Estimating a Portugal NAIRU

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1. Introduction

Interest in the Unemployment-inflation relationship has greatly risen during the last two decades. Gordon (2013) and Watson (2014) have found that cyclical unemployment is a leading indicator of inflation in the US. However, in the European Union, mainly policymakers have estimated structural unemployment and analysis of the concept itself has been scarce.

In this study, I have both estimated the NAIRU in the Portuguese economy and addressed the “flattening Phillips Curve”. In particular, I have included inflation targeting and taken into account the change in the labour market structure in Portugal. I find that the anchored inflation specification improves the traditionally used backward looking specification and that measures of unemployment including part-time and marginally attached workers outperform the official u3 unemployment rate.

2. Literature Review

2.1 NAIRU

The non-accelerating inflation rate of unemployment (NAIRU) is the rate of unemployment at which inflation stabilizes in the absence of any wage-price surprises. Friedman first argued in 1968 that inflation accelerates (decelerates) when actual unemployment falls below (rises above) a Natural Rate of Unemployment, but only on a temporary basis because the relationship is based not on inflation itself, but on an “unanticipated” inflation (Friedman, 1968). Once unemployment returns to the NRU, inflation will stabilize at a permanent higher (lower) level.

Since the NAIRU is unobservable, it needs to be estimated for policy analysis. Estimation methods can be divided into three categories: structural, statistical and
Reduced-form methods (Gianella et al., 2008).

- Structural methods: model the NAIRU as a function of labour and product market variables and involve estimating a system of equations explaining the wage setting and price setting behaviour (see, for example, Morrow & Roeger, 2000). This methodology identifies the specific determinants of structural unemployment. However, disagreements about the appropriate structural model, specification issues and statistical identification issues make the estimates too uncertain.

- Purely statistical methods: split unemployment into a trend component, identified as the NAIRU, and a cyclical component. The identification of the two components can be based either on filtering techniques, the most widely used being the HP filter (Hodrick Prescott, 1997) and the band pass filter (Baxter & King, 1995), or on statistical methods assuming that trend unemployment follows a random-walk (Watson, 1986). It is uncorrelated with inflation because unemployment is the only series used, around which the NAIRU moves, and the decomposition is based on arbitrary assumptions.

- Reduced-form approach: obtains the NAIRU by identifying the rate of unemployment consistent with stable inflation in a Phillips Curve equation. In addition, statistical constraints are imposed in the path of the NAIRU. It is the most accepted method due to the given unemployment-inflation relationship and the feasibility of the estimates across countries and periods.

The temporary trade off described by Friedman was captured by the “triangle model”, which measured the NAIRU in the reduced-form approach by identifying three factors affecting inflation in the Phillips Curve framework: expectations/inertia, demand
pressures and supply shocks (Gordon, 1997).

\[ \pi - \pi^e = A(L)(\pi_{t-1} - \pi^e_{t-1}) + \gamma(L)(U_t - U^*_t) + \delta(L)z_t + e_t, \quad (1) \]
\[ U^*_t = U^*_{t-1} + v_t, \quad (2) \]

where \( \pi \) is actual inflation, \( \pi^e \) is expected inflation, \( U_t \) is the actual rate of unemployment, \( U^*_t \) represents the NAIRU, \( z_t \) is a vector of supply shock variables, and \( e_t \) and \( v_t \) are serially uncorrelated error terms.

The model is a common benchmark to compute the NAIRU. However, the characteristics of the different factors may vary depending on the theoretical specification as reflected by different studies:

Expectations have traditionally been based on a backward-looking Phillips curve, where inertia is formed using recent inflation outcomes (see, for example, Fabiani & Mestre, 2004; Centeno et al., 2010). The assumption of adaptive expectations derives from the fact that inflation has followed a random walk during the last decades. However, the OECD has updated its model to incorporate the notion that expectations are anchored around the central bank inflation objective, which reduces the effect of changes in the unemployment gap on inflation (Clifton et al., 2001). In fact, the unemployment gap of most of the OECD countries incorporating anchored expectations has higher statistical significance (Rusticelli et al., 2015).

The NAIRU, determining demand pressures, follows a random walk. This implies a recognition of the hysteresis effect, which lies on the deterioration of human capital after prolonged periods of unemployment; reducing their influence on wage bargaining and, as a consequence, on unemployment (Blanchard & Summers, 1986 and Rusticelli & Guichard, 2011).
The measured NAIRU is key for policymakers, as acknowledged in the existing literature. Economists analyze inflationary developments, the sustainability of fiscal policy and the need for structural policy using the NAIRU as a benchmark of sustainable trends in output and employment.

2.2 Alternative measures of unemployment

Existing literature quantifying the Natural Rate of Unemployment has used the official measure of the unemployment rate, U-3. According to the OECD, “to be considered unemployed an individual must be during the reference period without a job, taking measures to get work during a specific time period (job search) and available to start work usually immediately”.

