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# COVID CRASH: A STUDY OF VOLATILITY SPILLOVERS FROM STOCKS TO US INDEXES AND FROM US INDEXES TO CRYPTOCURRENCIES

**Pedro Maria Fragoso de Almeida Carvalho**

Dissertation presented as partial requirement for obtaining the Master's degree in Data Science and Advanced Analytics, with major in Data Science

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**Adviser**: Bruno Damásio

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*To OT.*

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*"To make this hideous poverty worse, it exists side by side with wanton luxury." (Sir Thomas More, Utopia, 1516)*

# Abstract

During market crashes, panic ensues, investors run to cover and usually only in the aftermath questions arise: How? How much? In which markets? Where did it start? This work seeks to answer some of these questions by quantifying the volatility spillovers from stocks to indexes, but also indexes to cryptocurrency. Not only there is a quantification by variance ratios of the contribution of one asset to another but as well a demonstration of the irregularity and uniqueness of these spillovers compared to other moments in the market's history. The results from this work shed some light in the intricate relationship of causal effects in volatility such as which stocks contribute the most to the variance of each index and how much of a grip do US indexes have on the overall stability of the cryptocurrencies market.

To achieve this, a relatively recent method was employed, described by Christiansen and Bekaert et al., but with some customization to adjust to the particular cases, allowing to perform variance decomposition based on multiple applications of AR-GARCH family models. Since the method relies on sums of decomposed variance, the greatest challenge is to produce variables (time series) independent from each other, and thereby, ignore covariance terms that would interfere with the validity of the analysis.

On US indexes to cryptocurrency, the dummy variables were found significant and relevant to the overall modelling of the data, indicating that the period of the market crash is important to future models which otherwise would incur in significant bias that would change the results by orders of magnitude on the variance ratio.

On stocks to US indexes, the dummy variables were found to be insignificant at the final results level (conditional correlations and variance ratios), however, not on most individual prices and returns. The period of the market crash is clear graphically by finding a major dip of all conditional correlations, which indicates that no single stock had a major influence during this time and that it was an phenomenon which affected all stocks. Stocks with individual movements that had more influence on the index continued to have the relevance during the market crash. During market crashes, this behavior is expected and this work adds confirmation.

By comparing results from both cases, a relationship between eventual spikes in stocks and trend changes in cryptocurrency becomes apparent. This work does not clarify the reason as to why it happens but only its consequence and the cause seems to be exogenous to both cryptocurrency, indexes and possibly stocks. Further work is needed to explain it.

**Keywords:** volatility spillovers, cryptocurrency, index, variance decomposition, autoregression, heteroskedasticity, GARCH, conditional variance, conditional correlation, market crash, Covid-19

# Resumo


Durante quedas de mercado, existe uma instalação generalizada de pânico e normalmente apenas depois de terminado começam a surgir questões: Como? Quanto? Em que mercados? Onde começou? Esta dissertação é procura explorar estes tópicos e responder a algumas destas questões, quantificando a volatility spillovers de acções para índices e de índices para crptomoeda. Não só existe uma quantificação através de uma rácio de variância das contribuições de um valor para o outro, mas há também uma demonstração da irregularidade e singularidade destes spillovers comparado com outros momentos no histórico dos mercados. Os resultados desta dissertação dão alguma luz de uma forma detalhada sobre a relação de causalidade de volatilidade na contribuição de cada acção para o índice e quanta força é capaz de exercer o mercado de valores sobre o de criptomoedas usando os índices como meio.

Para conseguir isto, um método relativamente recente foi utilizado, descrito por Christiansen e Bekaert et al. com algumas alterações que ajustem o método para os casos em concreto e assim decompor a variância utilizando múltiplas aplicações de modelos AR-GARCH. O maior desafio foi produzir variáveis (séries temporais) independentes entre elas e assim ignorar completamente os termos de covariância que poderiam interferir com a validade desta análise.

Em índices para criptomoedas, as variáveis *dummy* são significantes e relevantes para a modelação, indicando que o período da queda de mercado é importante para modelos futuros, que caso não seja utilizado, possa fornecer estimativas insuficientes e que alterem os resultados finais em ordens de magnitude.

