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The stability of government bond markets' equilibrium and the interdependence of lending rates

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Abstract

In this paper, we introduce test procedures for no fractional cointegration against possible breaks to a fractional cointegrating relationship in a segment of the data. We base the proposed tests on the supremum of the Hassler and Breitung (Econom Theor 22(6):1091–1111, 2006) test statistic for no cointegration over possible breakpoints in the long-run equilibrium. We show that the new tests correctly standardized converge to the supremum of a Chi-squared distribution and that this convergence is uniform. An in-depth Monte Carlo analysis provides results on the finite sample performance of our tests. We then use the new procedures to investigate whether there was a dissolution of fractional cointegrating relationships between the yields of government bonds of eleven EMU countries (Spain, Italy, Portugal, Ireland, Greece, Belgium, Austria, Finland, the Netherlands, Germany and France) as a consequence of the European debt crisis and to understand the degree of interdependence of lending rates to non-financial corporations across these eleven countries.

Keywords Fractional cointegration · Persistence breaks · Hassler–Breitung test · Changing long-run equilibrium

JEL Classification C12 · C32

1 Introduction

The European government bond market has gone through important developments and profound changes over the last 2 decades. The introduction of a common currency

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promoted convergence of interest rates and eliminated exchange rate risk. This was an important milestone given that market convergence is of importance for policymakers and market participants (see, e.g., Abad and Chuliá 2014), and market stability is paramount for well-functioning economies and financial systems. However, convergence of government bonds has also contributed to the intensification of competition among government issuers; and to changes in the composition of investors in government bonds, with more weight being attributed to foreign investors; Baele et al. (2004).

The literature on the analysis of European government bond markets' convergence can be divided into two periods,[1]: (i) the period from the introduction of the common currency to either the financial crisis of 2008 or the sovereign debt crisis of 2010 and (ii) the post-crisis period. In the first period, literature has striven to understand the importance of the EMU for the convergence process of the European government bond markets. The elimination of exchange-rate uncertainty and the transition to a single monetary policy contributed significantly to the reduction of the euro-area member states' government bond yield differentials, and to a stronger degree of convergence between EMU countries; see, for instance, Christiansen (2014). However, yield differentials under EMU continued to exist (Favero et al. 2010). For instance, Geyer et al. (2004) and Pagano and Thadden (2004) attribute these differentials to a common risk factor; and Beber et al. (2009) to differences in credit quality and liquidity. Abad et al. (2010) suggest that euro area countries show only partial convergence and differ in terms of market liquidity and default risk[2] (see, e.g., Abad and Chuliá 2014 and Schaeffer and Ramirez 2017 for a more detailed overview).

In the second period, focus shifted to the impacts of the financial and the sovereign debt crises on the European government bond markets' convergence. Fiscal imbalances and greater international risk aversion—a higher common risk factor in spreads substantially amplified the yield differentials (von Hagen et al. 2011; Pozzi and Wolswijk 2012; Abad et al. 2014). Christiansen (2014) and Abad and Chuliá (2014) indicate that convergence among EMU members decreased during the recent crisis. Abad and Chuliá (2014) suggest that among other things, the level of convergence may have decreased because of increased uncertainty resulting from unexpected news releases by the ECB and of substitution effects between bond and money market instruments.

Most approaches used in the literature to analyze the European government bond market convergence apply procedures developed under the conventional I(1)/I(0) framework, where I(1) and I(0) stand for integration of order 1 (unit root non-stationarity) and 0 (weak stationarity), respectively. However, the discrete I(1) and I(0) context of analysis may be too restrictive, since equilibrium errors can be (fractionally) integrated of order d, I(d), where d is allowed to take on non-integer values, i.e., may display long-memory (see, e.g., Dueker and Startz 1998; Leschinski et al. 2018; Basse et al. 2018; Caporale and Gil-Alana 2019). Thus, modeling yield differentials as I(1)/I(0) variables when the true data-generating process exhibits long

[1] For results on market convergence and diversification using European countries before the introduction of the common currency see, *inter alia* Clare et al. (1995), Taylor and Tonks (1989), Favero et al. (1997), Clare and Lekkos (2000), Baum and Barkoulas (2006) and Swanson (2008).

[2] Gómez-Puig (2009a, b) find that differentials in the 10-year yield spread between Germany and the other EMU countries was mainly driven by domestic risk factors.

memory may give rise to misleading conclusions regarding its persistence; see, e.g., Sun (2006).

Long memory is characterized by long range dependence in the sense that the impact of past shocks die out at a slower hyperbolic rate (Hassler and Kokoszka 2010, Prop.2.1) than the usual exponential decay observed in I(0) series. Evidence of long memory has been reported for many financial and economic time series. In particular, interest rates and government bond yield spreads do display long memory type dependence and this needs to be accounted for in empirical work (see, for example, Golinski and Zaffaroni 2016; Baum and Barkoulas 2006; Busch and Nautz 2010; Sibbertsen et al. 2014; Wegener et al. 2019 and Kruse and Wegener 2019).

Moreover, since the impact of shocks to yield differentials can vary depending on whether the shocks are positive or negative, as well as on the underlying factors driving those shocks and the market conditions at the time, it is important to analyze their impact on the persistence of time series. A positive shock, such as an unexpected improvement in economic conditions or a positive policy announcement, can lead to an increase in yield differentials, but the impact on persistence may depend on the sustainability of the positive factors driving the shock. If the improvement is perceived as temporary or unsustainable, the impact on yield differentials may be short-lived, and they could revert to previous levels once the positive factors dissipate. On the other hand, if a negative shock occurs, such as an economic downturn or a geopolitical crisis, and the negative factors driving the shock persist or worsen over time, the amplification of yield differentials may be more prolonged, as investors remain cautious and risk-averse in response to ongoing uncertainties. As a result, the long-memory parameter that characterizes the long-term dynamics of the yield differential process may change. Since, the severity and impact of the financial and/or the sovereign debt crisis on the European government bond markets has likely triggered changes in the equilibrium relations, suitable approaches, such as the ones introduced in this paper, are necessary to determine the existence of segmented long-run equilibrium relationships in the European government bond market.

Long-run equilibria are commonly modeled by cointegration relationships. Since the seminal works of Engle and Granger (1987) and Johansen (1988) cointegration testing has become an important topic of research, both theoretically as well as empirically. The equilibrium relationship between economic and financial variables postulated by many economic theories is typically assumed to be constant over time. However, this assumption may be too restrictive. A constant long-run equilibrium may be questionable in light of the growing empirical evidence that economic and financial time series may display persistence changes over time (see, inter alia, Kim 2000; Kim et al. 2002; Busetti and Taylor 2004; Harvey et al. 2006, for tests when the order of integration is integer; and Giraitis and Leipus 1994; Beran and Terrin 1996, 1999; Sibbertsen and Kruse 2009; Hassler and Scheithauer 2011; Hassler and Meller 2014; Martins and Rodrigues 2014, for tests when the order of integration is some real number). Hence, it is natural to expect that changes in the persistence of economic and financial time series may also originate changes in the long-run equilibrium. In recent years a vast literature documenting changes in the historical behavior of economic and financial variables substantiated this; see among others, McConnell and Perez-Quiros (2000), Herrera and Pesavento (2005), Cecchetti et al. (2006), Kang et al. (2009) and Halunga et al. (2009).

The impact of structural breaks in the deterministic kernels on cointegration has been widely analyzed (see, e.g., Quintos and Phillips 1993; Hao 1996; Andrews et al. 1996; Gregory and Hansen 1996; Bai and Perron 1998; Kuo 1998; Inoue 1999; Johansen et al. 2000; Lütkepohl et al. 2003), but less attention has been given to the impact of changes in the actual long-run equilibrium; see Martins and Rodrigues 2018). Hence, given the empirical relevance of this feature, the focus of this paper is to propose new tests capable of detecting changes in fractional cointegration relationships. Or to be more precise we consider what can be called segmented fractional cointegration as we consider changes from cointegration to non-cointegration and vice versa. We introduce procedures designed to detect changes in the long-run equilibrium between time series based on rolling, recursive forward and recursive reverse estimation of the Hassler and Breitung (2006) test, in the spirit of the approaches proposed by, e.g., Davidson and Monticini (2010). We derive asymptotic results and evaluate the performance of the new tests in an in-depth Monte Carlo exercise. In particular, special attention is devoted to the case of unknown orders of integration of the variables involved due to its empirical relevance. Segmented cointegration in the $I(0)/I(1)$ case has been considered by Kim (2003). We decided to base our test on the Hassler and Breitung (2006) approach because of the normal limiting distribution of their test statistic which allows a combination with the Davidson and Monticini (2010) test.

To illustrate the empirical importance of the test procedures introduced in this paper, we provide an in-depth analysis of the long-run equilibrium in government bond markets of the European Monetary Union (EMU) finding evidence of segmented fractional cointegration with breaks at the beginning of the sovereign debt crisis. In addition, and related to the dynamics of the government bond markets, we also provide an analysis of possible changes in the degree of interdependence of lending rates to non-corporate firms. Also in this case do the tests show that over the period analyzed there have been episodes of interdependence between the lending rates of the EMU countries considered. The tests provide evidence of stronger interdependence between the lending rates after the introduction of the Euro.

In this paper, we thus use fractional cointegration methods accounting for long-memory equilibrium processes as a tool to measure nominal and real convergence or divergence since the start of EMU. In this set-up, economic convergence does not necessarily imply convergence in levels. Rather, the existence of a stable long-run equilibrium between macroeconomic variables across EMU countries is regarded as evidence of close and stable macroeconomic relationships among those countries. This is what we term macroeconomic convergence. The concept of segmented fractional cointegration thus identifies times with and without such a long-run equilibrium or investigates changes in the speed of convergence to such a long-run equilibrium, respectively.

This paper is organized as follows. Section 2 presents the model specification and assumptions; introduces the tests for no cointegration under persistence breaks, a break point estimator, and corresponding asymptotic theory; Sect. 3 discusses the results of a Monte Carlo analysis on the finite sample properties of the new tests; Sect. 4 provides an in-depth analysis of the long-run equilibrium between eleven EMU government bond markets and between the lending rates to non-financial corporations of these countries, using the new approaches introduced in this paper; Sect. 5 concludes the paper and finally an appendix collects all the proofs, and Supplementary Figures from the empirical analysis in Sect. 4.

2 Testing for no fractional cointegration

Consider an m-dimensional process $\mathbf{x}_t$ integrated of order $d > 0.5$, $I(d)$, and let y_t be an one-dimensional $I(d)$ process as well. The condition of $d > 0.5$ is strictly not necessary but is imposed here as it is the usual condition for fractional cointegration. We say the processes $\mathbf{x}_t$ and y_t are fractionally cointegrated if, considering the regression,

$$y_t = \mathbf{x}_t' \beta + u_t, \qquad t = 1, \ldots, T, \tag{1}$$

u_t is integrated of order $I(d - b)$ with $b > 0$.

In what follows the focus lies in testing the null hypothesis of no fractional cointegration, $H_0 : b = 0$. The usual alternative in this setting is to have fractional cointegration over the whole range of observations, $H_1 : b > 0$. However, we are interested in testing for segmented fractional cointegration. This means that the fractional cointegration relationship may hold only in subsamples of the period under analysis. Therefore, our alternative hypothesis is $H_1 : b_t > 0$ for $t = \lfloor \lambda_1 T \rfloor + 1, \ldots, \lfloor \lambda_2 T \rfloor$ and $b_t = 0$ elsewhere, with $0 \leq \lambda_1 < \lambda_2 \leq 1$ and $\lfloor \lambda_1 T \rfloor < \lfloor \lambda_2 T \rfloor$.

We base our proposed test statistics on the approach of Hassler and Breitung (2006), who provide a regression-based test for the null of no fractional cointegration on the residuals, $\hat{u}_t$, of a model as in (1). Before presenting the relevant test statistics let us make the following assumptions:

Assumption 1 Let y_t and $\mathbf{x}_t$ be fractionally integrated of order d with $y_t = 0$ and $\mathbf{x}_t = 0$ for $t \leq 0$. Thus, we assume type II long memory processes in the sense of Marinucci and Robinson (1999).

