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Tax competition with commuting in asymmetric cities

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Contents

**Introduction**

1 Commuting-induced spillovers and the case for efficiency-enhancing local wage taxes 5
   1.1 Introduction .................................................. 6
   1.2 The Model .................................................... 9
      1.2.1 The choice of the workplace ............................ 11
   1.3 First Best ................................................... 12
   1.4 The Equilibrium with Residence Taxes .................... 14
      1.4.1 Median voter of L works in L .......................... 16
      1.4.2 Median voter of L works in H .......................... 17
   1.5 The Tax Competition Equilibrium with Residence and Wage Taxes 19
      1.5.1 Median voter of L works in L .......................... 20
      1.5.2 Median voter of L works in H .......................... 21
   1.6 Only Residence Tax vs. Residence and Wage Taxes . . . . . 23
   1.7 Conclusion .................................................. 25
   1.8 Appendix .................................................... 26

2 Endogenous asymmetric transportation costs and the impact of toll usage 31
   2.1 Introduction .................................................. 32
   2.2 The Model .................................................... 35
      2.2.1 The choice of the workplace ............................ 37
   2.3 First Best ................................................... 38
   2.4 The Equilibrium with Transportation Infrastructure Funded by Residence Taxes 40
   2.5 The Equilibrium with Transportation Infrastructure Funded by Residence Taxes and a Toll 44
      2.5.1 Welfare analysis ......................................... 47
   2.6 Conclusion .................................................. 48
### Chapters

#### 3 Optimal fiscal instruments for tax decentralization in a city with congestion

- **3.1 Introduction** .......................... 54
- **3.2 The Model** ............................. 57
  - 3.2.1 The choice of the workplace .......... 60
- **3.3 First Best** ............................. 61
- **3.4 The Equilibrium with Residence Taxes** .......................... 63
- **3.5 The Equilibrium with Residence Taxes and a Toll** ............. 65
- **3.6 The Equilibrium with Residence Taxes and Wage Taxes** ........ 67
  - 3.6.1 Median voter of L works in L .......... 69
  - 3.6.2 Median voter of L works in H .......... 70
- **3.7 Welfare analysis** ...................... 72
- **3.8 Conclusion** ........................... 74
- **3.9 Appendix** ............................. 76

### Bibliography

- **Bibliography** .............................. 83
Introduction

Local governments have to deal with the benefits and problems arising from high daily flows of commuters. Commuting has been intensified in the last decades, both in terms of increased number of commuters and longer traveled distances, with consequent increase in the time spent in the journeys, specially during rush hours. According to the United Kingdom Department for Transport annual publication "Transport Statistics Great Britain" 2013, over 1.1 million persons enter central London daily during the morning peak, 18% more than in 1996. In 2012, commuters spent an average of 54 minutes to travel to central London, and 46 minutes to work in the remaining areas of inner London. In the third quarter of 2014 the average traffic speed in central London between 7 a.m. and 7 p.m. was of 8.4 mph, i.e., 13 km/h (Transport for London Streets Performance Report, quarter 3 2014/2015).

Intense commuting, namely from one city or jurisdiction to another, puts pressure in the transportation infrastructures of the cities and generates congestion problems, such as traffic, pollution or crime, which are negative externalities. In order to deal with such problems, some local governments introduced tolls that have to be paid by agents that want to drive in city centers. Examples include Singapore, London, Stockholm and Milan. Those tolls pretend to reduce the levels of congestion in those areas and fund the construction, maintenance and operation of transportation infrastructure or transportation services.

Although commuting may originate problems, it also makes agents contact with more than one city or jurisdiction. This means that commuters are exposed to the goods and services provided in both the city where they live and the one where they work. Public goods provided by local governments (e.g., transportation infrastructure, free parks, street lighting, etc.) will, therefore, benefit not just the residents of a city, but also its workers, independently of their residence. Thus, local public goods may have a positive spillover effect for the commuters that live in other cities.
Our goal is to analyse the impact of decision decentralizing versus the decision of a benevolent social planner in a framework that includes positive or negative externalities. We consider a linear city divided in two jurisdictions with different productivities (and, thus, different wages) and let agents decide where they want to work.

The different chapters and their respective contributions are summarized hereafter.

Chapter 1: Commuting-induced spillovers and the case for efficiency-enhancing local wage taxes

This chapter presents a model of a duo-centric linear city where agents choose their workplace. Jurisdictions are unequally productive and local governments use a head tax and, possibly, a source-based wage tax, to finance a local public good. Each agent derives utility from both the local public good of the jurisdiction where he works and where he lives; thus, interjurisdictional commuting generates endogenous spillovers. We analyze the tax competition equilibrium when local governments only use the head tax or both the head and the wage tax and compare it to the utilitarian benchmark. We show that local public goods are always underprovided in the most productive jurisdiction, but may be overprovided in the least productive one. We also show that distortive source-based wage taxation may improve upon the equilibrium with residence taxes alone, as it allows to charge commuters with part of the cost of the public good they enjoy.

Chapter 2: Endogenous asymmetric transportation costs and the impact of toll usage

This chapter presents a model of a duo-centric linear city where agents choose in which jurisdiction they want to work. The transportation cost in each jurisdiction depends on the investment of each local government on a local public good. Local governments can use a head tax and, possibly, a toll, to finance the local public good provision. We analyze the tax competition equilibrium when local governments only use the head tax or both the head tax and the toll and compare it to the utilitarian benchmark. We show that the local public good is always overprovided in the less productive jurisdiction, but may be under or overprovided in the more productive one. We also show that the usage of a toll may improve
the provision of the public good upon the equilibrium with residence
taxes alone, but a simulation shows that it will have a negative impact
on overall utility.

Chapter 3: Optimal fiscal instruments for tax de-
centralization in a city with congestion

This paper presents a model of a duo-centric linear city. Jurisdictions
are unequally productive and agents choose where they want to work,
knowing there is a congestion cost that depends on the number of workers
in each jurisdiction. Local governments can use a head tax, a toll and
a wage tax, to finance the local public good provision. We analyze the
tax competition equilibrium when local governments use (i) only the
head tax, (ii) both the head tax and the toll or (iii) both the head and
wage taxes, and compare it to the utilitarian benchmark. We compare
the three fiscal mechanisms, showing that if the transportation cost is
high enough or the productivity asymmetry is high enough, the head tax
alone is the best choice, while if the transportation cost is low enough:
(i) the wage tax is best for low productivity asymmetries but (ii) the toll
is preferable for mild productivity asymmetries.
Chapter 1

Commuting-induced spillovers and the case for efficiency-enhancing local wage taxes

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1.1 Introduction

Agents spend most of their time in the place where they live and where they work, making them consume the public goods provided in those places. This means that local public good provision may have a spillover effect for non residents of a certain region. However, among the several types of public goods provided by municipalities or jurisdictions we can find some that are mostly used by the inhabitants of the municipality (such as garbage collection, gas supply, parks or monuments with free entrance for locals, etc.) while others are used by both the inhabitants and the commuters that work there (road construction and maintenance, free parks, public transportation, street lighting, etc.).

Most types of public goods usually provided by municipalities or jurisdictions are only consumed by agents if they actually go to that municipality. Suppose the nearby town (where the agent seldom goes) now offers, for example, a better garbage collection service. The residents of that city enjoy higher utility due to cleaner streets, but it is difficult to argue that someone with seldom contact with that city is now better off.

The literature usually treats public goods spillovers as exogenous, i.e., agents “automatically” get utility from the public goods provided in other jurisdictions\footnote{This is the Oates’s tradition spillover that we can find for example in Besley and Coate (2003).}. However, in a commuting setup spillovers actually arise in a quite natural and endogenous way, namely by travelling on a daily basis across jurisdictions the agents enjoy public goods in both the municipality they live and in the one where they work.

The framework we are considering encompasses a large variety of possible forms of local governments, from different jurisdictions in one metropolitan area to neighboring cities or even states with common borders as long as it makes sense to have agents commuting from one to the other. As stated in Peralta (2007) “there is extensive evidence of the increasing importance of inter-jurisdictional commuting, possibly fostered by the improvement in transportation technologies”. Such increasing importance is documented for example in Shields and Swenson (2000), Glaeser et al. (2001) and Renkow (2003) using US data, by Van Ommeren et al. (1999) for The Netherlands or Cameron and Muellbauer (1998) for Great Britain. In all this papers we can find clear evidence that both the number of commuters and the commuting distance has been increasing in the last 40 or 50 years.

Local governments worldwide have different levels of autonomy, namely
when it comes to tax collection, and access to different kinds of taxes. Such taxes include residence-based wealth taxes, pure residence-based income taxes, pure source-based income taxes, or “hybrid” ones. These taxes are usually combined with grants or transfers from the central governments to form the total local budget.

Examples of residence-based wealth taxes are mostly residential and business property taxes which, in the United States, “are the most important source of local government tax revenue” (Braid, 2005). Pure residence-based income taxes charged by local governments can be found in Baltimore (according to Braid, 2009) or in Portugal.

As stated in Braid (2009) U.S. cities like San Francisco, Los Angeles, Newark (New Jersey) and Birmingham (Alabama) have pure source-based wage taxes or payroll taxes that apply uniformly to citizens depending only on their workplace: “a central-city’s wage tax applies at the same rate to central-city and suburban residents working in the central city, but not to central-city and suburban residents working in the suburbs” (Braid, 2009).

Examples of cities using “hybrid” income taxes are also presented in Braid (2009). In these cases all central city residents are taxed at a rate, irrespective of where they work, while residents in the suburbs who work in the central city can be taxed at a different rate. Kansas City, St. Louis, Wilmington, Detroit, New York City and Philadelphia are the provided examples. But the use of wage or income taxes by local governments is not confined to the U.S. As we can read in Peralta (2007) Mexico and “several OECD countries have payroll taxes at the state or local level: Australia, Austria, France and Greece”. Also Korea has source based income taxes (Chu and Norregaard, 1997). Besides these, Braid (2005) points the use of such taxes also on Sweden, Denmark, France, Germany, Japan and Spain.

When we think about the fiscal autonomy of local governments the problem of centralized vs. decentralized decision immediately arises. We traveled a long way since the pioneering work of Oates (1972) who formalized the standard approach for this question and reached the Oates’s Decentralization Theorem that states that decentralization is preferred in the absence of spillover effects while otherwise there is a trade-off due to the incapability of the central government to follow different public policies in different regions. This assumption of uniformity of the centralized policy is used in many other papers on fiscal federalism to impose a cost on centralization. The arguments in favor of local governments

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2For a thorough analysis of the fiscal autonomy of local governments please check the OECD (2009) study.

3For example in Alesina and Spolaore (1997) when studying the size of nations or
are usually justified by some kind of informational advantage on the features of their regions (they are “closer to the people”, which allows to better respond to the agents’ needs) but the decentralization comes to a cost due to the failure to internalize tax and expenditure spillover effects (Oates, 1999).

In this paper we look at the impact of local level fiscal decentralization on the provision of public goods in a framework where agents commute and local governments provide public goods (or publicly provided private goods) with an endogenous spillover effect. Our purpose is to analyze the majority voting decentralized equilibrium against the benchmark of a first-best benevolent social planner solution. The spillovers we want to analyze are due to the fact that agents reside in one place but can work in a different one and therefore can be subject to two different local governments. The reality we are trying to model is clearly pointed by Fisher (1996, p.6) when he writes that “Many individuals live in one city, work in another, and do most of their shopping at stores or a shopping mall in still another locality”.

Our model introduces public goods with an endogenous spillover effect in the framework of a linear city used by, e.g., Peralta (2007) and Braid (2000) to tackle interjurisdictional tax spillovers. The city is divided into two jurisdictions and agents choose where they want to work. Productivity, and thus wages, differ across regions and individuals trade-off the advantages (i.e., wage and working conditions) of a given job against travel costs (distance, time, and money) when choosing their work place. Our main contribution is to allow individuals to enjoy public goods in the work place. We do not, however, model the residence choice of agents, assuming that residence and working choices are independent, as argued by Wildasin (1986) and supported by empirical evidence provided by Rouwendal and Meijer (2001), Glaeser et al. (2001) and Zax (1991 and 1994). For an analysis of the residence decision refer to Wrede (2009) where land is included and agents can choose their residence location according to a bid-rent function.

The contribution of this paper is twofold. On the one hand, it introduces public good spillovers in a linear city tax competition model with commuting in the line of Peralta (2007). On the other hand, it introduces a distortive wage tax on a model with spillovers.

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\footnote{in Bolton and Roland (1997) analysing the threat of secession.}

\footnote{The use of such equilibrium in tax competition scenarios can also be found in Fuest and Hubber (2001) and Grazzini and van Ypersele (2003) who show that centralized decision regarding capital taxes can make the median voter worse off.}

\footnote{An approach similar to the one used in Fernandez (2004) based on Wheaton (1977).}
With this reality in mind, we need a representative agent who derives utility from both the public good provided in the residence location and in the working place. This means that agents only get utility from the public good supplied in the other jurisdiction if they choose to work there. Otherwise they only get utility from the one provided in their own jurisdiction. This formulation allow us to have an endogenous spillover effect instead of the traditional exogenous one.

Naturally agents might enjoy the public goods in other jurisdictions if they go there for leisure or shopping and therefore use the public goods provided even without working there. However, such use is occasional and most of the goods and services from which individual get utility in those cases are privately provided ones (hotels, restaurants, leisure facilities, shopping malls, theaters, etc.). As such, we chose to disregard these situations and concentrate on the commuters for work case.

We prove that in the tax competition equilibrium the public good of the most productive region is always underprovided, due to the externality, while that of the least productive region can be under or overprovided. We also show that tax competition leads to a less than efficient number of commuters in some cases. Interestingly, we show that the introduction of a distortive wage tax improves the provision of the public goods, when compared to a situation where local governments only use a lump sum residence tax. The use of the distortive wage tax is therefore, a second-best result, as it partially offsets the distortion generated by the endogenous spillover of the public goods.

This paper is organized as follows. In Section 2 we present the model. Section 3 computes the first best, which is then used as a benchmark to compare the results obtained in Sections 4 and 5, i.e., the equilibrium where only a lump sum tax is used, and where both a lump sum and a distortive tax are used, respectively. Section 6 compares the two equilibria found before, and Section 7 concludes.

1.2 The Model

We consider a linear city divided into two jurisdictions with the same size. Each jurisdiction has an employment center where agents can work. The total number of residents of the city is normalized to 1, as well as the city size, with extreme points of the segment \(-1/2\) and \(1/2\). Inhabitants are uniformly distributed across the city and cannot choose their residence location. Each agent is indexed by his residence place, \(x\).

Let \(n(x)\) and \(N(x)\) denote the density and distribution function, respectively, so that
\[ n(x) = 1 \quad \text{and} \quad N(x) = x + \frac{1}{2} \]

Since the two jurisdictions have the same size and residents are uniformly distributed, both have the same number of inhabitants, \( \bar{N} = 1/2 \). The median resident of each jurisdiction coincides with the geographic center of the jurisdiction, i.e., \( m_H = -1/4 \) and \( m_L = 1/4 \). The employment centers are assumed to be symmetrically located in \( \gamma \) and \(-\gamma\) and located outwards from the median resident (\( \gamma > 1/4 \)). This opens up the possibility for a majority of residents of one jurisdiction to commute to the other one.

Firms located at the employment centers produce an homogeneous good according to a linear technology \( Y_i = \alpha_i N_i \), where \( Y_i \) is the output and \( N_i \) is the number of workers in jurisdiction \( i \). The two jurisdictions have unequal productivities. We use \( H \) to denote the high-productivity jurisdiction and \( L \) for the low-productivity one, with \( \alpha_H > \alpha_L \).

The government of each jurisdiction collects a head tax (\( T_i \)) paid by all its residents and, possibly, an ad-valorem source-based tax on wages (\( \tau_i \)) paid by all workers in the employment center of jurisdiction \( i \) to finance a public good budget \( G_i \).

The local government budget constraint is therefore
\[
G_i = T_i \bar{N} + \alpha_i \tau_i N_i
\]
where \( \alpha_i \) is the gross wage earned by workers in the employment center of jurisdiction \( i \) and \( N_i \) is the number of workers in that jurisdiction.

Agents support a per-mile commuting cost \( c \) and can choose to which employment center they want to commute (i.e., where they want to work). Commuting to the jurisdiction where they do not live is, therefore, more costly than commuting to the one where they live since the

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\[ ^6 \text{As stated in Peralta (2007) the assumption of a linear technology is not essential} \]

\[ ^7 \text{Note that in our setup the head tax} \ T_i \text{can be seen as land or residential property tax with fixed house size; since residence place is not chosen by agents this is a lump-sum tax.} \]
distance they must travel is higher. Each individual provides one unit of labor and pays a wage tax at the source so that the net wage earned by an individual working in \( j \) is \( \omega_j = \alpha_j(1 - \tau_j) \). All agents have a revenue \( W \) from other sources which is assumed to be high enough such that everyone can always pay his tax bill. Agents get utility both from private consumption and from the public good provided.

We follow Peralta (2007) and Braid (2000) and assume a quasi-linear utility function; however, differently from those authors, we allow individuals to enjoy both the public goods of their residence and work place. The utility enjoyed by individual \( x \), who lives in \( i \) and works in \( j \) is given by:

\[
\begin{align*}
 u_{ij}(x; \tau; G_i; G_j) &= \omega_j - T_i + W - c|x - EC_j| + (1 - k)v(G_i) + k\upsilon(G_j) \\
 i, j &= H, L
\end{align*}
\]  

where \( EC_j \) is the location of the employment center where the agent chooses to work (\( \gamma \) or \( -\gamma \)), \( G_i \) is the public good provided in the jurisdiction where he lives, \( G_j \) is the public good provided in the jurisdiction where he works and \( \upsilon(G) \) is an increasing concave function. We sometimes use the function \( \upsilon(G) = \sqrt{G} \) to illustrate some of our results. The intensity of the spillover effect due to having individuals deriving utility from the public goods provided in both jurisdictions is measured by the constant \( k \), where \( 0 \leq k \leq 1 \). When \( k \leq 1/2 \), \( G_i \) is more important than \( G_j \), i.e., agents care more for the public good provided in the jurisdiction where they live than for the one provided in the jurisdiction where they work.

Again, notice that this is not the standard spillover effect we can find in the literature. In our case agents only get utility from the public good provided in the other jurisdiction if they decide to work there, i.e., the spillover is endogenous. When they decide the working location they are also choosing the public good mix they want to consume.

### 1.2.1 The choice of the workplace

An agent works in the jurisdiction where he lives if \( u_{ii}(x; \tau; G_i; G_H) \) \(-\chi\) \( u_{ij}(x; \tau; G_i; G_L) \geq 0 \) and commutes to the other jurisdiction otherwise.

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*This is what is considered, for example, in Besley and Coate (2003) and would fit our model since we argue that agents are able to get utility from a wider variety of public goods provided in their residence place. However, the assumption of these boundaries for \( k \) is not necessary to reach the results of this paper so we choose not to impose them and leave the problem as general as possible.*
Looking at this utility difference we can calculate the marginal interjurisdictional commuter, denoted \( \hat{x} \). From (1) we can see that the difference between the utility obtained working in H and the one obtained by working in L is:

\[
    u_{iH} - u_{iL} = \begin{cases} 
    \omega_H - \omega_L + 2\gamma_c + k[v(G_H) - v(G_L)] & \text{if } x \leq -\gamma \\
    \omega_H - \omega_L + 2xc + k[v(G_H) - v(G_L)] & \text{if } -\gamma < x < \gamma \\
    \omega_H - \omega_L - 2\gamma c + k[v(G_H) - v(G_L)] & \text{if } x \geq \gamma 
    \end{cases}
\]

If \( u_{iH}(x; \tau) - u_{iL}(x; \tau) \) is positive the agent chooses to work in H, otherwise he chooses to work in L. Note that for \( |x| > \gamma \) the utility difference is independent from x which means that if one agents that lives between the employment center of a jurisdiction and its outer limit wants to commute to the other one, every agent wants to do the same. We assume away such non-interesting cases and focus on the situation where \(-\gamma < \hat{x} < \gamma\). The marginal ij-commuter \( \hat{x} \) is the one indifferent between working in H or L, therefore

\[
    \hat{x} = \frac{\omega_H - \omega_L + k[v(G_H) - v(G_L)]}{2c} \quad (1.2)
\]

This marginal interjurisdictional commuter \( \hat{x} \) defines a commuting equilibrium where all \( x < \hat{x} \) work in H and all \( x > \hat{x} \) work in L.