Em relação a acções e índices, as variáveis *dummy* são insignificantes nas correlações condicionais e nos rácios de variância, no entanto, são significantes tanto nos resíduos dos preços e os seus retornos. O período da queda de mercado é claro nos gráficos mostrando uma lacuna em todas as correlações condicionais, o que indica que nenhuma acção em particular teve um efeito destacado em relação às outras e que foi um fenómeno que afectou todos os componentes dos índices. Acções com movimentos individuais que tinham maior influência fora da queda continuaram a exercer a mesma pressão de relativamente às outras durante a queda. Este comportamento era experado

e a presente dissertação fornece confirmação.

Comparando resultados de ambos os casos, uma relação entre alguns choques em acções e alterações de *trends* emerge. Esta dissertação não clarifica a causa e apenas documenta uma consequência da mesma pois aparenta ser externa a criptomoedas, acções e índices.

**Palavras-chave:** volatility, spillovers, criptomoedas, indices, decomposição da variância, autoregressão, heterosquedacidade, GARCH, variância condicional, correlação condicional, queda de mercado, COVID-19

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# Glossary

| | |
|---|---|
| **AR(k)** | Autoregressive mean model with k lags. 12, 13 |
| **AR(k)-TGARCH** | Autoregressive mean model with k lags with a Threshold Generalized Autoregressive Conditional Heteroskedasticity vaiance model with $p = 1, q = 1$. Similar to gjrGARCH, also designed to handle asymmetry. 13, 15 |
| **AR(k)-gjrGARCH** | Autoregressive mean model with k lags with a Glosten-Jagannathan-Runkle Generalized Autoregressive Conditional Heteroskedasticity vaiance model with $p = 1, q = 1$. Includes a dummy variable to discriminate asymmetry. 13, 14 |
| **AR(k)-GARCH** | Autoregressive mean model with k lags with a Generalized Autoregressive Conditional Heteroskedasticity vaiance model with $p = 1, q = 1$. 4, 13, 14, 15 |
| **CCC** | Constant Conditional Correlation 2 |
| **DCC** | Dynamic Conditional Correlation 2 |
| **LSTM** | Long short-term memory, a recursive neural network model 13, 14 |
| **SARIMA** | Seasonal Autoegressive Integrated Moving Average 13 |
| **TBATS** | Trigonometric regressors, Box-Cox, ARMA error, trend, seasonality model 13, 14 |

**VAR-GARCH** Vector Autoregressive mean model with k lags with a Generalized Autoregressive Conditional Heteroskedasticity vaiance model with $p = 1, q = 1$. A multivariate model family which can include CCC and DCC-GARCH. 13, 15

# Acronyms

**BTC** Bitcoin 9, 19

**DJI** Dow Jones Industrial Index 9, 11, 12, 16, 17, 18, 19

**ETH** Ethereum 9

**GSPC** Standard & Poor's 500 Index (SP500) 9, 11, 16

**NDX** Nasdaq 100 Index 9, 11, 16, 17, 18, 19

**XMR** Monero 9

**XRP** Ripple 9

# 1

# Introduction

## 1.1 Structure

This chapter presents an introduction to the topics and contextualization of the work and scientific production incorporated into this document. This dissertation displays two related cases on which a variation of the same methodology by Christiansen was applied. The approach originally to determine volatility spillovers during a major idiosyncratic shock in overnight returns in US, European and EU countries bonds markets (introduction of the Euro) but here it is applied to, in a novel application of this method, US indexes and major cryptocurrencies and major US indexes and the stocks that comprise them and during the Feb-Apr 2020 market crash.

The structure of e thesis is as follows: each section will be comprised of two subsections explaining details of each case: spillover of US indexes to cryptocurrencies and spillover of stocks to indexes. Section 2 will be concerned with previous work, scientific basis and the methodology employed on each case. Section 3 will explain the procedure and execution of the analysis and address specific problems encountered in data preparation and crucial details of the methodology along with solutions and concessions made for each issue. At the end of Section 3, there is a subsection on result interpretation and discussion for both cases and the relationships between them. Section 4 is a conclusion which summarizes the work and the conclusions achieved from it. All of the tables and figures will be in Appendix 1 (US indexes to cryptocurrency) and Appendix 2 (stocks to US indexes).