This measure has generally been used because it is an indicator of labour market slack that involves no value judgement by simply requiring an individual to be searching for a work and allows employment to measure production including all labour inputs. Apart from U-3 unemployment, labour market slack is composed of time-related unemployment and discouragement (OECD, Employment Outlook):

- Labour market slack: measures the insufficiency of the volume of work.
- Time-related unemployment: amount of workers willing and available to work more hours, as long as they are working less than a specified number of hours. They are structured in three groups: first, individuals who usually work full time but are working part-time because of economic slack; second, individuals who work part time but are working fewer hours because of economic slack; and those who are working part time because full time could not be found. While the two first groups are expected to be correlated with the economic cycle, the third group could arise due to either a structural or cyclical aggregate demand
Discouraged workers: subset of persons marginally attached to the labour force. The marginally attached are those persons who are available and willing to work, and who have looked for a job in the last 6 months, but have not been actively seeking for the last 4 weeks. As output expands, some of them will join the labour force; and as unemployment increases some of them will leave the labour force. This means that, during a recession, the labour resources not being utilized in the economy might be understated. Among the marginally attached, the discouraged workers have given a job-market related reason for not looking for a job.

Although U-3 unemployment has traditionally been a good proxy of labour market slack in Portugal, the onset of the crisis has varied the structure of the labour market. More specifically, “while conventional unemployment has almost doubled since 2008, the number of involuntary short-term workers has almost tripled and the number of discouraged workers has quadrupled since 2008” (IMF, 2013).
Then, as confirmed by the graph above, U-3 unemployment is no longer a good proxy for labour market slack, and other measures of unemployment should be analyzed. The alternative unemployment rates produced by the BLS are the following:

Table 1 – Alternative Measures of Labour Underutilization

<table>
<thead>
<tr>
<th>Measure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-1</td>
<td>Persons unemployed 15 weeks or longer, as a percent of the civilian labour force</td>
</tr>
<tr>
<td>U-2</td>
<td>Job losers and persons who completed temporary jobs, as a percent of the civilian labour force</td>
</tr>
<tr>
<td>U-3</td>
<td>Total unemployed, as a percent of the civilian labour force (official unemployment rate)</td>
</tr>
<tr>
<td>U-4</td>
<td>Total unemployed plus discouraged workers, as a percent of the civilian labour force plus discouraged workers</td>
</tr>
<tr>
<td>U-5</td>
<td>Total unemployed, plus discouraged workers, plus all other persons marginally attached to the labour force, as a percent of the civilian labour force plus all persons marginally attached to the labour force</td>
</tr>
<tr>
<td>U-6</td>
<td>Total unemployed, plus all persons marginally attached to the labour force, plus total employed part time for economic reasons, as a percent of the civilian labour force plus all persons marginally attached to the labour force</td>
</tr>
</tbody>
</table>
Given the enormous increase of discouraged and part-time workers in Portugal, the author considers it necessary to quantify the NAIRU using the U-6 and U-5 rates and compare them to the official unemployment rate.

3. The use of the NAIRU in a policy context

“Economists analyze future inflation trends, the sustainability of fiscal positions and the need to undertake structural reforms to permanently reduce unemployment and for this purpose they need a benchmark to identify and distinguish sustainable and unsustainable trends in output and unemployment. The NAIRU concept provides such a benchmark.” (Richardson et al., 2000)

- Monetary policy and inflation:

Structural unemployment is useful for monetary policy if it helps to forecast inflation developments in the short run. After the 2008 economic crisis, many economists have tried to solve the so-called “missing deflation puzzle”. They argued that, given the increase in unemployment in those years, inflation should have decreased much more than it did to be consistent with the Phillips Curve framework. In fact, the IMF (2013) stated that the relationship between cyclical unemployment and inflation had become muted. To explain the missing deflation, Ball and Mazumder (2015) reasoned that inflation expectations have become more anchored and that inflation development depends on short-term unemployment, which increased less than total unemployment.

In addition, Gordon (2013) argued that, although the Phillips Curve has not been able to capture inflation developments, the triangle model has. Specifically, he forecasts inflation in the US since 1996 and perfectly captures the actual series. Gordon explains
that supply shocks have a big influence on inflationary developments but are not included in the Phillips Curve. This means that, as explained in the literature review, one should consider expectations, supply shocks and demand shocks when analyzing inflation developments.

- Fiscal policy and medium term assessment:

The general government balance, measured by the difference between revenues and expenditures, varies with the business cycle. On the revenues side, almost all tax categories are affected by cyclical fluctuations while the expenditures side tends to compensate domestic demand changes. This implies that during an economic slowdown expenditures are higher and taxes lower.

Hence, a measure of the government balance in a cyclically normal situation is needed. This structural budget balance is calculated using the NAIRU estimation as a measure of structural labour utilization. The NAIRU determines the output gap as expressed by Okun’s law. This output gap together with revenue and expenditure elasticities quantifies the structural budget balance.

- Study determinants of the NAIRU and the scope for future reforms:

By regressing the obtained NAIRU on its potential determinants, one can obtain the contribution of these to the NAIRU estimates and learn how to decrease this rate. Some of these factors are the tax wedge, the real interest rate, the average unemployment benefit replacement rate and union density. Hence, measuring the determinants of the reduced form measured NAIRU is a key step for policy recommendations and in deriving the scope for future reforms. This step has been taken
by Gianella et al (2009), who found that, for a sample of 19 OECD countries including Portugal, the most significant variables in determining the NAIRU are the tax wedge, the PMR (product market regulation) indicator, and the real interest rate.