### 1.3 First Best

We now compute the utilitarian first best to use as a benchmark for the tax competition equilibrium analysis, i.e., the decision of a benevolent social planner that chooses the wage taxes, the residence taxes, the level of public good provided in each jurisdiction and allocates workers to an employment center so that overall utility is maximized.

The planner thus faces an overall budget constraint such that the provision of public goods must be fully paid by the wage and head taxes, i.e.,

\[
    G_H + G_L = \tau_H \alpha_H (\bar{N} + \hat{x}) + \tau_L \alpha_L (\bar{N} - \hat{x}) + \bar{N} (T_H + t_L) \quad (1.3)
\]

The problem faced by the social planner is therefore to maximize the overall utility of the population, which is equal to the sum of the
utility of all inhabitants of jurisdiction H \((U_H)\) and of all inhabitants of jurisdiction L \((U_L)\), subject to the budget constraint (3), by choosing \(\hat{x}, G_H\) and \(G_L\). It is never optimal to have H-residents commuting to L since their commuting cost is higher than if they work in H and their productivity is lower. Therefore, we can only have L residents commuting to H, i.e., \(\hat{x} \geq 0\), which allow us to calculate \(U_H\) and \(U_L\) as:

\[
U_H = \int_{-\frac{1}{2}}^{0} u_{HH} dx \quad (1.4)
\]

\[
U_L = \int_{0}^{\hat{x}} u_{LH} dx + \int_{\hat{x}}^{\frac{1}{2}} u_{LL} dx \quad (1.5)
\]

Denoting by \(C_i\) the total commuting costs of all the residents of jurisdiction \(i\), we have

\[
C_H = c \left[ \int_{-\frac{1}{2}}^{-\gamma} (-\gamma - x) dx + \int_{-\gamma}^{0} (x + \gamma) dx \right] = c \left( \frac{1}{8} + \gamma^2 - \frac{\gamma}{2} \right) \quad (1.6)
\]

\[
C_L = c \left[ \int_{0}^{\hat{x}} (x + \gamma) dx + \int_{\hat{x}}^{\gamma} (\gamma - x) dx + \int_{\gamma}^{\frac{1}{2}} (x - \gamma) dx \right] = C_H + c (\hat{x}^2) \quad (1.7)
\]

where the last term in \(C_L\) is the increase in commuting costs due to the interjurisdictional commuters which must travel a longer distance.

Total utility in each jurisdiction is therefore given by:

\[
U_H = \bar{N} [\omega_H - T_H + W + v(G_H)] - C_H \quad (1.8)
\]

\[
U_L = \bar{N} [\omega_L - T_L + W + v(G_L)] - C_H + \hat{x} [\omega_H - \omega_L + k\Delta(v)] - c (\hat{x}^2) \quad (1.9)
\]

where \(\Delta(v) = v(G_H) - v(G_L)\) and \(k\Delta(v)\) is the impact on utility of the consumption of the public good provided in jurisdiction H rather the one provided in L to interjurisdictional commuters.

Note that the two last terms of \(U_L\) are the gain to L of having interjurisdictional commuters. The novelty of our analysis is reflected on the term \(\Delta(v) = v(G_H) - v(G_L)\) generated by the spillover effect of the public goods: agents near the border of jurisdiction L now have two effects on utility when commuting to H: the difference in wage and the difference
in the level of public goods provided (weighted by \( k \) since they always get utility \((1 - k)\) from \( G_L \), the public good provided in the jurisdiction where they live).

Solving the social planner problem presented previously we can ignore the choice of the wage and head tax. Since the planner can allocate the workers to any of the employment centers, the fiscal instruments used to finance the public good is not relevant. The only thing that must be ensured is that the budget constraint is satisfied with these taxes. We can then assume \( \tau_i = 0 \) and finance the public goods exclusively with the head (lump-sum) taxes, which ensures the absence of any type of interjurisdictional transfers. Remember that the purpose of the calculation of the first best is to use it as a benchmark to compare with the tax competition equilibrium and so we want to keep it as neutral as possible.

The relevant first order conditions are therefore:

\[
\frac{\partial}{\partial \hat{x}} = 0 \iff \hat{x}^o = \frac{\alpha_H - \alpha_L + k \Delta(v)}{2c} \tag{1.10}
\]

\[
\frac{\partial}{\partial G_H} = 0 \iff \nu'(G_H) \left( \frac{1}{2} + \hat{x}k \right) = 1 \tag{1.11}
\]

\[
\frac{\partial}{\partial G_L} = 0 \iff \nu'(G_L) \left( \frac{1}{2} - \hat{x}k \right) = 1 \tag{1.12}
\]

Equation (10) gives the optimal interjurisdictional commuter \( \hat{x}^o \), which results from the trade-off between commuting costs and productivity gains and the public good level. \[9\]

Equations (11) and (12) express the Samuelson condition for the optimal provision of public goods. Since \( G_H \) provides \( k \)-weighted utility also to \( \hat{x} \) residents of \( L \), the marginal benefit of \( G_H \) is higher than without the spillover (reflected by the term \( \hat{x}k \)) while the inverse applies to \( G_L \).

\[10\]

### 1.4 The Equilibrium with Residence Taxes

Having calculated the conditions that define the first best, we can now compute the tax competition equilibrium and compare it to the utili-

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\[9\] As pointed in Peralta (2007).

\[10\] Comparing this condition with the one obtained in Peralta (2007) we can see that the difference lies exactly on the presence of the term \( k \Delta(v) \); the spillover makes agents consider the difference in public goods provision when deciding the work place since their utility depend on \( G_j \).
tarian optimum. In this section we assume that a government elected by majority rule in each jurisdiction decides the taxes and public goods levels. The elected policy is the one preferred by the median voter of each jurisdiction which in our model coincide with the median resident, i.e., $m_H = -1/4$ and $m_L = 1/4$.

In this section we compute the tax competition equilibrium when local governments only have access to the residence tax, $T_i$. We can then use this equilibrium to compare with the one resulting from the use of a distortive wage tax combined with a lump-sum tax, which is computed in the next section.

Each local government maximizes the utility of the median voter by choosing $G_i$ and $T_i$ subject to the commuting equilibrium, $\hat{x}$, in (2) and to the budget constraint of the jurisdiction when $\tau = 0$, i.e., $G_i = \hat{N}T_i$.

For the utility of the median voter we must separate the case where he commutes to the other jurisdiction from the case where he commutes to the employment center of his own jurisdiction. We present two cases, depending on whether the median voter of L works in L or H. While we cannot ensure in general that H does not commute to region L, we show in the appendix that this can never happen with $v(G) = \sqrt{G}$. The intuition for this resides on the fact that the gross wage earned in L is lower and the traveled distance by $m_H$ is much higher than if he decides to work in the employment center of H. For the median voter of L it can make sense to commute to H thanks to the increase in productivity.

The median voter of H thus enjoys an utility of:

$$u_{m_H} = \alpha_H + W - T_H - c\left(\frac{1}{4} + \gamma\right) + v(G_H)$$

For the median voter of L we must separate the case where he works in L from the case where he commutes to work in H. In the former case, since the only public good he consumes is $G_L$, we can compute his utility as:

$$u_{m_L} = \alpha_L + W - T_L - c\left(\gamma - \frac{1}{4}\right) + v(G_L)$$

However, if he decides to work in H he gets utility both from $G_L$ (weighted by $1-k$) and $G_H$ (weighted by $k$) and his utility is, therefore, given by:

$$u_{m_L} = \alpha_H + W - T_L - c\left(\frac{1}{4} + \gamma\right) + (1-k)v(G_L) + kv(G_H)$$
1.4.1 Median voter of L works in L

Let us first assume that the median voter of L works in the employment center of L, which happens when $\hat{x} < 1/4$. Solving the utility maximization problem for $m_H$ and $m_L$ we have the equilibrium levels of $G_H$ and $G_L$ implicitly defined by:

\[ v'(G^*_H) = 2 \]  
\[ v'(G^*_L) = 2 \]

(1.13)  
(1.14)

These conditions express the usual equality between marginal benefit and marginal cost. Since the population mass of each jurisdiction is 1/2, the marginal cost borne by the median voter to provide an additional unit of public good is 2.

If we compare the tax competition equilibrium obtained with the first-best we can state the following proposition:

**Proposition 1:** In the tax competition equilibrium where only the residence tax is available and both median voters work in their own jurisdictions:

(i) The local public good in jurisdiction H is underprovided and the one in jurisdiction L is overprovided;

(ii) There is undercommuting of agents.

*Proof.* See appendix.

The median voter of H does not take into account the spillover effect of the public good provided in H on the L-residents that commute to his jurisdiction and, therefore, considers a lower marginal benefit of $G_H$ when compared to the first-best. This leads to a situation of underprovision of this public good. Similarly, the median voter of L does not consider that a fringe $\hat{x}$ of the residents of L commute to H and, thus, get utility from $G_L$ weighted by k, leading to overprovision of $G_L$.

Since only the residence tax is being used agents decide their work place considering the gross wage earned and the public good provided in each jurisdiction. Knowing that $G_H$ is underprovided, jurisdiction H is less attractive than in the first-best solution, while jurisdiction L is more attractive due to the overprovision of $G_L$. Therefore, the number of agents commuting from L to H is lower than in the first best case.

Finally, we check that the equilibrium obtained respects the condition $\hat{x} < 1/4$, i.e., the median voter of L works in L.
From the first order conditions (13) and (14) we get that the equilibrium levels of public goods are the same in both jurisdictions:

\[ v'(G_H^*) = v'(G_L^*) = 2 \Leftrightarrow G_H^* = G_L^* \]

Thus, the marginal interjurisdictional commuter is

\[ \hat{x}^* = \frac{\alpha_H - \alpha_L}{2c} \]

Since the median voter of L works in L,

\[ \hat{x}^* = \frac{\alpha_H - \alpha_L}{2c} < \frac{1}{4} \Leftrightarrow \alpha_H - \alpha_L < \frac{c}{2} \]

Therefore, this is the condition that guarantees that the equilibrium exists.

### 1.4.2 Median voter of L works in H

If the median voter of L works in H he earns gross wage \( \alpha_H \) and gets utility from public goods provided by both jurisdictions. For the median voter of H everything remains the same, thus \( G_H \) is implicitly defined by (13). The key change for \( m_L \) is that the utility enjoyed thanks to the public good provided in his own jurisdiction is weighted by \( (1-k) \), hence \( G_L \) is implicitly defined by:

\[ (1-k)v'(G_L^*) = 2 \quad (1.15) \]

The level of public good provided in jurisdiction L is therefore lower than in the previous case since the marginal benefit of \( G_L \) to \( m_L \) is smaller.

This allows us to establish the following result.

**Proposition 2:** *In the tax competition equilibrium where only the residence tax is available and both median voters work in the high-productivity jurisdiction, both local public goods are underprovided.*

**Proof.** See appendix. \( \square \)

As in the previous case, the median voter of H does not take into account the spillover effect of the public good provided in H on the L-residents that commute to his jurisdiction, which leads to the under-provision of \( G_H \). Regarding \( G_L \), since the median voter of L works in
H he does not take into consideration that part of the residents in L get utility \(v(G_L)\) from it instead of \((1-k)v(G_L)\), which results in the underprovision of this public good.

In this case, we may have both under or overcommuting at the tax competition equilibrium. This stems from the fact that both jurisdiction are less attractive than they are in the first best solution due to the underprovision of both public goods.

We now check that the equilibrium obtained respects the condition \(\hat{x} > 1/4\), i.e., the median voter of L commutes to H.

Since we are unable to provide general conditions for this, we choose to illustrate it with a particular utility function, \(v(G) = \sqrt{G}\), thereby showing that this equilibrium is possible.

\[
v'(G_i) = \frac{1}{2\sqrt{G_i}}
\]

From the first order conditions (13) and (15) we get the equilibrium levels for \(G_H\) and \(G_L\)

\[
\frac{1}{2\sqrt{G_H^*}} = 2 \Leftrightarrow G_H^* = \frac{1}{16}
\]

\[
\frac{1}{2\sqrt{G_L^*}} = \frac{2}{1-k} \Leftrightarrow G_L^* = \frac{(1-k)^2}{16}
\]

Thus, the marginal interjurisdictional commuter is

\[
\hat{x}^* = \frac{\alpha_H - \alpha_L + k}{2c} \left[ \sqrt{\frac{1}{16}} - \sqrt{\frac{(1-k)^2}{16}} \right] = \frac{\alpha_H - \alpha_L + k^2}{2c}
\]

For the median voter of L to work in H,

\[
\hat{x}^* = \frac{\alpha_H - \alpha_L + k^2}{4} > \frac{1}{4} \Leftrightarrow \alpha_H - \alpha_L + \frac{k^2}{4} < \frac{c}{2}
\]

Which is the condition that guarantees that, when \(v(G) = \sqrt{G}\), the equilibrium exists.
1.5 The Tax Competition Equilibrium with Residence and Wage Taxes

We now focus on the tax competition equilibrium attained when local governments can use both the residence (lump-sum) and the wage (distortive) tax.

As a matter of fact, agents are now concerned with the net wage they earn in each employment center rather than the gross wage dictated by their productivity. This means that local governments, when deciding the wage tax level, face a trade-off between financing the public good and reducing the number of interjurisdictional commuters due to the reduction of the net wage in the jurisdiction.

As in the previous framework, each local government maximizes the utility of the median voter by choosing \( G_i, \tau_i \) and \( T_i \), subject to the commuting equilibrium \( \hat{x} \) in (2) and to the budget constraint of the jurisdiction \( G_i = \bar{N}T_i + \tau_i \alpha_i N_i \).

Remember that \( N_i \) is the number of agents working in the employment center of jurisdiction \( i \) so that \( N_H = \frac{1}{2} + \hat{x} \) and \( N_L = \frac{1}{2} - \hat{x} \).

Again, for the utility of the median voter we must separate the case where he commutes to the other jurisdiction from the case where he commutes to the employment center of his own jurisdiction. For the median voter of H, and as we did in the previous section, we know that he never commutes to jurisdiction L since the gross wage is lower and the traveled distance is much higher than if he decides to work in the employment center of H. For the median voter of L it can make sense to commute to H thanks to the increase in productivity. Therefore, the median voter of H enjoys an utility of:

\[
 u_{mH} = (1 - \tau_H) \alpha_H + W - T_H - c\left(\gamma - \frac{1}{4}\right) + \nu(G_H)
\]

For the median voter of L, his utility when he works in his own jurisdiction is given by:

\[
 u_{mL} = (1 - \tau_L) \alpha_L + W - T_L - c\left(\frac{1}{4} + \gamma\right) + \nu(G_L)
\]

If he decides to work in H he gets utility both from \( G_L \) (weighted by \( 1 - k \)) and \( G_H \) (weighted by \( k \)) and his utility is, therefore, given by:

\[
 u_{mL} = (1 - \tau_H) \alpha_H + W - T_L - c\left(\frac{1}{4} + \gamma\right) + (1 - k)\nu(G_L) + kv(G_H)
\]
1.5.1 Median voter of $L$ works in $L$

Let us first assume that the median voter of $L$ works in the employment center of $L$, which happens when $\hat{x} < 1/4$. Solving the utility maximization problem for $m_H$ and $m_L$ we have the equilibrium levels of $G_H$ and $G_L$ implicitly defined by:

$$v'(G_{H}^{**}) = 2 \left[ 1 - \tau_H \alpha_H \frac{k}{2c} v'(G_{H}^{**}) \right]$$  \hspace{1cm} (1.16)

$$v'(G_{L}^{**}) = 2 \left[ 1 - \tau_L \alpha_L \frac{k}{2c} v'(G_{L}^{**}) \right]$$  \hspace{1cm} (1.17)

These conditions express the usual equality between marginal benefit and marginal cost. Note that the marginal cost is affected by the term $\tau_i \alpha_i (k/2c) v'(G_i)$, which is the impact of the level of public good on the government budget due to interjurisdictional commuters, whose choice of working place is driven by public good provision. This means that increasing the provision of the public good increases the number of workers subject to the wage tax, affecting the cost borne by the median voter.

If we now look at the first order conditions that define reaction functions on $\tau_H$ and $\tau_L$ and combine them we obtain the equilibrium levels of the wage taxes given by\textsuperscript{11}

$$\tau_H^{**} = \frac{\alpha_H - \alpha_L + k \Delta^{**}(v)}{3 \alpha_H}$$

$$\tau_L^{**} = -\left( \frac{\alpha_H - \alpha_L - k \Delta^{**}(v)}{3 \alpha_L} \right)$$

which yields the equilibrium marginal interjurisdictional commuter:

$$\hat{x}^{**} = \frac{\alpha_H - \alpha_L + k \Delta^{**}(v)}{6c}$$

With these expression we can show that, in equilibrium, $\tau_H^{**} \alpha_H > \tau_L^{**} \alpha_L$ and $G_H^{**} > G_L^{**}$ since the opposite relations are ruled-out by the condition $\hat{x}^{**} > 0$\textsuperscript{12}

The characterization of the tax competition equilibrium is provided in the following proposition:

\textsuperscript{11}The full first order conditions can be found in the appendix.

\textsuperscript{12}The proof can be found in the appendix.
Proposition 3: In the tax competition equilibrium where both the residence and the wage taxes are available and both median voters work in their own jurisdictions:

(i) The wage is taxed in H and subsidized in L;
(ii) The local public good in jurisdiction H is underprovided while the one in jurisdiction L is overprovided;
(iii) There is undercommuting of agents.

Proof. See appendix.

The result that region H taxes wages while region L subsidizes them is also obtained by Peralta(2007): H residents are exporting part of their tax burden to the interjurisdictional commuters from region L using the wage tax and since the median voter of L works in L, he uses the head tax to impose a higher tax burden to the interjurisdictional commuters, which does not receive the wage subsidy. What we are seeing is a transfer of income from the interjurisdictional commuters to everyone else.

As for the provision of public goods, agents in H have a marginal cost of $G_H$ lower than those in L. Since both the median voters of H and L are exporting part of the tax burden to the L interjurisdictional commuters we have two effects: for H residents, $G_H$ is less expensive due to the tax export and due to the fact that by increasing $G_H$ the number of such commuters increase, which makes it even less expensive; for the median voter of L increasing $G_L$ decreases the number of commuters, which increases the marginal cost.

Comparing the levels of public good provided in each jurisdiction with the first-best solution, we reach an intuitive underprovision of the public good of jurisdiction H and overprovision of the one of jurisdiction L, as in the previous section.

All these distortions lead to undercommuting. This is easily explained by the fact that jurisdiction H is less attractive, while jurisdiction L is more attractive than in the first best case. A lower net wage earned in the employment center of H (due to the positive wage tax $\tau_H$) and a lower level of $G_H$ make jurisdiction H not so appealing while the opposite happens for L (with subsidized wages and higher provision of $G_L$).

1.5.2 Median voter of L works in H

We shall now analyse the Nash equilibrium where the median voter of L works in the employment center of H, i.e., he $ij$-commutes. Note that
the problem for the median voter of H remains unchanged, and \( G_H \) is implicitly defined by (16). However, for \( m_L \) his utility is now given by:

\[
u_{m_L} = (1 - \tau_H) \alpha_H + W - T_L - c \left( \frac{1}{4} + \gamma \right) + (1 - k)u(G_L) + ku(G_H)
\]

Recall that the difference to the previous case is that the median voter of L gets \((1 - k)\)-weighted utility from \( G_L \) and \( k \)-weighted utility from \( G_H \), the public good provided where he works.