### 1.1.1 US Indexes - Cryptocurrencies

It is the candidate's belief that comparable studies to this one will be much more common and recurrent in the next few years since cryptocurrency is becoming increasingly accepted and traded in most financial markets. It is of utmost importance to understand the behavior of what can be considered one of the most unstable and unpredictable type of assets (asset as it is not widely used as currency, so it can fall under the category of speculative asset). This study seeks to understand, quantify and compare the volatility spillovers of three very mature and established US stock indexes (Dow Jones, Nasdaq 100 and Standard & Poor's 500) and the four cryptocurrencies (Bitcoin, Ethereum, Ripple and Monero) during the Covid market crash and quantify the financial contagion effects between these assets which are very different in nature but seemingly interconnected as it will be demonstrated.

### 1.1.2 Stocks - US Indexes

The evolution of the major US indexes are a sign of the state and well-being of the overall US (and potentially global) market. It is relevant to study and recognize moments of stress as soon as possible and introduce novel methodologiesproviding new observation angles, allowing reliable information to come to light. The recent pandemic has been a stress test to the resilience of the society, including the economic engines', companies, and their ability to adapt to the new reality. This has translated to instability and increased market volatilityl, leading to an extremely significant value drop of the indexes in March 30th 2020. By quantifying this instability, the market can absorb the shock and become more resilient, whether by making bold adjustments to the function of the companies and relationships to financial institutions or through more conservative approaches to preserve economic stability that may still be currently, as of November 2021, at risk. The Stocks-US Indexes section focuses primarily on unexpected volatility of indexes and identification of the companies that originated it, as well as its quantification. The methodology proposed to achieve this goal, in based on decomposing volatility according to the various indexes' stocks relying on the ideas of Bekaert et al. (2005) and Christiansen (2007). Although there is much research on this topic, it is the author's belief that methods typically used, such as CCC-GARCH and DCC-GARCH models can be improved, as this method provides dynamic conditional correlations (which are impossible with CCC-GARCH).

# 2

# Methodology

## 2.1 Previous Works and Scientific Basis

The typical approach to this problem is with multivariate GARCH model based solutions, in particular DCC-GARCH models. Similar works have been seen in literature using these models as a basis but their use is discouraged by McAleer to some extent by providing evidence that DCC does not satisfy the condition for a conditional correlation matrix and that no underlying stochastic process leads to the DCC model. This methodology can present a more consistent option and a manner to address the issues arisen in the aforementioned literature (McAleer, 2019). An extremely similar work to part of the present dissertation includes Malhotra and Gupta (2019), where spillovers are quantified between Asian equity markets and cryptocurrency, in part using the aforementioned models. There is as well a work by Umer et al., where volatility spillovers among EAGLE markets (developing countries) are described using DCC (Engle, 2002) and BEKK-GARCH (Engle & Kroner, 2000) models to analyse the 2008 subprime crisis (Umer et al., 2018).

The basis for method in the present dissertation is proposed in Bekaert et al. (2005) to determine volatility spillovers between various markets paying close attention to regional integration of volatility spillovers from major globalized to regional ones, consisting of a two-step model framework. This could be an assumption of causality, an assumption that Christiansen, has taken and used, in a Granger sense (Granger, 1969), to study volatility spillovers in bonds markets and described as essential. In that work, are described and studied volatility spillovers from US markets to European to regional (country level) markets during the introduction of the Euro with dummy variables and other similar exogenous means of retrospective correction and testing (Christiansen, 2007). Both these works are based on the orthogonalization of variables through regression, taking advantage of the regression property of orthogonal residuals to their regressors, causing their independence and so, consequently, volatility decomposition is a simple process by eliminating the need of conditional correlation models such as DCC and BEKK.

The major difference between both cases is how the multiple variables are organized in causality flow. In the cryptocurrency case, a time series (index) has influence over other time series (cryptocurrency). In stocks, multiple time series (stocks) have influence to a single one (index) which is an understatement since the index is built by definition through co-integration of stocks. Of course that the weight in each stock may have importance, but it does not fully describe the variance spillovers and a more complete stochastic analysis is required.

