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The determinants of political selection: a citizen-candidate model with valence signaling and incumbency advantage

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Abstract

We expand the theory of politician quality in electoral democracies with citizen candidates by supposing that performance while in office sends a signal to the voters about the politician's valence. Individuals live two periods and decide to become candidates when young, trading off against type-specific private wages. The valence signal increases the reelection chances of high valence incumbents (screening mechanism of reelection), and thus their expected gain from running for office (self-selection mechanism). Since self-selection improves the average quality of challengers, voters become more demanding when evaluating the incumbent's performance. This complementarity between the self-selection and the screening mechanisms may lead to multiple equilibria. We show that more difficult and/or less variable political jobs increase the politicians' quality. Conversely, societies with more wage inequality have lower quality polities. We also show that incumbency advantage blurs the screening mechanism by giving incumbents an upper-hand in electoral competition and may wipe out the positive effect of the screening mechanism on the quality of the polity.

Keywords Endogenous candidates · Political accountability · Incumbency advantage

JEL Classification C72 · D72

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1 Introduction

The quality of politicians matters a lot for the prosperity of countries and the well-being of the populations. Besley et al. (2011) show that the departure of a post-graduated leader leads to a 0.713 percentage point reduction in growth *per annum*, contrasting with just 0.05 percentage points when the leader is non educated. Using comprehensive data on Swedish municipal politicians, that includes pre-politics wages and measures of cognitive abilities, Dal Bó et al. (2017) show that they are, on average, *significantly smarter and better leaders* than the average citizen, which the authors contrast with models that *"suggest that the less able have a comparative advantage at entering public life due to free-riding and lower opportunity costs"*. In this paper, we provide a theory of the determinants of politician's quality that sheds light on why *smarter and better* individuals decide to enter the political market.

We use an overlapping generations citizen-candidate model with high and low valence individuals. Valence (Stokes, 1963) is a characteristic that all voters value, independently of ideology. We follow Bernhardt et al. (2011) and focus on valence dimensions which are signaled during the politician's tenure in office, as opposed to the ones that are observable before election, such as charisma and rhetorical skills.

Individuals live for two periods and make a candidacy decision at the beginning of their lives. All the candidates face the same probability of election. The incumbent performs a political job with a random outcome, whose realization is observed by the voters.[1] High-valence politicians deliver better political outcomes, on average, than low valence ones. Therefore, the political office conveys a valence signal to the voters, whose screening power depends on the its characteristics. The power of the valence signal increases with the job's *difficulty*, i.e., the difference in value delivery of high and low-valence politicians, and decreases with its *randomness*. Voters reelect the incumbent if the updated probability of her being high-valence is higher than the average valence of the young first-time candidates. Hence, high-valence individuals face higher reelection prospects than the low-valence ones, which increases their expected payoff from entering the political market.

We analyze the interaction of these two mechanisms: the *screening* mechanism of reelections and the *self-selection* mechanism of entry. When self-selection improves the average quality of political candidates, voters become more demanding when evaluating an incumbent's performance, since challengers are better on average. Therefore, the two mechanisms reinforce each other, a complementarity that may lead to multiple equilibria. We characterize the quality of the polity, that is, the equilibrium share of high-valence candidates in politics, as a function of the power of the valence signal, the ego-rents and type-specific private wages. We show that a more powerful valence signal increases the quality of the polity. We then introduce noise in the *screening effect* in the form of incumbency advantage, that gives incumbents

[1] For analysis that assume valence as an observable characteristic, see Ansolabehere and Snyder (2000); Aragones and Palfrey (2002); Groseclose (2001).

an *a priori* upper hand in electoral competition *vis-à-vis* the challenger.[2] This noise creates a feedback loop between the self-selection and screening mechanisms, which leads to multiple equilibria. Incumbency advantage, by blurring the screening mechanism, may wipe out the positive effect of screening on the quality of the polity and a bad politicians equilibrium may arise, in which the political market is populated by low-valence politicians that prefer the ego-rent to their outside option, i.e., the (low) market wage.

According to Besley (2007), political selection depends on the interplay of the type-specific pure motivation of holding office, private wages, and reelection and election probabilities. We focus on the first three, and assume away any information revelation of political campaigns. Therefore, our analysis complements (Caselli & Morelli, 2004), who focus on election probabilities and do not model retrospective voting. These authors highlight the comparative advantage of bad politicians, given their lower market wage, which is traded-off against a lower election probability, assuming that voters extract information from electoral campaigns. They show that if ego-rents depend positively on the quality of the polity, the economy gets stuck in a *bad politicians* equilibrium. Another mechanism for bad candidates is analyzed by Messner and Polborn (2004), who provide a citizen-candidate model with symmetric information about political skills, in which individuals bear a cost to perform the public service, which may lead good candidates to free-ride on the willingness of bad ones to fill the ruling job. The symmetric information setup fits an election in a small organization.

Mattozzi and Merlo (2008) also explain why good politicians may prevail and focus on both self-selection and screening through the reelection mechanism. However, their analysis differs from ours in several dimensions: political ability is perfectly revealed by political performance, politicians may opt out of standing for reelection after the first term in office, and citizens know their political ability but not the market wage.[3] In this case, politicians are always a positive selection of the population, thanks to the fully informative signal. However, the highest quality incumbents step out after one term, because political ability and the market wage are positively correlated, hence the first-period performance changes the incumbent's outside option in the private market. Conversely, the left-tail of first-term incumbents would like to be reelected but are ousted by the voters. The remaining ones stay in office for two terms. In contrast to Mattozzi and Merlo (2008)'s assumption of a fully revealing first term in office, we model the valence signal explicitly, which allows us to shed light on the determinants of its power and, therefore, the likelihood of a good candidates type of outcome.

Complementary analysis studying the role of political parties in mediating candidacy decisions are provided by Poutvaara and Takalo (2007); Carrillo and Mariotti (2001), and Mattozzi and Merlo (2015). There is also an extensive empirical

[2] See Erikson (1971), for an early discussion of the phenomenon. Recent causal evidence about incumbency advantages been obtained, e.g., by Lee (2008), and Lopes da Fonseca (2017).

[3] Bernhardt et al. (2011) also analyze a model in which valence is fully revealed by political performance; however, in their model, politicians differ both in ideology and valence.

literature about the determinants of politicians' quality. Two features of our model that have been shown empirically to matter for politicians' quality are the political wage (Ferraz & Finan, 2009; Gagliarducci & Nannicini, 2013; Kotakorpi & Poutvaara, 2011; Dal Bó et al., 2013; Fisman et al., 2015), and the outside option Gagliarducci et al. (2010); Fedele and Naticchioni (2016); Grossman and Hanlon (2014)),[4]

Our main contributions are as follows. First, we show that the quality of the polity is increasing in the power of the valence signal. Moreover, our explicit modeling of the valence signal allows us to pinpoint characteristics of the political office that make this positive result more likely to occur: a more difficult political job, in which the difference in the expected performance of high and low-valence politicians is high, and/or one with less variable performance. Second, very unequal societies (in the sense of the private market wages) are such that low-valence candidates outnumber high-valence ones, independently of the power of the valence signal. Third, when inequality is not high, there is always a threshold value of the power of the valence signal above which the polity contains a majority of high-valence candidates. Fourth, when the screening mechanism of reelections is blurred by a sufficiently high level of incumbency advantage, the equilibrium in which the screening mechanism plays no role (i.e., a bad candidates equilibrium) always exists. This is true even when incumbency advantage is intermediate, in which case this bad candidates equilibrium co-exists with others that generate better polities.

The remainder of the paper is organized as follows. In Sect. 2, we present the model; Sect. 3 discusses the determinants of the quality of the polity; Sect. 4 computes the expected life-time utility of a voter. Finally, Sect. 5 concludes.

2 The model

The economy is populated by κ individuals of each type, the high- and low-valence, denoted $i = L, H$. Individuals decide whether to run for office at the beginning of their lives; they live for two periods and do not discount the future. Valence is private information. The office consists of one task, whose quality depends on the valence of the office holder. More precisely, its quality is a normal random variable with variance σ^2 and expectation λ_i, $i = L, H$, with $\lambda_H \geq \lambda_L$, for high- and low-valence politicians. Therefore, valence changes the expected quality of a task, but not its variability. The impact of valence on the quality of the task depends on its nature. A simpler task does not suffer much from being undertaken by low-valence politicians—therefore, $(\lambda_H - \lambda_L)$ measures the task's *difficulty*.