4. Approaches for estimating the NAIRU

As recognized by Staiger, Stock and Watson (1996), the estimation of the NAIRU is imprecise because the NAIRU is not observable and varies over time. This uncertainty is explained by three reasons: firstly, the estimation of the parameters is uncertain; secondly, the NAIRU is stochastic and its determinants are not known with precision; and lastly, there are a number of potential models to estimate it. In this paper, three different measurement versions are going to be tested: two of them based on the reduced form approach and the other based on the statistical approach.

- Reduced-form:

The measurement of the NAIRU proposed in this study is based on the unobserved components model (see annex 1), which obtains its time-path from the information contained in a reduced-form Phillips curve. Additionally, the behaviour of the unobserved variable is defined (see equations 1 and 2).

Following Gordon, the Phillips curve, represented by the “triangle model”, specifies that inflation changes depend on three factors: supply shocks, demand shocks and inertia. As recognized by the OECD, the dependent variable could be either price inflation or wage inflation. Also, Rusticelli and Guichard (2011) propose an alternative specification for Southern European Countries in which “price inflation barely responds to unemployment”. They argue that although excessive labour demand might have had little impact on prices before the crisis, unit labour costs could have increased. Both this
proposition and services inflation have been tested and no improvement in the fit has been found.

Supply shocks are represented by the change in the inflation rate of food and energy contemporaneously and the rate of inflation of food and energy with respect to headline inflation in the previous period. Other possible proxies are the change in real oil prices and the change in real import prices, but have not been considered due to their lower explanatory power as regressors of inflation developments. Additionally, the demand shocks are measured by the difference between the actual rate of unemployment and the natural rate, where the natural rate of unemployment is allowed to change over time. Macroeconomic theory emphasizes that in case unemployment is higher than the natural rate, workers searching for a job will push wages down, thereby decreasing unemployment. Lastly, inertia measures the effect of previous inflation on current inflation. Given that, in the long run, inflation is assumed to depend only on nominal factors, the coefficients of lagged inflation have to be constrained to sum to 1. This implies that in the absence of supply and demand shocks inflation will remain constant. However, in this model only 3 lags are included, specification obtained by dropping insignificant lags after applying the information criteria.

In this study, I have tested the specification proposed by Gordon using the number of lags determined by the selection criteria for the supply and demand shocks and 24 lags for inflation which sum equals to 1 but then lagged inflation suppresses the explanatory power of the shocks\(^1\). I have also tested the model using the lags proposed by the information criteria for inertia, supply and demand shocks, being the NAIRU measured robust (see annex 2). Due to this, I have followed the policymakers’

\(^1\) *“When serial correlation is high and the exogenous variables are heavily trended, the lagged variable will falsely dominate the regression and suppress the legitimate effect of the other variables”* Achen, 2001
specification by simplifying the model in terms of the number of variables included, obtaining a structural approach.

Regarding the equation defining the law of motion of the unobservable variable, the NAIRU is assumed to follow a random walk. A second law of motion is imposed in Model 2 capturing the behaviour of the gap between the unemployment rate and the NAIRU. In both models the path of the NAIRU is derived from the information contained in the Phillips Curve by means of the Kalman Filter (see annex 1). Then, we are going to consider two models:

Model 1

To measure the NAIRU we depart from the basic formulation, which includes the Phillips Curve and the NAIRU equation.

\[
\Delta \pi_t = A(L)\Delta \pi_{t-1} + \beta(U_t - U^*_t) + \gamma z_t + \epsilon_t, \tag{3}
\]
\[
U^*_t = U^*_{t-1} + \nu_t, \tag{4}
\]

where \(\Delta\) is the first difference operator, \(\pi\) is the headline inflation, \(U_t\) is the actual rate of unemployment, \(U^*_t\) represents the NAIRU, \(z_t\) is a vector of supply shock variables, \(\epsilon_t\) is a serially uncorrelated error term with zero mean and variance \(\sigma^2\), and \(\nu_t\) is a serially uncorrelated error term with zero mean and variance \(\sigma^2\).

The Kalman Filter allows for the estimation of the behaviour of the latent variables and the coefficients simultaneously by maximum likelihood. However, as recognized by Gordon (2013) and Watson (2014), the variance of the trend equation error term has to be set artificially. This value is very important because it determines the signal-to-noise ratio \((\sigma^2/\sigma^2)\), the smoothness of the series. Logically, the higher the variance of the error term of the transition equation the less smooth the series will be.
Gordon (1997) suggested choosing a signal-to-noise ratio that allows the NAIRU to move freely but rules out sharp quarter-to-quarter zigzags. Specifically, he proposed $\sigma_v^2 = 0.09$. However, other authors have proposed different values: for instance, Müller (2007) estimated that $\sigma_v^2 = 0.28$. In this study, I impose $\sigma_v^2 = 0.10$, consistent with Gordon propositions. Higher variance implies an inappropriately high volatility of the NAIRU. Then, the smoothness of the NAIRU achieved by this specification is an assumption, making it convenient to compare the result with other models.