### 2.1.1 US Indexes - Cryptocurrencies

There is a remarkable difference not only in the assets themselves but in how these markets' time frames operate. Simply using one lag might not properly take into account past information and make the models poorly estimated during some particular moments, especially weekends and Mondays. The variables must be under the usual constraints of stationarity and normality as well as causality in a Granger sense (Granger, 1969). Two types of models were applied, an AR(k)-GARCH for the indexes and an AR(1)-gjrGARCH, a model described in Glosten et al., 1993 for the cryptocurrencies, where $k$ is the lag at which Granger causality is significant, but over the all the iterations of the pairs of indexes and cryptocurrencies, initially, the lag $k = 4$ was the only one used since it is significant for Bitcoin and Ethereum and it is also the lag that minimizes the p-value of Ripple and Monero. However, this only proved effective when the synchronization of the indexes' and cryptocurrencies' time series was based on a forward fill method. This brought nonsensical changes in the data, in particular volatility itself and as such the method had to be rethinked. After changing the method to deal with the synchronization, the Granger causality was found significant at lag 2. Models were estimated on lag 1 and 2, and very minor differences were found between them, as such, a model with lag 1 was preferred since it is the simpler solution.

Since this work only has two steps, indexes into cryptocurrencies, some of the modelling framework from previous works is not applied. And so, it begins with a model specification of the index's log returns:

$$R_{I,t} = c_{0,I} + c_{1,I} R_{I,t-1} + e_{I,t} \tag{2.1}$$

The error term $e_{I,t}$ is normally distributed with mean 0 and it is assumed to follow a GARCH(1,1) on the residuals:

$$\sigma^2_{I,t} = \omega_I + \alpha_I e^2_{I,t-1} + \beta_I \sigma^2_{I,t-1} \tag{2.2}$$

The next equation presents the AR(1) estimation of the cryptocurrencies, using index returns and the previous model's residuals. So we can assume that an orthogonalization between the indexes and cryptocurrencies will occur which will be essential later on, for it will be the only way that the variation ratios will make sense. The $e_{C,t}$ term shows ARCH effects, possibly making the model an AR(1)-GARCH but there is some asymmetry in the data for cryptocurrencies. A solution is provided by Chu and Chan, where a gjrGARCH is one of the two most viable models to estimate with and offers a solution to this asymmetry (Chu & Chan, 2017). The AR(1)-gjrGARCH model with the external regressors from the previous index estimation can be fully explained now:

$$R_{C,t} = c_{0,C} + c_{1,C} R_{C,t-1} + \zeta_{C,1} R_{I,t-1} + \phi_C e_{I,t} + e_{C,t} \tag{2.3}$$

where the gjrGARCH(1,1),

$$\begin{aligned} \sigma^2_{C,t} &= \omega_C + (\alpha_C + \gamma_C I_{C,t-1}) e^2_{C,t-1} + \beta_C \sigma^2_{C,t-1} \\ I_{C,t-1} &:= \begin{cases} 0 & \text{if } R_{C,t-1} \geq \mu \\ 1 & \text{if } R_{C,t-1} < \mu \end{cases} \end{aligned} \tag{2.4}$$

It is simple to realize how the gjrGARCH operates by separating the returns by above or below the conditional mean and helping to properly estimate when there is asymmetry. These models culminate in independent idiosyncratic shocks but not independent unexpected returns:

$$\varepsilon_{US,t} = e_{US,t} \tag{2.5}$$

$$\varepsilon_{C,t} = \phi_C e_{I,t} + e_{C,t} \tag{2.6}$$

From the previous equations, the conditional variance of the unexpected return of the cryptocurrencies can be deduced, and this is only possible because $e_{I,t-k}$ is orthogonal due to the AR($k = 1$) specification from equation 2.1.

$$h_{C,t} = E[\epsilon^2_{C,t} | I_{t-1}] = \phi^2_C \sigma^2_{I,t} + \sigma^2_{C,t} \tag{2.7}$$