In all the periods in which an individual is not serving as the elected politician—the first and second periods for those who either do not enter the political market, or enter, but are not elected, and the second period for non-reelected incumbents—she

[4] Other quality determinants analyzed in empirical contributions include political competition (Galasso & Nannicini, 2011) monitoring institutions (Grossman & Hanlon, 2014), electoral rules (Beath et al., 2016), and gender quotas (Baltrunaite et al., 2014).

earns a type-specific private market wage, $w_H \geq w_L$. The elected politician enjoys an ego-rent, or political wage, of $\mu > w_H$.

Individuals pay an idiosyncratic campaign cost between 0 and 1 to contest the political job, given by γ. We assume hereafter that the number of individuals with an entry cost below a given level γ can be approximated by the uniform distribution $F(\gamma) = \gamma$. This is clearly an approximation, as we have a discrete number of potential candidates.[5] The political campaign may convey some information about the political valence; however, it is reasonable to assume that one's record as a politician is a much better signal of one's valence than the campaign. We capture this feature by assuming that the campaign is uninformative about valence; hence, all non-incumbent candidates face an (endogenous) equal chance of winning the election, which we denote q. The entry process generates an endogenous proportion of high-valence candidates, denoted β.

Voters observe the outcome of the political job, x. Given the normality assumption, the probability that a politician with valence $i = L, H$ generates x is

$$p(x \mid \lambda_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\lambda_i)^2}{2\sigma^2}} \tag{1}$$

2.1 The reelection stage

Politicians are term-limited, and can only be re-elected once. We assume that there are unforeseen events that are not correlated with one's performance in office, e.g., a corruption or political scandal, that make the voters oust the incumbent, or lead to an early voluntary retirement from politics. This happens with probability α. In the remaining cases, at the end of the first period, the voters compute the posterior probability of a high-valence politician, given the observed performance, and the prior probability that the incumbent is high-valence, β_{t-1}. We denote this updated probability $p_t(i = H \mid x)$. The incumbent is re-elected if this probability is higher than the probability that the challenger is high-valence, β_t, according to (2).[6]

$$p_t(i = H \mid x) = \frac{p(x \mid \lambda_H)\beta_{t-1}}{p(x \mid \lambda_H)\beta_{t-1} + p(x \mid \lambda_L)(1 - \beta_{t-1})} \geq \beta_t \tag{2}$$

For $0 < \beta_t < 1$, the updated probability of facing a high-valence politician is greater than the prior if and only if

$$(1 - \beta_t)\beta_{t-1}p_t(x \mid \lambda_H) > (1 - \beta_{t-1})\beta_t p_t(x \mid \lambda_L)$$

[5] This is compatible with κ candidates having campaign costs equidistantly distributed on [0, 1], i.e., exactly one individual of each type $j = g, b$ at $\gamma = 0, 1/\kappa, 2/\kappa, \dots, 1$ and κ large enough.

[6] This reelection outcome is compatible with a weak majority of the old generation. Should the median voter be young, she could be interested in electing a newcomer with the same expected quality as the incumbent, because she comes with a (non negative) option value attached. Indeed, she may reveal being of the high type during her term.

which, after simplification, becomes

$$x > \frac{\lambda_H + \lambda_L}{2} + \frac{\sigma^2}{\lambda_H - \lambda_L} \ln\left(\frac{1-\beta_{t-1}}{\beta_{t-1}} \frac{\beta_t}{1-\beta_t}\right) \tag{3}$$

Let s denote the power of the valence signal conveyed by the political office, defined as follows:

$$s = \frac{1}{\sigma} \frac{\lambda_H - \lambda_L}{2} \tag{4}$$

The power of the signal increases with the tasks' difficulty $\lambda_H - \lambda_L$; conversely, it decreases with the randomness of task quality, σ.[7]

We obtain $P_H(s;\theta_t/\theta_{t-1})$This result shows that the as the probability of (3)

$$P_H(s;\theta_{t-1}/\theta_t) = 1 - \Phi\left(-s + \frac{\ln\left(\frac{1-\beta_{t-1}}{\beta_{t-1}} \frac{\beta_t}{1-\beta_t}\right)}{2s}\right) = \Phi\left(s - \frac{\ln\left(\frac{\theta_{t-1}}{\theta_t}\right)}{2s}\right), \tag{5}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standardized normal distribution and

$$\theta_t = \frac{1-\beta_t}{\beta_t}$$

is an odds ratio of low- to high-valence politicians who enter the market at time t. The higher the expected valence of the polity, the lower the θ_t. Using (3), the probability of reelection for a high-valence politician is given by

$$\rho_H(s;\theta_{t-1}/\theta_t) = (1-\alpha)P_H(s;\theta_{t-1}/\theta_t)$$

Analogously, the probability that a low-valence politician is re-elected is $\rho_L(s;\theta_{t-1}/\theta_t) = (1-\alpha)P_L(s;\theta_{t-1}/\theta_t)$, with

$$P_L(s;\theta_{t-1}/\theta_t) = \Phi\left(-s - \frac{\ln\left(\frac{\theta_{t-1}}{\theta_t}\right)}{2s}\right).$$

Therefore, the probability of reelection of an incumbent is decreasing with θ_{t-1}, and increasing with θ_t, i.e., when the ex-ante expected valence of the incumbent increases (resp., the ex-ante expected valence of the challenger decreases) reelection

[7] If the political job comprises several iid tasks, the variance of the valence signal decreases in the number of tasks. In such a natural extension of the model, increasing the number of tasks of the political job sharpens the valence signal and improves the screening mechanism.

probabilities are higher. The *screening effect* is embodied in the fact that high-valence politicians are more likely to be reelected, since $\lambda_H \geq \lambda_L$, i.e.

$$P_H(s;\theta_{t-1}/\theta_t) \geq P_L(s;\theta_{t-1}/\theta_t)$$

2.2 The entry decision

From the discussion above, it is immediate that the decision to enter into the political market in a given period $t-1$ depends on the expectation about the valence of the polity in the next period, summarized in the odds ratio θ_t.

The expected utility of running for office at $t-1$ for a candidate of valence $i = L, H$ is computed as follows. With probability q, the candidate is elected for a first period in office, and then re-elected for a second period with probability $\rho_i(s;\theta_{t-1}/\theta_t)$, amounting to an expected gain of $q(1+\rho_i(s;\theta_{t-1}/\theta_t))\,\mu$. The total expected payoff of running for office is thus $q(1+\rho_i(s;\theta_{t-1}/\theta_t))\mu + q(1-\rho_i(s;\theta_{t-1}/\theta_t))w_i + (1-q)2w_i - \gamma$. Not running pays off w_i in both periods. We compute the difference between the two, set it equal to 0, and solve for the type-specific cutoff entry cost $\hat{\gamma}_i(q, s;\theta_{t-1}/\theta_t)$, obtaining

$$\hat{\gamma}_i(s;\theta_{t-1}/\theta_t)) = q(1+\rho_i(s;\theta_{t-1}/\theta_t))(\mu - w_i),\ i = L, H \tag{6}$$

The cutoff entry cost is increasing in both the election and reelection probabilities, i.e., better election prospects lead individuals with higher costs to enter the political market. Therefore, when the ex-ante expected valence of the incumbent increases, so does the cutoff entry cost, and more individuals become politicians. The opposite is true for the expected valence of the challenger.

High-valence individuals face higher reelection prospects because $\rho_H(s;\theta_{t-1}/\theta_t) > \rho_L(s;\theta_{t-1}/\theta_t)$, hence increasing their cutoff campaign cost. Therefore, the *screening effect* of reelections reinforces the *self-selection effect* of endogenous entry. However, they have higher market wages, which decreases the cutoff campaign cost.

From (6), we obtain the expected number of political candidates in the market: $\kappa\hat{\gamma}_H(s;\theta_{t-1}/\theta_t)$ high-valence individuals, and $\kappa\hat{\gamma}_L(s;\theta_{t-1}/\theta_t)$ low-valence ones, respectively.