The decision on \( G_L \) is given by:

\[
(1 - k)u'(G_{L}^{**}) = 2 \left[ 1 - \tau_L \alpha_L \frac{k}{2c} u'(G_{L}^{**}) \right]
\]

The marginal benefit of \( G_L \) for the median voter of L is weighted by \((1 - k)\) instead of 1, since he works in H and therefore gets \( k \)-weighted utility from \( G_H \). The expression for the marginal cost is the same as before.

Regarding the wage taxes, combining the first order conditions from the problems of \( m_H \) and \( m_L \) we get the following equilibrium expressions:\[13\]

\[
\tau_H^{**} = \frac{\alpha_H - \alpha_L + c + k \Delta^{**}(v)}{3 \alpha_H}
\]

\[
\tau_L^{**} = \frac{2c - (\alpha_H - \alpha_L) - k \Delta^{**}(v)}{3 \alpha_L}
\]

which yields the equilibrium marginal interjurisdictional commuter:

\[
\hat{x}^{**} = \frac{\alpha_H - \alpha_L + k \Delta^{**}(v)}{6c} + \frac{1}{6}
\]

The next proposition characterizes the tax competition equilibrium:

**Proposition 4:** In the tax competition equilibrium where both the residence and the wage taxes are available and both median voters work in the high-productivity jurisdiction:

(i) The wages are taxed in H and in L;

(ii) The public good in jurisdiction H is underprovided;

\[13\]The full first order conditions can be found in the appendix.
Proof. See appendix.

In this situation no jurisdiction is willing to subsidize wages. The median voter of L is not willing to subsidize the wage in L due to the fact that he is not working in that jurisdiction. Since he is now one of the interjurisdictional commuters he wants to use $\tau_L$ to finance the budget of L because he is not subject to such tax.

As for the provision of public goods, the intuition is basically the same as in the previous case, with the additional fact that on the choice of $G_L$ the marginal benefit for $m_L$ is now smaller since it is weighted by $(1 - k)$.

We can still show that $G_H$ is underprovided, but regarding the public good of jurisdiction L and the number of commuters we cannot be sure how the equilibrium levels compare with the first best. We can only say that if we have overprovision of $G_L$ we have undercommuting (jurisdiction L is more attractive than it should) and if we have overcommuting we have underprovision of $G_L$. However, the opposite implications are not valid.

The intuition for the underprovision of $G_H$ is the same as before, but now for $G_L$ all we know is that, comparing to the case where the median voter of L works in L and we are able to say that it is overprovided, the marginal benefit is now lower due to the $(1 - k)$ weight. This implies that the $G_L$ level chosen by $m_L$ is lower, but we cannot be sure if this reduction is such that it is not overprovided: it can be above the first best or it can be below the first best. This uncertainty about the under or overprovision of $G_L$ extends to the commuting level since it is a determinant of the desirability of working in L.

1.6 Only Residence Tax vs. Residence and Wage Taxes

In this section we compare the tax competition equilibrium obtained when local governments only use the lump-sum head tax to the one when both the lump-sum head tax and the distortive wage tax are used.

Following the structure of the previous sections, we first focus on the case where the median voter of L works in L. Comparing the two tax competition equilibria we achieve a second-best result induced by the use of the distortive tax:

---

14 The proof can be found in the appendix.
Proposition 5: When both median voters work in their own jurisdictions, the use of the distortive tax enhances the provision of the public goods vis-a-vis the case where only the lump-sum tax is used.

Proof. See appendix.

As a matter of fact, the proof shows that:

\[ G_H^O > G_H^{***} > G_H^* \]
\[ G_L^O < G_L^{***} < G_L^* \]

The distortion introduced by the wage tax partially offsets the inefficiency created by the tax competition equilibrium due to the spillover effect of the public goods to the interjurisdictional commuters. This is a typical second-best result where the introduction of two distortions (the wage tax and the inter-jurisdictional externalities) improves upon the case where only one distortion is present. The tax export generated by the wage tax on H reduces the marginal cost to the policy-maker in H, thus leading him to provide a higher level of \( G_H \), thus getting closer to the optimal provision. The reverse applies to L where the overprovision is reduced by the introduction of the wage subsidy that increases the cost of provision to \( m_L \).

When we look at the case where the median voter of L works in H the achieved result is not so strong:

Proposition 6: When the median voter of H works in H while the median voter of L is an interjurisdictional commuter, the use of the distortive tax increases the level of public goods provided in both jurisdictions vis-a-vis the case where only the lump-sum tax is used.

Proof. See appendix.

The proof shows that:

\[ G_H^O > G_H^{***} > G_H^* \]
\[ G_L^{***} > G_L^* \]

Note that we can no longer say for sure that the provision of both public goods is enhanced with the introduction of the distortive wage tax. We can be sure of such enhancement regarding \( G_H \), but when we look at \( G_L \) we may be facing an increase that changes the situation of
underprovision to overprovision, since we are not sure of the comparison between $G^{*}_L$ and $G^{o}_L$ as seen in the previous section.

1.7 Conclusion

This paper introduces commuting-related spillovers in a duo-centric linear city where local governments provide public goods and agents choose in which region they want to work.

We show that in the tax competition equilibrium the public goods provided in the most productive region is always underprovided and the one provided in the less productive region can be under or overprovided. Furthermore, we showed that the use of the distortive tax tends to be preferred to the single use of a lump sum tax in terms of the provision of the public goods as it partially offsets the distortion introduced by the endogenous spillover effect.

Since the results were obtained using very general assumptions they are quite robust since they do not depend on explicit functional forms for, e.g., $v(G_i)$. The two kinds of taxes considered are also currently used in real world countries, such as U.S. states as referred in the introduction and their application is, therefore, reasonable and feasible.
1.8 Appendix

Proof of Proposition 1.

(i) $v'(G^o_H) - v'(G'^*_H) = \frac{2}{1+2\hat{x}^o k} - \frac{2}{1} < 0$ since $1 + 2\hat{x}^o k > 1$. Thus, $G^o_H > G'^*_H$.

$\therefore v'(G^o_L) - v'(G'^*_L) = \frac{2}{1-2\hat{x}^o k} - \frac{2}{1} > 0$ since $1 - 2\hat{x}^o k < 1$. Thus $G^o_L < G'^*_L$.

(ii) $\hat{x}^o - \hat{x}^* = \frac{\alpha_H - \alpha_L + k[v(G^o_H) - v(G^*_H)]}{2c} - \frac{\alpha_H - \alpha_L + k[v(G^o_L) - v(G^*_L)]}{2c}$

Since $G^o_H > G^*_H$ and $G^o_L < G^*_L$, $\hat{x}^o - \hat{x}^* > 0 \iff \hat{x}^o > \hat{x}^*$ 

Proof of Proposition 2.

For $G_H$ please check the proof of proposition 1 as the problem is the same.

$v'(G^o_L) - v'(G^*_L) = \frac{2}{1-2\hat{x}^o k} - \frac{2}{1-\hat{x}^o}$

Using the fact that $\hat{x}^o \in (0; \frac{1}{2})$, we can state that $1 - k < 1 - 2\hat{x}^o k$, i.e., $v'(G^o_L) - v'(G^*_L) < 0$. Thus, $G^o_L > G^*_L$

Proof of Proposition 3.

(i) $\tau_H^{**} = \frac{\alpha_H - \alpha_L + k[v(G^o_H) - v(G^*_H)]}{3\alpha_H}$

Therefore, $\tau_H^{**} > 0$ since $G^{**}_H > G^*_H$.

$\tau_L^{**} = \frac{-3(\alpha_H - \alpha_L - k[v(G^o_L) - v(G^*_L)]}{3\alpha_L}$

Therefore, $\tau_L^{**} < 0$ since $G^{**}_H > G^*_L$.

(ii) $v'(G^o_H) - v'(G^*_H) = \frac{2}{1+2\hat{x}^{**} k} - \frac{2}{1+2\hat{x}^o k}$

Thus, $G^o_H > G^*_H$ since $\hat{x}^o > \hat{x}^{**}$.

$v'(G^o_L) - v'(G^*_L) = \frac{2}{1-2\hat{x}^o k} - \frac{2}{1-2\hat{x}^{**} k}$

Thus, $G^o_L < G^*_L$ since $\hat{x}^o > \hat{x}^{**}$.

(iii) $\hat{x}^o - \hat{x}^{**} = \frac{\alpha_H - \alpha_L + k[v(G^o_H) - v(G^*_H)]}{2c} - \frac{\alpha_H - \alpha_L + k[v(G^o_L) - v(G^*_L)]}{2c}$

Therefore, $\hat{x}^o > \hat{x}^{**}$ since $G^o_H > G^{**}_H$ and $G^o_L < G^*_L$ 

□
Proof of Proposition 4.

(i) $\tau_H^{**} \alpha_H^{**} = \frac{\gamma}{3} + 2c (\hat{x}^{**} - \frac{1}{6})$

Since $\hat{x}^{**} \in \left(\frac{1}{4}; \frac{1}{2}\right)$, $\tau_H^{**} > 0$

$\tau_L^{**} \alpha_L^{**} = \frac{2}{3}c - 2c (\hat{x}^{**} - \frac{1}{6})$

Since $\hat{x}^{**} \in \left(\frac{1}{4}; \frac{1}{2}\right)$, $\tau_L^{**} \alpha_L^{**} \in \left(0; \frac{1}{2}\right)$, which allow us to state $\tau_L^{**} > 0$

(ii) For $G_H$ please check the proof of proposition 3 as the problem is the same.

\[\square\]

Proof of Proposition 5.

\[v'(G_H^*) - v'(G_H^{**}) = 2 - \frac{2}{1 + 2^{2\hat{x}^{**}} - 1}\]

Since $1 + 2\hat{x}^{**} > 1$, we can state that $v'(G_H^*) - v'(G_H^{**}) > 0$, i.e., $G_H^* < G_H^{**}$

\[v'(G_L^*) - v'(G_L^{**}) = \frac{2}{1-k} - \frac{2}{1+k(2\hat{x}^{**} - \frac{1}{3})}\]

Since $1 - 2\hat{x}^{**} < 1$, we can state that $v'(G_L^*) - v'(G_L^{**}) < 0$, i.e., $G_L^* > G_L^{**}$

\[\square\]

Proof of Proposition 6.

\[v'(G_H^*) - v'(G_H^{**}) = \frac{2}{1-k} - \frac{2}{1+k(\frac{4}{3} - 2\hat{x}^{**})}\]

Since $\hat{x} \in \left(\frac{1}{4}; \frac{1}{2}\right)$, we can state that $1 + k (\frac{4}{3} - 2\hat{x}^{**}) > 1 - k$, thus allowing to conclude that $v'(G_L^*) - v'(G_L^{**}) > 0$, i.e., $G_L^* < G_L^{**}$

\[\square\]

The median voter of $H$ is not willing to work in $L$ in section 4.

If the median voter of $H$ commutes to $L$, $\hat{x} < -1/4$ and the local government in $H$:

\[
\begin{align*}
\max_{G_i, T_i} u_{mH} & = \alpha_L + W - T_H - c \left(\frac{1}{4} + \gamma\right) + (1-k)\sqrt{G_H} + k\sqrt{G_L} \\
\text{s.t.} & \quad \hat{x} = \frac{\alpha_H - \alpha_L + k (\sqrt{G_H} - \sqrt{G_L})}{2c} \\
& \quad G_H = \frac{1}{2}T_H
\end{align*}
\]
FOC:
\[ \frac{1}{2\sqrt{G_H}} = \frac{2}{1-k} \iff \sqrt{G_H^*} = \frac{1-k}{4} \]

The first order condition in \( G_L \) is the same as (14) which leads to \( \sqrt{G_L^*} = 1/4 \).

The commuting equilibrium is therefore defined by:
\[ \hat{x} = \alpha_H - \alpha_L + k \left( \frac{1+k}{4} - \frac{1}{4} \right) \]

and since \( k \in (0; 1) \)
\[ \hat{x} \in \left( \frac{\alpha_H - \alpha_L}{2c}, \frac{\alpha_H - \alpha_L}{2c} \right) > -\frac{1}{4} \]

Thus, and as expected, it is impossible to have the median voter of H commuting to L since he would bear a higher commuting cost and earn a lower wage.

□

FOC of the utility maximization problems is section 5.1.

\[ \frac{\partial U^*_{mH}}{\partial G_H} = 0 \iff -2 \left[ 1 - \tau_H \alpha_H \frac{\partial \hat{x}}{\partial G_H} \right] + u'(G_H) = 0, \text{ which after straightforward computations leads to } u'(G_H^*) = \frac{2c}{\tau_H \alpha_H k + c}. \]

\[ \frac{\partial U^*_{mH}}{\partial \tau_H} = 0 \iff -\alpha_H - 2 \left[ -\alpha_H \left( \frac{1}{2} + \hat{x} \right) + \tau_H \alpha_H \frac{\partial \hat{x}}{\partial \tau_H} \right] = 0, \text{ which after straightforward computations leads to } \tau_H^* = \frac{\alpha_H - (1-\tau_L) \alpha_L + k \Delta(u)}{2\alpha_H}. \]

\[ \frac{\partial U^*_{mL}}{\partial G_L} = 0 \iff -2 \left[ 1 - \tau_L \alpha_L \frac{\partial \hat{x}}{\partial G_L} \right] + u'(G_L) = 0, \text{ which after some algebra results in } u'(G_L^*) = \frac{2c}{\tau_L \alpha_L k + c}. \]

\[ \frac{\partial U^*_{mL}}{\partial \tau_L} = 0 \iff -\alpha_L - 2 \left[ -\alpha_L \left( \frac{1}{2} + \hat{x} \right) - \tau_L \alpha_L \frac{\partial \hat{x}}{\partial \tau_L} \right] = 0, \text{ which after some algebra results in } \tau_L^* = \frac{\alpha_L - (1-\tau_L) \alpha_H + k \Delta(u)}{2\alpha_L}. \]

Combining the two reaction functions on the wage tax we get:
\[ \tau_H^* = \frac{\alpha_H - \alpha_L + k [u(G_H^*) - u(G_L^*)]}{3\alpha_H} \]
\[ \tau_L^* = \frac{-(\alpha_H - \alpha_L) - k [u(G_H^*) - u(G_L^*)]}{3\alpha_L} \]

28
Proof that $\tau^*_H \alpha_H > \tau^*_L \alpha_L$ and $G^*_H > G^*_L$ in section 5.1.

$$
\tau^*_H \alpha_H = \frac{\alpha_H - \alpha_L + k(v(G^*_H) - v(G^*_L))}{3} \\
\tau^*_L \alpha_L = \frac{-\alpha_H - \alpha_L - k(v(G^*_H) - v(G^*_L))}{3} \\
\tau_H^* \alpha_H - \tau_L^* \alpha_L = \frac{2}{3} (\alpha_H - \alpha_L + k [v(G^*_H) - v(G^*_L)])
$$

If $\tau^*_H \alpha_H > \tau^*_L \alpha_L$, $G^*_H > G^*_L$, thus meaning that, $\tau^*_H \alpha_H > \tau^*_L \alpha_L$ and $x^{**} > 0$, which is ok.

If $\tau^*_H \alpha_H < \tau^*_L \alpha_L$, $k [v(G^*_H) - v(G^*_L)] > \alpha_H - \alpha_L$, thus implying that $x^{**} < 0$, which is impossible.

\[\square\]

FOC of the utility maximization problems in section 5.2.

The FOC on $U_{mH}$ are the same as in section 5.1.

$$
v'(G^*_H) = \frac{2c}{\tau_H \alpha_H k + c} \\
\tau_H^* = \frac{\alpha_H - (1-\tau) \alpha_L + k \Delta(v)}{2a_H}
$$

For $U_{mL}$ we have:

$$
\frac{\partial U_{mL}}{\partial t_L} = 0 \iff -2 \left(1 - \tau_L \alpha_L \frac{\partial \hat{x}}{\partial t_L} \right) + (1 - k)v'(G_L) = 0, \text{ which results in } v'(G^*_L) = \frac{2c}{\tau_L \alpha_L k + (1-k)c} \\
\frac{\partial U_{mL}}{\partial t_L} = 0 \iff 2 \left[\alpha_L \left(\frac{1}{2} - \hat{x}\right) + \tau_L \alpha_L \frac{\partial \hat{x}}{\partial t_L} \right] = 0, \text{ which results in } \tau^*_L = \frac{c+\alpha_L - (1-\tau_H) \alpha_H - k \Delta(v)}{2a_L}
$$

Combining the two reaction functions on the wage tax we get:

$$
\tau_H^* = \frac{c+(\alpha_H - \alpha_L) + k [v(G^*_H) - v(G^*_L)]}{3a_H} \\
\tau_L^* = \frac{2c - (\alpha_H - \alpha_L) - k [v(G^*_H) - v(G^*_L)]}{3a_L}
$$

Proof that, in section 5.2,

(i) Overprovision of $G_L$ implies undercommuting of agents;

(ii) Overcommuting of agents implies underprovision of $G_L$.

$$
v'(G^*_L) - v'(G^*_L) = \frac{2}{1-2x^{**}k} - \frac{2}{1-2x^{**}k - \frac{2k}{k}}
$$

Since the numerators of both fractions are the same we can compare just
the denominators:

\[ [1 - 2\hat{x}^o k] - [1 - 2\hat{x}^{**} k - \frac{2}{3} k] = 2k \left[ \frac{1}{3} - (\hat{x}^o - \hat{x}^{**}) \right] \]

Overprovision of \( G_L \), i.e., \( G_L^o < G_L^{**} \), implies that \( v'(G_L^o) > v'(G_L^{**}) \).

Therefore, \( 2k \left[ \frac{1}{3} - (\hat{x}^o - \hat{x}^{**}) \right] < 0 \). For this to be true, \( \hat{x}^o > \hat{x}^{**} \).

Overcommuting of agents, i.e., \( \hat{x}^o < \hat{x}^{**} \), implies that \( 2k \left[ \frac{1}{3} - (\hat{x}^o - \hat{x}^{**}) \right] > 0 \). Therefore, \( v'(G_L^o) < v'(G_L^{**}) \), i.e., \( G_L^o > G_L^{**} \). 

\[ \square \]
Chapter 2

Endogenous asymmetric transportation costs and the impact of toll usage

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2.1 Introduction

In the last decades several cities worldwide implemented tolls paid by drivers who want to drive in the city center during working days between early morning and late afternoon. Singapore was the first city to effectively implement a congestion pricing scheme - the Singapore Area Licensing Scheme - in 1975, based on paper licenses; this scheme was later replaced by an electronic charging scheme - the Electronic Road Pricing (ERP). An impact of the successes and shortcomings of the Singapore road pricing can be found in Phang and Toh (2004). London introduced in 2003 a congestion charge which is levied electronically on vehicles traveling on the London Congestion Charge Zone (CCZ), which results have been analyzed for example by Prud’homme and Bocarejo (2005), Leape (2006), Givoni (2011) or Dudley (2013). Stockholm implemented in 2007 the Stockholm Congestion Tax which is levied electronically on vehicles entering and exiting central Stockholm. Milan charges vehicles traveling on the traffic restricted zone (ZTL) with a scheme which started in 2008 with the name “Ecopass" and then took the name “Area C" in 2012. Göteborg implemented in 2013 a toll similar to the one charged in Stockholm.