Now, we have all the ingredients required to determine variance ratios from indexes into cryptocurrencies:

$$VR_t = \frac{\phi^2_C \sigma^2_{I,t}}{h_{C,t}} \tag{2.8}$$

The previous equation takes values from 0 to 1. This value demonstrates influence of the index's variance on cryptocurrency's. The remainder of the value up to 1 corresponds to own volatility movements and pure cryptocurrency shocks for which the index cannot explain. Although, this equation demonstrates constant spillovers to which the coefficients of the equation 2.5 demonstrate the spillover effects:

$$\begin{aligned} \zeta_{C,i} &= \zeta_C \forall_t \\ \phi_{C,i} &= \phi_C \forall_t \end{aligned} \tag{2.9}$$

To understand the phenomena during the Covid market crash, the parameters were set in this fashion:

$$\begin{aligned}\zeta_{C,i} &= \zeta_{C,0,i} + \zeta_{C,1,i} D_t \\ \phi_{C,i} &= \phi_{C,0,i} + \phi_{C,1,i} D_t\end{aligned} \tag{2.10}$$

$D_t$ is a dummy variable that takes unit value during the period in which the market crash occurred, February 20th 2020 and April 7th of the same year. This specification was introduced in equation 2.3 and then retraced all the subsequent steps to retrieve the new variance ratios.

### 2.1.2 Stocks - US Indexes

The daily returns may be represented as

$$R_{I,t} = \sum_{i=1}^{k} \beta_i R_{s,t} + e_{I,t} \tag{2.11}$$

where $R_{s,t}$ is the daily returns of stock $s$ at moment $t$, $\beta_s$ is the weight of stock $s$ and $e_{I,t}$ is an error term, which can be a measurement of model accuracy and variability of $\beta_s$ over time. $q$ is the total number of stocks in each index. Let $F_{t-1}$ be the information set generated by the available information until $t-1$. Given that

$$Var(I_t|F_{t-1}) = \sum_{i=1}^{q} \beta_s^2 Var(R_{s,t}|F_{t-1}) + 2\sum_{s=1}^{q-1}\sum_{j=s+1}^{q} Cov(R_{s,t}, R_{j,t}|F_{t-1}) \tag{2.12}$$

it is clear that the index's variance cannot be decomposed using solely the terms $Var(R_{s,t}|F_{t-1})$. To overcome this problem, a method was developed based on a methodology by Christiansen, which was used to extract the volatility spillover effects in bond markets. It is assumed that the returns can be expressed as

$$R_{I,t} = \zeta_t + \sum_{s=1}^{q} \eta_s e_{s,t} + \upsilon_t \tag{2.13}$$

where $\zeta_t$ is $F_{t-1}$-measurable and may include a constant, seasonal dummies and dynamic terms such as ARMA family models and other stock returns. $e_{s,t}$ are orthogonal shocks associated with each stock and $\upsilon_t$ is the error term. And so, the conditional variance of $R_{I,t}$ can be decomposed into the sum of individual shocks of each stock plus the variance of the idiosyncratic term $\upsilon_t$,

$$Var(R_{I,t}|F_{t-1}) = \sum_{s=1}^{q} \eta_s^2 Var(e_{s,t}|F_{t-1}) + Var(\upsilon_t|F_{t-1}) \tag{2.14}$$

This is the expression to used to decompose the variance. It permits the examination of the relationship and volatility spillovers from stocks to indexes as the conditional variance of each stock can be directly compared to the conditional variance of the index. To enable this expression, two issues still have to be resolved: 1. the shocks need to be estimated 2. they must be orthogonal.

The shock $e_{s,t}$ is unknown but it can be estimated from the residuals of $R_{s,t}$ on $\mu_{s,t}$:

$$R_{s,t} = \mu_{s,t} + e_{s,t} \tag{2.15}$$

The term $\mu_{s,t}$ is $F_{t-1}$ mensurable and may include a constant, seasonal dummies, or dynamic terms, and any other terms that may be correlated with $R_{s,t}$. This procedure aims to remove all the effects defined in $\mu_{s,t}$ from $R_{s,t}$ and other effects so that $e_{s,t}$ is uncorrelated with $e_{j,t}$ $(j \neq s)$.