We now compute election probabilities. To do so, we enumerate the possible strategic situations that potential candidates face when running for office. In all periods after the initial one, there may be an incumbent who is ending her second term or an incumbent with a bad record—in both cases, the incumbent is not re-elected. Alternatively, there may be an incumbent with a good record who is up for reelection, and can be ousted with probability α. The unforeseen events that lead to this unexpected departure from office unfold after the potential entrants make their candidacy decisions. Therefore, the individuals incorporate the probability that the candidate is ousted into their entry decisions.

The entry decision varies slightly between the two cases. In the first one, the election probability is equal to the inverse of the number of candidates,

$$q_1(s;\theta_{t-1}/\theta_t) = \frac{1}{\kappa\left(\hat{\gamma}_H(s;\theta_{t-1}/\theta_t) + \hat{\gamma}_L(s;\theta_{t-1}/\theta_t)\right)}, \tag{7}$$

while in the second, the election is lost for sure with probability $1-\alpha$. With probability α, it is given by (7), hence the expected election probability is

$$q_2(s;\theta_{t-1}/\theta_t) = \frac{\alpha}{\kappa\left(\hat{\gamma}_H(s;\theta_{t-1}/\theta_t) + \hat{\gamma}_L(s;\theta_{t-1}/\theta_t)\right)}, \tag{8}$$

Using (6) in (7) and (8), we get

$$\begin{aligned} q_1^*(s;\theta_{t-1}/\theta_t) =& \kappa^{-1/2}\big[(1+\rho_H(s;\theta_{t-1}/\theta_t))(\mu - w_H) \\ &+ (1+\rho_L(s;\theta_{t-1}/\theta_t))(\mu - w_L)\big]^{-1/2} \end{aligned} \tag{9}$$

and

$$q_2^*(s;\theta_{t-1}/\theta_t) = \sqrt{\alpha}\, q_1^*(s;\theta_{t-1}/\theta_t)$$

The election probability is given by $q(s;\theta_{t-1}/\theta_t) = q_1^*(s;\theta_{t-1}/\theta_t)$, or $q(s;\theta_{t-1}/\theta_t) = q_2^*(s;\theta_{t-1}/\theta_t)$, for each of the two possible cases. It is smaller than one because the high and low-valence individuals with $\gamma = 0$ enter the market for sure. Moreover, it is readily obtained that it *(i)* decreases in κ, reflecting the natural effect of a bigger population on the number of candidates in the market, and *(ii)* depends on the power of the valence signal, s. This ultimately shapes the quality of the polity. We address these important topics in the next section.

2.3 The quality of the polity

Individual entry decisions determine the share of high-valence individuals in the political market—the *quality of the polity*—, and the number of politicians, a measure of political market thickness.

Given the threshold entry costs and the uniform assumption, the quality of the polity is

$$\beta_{t-1} = \frac{\hat{\gamma}_H(s;\theta_{t-1}/\theta_t)}{\hat{\gamma}_H(s;\theta_{t-1}/\theta_t) + \hat{\gamma}_L(s;\theta_{t-1}/\theta_t)} \tag{10}$$

Using (6) in (10), one readily obtains

$$\beta_{t-1} = \frac{1}{1 + \frac{1+\rho_L(s;\theta_{t-1}/\theta_t)}{1+\rho_H(s;\theta_{t-1}/\theta_t)}\frac{\mu - w_L}{\mu - w_H}} \tag{11}$$

Note that β_{t-1} depends on reelection probabilities and not on the election probability q. This is a natural consequence of the fact that, in our model, the campaign is non-informative, while reelection is contingent upon performance in office. This feature ensures that the quality of the polity resulting from entry at period $t-1$ is independent both of the updated valence of the period $t-1$ incumbent and of her tenure in office.

To better understand the effects that drive the results, note that (11) can be written as the solution to

$$\theta_{t-1} = \frac{\mu - w_L}{\mu - w_H} \frac{1 + \rho_L(s;\theta_{t-1}/\theta_t)}{1 + \rho_H(s;\theta_{t-1}/\theta_t)} \tag{12}$$

Expression (12) defines the dynamics of the entry game: it gives the value (or values) of θ_t, the expected odds-ratio of the challengers' valence, that can sustain a given θ_{t-1} in the current period, which is a consequence of entry decisions at $t-1$. It shows the interaction of the *self-selection effect*, that determines θ_{t-1}, and the *screening effect*, through the reelection probabilities on the right hand side.

Without screening, reelection prospects become type independent $\rho_L(s;\theta_{t-1}/\theta_t) = \rho_H(s;\theta_{t-1}/\theta_t)$ and self-selection is driven only by the relative disadvantage of the high type stemming from her outside option. The low-valence politicians would then outnumber the high-valence ones in the polity. Clearly, thanks to the screening effect, there will be a lower odds ratio of low to high-valence politicians (θ_{t-1}) than in the pure outside option case.

In this setup, an equilibrium is a sequence of θ_t, for all t, such that: *(i)* the expected θ_t determines θ_{t-1}, according to (12); and *(ii)* the same value of θ_t is realized tomorrow, given the expected θ_{t+1}. In order to characterize this equilibrium, we begin by showing that there exists an equilibrium in which the odds ratio is reproduced in all periods, that is, the equilibrium sequence is $\theta_t = \theta$, $\forall t$. We call it the stationary equilibrium. We discuss the possibility of a non-stationary equilibrium in the appendix.

3 Good politicians?

When the quality of the polity is the same across all periods, i.e., given by $\theta_{t-1} = \theta_t$, we readily obtain that $\rho_L(s;\theta_{t-1}/\theta_t) = (1-\alpha)\Phi(-s)$, and $\rho_H(s;\theta_{t-1}/\theta_t) = (1-\alpha)\Phi(s)$, therefore, both are independent from the odds ratio in both periods. In other words, when the quality of the polity is reproduced in all periods, it no longer influences reelection probabilities, since the positive impact (higher prior for the incumbent) is canceled out by the negative impact (higher valence of the challenger).

From (12), the equilibrium θ is given by

$$\theta = \frac{\mu - w_L}{\mu - w_H} \frac{1 + \rho_L(s;1)}{1 + \rho_H(s;1)}$$

The above equation shows that a higher value of s decreases the equilibrium θ, and therefore increases the quality of the polity. This is due to the impact of the power of the valence signal on reelection probabilities, which unambiguously increases the relative attractiveness of the political job for high valence individuals.

It is also straightforward to obtain that the total *number* of candidates decreases as the valence signal becomes stronger. Indeed, $\hat{\gamma}_H(s;1) + \hat{\gamma}_L(s;1)$ is decreasing in s.[8]

The intuition for these results is as follows. Better signaling increases the attractiveness of politics for high-valence individuals and decreases it for low-valence ones. This effect is discounted by outside option differences, and is thus amplified for low-valence candidates. Therefore, the market becomes thinner and the election probability increases. Since the re-election probability also increases for high-valence candidates, not surprisingly, more of them enter politics. Conversely, low-valence individuals face a higher election probability, and a lower re-election one. The combined effect leads to a lower number of low-valence individuals in the market.

We summarize these results in the following proposition.

Proposition 1 *When the power of the valence signal, s, increases, the number of low-valence candidates decreases, while that of high-valence ones increases. Overall, less citizens become candidates, and the election probability increases.*

Proof See Appendix.

This result shows that the characteristics of the political job that allow the voters to extract information about the politician's valence have a direct impact on the quality of the polity. From the definition of s in (1), more difficult (i.e., with higher $\lambda_H - \lambda_L$) and/or less random (i.e., with lower σ) political jobs attract more high-valence candidates. Interestingly, better polities are also smaller.

In order to better understand the result in Proposition 1, it is instructive to suppose that both types face the same outside option, i.e.,$w_L = w_H$ and let $\alpha \to 0$. If the valence signal is non-informative, both types are equally likely to be reelected, i.e., there is no *screening effect*. When the valence signal is perfectly informative, i.e., $s \to \infty$, low-valence politicians are never reelected, while high-valence ones are reelected for sure, i.e., the *screening effect* is fully discriminant. In the latter case, the equilibrium θ is equal to 1/2, while in the former it is equal to 1. Therefore, β, the equilibrium quality of the polity, varies between 1/2 and 2/3, increasing in the power of the valence signal, as one would expect. When one shuts down the outside option difference, there are on average more high than low-valence candidates because the valence signal effect favors the high type. This shows the importance of the screening effect, which explains the difference between our results and those of Caselli and Morelli (2004). However, there is always a positive share of low-valence candidates, due the distribution of campaign costs.