**Model 2:**

This approach relies on the equations defined in the previous specification, the Phillips curve and the law of motion describing the NAIRU, and adds a second transition equation specifying the time-path of the unemployment gap. This additional equation guarantees that the NAIRU does not deviate permanently from the actual unemployment rate, consistent with a sticky labour market.

\[ U_t - U_t^* = \psi(L)(U_{t-1} - U_{t-1}^*) + \zeta_t, \]  

(5)

where $U_t - U_t^*$ is the unemployment gap and $\zeta_t$ is a serially uncorrelated error term with zero mean and variance $\sigma_\zeta^2$. Following Jaeger and Parkinson (1994), two lags of the unemployment gap are included. The sum of the autoregressive parameters varies between 0.75 and 0.9 (Rusticelli & Guichard, 2011), having imposed in this study that the sum equals to 0.85.

With respect to the smoothness of the series, it is now determined by both the relative variance of the error of the NAIRU transition equation with respect to the variance of the error of the Phillips Curve equation and the relative variance of the error of the unemployment gap equation relative to the variance of the Phillips curve.
equation. The higher the latter the more volatile the NAIRU series will be. The Kalman filter methodology has been used to quantify the parameters and the unobserved variables by maximum likelihood.

-Statistical

HP filter:

Numerous papers have tried to explain inflationary developments using the Hodrick-Prescott (HP) filter to statistically measure the trend of the unemployment rate and have used the difference between the actual unemployment rate and the HP filtered trend unemployment rate as the demand shock in equation 3. In this paper, the HP filter NAIRU estimates are derived using a $\lambda$ of 6400. Although typically the parameter is assumed to be 1600 when using quarterly data, I assume 6400 following Gordon because it yields a smoother trend.

Seeing as the unemployment series is composed of a trend component $tr_t$ and a cyclical component $c_t$, Hodrick and Prescott (1997) argue that we can obtain the trend component by solving the following problem:

$$Min_{\{tr_t\}} \left( \sum_{t=1}^{T} (u_t - tr_t)^2 + \lambda \sum_{t=2}^{T-1} (\nabla^2 tr_{t+1})^2 \right),$$

where $\nabla$ is the difference operator and $\lambda$ determines the smoothness of the trend estimate.
5. **Empirical results**

The results obtained in all the models are consistent with economic theory: the unemployment gap is negatively related to inflation. The goodness of fit of the models has been evaluated using the Akaike Information Criteria\(^2\). We see in table 2 that in model 2 the NAIRU performs better than in the other two specifications, which means that it explains the data better.

### Table 2 - Estimated Phillips Curve using the alternative models

unemployment gaps

<table>
<thead>
<tr>
<th></th>
<th>(1) Model 1</th>
<th>(2) Model 2</th>
<th>(3) HP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta \pi(-1))</td>
<td>-0.678***</td>
<td>-0.666***</td>
<td>-0.685***</td>
</tr>
<tr>
<td></td>
<td>(0.0382)</td>
<td>(0.0376)</td>
<td>(0.0364)</td>
</tr>
<tr>
<td>(\Delta \pi(-2))</td>
<td>-0.589***</td>
<td>-0.500***</td>
<td>-0.504***</td>
</tr>
<tr>
<td></td>
<td>(0.0544)</td>
<td>(0.0544)</td>
<td>(0.0526)</td>
</tr>
<tr>
<td>(\Delta \pi(-3))</td>
<td>-0.548***</td>
<td>-0.538***</td>
<td>-0.547***</td>
</tr>
<tr>
<td></td>
<td>(0.0328)</td>
<td>(0.0323)</td>
<td>(0.0314)</td>
</tr>
<tr>
<td>(\Delta \pi)</td>
<td>0.283***</td>
<td>0.292***</td>
<td>0.280***</td>
</tr>
<tr>
<td></td>
<td>(0.0375)</td>
<td>(0.0365)</td>
<td>(0.0370)</td>
</tr>
<tr>
<td>(\pi(-1)-\pi(-1))</td>
<td>0.328***</td>
<td>0.353***</td>
<td>0.310***</td>
</tr>
<tr>
<td></td>
<td>(0.0407)</td>
<td>(0.0421)</td>
<td>(0.0404)</td>
</tr>
<tr>
<td>(\Delta \pi)</td>
<td>0.112***</td>
<td>0.115***</td>
<td>0.116***</td>
</tr>
<tr>
<td></td>
<td>(0.0146)</td>
<td>(0.0148)</td>
<td>(0.0141)</td>
</tr>
<tr>
<td>(\pi(-1)-\pi(-1))</td>
<td>0.137***</td>
<td>0.145***</td>
<td>0.141***</td>
</tr>
<tr>
<td></td>
<td>(0.0190)</td>
<td>(0.0180)</td>
<td>(0.0192)</td>
</tr>
<tr>
<td>Unemployment Gap</td>
<td>-0.0739*</td>
<td>-0.0881**</td>
<td>-0.119**</td>
</tr>
<tr>
<td></td>
<td>(0.0285)</td>
<td>(0.0292)</td>
<td>(0.0358)</td>
</tr>
<tr>
<td>adj. R-sq</td>
<td>0.896</td>
<td>0.901</td>
<td>0.900</td>
</tr>
<tr>
<td>AIC</td>
<td>124.8</td>
<td>118.7</td>
<td>120.1</td>
</tr>
</tbody>
</table>