Assumption 2 The vector $\mathbf{v}_t' := (v_{1,t}, \mathbf{v}_{2,t}') = (\Delta_+^d y_t, \Delta_+^d \mathbf{x}_t')$, is a stationary vector autoregressive process of order p of the form,

$$\mathbf{v}_t = A_1 \mathbf{v}_{t-1} + \cdots + A_p \mathbf{v}_{t-p} + \varepsilon_t, \tag{2}$$

where $\Delta_+^d y_t := (1 - L)^d y_t I(t > 0)$, $\Delta_+^d \mathbf{x}_t := (1 - L)^d \mathbf{x}_t I(t > 0)$, $I(\cdot)$ is the indicator function, L denotes the usual backshift or lag operator and the error process ε_t is independent and identically distributed (iid) with mean zero and covariance matrix,

$$\mathbf{\Sigma} := \begin{pmatrix} \sigma_{11}^2 & \sigma_{21}' \\ \sigma_{21} & \mathbf{\Sigma}_{22} \end{pmatrix}.$$

As in Hassler and Breitung (2006) the cointegrating vector β in (1) is not identified under the null hypothesis of no cointegration. Thus, we use the following regression model,

$$\Delta^d y_t = \Delta^d \mathbf{x}_t' \beta + \Delta^d u_t, \qquad \beta := \boldsymbol{\Sigma}_{22}^{-1} \sigma_{21}, \tag{3}$$

where $e_t = \Delta^d u_t := v_{1,t} - \mathbf{v}_{2,t}' \boldsymbol{\Sigma}_{22}^{-1} \sigma_{21}$, and apply the LM test for no cointegration on the OLS residuals, $\hat{e}_t$, obtained from (3), i.e.,

$$\hat{e}_t := e_t - \sum_{\tau=1}^{T} \mathbf{v}_{2,\tau}' e_\tau \left(\sum_{\tau=1}^{T} \mathbf{v}_{2,\tau} \mathbf{v}_{2,\tau}' \right)^{-1} \mathbf{v}_{2,\tau}.$$

Specifically, to implement the tests proposed by Hassler and Breitung (2006), which is the approach followed in this paper, we consider a regression framework, viz.,

$$\hat{e}_t = \phi \hat{e}_{t-1}^* + \sum_{i=1}^{p} \gamma_i \hat{e}_{t-i} + a_t, \qquad t = 1, \ldots, T, \tag{4}$$

where $\hat{e}_{t-1}^* := \sum_{j=1}^{t-1} j^{-1} \hat{e}_{t-j}$ and a_t is a martingale difference sequence. Equation (4) is used to test the null $H_0 : \phi = 0$ $(b = 0)$ against the alternative $H_1 : \phi < 0$ $(b > 0)$.

Remark 2.1 Under local alternatives of the form $\mathrm{H}_1 : b = c/\sqrt{T}$ with a fixed $c > 0$, it can be shown that $\phi = -c/\sqrt{T} + O\left(T^{-1}\right)$ and that $\{a_t\}$ is a fractionally integrated noise component. As a result, the heterogenous behavior of ϕ and the different stochastic properties of a_t provide a sound statistical basis for the identification of the order of fractional integration of $\left\{\hat{e}_t\right\}$. Despite the apparent theoretical simplicity of this framework, the fact that, under the null hypothesis and Assumption 1, $\hat{e}_{t-1}^*$ converges in mean square sense to $e_{t-1}^{**} := \sum_{j=1}^{\infty} j^{-1} e_{t-j,d}$ with $\left\{e_{t-1}^{**}\right\}$ being a stationary linear process with non-absolutely summable coefficients, poses major technical difficulties for the asymptotic analysis in this context; see, e.g., Hassler et al. (2009). □

Remark 2.2 Demetrescu et al. (2008) and Hassler et al. (2009) derive the asymptotic theory of the fractional integration tests under least-squares (LS) estimation of the set of parameters $\boldsymbol{\kappa} := \left(\phi, \gamma_1, \ldots, \gamma_p\right)'$ of a regression as in (4), and show that these are $\sqrt{T}$-consistent and asymptotically normal under fairly general conditions. As a result, in a conventional setting as in (4) $\mathrm{H}_0 : \phi = 0$ can be tested by means of a standard t-ratio, or some measurable transformation such as its squares. If our assumptions are strengthened such that $a_t \sim iid\mathcal{N}\left(0, \sigma^2\right)$, the specific harmonic weighting upon which $\left\{e_{t-1}^*\right\}$ is constructed in (4) also ensures efficient testing. □

Remark 2.3 The test regression in Eq. (4) generalizes the score type long memory test of Breitung and Hassler (2002) and Agiakloglou and Newbold (1994) to fractional cointegration. □

Remark 2.4 The test regression in Eq. (4) allows for an autoregressive term of order p. In the case $p \geq 1$ Hassler and Breitung (2006) propose prewhitened residuals before applying the test. They derive the asymptotic standard normal limit distribution of the test statistic for prewhitened residuals which is the ingredient we need in our approach. An alternative procedure to prewhitening is parametric augmentation as suggested in

Demetrescu et al. (2008). The advantage of parametric augmentation is that p does not need to be known in advance and can increase with sample size though slower than the sample size. Demetrescu et al. (2008) prove asymptotic standard normality of the t-test under parametric augmentation. Throughout this paper, we use the parametric augmentation by Demetrescu et al. (2008) for our test on segmental cointegration. □

2.1 The test statistics

As we are interested in testing for no fractional cointegration against the alternative of segmented fractional cointegration, we apply the Hassler and Breitung (2006) test on a subinterval defined by the truncation points λ_1 and λ_2, with $0 \leq \lambda_1 < \lambda_2 \leq 1$.

For λ_1 and λ_2 fixed we consider the statistic,

$$t(\hat{e}(\lambda_1, \lambda_2)) = \frac{\sqrt{\lfloor \lambda_2 T \rfloor - \lfloor \lambda_1 T \rfloor} \sum_{t=\lfloor \lambda_1 T \rfloor + 1}^{\lfloor \lambda_2 T \rfloor} \hat{e}_t(\lambda_1, \lambda_2) \hat{e}^*_{t-1}(\lambda_1, \lambda_2)}{\sqrt{\sum_{t=\lfloor \lambda_1 T \rfloor + 1}^{\lfloor \lambda_2 T \rfloor} \hat{e}^{*2}_{t-1}(\lambda_1, \lambda_2)} \sqrt{\frac{1}{T-1} \sum_{t=\lfloor \lambda_1 T \rfloor + 1}^{\lfloor \lambda_2 T \rfloor} \hat{e}^2_t(\lambda_1, \lambda_2)}}, \tag{5}$$

where $\hat{e}_t(\lambda_1, \lambda_2)$ are the subsample based residuals and $\hat{e}^*_{t-1}(\lambda_1, \lambda_2)$ the corresponding harmonic weighted residuals as defined in (4).

However, since the breakpoints, λ_1 and λ_2, are usually unknown we adopt the split sample testing approach proposed by Davidson and Monticini (2010), and define the following sets on which the tests will be performed:

$$\Lambda_S = \left\{ \left\{ 0, \frac{1}{2} \right\}, \left\{ \frac{1}{2}, 1 \right\} \right\}; \tag{6}$$

$$\Lambda_{0f} = \{\{0, s\} : s \in [\lambda_0, 1]\}; \tag{7}$$

$$\Lambda_{0b} = \{\{s, 1\} : s \in [0, 1 - \lambda_0]\}; \tag{8}$$

$$\Lambda_{0R} = \{\{s, s + \lambda_0\} : s \in [0, 1 - \lambda_0]\}, \tag{9}$$

where Λ_S represents a simple split sample with just two elements; Λ_{0f} and Λ_{0b} denote forward- and backward-running incremental samples, respectively, of minimum length $\lfloor \lambda_0 T \rfloor$ and maximum length T; Λ_{0R} defines a rolling sample of fixed length $\lfloor \lambda_0 T \rfloor$, and finally $\lambda_0 \in (0, 1)$ is fixed and needs to be chosen by the practitioner. Davidson and Monticini (2010) consider two additional sets, namely $\Lambda^*_S = \Lambda_S \cup \{0, 1\}$ and $\Lambda^*_{0R} = \Lambda_{0R} \cup \{0, 1\}$.

Therefore, considering (6) to (9), our proposed test procedures against breaks in the fractional cointegration relation are the split sample tests,

$$\mathcal{T}_S := \max_{\{\lambda_1, \lambda_2\} \in \Lambda_S} t^2(\hat{e}(\lambda_1, \lambda_2)); \tag{10}$$

$$\mathcal{T}_{S^*} := \max_{\{\lambda_1, \lambda_2\} \in \Lambda_{S^*}} t^2(\hat{e}(\lambda_1, \lambda_2)); \tag{11}$$

the incremental (recursive) tests

$$\mathcal{T}_{I_f}(\lambda) := \max_{\lambda_0 \leq \lambda \leq 1} t^2(\hat{e}(0, \lambda)); \tag{12}$$

$$\mathcal{T}_{I_b}(\lambda) := \max_{0 \leq \lambda \leq 1-\lambda_0} t^2(\hat{e}(\lambda, 1)); \tag{13}$$

and the rolling sample tests

$$\mathcal{T}_R(\lambda) := \max_{0 \leq \lambda \leq 1-\lambda_0} t^2(\hat{e}(\lambda, \lambda + \lambda_0)); \tag{14}$$

$$\mathcal{T}_{R^*}(\lambda) := \max_{\{\lambda_1, \lambda_2\} \in \Lambda_{0R^*}} t^2(\hat{e}(\lambda_1, \lambda_2)). \tag{15}$$

We can state these statistics in general form as,

$$\mathcal{T}_K(\lambda_1, \lambda_2) := \max_{\lambda_1 \in \Lambda_1, \lambda_2 \in \Lambda_2} t^2(\hat{e}(\lambda_1, \lambda_2)),\ K = S, S^*, I_f, I_b, R, R^*. \tag{16}$$

2.2 Asymptotic results

To characterize the asymptotic behavior of the test statistics in (10)–(15), consider first Theorem 1 which states the asymptotic normality of the test statistic in (5) and which is the main building block of $\mathcal{T}_K(\lambda_1, \lambda_2),\ K = S, S^*, I_f, I_b, R, R^*$.

Theorem 1 *Assuming that the data are generated from* (1) *and that Assumptions 1 and 2 hold, it follows under the null hypothesis of no fractional cointegration that, as* $T \to \infty$,

$$t(\hat{e}(\lambda_1, \lambda_2)) \Rightarrow N(0, 1), \tag{17}$$

where $\Rightarrow$ *denotes weak convergence.*

Theorem 1 implicitly assumes that the fractional integration parameter d is known. However, the result still holds true if a $\sqrt{T}$ consistent estimator of d is used. The spectral-based maximum likelihood estimator of Fox and Taqqu (1986) is one possible choice. The result is stated in the following corollary.

Corollary 1 *Assuming that the data are generated from* (1) *and that Assumptions 1 and 2 hold, but considering* d *unknown, the limit result of Theorem* 1 *holds, if a* $\sqrt{T}$ *consistent estimator* $\hat{d}$ *of* d *fulfilling* $\sqrt{T}(\hat{d} - d) = O_p(1)$ *is used.*

Hence, based on the results of Theorem 1 and Davidson and Monticini (2010) we can now state the limit results for the test statistics introduced in (10)–(15).