Although the main reasons invoked to charge these city tolls are the reduction of congestion and environmental concerns, the revenue levied with the tolls is usually directed to the operation, maintenance, and construction of transportation infrastructure. For example, the London Congestion Charge is one of the sources of revenue of Transport for London, having generated a revenue of £222 million in 2013 (about 5% of the total revenue, which is mainly composed of public transportation fares, according to the 2013 Transport for London Annual Report). But besides the cases where the main argument behind a toll is congestion, many tolls are charged with the sole purpose of financing the expenditures generated by an infrastructure, e.g., a highway, a bridge or a tunnel. In Portugal, almost all highways are fully (or at least partially) tolled. In Lisbon, the two available bridges that can be crossed by travelers arriving from Tagus river south bank are tolled, and the proceeds are used to fund the maintenance and operation of the bridges.

Daily commuters are the main users of these infrastructures: they often live in one city and work in another, therefore needing to travel daily for several miles in order to reach their employment. The commuting distance traveled by workers on a daily basis can be quite long, as well as the time spent in commuting. Data for Great Britain indicate that average distance commuted to work in inner London increased from 8.8 km in 2001 to 11.2 km in 2011 (UK Office for National Statistics...
Census 2011), while the average time taken to travel to work in central London was, in 2012, of 54 minutes (UK Department for Transport annual publication “Transport Statistics Great Britain” 2013). These facts put pressure on local governments to improve the transportation infrastructure in order to reduce the transportation costs for their voters (monetary cost and/or time spent in travels), while also meaning that commuters use the transportation infrastructure of more than one jurisdiction. Therefore, whenever a local government improves the transportation infrastructure, it is serving all agents using that infrastructure, thus generating positive spillovers to the residents of nearby cities.

Spillovers are, thus, generated by commuting from one region to another, making agents enjoy public goods in both the municipality where they live and where they work. In this paper we look at the impact of local level fiscal decentralization on the provision of public goods that determine transportation cost in a framework where agents commute. We may consider several types of local governments, from jurisdictions in one city to neighboring cities or even states, as long as it is expectable that agents commute from one to the other.

The autonomy of local governments regarding tax collection is far from being homogeneous. Different local governments have access to different types of taxes, e.g., residence-based wealth or income taxes, source-based income taxes, or “hybrid” taxes. These taxes are usually combined with grants or transfers from the central governments to form the total local budget.

Tolls to have access to the city center are in practice very close to the “hybrid” taxes used, for example, in Kansas City, St. Louis, Wilmington, Detroit, New York City or Philadelphia (Braid, 2009). These “hybrid” taxes are characterized by the fact that all central city residents are taxed at a rate, irrespective of where they work, while residents in the suburbs who work in the central city can be taxed at a different rate. This is precisely the effect of a toll at the entrance of a city or jurisdiction: while the residents are not subject to its payment, the residents in the adjoining cities (or in the suburbs) must pay it, therefore resulting in different amounts being charged to each type of worker.

The contribution of this paper is the introduction of endogenous and asymmetric transportation costs in the framework of a linear city allowing, simultaneously, to study the impact of charging a toll on the pro-

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1 Unlike the case of the exogenous spillover effect considered, e.g., in Besley and Coate (2003) that follows the traditional Oates’s spillover effect.

2 The OECD (2009) study regarding the fiscal autonomy of sub-central governments provides a thorough analysis of this topic. Braid (2005 and 2009) also provide several examples of taxes and states or countries that use each kind of tax.
vision of transportation infrastructure on cities. The linear city model is the one used by, e.g., Braid (2000) and Peralta (2007) to tackle interjurisdictional tax spillovers. The city is divided into two jurisdictions and agents choose where they want to work. Productivity, and thus wages, differ across regions and individuals trade-off the advantages (i.e., wage and working conditions) of a given job against travel costs (distance, time, and money) when choosing their workplace. The travel cost differs from one jurisdiction to the other since it depends on the level of local public good (which can be understood as transportation infrastructure) being provided in each jurisdiction. This formulation implicitly introduces a spillover effect on the public good provided in a jurisdiction that provides employment to the residents of the other one. We take the residence of agents as given, assuming independence between residence and working choices. This independence is argued by Wildasin (1986) and supported by empirical evidence provided by Rouwendal and Meijer (2001), Glaeser et al. (2001) and Zax (1991 and 1994).

We take a representative agent who derives utility from both the public good provided in the residence location and in the working place. This means that agents only benefit from the transportation cost resulting from the public good supplied in the other jurisdiction if they choose to work there. Otherwise they are only interested in the transportation infrastructure provided in their own jurisdiction. We abstract from other forms of interjurisdictional commuting, e.g., for leisure or shopping. A worker uses the transportation infrastructure daily and is therefore much more important.

Our main results are as follows. Firstly, the toll distorts the number of interjurisdictional commuters. However, it may improve the provision of the public good in the high productivity jurisdiction, where the transportation infrastructure is underprovided if funded only by residence taxes, as it decreases the marginal cost faced by the median voter of that jurisdiction. A simulation shows that, even though the provision of the public in the high productivity jurisdiction may be closer to the optimal, the overall utility is decreased by the introduction of the toll due to the reduction of interjurisdictional commuters that it imposes.

This paper is organized as follows. In Section 2 we present the model. Section 3 computes the first best, which is then used as a benchmark to compare the results obtained in Sections 4 and 5, i.e., the tax competition equilibrium where only a lump sum residence tax is used, and where both the residence tax and a toll on interjurisdictional commuters are used, respectively. Section 6 concludes.
2.2 The Model

We consider a linear city divided into two jurisdictions with the same size. Each jurisdiction has an employment center where agents can work. The total number of residents of the city is normalized to 1, as well as the city size, with extreme points of the segment -1/2 and 1/2. Inhabitants are uniformly distributed across the city and cannot choose their residence location. Each agent is indexed by his residence place, $x$.

Let $n(x)$ and $N(x)$ denote the density and distribution function, respectively, so that

$$n(x) = 1 \quad \text{and} \quad N(x) = x + \frac{1}{2}$$

Since the two jurisdictions have the same size and residents are uniformly distributed, both have the same number of inhabitants, $\bar{N} = 1/2$. The employment centers are assumed to be symmetrically located in the center of each jurisdiction, i.e., $-1/4$ and $1/4$. The maximum travel distance from a point in the jurisdiction and the employment center is, therefore, $1/4$. Since we are analyzing the choice of a local public good with impact on the transportation costs, the median voter of each jurisdiction is the one that travels the median distance, i.e., $m_H = -1/8$ and $m_L = 1/8$.

Firms located at the employment centers produce an homogeneous good according to a linear technology $Y_i = \alpha_i N_i$, where $Y_i$ is the output and $N_i$ is the number of workers in jurisdiction $i$. The two jurisdictions have unequal productivities. We use $H$ to denote the high-productivity jurisdiction and $L$ for the low-productivity one, with $\alpha_H > \alpha_L$.

The government of each jurisdiction collects a head tax ($T_i$) paid by all its residents and, possibly, a toll situated at point 0 paid by the interjurisdictional commuters that are arriving at the jurisdiction, to finance a public good $G_i$.

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3 As a matter of fact, if we assume that the median voter works in his own jurisdiction, there are two agents that travel the same distance from their residence to the employment center: for the high productivity jurisdiction, both the one that stands at $-3/8$ and the one that stands at $1/8$ must travel a distance of $1/8$, while for the low productivity jurisdiction both the agents at $1/8$ and $3/8$ travel the same distance. We can therefore use either one or the other, but if we open the possibility of the median voter to commute to the employment center of the other jurisdiction, only the one that stand at $-1/8$ (in H) or $1/8$ (in L) fit the median distance of $1/8$.

4 As stated in Peralta (2007) the assumption of a linear technology is not essential and the obtained results would remain unchanged if we introduce perfectly mobile capital in the model with a constant returns to scale production function and a small region assumption such that $f'(k)$ is given.

5 Note that in our setup the head tax $T_i$ can be seen as land or residential property...
The local government budget constraint is therefore

$$G_i = T_i \bar{N} + P_i (\max(N_i - \bar{N}; 0))$$

where $P_i$ is the toll collected at the border of jurisdiction $i$ and $N_i$ is the number of workers in that jurisdiction. Note that the toll is only paid by the interjurisdictional commuter that are arriving at the region, i.e., the number of workers that exceed the residents of that jurisdiction $(N_i - \bar{N})$; if there is no interjurisdictional commuting towards that region, the number of workers paying the toll is null.

To travel on each jurisdiction, agents support a per-mile commuting cost $c_i$ which depends on the local public good provision in that jurisdiction, i.e., $c_i(G_i)$. Agents can choose to which employment center they want to commute (i.e., where they want to work). Commuting to the jurisdiction where they do not live is, therefore, more costly than commuting to the one where they live since the travel distance is higher. Each individual provides one unit of labor and earns a wage $\omega_j = \alpha_j$ for working in the employment center located in $j$. All agents have a revenue $W$ from other sources which is assumed to be high enough such that everyone can always pay his tax bill.

Agents get utility from private consumption, following a linear utility function similar to the quasi-linear utility function used by Braid (2000) and Peralta (2007), but where the utility arising from the public good is indirectly captured by the reduction in the transportation cost. Since the transportation cost is not symmetric in the two jurisdictions it is important to write the utility function in two cases: (i) when the agent lives and works in the same jurisdiction, and (ii) when the agent is an interjurisdictional commuter. The utility enjoyed by individual $x$, who lives and works in $i$, is given by:

$$u_{xi}(x; G_i) = \omega_i - T_i + W - c(G_i) |x - EC_i|, \quad i, j = H, L$$

where $EC_i$ is the location of the employment center ($-1/4$ or $1/4$) tax with fixed house size; since residence place is not chosen by agents this is a lump-sum tax.
and \(c(G_i)\) is the transportation cost per mile, which depends on the public good provided in the jurisdiction where the agent lives \((G_i)\).

The utility enjoyed by individual \(x\), who lives in \(i\) and works in \(j\) (with \(i \neq j\)) is given by:

\[
u_{ij}(x; P_j; G_i; G_j) = \omega_j - T_i + W - c(G_i)|x| - c(G_j)|0 - EC_j| - P_j, \quad (2.2)
\]

\[i, j = H, L\]

where \(EC_j\) is the location of the employment center where the agent chooses to work \((-1/4\) or \(1/4\)), \(c(G_i)\) is the transportation cost per mile in the jurisdiction where the agent lives, which depends on the public good provided in that jurisdiction \((G_i)\), \(c(G_j)\) is the transportation cost per mile in the jurisdiction where the agent works, which depends on the public good provided there \((G_j)\), and \(P_j\) is the toll charged when the agent enters jurisdiction \(j\).

The transportation cost per mile in each jurisdiction is given by \(c(G_i)\), which is a decreasing convex function, i.e.,

\[
c_i' = \frac{\partial c}{\partial G_i} < 0 \quad \text{and} \quad c_i'' = \frac{\partial^2 c}{\partial G_i^2} > 0
\]

### 2.2.1 The choice of the workplace

An agent works in the jurisdiction where he lives if

\[
u_{ii}(x; P; G_i; G_H) - u_{ij}(x; P; G_i; G_L) \geq 0
\]

and will commute to the other jurisdiction otherwise. Given that wage in \(H\) is greater than the wage in \(L\) and the travel distance is higher if the agent decides to commute to the other jurisdiction rather than work on his own jurisdiction, it is possible to show that the tax competition equilibrium never involves commuting from \(H\) to \(L\). Therefore, we focus on the choice of the workplace of the agents who live in \(L\).

Computing the utility difference we can calculate the marginal interjurisdictional commuter, denoted \(\hat{x}\). Using (1) and (2), the difference between the utility obtained working in \(H\) and the one obtained by working in \(L\) for a resident on \(L\) is:

\[
u_{iH} - u_{iL} = \begin{cases} 
\omega_H - \omega_L + \frac{x}{4}(c_L - c_H) - P_H - 2xc_L & \text{if } -\frac{1}{4} < x < \frac{1}{4} \\
\omega_H - \omega_L - \frac{x}{4}(c_L + c_H) - P_H & \text{if } x \geq \frac{1}{4}
\end{cases}
\]
Note that for \(|x| > 1/4\) the utility difference is independent from \(x\), implying that if one agent that lives between the employment center of a jurisdiction and its outer limit wants to commute to the other one, every agent will want to do the same. We assume away such non-interesting cases and focus on the situation where \(\hat{x} < 1/4\). The marginal \(ij\)-commuter \(\hat{x}\) will be the one indifferent between working in H or L, therefore

\[
\hat{x} = \frac{\omega_H - \omega_L + \frac{P_H}{c_L}(c_L - c_H) - P_H}{2c_L}
\]  

(2.3)

This marginal interjurisdictional commuter \(\hat{x}\) defines a commuting equilibrium where all \(x < \hat{x}\) work in H and all \(x > \hat{x}\) work in L.

### 2.3 First Best

We now compute the utilitarian first best to use as a benchmark for the tax competition equilibrium analysis, i.e., the decision of a benevolent social planner that chooses the residence taxes, the toll, the level of public good provided in each jurisdiction and allocates workers to an employment center so that overall utility is maximized.

The planner faces an overall budget constraint such that the provision of public goods must be fully paid by the head taxes and the toll, i.e.,

\[
G_H + G_L = \hat{N} (T_H + T_L) + P \hat{x}
\]  

(2.4)

The problem faced by the social planner is therefore to maximize the overall utility of the population \((U)\), which is equal to the sum of the utility of all inhabitants of jurisdiction H \((U_H)\) and of all inhabitants of jurisdiction L \((U_L)\), subject to the budget constraint \(G_H + G_L = \hat{N} (T_H + T_L) + P \hat{x}\), by choosing \(\hat{x}, G_H, G_L, T_H, T_L\) and \(P\). Recall that it is never optimal to have H-residents commuting to L since their commuting cost is higher than if they work in H and their productivity is lower. Therefore, we can only have L residents commuting to H, i.e., \(\hat{x} \geq 0\), which allow us to calculate \(U_H\) and \(U_L\) as:

\[
U_H = \int_{-\frac{1}{2}}^{0} u_{HH} dx
\]  

(2.5)

\[
U_L = \int_{0}^{\frac{1}{2}} u_{HH} dx + \int_{\hat{x}}^{\frac{1}{2}} u_{LL} dx
\]  

(2.6)
Denoting by $C_i$ the total commuting costs of all the residents of jurisdiction $i$, we have

$$C_H = c_H \left[ \int_{-\frac{1}{4}}^{\frac{1}{2}} \left( \frac{1}{4} - x \right) dx + \int_{-\frac{1}{4}}^{0} \left( x + \frac{1}{4} \right) dx \right] = c_H \left( \frac{1}{16} \right) \quad (2.7)$$

$$C_L = c_L \left[ \int_{-\frac{1}{4}}^{\frac{1}{4}} \frac{1}{2} x \left( \frac{1}{4} - x \right) dx + \int_{\frac{1}{4}}^{\frac{1}{2}} \left( x - \frac{1}{4} \right) dx \right] + c_H \frac{1}{4} \hat{x} + \hat{x}P_H =
$$

$$= c_L \left( \frac{1}{16} + \frac{\hat{x}^2 - \hat{x}}{4} \right) + c_H \left( \frac{\hat{x}}{4} \right) + \hat{x}P_H \quad (2.8)$$

where the last two terms in $C_L$ are the amounts paid by the interjurisdictional commuters in the high productivity jurisdiction (the transportation cost and the toll to enter $H$).

If all $L$ residents worked in $L$ the total commuting cost of all the residents of this jurisdiction, $C_L$, would be $c_L(1/16)$, which means that the existence of interjurisdictional commuters, that must travel a longer distance and pay a toll to get to their employment center, increases the total commuting cost of the residents of $L$ in $(c_L (\hat{x}^2 - \hat{x}) + c_H (\frac{\hat{x}}{4}) + \hat{x}P_H)$.

Total utility in each jurisdiction is given by:

$$U_H = \bar{N} [\omega_H - T_H + W] - C_H \quad (2.9)$$

$$U_L = \bar{N} [\omega_L - T_L + W] + \hat{x} [\omega_H - \omega_L] - C_L \quad (2.10)$$

Note that the gain to $L$ of having interjurisdictional commuters is the difference between the increase in wage earned by this agents and the increase in the commuting costs referred before.

Solving the social planner problem formalized previously we can easily see that the planner is indifferent between using the toll or the head tax, since he can allocate the workers to any of the employment centers. Therefore the choice of $P_H, T_H$ and $T_L$ is irrelevant for our analysis. The only thing that must be ensured is that the budget constraint is satisfied with these taxes. We can then assume $P_H = 0$ and finance the public goods exclusively with the head (lump-sum) taxes. This has the merit of ensuring that we are not implicitly performing any type of interjurisdictional transfers.\(^6\) Remember that the purpose of the calculation of the

\(^6\) As pointed in Peralta (2007).
first best is to use it as a benchmark to compare with the tax competition equilibrium and so we want to keep it as neutral as possible.

The relevant first order conditions are therefore:

\[
\frac{\partial(\hat{x})}{\partial \hat{x}} = 0 \iff \hat{x}^o = \frac{\omega_H - \omega_L + \frac{1}{4}(c_L - c_H)}{2c_L} \quad (2.11)
\]

\[
\frac{\partial(\hat{x})}{\partial G_H} = 0 \iff \frac{\partial c_H}{\partial G_H} \left( \frac{1}{16} + \frac{1}{4} \hat{x} \right) = -1 \quad (2.12)
\]

\[
\frac{\partial(\hat{x})}{\partial G_L} = 0 \iff \frac{\partial c_L}{\partial G_L} \left( \frac{1}{16} + \hat{x}^2 - \frac{1}{4} \hat{x} \right) = -1 \quad (2.13)
\]

Equation (11) gives the optimal interjurisdictional commuter \( \hat{x}^o \), which results from the trade-off between commuting costs and productivity gains and the public good level\(^7\). Equations (12) and (13) express the Samuelson condition for the optimal provision of public goods. Since \( G_H \) provides a reduction in the commuting cost which is beneficial to the interjurisdictional commuters, the marginal benefit of \( G_H \) depends positively on \( \hat{x} \) while the inverse applies to \( G_L \).

2.4 The Equilibrium with Transportation Infrastructure Funded by Residence Taxes

Having calculated the conditions that define the first best, we can compute the decentralized equilibrium and compare it to the utilitarian optimum. In this section we will assume that a government elected by majority rule in each jurisdiction decides the taxes and public goods levels. The elected policy will then be the one preferred by the median voter of each jurisdiction.

In this section we compute the equilibrium when transportation infrastructure is funded exclusively by a residence tax, \( T_i \). We then use this equilibrium to compare with the one resulting from the use of a lump-sum tax combined with a toll, which is computed in the next section.

\(^7\)Comparing this condition with the one obtained in Peralta (2007) we can see that the difference lies on the presence of the term \((c_L - c_H)/4\) and the use of the travel cost of L in the denominator; when deciding the workplace, L residents also consider the difference in the travel costs between L and H, having as benchmark the commuting cost of their own jurisdiction.
Each local government maximizes the utility of the median voter, \( u_m \), subject to the commuting equilibrium, \( \hat{x} \), given by (3) when \( P_H = 0 \) and to the budget constraint of the jurisdiction, i.e., \( G_i = NT_i \).

As stated previously, the median voter of H stands at \(-1/8\) and the median voter of L at \(1/8\) both if he decides to work in L or to be an interjurisdictional commuter. Therefore, the distance traveled by the median voter in L is always \(1/8\), even though the total commuting cost will differ if he decides to work in H since he must also travel \(1/4\) miles in H. We should, then, separate the case where he works in L from the case where he commutes to work in H.