*Standard errors in parentheses
1965:Q1 to 2015:Q3
* p<0.05, ** p<0.01, *** p<0.001

\(^2\) The AIC determines the best model in terms of the likelihood function adjusted by the number of parameters. The best model is the one with the lowest AIC.
The HP-filter specification measures an unemployment gap that is statistically significant at the 95% level. However, it is based on arbitrary assumptions commoving excessively with the actual unemployment rate (see Figure 2). We see in Figure 3 that demand pressures in the aftermath of the recession were lower according to the HP filter. It seems more plausible that the macroeconomic adjustment implied higher demand pressures consistent with the Kalman Filter specifications.

Figure 2 - NAIRU estimates from the three models

Figure 2 depicts the observed unemployment rate and the estimated NAIRU. The NAIRU varies considerably less than the Unemployment rate and displays an upward tendency beginning in 2000. Specifically, model 2 indicates that it increased from 6% in 2000 to 8.45% in 2007 to 12.3% in 2013. The increase of the NAIRU in the period 2007-2013 is remarkable. Carneiro, Portugal and Varejao (2013) identify three main channels that might have amplified the employment response to the great recession in
Portugal: credit constraints faced by Portuguese firms, rigidity of wages to respond to negative demand shocks and segmentation of the labour market being those employed with temporary contracts the most affected by the recession.

We see in Figure 3 that also demand pressures increased a lot during the period 2007-2013, which means that the Unemployment rate increased more than the NAIRU. However, since then the Unemployment rate started to decrease while the NAIRU has remained around 12%.

Figure 3- Unemployment Gap estimates from the three models

6. The Missing Deflation puzzle

Comparing results in 1985-1999 and 1999-2015, we see that the effect of the shocks is generally lower in the second period, and specifically, that the unemployment gap coefficient decreases from -0.105, significant at the 90% level, to -0.0637, not significant (see table 3). As explained before, it has been argued that the Phillips Curve
has flattened and this result would confirm this theory. Then, I have included the recent developments in Portugal and compared the results. First, I have incorporated anchored expectations to the model in the context of the European Monetary Union’s 2% inflation target; and then I have considered alternative measures of labour slack consistent with increasing discouraged and part-time workers in the Portuguese economy.

Table 3- Comparison of different periods estimation of the Phillips Curve

<table>
<thead>
<tr>
<th></th>
<th>(1) 1985-1999</th>
<th>(2) 1999-2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \pi(-1)$</td>
<td>-0.468***</td>
<td>-0.733***</td>
</tr>
<tr>
<td></td>
<td>(0.0655)</td>
<td>(0.0698)</td>
</tr>
<tr>
<td>$\Delta \pi(-2)$</td>
<td>-0.467***</td>
<td>-0.419***</td>
</tr>
<tr>
<td></td>
<td>(0.0642)</td>
<td>(0.0957)</td>
</tr>
<tr>
<td>$\Delta \pi(-3)$</td>
<td>-0.377***</td>
<td>-0.509***</td>
</tr>
<tr>
<td></td>
<td>(0.0564)</td>
<td>(0.0664)</td>
</tr>
<tr>
<td>$\Delta \pi_f$</td>
<td>0.378***</td>
<td>0.240***</td>
</tr>
<tr>
<td></td>
<td>(0.0517)</td>
<td>(0.0470)</td>
</tr>
<tr>
<td>$\pi_f(-1)-\pi(-1)$</td>
<td>0.371***</td>
<td>0.261***</td>
</tr>
<tr>
<td></td>
<td>(0.0542)</td>
<td>(0.0528)</td>
</tr>
<tr>
<td>$\Delta \pi_e$</td>
<td>0.116*</td>
<td>0.127***</td>
</tr>
<tr>
<td></td>
<td>(0.0432)</td>
<td>(0.0137)</td>
</tr>
<tr>
<td>$\pi_e(-1)-\pi(-1)$</td>
<td>0.0346</td>
<td>0.164***</td>
</tr>
<tr>
<td></td>
<td>(0.0580)</td>
<td>(0.0189)</td>
</tr>
<tr>
<td>Unemployment Gap</td>
<td>-0.105*</td>
<td>-0.0637</td>
</tr>
<tr>
<td></td>
<td>(0.0449)</td>
<td>(0.0359)</td>
</tr>
<tr>
<td>adj. R-sq</td>
<td>0.921</td>
<td>0.920</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

6.1 Anchored expectations

One possible explanation for the change in the explanatory power of demand and supply shocks on inflation developments is inflation targeting. Since the creation of the
European Monetary Union in 1999, policymakers have given high emphasis to keeping inflation at the 2% level. This should be considered by including the difference between the Euro-area inflation rate and an inflation target in the Phillips Curve:

$$\Delta \pi_t = A(L)\Delta \pi_{t-1} + \beta (U_t - U^*_t) + \gamma z_t + \theta DU(\pi_\text{ea}_{t-1} - TAR) + e_t, \quad (7)$$

where DU is a dummy taking the value of 0 before inflation targeting started and 1 after it is anchored around the central bank target (1999), TAR is the central bank inflation target and $\pi_\text{ea}$ is the euro area inflation rate.