Theorem 2 *Assuming that the data are generated from* (1) *and that Assumptions 1 and 2 hold, under the null hypothesis of no fractional cointegration it follows, as* $T \to \infty$, *that*

$$\mathcal{T}_K(\lambda_1, \lambda_2) \Rightarrow \sup_{\lambda_1 \in \Lambda_1, \lambda_2 \in \Lambda_2} \chi_1^2(\lambda_1, \lambda_2),\ K = S, S^*, I_f, I_b, R, R^*. \tag{18}$$

As a next step, we provide an estimator of the break point τ under the alternative. The estimator basically consists of minimizing the sum of squared residuals of a regression as in (3). Thus, our break point estimator is,

$$\hat{\tau} = \underset{\tau \in \Delta}{arg\ min} \lfloor \tau T \rfloor^{-2} \sum_{t=1}^{\lfloor \tau T \rfloor} \hat{e}_t^2(\tau), \tag{19}$$

where $\Delta := (\delta; (1-\delta))$ with $0 < \delta < 0.5$ is an interval eliminating the first and last observations for the break point estimation. For this statistic, the following consistency result can be stated:

Theorem 3 *Assuming that the break is from the non-cointegrated subsample to the cointegrated subsample and that Assumptions 1 and 2 hold, as $T \to \infty$, then*

$$\hat{\tau} \to \tau_0, \tag{20}$$

where τ_0 denotes the true break fraction.

Remark 2.5 If the break is from the cointegrated to the non-cointegrated sample then the reversed sum of squared residuals, from T to $\lfloor \tau T \rfloor$, is a consistent estimator of the break fraction τ_0. □

3 Monte Carlo study

3.1 Data generation process and test implementation

In this Section, we analyze the finite-sample properties of the residual-based tests for segmented fractional cointegration introduced above by means of Monte Carlo simulations. The data generation process (DGP) considered for the empirical size and power analysis are,

$$y_t = x_t + e_t, \qquad t = 1, \ldots, T \tag{21}$$

$$x_t = x_{t-1} + v_t, \tag{22}$$

$$(1-L)^{(1-b_t)} e_t = a_t, \tag{23}$$

where

$$\begin{pmatrix} v_t \\ a_t \end{pmatrix} \sim iidN\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right), \text{ and } \rho \in \{0, 0.8\}.$$

For $\rho = 0$, x_t is strictly exogenous whereas for $\rho \neq 0$, x_t is correlated with e_t (i.e., endogenous).

For implementation of the tests, we first estimate a model as in (3), i.e.,

$$\Delta^d y_t = \alpha + \beta \Delta^d x_t + e_t, \tag{24}$$

Table 1 Critical values for subsample tests

	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$
$T = 250$				
1%	9.438	7.722	7.699	7.172
5%	5.960	4.458	4.471	4.112
10%	4.470	3.130	3.133	2.867
$T = 500$				
1%	8.888	7.387	7.405	6.862
5%	5.737	4.293	4.296	3.955
10%	4.381	3.000	3.006	2.767

For implementation of the tests we considered $\lambda_0 = 0.5$ and all results are based on 5000 Monte Carlo replications

and using the resulting OLS residuals, $\hat{e}_t = y_t - \hat{\alpha} - \hat{\beta} x_t$, we compute $\mathcal{T}_S^*$, $\mathcal{T}_{I_f}(\lambda_0)$, $\mathcal{T}_{I_b}(\lambda_0)$, and $\mathcal{T}_R(\lambda_0)$, as well as the full sample test proposed by Hassler and Breitung (2006), which we denote as $\mathcal{T}_{\mathrm{HB}}$. All results reported are for a 5% significance level and are based on 5000 Monte Carlo replications. We present results for sample sizes $T = \{250, 500\}$.

For benchmarking purposes, we consider the test statistics computed either for iid innovations as in Breitung and Hassler (2002) or using Eicker–White's correction against heteroskedasticity as in Demetrescu et al. (2008).

To compute the critical values for the tests, we generate data from

$$y_t = x_t + e_t, \qquad t = 1, \ldots, T \tag{25}$$

$$(1 - L)^d x_t = v_t, \tag{26}$$

$$(1 - L)^d e_t = a_t, \tag{27}$$

with $d = \{0.5, 0.6, \ldots, 1\}$ and computed the critical values as the average of the critical values obtained for each d considered at a specific significance level. Table 1 reports the critical values for samples $T = 250$ and $T = 500$ at the 1%, 5% and 10% significance levels.

3.2 Empirical rejection frequencies

For the analysis of the finite sample rejection frequencies under the null and alternative hypothesis, we consider three experiments:

Experiment 1: Constant cointegration relation over the whole sample.

Experiment 2: Spurious regime in the first part of the sample and a fractional cointegrated regime in the second part, i.e.,

$$\begin{cases} b_t = 0 & \text{for } t = 1, \ldots, \lfloor \lambda T \rfloor \\ b_t > 0 & \text{for } t = \lfloor \lambda T \rfloor + 1, \ldots, T \end{cases}. \tag{28}$$

Experiment 3: Fractional cointegrated regime in the first part of the sample and a spurious regime in the second part of the sample, i.e.,

$$\begin{cases} b_t > 0 & \text{for } t = 1, \ldots, \lfloor \lambda T \rfloor \\ b_t = 0 & \text{for } t = \lfloor \lambda T \rfloor + 1, \ldots, T \end{cases}, \tag{29}$$

with $\lambda \in \{0.3, 0.5, 0.7\}$ in both experiments 2 and 3.

In the case of Experiment 1, data are generated from (21) to (23), where y_t and x_t are both I(1) variables and $b_t = b = \{0, 0.05, 0.10, \ldots, 0.50\}$ which allows us to look at the empirical rejection frequencies under the null hypothesis (empirical size, $b = 0$) as well as under the alternative (finite sample power, $b_t > 0$). The first observation we can make from the upper panel of Table 2 is that for $T = 250$, with the exception of $\mathcal{T}_{\mathrm{HB}}$ (which displays an empirical size of 8.4%), all other tests have acceptable finite sample size (ranging between 5.2% and 6.1%). As the sample size increases to $T = 500$ all tests improve in size (for $\mathcal{T}_{\mathrm{HB}}$ the empirical rejection frequency under the null hypothesis reduces to 6.4% whereas for the other subsample tests it ranges between 4.5% and 4.9%). Also in terms of power an improvement is observed. In the lower panel with endogenous x_t, we observe lower empirical sizes for $T = 250$ compared to the exogenous case and slightly higher empirical sizes for $T = 500$. The power is always better than with exogenous x_t. Overall, all tests are relatively robust to endogeneity. Note, that of the set of sequential tests proposed, the best performing in both cases are the recursive tests, $\mathcal{T}_{I_f}(\lambda_0)$ and $\mathcal{T}_{I_b}(\lambda_0)$, although, as expected, $\mathcal{T}_{\mathrm{HB}}$ displays in the case of Experiment 1 the overall best performance.

In the case of Experiment 2, the sample is divided into two sub-periods where in the first sub-period there is no cointegration ($b = 0$) and in the second the variables are cointegrated ($b > 0$). We allow the change into the cointegrated regime to be early in the sample ($\lambda = 0.3$), in the middle of the sample ($\lambda = 0.5$) and late in the sample ($\lambda = 0.7$). We consider a similar exercise in Experiment 3 except that the first sub-period corresponds to cointegration ($b > 0$) and the second to a spurious regression ($b = 0$). From Table 4, we observe first that the overall best performing test of the sequential tests introduced is $\mathcal{T}_S^*$ followed by $\mathcal{T}_{I_f}(\lambda_0)$. The overall test $\mathcal{T}_{\mathrm{HB}}$, although slightly oversized, also displays interesting power performance. The good behavior of $\mathcal{T}_S^*$ is clearly observable in the larger sample ($T = 500$) where it stands out particularly for $\lambda = 0.5$ and $\lambda = 0.7$. For $\lambda = 0.3$ the difference of $\mathcal{T}_S^*$ with regards to $\mathcal{T}_{\mathrm{HB}}$ is not as marked.

Table 3 reports results for the case where there is cointegration in the first sub-period and in the second sub-period there is no cointegration. In this case the rolling approach $\mathcal{T}_R(\lambda_0)$ displays interesting behavior, particularly for $b_t > 0.15$ and $T = 250$, and for $b_t > 0.1$ when $T = 500$. The $\mathcal{T}_S^*$ statistic also displays good power performance.[3]

We also apply the break point estimator to data from Experiment 3 and residuals from a regression without constant in order to detect a break from cointegration to no cointegration. Table 5 shows the estimated break fraction for different choices of δ. This

[3] We have also performed simulations using Eicker–White robust standard errors in the implementation of the statistics, however, since the results are qualitatively similar to those reported in Tables 2 and 3 we have decided not to include them in the paper for the sake of space. These can, however, be obtained from the authors.

Table 2 Empirical rejection frequencies of the fractional cointegration tests under Experiment 1 at a 5% significance level, with $\lambda_0 = 0.5$ and $d = 1$

$\rho = 0$

	$T = 250$					$T = 500$				
b	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{\mathrm{HB}}$	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{\mathrm{HB}}$
0.00	0.061	0.061	0.058	0.052	0.084	0.049	0.046	0.045	0.048	0.064
0.05	0.154	0.183	0.181	0.115	0.226	0.208	0.252	0.256	0.158	0.307
0.10	0.384	0.445	0.448	0.258	0.514	0.625	0.695	0.694	0.431	0.752
0.15	0.691	0.756	0.758	0.486	0.811	0.932	0.959	0.959	0.758	0.970
0.20	0.899	0.939	0.939	0.697	0.961	0.996	0.998	0.998	0.944	0.999
0.25	0.988	0.995	0.995	0.879	0.997	1	1	1	0.995	1
0.30	0.999	1	1	0.957	1	1	1	1	0.999	1
0.35	1	1	1	0.992	1	1	1	1	1	1
0.40	1	1	1	0.998	1	1	1	1	1	1
0.45	1	1	1	1	1	1	1	1	1	1
0.50	1	1	1	1	1	1	1	1	1	1

Table 2 continued

$\rho = 0$										
	$T = 250$					$T = 500$				
b	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{\mathrm{HB}}$	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{\mathrm{HB}}$
$\rho = 0.8$										
	$T = 250$					$T = 500$				
b	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{\mathrm{HB}}$	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{\mathrm{HB}}$
0.00	0.048	0.054	0.051	0.039	0.080	0.052	0.055	0.056	0.047	0.066
0.05	0.159	0.199	0.205	0.095	0.264	0.314	0.375	0.365	0.149	0.403
0.10	0.455	0.547	0.549	0.217	0.626	0.797	0.853	0.850	0.426	0.862
0.15	0.798	0.872	0.868	0.419	0.907	0.983	0.989	0.990	0.748	0.990
0.20	0.965	0.981	0.980	0.630	0.988	1	1	1	0.939	1
0.25	0.996	0.999	0.999	0.789	1	1	1	1	0.988	1
0.30	1	1	1	0.915	1	1	1	1	0.999	1
0.35	1	1	1	0.967	1	1	1	1	1	1
0.40	1	1	1	0.986	1	1	1	1	1	1
0.45	1	1	1	0.994	1	1	1	1	1	1
0.50	1	1	1	0.999	1	1	1	1	1	1

Table 3 Empirical rejection frequencies of the fractional cointegration tests under Experiment 2 at a 5% significance level, with $\lambda_0 = 0.5$ and $d = 1$