It is assumed $e_{s,t}|F_{t-1} \sim N(0, \sigma^2_{s,t})$ and $v_t|F_{t-1} \sim N(0, \sigma^2_{v,t})$. A test based on Lagrange multiplier principle confirms that most the errors display GARCH type effects.

To confirm the orthogonalization of the stock residuals, a Lagrange multiplier statistic is considered $\lambda_{LM} = n\sum_{s=2}^{M}\sum_{j=1}^{s-1}\hat{\rho}^2_{s,j}$, for which $\hat{\rho}_{s,j} = Corr(e_s, e_j)$. Under the null, $H_o : \hat{\rho}_{i,j} = 0$, $\lambda_{LM}$ has a limiting chi-squared distribution with $M(M-1)/2$ degrees of freedom.

Let $y_t = (r_{1,t}, ..., r_{q,t}, I_t)$, $\mu_t = (\mu_{1,t}, ..., \mu_{q,t}, \omega_t)$, $u_t = \Phi e_t$ where $\Psi$ and $e_t$ are defined as follows

$$u_t = \underbrace{\begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \\ \eta_1 & \eta_2 & \dots & \eta_q & 1 \end{bmatrix}}_{\Psi} \underbrace{\begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_{q,t} \\ v_t \end{bmatrix}}_{e_t} \tag{2.16}$$

The model can be succinctly written as $y_t = \mu_t + u_t$ where $u_t = \Psi e_t$ (note that $\Psi$ is a triangular matrix). The conditional variance of $y_t$ is

$$H_t = Var(y_t|F_{t-1}) = Var(u_t|F_{t-1}) = \Psi\Sigma_t\Psi' =$$
$$= \begin{bmatrix} \sigma^2_{1,t} & 0 & \dots & 0 & \eta_1\sigma_{1,t} \\ 0 & \sigma^2_{2,t} & \dots & 0 & \eta_2\sigma_{2,t} \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & \sigma^2_{1,t} & \eta_q\sigma_{q,t} \\ \eta_1\sigma_{1,t} & \eta_1\sigma_{2,t} & \dots & \eta_q\sigma_{q,t} & \sum_{s=1}^{q}\eta_i^2\sigma^2_{s,t} + \sigma^2_{v,t} \end{bmatrix} \tag{2.17}$$

where $\Sigma_t := Var(e_t|F_{t-1})$. It follows that

$$Var(I_t|F_{t-1}) = \sum_{i=1}^{q}\eta_i^2\sigma^2_{i,t} + \sigma^2_{v,t} \tag{2.18}$$

$$Corr(R_{s,t}, I_t|F_{t-1}) = \frac{\eta_i\sigma^2_{s,t}}{\sqrt{\sigma^2_{s,t}(\sum_{s=1}^{q}\eta_i^2\sigma^2_{s,t} + \sigma^2_{v,t})}} = \frac{\eta_i\sigma_{s,t}}{\sqrt{\sum_{s=1}^{q}\eta_i^2\sigma^2_{s,t} + \sigma^2_{v,t}}} \tag{2.19}$$

To finalize these effects, and in a similar fashion to the previous case with cryptocurrencies, a dummy variable will be inserted and its significance verified. The simplest and an effective way is to apply an AR(1) model to conditional variances, variance ratios and conditional correlations with a dummy variable as exogenous with unitary value during the Covid market crash and zero during all remaining time.

3

# Data, Empirical Strategy and Results

## 3.1 Data

### 3.1.1 US Indexes - Cryptocurrencies

The data used for this case has been comprised of three indices:

- Dow Jones Industrial (Dow Jones Industrial Index (DJI))
- Standard & Poor's 500 (SP500 or Standard & Poor's 500 Index (SP500) (GSPC))
- NASDAQ 100 (Nasdaq 100 Index (NDX))

And also included four cryptocurrencies:

- Bitcoin (Bitcoin (BTC))
- Ether, colloquially known by the chain name Ethereum (Ethereum (ETH))
- Ripple (Ripple (XRP))
- Monero (Monero (XMR))

All the aforementioned data was retrieved by making API requests to yahoo finance data of close adjusted prices. The indexes' data was retrieved from 2002 and the cryptocurrencies from their earliest record listing on yahoo finance. The data was processed into daily logarithmic returns and all data was casted into a time series data structure but here is presented an uncommon challenge: the data does not match temporally. This happens because while the stock markets are not open during weekends and holidays, cryptocurrencies are permanently open and so this posed a challenge on how to synchronize the data. A solution is clear: joining the data from each index and each cryptocurrency so that not only they have the same beginning (there were not cryptocurrencies in 2002), but also the indexes' time series will display a missing value on non-business days, and hence accounting for missing days in the original data, while the cryptocurrencies' will display a value. A method was to be considered to fill these missing values in order to not lose data.