Extending the intuition above for the more general case of a positive value of α, and reintroducing the outside option differences, one readily obtains lower and upper bounds for the quality of the polity.

When screening is not operating ($s = 0$), the equilibrium $\tilde{\theta}$ is given by

[8] Just use the fact that $\frac{d\rho_H(s;1)}{ds} = -\frac{d\rho_L(s;1)}{ds}$ to obtain $\frac{d(\hat{\gamma}_H+\hat{\gamma}_L)}{ds} = \frac{d\rho_H(s;1)}{ds}(w_L - w_H) < 0$.

$$\tilde{\theta} = \frac{\mu - w_L}{\mu - w_H},$$

which we call the *bad* equilibrium hereafter.

Conversely, when screening is fully discriminant, ($s \to \infty$), the equilibrium is given by

$$\theta^* = \frac{\mu - w_L}{(2-\alpha)(\mu - w_H)}$$

In this equilibrium, screening is fully discriminant, therefore a high-valence incumbent is re-elected with probability $1 - \alpha$, while a low-valence incumbent is ousted for sure. In other words, this equilibrium perfectly selects good from bad incumbents and, not surprisingly, the quality of the polity is higher in this case than in the $\tilde{\theta}$ one. For this reason, we call it the *good* equilibrium.

These equilibria may equivalently be defined as

$$\tilde{\beta} = \frac{\mu - w_H}{\mu - w_L + \mu - w_H} \quad \text{and} \quad \beta^* = \frac{(2-\alpha)(\mu - w_H)}{\mu - w_L + (2-\alpha)(\mu - w_H)} \tag{13}$$

Clearly, there are values of the parameters for which even the *good* equilibrium contains more low than high-valence candidates. In fact, solving for $\theta^* = 1$ one may establish the following result. □

Proposition 2 *Suppose that* $w_H - w_L > (1-\alpha)(\mu - w_H)$. *Then, there are more low-valence than high-valence candidates, independently of the power of the valence signal. Therefore, unequal societies are more likely to generate low quality polities.*

This proposition shows that when the outside options are sufficiently different, the economy is stuck in Caselli and Morelli's *bad politicians* equilibrium. When $\alpha \to 0$, the inequality in Proposition 2 becomes

$$w_H - w_L > \mu - w_H,$$

i.e., the *bad candidates* equilibrium can only be avoided if the premium of joining the political market, for high-valence individuals, is higher than the skill premium in the private market.

However, there is a whole range of outside option values for which the signaling of the political office may improve upon this undesirable equilibrium. We tackle this in the next proposition.

Proposition 3 (Good Politicians) *Suppose that* $w_H - w_L \leq (1-\alpha)(\mu - w_H)$, *i.e., the society is not too unequal. Then, if the power of the valence signal,* s, *is high enough, there are more high-valence than low-valence candidates.*

This result may shed light on the empirical results in Dal Bó et al. (2017). The authors highlight that Sweden is a *quintessential advanced democracy* which has scored a perfect 10 in the −10 to 10 Polity-IV scale for a long period. In such a well-established democracy, it is likely that the non-financial rewards from holding office are high, making μ high enough that politicians are, on average, quite good.

Our analysis identifies the following drivers of the quality of the polity. Higher ego-rents, which may also be interpreted as the politician's salary, increase the average valence of the politicians. The same happens when the private wages of the low valence increases or that of the high-valence decreases. This implies that societies with lower inequality have better polities. The power of the valence signal, determined by the difficulty and randomness of the political job, also increases the quality of the polity.

Finally, note that the equilibria just discussed are all stable. Suppose that there is a mistake around the equilibrium value of θ in period $t-1$, i.e., the pool of entrants induces a deviation from the equilibrium quality of the stationary equilibrium. For the sake of the argument, suppose that θ is smaller, i.e., more high valence individuals enter the market. The voters will then be more likely to reelect the incumbent at period t, since this mistake increases the posterior probability that she is high valence. However, recall that the incumbent is ousted with an exogenous probability of α despite this higher posterior probability. This implies that, when the new generation decides entry at time t, the higher reelection probability of the incumbent has no impact on the entrants' decision, who still face a positive probability of election. More precisely, the quality θ_{t-1} has no impact on the expected payoff of a citizen candidate making her decision at time t, nor on the type-specific cutoff entry costs, as given by (6), which only depend on the expected valence of the following generation, i.e., the one who will decide whether to enter the market at $t+1$. This shows that mistakes around the equilibrium do not propagate to the future.

4 Incumbency advantage and the quality of the polity

Incumbency advantage has been shown to matter for electoral results (Lee, 2008; Lopes da Fonseca, 2017). The screening effect we consider has so far been purely bayesian, in the sense that the voters do not have any bias in their choice. As it will become clear, blurring the signal with incumbency advantage changes the screening effect and therefore has an impact on the quality of the polity.

4.1 Incumbency advantage: preliminary facts

Introducing incumbency advantage amounts to changing (2) such that the incumbent is not reelected only if $\beta_t - p_t(i = H \mid x) > \delta$, with $p_t(i = H \mid x)$ given by (2). Incumbency advantage introduces a wedge in the comparison between the challenger's expected valence and the updated probability that the incumbent is high valence. Incumbency advantage may stem from distortions in the political process, such as pork barrel, but it may also be due to voter bias in favor of political experience. Note that when $\beta_t \leq \delta$ the incumbent is always re-elected because the incumbency advantage outweighs the expected valence of the challenger who replaces the ousted incumbent. Similarly, when $\beta_{t-1} = 1$, the incumbent is always re-elected because there are only high-valence politicians in the market.

Introducing the incumbency advantage parameter, δ, in (5) yields the following type-specific reelection probability

$$P_H^{\mathcal{I}}(s;\beta_{t-1},\beta_t,\delta) = 1 - \Phi\left(-s + \frac{\ln\left(\frac{1-\beta_{t-1}}{\beta_{t-1}}\frac{\beta_t-\delta}{1-\beta_t+\delta}\right)}{2s}\right) \tag{14}$$

In order to focus on the comparison with the equilibrium without incumbency advantage, we focus on the stationary equilibrium, in which $\beta_t = \beta, \forall t$.[9] It is immediate from (14) that, in this case, the expression

$$\frac{1-\beta_{t-1}}{\beta_{t-1}}\frac{\beta_t-\delta}{1-\beta_t+\delta}$$

no longer boils down to 1. This is because incumbency advantage *de facto* introduces a wedge between the prior probability of being good for the incumbent and the challenger. Therefore, the odds ratios considered for the reelection probability, modified by this wedge, no longer cancel out, despite a common β for both generations.

When $\beta > \delta$, simplification of (14) yields the type-specific reelection probabilities, which are given by $\rho_i(s,\mathcal{I}) = (1-\alpha)P_i^{\mathcal{I}}(s,\mathcal{I})$, $i = L, H$, with

$$P_H^{\mathcal{I}}(s,\mathcal{I}) = \Phi\left[s + \frac{\mathcal{I}}{2s}\right], \text{ and, } P_L^{\mathcal{I}}(s,\mathcal{I}) = \Phi\left[-s + \frac{\mathcal{I}}{2s}\right], \tag{15}$$

where

$$\mathcal{I} = \ln\left(1 + \frac{\delta}{(1-\beta)(\beta-\delta)}\right)$$

is the noise introduced in the screening mechanism due to incumbency advantage. It is immediate that with $\delta = 0$, re-election probabilities are the same as in Sect. 2. Interestingly, high-valence politicians still face higher reelection prospects than low-valence ones, but the difference between the two is now lower, since incumbency advantage induces a relatively higher benefit for low-valence politicians.[10]

The term $\mathcal{I}$ is, not surprisingly, increasing in δ. The impact of β on $\mathcal{I}$ is non-monotonic, reaching a minimum when $\beta = \dfrac{1+\delta}{2}$. Recall that at the two limiting cases of $\beta = \delta$ (i.e., when the difference between the challenger's and the incumbent's reputation just cancels the reputation of the former) and $\beta = 1$ (only high-valence politicians in the polity), the incumbent is always reelected. Non-reelection

[9] Note from (14) that it is no longer possible to simplify the reelection probability as a function of θ, as was done in the previous section.