	$T = 250$					$T = 500$				
b	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{\mathrm{HB}}$	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{\mathrm{HB}}$
$\lambda = 0.3$										
0	0.061	0.061	0.058	0.052	0.084	0.058	0.055	0.054	0.050	0.076
0.05	0.111	0.137	0.128	0.115	0.171	0.152	0.169	0.165	0.162	0.208
0.10	0.257	0.273	0.270	0.258	0.326	0.399	0.401	0.394	0.416	0.462
0.15	0.438	0.432	0.426	0.479	0.504	0.709	0.661	0.655	0.753	0.717
0.20	0.653	0.617	0.603	0.682	0.677	0.915	0.832	0.832	0.934	0.868
0.25	0.830	0.741	0.735	0.863	0.788	0.980	0.910	0.908	0.988	0.929
0.30	0.927	0.828	0.819	0.940	0.864	0.995	0.939	0.937	0.998	0.954
0.35	0.962	0.873	0.865	0.978	0.899	0.998	0.958	0.954	0.998	0.966
0.40	0.986	0.908	0.902	0.993	0.933	1.000	0.974	0.973	1.000	0.979
0.45	0.995	0.926	0.920	0.997	0.944	0.999	0.982	0.980	1.000	0.986
0.50	0.997	0.948	0.943	0.998	0.961	0.999	0.981	0.979	1.000	0.986
$\lambda = 0.5$										
0	0.058	0.059	0.061	0.057	0.081	0.049	0.048	0.050	0.051	0.069
0.05	0.097	0.095	0.093	0.230	0.123	0.115	0.112	0.114	0.360	0.152
0.10	0.193	0.169	0.163	0.509	0.222	0.311	0.237	0.229	0.686	0.288
0.15	0.350	0.250	0.243	0.726	0.305	0.591	0.365	0.363	0.879	0.425
0.20	0.529	0.344	0.334	0.845	0.406	0.823	0.495	0.494	0.962	0.556
0.25	0.702	0.430	0.413	0.926	0.494	0.934	0.602	0.593	0.987	0.651
0.30	0.828	0.516	0.504	0.965	0.574	0.970	0.678	0.675	0.997	0.724
0.35	0.888	0.560	0.551	0.983	0.623	0.980	0.752	0.746	0.996	0.789
0.40	0.937	0.633	0.623	0.991	0.684	0.989	0.780	0.773	0.998	0.820
0.45	0.953	0.673	0.664	0.994	0.721	0.989	0.813	0.817	0.998	0.845

Table 3 continued

b	$T=250$ $\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{\mathrm{HB}}$	$T=500$ $\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{\mathrm{HB}}$
0.50	0.967	0.711	0.703	0.996	0.756	0.991	0.849	0.848	0.999	0.877
$\lambda=0.7$										
0	0.058	0.057	0.055	0.057	0.080	0.051	0.052	0.050	0.054	0.076
0.05	0.071	0.079	0.072	0.079	0.104	0.077	0.085	0.077	0.095	0.113
0.010	0.108	0.107	0.104	0.123	0.139	0.120	0.123	0.117	0.155	0.154
0.15	0.136	0.135	0.129	0.158	0.172	0.181	0.165	0.154	0.223	0.206
0.20	0.163	0.161	0.155	0.197	0.202	0.241	0.222	0.208	0.292	0.269
0.25	0.205	0.191	0.183	0.238	0.245	0.285	0.262	0.250	0.340	0.314
0.30	0.230	0.217	0.212	0.268	0.272	0.351	0.310	0.296	0.411	0.357
0.35	0.263	0.249	0.241	0.306	0.306	0.402	0.359	0.353	0.456	0.418
0.40	0.291	0.274	0.265	0.341	0.328	0.436	0.398	0.388	0.485	0.462
0.45	0.347	0.321	0.314	0.386	0.376	0.496	0.447	0.444	0.543	0.504
0.50	0.368	0.341	0.332	0.413	0.401	0.527	0.484	0.482	0.566	0.543

Table 4 Empirical rejection frequencies of the fractional cointegration tests under Experiment 3 at a 5% significance level, with $\lambda_0 = 0.5$ and $d = 1$

	$T = 250$					$T = 500$				
b	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{\text{HB}}$	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{\text{HB}}$
$\lambda = 0.3$										
0	0.055	0.058	0.061	0.054	0.076	0.058	0.055	0.056	0.056	0.065
0.05	0.079	0.077	0.079	0.051	0.104	0.082	0.083	0.083	0.057	0.096
0.10	0.101	0.100	0.103	0.050	0.128	0.133	0.125	0.136	0.052	0.141
0.15	0.134	0.129	0.144	0.051	0.166	0.189	0.173	0.182	0.053	0.191
0.20	0.161	0.151	0.167	0.054	0.189	0.254	0.222	0.238	0.051	0.243
0.25	0.202	0.178	0.194	0.053	0.221	0.311	0.265	0.272	0.052	0.293
0.30	0.237	0.210	0.230	0.050	0.257	0.375	0.325	0.339	0.050	0.351
0.35	0.281	0.247	0.262	0.051	0.298	0.453	0.393	0.410	0.052	0.420
0.40	0.310	0.275	0.293	0.056	0.324	0.499	0.424	0.437	0.052	0.454
0.45	0.353	0.307	0.313	0.049	0.359	0.537	0.467	0.473	0.058	0.493
0.50	0.397	0.341	0.353	0.055	0.393	0.594	0.514	0.527	0.052	0.543
$\lambda = 0.5$										
0	0.051	0.060	0.060	0.048	0.078	0.054	0.059	0.057	0.054	0.069
0.05	0.092	0.094	0.099	0.063	0.126	0.114	0.115	0.118	0.090	0.132
0.10	0.159	0.147	0.155	0.091	0.182	0.279	0.224	0.239	0.164	0.248
0.15	0.269	0.207	0.227	0.142	0.260	0.474	0.331	0.345	0.228	0.361
0.20	0.411	0.285	0.298	0.204	0.336	0.658	0.426	0.436	0.283	0.454
0.25	0.530	0.345	0.358	0.240	0.400	0.775	0.532	0.531	0.344	0.560
0.30	0.640	0.409	0.418	0.267	0.463	0.832	0.593	0.594	0.373	0.619
0.35	0.727	0.462	0.473	0.297	0.518	0.871	0.657	0.654	0.404	0.676
0.40	0.770	0.515	0.518	0.325	0.565	0.894	0.707	0.700	0.425	0.727

Table 4 continued

	$T=250$					$T=500$				
b	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{\mathrm{HB}}$	$\mathcal{T}_S^*$	$\mathcal{T}_{I_f}(\lambda_0)$	$\mathcal{T}_{I_b}(\lambda_0)$	$\mathcal{T}_R(\lambda_0)$	$\mathcal{T}_{\mathrm{HB}}$
0.45	0.811	0.565	0.566	0.328	0.618	0.906	0.739	0.732	0.432	0.757
0.50	0.832	0.611	0.612	0.348	0.653	0.924	0.766	0.766	0.452	0.783
$\lambda=0.7$										
0	0.060	0.062	0.058	0.053	0.085	0.056	0.056	0.058	0.054	0.066
0.05	0.114	0.128	0.133	0.072	0.166	0.154	0.167	0.178	0.080	0.188
0.10	0.207	0.216	0.232	0.090	0.266	0.342	0.360	0.364	0.119	0.386
0.15	0.346	0.344	0.347	0.105	0.398	0.583	0.546	0.538	0.137	0.572
0.20	0.509	0.465	0.467	0.125	0.518	0.739	0.657	0.646	0.181	0.679
0.25	0.625	0.534	0.542	0.145	0.592	0.847	0.741	0.724	0.210	0.757
0.30	0.726	0.619	0.613	0.159	0.660	0.887	0.780	0.766	0.239	0.796
0.35	0.798	0.669	0.664	0.177	0.714	0.912	0.818	0.810	0.279	0.831
0.40	0.837	0.710	0.700	0.197	0.738	0.927	0.837	0.825	0.311	0.850
0.45	0.871	0.742	0.735	0.227	0.774	0.933	0.843	0.832	0.336	0.854
0.50	0.884	0.761	0.751	0.237	0.789	0.946	0.868	0.854	0.369	0.878

Table 5 Break point estimates with $T = 1000$ and 5000 Monte Carlo replications

δ	0.05			0.1			0.15		
$b\backslash\lambda$	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
0.10	0.564	0.604	0.688	0.559	0.598	0.676	0.550	0.589	0.659
0.15	0.503	0.558	0.667	0.509	0.560	0.666	0.514	0.559	0.665
0.20	0.461	0.526	0.661	0.458	0.526	0.658	0.472	0.524	0.660
0.25	0.424	0.499	0.655	0.437	0.501	0.657	0.436	0.503	0.658
0.30	0.410	0.483	0.654	0.412	0.488	0.656	0.414	0.494	0.659
0.35	0.389	0.470	0.653	0.397	0.473	0.656	0.404	0.478	0.656
0.40	0.373	0.458	0.655	0.381	0.461	0.655	0.392	0.470	0.656
0.45	0.365	0.446	0.648	0.374	0.457	0.651	0.387	0.463	0.653
0.50	0.358	0.448	0.647	0.375	0.453	0.648	0.380	0.458	0.653
no break	0.938			0.890			0.842		

choice does not have any influence on the results. Therefore, for practical purposes, a small δ is recommended in order to keep a large part of the data in the analysis. With small b, there is a tendency to locate the break in the middle of the sample, but the results improve as the cointegrating strength b increases and for the largest b the accuracy is good. Hence, with strong cointegrating relations, the break point estimator delivers reliable results. If there is permanent cointegration, the break is estimated at the end of the admissible window. If the data are generated from Experiment 2, the regression residuals are reversed before applying the break point estimator. The results remain the same and are available upon request.

Note that throughout this section we have assumed that y_t and x_t are integrated of order one, $d = 1$; however, this assumption is without loss of generality as the results hold regardless of the value of d. We have also assumed that d is known, which, empirically is not a reasonable assumption, but also in this case, as long as a suitable estimator of d is used, this assumption is of no consequence. To illustrate both claims, in the appendix we provide simulation results on the size and power of the tests (Tables B1–B3), which are obtained from data generated as described in this section, but where y_t and x_t are integrated of order $d = 0.9$ and d is estimated prior to the application of the tests, using the spectral maximum likelihood estimator of Fox and Taqqu (1986).

4 Empirical application

4.1 Government bond markets' equilibrium

In our first empirical analysis, we apply the tests introduced in Sect. 2 to benchmark government bonds of countries that are part of the EMU. The analysis is based on daily observations between 01.01.1999 and 08.08.2017 (about 4800 observations per country) of 10-year-to-maturity government bonds of eleven EMU countries (Spain

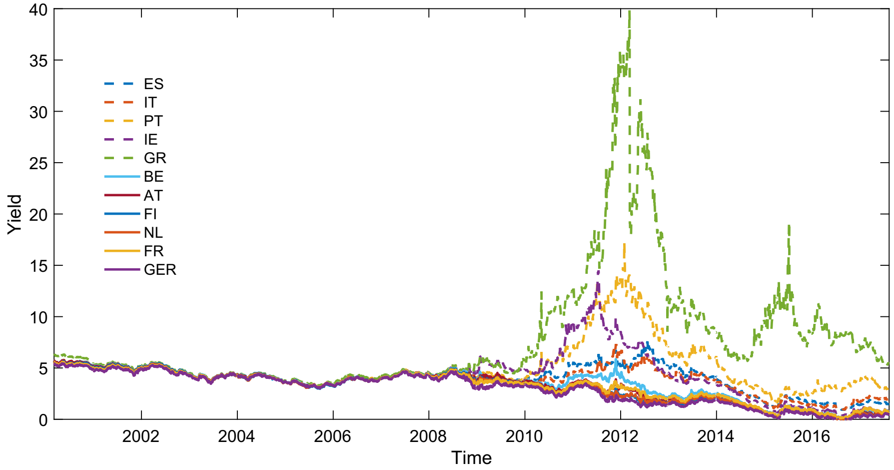

Fig. 1 Yields of EMU government bonds

(ES), Italy (IT), Portugal (PT), Ireland (IE), Greece (GR), Belgium (BE), Austria (AT), Finland (FI), the Netherlands (NL), France (FR) and Germany (GER)). The data are obtained from Thomson Reuters Eikon.

In general, the convergence of national yields to a stable level with reduced risk aids the overall economy, by allowing cheaper access to debt financing with less uncertainty regarding the value of such funds over time. This, in turn, stimulates investment and output within converging countries. The expansion of the euro-zone bond market is one beneficial outcome of this process (Hartmann et al. 2003 and Abad et al. 2010).