The median voter of H thus enjoys a utility of:

\[
u_{mH} = \omega_H - T_H + W - \frac{1}{8}c_H\]

When the median voter of L works in L, he enjoys a utility of:

\[
u_{mL} = \omega_L - T_L + W - \frac{1}{8}c_L\]

If the median voter of L decides to work in H, he enjoys a utility of:

\[
u_{mL} = \omega_H - T_L + W - \frac{1}{8}c_L - \frac{1}{4}c_H\]

However, since the median voter of L has no power to decide the level of public good in H, the choice of \( G_L \) is independent of his choice of the workplace. It only depends on the distance traveled on L, which is \(1/8\) in both cases. We can then state the following Lemma:

**Lemma 1:** Suppose that jurisdictions fund the transportation infrastructure with a residence tax. Then, the choice, by the median voter of L, of the level of public good provided in L does not depend on his choice of the employment center where he works, i.e., the choice of \( G_L \) is the same when the median voter of L works in L or in H.

**Proof.** When the median voter of L works in L, the first order conditions of his utility maximization problem with respect to \( G_L \) are:

\[
\frac{1}{8}c_L(G_L) = -2 \iff \left( \frac{\partial c_L}{\partial G_L} \right)^* = -16
\]

When the median voter of L works in H, the first order conditions of his
utility maximization problem with respect to $G_L$ are:

$$\frac{1}{8}c_L(G_L) = -2 \Leftrightarrow \left( \frac{\partial c_L}{\partial G_L} \right)^* = -16$$

Therefore, the level of $G_L$ chosen by the median voter of $L$ in the tax equilibrium is the same no matter if he chooses to work in $L$ or in $H$. □

The first order conditions presented in the proof of Lemma 1 implicitly define the equilibrium level of $G_L$. Computing also the solution of the utility maximization problem for $m_H$ (which will provide the equilibrium level of $G_H$), we find out that both $G_H$ and $G_L$ are implicitly defined by the same condition:

$$\frac{1}{8} \left( \frac{\partial c_H}{\partial G_H} \right) = -2 \Leftrightarrow \left( \frac{\partial c_H}{\partial G_H} \right)^* = -16 \quad (2.14)$$

$$\frac{1}{8} \left( \frac{\partial c_L}{\partial G_L} \right) = -2 \Leftrightarrow \left( \frac{\partial c_L}{\partial G_L} \right)^* = -16 \quad (2.15)$$

These equations express the usual Samuelson condition of equality between marginal benefit and marginal cost. The marginal benefit is weighted by the distance traveled by the median voter, 1/8, and since the population mass of each jurisdiction is 1/2, the marginal cost paid by each agent to provide an additional unit of public good is 2.

If the first order conditions are equal, the equilibrium level must also be the same, which leads us to the second result:

**Proposition 1:** In the equilibrium where the transportation infrastructure is funded with a residence tax, the levels of public good provided in $H$ and in $L$ are the same.

Since the median voter has no mechanism to change the cost of provision of the public good, namely by transferring the cost to the residents of the other jurisdiction, the only determinant of his choice is the distance traveled in his jurisdiction. Both the median voter of $L$ and of $H$ travel 1/8 miles in their jurisdiction, so the equilibrium choice is the same.

If we compare the equilibrium obtained with the first-best we can state the following proposition:

**Proposition 2:** Suppose that jurisdictions fund the transportation infrastructure with a residence tax. Then, the level of public good is the
same in both jurisdictions, and it is over provided in the high productivity jurisdiction, and under provided in the high productivity jurisdiction. Moreover, there is undercommuting in equilibrium.

Proof. See appendix.

The median voter of H does not consider the commuters that reside in L and chose to work in H. These agents also benefit from a lower transportation cost in H, fact that is disregarded by the median voter of H, which considers a lower marginal benefit of $G_H$ when compared to the first-best. This situation results in underprovision of $G_H$. Similarly, the median voter of L does not take into account that $\hat{x}$ residents of L commute to H to work and, thus, make a lower use of the transportation infrastructure of jurisdiction L, therefore resulting in overprovision of $G_L$.

In this case, when only residence is being taxed, agents decide their workplace considering the gross wage earned and the transportation costs in each jurisdiction. Knowing that $G_H$ is underprovided, jurisdiction H is less attractive than in the first-best solution since the transportation cost is higher than the optimal, while jurisdiction L is more attractive due to the overprovision of $G_L$, which reduces the transportation cost. As a result, less agents than optimal are commuting from L to H.

Finally, we check that the equilibrium obtained respects the condition $\hat{x} < 1/4$, i.e., some residents of L commute to the employment center of L.

From the first order conditions (14) and (15) we get that the equilibrium levels of public goods are the same in both jurisdictions, so the marginal interjurisdictional commuter is:

$$\hat{x}^* = \frac{\omega_H - \omega_L}{2c_L}$$ (2.16)

To avoid that all population of L commutes to work in the employment center of H,

$$\hat{x}^* = \frac{\omega_H - \omega_L}{2c_L} < \frac{1}{4} \iff \omega_H - \omega_L < \frac{c_L}{2}$$

Therefore, if the equation above is respected, the equilibrium satisfies the conditions imposed.
2.5 The Equilibrium with Transportation Infrastructure Funded by Residence Taxes and a Toll

We now analyze the tax competition equilibrium obtained when local governments can use both the residence tax (lump-sum) and a toll charged when agents arrive at the jurisdiction. As a result, when agents decide to commute to a jurisdiction different from the one they live, besides taking into account the wage that is paid at the employment center of that jurisdiction and the transportation cost there, must also consider that there is an additional cost: the toll. This means that local governments, when deciding the toll, face a trade-off between financing the public good and reducing the number of interjurisdictional commuters due to the increase of the commuting costs of the agents that come from the other jurisdiction.

The decision process consists of two stages: first, the local governments choose \( G_i \) and then, on the second step, \( P_i \) is chosen. \( T_i \) ensures the budget balance. This fact is only relevant for the choice of the median voter of \( H \) since that is the only jurisdiction where the toll will be charged.

Just as in the previous section, the median voter of \( H \) enjoys a utility of:

\[
 u_{mH} = \omega_H - T_H + W - \frac{1}{8}c_H 
\]  

(2.17)

Regarding the median voter of \( L \), if he decides to works in \( L \), he also enjoys the same utility as in the case where only the residence tax was available:

\[
 u_{mL} = \omega_L - T_L + W - \frac{1}{8}c_L 
\]  

but if the median voter of \( L \) decides to work in \( H \), he must now pay a toll to enter jurisdiction \( H \), so his utility becomes:

\[
 u_{mL} = \omega_H - T_L + W - \frac{1}{8}c_L(G_L) - \frac{1}{4}c_H - P_H 
\]  

(2.19)

Each local government maximizes the utility of the median voter subject to the commuting equilibrium \( \hat{x} \) and to the budget constraint of the jurisdiction. For the median voter of \( L \), since there are no agents commuting from \( H \) to \( L \), \( P_L = 0 \) and he maximizes \( u_{mL} \) given by (18) or (19) subject to the commuting equilibrium in (3) and the budget constraint \( G_L = \frac{1}{2}T_L \), by choosing \( G_L \) and \( T_L \).
Furthermore, since the median voter of L has no power to decide neither the level of public good in H, nor the toll being charged there, the choice of \( G_L \) is independent of his choice of the workplace, and Lemma 1 applies. The first order conditions are the same as in the equilibrium with residence taxes, and so is the level of \( G_L \).

Regarding the choice in the high productivity jurisdiction, since there are agents commuting from L to H, the median voter of H has the incentive to choose a positive amount for the toll, so that part of the cost of providing \( G_H \) is paid by the residents of L. Knowing that the decision process is in two stages, we use backward induction to find the optimal choice of \( G_H \) and \( P_H \) by the median voter of H.

We begin by characterizing the choice of \( P_H \). We maximize \( u_{m_H} \) subject to the commuting equilibrium \( \hat{x} \) in (3) and to the budget constraint \( G_H = \frac{1}{2}T_H + \hat{x}P_H \) by choosing \( P_H \), yielding the following equilibrium value of the toll.

\[
P_{H}^{**} = \frac{\omega_H - \omega_L + \frac{1}{4}(c_L - c_H)}{2}
\]

We now use (20) to solve the first stage of the game, in which jurisdictions choose \( G_H \) and \( G_L \) simultaneously. Jurisdictions maximize the utility of the median voter given by (17) and (18), subject to the commuting equilibrium \( \hat{x} \) in (3), the budget constraint \( G_H = \frac{1}{2}T_H + \hat{x}P_H \), and the toll obtained in (20).

Solving the utility maximization problem we find the equilibrium level of \( G_H \) implicitly defined by:

\[
\frac{1}{8} \frac{\partial c_H}{\partial G_H} = -2 \left( 1 - \frac{\partial (\hat{x}P_H)}{\partial G_H} \right)
\]

This condition reflects the usual equality between marginal benefit and marginal cost. The median voter now has a mechanism to charge part of the cost of providing the public good to the residents of the other jurisdiction. Therefore, the marginal cost of providing \( G_H \) which is perceived by the median voter of H is now reduced by the toll paid by the interjurisdictional commuters at the entrance of jurisdiction H (reflected on the derivative \( \frac{\partial (\hat{x}P_H)}{\partial G_H} \)).

Solving (21) yields the following implicit reaction function:

\[
\left( \frac{\partial c_H}{\partial G_H} \right)^{**} = \frac{-16c_L}{\omega_H - \omega_L + \frac{1}{4}(c_L - c_H) + c_L}
\]

From this reaction function we can conclude that the local public
goods of jurisdictions L and H are strategic complements\[^9\]. The intuition for this result is as follows: an increase in \( G_L \) reduces \( c_L \), therefore making jurisdiction L more attractive; since the median voter of H can now charge a toll to the interjurisdictional commuters, he has an incentive to compete for commuters, which will pay part of the tax burden in H. He does so by increasing \( G_H \).

If we compare the tax competition equilibrium obtained with the first-best we can state the following proposition:

**Proposition 3:** In the equilibrium where the transportation infrastructure is funded with a residence tax and a toll, the high productivity jurisdiction charges a positive toll to the interjurisdictional commuters arriving from L. The local public good in jurisdiction L is overprovided.

**Proof.** See appendix.

The intuition behind the fact that region H charges a positive toll to L residents that work in H is analogous to the one behind the use of a wage taxes presented in Peralta(2007): H residents are exporting part of their tax burden to the interjurisdictional commuters from region L. We can therefore identify a transfer of income from the interjurisdictional commuters to the residents of the high productivity jurisdiction.

Regarding the provision of public good in jurisdiction L, and as before, the median voter of L does not take into account that \( \hat{x} \) residents of L commute to H to work and, thus, use less of the transportation infrastructure of jurisdiction L, therefore resulting in overprovision of \( G_L \).

As for H residents, the marginal cost of providing \( G_H \) is now lower than the one perceived by L residents, since the median voters of H is exporting part of the tax burden to the interjurisdictional commuters that are arriving from L; this fact increases the level of public good provided in H.

When we compare the tax competition equilibrium obtained when local governments only use the residence tax to the one when both the residence tax and the toll used, we can reach the following result:

**Proposition 4:** The use of the toll on the commuters that are arriving at jurisdiction H increases the level of public good provided in that jurisdiction vis-a-vis the case where only the residence tax is used, but has no impact on the local public good being provided in jurisdiction L.

\[^{9}\] (\( \frac{\partial c_H}{\partial G_H} \)) ** depends negatively on \( c_L \), which means that if \( c_L \) increases, \( c_H \) will also increase since \( \frac{\partial c_H}{\partial G_H} < 0 \), and vice-versa.
Proof. See appendix.

The distortion introduced by the toll partially offsets the inefficiency created by the tax competition equilibrium due to the positive externality of the public good provided in $H$ to the interjurisdictional commuters. The tax exporting generated by the toll reduces the marginal cost to the policy-maker in $H$, thus leading him to provide a higher level of $G_H$. Since the toll has no impact on the budget constraint of $L$, it has no impact on the decision being taken by the median voter of that jurisdiction, resulting in no change on the overprovision of $G_L$.

2.5.1 Welfare analysis

In order to analyze the impact of introducing the toll in the utility of the residents of each jurisdiction and on the overall welfare, we perform a simulation, assuming $c_i(G_i) = \beta - \gamma \ln(G_i)$, with $\beta$ high so that $c_i(G_i)$ is always positive. We conclude that, for the acceptable parameters of the model, i.e., such that the marginal interjurisdictional commuter is not located to the right of the employment center of $L$, the introduction of the toll decreases overall welfare in the city vis-a-vis the case where only residence taxes are used.

To understand the drivers of total welfare, it is useful to separate two effects: that of productive efficiency, captured by the extent of interjurisdictional commuting, and that of commuting costs. The toll level discourages commuting because its level is driven by the tax-exporting motives of the high productivity jurisdiction residents. Therefore, there is under-commuting, hence, from the viewpoint of productive efficiency, the introduction of the toll has a negative welfare impact. On the other hand, the higher public good level in jurisdiction $H$ decreases the commuting costs of all $H$ residents, as well as that of the inter-jurisdictional commuters. However, inter-jurisdictional commuters also bear the negative impact of the toll, which outweighs the positive one of the lower commuting cost – were this not the case, there would not be under-commuting. Therefore, the only positive impact is that on decreased commuting costs of the $H$ residents, and the overall impact of the toll is negative. Notwithstanding, it benefits the high productivity region residents, who can export part of their tax bill to the interjurisdictional

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10 In the following section we perform simulations assuming $c_i(G_i) = \beta - \gamma \ln(G_i)$ where it is possible to observe that, even though the provision of $G_H$ increases with the introduction of the toll, it is still underprovided.

11 In the simulations we used $\beta = 5$ and $\alpha_H = 10$, letting the wage gap $\alpha_H - \alpha_L$ vary.
commuters, on top of the lower commuting costs. These results do not depend on $\eta$, as it changes the scale of $G_H$ and $G_L$ but not the relations between the variables.\footnote{We provide in the Appendix simulation results for different levels of $\eta$.}

This simulation points out that, in the absence of other distortions besides the externality related to the commuting cost presented in this paper (e.g., congestion), it is better to use a lump sum tax alone, such as a residence or property tax, than to combine it with a distortive mechanism such as a toll.

### 2.6 Conclusion

This paper introduces endogenous asymmetric transportation costs in a duo-centric linear city where local governments provide public goods (which determine the transportation cost in each jurisdiction) and agents choose in which region they want to work.

We show that in the tax competition equilibrium the public good provided in the low productive region is always overprovided and the one provided in the high productive region can be under or overprovided, if a toll is used. Furthermore, we show that the use of a toll tends to be preferred to the single use of a lump sum tax in terms of the provision of
the public goods as it partially offsets the distortion introduced by the positive externality generated by the public good provided in jurisdiction H to the residents of L that commute to work in H.

Nevertheless, a simulation shows that, in the absence of other distortions, it is better to use a lump sum tax alone than to combine it with a distortive mechanism such as a toll.
2.7 Appendix

Proof of Proposition 2.

(i) \( \left( \frac{\partial c_H}{\partial G_H} \right)^o - \left( \frac{\partial c_H}{\partial G_H} \right)^* = -\frac{16}{1 + 4\hat{x}^o} - (-16) \)

Since \( \hat{x}^o > 0 \) \( \Rightarrow \left( \frac{\partial c_H}{\partial G_H} \right)^o - \left( \frac{\partial c_H}{\partial G_H} \right)^* > 0 \Leftrightarrow G^o_H > G^*_H \)

\( \left( \frac{\partial c_L}{\partial G_L} \right)^o - \left( \frac{\partial c_L}{\partial G_L} \right)^* = -\frac{16}{1 + 16\hat{x}^{o^2} - 4x^o} - (-16) \)

Since \( \hat{x}^o < \frac{1}{4} \) \( \Rightarrow \left( \frac{\partial c_L}{\partial G_L} \right)^o - \left( \frac{\partial c_L}{\partial G_L} \right)^* < 0 \Leftrightarrow G^o_L < G^*_L \)

(ii) \( \hat{x} = \frac{\omega_H - \omega_L + \frac{1}{4}(c_L - c_H)}{2c_L} \)

Since \( G_H \) is underprovided and \( G_L \) is overprovided \( \Rightarrow \hat{x}^o > \hat{x}^* \)

Proof of Proposition 3.

(i) \( P^{**}_H = \frac{\omega_H - \omega_L + \frac{1}{4}(c_L - c_H)}{2} \)

Since \( (\omega_H - \omega_L) > 0 \) and \( (c^o_L - c^o_H) > 0 \), \( P^{**}_H > 0 \)

(ii) For \( G_L \) please check the proof of proposition 3. as the problem is the same.

Proof of Proposition 4.

\( \left( \frac{\partial c_H}{\partial G_H} \right)^* - \left( \frac{\partial c_H}{\partial G_H} \right)^{**} = -16 - \left( -\frac{16c^{**}_L}{\omega_H - \omega_L + \frac{1}{4}(c^*_L - c^*_H)} \right) = -\frac{\hat{x}^{**}}{16x^{**} + 1} < 0 \), thus \( G^{**}_H > G^*_H \)

□
Figure 2.3: Simulation results (1)
Figure 2.4: Simulation results (2)
Chapter 3

Optimal fiscal instruments for tax decentralization in a city with congestion⁰

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3.1 Introduction

Congestion is a major issue that large cities must deal with. Average time taken to travel to work in central London was, in 2012, 54 minutes and to work in the rest of inner London 46 minutes, according to the United Kingdom Department for Transport annual publication “Transport Statistics Great Britain” 2013. According to the same publication, in 2012 over 1.1 million persons entered central London daily during the morning peak (i.e., from 7 a.m. until 10 a.m.), 18% more than in 1996.

According to Leape (2006), by the 1990s the average speed of trips across London was below that at the beginning of the twentieth century, as traffic speeds in central London decreased almost 30 percent from 1975, when the average speed during the morning peak period reached 14.2 mph (23 km/h), until 1998, when the average speed was of 10 mph (16 km/h). After 1998 this value felt even more: in the third quarter of 2014 the average traffic speed in central London between 7 a.m. and 7 p.m. was of 8.4 mph (13 km/h), according to Transport for London Streets Performance Report (quarter 3 2014/2015).

Travel time is only one of several examples of congestion costs associated with increased commuting, to which we may add e.g. pollution and crime, all representing negative externalities for the commuters. As pointed in Peralta (2007) “there is extensive evidence of the increasing importance of inter-jurisdictional commuting, possibly fostered by the improvement in transportation technologies”. This importance is presented for example in Shields and Swenson (2000), Glaeser et al. (2001) and Renkow (2003) for the US, by Van Ommeren et al (1999) for the Netherlands or Cameron and Muellbauer (1998) for the United Kingdom. All these papers find clear evidence that both the number of commuters and the commuting distance have been increasing in the last decades. This evidence is corroborated by the 2011 England and Wales census, where it is observable that in these two regions, the average distance commuted to work increased by 1.6 km since 2001 (Office for National Statistics UK). Considering inner London workers, the distance increase is higher, from 8.8 km in 2001 to 11.2 km in 2011, according to the same census.

In order to deal with increased commuting and resulting congestion costs, several cities implemented tolls at its entrance. In those cities,
agents must pay a fee if they want to drive in some areas (typically the city center) and usually during the period that affects the commuters: working days between early morning and late afternoon. Examples of cities where tolls are charged are Singapore, London, Stockholm, Milan or Gothenburg. Singapore was the first city to implement a congestion pricing scheme, in 1975, which was originally made of paper licenses and is currently an electronic mechanism (the Electronic Road Pricing). London introduced, in 2003, the London Congestion Charge Zone which electronically charges vehicles traveling on the city center. From 2007 onwards, the Stockholm Congestion Tax electronically charges vehicles entering and exiting central Stockholm area, a scheme followed by Gothenburg in 2013. In Milan, vehicles traveling on the traffic restricted zone are also charged since 2008\footnote{For analysis of the implementation of these tolling schemes check, e.g., Phang and Toh (2004) for the Singapore case; Prud’homme and Bocarejo (2005), Leape (2006), Givoni (2011) or Dudley (2013) for London.}

By introducing tolls, local governments decrease congestion, but may also be influencing production efficiency by reallocating workers from a more productive employment center to less productive one. Arnott (2007) shows that in a framework with one negative externality (traffic congestion), and one positive externality (external economies of scale), and where congestion tolling is feasible but wage subsidizing is not, the imposition of even a low toll might reduce efficiency. Another problem arising when implementing congestion tolling is to define the optimal toll level and location. Ekström et al. (2014) analyse the Stockholm case to conclude that tuning these decisions may lead to a significant increase in the welfare gain generated by the toll. They also show that by optimizing both toll locations and levels, a congestion pricing scheme with welfare gain close to what can be achieved by marginal social cost pricing can be designed even without tolling all possible tollable links.