[10] We show in the Appendix that $\dfrac{dP_L(s,\mathcal{I})}{d\mathcal{I}} > \dfrac{dP_H(s,\mathcal{I})}{d\mathcal{I}} > 0.$

occurs for intermediate values of β, for which the challenger's reputation is above the incumbency advantage, and the voters are not sure as to whether the incumbent is high-valence. The reason for the non-monotonicity, as discussed above, is the wedge introduced by incumbency advantage between the incumbent's and the challenger's reputations.

Before proceeding, we take a closer look at the interaction of self-selection and screening. Better screening increases the power of the valence signal, which improves the quality of the polity via self-selection. The quality of the candidates changes the pool from which nature draws a replacer for an ousted incumbent. This increases the standard against which voters evaluate the incumbent. Since incumbency advantage is *de facto* a wedge between the challenger and the incumbent's probability of being of the high type, screening works better when self-selection is improved. The introduction of incumbency advantage creates room for this feedback effect, and leads to multiple equilibria, more likely to arise when the screening effect, measured by s, is sufficiently informative. As for the incumbency advantage, δ, as it will become clear, it cannot be too strong, nor too weak.

4.2 Equilibria with incumbency advantage

The equilibrium quality of the polity, $\hat{\beta}(\delta)$, solves a similar Equation to (12), with $F(s)$ augmented to $M(s, \beta, \delta)$, i.e.,

$$G(\beta) \equiv \frac{1-\beta}{\beta}\frac{\mu - w_H}{\mu - w_L} = \frac{1+\rho_L(s,\mathcal{I})}{1+\rho_H(s,\mathcal{I})} \equiv M(s,\beta,\delta) \tag{16}$$

As in (12) above, (16) includes the *screening effect* effect, captured by $M(\cdot)$, scaled down by the outside option difference, given by $G(\beta)$. The function $M(\cdot)$, is bounded above by 1, since the low type is never more likely to be reelected than the high type. It is also clear from (16) that, as expected, incumbency advantage blurs the screening effect. The two functions are depicted in Fig. 1 for the case of three equilibria, with $\tilde{\beta}$ and β^* defined by (13).

In fact, we are sure that at least one equilibrium exists; uniqueness, in turn, arises under special circumstances. We readily obtain that $G(\beta)$ is decreasing and convex.[11] As regards $M(\cdot)$, recall that when the quality of the polity is smaller than the incumbency advantage, the incumbent is re-elected for sure, independently of her valence type. Therefore, for $\beta \leq \delta$, both types face the same reelection probabilities. Moreover, the power of the valence signal inherits the non-monotonicity of $\mathcal{I}$, i.e., it is u-shaped in β, decreasing when $\beta < (\delta + 1)/2$, and increasing otherwise.[12]

We turn to the analysis of the two limit cases of $s = 0$ and $s \to \infty$, represented in Fig. 2. Recall from Sect. 2.3 that, without incumbency advantage, a non-informative

[11] With $G(0) \to \infty$, and $G(1) = 0$.

[12] Existence is easily established: $M(s, 0, \alpha) = 1 < G(0)$. Moreover, $M(s, 1, \alpha) = 1 > G(0)$. Therefore, by continuity, the two functions cross at least once and an equilibrium exists.

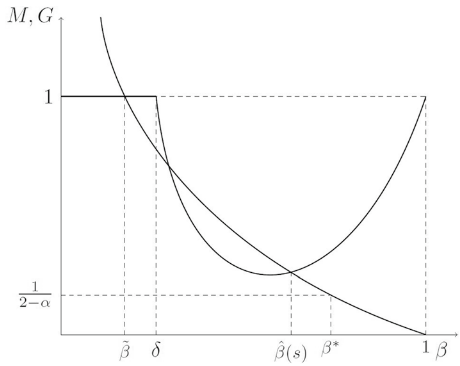

Fig. 1 Possibility of multiple equilibria with incumbency advantage

signal leads to the *bad* equilibrium, whereas the perfectly informative signal generates the *good* equilibrium, both given by (13).

Since incumbency advantage blurs the valence signal, it is not surprising that it does not change the nature of the equilibrium in its absence, i.e., when $s = 0$. The equilibrium in this case is the same as without incumbency advantage, and is depicted in Fig. 2.

When the valence signal is powerful, incumbency advantage has a non-trivial impact on the nature of the game: not only does it create room for multiple equilibria, as it allows for the possibility that the bad equilibrium arises, and even that the good one disappears. In order to grasp why this is the case, let us think about the shape of the ratio of reelection probabilities. If the incumbency advantage δ is higher than the expected valence of the challenger β, the incumbent is reelected independently of her type, just as when $s = 0$, and the ratio is equal to 1. When there are only high valence politicians in the market, i.e., $\beta = 1$, reelection also occurs independently of the incumbent's type, and again $F(\cdot) = 1$. For all other values of β, high valence individuals are reelected with a probability of $1 - \alpha$ and low valence individuals are not reelected; therefore, the ratio is equal $(2 - \alpha)^{-1}$, just as in Sect. 2.3. Therefore, the valence reaches its maximum power only when the quality of the polity is in the interval $\delta < \beta < 1$. This is depicted in Fig. 2.

As Fig. 2 shows, when incumbency advantage is not too strong (left panel), it plays no role, and everything works as in Sect. 2.3, i.e., the unique equilibrium of the game is the *good* one, β^*. Conversely, when incumbency advantage is very strong (right panel), it completely eliminates the screening effect, and the equilibrium is driven by outside option differences, as when $s = 0$, i.e., the unique equilibrium is the *bad* one, $\tilde{\beta}$. For intermediate values of the incumbency advantage (central panel), the two equilibria co-exist.

Based on these preliminary facts, we establish the following propositions.

Proposition 4 *If the political task is not informative at all, i.e.,* $s = 0$, *the unique equilibrium is the bad one.*

Unsurprisingly, without political signaling, the *bad candidates* result prevails.

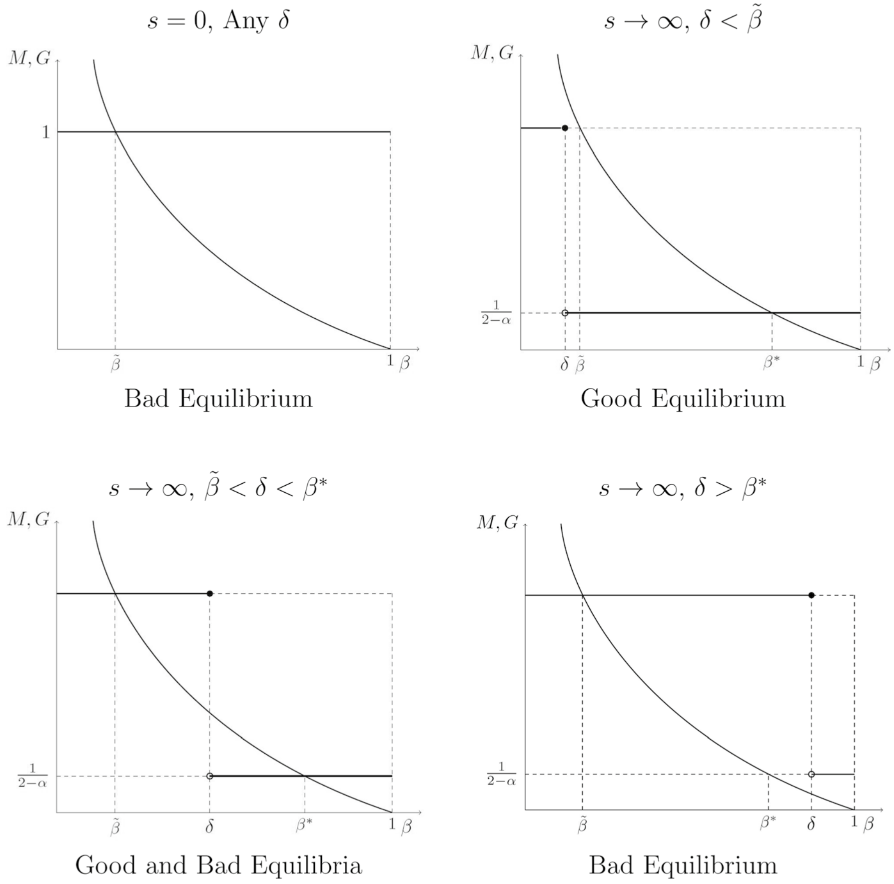

Fig. 2 Equilibria with non- and fully-informative valence signal

Proposition 5 *If the political task is perfectly informative of the incumbent's valence type, i.e., $s \to \infty$, then,*

(i) *if $\delta > \beta^*$, the unique equilibrium is the bad one*;
(ii) *if $\tilde{\beta} \leq \delta \leq \beta^*$, both the bad and the good equilibria co-exist*;
(iii) *if $\delta < \tilde{\beta}$, the unique equilibrium is the good one.*

This proposition shows that when incumbency advantage is sufficiently low, it does not play any role in the definition of the equilibrium of the game with a perfectly informative signal. By contrast, when it is very strong, it completely eliminates the advantage of the valence signal, and the equilibrium boils down to the case of the non-informative game. For intermediate values of the incumbency advantage, the game has multiple equilibria, and the polity may have the highest or the lowest possible value.

