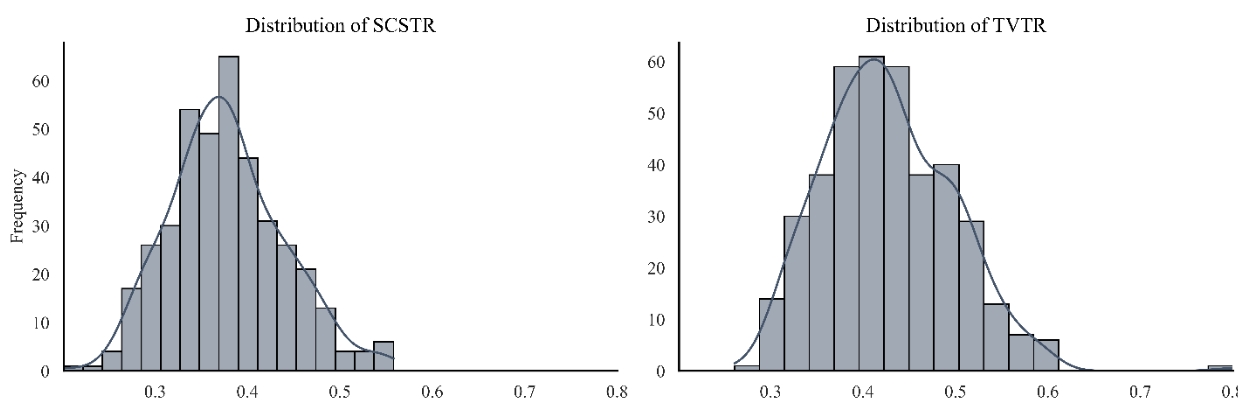

Figure 5: Distributions of SCSTR and TVTR as calculated by Hill 05.

Interestingly, as can be seen from Table 3, SCSTR performs better than TVTR as a predictor of future returns on the short horizons, which were the revelations of Faias (2023) in the American markets. For the equally weighted excess returns, SCSTR is positive and significant at the 10% level for the one-month horizon, where TVTR was not. For the equally- and value-weighted results, SCSTR is significant at the 5% level for one-, two- and three-year horizon and at the 1% level for the five-year horizon. The discussion regarding the difference between equally weighted and value weighted returns can be extended from the prior analysis. As with TVTR, the out-of-sample analysis reveals that SCSTR performs significantly better than the historical mean (Tabel 4).

	Equally Weighted Excess Returns			Value Weighed Excess Returns		
	β	$t\ stat$	R^2	β	$t\ stat$	R^2
1 month	10.21*	1.90	1.34	3.47	0.91	0.24
1 year	13.50**	2.44	10.81	6.76**	2.24	6.34
2 years	9.67**	2.39	12.26	6.52**	2.27	9.87
3 years	9.61**	2.09	14.87	6.48**	1.90	12.63
5 years	8.82***	3.00	21.56	6.33***	2.63	18.51

Table 3: Regression output from in-sample predictive analysis of SCSTR on future excess market returns over one-month, one-year, two-years, three-years and five-years. SCSTR is standardized. Market returns are in percentage and the coefficients are annualized. R^2 is in percentage. Hodrick's corrected standard errors are used to calculate t-statistics. Asterisk (***), (**) and (*) indicate significance at 1%, 5% and 10%, respectively.

	Equally Weighted Returns	Value Weighted Returns
1 month	3.13*	5.04*
1 year	0.46*	2.91*
2 years	10.95*	11.62*
3 years	16.30*	15.07*
5 years	23.13*	23.00*

Table 4: Out-of-sample R^2 in % for market returns over one-month, one-year, two-years, three-years and five-years. R^2 represents improvement of SCSTR compared to the historical mean market return. Asterisk (*) represents significance of 5%, based on Clark and McCraken (2001) ENC-NEW test for out-of-sample predictability.

4.7 Cross Sectional Analysis

Lastly, I analyze whether tail risk exposure helps explain differences in returns of stocks. The hypothesis argues that, for risk averse investors, stocks with high exposure to tail risk should be compensated with higher returns. In contrast, stocks that have less predictive loading, or even negative loading, would serve as tail risk hedges, and thus should experience lower returns. This is consistent with the previous analysis of aggregate market returns.