The autonomy of local governments is not constant around the world. For example, when it comes to taxes, Braid (2005) points out that residential and business property taxes, which are residence-based wealth taxes, are the most important source of local government tax revenue in the United States. But some local governments also use other types of taxes, such as pure source-based wage taxes or payroll taxes, which must
be paid by agents who work in, e.g., San Francisco, Los Angeles, Newark (New Jersey) or Birmingham (Alabama).

Besides the examples referred in the U.S., local governments worldwide use several wage or income taxes: Mexico and several OECD countries have payroll taxes at the state or local level: Australia, Austria, France and Greece, as referred in Peralta (2007). Source based income taxes can be found in Sweden, Denmark, France, Germany, Japan and Spain (Braid, 2005).

The tolls levied to enter some city centers are similar to hybrid taxes charged in some US cities. As stated in Braid (2009), Kansas City, St. Louis, Wilmington, Detroit, New York City or Philadelphia charge “hybrid” taxes where central city residents are subject to a tax rate, no matter where they work, and agents that live outside the center but work there are taxed at a different rate. A toll at the border of a city or jurisdiction has the same practical result: the residents do not pay it, but the commuters arriving from the suburbs do.

This paper analyses a linear city with commuting and congestion costs in which the jurisdictions may use tolls to fund local public goods while, at the same time, discouraging commuting hence decreasing congestion costs. We analyse the majority voting decentralized equilibrium against the benchmark of a first-best benevolent social planner solution. The contribution of this paper is the introduction of congestion costs in the framework of a linear city allowing, simultaneously, to study the impact of using different fiscal mechanism to deal with congestion on commuting, productive efficiency and welfare.

The linear city model is used, for example, by Braid (2000) and Peralta (2007) to tackle interjurisdictional tax spillovers. The linear city is divided into two jurisdictions and agents choose where to work. Productivity, and thus wages, differ across regions, and agents trade-off the advantages of a given job (namely, the wage) against travel costs (distance, time, and money) and congestion when choosing their workplace. The congestion differs from one jurisdiction to the other since it depends on the number of employees that decided to work in each jurisdiction. This set up introduces a negative externality imposed by each worker on everyone else that decides to work in the same employment center. We assume that residence and working choices are independent, taking residence location as exogenous.

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4 As analysed in Braid (2009).
5 Check the OECD (2009) study for a thorough analysis of the fiscal autonomy of local governments.
6 This claim finds empirical evidence in Rouwendal and Meijer (2001), Glaeser et al. (2001) and Zax (1991 and 1994). For an analysis of the residence decision check,
Our main results are as follows: when local governments only use a head tax we are in the presence of a pure negative externality, leading to a situation of overcommuting of agents, but being the best decentralized alternative if the transportation cost or the productivity asymmetry are high. The introduction of a toll distorts the number of interjurisdictional commuters, resulting in undercommuting of agents since the local government of the high productivity jurisdiction uses it both for reducing congestion and collecting revenue. Nevertheless, the toll is the fiscal mechanism that presents higher overall utility in a decentralized equilibrium if the transportation cost is low and the productivity asymmetry is mild. Furthermore, we show that it is possible to replicate the first best by allocating the decision of the toll amount to a benevolent central government while leaving the decision regarding the public goods provision to the local governments. Finally, the introduction of wage taxes, which can be used by both local governments, may lead to under or overcommuting, being the best decentralized equilibrium if both the transportation cost and the productivity asymmetry are low.

This paper is organized as follows: Section 2 presents the model. Section 3 computes the first best, the benchmark that is compared with the decentralized equilibria obtained in the following sections: Section 4, i.e., the equilibrium where only a lump sum residence tax is used; Section 5, where local governments use both the residence tax and a toll on interjurisdictional commuters; and Section 6, where both residence and source-based wage taxes are used. Section 7 presents a welfare analysis of the three tax competition equilibria and Section 8 concludes.

3.2 The Model

We consider a linear city divided into two jurisdictions with the same size. Each jurisdiction has an employment center where agents can work. The total number of residents of the city is normalized to 1, as well as the city size, with extreme points of the segment \(-1/2 \leq x \leq 1/2\). Inhabitants are uniformly distributed across the city and their residence location is exogenous. Each agent is indexed by his residence place, \(x\).

Let \(n(x)\) and \(N(x)\) denote the density and distribution function, respectively, so that
\[
n(x) = 1 \quad \text{and} \quad N(x) = x + \frac{1}{2}
\]

\(n(x) = 1\) and \(N(x) = x + \frac{1}{2}\). E.g., Wrede (2009) where agents choose their residence location according to a bid-rent function. The impact of congestion tolling on the concentration of cities is analyzed by Arnott (1998). The impact of zoning policies in cities with traffic congestion is discussed by, e.g., Anas and Pines (2013) and Rhee et al. (2014).
Since the two jurisdictions have the same size and residents are uniformly distributed, both have the same number of inhabitants, $\bar{N} = 1/2$. The median resident of each jurisdiction coincides with the geographic center of the jurisdiction, i.e., $m_H = 1/4$ and $m_L = 1/4$. The employment centers are assumed to be symmetrically located in $\gamma$ and $-\gamma$ and located outwards from the median resident ($\gamma > 1/4$).

Firms located at the employment centers produce an homogeneous good according to a linear technology $Y_i = \alpha_i N_i$, where $Y_i$ is the output and $N_i$ is the number of workers in jurisdiction $i$.

The two jurisdictions have unequal productivities. We use $H$ to denote the high-productivity jurisdiction and $L$ for the low-productivity one, with $\alpha_H > \alpha_L$.

To finance a public good ($G_i$), the government of each jurisdiction collects a head tax ($T_i$) paid by all its residents and, possibly, a toll ($P_i$) paid by the interjurisdictional commuters that cross the jurisdiction border, or an ad-valorem source-based tax on wages ($\tau_i$) paid by all workers in the employment center of that jurisdiction (wage tax). The local government decisions are based on majority voting, thus the selected outcome is the one preferred by the median voter.

The local government budget constraint is therefore

$$G_i = T_i \bar{N} + P_i (\text{max}(N_i - \bar{N}; 0)) + \alpha_i \tau_i N_i$$

where $\alpha_i$ is the gross wage earned by workers in the employment center of jurisdiction $i$, and $N_i$ and $\bar{N}$ are, respectively, the number of workers and the number of inhabitants of that jurisdiction. Note that the toll ($P_i$) is only paid by the interjurisdictional commuter that cross the border, i.e., the number of workers that exceed the residents of the jurisdiction ($N_i - \bar{N}$). If there is no interjurisdictional commuting towards that region, the number of workers paying the toll is null.

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7As stated in Peralta (2007) the assumption of a linear technology is not essential and the obtained results would remain unchanged if we introduce perfectly mobile capital in the model with a constant returns to scale production function and a small region assumption, ensuring that $f'(k)$ is given.

8Note that in our setup the head tax $T_i$ can be seen as land or residential property tax with fixed house size; since residence place is not chosen by agents this is a lump-sum tax.
To travel on each jurisdiction, the agents support a constant per-mile commuting cost, $c$. Agents can choose to which employment center they want to commute (i.e., where they want to work). Commuting to the jurisdiction where they do not live is, therefore, more costly than commuting to the one where they live since the travel distance is higher. Additionally, if a toll is at place, interjurisdictional commuters must also pay that additional cost.

Each individual provides one unit of labor and earns a gross wage $\alpha_j$ for working in the employment center located in $j$. If the wage tax is being used, agents must pay a tax at the source of the income, so that the net wage earned by an individual working in $j$ is $\omega_j = \alpha_j(1 - \tau_j)$. All agents have a revenue $W$ from other sources which is assumed to be high enough such that everyone can always pay his tax bill.

Agents get utility from private consumption, following a linear utility function similar to the quasi-linear utility function used by Braid (2000) and Peralta (2007), but we introduce a congestion cost which positively depends on the number of agents working at the employment center of the jurisdiction where the agent is employed ($z(N_i)$). This congestion cost may stem from increased traveling time, difficulty in finding parking space close to the employment center or pollution.

We now write the utility function in two different cases: (i) when the agent lives and works in the same jurisdiction, and (ii) when the agent is an interjurisdictional commuter.

The utility enjoyed by individual $x$, who lives and works in $i$, is given by:

$$u_{ii}(x; \tau_i; G_i) = \omega_i - T_i + W - c|x - EC_i| - z(N_i) + v(G_i)$$ (3.1)

where $EC_i$ is the location of the employment center ($\gamma$ or $-\gamma$), $G_i$ is the public good provided in the jurisdiction where he lives and $v(G)$ is an increasing concave function.

The utility enjoyed by individual $x$, who lives in $i$ and works in $j$ (with $i \neq j$) is given by:

$$u_{ij}(x; \tau_j; P_j; G_i) = \omega_j - T_i + W - c|x - EC_j| - P_j - z(N_j) + v(G_i)$$ (3.2)

$\text{Recall that } x \text{ is the point of residence of the agent in the city, i.e., } x \in [-1/2; 1/2].$ If $x \in [-1/2; 0]$, the agent lives in the high productivity jurisdiction. If $x \in (0; 1/2]$, he lives in the low productivity one.
The congestion cost in each jurisdiction is given by \( z(N_i) \), which is an increasing convex function, i.e.,

\[
    z' = \frac{\partial z}{\partial N_i} > 0 \quad \text{and} \quad z'' = \frac{\partial^2 z}{\partial N_i^2} > 0
\]

For simplicity, we assume that \( z(N_i) = \eta N_i^2 / 2 \), where \( \eta \) reflects the intensity of the congestion cost, i.e., the higher it is, the more costly the congestion becomes.\(^{10}\) If \( \eta = 0 \) there is no congestion cost and we have the model analyzed in Peralta (2007).

3.2.1 The choice of the workplace

An agent works in the jurisdiction where he lives if

\[
    u_{ii}(x; \tau_i; G_i) - u_{ij}(x; \tau_j; G_i) \geq 0
\]

and commutes to the other jurisdiction otherwise. We can show that there is only commuting from the low productivity jurisdiction to the high productivity one and not vice-versa. A resident in H that decides to commute to L earns a lower wage, faces a higher transportation cost (since the travel distance is higher) and eventually pays a toll. Each of these facts reduces the utility of the agent. Even in the case where wage taxes are used, we may show that the net wage earned in H is always higher than the net wage in L, thus ensuring there is no incentive for H residents to work in L.\(^{11}\) If there were agents commuting from H to L, the congestion in L would also be higher than in H, which would again reduce the utility of the agent. Therefore, we focus on the choice of the workplace of the agents who live in L.

Computing the utility difference we obtain the marginal interjurisdictional commuter, denoted \( \hat{x} \). Using (1) and (2) it is straightforward that, for a resident in L, the difference between the utility attained when working in H rather than in L is:

\[
    u_{iH} - u_{iL} = \begin{cases} 
    \omega_H - \omega_L - P_H + 2xc - (z(N_H) - z(N_L)) & \text{if } x < \gamma \\
    \omega_H - \omega_L - 2\gamma c + (z(N_H) - z(N_L)) & \text{if } x \geq \gamma 
    \end{cases}
\]

When deciding to commute, an individual takes the number of workers in a jurisdiction (\( N_H \) and \( N_L \)) as given. Note that for \( |x| > \gamma \) the utility difference is independent from \( x \), implying that if one agent that lives

\(^{10}\) Our results do not depend qualitatively on this assumption.

\(^{11}\) Computations in appendix.
between the employment center of a jurisdiction and its outer limit commutes to the other one, every agent does. The marginal interjurisdictional commuter \( \hat{x} \) is the one indifferent between working in H or L, therefore

\[
\hat{x} = \frac{\omega_H - \omega_L - P_H - (z(N_H) - z(N_L))}{2c} \quad (3.3)
\]

This marginal interjurisdictional commuter \( \hat{x} \) defines a commuting equilibrium where all \( x < \hat{x} \) work in H and all \( x > \hat{x} \) work in L.

### 3.3 First Best

We now compute the utilitarian first best to use as a benchmark for the tax competition equilibrium analysis, i.e., the decision of a benevolent social planner that chooses the residence taxes, the toll, the wage tax and the level of public good provided in each jurisdiction and allocates workers to an employment center so that overall utility is maximized.

The planner faces the following budget constraint:

\[
G_H + G_L = \bar{N} (T_H + T_L) + P_H \hat{x} + \tau_H \alpha_H N_H + \tau_L \alpha_L N_L \quad (3.4)
\]

which translates the fact that the provision of public goods must be fully paid by the head taxes, the toll and the wage tax.

The problem faced by the social planner is therefore to maximize the overall utility of the population \( U \), which is equal to the sum of the utility of all inhabitants of jurisdiction H \( U_H \) and of all inhabitants of jurisdiction L \( U_L \), subject to the budget constraint (4), by choosing \( \hat{x} \), \( G_H \) and \( G_L \).

It is never optimal to have H-residents commuting to L since their commuting cost is higher than if they work in H and their productivity is lower. Therefore, we can only have L residents commuting to H, i.e., \( \hat{x} \geq 0 \), which allow us to calculate \( U_H \) and \( U_L \) as:

\[
U_H = \int_{-\hat{x}}^{0} u_{HH}(\hat{x}; G_H; \tau_H; P_H) \, dx
\]

\[
U_L = \int_{0}^{\hat{x}} u_{LH}(\hat{x}; G_L; \tau_H; P_H) \, dx + \int_{\hat{x}}^{\frac{1}{2}} u_{LL}(\hat{x}; G_L; \tau_L) \, dx
\]

Denoting by \( C_i \) the commuting costs of all the residents of jurisdiction
including the eventual toll that has to be paid by interjurisdictional commuters, we have

\[
C_H = c \left[ \int_{-\gamma}^{-\gamma - \frac{\gamma}{2}} (-\gamma - x)dx + \int_{-\gamma}^{0} (x + \gamma)dx \right] = c \left( \frac{1}{8} + \gamma^2 - \frac{\gamma}{2} \right)
\]

\[
C_L = c \left[ \int_{0}^{\frac{x}{2}} (x + \gamma)dx + \int_{\frac{x}{2}}^{\gamma} (\gamma - x)dx + \int_{\gamma}^{\frac{1}{2}} (x - \gamma)dx \right] + \hat{x}P_H = C_H + c (\hat{x}^2) + \hat{x}P_H
\]

where the two last terms in \( C_L \) are the increase in commuting costs due to the interjurisdictional commuters which must travel a longer distance, and pay a toll to enter H.

Total utility in each jurisdiction is given by:

\[
U_H = \bar{N} \left[ \omega_H - T_H + W - z(N_H) + v(G_H) \right] - C_H
\]

\[
U_L = \bar{N} \left[ \omega_L - T_L + W - z(N_L) + v(G_L) \right] + \hat{x} [\omega_H - \omega_L] - \hat{x} [z(N_H) - z(N_L)] - C_L
\]

Note that the gain to L of having interjurisdictional commuters is the difference between (i) the increase in the net wage earned by this agents and (ii) the increase in the congestion cost faced by this individuals, \( z(N_H) - z(N_L) \), plus (iii) the increase in the commuting costs stated before.

When solving the social planner problem formalized previously, we can see that the planner chooses the level of public good provided in jurisdiction H (\( G_H \)) and in jurisdiction L (\( G_L \)), as well as the marginal interjurisdictional commuter (\( \hat{x} \)), i.e., the allocation of workers to the employment centers.

Therefore, the relevant first order conditions are\[12\]

\[
\frac{\partial U}{\partial \hat{x}} = 0 \iff \hat{x}^o = \frac{(\alpha_H - \alpha_L) - (z_H - z_L) - \frac{\hat{x}^2}{2}}{2c + z_H + z_L} \quad (3.5)
\]

\[
\frac{\partial U}{\partial G_H} = 0 \iff \frac{1}{2} v'(G_H) = 1 \quad (3.6)
\]

\[
\frac{\partial U}{\partial G_L} = 0 \iff \frac{1}{2} v'(G_L) = 1 \quad (3.7)
\]

Equations (6) and (7) express the Samuelson condition for the op-

\[12\]We checked the second order conditions of this problem, as well as all other optimization problems present in this paper.
timal provision of public goods. Equation (5) provides the optimal marginal interjurisdictional commuter $\hat{x}^o$, which results from the trade-off between productivity gains, and the increase in commuting and congestion costs.\textsuperscript{13}

The budget constraint (4) must be respected, which means that the public goods $G_H$ and $G_L$ must be funded by a combination of head taxes, wage taxes and a toll. However, the set of fiscal instruments used by the social planner is not relevant in terms of overall welfare since the allocation of workers is already decided. Since the purpose of computing the first best is to use it as a benchmark to compare with the decentralized equilibrium, we want to keep it as neutral as possible. For this reason we assume $P_H = 0$, $\tau_H = 0$ and $\tau_L = 0$ and finance the public goods exclusively with the head (lump-sum) taxes. This satisfies the budget constraint and has the merit of ensuring that we are not implicitly performing any type of interjurisdictional transfers, as pointed in Peralta (2007).

Assuming $z(N_i) = \eta \frac{N^2_i}{2}$, $i = H, L$, we get the optimal marginal interjurisdictional commuter given by:

$$\hat{x}^o = \frac{(\alpha_H - \alpha_L)}{2c + 3\eta}$$

(3.8)

The higher is $\eta$, the more costly the congestion cost becomes, leading to a reduction in the optimal number of interjurisdictional commuters.

### 3.4 The Equilibrium with Residence Taxes

Knowing the conditions that define the utilitarian first best, we can compute the tax competition equilibrium and compare it to this benchmark. In this section we assume that a government elected by majority rule in each jurisdiction decides the taxes and public goods levels. The elected policy is, therefore, the one preferred by the median voter of each jurisdiction.

In this section we compute the tax competition equilibrium when local governments only have access to the residence tax, $T_i$. We then

\textsuperscript{13}Comparing this condition with the one obtained in Peralta (2007) we can see that the difference lies on the presence of the terms concerning the congestion costs. When deciding the workplace, the residents in L also consider the impact of facing higher congestion in H; the social planner also takes into account the marginal impact of increasing the number of workers in H and reducing the number of workers in L in the congestion cost of each jurisdiction.
use this equilibrium to compare with the one resulting from the use of a lump-sum tax combined (i) with a toll or (ii) with a wage tax, which are computed in the following sections.

Each local government maximizes the utility of the median voter, $u_{m_i}$, subject to the budget constraint of the jurisdiction, i.e., $G_i = NT_i$, and to the commuting equilibrium, $\hat{x}$, given by equation (3) when $P_H = 0, \tau_H = 0$ and $\tau_L = 0$.

The median voter of H thus enjoys a utility of:

$$u_{m_H} = \alpha_H - T_H + W - z\left(\frac{1}{2} + \hat{x}\right) + v(G_H) - c\left(-\frac{1}{4} + \gamma\right) \quad (3.9)$$

Regarding the median voter of L, if he works in L, his utility is:

$$u_{m_L} = \alpha_L - T_L + W - z\left(\frac{1}{2} - \hat{x}\right) + v(G_L) - c\left(\gamma - \frac{1}{4}\right) \quad (3.10)$$

If the median voter of L decides to work in H, his utility becomes:

$$u_{m_L} = \alpha_H - T_L + W - z\left(\frac{1}{2} + \hat{x}\right) + v(G_L) - c\left(\gamma + \frac{1}{4}\right) \quad (3.11)$$

The marginal interjurisdictional commuter, $\hat{x}$, does not depend on the head tax $T_L$, which is determined such that its revenue pays the public good, $G_L$. As a consequence, the choice of $G_L$ and $T_L$ is independent of the median voter’s choice of the workplace. Furthermore, the only choice to be taken by the median voter of each jurisdiction is the level of public good provided in that region. The equilibrium values for $G_H$ and $G_L$ are therefore the same as in the first best, implicitly defined by (6) and (7).