We thus have fully characterized the equilibria in the limiting cases of a perfectly informative or not informative at all valence signal. The results suggest that the combined effect of a sufficiently powerful signal with intermediate values of the incumbency advantage create room for multiplicity of equilibria. We provide a formal result in what follows.

Proposition 6 *When $\tilde{\beta} < \delta \leq \beta^*$, there always exists $\tilde{s}$ such that there are multiple equilibria iff $s > \tilde{s}$. All equilibria $\hat{\beta}(s)$ are such that $\hat{\beta}(s) < \beta^*$.*

Proof See Appendix.

As expected, multiple equilibria arise under two conditions. The first is that the incumbency advantage cannot be too low or too high. Indeed, under a very high incumbency advantage, incumbents are always re-elected and the *bad* equilibrium $\tilde{\beta}$, driven by the outside option difference, prevails. When the incumbency advantage is low, the feedback loop between screening and self-selection is not strong enough to allow for multiplicity of equilibria. In this case, there is an equilibrium $\hat{\beta}$ which lies between the bad, $\tilde{\beta}$, and the good one, β^*. However, for intermediate values of the incumbency advantage, multiple equilibria may arise, if the valence signal is sufficiently powerful. Besides the *bad* equilibrium $\tilde{\beta}$, there is another equilibrium with a better quality of the polity. Indeed when s is strong, the screening mechanism is good enough that it pays to have a good pool of politicians via self-selection. In this case, the two mechanisms reinforce each other and there are several equilibria. This corresponds to the case depicted in Fig. 1.

5 Conclusion

We use a citizen-candidate model to shed light on why smarter and better individuals may decide to enter the political market. The performance of elected politicians, while in office, is used by the voters on their reelection decisions. This generates a *screening effect* which increases the reward of running for office for high-valence individuals. Therefore, the screening effect of reelection is complementary to the self-selection one of entry into the political market. Improved selection makes the voters more demanding in their reelection decision, because they understand that the average quality of the challengers is higher. At the same time, improved screening makes the political market more attractive for high-valence individuals, thus improving self-selection. This complementarity between the self-selection and the screening mechanisms may lead to multiple equilibria.

Our explicit model of random political performance allows us to characterize the features of the political office that improve the screening mechanism (or the power of the valence signal), namely, the job's difficulty–i.e., the difference between the expected delivery by a high- and a low-valence incumbent–and its randomness. More difficult and less random jobs improve the voters' ability to oust low-valence incumbents and therefore make it more likely that the polity is a positive selection of the citizens. When the valence signal is sufficiently powerful, we obtain a *good candidates* equilibrium, in which high-valence individuals outnumber the low-valence ones. We study how incumbency advantage blurs the screening mechanism by giving incumbents an upper-hand in electoral competition. This may wipe out the positive effect of the screening mechanism on the quality of the polity.

An important implication of this result is that giving more responsibility to the politicians in a way that makes the outcomes of their actions more dependent on their valence can increase the quality of the polity. This can be somehow counter-intuitive if the society is stuck in a bad candidates equilibrium, in which a more static reasoning would advice against tasking them with difficult political jobs.

However, if incumbency advantage is strong enough, it completely wipes out the signaling mechanism, i.e., the quality of the polity is fully determined by the difference in the outside options, which favors low-valence candidates. More worrisome is the fact that this same equilibrium co-exists with better ones when the incumbency advantage is only intermediate. This shows that political institutions (for instance, lack of press independence) that blur the valence signal are potentially bad for accountability.

Appendix

Dynamics of the equilibrium

We now briefly discuss why there is no sequence of θ_t that solves the model and changes for every t. In what follows, let $r_t = \theta_{t-1}/\theta_t$; using (12) for θ_{t-1} and θ_t and computing the ratio, we may readily obtain that a sequence of equilibrium values must respect the following necessary condition:

$$r_t - \frac{1+\rho_L(s;r_t)}{1+\rho_H(s;r_t)} \frac{1+\rho_H(s;r_{t+1})}{1+\rho_L(s;r_{t+1})} = 0 \tag{17}$$

The recursive Eq. in (17) gives the locus of the pairs (r_t, r_{t+1}) such that the expected r_{t+1} can sustain entry in period t that generates r_t.

For further reference, we use the implicit function theorem on (17) to establish that (after some cumbersome algebra):

$$\frac{dr_{t+1}}{dr_t} = -\frac{1-\frac{d\frac{1+\rho_L(s;r_t)}{1+\rho_H(s;r_t)}}{dr_t}\frac{1+\rho_H(s;r_{t+1})}{1+\rho_L(s;r_{t+1})}}{-\frac{1+\rho_L(s;r_t)}{1+\rho_H(s;r_t)}\frac{d\frac{1+\rho_H(s;r_{t+1})}{1+\rho_L(s;r_{t+1})}}{dr_{t+1}}}$$

$$= \frac{1-\frac{1-\alpha}{sr_t}\frac{\phi_H^t}{1+\rho_H(s;r_t)}\left(\frac{1+\rho_L(s;r_t)}{1+\rho_H(s;r_t)}-\frac{\phi_L^t}{\phi_H^t}\right)\frac{1+\rho_H(s;r_{t+1})}{1+\rho_L(s;r_{t+1})}}{\frac{1-\alpha}{sr_{t+1}}\frac{\phi_L^{t+1}}{1+\rho_L(s;r_{t+1})}\left(\frac{1+\rho_H(s;r_{t+1})}{1+\rho_L(s;r_{t+1})}-\frac{\phi_H^{t+1}}{\phi_L^{t+1}}\right)\frac{1+\rho_L(s;r_t)}{1+\rho_H(s;r_t)}}, \tag{18}$$

with $\phi_L^t = \phi\left(-s - \frac{\ln(r_t)}{2s}\right)$ and $\phi_H^t = \phi\left(s - \frac{\ln(r_t)}{2s}\right)$.

A few facts are apparent from (17).

(i) $r_t = r_{t+1} = 1$ is a solution. This is the stationary equilibrium discussed in Sect. 3. At this equilibrium, and using the fact $\phi_L^t = \phi_H^t = \phi_L^{t+1} = \phi_H^{t+1} = \phi_L$, (18) boils down to

$$\left.\frac{dr_{t+1}}{dr_t}\right|_{r_t=1} = 1 + s\frac{(1+\rho_H(s;1))(1+\rho_L(s;1))}{(1-\alpha)\phi_L\left[\rho_H(s;1)-\rho_L(s;1)\right]} > 1$$

(ii) It is straightforward to obtain that (17) has no solution for r_t in the vicinity of $r_t = 0$, since $\frac{1+\rho_L(s;0)}{1+\rho_H(s;0)} = 1$, and $\frac{1+\rho_H(s;r_{t+1})}{1+\rho_L(s;r_{t+1})} \geq 1,\ \forall r_{t+1}$.

Since (17) is a continuous function of (r_t, r_{t+1}), and it has at least one root $r_t = r_{t+1} = 1$, then there must be a minimum value of r_t for which it has a solution. We denote $\underline{r}_t \leq 1$ that value which is implicitly given by

$$r = \frac{1+\rho_L(s;r)}{1+\rho_H(s;r)}$$

which is the value of r_t that solves (17) when $\frac{1+\rho_H(s;r_{t+1})}{1+\rho_L(s;r_{t+1})}$ is minimum (i.e., equal to 1). Therefore, $r_t = \underline{r}$ can be sustained by two values of r_{t+1}, namely, $r_{t+1} = 0$ and $r_{t+1} \to \infty$.