Every month, I calculate the previously estimated aggregated market tail risk (Hill 05) loading for every stock trading on the Norwegian Stock Exchange. The regression of the stocks' tail risk sensitivity is $E[r_{i,t+1}] = \mu_i + \beta_{i,t}\lambda_t^{Hill}$ conditional on the information at time t, where $\beta_{i,t}$ is the monthly tail risk loading of stock i. For every month, the data used is that available to investors, simulating a realistic portfolio composition decision. The estimation uses a rolling 120 period window of returns, with a lower threshold of 36 periods to ensure a meaningful sensitivity measure. Based on each stock's respective tail risk loading, they are sorted into 5 quintile portfolios, which are re-composed every month.

	Low	2	3	4	High	**High-Low**	**t-stat**
Average Returns	0.210	0.102	-0.053	0.400	0.831	**0.620**	**2.36**
CAPM Alpha	-1.005	-1.201	-1.339	-0.729	-1.104	**0.596**	**2.40**
FF Alpha	-1.016	-1.248	-1.327	-0.522	0.115	**0.827**	**3.24**
Carhart 4F Alpha	-1.032	-1.206	-1.280	-0.558	0.082	**0.806**	**3.08**

Table 5: Cross-sectional analysis results. Equally weighted portfolios of stocks sorted on TVTR loading's returns and alphas in percentage. The portfolio 'High-Low' report results from the high minus low zero net investment that is long portfolio 'High' and short portfolio 'Low'. Associated t-statistics are reported.

Table 5 presents the results of the analysis. The average returns of the portfolios do not mechanically increase, but portfolios 4 and 5 are significantly larger than 1 and 2. The 'High-Low' portfolio's average return is 0.62%, significant at the 5% level with an associated t-statistic of 2.36.

Alphas from CAPM, Fama French three factor model and Carhart four factor model report similar positive alphas of 0.6%, 0.8% and 0.8%, statistically significant at the 5%, 1% and 1% level, respectively. Results from this analysis suggest that there is compensation for holding stocks exposed to market tail risk.

5 Limitations

This work project offers valuable insights into tail risk's impact on asset pricing in the Norwegian stock market. However, there are limitations in the analysis that might affect the findings' interpretation and generalizability. Recognizing these limitations is vital as they contextualize the findings and create opportunities for further expanding research.

This work project focuses almost exclusively on tail risk as the sole variable predicting market returns and cross-sectional returns. While the analysis does incorporate various versions of tail risk, such as the different threshold selections and the smooth version of Kelly and Jiang's TVTR, SCSTR, it does not incorporate other potential predictors.

Asset pricing is influenced by a variety of factors. Although tail risk has been shown in this work project to be influential, there could be numerous market factors that play a role in predicting returns, for example, macroeconomic indicators like GDP growth, inflation rates and interest rates or firm-specific factors such as EPS, P/E ratio and financing ratios. Moreover, financial risk is a vast area of research and not limited to tail risk. Other types of risk might significantly impact asset pricing.

These alternative variables might not only explain the predictability of tail risk but could also be superior predictors. In not considering additional predictors, the work project might overlook important aspects of asset pricing dynamics on the Norwegian stock exchange.

6 Conclusion

This work project has investigated the implications of tail risk on asset pricing on the Norwegian stock market. The main framework of this analysis is Kelly and Jiang's (2014) time-varying tail risk (TVTR), Faias' (2023) smooth cross-sectional tail risk (SCSTR) and the Hill (1975) estimator in which the two are built upon.

The analysis revealed several findings. Firstly, TVTR was found to be persistent. Secondly, novel findings of tail risk's positive and statistically significant predictive power on in-sample aggregate market returns for time horizons for and over one-year for TVTR and in the case of SCSTR, for all time horizons. Thirdly, TVTR and SCSTR were found to significantly outperform market return prediction out-of-sample compared to the historical mean. Lastly, through cross-sectional analysis, portfolios of stocks with high tail risk loading achieved, in comparison to the portfolio with low tail risk loading, significantly positive returns as well as positive and significant CAPM-, Fama-French 3-factor- and Carhart 4-factor alpha.

The evidence in this work project suggests that tail risk is a meaningful measure of risk on the Norwegian stock market, that tail risk is a valuable predictor of future market returns and that tail risk exposure is compensated for with higher associated future returns.

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