Regarding the number of commuters, in the presence of an externality the decentralized equilibrium with lump sum taxes cannot be optimal. When only the residence tax is used, the marginal interjurisdictional commuter is given by equation (3) with the distortive taxes set to zero, i.e.,

$$\hat{x} = \alpha_H - \alpha_L - (z_H - z_L) \quad \frac{2c}{2}$$

Assuming $z(N_i) = \eta N_i^2$, $i = H, L$, we get:

$$\hat{x}^* = \frac{(\alpha_H - \alpha_L)}{2c + \eta} \quad (3.12)$$
For an interior solution $\hat{x}^* < \frac{1}{2}$, i.e., $c > (\alpha_H - \alpha_L) - \frac{\eta}{2}$.

If we compare the tax competition equilibrium obtained with the first-best we can state the following proposition:

**Proposition 1:** In the equilibrium where only the residence tax is used there is overcommuting of agents.

*Proof.* See appendix.

Since congestion is a negative externality, there is overcommuting: residents of L do not take into account their impact on the congestion cost in H to the other workers of that employment center. As a result, more agents than optimal are commuting from L to H.

### 3.5 The Equilibrium with Residence Taxes and a Toll

We now analyze the tax competition equilibrium obtained when local governments can use both the residence tax (lump-sum) and a toll charged when agents cross the jurisdiction border. As a result, interjurisdictional commuters bear an additional cost, the toll, meaning that local governments face a trade-off between increasing the revenue obtained with each toll payer and reducing the number of interjurisdictional commuters.

Similarly to the previous section, each local government maximizes the utility of the median voter, $u_{m_i}$, subject to the budget constraint of the jurisdiction, i.e., $G_i = NT_i + P_i\hat{x}$, and to the commuting equilibrium, $\hat{x}$, given by equation (3) when $\tau_H = 0$ and $\tau_L = 0$.

Notice that the utility enjoyed by the median voter of H and by the median voter of L when he works in his own jurisdiction are the same as in the previous sections, given by (9) and (10), respectively.

However, if the median voter of L works in H, his utility is:

$$u_{m_L} = \alpha_H - T_L + W - z \left( \frac{1}{2} + \hat{x} \right) + v(G_L) - c \left( \gamma + \frac{1}{4} \right) - P_H \quad (3.13)$$

In the low productivity jurisdiction, since there are no agents commuting from H to L, there is no possible revenue from a toll. The median voter of L maximizes $u_{m_L}$ given by (10) or (13) subject to the commuting equilibrium in (3) and the budget constraint $G_L = \frac{1}{2}T_L$, by choosing $G_L$ and $T_L$. 

65
When the toll is being used, the marginal interjurisdictional commuter is given by equation (3) with the wage taxes equal to zero, i.e.,

\[
\hat{x}^{**} = \frac{\alpha_H - \alpha_L - (z_H - z_L) - P_H}{2c}
\]  

(3.14)

Regarding the choice in the high productivity jurisdiction, since there are agents commuting from L to H, the median voter of H has the incentive to choose a positive amount for the toll, so that part of the local budget is paid by the residents of L. The median voter of H thus maximizes his utility (9), subject to the commuting equilibrium (14) and to the budget constraint given by \( G_H = T_H(1/2) + P_H \hat{x} \), by choosing \( G_H, T_H \) and \( P_H \).

From the first order condition on \( P_H \) we get:

\[
P_H = 2c \hat{x} + \frac{z_H'}{2} = \frac{\alpha_H - \alpha_L - (z_H - z_L) + \frac{z_H'}{2}}{2}
\]

Since some L residents are commuting to work in H, \( \hat{x} \) is positive and since \( z \) is an increasing function, \( z' \) is always positive, allowing us to state the following Lemma:

**Lemma 1:** When local governments have the possibility of charging a toll at the entrance of the jurisdiction, the high productivity jurisdiction charges a positive toll.

The high productivity jurisdiction charges a positive amount to allow L residents to enter in H, thus exporting part of the tax burden of H to the interjurisdictional commuters, and simultaneously reducing the congestion in H. The amount of the toll trades-off the revenue per commuter with the reduction of commuters resulting from the toll, the latter having a negative impact on the revenue but a positive impact on the congestion cost.

The marginal interjurisdictional commuter becomes:

\[
\hat{x}^{**} = \frac{\alpha_H - \alpha_L - (z_H - z_L) - \frac{z_H'}{2}}{4c}
\]

Assuming \( z(N_i) = \eta \frac{N_i^2}{2}, i = H, L \), we get:

\[
\hat{x}^{**} = \frac{\alpha_H - \alpha_L - \frac{1}{3} \eta}{4c + \frac{2}{3} \eta}
\]

(3.15)

For feasibility, \( \hat{x}^{**} \geq 0 \), i.e., \( \alpha_H - \alpha_L > \frac{\eta}{4} \).

If we compare the tax competition equilibrium obtained in (15) with
the first-best we can state the following proposition:

**Proposition 2:** In the equilibrium where both the residence tax, and a toll at the entrance of H are used, there is undercommuting of agents.

*Proof.* See appendix.

The introduction of the toll sufficiently increases the commuting cost that it outweighs the externality effect, moving the equilibrium from overcommuting to undercommuting. The distortion in the commuting costs introduced by this fiscal mechanism more than offsets the congestion externality that led to the overcommuting situation in the previous case. This is due to the fact that the median voter of H is using the toll with two purposes: reducing the overcongestion in H and, simultaneously, getting revenue from the interjurisdictional commuters.

It is possible to replicate the first best by setting the toll such that it leads to the optimal commuting level defined in (8), and decentralizing the decision on the provision of the public good, which we already know that is optimally provided.

**Proposition 3:** The following optimal toll

\[
P_{H}^{*o} = \frac{2\eta}{2c + 3\eta}(\alpha_{H} - \alpha_{L})
\]

combined with decentralized decision about public good and residence taxes decentralizes the first best.

*Proof.* See appendix.

In this case it is not possible to decentralize the first best allowing the jurisdiction to price the externality, as predicted by Coase Theorem, as the externality is between agents, not between jurisdictions, while the price of the externality is being decided by the jurisdiction.

### 3.6 The Equilibrium with Residence Taxes and Wage Taxes

We now analyze the tax competition equilibrium obtained when local governments can use both the residence tax (lump-sum) and the wage tax. As a result of this tax, agents are now concerned with the net wage \( \omega_{i} = \alpha_{i}(1 - \tau_{i}) \) earned in each employment center, rather than the gross wage determined by their productivity. Consequently, the choice of the
wage tax results from a trade-off between increasing the revenue per worker and reducing the number of workers in the employment center (and, therefore, the number of tax payers) due to the reduction of the net wage in the jurisdiction.

When both the residence and the wage tax are used, the marginal interjurisdictional commuter is given by (3) with the toll equal to zero, i.e.,

$$x^{**} = \frac{\omega_H - \omega_L - (z_H - z_L)}{2c} \quad (3.16)$$

where $\omega_H = \alpha_H (1 - \tau_H)$ and $\omega_L = \alpha_L (1 - \tau_L)$.

Similarly to the previous sections, each local government maximizes the utility of the median voter, $u_{m_i}$, subject to the commuting equilibrium, $x$, given by equation (3) when $P_H = 0$, and to the budget constraint of the jurisdiction, i.e., $G_i = NT_i + N_i \tau_i \alpha_i$, where $N_i$ is the number of workers in the employment center of jurisdiction $i$, $\alpha_i$ is the gross wage paid in $i$ and $\tau_i$ is the tax rate charged on the wage that is paid in that jurisdiction.

The median voter of $H$ enjoys a utility of:

$$u_{m_H} = \omega_H - T_H + W - z \left( \frac{1}{2} + \hat{x} \right) + v(G_H) - c \left( -\frac{1}{4} + \gamma \right) \quad (3.17)$$

Regarding the median voter of $L$, if he works in $L$, his utility is:

$$u_{m_L} = \omega_L - T_L + W - z \left( \frac{1}{2} - \hat{x} \right) + v(G_L) - c \left( \gamma - \frac{1}{4} \right) \quad (3.18)$$

If the median voter of $L$ decides to work in $H$, he receives the net wage paid in that employment center, and must face the congestion of $H$. Therefore, he enjoys a utility of:

$$u_{m_L} = \omega_H - T_L + W - z \left( \frac{1}{2} + \hat{x} \right) + v(G_L) - c \left( \gamma + \frac{1}{4} \right) \quad (3.19)$$

Unlike the case in sections 4 and 5, the median voter of $L$ has now an instrument to influence the number of commuters (and, therefore, the congestion costs faced in each jurisdiction). We must separate the case in which the median voter of $L$ works in $L$ (and has to pay the wage tax of $L$, which is decided by himself) from the case where the median voter of $L$ works in $H$. 
Regarding the choice in the high productivity jurisdiction, the median voter of H maximizes his utility as given by (18), subject to the commuting equilibrium (17) and to the budget constraint given by \( G_H = T_H(1/2) + \tau_H \alpha_H (1/2 + \hat{x}) \), by choosing \( G_H, \tau_H \) and \( T_H \).

### 3.6.1 Median voter of L works in L

Solving for the equilibrium in wage taxes we obtain\(^{14}\)

\[
\tau_H^{**} = \frac{\alpha_H - \alpha_L - (z_H - z_L) + z'_H + \frac{1}{2}z'_L}{3\alpha_H}
\]

\[
\tau_L^{**} = \frac{-(\alpha_H - \alpha_L) + (z_H - z_L) + z'_L + \frac{1}{2}z'_H}{3\alpha_L}
\]

These two equations yield the equilibrium marginal interjurisdictional commuter:

\[
\hat{x}^{***} = \frac{\alpha_H - \alpha_L - (z_H - z_L) - \frac{1}{2}(z'_H - z'_L)}{6c}
\]

Assuming \( z(N_i) = \eta \frac{N^2_i}{2} \), \( i = H, L \), we get:

\[
x^{***} = \frac{\alpha_H - \alpha_L}{6c + 2\eta}
\] (3.20)

\[
\tau_H^{***} = \frac{(12c + 3\eta)(\alpha_H - \alpha_L) + 9c\eta + 3\eta^2}{12(3c + \eta)\alpha_H}
\] (3.21)

\[
\tau_L^{***} = \frac{-(12c + 3\eta)(\alpha_H - \alpha_L) + 9c\eta + 3\eta^2}{12(3c + \eta)\alpha_L}
\] (3.22)

For the median voter of L to work in L \( \hat{x} < 1/4 \), i.e., \( \alpha_H - \alpha_L < \frac{3c + \eta}{2} \).

The next proposition characterizes the equilibrium:

**Proposition 4:** In the equilibrium where both the residence and the wage taxes are available, and both median voters work in their own jurisdictions:

(i) The wage is taxed in H;

(ii) The wage in L is subsidized if the productivity gap between jurisdictions is high, and taxed if the productivity gap is low.

\(^{14}\)The full first order conditions can be found in the appendix.
(iii) There can be undercommuting, if the commuting cost is high \((c > \eta/4)\), or overcommuting of agents, if the commuting cost is low \((c < \eta/4)\). If \(c = \eta/4\) the optimal commuting level is achieved.

Proof. See appendix. \(\square\)

The result that region H taxes wages is also obtained in Peralta (2007): H residents are exporting part of their tax burden to the interjurisdictional commuters from region L using the wage tax. In this case the tax on the wage is also a way of reducing the incentive to commute, which decreases the congestion faced by the residents in H.

Regarding the wage tax in region L we notice that, if the productivity gap is high enough, i.e., if the number of interjurisdictional commuters is high, the wage in L is subsidized. The median voter, who works in the employment center of L, is therefore exporting part of the tax burden to the interjurisdictional commuters by increasing the residence tax and distributing part of the amount levied with this tax to the workers in L.

However, if the productivity gap is low, leading to a low number of interjurisdictional commuters, the median voter of L taxes the wage. In this case, since there is a higher number of workers in the employment center of L, the median voter wants to use the wage tax as a way to make that employment center less attractive and, that way, reduce the congestion he faces to reach his workplace.

In terms of commuters, we may have either under or overcommuting, depending on the commuting cost per mile, \(c\), faced by the agents and the intensity of congestion, \(\eta\). If the commuting cost per mile is \(\eta/4\), the decentralized equilibrium leads to the optimal commuting level. \(c\) above that threshold leads to undercommuting, and below it to overcommuting.

### 3.6.2 Median voter of L works in H

We now analyse the Nash equilibrium where the median voter of L works in the employment center of H, i.e., he is an interjurisdictional commuter. Note that the problem for the median voter of H remains unchanged. However, for \(m_L\) the first order conditions of the utility maximization problem change. As a consequence, the reaction functions on \(\tau_H\) and \(\tau_L\) become\textsuperscript{15}

\[
\tau_H^{**} = \frac{\alpha_H - \alpha_L - (z_H - z_L) + \frac{1}{2} z_H' + c}{3\alpha_H}
\]

\textsuperscript{15}as in the previous section, the full first order conditions can be found in the appendix.
\[ \tau_{L}^{***} = -\frac{(\alpha_H - \alpha_L) + (z_H - z_L) - \frac{1}{2}z_H' + 2c}{3\alpha_L} \]

These equations lead to the equilibrium marginal interjurisdictional commuter:

\[ \hat{x}^{***} = \frac{\alpha_H - \alpha_L - (z_H - z_L) - z_H' + c}{6c} \]

Assuming \( z(N_i) = \eta N_i^2 \), \( i = H, L \), we get:

\[ \hat{x}^{***} = \frac{\alpha_H - \alpha_L + c - \frac{1}{2}\eta}{6c + 2\eta} \] (3.23)

\[ \tau_{H}^{***} = \frac{(\alpha_H - \alpha_L)(12c + 3\eta) + c(12c + 6\eta) + \frac{3}{2}\eta^2}{12(3c + \eta)\alpha_H} \] (3.24)

\[ \tau_{L}^{***} = \frac{-(\alpha_H - \alpha_L)(12c + 3\eta) + c(24c + 6\eta) - \frac{3}{2}\eta^2}{12(3c + \eta)\alpha_L} \] (3.25)

For the median voter of L to work in H \( \hat{x} > \frac{1}{4} \), i.e., \( \alpha_H - \alpha_L > \frac{c + 2\eta}{2} \).

The next proposition characterizes this tax competition equilibrium:

**Proposition 5:** In the equilibrium where both the residence and the wage taxes are available, and both median voters work in the high productivity jurisdiction:

(i) The wage is taxed in H;

(ii) The wage in L is subsidized if the productivity gap between jurisdictions is high enough, and taxed if the productivity gap is low enough;

(iii) There is undercommuting of agents.

**Proof.** See appendix. \( \square \)

The results are analogous to the ones reached when both median voters work in their own jurisdictions. The intuition exposed previously applies with the necessary adaptations: H residents are exporting part of their tax burden to the interjurisdictional commuters from region L using the wage tax, which incidently decreases the congestion in H.

Regarding the wage tax in region L we notice that, unlike the case where there is no congestion cost, the median voter may be willing to
subsidize the wage paid in the employment center of \( L \) even though he does not work there. That occurs in cases where the number of interjurisdictional commuters is high, therefore imposing a congestion problem of such magnitude in \( H \) that even the median voter of \( L \) is willing to pay to reduce the number of workers in that employment center.

Regarding the number of commuters, we see that in this case the decentralized equilibrium always results in undercommuting. Since both median voters work in the business center of \( H \), both have the incentive to reduce congestion there, thus leading to number of interjurisdictional commuters below the optimal.

### 3.7 Welfare analysis

In this section we compare the equilibria computed previously, in order to understand what is the best choice of fiscal mechanisms to apply in a linear city divided in two jurisdictions where agents face congestion costs in the employment centers. We compare the overall utility achieved in each case by looking at the module of the difference between the level of commuting achieved and the benchmark of the first best. Besides working as a productivity efficiency indicator, the absolute gap is a valid approach for overall utility since, using our assumption that \( z_i = \eta N_i^2 / 2, i = H, L \), overall utility is a quadratic function of the marginal interjurisdictional commuter.\(^{16}\) We perform pair comparisons of the equilibria computed in the previous sections to understand which one is preferable in each case.

Comparing the equilibrium when only the residence taxes are available with the one when the median voter uses both the residence taxes and the toll, we find that if the commuting cost is high enough, the use of the residence tax alone is better, while if the commuting cost is low enough, it is preferable to combine the residence taxes with a toll.

Recall that the use of the residence taxes alone leads to overcommuting of agents, while the combination of those taxes with a toll leads to undercommuting. If the commuting cost is low, the number of interjurisdictional commuters becomes too high in the case where only residence taxes are used, therefore imposing a high congestion cost in the high productivity jurisdiction. In this case, the use of the toll reduces the number of commuters to a figure closer to the optimum. However, if the commuting cost is high, the number of interjurisdictional commuters is low and, if the toll is used, that number of commuters is reduced to a

\(^{16}\)For simplicity, we assume \( \eta = 1 \) in this section.
figure that lies farther away from the first best than the one obtained with the residence taxes alone.

Comparing the equilibrium when only the residence taxes are available with the one obtained when the median voter uses both residence and wage taxes, we find a similar conclusion: if the commuting cost is high enough, the use of the residence tax alone is better, while if the commuting cost is low enough it is preferable to combine residence taxes with wages taxes. Unlike the toll, the wage taxes can lead to under or overcommuting, but the intuition stated for the case of the toll is valid in this case as well.

From the two comparisons performed so far we know that for low commuting costs it is preferable to use a distorting fiscal mechanism (toll or wage tax) rather than just a lump-sum tax (residence tax). The distortion generated by the toll or by the wage tax partially offsets the negative externality present in the congestion cost. Notice that, for the commuting equilibria to make sense, the marginal interjurisdictional commuter $\hat{x}$ must be inside the city, so between $-1/2$ and $1/2$. As a consequence, the cases when the commuting cost $c$ is very small are only feasible if the productivity gap $\alpha_H - \alpha_L$ is also small. On the other hand, if the wage gap is very high, the commuting cost must also be high. This means that, for high levels of productive asymmetry, the commuting cost must be high and, therefore, the residence tax alone is the best choice.

We shall now compare the two distortive taxes to find which one is better. This comparison lead us to conclude that, for low levels of productivity gap, the wage tax leads to a better output, while for higher levels of wage gap the toll is preferable. Once again, recall that the toll always leads to undercommuting while the wage tax can lead to under or overcommuting, depending on the transportation cost and on the productivity gap. If the productivity gap is very small, the effect of the toll is too strong, reducing the number of commuters to a figure that is father away from the first best than the wage tax, which is softer. However, if the productivity gap is mild, i.e., if the number of interjurisdictional commuters is higher, the toll is more efficient in reducing that number to a figure closer to the first best than the wage tax.

The following proposition and picture summarize our findings:

**Proposition 6:** If local governments can choose to use a residence tax alone, a combination of a residence tax and a toll, or a combination of a residence tax and a wage tax, the optimal fiscal mechanism to use is:

(i) The residence tax alone if the transportation cost is high enough or
if the productivity asymmetry is high enough;

(ii) The residence tax and the wage tax if the transportation cost is low and the productivity asymmetry is low;

(iii) The residence tax and the toll if the transportation cost is low and the productivity asymmetry is mild.

Proof. See appendix.