(iii) It is also straightforward to obtain that (17) has no solution when $r_t \to \infty$, since $\lim_{r_t\to\infty}\frac{1+\rho_L(s;r_t)}{1+\rho_H(s;r_t)} = 1$, and, as shown below in point (v), $\frac{1+\rho_H(s;r_{t+1})}{1+\rho_L(s;r_{t+1})} \leq \tilde{r}^2,\ \forall r_{t+1}$, i.e. it has a finite maximum. An analogous continuity argument establishes that there exists $\bar{r}_t \geq 1$ above which (17) has no solution, implicitly given by

$$\bar{r} = \frac{1+\rho_L(s;\bar{r})}{1+\rho_H(s;\bar{r})}\frac{1+\rho_H(s;\tilde{r})}{1+\rho_L(s;\tilde{r})}$$

Therefore, $r_t = \bar{r}$ can be sustained by a single value $r_{t+1} = \tilde{r}$.

(iv) When $r_{t+1} = 0$ or $r_{t+1} \to \infty$, $\frac{1+\rho_H(s;r_{t+1})}{1+\rho_L(s;r_{t+1})} = 1$, since $\rho_H(s;r_{t+1}) = \rho_L(s;r_{t+1})$ in both cases. Recalling that $\rho_H(s;r_{t+1}) \geq \rho_L(s;r_{t+1})$, this defines the minima of the ratio, which is therefore bell shaped.

(v) Moreover, we can derive $\frac{1+\rho_H(s;r_{t+1})}{1+\rho_L(s;r_{t+1})}$ and use a bit of algebra, together with the properties of the density of the normal distribution, to obtain the following equation, which implicitly defines the maximand of this ratio, $\tilde{r}$

$$\tilde{r} = \sqrt{\frac{1+\rho_H(s;\tilde{r})}{1+\rho_L(s;\tilde{r})}} > 1$$

Therefore, for all $r_t \in [r_t, \bar{r}_t[$ (17) has two roots in r_{t+1}, one below, and the other above $\tilde{r}$.

Moreover, when $r_t < 1$ (resp. $r_t > 1$), one of the roots in r_{t+1} is smaller (resp., larger) than r_t. This is immediate from (17), since when $r_{t+1} > r_t$ (resp., smaller), we have $\dfrac{1+\rho_L(s;r_t)}{1+\rho_H(s;r_t)} \dfrac{1+\rho_H(s;r_{t+1})}{1+\rho_L(s;r_{t+1})} > 1$ (resp., smaller than 1), hence it cannot be equal to $r_t < 1$ (resp., $r_t > 1$).

Figure 3 plots the correspondence defined by (17). It shows two sequences of r_t, one below 1 and the other above 1, which diverge into values of r_t outside the feasibility range. Moreover, note that, as $\tilde{r} > \bar{r}$, the roots in the upper branch of the correspondence diverge in just one period to values outside the feasible range.

Proofs of propositions

Proof of proposition 1

Differentiating (9) when $\theta_t = \theta_{t+1} = \theta$,

$$\begin{aligned} \frac{dq_1^*}{ds} =& -\frac{1}{2}\left[\left(1+\rho_H(s;1)(\mu - w_H) + (1+\rho_L(s;1)(\mu - w_L)\right)\right]^{-3/2} \\ & \kappa(w_H - w_L)\frac{d\rho_L(s;1)}{ds} \\ =& -\frac{1}{2}(w_H - w_L)\frac{d\rho_L(s;1)}{ds}\frac{q_1^{*3}}{\kappa^2} > 0, \end{aligned} \tag{19}$$

since $d\rho_L(s;1/ds = -d\rho_H(s;1/ds < 0$.

Differentiating (6) and using (19), we get

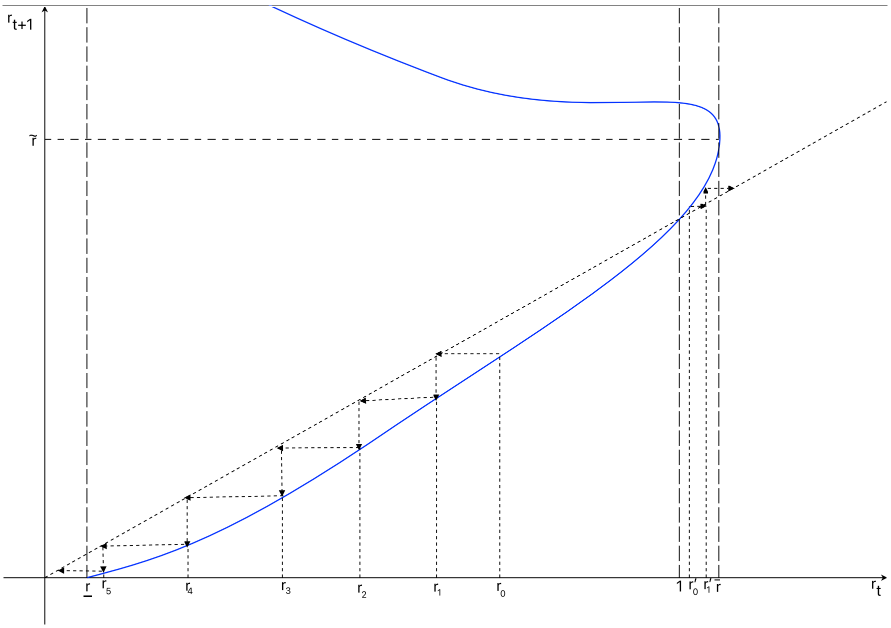

Fig. 3 The dynamics of entry

$$\begin{aligned}\frac{d\gamma_L}{ds} &= q_1^*(\mu - w_L)\frac{d\rho_L(s;\theta_{t-1}/\theta_t)}{ds}\left(1 - \frac{1}{2}q_1^{*^2}\kappa(w_H - w_L)(1 + \rho_L(s;1)\right)\\ &= q_1^*(\mu - w_L)\frac{d\rho_L(s;1}{ds}\left(1 - \frac{1}{2}\frac{(w_H - w_L)(1 + \rho_L(s;1)}{(1 + \rho_H(s;1)(\mu - w_H) + (1 + \rho_L(s;1)(\mu - w_L)}\right) < 0,\end{aligned}$$

where we use the fact that $\mu - w_L \geq \mu - w_H$ and $\frac{d\rho_L(s;1)}{ds} < 0$.

To see that $\frac{d\gamma_H}{ds} > 0$, it is enough to note that both q_1^* and $\rho_H(s;1)$ are increasing in s.

The computations for $q_2^* = \sqrt{\alpha}q_1^*$ are analogous, up to the multiplicative constant $\sqrt{\alpha}$.□

Proof of proposition 5

Behavior of the $M(s, \beta, \delta)$ **function** We begin by establishing important facts about $M(s, \beta, \delta)$.

- Derivative with respect to δ

  Straightforward derivation using (16) yields

$$\frac{\partial M(s,\beta,\delta)}{\partial \delta} = \frac{\frac{\partial \Phi\left(-s+\frac{\mathcal{I}}{2s}\right)}{\partial \mathcal{I}}\left(1+\rho_H\right) - \frac{\partial \Phi\left(s+\frac{\mathcal{I}}{2s}\right)}{\partial \mathcal{I}}\left(1+\rho_L\right)}{(1+\rho_H)^2}\frac{\partial \mathcal{I}}{\partial \delta}(1-\alpha)$$

$$= \frac{1}{2s}\frac{\phi\left(-s+\frac{\mathcal{I}}{2s}\right)\left(1+\rho_H\right) - \phi\left(s+\frac{\mathcal{I}}{2s}\right)\left(1+\rho_L\right)}{(1+\rho_H)^2}\frac{\partial \mathcal{I}}{\partial \delta}(1-\alpha) > 0,$$

where the inequality follows from the fact that $\rho_H \geq \rho_L$, and $\phi\left(s+\frac{\mathcal{I}}{2s}\right) < \phi\left(-s+\frac{\mathcal{I}}{2s}\right)$. The last inequality holds, as when $-s+\frac{\mathcal{I}}{2s} < 0$,

$$\phi\left(s+\frac{\mathcal{I}}{2s}\right) < \phi(s) = \phi(-s) < \phi\left(-s+\frac{\mathcal{I}}{2s}\right);$$

when $-s+\frac{\mathcal{I}}{2s} > 0$, $\phi\left(s+\frac{\mathcal{I}}{2s}\right) < \phi\left(-s+\frac{\mathcal{I}}{2s}\right)$ holds as $\phi(z)$ is decreasing when $z > 0$ and $s+\frac{\mathcal{I}}{2s} > -s+\frac{\mathcal{I}}{2s} > 0$.