Table 4 – Estimated Phillips Curve incorporating Anchored Expectations

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$-0.733^{***}$</td>
<td>$-0.776^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.0655)$</td>
<td>$(0.0598)$</td>
<td>$(0.0597)$</td>
</tr>
<tr>
<td>$\Delta \pi(-2)$</td>
<td>$-0.467^{***}$</td>
<td>$-0.419^{***}$</td>
<td>$-0.501^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.0642)$</td>
<td>$(0.0597)$</td>
<td>$(0.0594)$</td>
</tr>
<tr>
<td>$\Delta \pi(-3)$</td>
<td>$-0.377^{***}$</td>
<td>$-0.509^{***}$</td>
<td>$-0.471^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.0564)$</td>
<td>$(0.0564)$</td>
<td>$(0.0537)$</td>
</tr>
<tr>
<td>$\Delta \pi_f$</td>
<td>$0.378^{***}$</td>
<td>$0.240^{***}$</td>
<td>$0.218^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.0517)$</td>
<td>$(0.0470)$</td>
<td>$(0.0388)$</td>
</tr>
<tr>
<td>$\pi_f(-1)-\pi(-2)$</td>
<td>$0.371^{***}$</td>
<td>$0.261^{***}$</td>
<td>$0.328^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.0542)$</td>
<td>$(0.0529)$</td>
<td>$(0.0545)$</td>
</tr>
<tr>
<td>$\Delta \pi_e$</td>
<td>$0.116^{*}$</td>
<td>$0.127^{***}$</td>
<td>$0.118^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.0432)$</td>
<td>$(0.0137)$</td>
<td>$(0.0141)$</td>
</tr>
<tr>
<td>$\pi_e(-1)-\pi(-2)$</td>
<td>$0.0346$</td>
<td>$0.164^{***}$</td>
<td>$0.170^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.0580)$</td>
<td>$(0.0189)$</td>
<td>$(0.0181)$</td>
</tr>
<tr>
<td>Unemployment Gap</td>
<td>$-0.105^{*}$</td>
<td>$-0.0837$</td>
<td>$-0.0887^{*}$</td>
</tr>
<tr>
<td></td>
<td>$(0.0449)$</td>
<td>$(0.0359)$</td>
<td>$(0.0278)$</td>
</tr>
<tr>
<td>$\pi(\text{ea},-1)-\pi(\text{target})$</td>
<td></td>
<td>$-0.231^{***}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(0.0448)$</td>
<td></td>
</tr>
</tbody>
</table>

Adj. R-sq | 0.921 | 0.920 | 0.930 |
AIC | 50.21 | 48.89 | 32.30 |

Standard errors in parentheses
BE: backward-looking expectations
AE: anchored expectations
* p<0.05, ** p<0.01, *** p<0.001
The difference between the euro area inflation rate and the target set by the ECB is statistically significant. In addition, when I take into account inflation targeting the coefficient of the demand shock increases from -0.0637 to -0.0687 in the period 1999-2015 and is significant at the 90% confidence level. This partly explains why inflation did not fall more during the crisis after the high increase in unemployment: inflation targeting offset the effect of the increase in unemployment. In fact, “better anchored expectations has been recognized as the main reason for more stable inflation and the absence of disinflation in the aftermath of the financial crisis” (Rusticelli et al., 2015).

The NAIRU estimated using this alternative specification has been lower than the one estimated using backward-looking inflation expectations since the beginning of the crisis (see Figure 4). This provides evidence that demand pressures might have been underestimated due to inflation targeting.

Figure 4 – NAIRU estimates with Anchored expectations
6.2. Labour Market Slack

As I have remarked in the first section, labour market slack refers to the insufficiency of work and not only covers those who are not working and searching for a job but also those who are working part-time for economic reasons and those who are not searching for a job but would like to work. These have to be tested because they are important for policy recommendations and might provide a more precise inflation forecast.

Part-time workers are expected to exert a higher pressure on wages than the unemployed, and Centeno, Maria and Novo (2010) have shown that, in Portugal, the probability of a marginally attached worker obtaining a job is almost the same as that of an unemployed worker, and attribute the difference in the probability of transitioning to inactivity (see Figure 5). For this reason, discouraged workers should also be included when using the NAIRU to define future reforms.
The graphs show the probability of employed (E), unemployed (U), marginally attached (M), and non-activity workers (N) getting to each of the other states between 1999 and 2009.