The existence of a (fractional) cointegrating relationship between the EMU countries' government bond markets is a strong indication of alignment of the markets. This conclusion is supported by Fig. 1 which shows how the bond yields co-move up to 2008. In specific, the small differentials observed up until the end of 2008 are consistent with available evidence of markets convergence. From the beginning of the financial crisis in 2008 onward, the yields of some countries start drifting apart suggesting a change in the long-run equilibrium of the government yields and consequently in the market convergence process. As a consequence of the financial crisis the yield differentials increased moderately in 2009, but surged in 2010 reaching unprecedented levels in 2011 (levels that were higher than those observed in the early 90 s). Hence, it is likely that as a consequence of this change in the long-run equilibrium, testing for no cointegration over the full sample does not allow us to reject the null hypothesis. However, with our new tests we expect to be able to detect sub-periods of cointegration in the sense that under the alternative we allow for segmented fractional cointegration, i.e., fractional cointegration in certain sub-samples and no cointegration elsewhere.

To start our analysis, we need first to determine the order of integration of the country yields under consideration. Since the order of integration of our data is unknown, we apply unit root and stationarity tests and compute 95% confidence intervals for the fractional parameter d (see Table 6). The ADF-test, augmented based on Schwert's

Table 6 p-values of ADF- and KPSS-tests, and estimates and 95% confidence intervals for the fractional parameter d

	ADF	KPSS	$\hat{d}$	$\mathrm{CI}_{95\%}(d)$	$\mathrm{CI}^{\mathrm{EW}}_{95\%}(d)$
ES	0.93	0.01	1.02	[0.92,1.12]	[0.84,1.19]
IT	0.93	0.01	1.09	[0.96,1.21]	[0.84,1.28]
PT	0.93	0.01	0.89	[0.84,0.94]	[0.75,1.03]
IE	0.92	0.01	1.02	[0.94,1.10]	[0.79,1.23]
GR	0.93	0.01	0.92	[0.84,1.00]	[0.63,1.17]
BE	0.93	0.01	0.92	[0.82,1.02]	[0.80,1.06]
AT	0.93	0.01	0.95	[0.81,1.06]	[0.80,1.09]
FI	0.93	0.01	0.95	[0.83,1.06]	[0.81,1.07]
NL	0.93	0.01	0.95	[0.82,1.06]	[0.81,1.07]
FR	0.93	0.01	0.97	[0.84,1.08]	[0.82,1.10]
GER	0.93	0.01	0.99	[0.86,1.09]	[0.83,1.11]

$\hat{d}$ denotes the estimate of the fractional parameter, and $\mathrm{CI}_{95\%}(d)$ and $\mathrm{CI}^{\mathrm{EW}}_{95\%}(d)$ denote 95% confidence intervals for d, with the former assuming homoskedastic errors and the latter using Eicker–White robust standard errors. The confidence intervals which include the true value of the fractional parameter with 95% asymptotic coverage are computed by inverting the non-rejection region of the fractional integration test of Demetrescu et al. (2008); see Hassler et al. (2016) for details

rule and including a drift, cannot reject the unit root hypothesis and the KPSS-test rejects stationarity suggesting that $d_i = 1$ for all country yields. The robust 95% confidence intervals for d_i, $\mathrm{CI}^{\mathrm{EW}}_{95\%}(d)$, support this result, suggesting that all yields are non-stationary. Although a unit root may not be plausible from an economic perspective, the finite sample behavior of these series is consistent with results available in the literature on fractional cointegration, confer for example Chen and Hurvich (2003) and Nielsen (2010). A further conclusion that can be drawn from the robust confidence intervals presented in Table 6 is the greater estimation uncertainty of d for ES, IT, PT, IE, GR and BE as these are the yields with the largest $\mathrm{CI}^{\mathrm{EW}}_{95\%}(d)$, and also correspond to the countries most strongly affected by the crisis.

Once the order of integration of the yields is determined, the cointegrating regressions are estimated in a bivariate setting where the yield of country i, y_{it}, is regressed on the yield of country k, y_{kt}, i.e.,

$$y_{it} = \beta_0 + \beta_1 y_{kt} + e_t, \text{ for } i \neq k = 1, \ldots, 11. \tag{30}$$

The residuals obtained from (30) are then used for application of the split, incremental and rolling sample versions of the tests where λ_0 is set to 0.2 and 0.5, respectively. We also compute the full sample Hassler–Breitung test ($\mathcal{T}_{\mathrm{HB}}$). To account for autocorrelation when implementing the tests, we chose the augmentation order p in (4) using Schwert's rule[4] as suggested in Demetrescu et al. (2008), and use Eicker–White (EW) heteroscedasticity-robust standard errors as these are more suitable in our empirical setting.

[4] In specific, in our analysis we consider $p = \lfloor 4(T/100)^{1/4} \rfloor$.

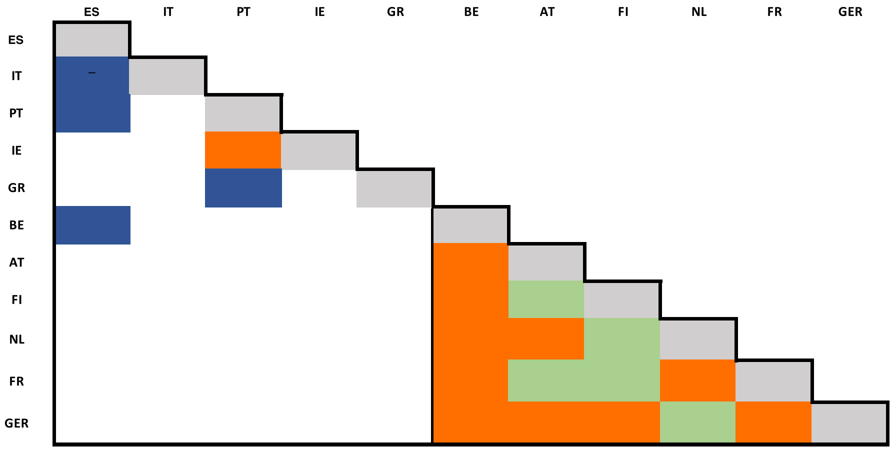

Fig. 2 Summary of fractional cointegration tests' results considering a 5% significance level and $\lambda_0 = 0.5$ in the implementation of the subsample test statistics. *Note:* Orange cells indicate rejection of the null hypothesis of no fractional cointegration at a 5% nominal significance level by more than one test procedure; blue cells indicate rejection by the rolling test only; the green cells indicate rejection of the null hypothesis of all tests including the Hassler–Breitung test; and white cells correspond to non-rejection of the null hypothesis

4.1.1 Full sample analysis

Considering the full sample (from 06-Mar-2000 to 08-Aug-2017) we estimate (30) for all country pair combination. Specifically, a total of 110 test regressions are performed. A summary of the results for $\lambda_0 = 0.5$ is given in Fig. 2 (see Figure B.1 in the appendix for results with $\lambda_0 = 0.2$).[5]

Figure 2 highlights the rejections of the null hypothesis of no pairwise fractional cointegration for BE, AT, FI, NL, FR and GER. The tests also reject the null of no fractional cointegration for the pairs (ES, IT), (ES, BE), (ES, PT), (PT, IE) and (PT, GR). No cointegration in the full sample is found between AT, FI, NL, FR, GER and ES, IT, PT, IE and GR.[6] The results in this figure correspond to the results obtained with the new tests introduced in Sect. 3 and with the Hassler–Breitung test, $\mathcal{T}_{\mathrm{HB}}$. The full sample test $\mathcal{T}_{\mathrm{HB}}$ only finds evidence of fractional cointegration in five country pairs: (AT, FI), (AT, FR), (FI, NL), (FI, FR) and (GER, NL). In addition to the $\mathcal{T}_{\mathrm{HB}}$ rejections, the new tests introduced in Sect. 3 further reject the null hypothesis for (ES, IT), (ES, PT), (ES, BE), (PT, IE), (BE, AT), (BE, FI), (BE, NL), (BE, FR), (BE, GER), (AT, NL), (BE, GER), (FI, GER), (NL, FR) and (FR, GER). Hence, the evidence in Fig. 2 points to the potential existence of two groups of countries: Group I—BE, AT, FI, NL, FR and GER; and Group II—ES, IT, PT, IE, GR. Group I displays strong evidence of cointegration within the group, and almost no evidence of an equilibrium

[5] The choice of $\lambda_0 = 0.5$ follows from Davidson and Monticini (2010) who recommend the use of $\lambda_0 = 0.5$ because a break must occur in either the first half of the sample or the second.

[6] Note that rejection of the null hypothesis is only considered when the tests applied to the residuals of the yields of country i on the yields of country j, and vice versa, reject the null.

	ES	IT	PT	IE	GR	BE	AT	FI	NL	FR	GER
ES											
IT	02/02/2006										
PT	28/03/2011 12/06/2012*	11/03/2011 17/12/2012*									
IE	12/08/2010	14/06/2010 03/04/2012*	14/10/2008								
GR	27/04/2011 17/07/2002*	18/04/2011 20/09/2012*	13/09/2011 22/09/2010*	09/09/2011							
BE	21/05/2010	08/07/2011	17/02/2011	11/08/2010 29/08/2013*	23/03/2011 28/11/2012*						
AT	26/05/2010	02/11/2010	23/08/2010	16/06/2010 06/12/2013*	23/06/2010 06/12/2012*	07/09/2010 03/06/2014*					
FI	21/05/2010	26/05/2010	17/06/2010	04/06/2010 10/02/2014*	16/06/2010 04/12/2012*	02/06/2010 13/03/2014*	15/12/2008 02/11/2012*				
NL	20/05/2010	01/06/2010	16/06/2010	04/06/2010 02/10/2013*	16/06/2010 05/12/2012*	03/06/2010 07/04/2014*	11/12/2008 24/01/2013*	22/11/2011			
FR	24/05/2010	16/08/2010	24/08/2010	16/06/2010 29/08/2013*	24/06/2010 28/11/2012*	04/06/2010 15/10/2012*	22/10/2008	13/06/2011	05/12/2008		
GER	21/05/2010	25/05/2010	04/06/2010	02/06/2010 02/12/2013*	27/04/2010 05/12/2012*	04/06/2010	30/10/2008	02/12/2002	23/10/2008	09/06/2011	

Fig. 3 Estimated break dates. *Note*: Date format dd/mm/yy. The dates on the top lines for each country correspond to breaks determined from the regression residuals and the dates on the lines below (indicated with an *) to the breaks determined from reversed residuals. Break dates are determined using the estimator introduced in (19) with $\delta = 0.05$ and imposing a minimum length of $\lfloor 0.1T \rfloor$ between the sequential estimated breaks

relationship with the countries of Group II. Furthermore, within Group II evidence of fractional cointegration seems to be small.

These results suggest, on the one hand, that fractional cointegration is mainly found for countries that were less affected by the sovereign debt crisis and no cointegration for those more strongly affected; and on the other hand, that the European yields were not cointegrated over the whole period. The rejections observed in Fig. 2 for (ES, IT), (ES, PT), (ES, BE) and (PT, GR) result from the rolling regression, therefore providing evidence for the existence of windows of fractional cointegration in the sample. Recall that for a given window width, tests based on a rolling sequence of statistics are designed to pick up a window of fractional cointegration, of (roughly) the same length, within the data. Note also that the finding of segmented cointegration does not contradict the rejection of the Breitung–Hassler-test as it also has power, albeit less, in the presence of segmented cointegration. The tests for segmented cointegration also have power when the cointegrating relation is permanent as they include the full sample as well.

4.1.2 Subsample analysis

To examine the potential existence of segmented fractional cointegration further, we proceed by estimating the break date with the break point estimator proposed in (19) based on residuals from a regression without a constant. We set $\delta = 0.05$ and impose a minimum length of $\lfloor 0.1T \rfloor$ between the sequentially estimated breaks. The results are given in Fig. 3.