- It follows that $\frac{\partial M}{\partial \delta}$ has the sign of $\frac{d\mathcal{I}}{d\delta} > 0$.
- Derivative with respect to β
  A similar reasoning holds for $\frac{\partial M}{\partial \beta}$, i.e., $\frac{\partial M}{\partial \beta} < 0$ if and only if $\beta < (\delta+1)/2$.
- Derivative with respect to s
  $M(s,\beta,\delta)$ is non-increasing in s. Indeed, when $\beta \leq \delta$, $M(s,\beta,\delta) = 1$, thus $\frac{\partial M(s,\beta,\delta)}{\partial s} = 0$. When $\beta > \delta$, straightforward derivation of (16) yields

$$\frac{\partial M(s,\beta,\delta)}{\partial s} = -(1-\alpha)\frac{1+\Phi\left(-s+\frac{\mathcal{I}}{2s}\right)(1-\alpha)}{1+\Phi\left(s+\frac{\mathcal{I}}{2s}\right)(1-\alpha)}\times$$

$$\left[\frac{\phi\left(-s+\frac{\mathcal{I}}{2s}\right)}{1+\Phi\left(-s+\frac{\mathcal{I}}{2s}\right)(1-\alpha)} + \frac{\phi\left(s+\frac{\mathcal{I}}{2s}\right)}{1+\Phi\left(s+\frac{\mathcal{I}}{2s}\right)(1-\alpha)}\right.$$

$$\left.+\frac{\mathcal{I}}{2s^2}\left(\frac{\phi\left(-s+\frac{\mathcal{I}}{2s}\right)}{1+\Phi\left(-s+\frac{\mathcal{I}}{2s}\right)(1-\alpha)} - \frac{\phi\left(s+\frac{\mathcal{I}}{2s}\right)}{1+\Phi\left(s+\frac{\mathcal{I}}{2s}\right)(1-\alpha)}\right)\right]$$

which is negative as $\dfrac{\phi\left(-s+\frac{\mathcal{I}}{2s}\right)}{1+\Phi\left(-s+\frac{\mathcal{I}}{2s}\right)(1-\alpha)} > \dfrac{\phi\left(s+\frac{\mathcal{I}}{2s}\right)}{1+\Phi\left(s+\frac{\mathcal{I}}{2s}\right)(1-\alpha)}$ when $\mathcal{I}\top 0$.

Proof of proposition 5 We use a series of claims to prove the Proposition.

Claim 1 *For all β and δ such that $0 < \delta < \beta < 1$, and for all $x \in]\frac{1}{2-\alpha}, 1[$, there exists $s(x)$ finite such that $M(s(x),\beta,\delta) = x$.*

Proof it directly follows from $\frac{\partial M}{\partial s} < 0$ and from the fact that for any β and δ such that $0 < \delta < \beta < 1$, we have that $\lim_{s \to 0} M(s, \beta, \delta) = 1$ and $\lim_{s \to \infty} M(s, \beta, \delta) = \frac{1}{2-\alpha}$. □

Claim 2 *The expression defined by (16) has at least one fixed point and, therefore, there is at least one equilibrium given by $\hat{\beta}$ such that $G(\hat{\beta}) = M(s, \hat{\beta}, \delta)$.*

Proof The function $M(s, \beta, \delta)$ is u-shaped in β, decreasing when $\beta < (\delta + 1)/2$, and increasing otherwise, with $M(s, \beta, \delta) = 1$, $\forall \beta \leq \delta$ and $M(s, 1, \delta) = 1$. Therefore, $M(s, 0, \delta) = 1 < G(0)$ and, $M(s, 1, \delta) = 1 > G(1)$ and, by continuity, the two functions cross at least once and an equilibrium exists. □

Recall from the discussion about the limit cases of $s = 0$ and $s \to \infty$ that $\tilde{\beta} : G(\tilde{\beta}) = 1$, is given by

$$\tilde{\beta} = \frac{\mu - w_H}{(\mu - w_H) + (\mu - w_L)}$$

Moreover, $\beta^* : G(\beta^*) = (2 - \alpha)^{-1}$ is given by

$$\beta^* = \frac{(2 - \alpha)(\mu - w_H)}{(2 - \alpha)(\mu - w_H) + (\mu - w_L)}$$

Claim 3 *Any equilibrium $\hat{\beta}$ of the game with finite s, that solves (16), is such that $\hat{\beta} < \beta^*$.*

Proof Note that $G(\beta^*) = (2 - \alpha)^{-1}$, and that, by Claim 1, $M(s, \beta, \delta) > (2 - \alpha)^{-1}$, $\forall \beta$. Moreover, for any $\beta \geq \beta^*$, we have that $M(s, \beta, \delta) > (2 - \alpha)^{-1} > G(\beta)$. Hence, there is no fixed point of (16) given by $\beta \geq \beta^*$, and the equilibrium must be such that $\hat{\beta} < \beta^*$. □

We now analyze separately the cases of $\delta < \tilde{\beta}$ and $\delta \geq \tilde{\beta}$.

Claim 4.1 *When $\delta < \tilde{\beta}$, the equilibrium $\hat{\beta}$ that solves (16) respects $\hat{\beta} > \delta$.*

Proof Take $\delta < \tilde{\beta}$, and $\beta < \delta$. Given that $G(\beta)$ is decreasing and $G(\tilde{\beta}) = 1$, it readily obtains that $G(\beta) > 1$. Moreover, from the fact that $\beta < \delta$, we have $M(s, \beta, \delta) = 1$. Therefore, there can be no equilibrium $\hat{\beta}$ such that $\hat{\beta} < \delta$. □

Claim 4.2 *When $\delta \geq \tilde{\beta}$, there is at least one equilibrium $\hat{\beta}$ that solves (16), with $\hat{\beta} = \tilde{\beta}$. Moreover, if $\delta < \beta^*$ and s is sufficiently large, there is at least one additional equilibrium, $\hat{\beta}'$.*

Proof Firstly, note that $M(s, \tilde{\beta}, \delta) = 1 = G(\tilde{\beta})$. Hence, $\tilde{\beta}$ is an equilibrium. If $\delta \geq \beta^*$, this equilibrium is unique. Indeed, we know from Claim 3 that any equilibrium respects $\hat{\beta} < \beta^*$, and $(2-\alpha)^{-1} < G(\beta) < M(s, \beta, \delta) = 1$ for any $\beta < \beta^* < \delta$.

Let us now analyze the case $\delta < \beta^*$. We will show that, under some conditions, there exists at least one second equilibrium $\hat{\beta}'$.

We know that for any $\beta \in]\delta, \beta^*[$, $G(\beta) \in \left]\frac{1}{2-\alpha}, 1\right[$ and from Claim 1, we have that there exists $\tilde{s}(\beta)$, such that $G(\beta) < M(s, \beta, \delta)$ if and only if $s < \tilde{s}(\beta)$.

Define $\tilde{s} = \min_\beta \tilde{s}(\beta)$. If $s < \tilde{s}$, then $G(\beta) < M(s, \beta, \delta)$for all $\beta > \delta$and therefore there can be no equilibrium $\hat{\beta}' \in]\delta, \beta^*]$. If $s > \tilde{s}$,we know that for $\beta_0 = \arg\min \tilde{s}(\beta)$, $G(\beta_0) > M(s, \beta_0, \delta)$. Moreover, since $G(1) = 0 < M(s, 1, \delta)$, we conclude, by continuity, that there is a $\hat{\beta}' > \beta_0$ such that $G(\hat{\beta}') = M(s, \hat{\beta}', \delta)$, which is a second equilibrium. □

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