Considering the analysis above, two additional labour slack measures are going to be used to measure the NAIRU: the U-5, that includes U-3 unemployed plus marginally attached workers, and the U-6, that considers U-3 unemployed, marginally attached and part-time workers.
The NAIRU measured with the alternative unemployment measures is the following:

Figure 6- Alternative Unemployment Measures

Figure 7- NAIRU estimates, alternative Unemployment measures
The effect of the demand shock on inflation developments is higher and more significant when measured with the U-6 Unemployment rate. Also, the Akaike Information Criteria indicates that U6 Unemployment captures the data better. This means that discouraged and part time workers might be considered in the Unemployment measurement both for a better explanation of the inflation progress and in order to include these in the scope for future reforms. Also, U-5 Unemployment, adding only marginally attached workers to the official rate, performs better than the official rate (see table 5).

| Table 5 - Estimated Phillips Curve using alternative measures of Unemployment |
|-----------------------------|-----------------------------|-----------------------------|
|                            | (1)            | (2)            | (3)            |
| Δn(-1)                     | -0.799***     | -0.825***     | -0.829***     |
|                            | (0.0703)      | (0.0705)      | (0.0702)      |
| Δn(-2)                     | -0.550***     | -0.582***     | -0.686***     |
|                            | (0.0867)      | (0.0841)      | (0.0839)      |
| Δn(-3)                     | -0.485***     | -0.495***     | -0.492***     |
|                            | (0.0589)      | (0.0547)      | (0.0546)      |
| Δnf                        | 0.240***      | 0.237***      | 0.238***      |
|                            | (0.0491)      | (0.0483)      | (0.0480)      |
| n(-1)-π(-1)                | 0.367***      | 0.352***      | 0.350***      |
|                            | (0.0093)      | (0.0090)      | (0.0091)      |
| Δne                        | 0.182***      | 0.181***      | 0.182***      |
|                            | (0.0174)      | (0.0171)      | (0.0172)      |
| n(-1)-π(-1)                | 0.187***      | 0.189***      | 0.190***      |
|                            | (0.0230)      | (0.0223)      | (0.0225)      |
| n(ea-1)-target             | -0.255***     | -0.248***     | -0.231***     |
|                            | (0.0467)      | (0.0431)      | (0.0425)      |
| Unemployment Gap           | -0.0772      | -0.0944**     | -0.0910**     |
|                            | (0.0308)      | (0.0332)      | (0.0318)      |

adj. R-sq 0.943 0.948 0.948
AIC 30.66 26.43 26.39

Standard errors in parentheses
2002:01 to 2015:03
* p<0.05, ** p<0.01, *** p<0.001
7. Conclusion

The NAIRU is a key concept in macroeconomic policy but unfortunately no consensus has been reached as to how it should be measured. Anyway, the “triangle model” proposed by Gordon has been widely accepted because it is very precise. This methodology has been applied to the Portuguese case and two alternatives to better capture inflation developments have been proposed: inflation anchoring and more complete measures of slack in the labour market.

The primary objective of the Eurosystem is to maintain price stability in the euro area and it is key to include inflation targeting in the estimation of the NAIRU. In fact, we have seen that inflation targeting is significant at the 99% level and determines a lower NAIRU which means that demand pressures were higher than initially computed. More workers with more influence have been affecting prices since the outburst of the crisis to a greater degree than has been expected by the traditional model because inflation targeting has kept inflation low.

Also, including part time and marginally attached workers in the labour insufficiency rate improves the results. The traditional specification was not capturing the development of the Portuguese labour market, characterized by a huge increase in discouraged and part time workers. The exclusion of part time workers might be acceptable due to the difficulty of measuring them but given the similar probability of transitioning to employment of marginally attached and unemployed workers, they should be included to measure the NAIRU.
Annex 1. State-space model

For Kalman filter estimation, the model is expressed in its state-space form, composed by a measurement and a state equation:

\[ Y_t = Dz_t + Fw_t + Gv_t, \]
\[ z_t = Az_{t-1} + C\epsilon_t, \]

where \( Y_t \) is an observed endogenous variable, \( w_t \) is a vector of observed exogenous variables, \( D, F, A, C \) and \( G \) are matrices of time-invariant parameters, \( z_t \) is a vector of unobserved parameters and \( v_t \) and \( \epsilon_t \) are white noise error terms.

The measurement equation derives from theoretical grounds; in this case, the Philips curve determines the relationship between unemployment and inflation. On the other hand, the transition equation contains atheoretical laws of motion describing the behaviour of the unobservable variable.

The Kalman filter is a recursive procedure for computing the optimal estimator (thus, minimizing the mean square error) at time \( t \) based on the information available at that time. Starting with an assumed initial unobservable variable \( z_t \), \( Y_t \) is predicted; then using the observed value of \( Y_t \) the prediction error is computed. Lastly, this prediction and the state equation errors are used to obtain the unobserved variable.