The first observation we can make from the break dates presented in Fig. 3 is that for most pairs of countries changes in the long-run equilibrium occurred between 2010 and 2011, with the exception of (FI, GER) which displays an early change (Dec, 2002),

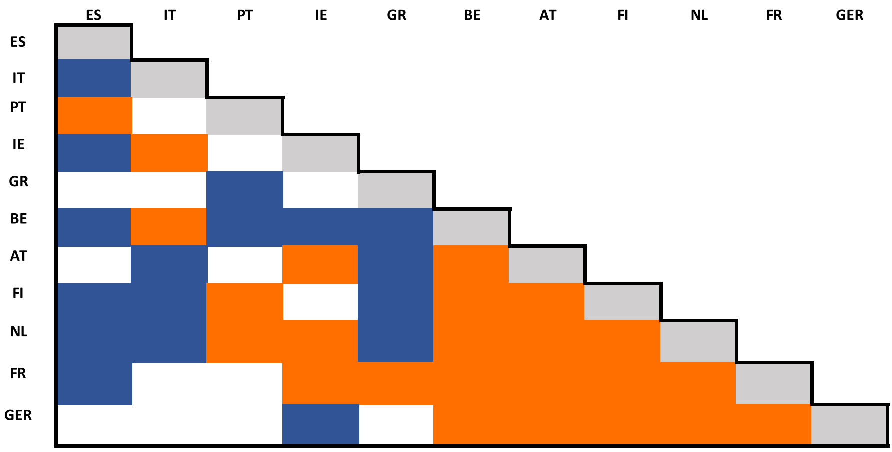

Fig. 4 Summary of fractional cointegration tests' results for the sub-period before the break considering a 5% significance level and $\lambda_0 = 0.5$ in the implementation of the subsample test statistics. *Note:* Orange cells indicate rejection of the null hypothesis of no fractional cointegration at a 5% nominal significance level by more than one test procedure; blue cells indicate rejection by the rolling test only; and white cells correspond to non-rejection of the null hypothesis

(ES, IT) in Feb. 2006, and (NL, GER), (NL, FR), (NL, AT), (FI, AT), (AT, GER), (IE, PT), (FR, AT) in the second half of 2008 (corresponding to the global financial crisis).

Although sovereign debt only substantially increased in a few eurozone countries, it was generally perceived as a problem for the Euro area as a whole. This crisis forced Greece, Spain, Ireland, Portugal and Cyprus to seek financial aid by the end of 2012. In mid-2012, due to successful fiscal consolidation and implementation of structural reforms in the countries being most at risk and various policy measures taken by EU leaders and the ECB, financial stability in the eurozone improved significantly and interest rates fell steadily and by 2014 most countries had regained market access. This is to a certain extent con-substantiated in the break dates computed from the reversed residuals (dates indicated in Fig. 3 with an *). In this case, most changes detected fall between the second semester of 2012 and the first semester of 2014, essentially consistent with the ending of the sovereign debt crisis.

We further computed additional break dates from sequential estimation in all sub-samples; however, most break dates detected were either in 2000/2001 or in 2016 which are dates close to the beginning or end of the sample. However, since the determination of breaks in small subsamples close to the edges is doubtful we decided not to consider them. Hence, in what follows the analysis only considered the break dates reported in Fig. 3.

If we consider the sample starting 1999 up to the first break (see Fig. 4), which generally corresponds to the pre-sovereign debt crisis period, we observe that there is more evidence of fractional cointegration between the pairs of countries than when the full sample is used, the exceptions are (ES, GR), (ES, AT), (ES, GER), (IT, PT), (IT, GR), (IT, FR), (IT, GER), (PT, IE), (PT, AT), (PT, FR), (PT, GER), (IE, GR), (IE, FI) and (GR, GER). These non-rejections of the null hypothesis are obtained in combinations of countries which involve ES, IT, PT, IE and GR. These non-rejections

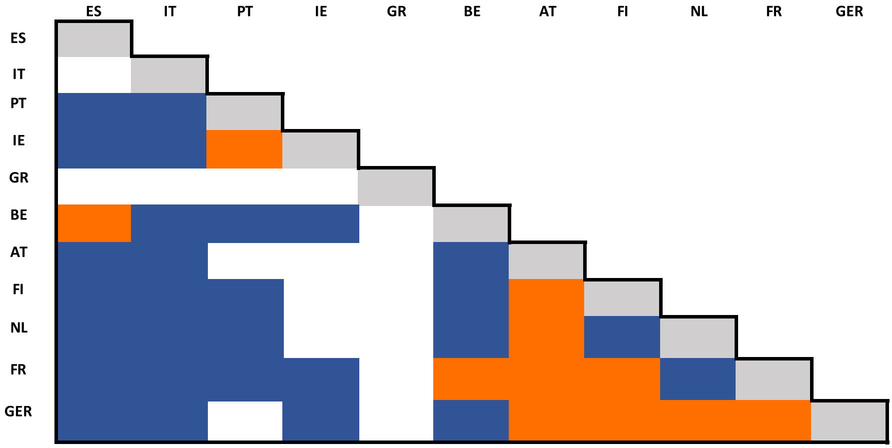

Fig. 5 Summary of fractional cointegration tests' results for the sub-period after the break considering a 5% significance level and $\lambda_0 = 0.5$ in the implementation of the subsample test statistics. *Note:* Orange cells indicate rejection of the null hypothesis of no fractional cointegration at a 5% nominal significance level by more than one test procedure; blue cells indicate rejection by the rolling test only; and white cells correspond to non-rejection of the null hypothesis

may result in part from the fact that these countries where at different economic stages when entering the common currency in 1999 and as a consequence the adjustment process may have led to structural breaks in the cointegrating relations between some of these countries. This is also clear from the cases for which rejection of the null is only attained by the rolling test which in contrast to the other procedures employed is able to find windows of fractional cointegration. This is the case for (ES, IT), (ES, IE), (ES, BE), (ES, FI), (ES, NL), (ES, FR), (ES, GER), (IT, AT), (IT, FI), (IT, NL), (IT, FR), (PT, GR), (PT, BE), (IE, BE), (IE, GER), (GR, BE), (GR, AT), (GR, FI) and (GR, NL), which support the evidence for segmented cointegration in this subsample.

Figure 5 presents the results when the subsample from the first break date until 2017 is considered. One first observation we can make is that in comparison with the full sample also in this case more cointegrating relationships are found. Interestingly, in contrast to the results for the pre-sovereign debt crisis period in Fig. 4, a lot of the rejections reported in this figure result from the rolling test (blue cells), suggesting again the existence of windows of fractional cointegration between countries in this period. This is an expected result as this second subsample includes the period of the sovereign debt crisis and for some countries also the period of the global financial crisis, and the presence of these crises events in the sample may affect the power of the other tests.

We redid the analysis for this second subperiod by removing the part of the sample between the two breaks. The results of this analysis provide evidence that in addition to the rejections in Fig. 5, the rolling test also rejects the null hypothesis of no fractional cointegration for (IE, AT), (IE, FI) and (IE, NL). In this second sub-period Greece does not seem to cointegrate with any of the other countries considered.

4.1.3 Robustness analysis

As a robustness check of our previous analysis, we divided the sample into three subsamples: Subsample I—06-Mar-2000 to 31-Dec-2009; Subsample II—01-Jan-2010 to 31-Dec-2012 and Subsample III - 01-Jan-2013 to 08-Aug-2017 and applied the tests on these subperiods. A summary of the results obtained is presented in Fig. 6.

The results for Subsamples I and III are generally in conformity with the results of the previous section. It is noted that in Subsample I there is strong evidence of fractional cointegration across most pairs of countries. As we can see from Fig. 6, more than one test rejects (orange cells) the null of no fractional cointegration for most cases (except for (BE, IT), (FR, IT), (GER, IT), (FI, IE) and (FI, GR)).

Similarly in Subsample III, where rejections are very much in line with the rejections observed in Fig. 5 and after removing the subsample between the break dates provided in Fig. 3. Also in this case no evidence of fractional cointegration for Greece is observed.

The summary of results obtained for the sovereign debt crisis period (Subsection II) clearly illustrates the impact that this crisis had on the long-run equilibrium relations of ES, IT, PT, IE and GR. It clearly highlights the breakdown of cointegration from Subperiod I to subperiod II and reinforces the importance of the new approaches introduced in this paper which allow for segmented cointegration.

A further important result, illustrated in this figure, is the consistent cointegrating relations between the countries of Group I (BE, AT, FI, NL, FR and GER) over the whole period (Subsample I, II and III), although BE seems to display some periods of segmented cointegration with GER (Subperiod I), with FI (Subperiod II) and with FI and NL (Subperiod III). Also FR seems to display segmented cointegration in Subperiod III with AT and FI.

All in all, we conclude that the yields of the countries considered were fractionally cointegrated after the introduction of the euro until the European debt crisis. The break point estimates suggest the dissolution of fractional cointegrating relationships and market convergence for most pairs of countries at the beginning of the European debt crisis in 2010 and the reestablishment of the cointegrating relationships in 2012/2013.

4.2 Firm interest rates heterogeneity

In this second empirical analysis, we employ the tests developed in this paper to determine the degree of interdependence among lending rates (Total Loans) to non-financial corporations (NFCs),[7] for the eleven EMU countries (Spain (ES), Italy (IT), Portugal (PT), Ireland (IE), Greece (GR), Belgium (BE), Austria (AT), Finland (FI), the Netherlands (NL), France (FR) and Germany (GER)) considered in the previous section, covering the period from 1980:01 to 2020:10. This question is of interest, for example, because of the importance attached to interest rate convergence in the Maastricht Treaty on European Monetary Union agreed to in December 1991.

[7] Specifically, we use time series on interest rates on total new business loans to non-financial corporations. The data are obtained from the ECB—https://sdw.ecb.europa.eu.

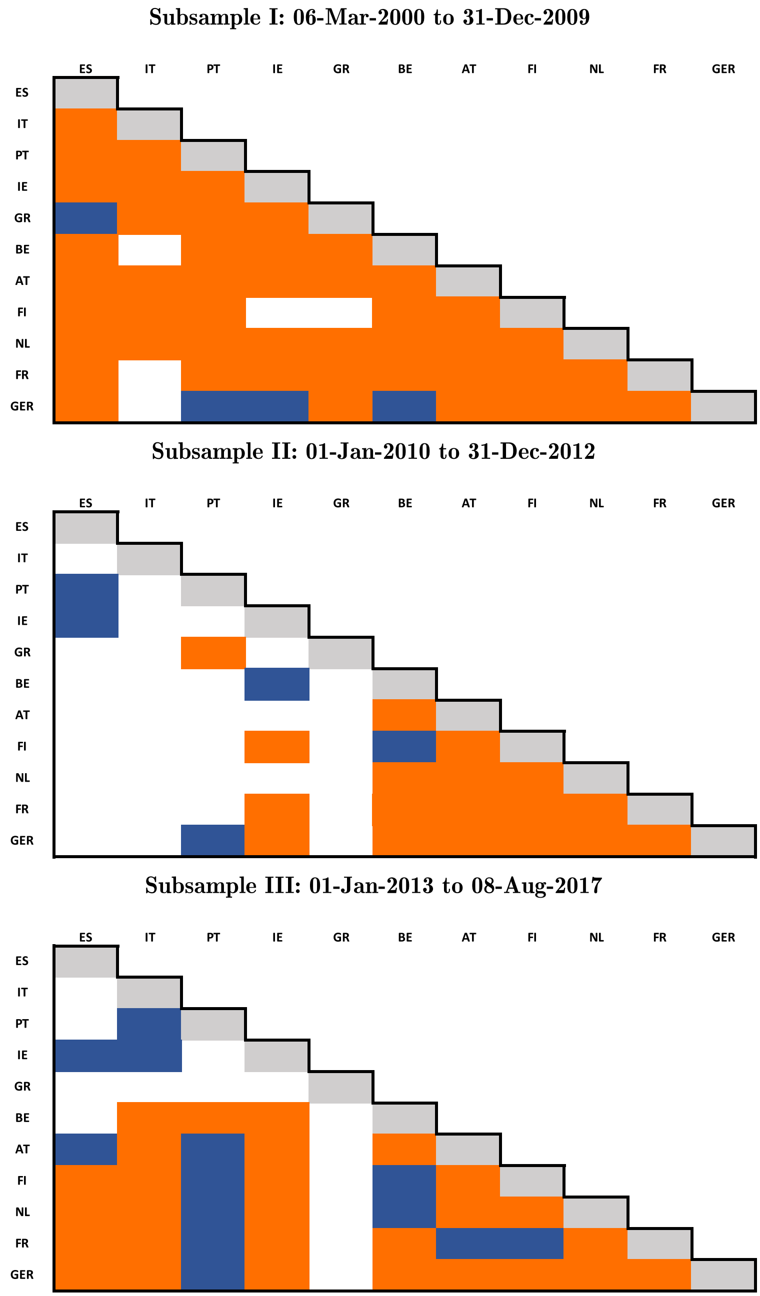

Fig. 6 Summary of fractional cointegration tests' results considering a 5% significance level and $\lambda_0 = 0.5$ in the implementation of the subsample test statistics. *Note:* Orange cells indicate rejection of the null hypothesis of no fractional cointegration at a 5% nominal significance level by more than one test procedure; blue cells indicate rejection by the rolling test only; and white cells correspond to non-rejection of the null hypothesis