$\Box$

Figure 1: Optimal fiscal instruments depending on the wage gap $(\alpha_H - \alpha_L)$ and on the travel cost $(c)$

3.8 Conclusion

This paper introduces congestion costs in linear city with two unequally productive jurisdictions where local governments provide public goods and agents choose in which region they want to work.

We look at three different fiscal mechanisms currently used in countries worldwide that can be used by local governments to finance their budgets: a residence tax alone, a combination of a residence tax and a toll or a combination of a residence tax and a wage tax. We show that the residence tax alone always leads to overcommuting of agents.
from the low productivity jurisdiction to the high productivity one, as
the individual agents do not take into account the negative externality
generated by the congestion cost on other individuals. The introduction
of a toll creates a high additional cost to interjurisdictional commuters,
therefore leading to undercommuting. The wage tax can either lead to
over or undercommuting, depending on the productivity asymmetry and
on the transportation cost.

Finally, we compare the three fiscal mechanisms, showing that if the
transportation cost is high enough or the productivity asymmetry is high
enough, the head tax alone is the best choice, while if the transportation
cost is low enough: (i) the wage tax is best for low productivity asym-
metries but (ii) the toll is preferable for mild productivity asymmetries.

We do not analyze the possibility of using all three tax instruments
simultaneously, but the outcome would be similar to the one obtained
when local governments use the residence taxes combined with wage
taxes. As a matter of fact, the wage tax and the toll charged in $H$
both allow the $H$ residents to export part of their tax bill to the inter-
jurisdictional commuters. For the residents in the low productivity region
that want to commute to $H$, what matters is the difference between the
net amount received in $H$ and in $L$. Both the wage tax and the toll are
perceived as a reduction in the net wage that $L$ residents receive when
working in $H$, so they are indifferent between paying one or the other.
As for $H$ residents, the same reasoning is valid: what matters is the total
revenue levied with these distortive taxes, which impact in the number
of interjurisdictional commuters $\hat{x}$ is identical.
3.9 Appendix

Proof of Proposition 1.

\[ \hat{x}^* - \hat{x}^o = \frac{(H - a_L)}{2c+\eta} - \frac{(H - a_L)}{2c+3\eta} \]

Since the numerator of both fractions is the same and \(2c + 3\eta > 2c + \eta\),
\[ \hat{x}^* > \hat{x}^o \]

\[ \square \]

Proof of Proposition 2.

\[ \hat{x}^{**} - \hat{x}^o = \frac{(H - a_L) - 0.25\eta}{4c+1.5\eta} - \frac{(H - a_L)}{2c+3\eta} = - \frac{(2(a_H - a_L) + \frac{\eta}{2})c - \frac{\eta}{2}(a_H - a_L - \frac{\eta}{2})}{2(2c+3\eta)^2} < \]

\[ \frac{(2(a_H - a_L) + \frac{\eta}{2})(a_H - a_L - \frac{\eta}{2})}{2(2c+3\eta)^2} = - \frac{(a_H - a_L - \frac{\eta}{2})^2}{(2c+3\eta)^2} < 0 \]

Where we have used the lower bound of \(c, a_H - a_L - 0.5\eta\).

\[ \square \]

Proof of Proposition 3.

Replacing \( P_{H^{*}} = \frac{2n}{2c+3\eta}(a_H - a_L) \) in the expression of \( x^{**} \) we reach
\[ \hat{x}^{**} = \frac{(a_H - a_L)}{2c+3\eta} = \hat{x}^o \]

\[ \square \]

FOC of the utility maximization problems in section 6.1.

\[ \frac{\partial U}{\partial \tau_H} = 0 \iff -a_H + 2a_L \left( \frac{1}{2} + \hat{x} \right) + 2\tau_H a_H \frac{\partial \hat{x}}{\partial \tau_H} - \frac{\partial \hat{x}}{\partial \tau_H} \frac{\partial \tau_H}{\partial \tau} z_H = 0 \]

where \( z_H' \) is the partial derivative of the congestion cost, \( z \), with respect to the number of workers, evaluated at the number of agents working at the high productivity jurisdiction, i.e., \( \frac{1}{2} + \hat{x} \).

Using \( \hat{x} = \frac{a_H(1-\tau_H) - a_L(1-\tau_L) + (z_H - z_L)}{2c} \),

\[ \tau_H^{**} = \frac{a_H - a_L(1-\tau_L)}{2a_L} + \frac{z_L'}{4a_L} \]

\[ \frac{\partial U}{\partial \tau_L} = 0 \iff -a_L + 2a_L \left( \frac{1}{2} - \hat{x} \right) - 2\tau_L a_L \frac{\partial \hat{x}}{\partial \tau_L} + \frac{\partial \hat{x}}{\partial \tau_L} z_L = 0 \]

where \( z_L' \) is the partial derivative of the congestion cost, \( z \), with respect to the number of workers, evaluated at the number of agents working at the low productivity jurisdiction, i.e., \( \frac{1}{2} - \hat{x} \).

Using \( \hat{x} = \frac{a_H(1-\tau_H) - a_L(1-\tau_L) + (z_H - z_L)}{2c} \),

\[ \tau_L^{**} = \frac{a_H - a_L(1-\tau_H)}{2a_L} + \frac{z_L'}{4a_L} \]

Combining the two reaction functions on the wage taxes we get:

\[ \tau_H^{**} = \frac{a_H - a_L(z_H - z_L) + z_L'}{3a_L} \]
Using $\tau z$ where $\tau$ to the number of workers, evaluated at the number of agents working at
The FOC on $\tau$ the same as in section 6.1.

\[ \tau^{**} = \frac{(\alpha_H - \alpha_L)(z_H - z_L) + \frac{1}{2}z_H + \frac{1}{2}z_H}{3\alpha_L} \]

**Proof of Proposition 4.**

(i) Assuming $z_i = \eta \frac{N^2}{2}$, $\tau^{**}_H = \frac{\alpha_H - \alpha_L + \frac{3}{2}\eta - \frac{1}{2}\eta^2}{3\alpha_H}$

Knowing $x^{**} = \frac{\alpha_H - \alpha_L}{6c + 2\eta}$, $\tau^{**} = \frac{(12c + 3\eta)(\alpha_H - \alpha_L) + 9c\eta + 3\eta^2}{12(3c + \eta)\alpha_H} > 0$

(ii) Assuming $z_i = \eta \frac{N^2}{2}$, $\tau^{**}_L = \frac{(\alpha_H - \alpha_L + \frac{3}{2}\eta - \frac{1}{2}\eta^2)}{3\alpha_L}$

Knowing $x^{**} = \frac{\alpha_H - \alpha_L}{6c + 2\eta}$, $\tau^{**} = \frac{-(12c + 3\eta)(\alpha_H - \alpha_L) + 9c\eta + 3\eta^2}{12(3c + \eta)\alpha_L}$

Which can be positive or negative, depending on the wage gap $(\alpha_H - \alpha_L)$, on the travel cost $c$, and on the intensity of the congestion cost $\eta$.

For instance, taking $\eta = 1$, $\tau^{**}_L < 0$ if $(\alpha_H - \alpha_L) > \frac{3c + 1}{4c + 1}$ or positive otherwise.

(iii) $\dot{x}^{**} - \dot{x}^o = \frac{\alpha_H - \alpha_L}{6c + 2\eta} - \frac{\alpha_H - \alpha_L}{2c + \eta}$

Since the numerator of both fractions is the same, we can look just at the denominator:

If $c < \frac{1}{4}\eta$, $\dot{x}^{**} > \dot{x}^o$. Otherwise, $\dot{x}^{**} < \dot{x}^o$

**FOC of the utility maximization problems in section 6.2.**

The FOC on $U_{m,H}$, and consequently the reaction function of $\tau_H$, is the same as in section 6.1.

\[ \tau^{**}_H = \frac{(\alpha_H - \alpha_L)(1 - \tau_L) - (z_H - z_L)}{2\alpha_H} + \frac{z_H}{4\alpha_H} \]

\[ \frac{\partial U_{m,L}}{\partial \tau_L} = 0 \iff 2\alpha_L \left( \frac{1}{2} - \dot{x} \right) - 2\tau_L \alpha_L \frac{\partial \dot{x}}{\partial \tau_L} - \frac{z'_H}{\tau_L} z'_H = 0 \]

where $z'_H$ is the partial derivative of the congestion cost, $z$, with respect to the number of workers, evaluated at the number of agents working at the high productivity jurisdiction, i.e., $\frac{1}{2} + \dot{x}$.

Using $\dot{x} = \frac{\alpha_H(1 - \tau_H) - \alpha_L(1 - \tau_L) - (z_H - z_L)}{c}$

\[ \tau^{**}_L = \frac{c + \alpha_L - \alpha_H(1 - \tau_L) + (z_H - z_L)}{2\alpha_L} - \frac{z'_H}{4\alpha_L} \]

Combining the two reaction functions on the wage tax we get:

\[ \tau^{**}_H = \frac{(\alpha_H - \alpha_L)(z_H - z_L) + \frac{1}{2}z'_H + c}{3\alpha_H} \]

\[ \tau^{**}_L = \frac{-(\alpha_H - \alpha_L)(z_H - z_L) - \frac{1}{2}z'_H + 2c}{3\alpha_L} \]
Proof of Proposition 5.

(i) Assuming \( z_i = \eta N_i^2 \), \( \tau_{\mu}^{**} = \frac{\phi_{H} - \phi_{L} + \frac{1}{3} \eta x + c}{3\phi_{H}} \)

Knowing \( x^{**} = \frac{\phi_{H} - \phi_{L} + \frac{1}{3} \eta x + c}{6c + 2\eta} \);

\[ \tau_{\mu}^{**} = \frac{(\phi_{H} - \phi_{L})(12c + 3\eta) + c(12c + 6\eta) + \frac{3}{2} \eta^2}{12(3c + \eta)\phi_{H}} > 0 \]

(ii) Assuming \( z_i = \eta N_i^2 \), \( \tau_{\mu}^{**} = \frac{-(\phi_{H} - \phi_{L}) + \frac{1}{3} \eta x - \frac{1}{4} \eta + 2c}{3\phi_{L}} \)

Knowing \( x^{**} = \frac{\phi_{H} - \phi_{L} + \frac{1}{3} \eta x - c}{6c + 2\eta} \);

\[ \tau_{\mu}^{**} = \frac{-(\phi_{H} - \phi_{L})(12c + 3\eta) + c(24c + 6\eta) - \frac{3}{2} \eta^2}{12(3c + \eta)\phi_{L}} \]

Which can be positive or negative, depending on the wage gap \((\phi_{H} - \phi_{L})\), on the travel cost \(c\), and on the intensity of the congestion cost \(\eta\).

For instance, taking \( \eta = 1 \), \( \tau_{\mu}^{**} < 0 \) if \( (\phi_{H} - \phi_{L}) > \frac{16c^2 + 4c - 1}{6c + 2} \) and \( \tau_{\mu}^{**} > 0 \) otherwise.

Nevertheless, the net wage gap is always positive, irrespective of \(\eta\).

\[ \omega_{H} - \omega_{L} = (\phi_{H} - \phi_{L}) \left( \frac{2c + \eta}{6c + 2\eta} \right) + \frac{3c^2 + \eta^2}{6c + 2\eta} > 0 \]

(iii) \( \hat{x}^{**} - \hat{x}^{o} = \frac{(\phi_{H} - \phi_{L})(\eta - 4c) + (2c^2 + 2c\eta - \frac{3}{2} \eta^2)}{(6c + 2\eta)(2c + 3\eta)} \)

Focusing on the sign of the numerator,

if \( (\eta - 4c) < 0 \), and knowing that \( \phi_{H} - \phi_{L} > \frac{c^2 + 2\eta}{2} \) for the median voter of L to work in H,

\( (\phi_{H} - \phi_{L})(\eta - 4c) + (2c^2 + 2c\eta - \frac{3}{2} \eta^2) < \frac{-3c^2 - \eta^2}{2} < 0 \), i.e., \( \hat{x}^{**} < \hat{x}^{o} \)

if \( (\eta - 4c) > 0 \), i.e., \( c < \frac{\eta}{4} \),

\( (\phi_{H} - \phi_{L})(\eta - 4c) + (2c^2 + 2c\eta - \frac{3}{2} \eta^2) < \langle (\phi_{H} - \phi_{L})(\eta - 4c) - \frac{15}{16} \eta^2 < 0 \), i.e., \( \hat{x}^{**} < \hat{x}^{o} \)

\( \square \)

Proof of Proposition 6.

Using the overall utility found in section 3. as \( U = U_{H} + U_{L} \), where \( U_{H} \) is the overall utility of all residents in jurisdiction H and \( U_{L} \) is the overall utility of all residents in jurisdiction L, and assuming \( z_i = \eta N_i^2 \),
with \( \eta = 1 \), we get:

\[
U = -\left(\frac{3}{2} + c\right) \hat{x}^2 + (\alpha_H - \alpha_L) \hat{x} + d
\]

with \( d = \frac{1}{2} \alpha_H + \alpha_L G_H - G_L W + \frac{1}{2} [v(G_H) + v(G_H)] - 2c \left( \frac{3}{8} + \gamma^2 - \frac{7}{4} \right) \)

i.e.,

\[
U = a \hat{x}^2 + b \hat{x} + d
\]

The difference in overall utility attained in two different levels of commuting, \( \hat{x}_1 \) and \( \hat{x}_2 \), is therefore:

\[
U(\hat{x}_1) - U(\hat{x}_2) = a(\hat{x}_1^2 - \hat{x}_2^2) + b(\hat{x}_1 - \hat{x}_2) = (\hat{x}_1 - \hat{x}_2)a(\hat{x}_1 + \hat{x}_2)
\]

Which can be written using the deviation of each commuting level to the optimal commuting, \( \hat{x} = \hat{x}_1 - \hat{x}_2 \):

\[
U(\hat{x}_1) - U(\hat{x}_2) = (\hat{x}_1 - \hat{x}_2)(2a\hat{x}^o + a(\hat{x}_1 + \hat{x}_2) + b)
\]

Which, using \( \hat{x}^o = -\frac{b}{2a} \) becomes

\[
U(\hat{x}_1) - U(\hat{x}_2) = a(\hat{x}_1 - \hat{x}_2)(\hat{x}_1 + \hat{x}_2)
\]

Since \( a = -\left(\frac{3}{2} + c\right) < 0 \), it immediately follows that when \( (\hat{x}_1 - \hat{x}_2)(\hat{x}_1 + \hat{x}_2) < 0 \), \( \hat{x}_1 \) is better, while with \( (\hat{x}_1 - \hat{x}_2)(\hat{x}_1 + \hat{x}_2) > 0 \), \( \hat{x}_2 \) is better. We now compute the deviations of each equilibrium to the optimal commuting:

\[
\hat{x}^* = \hat{x}^o - \hat{x}^o = \frac{2(\alpha_H - \alpha_L)}{(2c + 1)(2c + 3)} > 0
\]

\[
\hat{x}^{**} = \hat{x}^{**} - \hat{x}^o = -\frac{(\alpha_H - \alpha_L)(2c - \frac{3}{2}) + \frac{3}{4}}{2(2c + 3)^2} < 0
\]

\[
\hat{x}^{***} = \hat{x}^{***} - \hat{x}^o = \frac{(\alpha_H - \alpha_L)(1 - 4c)}{(6c + 2)(2c + 3)}
\]

For the cases of the residence tax alone, the residence tax and toll, and the residence tax and the wage tax, respectively, and using the results of Propositions 1 and 2.

We now compare welfare levels across the three possible scenarios.

(i) Residence taxes and a toll vs. residence taxes alone

First of all, notice that \( (\hat{x}^{**} - \hat{x}^*) < 0 \) since \( \hat{x}^{**} < 0 < \hat{x}^* \). Moreover, straightforward algebra allows us to write \( (\hat{x}^{**} + \hat{x}^*) = \frac{(c-r_1)(c-r_2)}{(4c+1.5)(2c+3)(2c+1)} \), where

\[
r_1 = \frac{k_1 + \sqrt{k_1^2 + k_2}}{k_3} > 0, r_2 = \frac{k_1 - \sqrt{k_1^2 + k_2}}{k_4} < 0, \text{ with } k_1 = 9(\alpha_H - \alpha_L) - 2 > 0, k_2 = 4[4(\alpha_H - \alpha_L) + 1][4.5(\alpha_H - \alpha_L) - 0.75] > 0
\]

79
since $\alpha_H - \alpha_L > \frac{1}{4}$, $k_3 = [4(\alpha_H - \alpha_L) + 1] > 0$ using the fact that $\alpha_H - \alpha_L > 1/4$.

Therefore, $U(x^{**}) > U(x^*)$, i.e., the combination of the residence taxes and the toll is preferable to the use of the residence taxes alone, if $c$ is low ($c < r_1$). Otherwise, the use of the residence tax alone is preferable.

(ii) Residence and wage taxes vs. residence taxes alone

Straightforward algebra allow us to show that

$$ (\tilde{x}^{***} - \tilde{x}^*) = \frac{(\alpha_H - \alpha_L)(-8c^2 - 14c - 3)}{(6c+2)(2c+1)(2c+3)} < 0 \quad \text{and} \quad (\tilde{x}^{***} + \tilde{x}^*) = -\frac{(\alpha_H - \alpha_L)}{(6c+2)(2c+1)(2c+3)} (c - r_3)(c - r_4),$$

where $r_3 = \frac{10 + \sqrt{260}}{16} > 0$ and $r_4 = \frac{10 - \sqrt{260}}{16} < 0$. 

Therefore, $U(x^{**}) > U(x^*)$, i.e., the combination of the residence and wage taxes is preferable to the use of the residence taxes alone, if $c$ is low ($c < r_3$). Otherwise, the use of the residence tax alone is preferable.

(iii) Residence and wage taxes vs. residence taxes and toll

Computations allow us to show that

$$ (\tilde{x}^{***} - \tilde{x}^*) = \frac{(\alpha_H - \alpha_L)(-56c^3 - 34c^2 - 10c - 1.5)}{(6c+2)(4c+1.5)(2c+3)} < 0, \quad \text{using} \quad \frac{\alpha_H - \alpha_L}{2c+1} < \frac{1}{2},$$

and $$(\tilde{x}^{***} - \tilde{x}^*) = -\frac{(\alpha_H - \alpha_L)}{6c+2}(c - r_3)(c - r_4),$$

where $r_5 = \frac{2(\alpha_H - \alpha_L) - 2}{6 - 8(\alpha_H - \alpha_L)} < 0$ and $r_6 = \frac{12(\alpha_H - \alpha_L) - 9}{6 - 8(\alpha_H - \alpha_L)} > 0$ if $(\alpha_H - \alpha_L) \in (0.75; 1)$.

Therefore, $U(x^{**}) > U(x^*)$, i.e., the combination of the residence and wage taxes is preferable to the use of the combination of residence taxes and toll, if $c < r_6$ and $(\alpha_H - \alpha_L) \in (0.75; 1)$. Otherwise, it is preferable to use the residence taxes combined with a toll.

(iv) Summarizing:

- The residence taxes alone are preferable when $c$ is high, i.e., $c > \frac{10 + \sqrt{260}}{16}$ if $(\alpha_H - \alpha_L) < 0.75$, or $c > r_1$ if $(\alpha_H - \alpha_L) > 0.75$.

- The combination of the residence and wage taxes is preferable when both $c$ and the wage gap are low, i.e., $c < \frac{10 + \sqrt{260}}{16}$ and $(\alpha_H - \alpha_L) < 0.75$.

- The combination of the residence taxes and a toll is preferable when $c$ is low and the wage gap is mild, i.e., $c < r_1$ and $(\alpha_H - \alpha_L) > 0.75$.

- The feasibility condition of $\hat{x}$ not being out of the city limits must be observed, i.e., if the productivity gap is very high, the
travel cost must also be high in order to have the marginal interjurisdictional commuter inside the city. That situation leads us to the case where $c$ is high and, therefore, the residence taxes alone are preferable.
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