The algorithm adopted for the system estimation was the SSPACE procedure available in the econometrics package STATA. The following state space systems have been estimated:
Model 1

Measurement equation:

$$
\Delta \pi_t = A(L) \Delta \pi_{t-1} + \beta (U - U^*) + yz_t + e_t,
$$

$$
U = U^* + (U - U^*),
$$

which can be expressed in the matrix explained above as:

$$
\begin{bmatrix}
\Delta \pi_{t-1} \\
\Delta \pi_{t-2} \\
\Delta \pi_{t-3} \\
\Delta \pi_f \\
(\pi_{t-1} - \pi_{t-1}) \\
\pi_{t-1} - \pi_{t-1}
\end{bmatrix}
= 
\begin{bmatrix}
0 & -\beta \\
1 & 1
\end{bmatrix}
\begin{bmatrix}
U^* \\
U_t - U^*_t
\end{bmatrix}
+ 
\begin{bmatrix}
a_1 & a_2 & a_3 & b_1 & d_1 & e_1 & f_1
\end{bmatrix}
\begin{bmatrix}
\pi_{t-1} \\
\pi_{t-2} \\
\pi_{t-3} \\
\pi_f \\
(\pi_{t-1} - \pi_{t-1}) \\
(\pi_{t-1} - \pi_{t-1})
\end{bmatrix}
+ 
\begin{bmatrix}
1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
e_t
\end{bmatrix}
$$

where the variance-covariance matrix of \(e_t\) is: \(\Sigma_{e_t} = \begin{bmatrix} (\sigma_t)^2 & 0 \\ 0 & 0 \end{bmatrix}\).

State equation:

$$
U^*_t = U^*_{t-1} + v_t,
$$

$$
U_t - U^*_t = \zeta_t,
$$

which can be expressed in the matrix form explained above as:

$$
\begin{bmatrix}
U^* \\
U - U^*
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
U^*_{t-1} \\
U_t - U^*_t
\end{bmatrix}
+ 
\begin{bmatrix}
0 & 1
\end{bmatrix}
\begin{bmatrix}
v_t \\
\zeta_t
\end{bmatrix}
$$

Where the variance-covariance matrix of \(v_t\) is: \(\Sigma_{v_t} = \begin{bmatrix} (\sigma_v)^2 & 0 \\ 0 & (\sigma_{\zeta})^2 \end{bmatrix}\).
Model 2:

Measurement equation:

\[ \Delta \pi_t = A(L)\Delta \pi_{t-1} + \beta(U - U^*) + \gamma z_t + e_t, \]

\[ U = U^* + (U - U^*), \]

which can be expressed in the matrix explained above as:

\[
\begin{bmatrix}
\Delta \pi_t \\
U
\end{bmatrix} =
\begin{bmatrix} 0 & -\beta & 0 \\
1 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
U^* \\
U_t - U_t^* \\
U_{t-1} - U_{t-1}^*
\end{bmatrix}
+ \begin{bmatrix} a_0 & a_2 & a_3 & b_1 & d_1 & e_1 & f_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta \pi_{t-1} \\
\Delta \pi_{t-2} \\
\Delta \pi_{t-3} \\
\Delta \pi^f \\
(\pi_{t-1}^f - \pi_{t-1}) \\
(\pi_{t-1} - \pi_{t-1})
\end{bmatrix}
+ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
e_t \\
0
\end{bmatrix}
\]

Where the variance-covariance matrix of \([e_t]_0^\pi\) is: \(\Sigma_e = \begin{bmatrix} (\sigma^\pi)^2 & 0 \\
0 & 0 \end{bmatrix}\).

State equations:

\[ U_t^* = U_{t-1}^* + \nu_t, \]

\[ U_t - U_t^* = \psi(L)(U_{t-1} - U_{t-1}^*) + \xi_t, \]

which can be expressed in the matrix form explained above as:

\[
\begin{bmatrix}
U^* \\
U_t - U_t^* \\
U_{t-1} - U_{t-1}^* \\
U_{t-2} - U_{t-2}^*
\end{bmatrix}
= \begin{bmatrix} 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
U_{t-1}^* \\
U_{t-1} - U_{t-1}^* \\
U_{t-2} - U_{t-2}^*
\end{bmatrix}
+ \begin{bmatrix} 0 & 0 & 0 & 0 \\
1 & 0 & 0 & \xi_t \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

Where the variance-covariance matrix of \([\nu_t]^\xi_t_0\) is: \(\Sigma_{\nu_t} = \begin{bmatrix} (\sigma^\nu)^2 & 0 & 0 \\
0 & (\sigma^\xi)^2 & 0 \\
0 & 0 & 0 \end{bmatrix}\).
Annex 2. Sensitivity of the results

Studies measuring the NAIRU differ in the number of lags selected for demand and supply shocks. There are three approaches: selection based on theoretical grounds, on the information criteria and select the number of lags proposed by the information criteria and drop the insignificant ones. Studies trying to validate the NAIRU concept use the second approach and use typically 4 lags of the inertia, supply and demand shocks (complete approach). However, most of the NAIRU estimation studies impose the demand shock on theoretical grounds selecting only 1 lag and the supply shocks using either the first or the last option.

I have selected in this study the first approach because of its feasibility for policy analysis and the robustness of the model under the different specifications. In fact, in the next graph I compare the NAIRU that I have estimated in the paper (structural) with the complete approach estimation and they are almost identical.

Figure A1- Sensitivity of the NAIRU
References


Fabiani, S. and Mestre, R. (2004). A system approach for measuring the euro area


IMF, (2013). *The Dog that didn't Bark: Has Inflation been muzzled or was it Just Sleeping*.