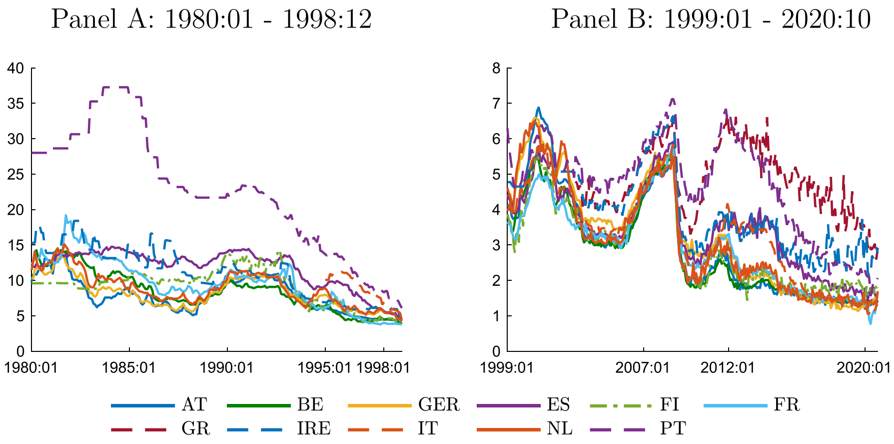

Fig. 7 Lending rates to NFCs-total loans. *Source*: ECB

The dynamics of bank lending rates on loans to non-financial corporations (NFCs) has been quite heterogeneous across countries over the last 4 decades (see Fig. 7). Cross-country heterogeneity can be the consequence of i) the different structures of financial systems across countries; (ii) country specific institutional factors, such as the fiscal and regulatory frameworks, enforcement procedures and differences in the degree to which loans are secured; (iii) business cycles and associated perceptions of credit risk may differ across countries; and (iv) the divergence in banks funding conditions.

For illustration and analysis, we split the sample period (1980:01-2020:10) into two parts as illustrated in Fig. 7. Panel A presents the dynamics of the lending rates to firms between 1980:01 and 1998:12, essentially since the beginning of the European Monetary System (which was set up in 1979) up to the introduction of the euro, and Panel B plots the lending rates from 1999:01 to 2020:10. This period includes the financial crisis and the European sovereign debt crisis. From Panel A we observe that there is considerable heterogeneity across countries in the lending rates between 1980 and 1990, which seems to decrease from 1990 onward. In Panel B we observe some heterogeneity up to around 2010, which clearly increases with the European sovereign debt crisis.

Following the onset of the Economic and Monetary Union (EMU), from 1999 until September 2008, the dispersion of the rates charged to non-financial corporations for new loans was low. Although heterogeneity in lending rates still persisted, the level of integration of financial markets was satisfactory.[8]

However, the financial crisis, fragmented the financial markets of the euro area. This originated difficulties in the assessment of the monetary policy transmission mechanism, since whereas in some countries, the loose monetary policy adopted by the ECB during the crisis reflected, more or less, the expected correspondence in the bank credit growth to non-financial private sectors, in other countries, this variable

[8] See https://www.bis.org/ifc/publ/ifcb39a.pdf.

recorded a much lower response compared to the foreseen results in periods prior to the crisis.

Pressures on banks funding conditions eased in 2012, following the cuts in the key ECB policy rates and the implementation of further non-standard monetary policy measures (namely the broadening of the Eurosystem collateral framework and the two three-year longer-term refinancing operations). These policy measures have contributed to lower lending rates across the euro area, which is also visible in Fig. 7 (Panel B). At the same time, sovereign debt tensions (as was also highlighted in the previous empirical analysis) also help explain country heterogeneity in the cost of lending to NFCs, to the extent that they translate into bank funding and balance sheet vulnerabilities.

Since the beginning of the European Monetary System (EMS) and the establishment of the Exchange Rate Mechanism (ERM) it is of popular view that the system operated in an asymmetric manner, with Germany being the center country and the remaining member countries bearing the burden of adjustment (see Baum and Barkoulas 2006). According to Baum and Barkoulas (2006) this view of an asymmetric system with the German central bank conducting monetary policy independently is referred to in the literature as the "German Dominance Hypothesis" (GDH). A direct implication of the GDH is that the interest rates of other EMS member countries are cointegrated with the German interest rate, with the latter playing the leading role.

A small number of studies has examined the linkages between nominal interest rates across countries. For instance, Karfakis and Moschos (1990), Katsimbris and Miller (1993), Hassapis et al. (1999), and Caporale et al. (1996), among others, do not find evidence of cointegration in the short-term interest rates of EMS countries with the German interest rate. The absence of a common trend in the bivariate systems of EMS and German interest rates refutes the monetary-policy objectives of the EMS, and suggests the absence of convergence of European monetary policies. However, these studies consider only integer orders of integration of the variables.

This section considers as in Baum and Barkoulas (2006) the possibility of fractional, long-memory, (co)integrating relationships between these time series. We analyze interest rate linkages between the eleven countries analyzed in the previous section. We provide evidence that the interest rate differentials relative to Germany are persistent, but display mean-reverting behavior with long-memory features.

Figure 8 summarizes the test results. The colored cells indicate rejection of the null hypothesis of no cointegration. Comparing the results for both periods we observe an increase in the number of rejections in the period between 1999:01 and 2020:10, indicating a potential increase in the alignment of interest rates across countries. To better understand these rejections we analyze two cases: The first relates to the long-run equilibrium between Germany (GER) and the other countries; and the second investigates the long-run equilibrium between periphery countries, such as Greece, Italy, Portugal and Spain, and the other countries.

When we apply the test statistics introduced in this paper to the full sample, essentially rejections are obtained with the rolling statistics and the backward recursive statistics. This result is in accordance with a change from no cointegration to cointegration.

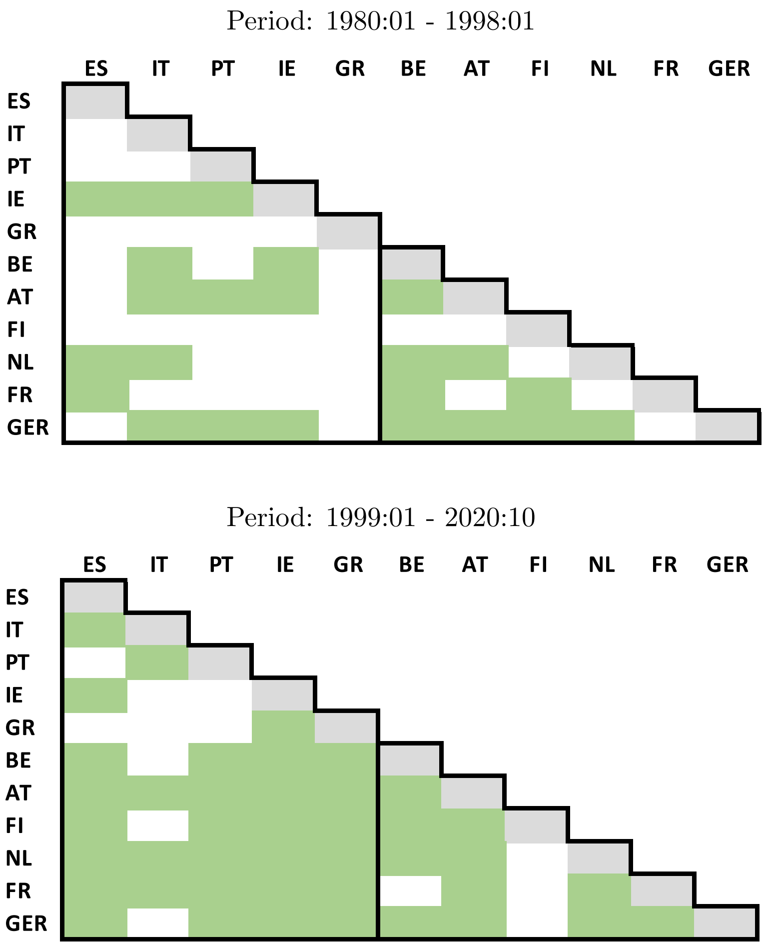

Fig. 8 Significant long-run relations-sub-periods

Recall that tests based on the forward and reverse recursive sequences of subsample statistics are designed to detect cointegration relationships (near) the beginning and the end of sample, respectively. Additionally, the reverse recursive-based tests could in addition be employed in an on-going monitoring exercise for the emergence of cointegration. On the other hand, tests based on a rolling sequence of statistics are designed to pick up a window of cointegration, of (roughly) the same length as the rolling window used, within the data.

Interestingly, if we plot the backward test statistics, these will be informative as to where the equilibrium is found between two countries. Figure 9 plots the backward recursive test statistics considering Germany as the benchmark country.

We observe, for instance, evidence of cointegration at the time of the introduction of the Euro for ES, FI, FR, IRE and the NL. This equilibrium between the interest rates of GER and these countries seems to have lasted longer for ES, FR and NL (until

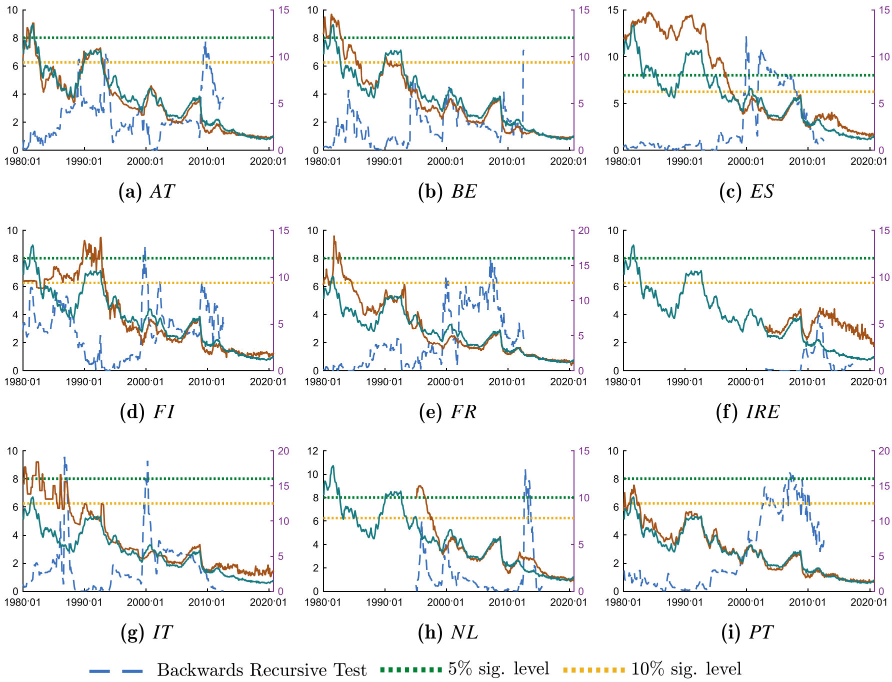

Fig. 9 Backward recursive test results for GERMANY (Blue Line) as reference country and other countries—Austria, Belgium, Spain, Finland, France, Ireland, Italy, Netherlands and Portugal—(Orange Line)

around the end of the financial crisis), whereas between GER and FI, and GER and IRE it seems to have been a short lived episode after the financial crisis.

Regarding the peripheral countries (Greece, Italy, Portugal and Spain) the recursive statistics plotted in Figures B.2–B.5 indicate that there is a small period of cointegration between the NFCs lending rates of the peripheral countries and the other countries considered. Specifically, from Figure B.2 we observe that there is evidence of cointegration from the forward recursive statistics around 2004 between the lending rates of GR and AT, BE, GER, ES, FI, FR, IT and NL and from the backward recursive statistics also at a later period (after the sovereign debt crisis) with AT, BE, GER and FI; from Figure B.3 we observe that the backward recursive statistics provide evidence of cointegration between the lending rates of IT and AT, BE, FI, NL and PT, after the European sovereign debt crisis, and similarly Figure B.4 provides evidence of cointegration between the lending rates of PT and AT, BE, ES, FI, FR, IT and NL, around the same period with the exception of ES, FI and NL. Finally, Figure B.5 shows that the lending rates in Spain cointegrate with AT and NL also around the end of the sovereign debt crisis and with FI and FR around or before the introduction of the Euro.

5 Conclusion

In this paper, we present tests for the null of no fractional cointegration against the alternative of segmented fractional cointegration. To do this, we develop new tests based on the procedure of Hassler and Breitung (2006) combined with ideas from Davidson and Monticini (2010). We introduce split sample, forward- and backward-running incremental sample and rolling sample tests for segmented cointegration. We show that the limit distribution of all of these statistics converge to the supremum of a Chi-squared distribution. Furthermore, a break point estimator based on minimizing the sum of squared residuals is also proposed.

A Monte Carlo study shows that our tests display adequate size and power properties in various situations. However, it turns out that the split sample test performs best in terms of power when the break occurs from the spurious to the fractionally cointegrated regime wherever the breakpoint is. On the other hand, if the break is from the fractionally cointegrated regime to the spurious regime, the rolling window test has the best power properties for all possible breakpoints. Therefore, we recommend application of both the split sample and the rolling window tests.

As segmented fractional cointegration is a very likely empirical situation, we investigated daily EMU government bonds between January 1999 and August 2017 and monthly lending rates (Total Loans) to non-financial corporations (NFCs) between January 1980 and October 2020.

Regarding the EMU government bonds, we find constant fractional cointegration between the bond yields of Austria, Finland, the Netherlands, France and Germany. For the other countries, namely Spain, Italy, Portugal, Greece, Ireland, and Belgium we find periods of segmented fractional cointegration before and after the sovereign debt crises. With respect to the lending rates to non-financial corporations, evidence seems to suggest a stronger interdependence after the introduction of the Euro (Fig. 8) and that there have been episodes of fractional cointegration between the peripheral countries and the other countries of the EMU.

A Technical Appendix

Before we prove the theorems define

$$\mathbf{e}'(\lambda_1, \lambda_2) := (e_{\lfloor \lambda_1 T \rfloor + 2}, \ldots, e_{\lfloor \lambda_2 T \rfloor})$$

and

$$\mathbf{e}^{*'}(\lambda_1, \lambda_2) := (e^*_{\lfloor \lambda_1 T \rfloor + 1}, \ldots, e^*_{\lfloor \lambda_2 T \rfloor}).$$

Proof of Theorem 1: From Lemma A in Hassler and Breitung (2006), we have directly:

$$\frac{1}{\lfloor\lambda_2 T\rfloor - \lfloor\lambda_1 T\rfloor}\mathbf{e}^{'}(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2) \xrightarrow{P} \sigma^2; \tag{A.1}$$

$$\frac{1}{(\lfloor\lambda_2 T\rfloor - \lfloor\lambda_1 T\rfloor)^{1/2}}\mathbf{e}^{'}(\lambda_1,\lambda_2)\mathbf{e}^{*}(\lambda_1,\lambda_2) \Rightarrow N\left(0;\sigma^4\frac{\pi^2}{6}\right); \tag{A.2}$$

$$\frac{1}{\lfloor\lambda_2 T\rfloor - \lfloor\lambda_1 T\rfloor}\mathbf{e}^{*'}(\lambda_1,\lambda_2)\mathbf{e}^{*}(\lambda_1,\lambda_2) \xrightarrow{P} \sigma^2\frac{\pi^2}{6}. \tag{A.3}$$

The rest of the proof follows exactly the lines of the proof of proposition 3 in Hassler and Breitung (2006) with the only difference that we localize their arguments to the interval $t = \lfloor\lambda_1 T\rfloor + 1, \ldots, \lfloor\lambda_2 T\rfloor$. For ease of readability we recall their arguments here.

Defining $\hat{e}_t(\lambda_1,\lambda_2) = e_t(\lambda_1,\lambda_2) - \mathbf{e}^{'}(\lambda_1,\lambda_2)\mathbf{V}_2(\lambda_1,\lambda_2)(\mathbf{V}_2^{'}(\lambda_1,\lambda_2)\mathbf{V}_2(\lambda_1,\lambda_2))^{-1}v_{2,t}(\lambda_1,\lambda_2)$ and $\hat{e}^{*}_{t-1}(\lambda_1,\lambda_2) = e^{*}_{t-1}(\lambda_1,\lambda_2) - \mathbf{e}^{'}(\lambda_1,\lambda_2)\mathbf{V}_2(\lambda_1,\lambda_2)(\mathbf{V}_2^{'}(\lambda_1,\lambda_2)\mathbf{V}_2(\lambda_1,\lambda_2))^{-1}v^{*}_{2,t-1}(\lambda_1,\lambda_2)$ we have

$$\begin{aligned}
\hat{\mathbf{e}}'(\lambda_1,\lambda_2)\hat{\mathbf{e}}(\lambda_1,\lambda_2) &= \mathbf{e}'(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2) - r_T'\mathbf{V}_2'(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2);\\
\hat{\mathbf{e}}^{*\prime}(\lambda_1,\lambda_2)\hat{\mathbf{e}}^{*}(\lambda_1,\lambda_2) &= \mathbf{e}^{*\prime}(\lambda_1,\lambda_2)\mathbf{e}^{*}(\lambda_1,\lambda_2) - 2r_T'\mathbf{V}_2^{*\prime}(\lambda_1,\lambda_2)\mathbf{e}^{*}(\lambda_1,\lambda_2)\\
&\quad + r_T'\mathbf{V}_2^{*\prime}(\lambda_1,\lambda_2)\mathbf{V}_2^{*}(\lambda_1,\lambda_2)r_T;\\
\hat{\mathbf{e}}^{*\prime}(\lambda_1,\lambda_2)\hat{\mathbf{e}}(\lambda_1,\lambda_2) &= \mathbf{e}^{*\prime}(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2) - r_T'\mathbf{V}_2^{*\prime}(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2)\\
&\quad - r_T'\mathbf{V}_2'(\lambda_1,\lambda_2)\mathbf{e}^{*}(\lambda_1,\lambda_2) + r_T'\mathbf{V}_2^{*\prime}(\lambda_1,\lambda_2)\mathbf{V}_2(\lambda_1,\lambda_2)r_T
\end{aligned}$$

with $r_T := (\mathbf{V}_2'(\lambda_1,\lambda_2)\mathbf{V}_2(\lambda_1,\lambda_2))^{-1}\mathbf{V}_2'(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2)$, $\mathbf{V}_2 := \left(\mathbf{V}_{2,2}', \ldots, \mathbf{V}_{2,T}'\right)$. By Assumption 2 and the iid assumption for v_t it holds that,

$$\begin{aligned}
\mathbf{V}_2'(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2) &= O_P(T^{1/2});\\
r_T &= O_P(T^{-1/2});\\
\mathbf{V}_2^{*\prime}\mathbf{e}^{*} &= O_P(T)
\end{aligned}$$

and

$$\begin{aligned}
\frac{1}{\lfloor\lambda_2 T\rfloor - \lfloor\lambda_1 T\rfloor}\mathbf{V}_2^{*\prime}(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2) &\to 0;\\
\frac{1}{\lfloor\lambda_2 T\rfloor - \lfloor\lambda_1 T\rfloor}\mathbf{V}_2'(\lambda_1,\lambda_2)\mathbf{e}^{*}(\lambda_1,\lambda_2) &\to 0.
\end{aligned}$$

From (A.1) we now have:

$$\begin{aligned}
&\frac{1}{\lfloor\lambda_2 T\rfloor - \lfloor\lambda_1 T\rfloor}\hat{\mathbf{e}}'(\lambda_1,\lambda_2)\hat{\mathbf{e}}(\lambda_1,\lambda_2)\\
&\quad = \frac{1}{\lfloor\lambda_2 T\rfloor - \lfloor\lambda_1 T\rfloor}\mathbf{e}'(\lambda_1,\lambda_2)\mathbf{e}(\lambda_1,\lambda_2) + o_P(1) \xrightarrow{P} \sigma^2;\\
&\quad \frac{1}{(\lfloor\lambda_2 T\rfloor - \lfloor\lambda_1 T\rfloor)^{1/2}}\hat{\mathbf{e}}'(\lambda_1,\lambda_2)\hat{\mathbf{e}}^{*}(\lambda_1,\lambda_2)
\end{aligned}$$

$$= \frac{1}{(\lfloor \lambda_2 T \rfloor - \lfloor \lambda_1 T \rfloor)^{1/2}} \mathbf{e}'(\lambda_1, \lambda_2)\mathbf{e}^*(\lambda_1, \lambda_2) + o_P(1) \Rightarrow N\left(0; \sigma^4 \frac{\pi^2}{6}\right);$$

$$\frac{1}{\lfloor \lambda_2 T \rfloor - \lfloor \lambda_1 T \rfloor} \hat{\mathbf{e}}^{*\prime}(\lambda_1, \lambda_2)\hat{\mathbf{e}}^*(\lambda_1, \lambda_2)$$

$$= \frac{1}{\lfloor \lambda_2 T \rfloor - \lfloor \lambda_1 T \rfloor} \mathbf{e}^{*\prime}(\lambda_1, \lambda_2)\mathbf{e}^*(\lambda_1, \lambda_2) + o_P(1) \xrightarrow{P} \sigma^2 \frac{\pi^2}{6}$$

which proves the theorem. □

Proof of Corollary 1: The proof of this corollary is identical to the proof of Corollary 2.2 in Martins and Rodrigues (2014) and is therefore omitted here. □

Proof of Theorem 2: The proof follows directly from the results in Theorem 1 and the arguments in Davidson and Monticini (2010). □

Proof of Theorem 3: Assume that the break is from non-cointegration to cointegration, i.e., the residuals are of integration order $-b$ after the break and of order 0 before the break.
Let us first consider $\tau < \tau_0$ and τ_0 is the break point. We have,

$$\tau^{-1} \lfloor \tau T \rfloor^{-1} \sum_{t=1}^{\lfloor \tau T \rfloor} \hat{e}_t^2(\tau) \Rightarrow \tau^{-1} var(\hat{e}_t),$$

where $var(\hat{e}_t)$ is fixed and finite. On the other hand for $\tau > \tau_0$ it is $\hat{\tau} = O_P(1)$. Thus, we have obtained that the limit function of $\hat{\tau}$ is given by $\tau^{-1} var(\hat{e}_t) 1_{\tau < \tau_0} + \infty 1_{\tau > \tau_0}$ which proves the theorem. □

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Declarations

Conflict of interest The authors have no conflict of interest.

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