Let index data be a time series of daily close adjusted prices with values only on business days. It can be considered that there are three options of filling: 1. fill with zero, 2. interpolate, 3. forward fill. Filling with zero presents a nonsensical shock and change of the time series in an artificial manner which would render most analysis useless, even with a dummy variable, so this method is unacceptable.

Interpolating the non-business days, using Friday and Monday values for example, could bring good results and tie the values in a somewhat continuous fashion during these days. But in a time series sense it is inappropriate, it means there is an information leak from future values for the price at $t$ ,becoming dependent on $t+k, k > 0$, creating a non-causal system and incur bias in later experimentation.

Considering the previous two issues, a forward fill seems to be the best option, avoids creating nonsense data which would happen with a zero fill, and allows the continuation of a causal system which would not happen with interpolation. This method of filling seems to be the one that minimizes impact on the price time series.

Although the paragraphs above contain methods to deal with the null values on price, the data is logarithmic returns, but the logic can be translated into this data. Since the return is a dimensionless coefficient of differentiation of price at $t$ and $t-1$, if there is no change in price with forward fill then the return is zero. And so, in dealing with returns with the aforementioned data, the null values were filled with zero. Later, it will be explained why this method was not used.

On the second iteration of models, the returns and residuals of the aforementioned equations were simply lagged and introduced in the AR(1)-gjrGARCH cryptocurrency models on the next step as exogenous variables in equations 2.3 and 2.4.

There is also a similar issue in the variances. Since the modelling of the indexes was not made to respect the time frame of the cryptocurrencies but of solely the indexes themselves, the variances would have missing values on non-business days similarly to price data. If the previous method was used, simply filling with zero, the variance ratio would not make sense because it would be zero as well on those same days, which is as if saying, for example, that any amount of volatility on Friday in indexes is not spilled during the weekend to cryptocurrencies, which is unacceptable. Although the prices are not observable, or even if the markets are closed, there is a change in the price and making suppositions can overwrite this and create disparities between the presented work and the phenomenon itself by introducing bias through the new data. Models were created using it and they had a quirky behavior by featuring a very high importance of AR(3) and AR(4) coefficients, which is somewhat strange. How can it be expected, in a high speed transaction environment, to find statistical significance from three to four days ago, probably due to weekend effects, on a Thursday or Friday in detriment of the immediately previous day?

The final solution is clear: any rows containing missing values must be dropped. This means that a synchronization had to be made at the price level and not at the returns' since it would pair index returns from Friday to Monday while cryptocurrencies would have from Sunday to Monday. Although there is some loss of data, there is no inserted data not only on the returns but also the volatility which was the most problematic data insertion. This was the final data used for this work.

### 3.1.2 Stocks - US Indexes

Here the approach is somewhat different than the previous case, but the indexes in study are still the same: DJI, GSPC and NDX. The data is as well daily logarithmic returns of close adjusted prices of indexes and all the stocks that comprise them which for the sake of brevity a complete listing will be withhold for there were used hundreds of unique stocks. But there were two issues with the data: 1. there was not a simple automated way to retrieve the all the stocks for each index 2. index composition changes over time which could render the method used useless since the data would not match with the index.

To solve the first issue, webscrapping was used. This allowed to retrieve the updated composition of each index and their respective tickers, and so the automated download of the data was possible.

For the second issue, a compromise had to be made. Instead of using data from 2002 similarly to the previous case, the data starts from 2018, and so, a great deal of major changes to the indexes were avoided but still retaining the objective of this work, since the market crash in study occurred only in the first and second quarters of 2020.

Despite the already vast amount of variables, more were added to mitigate issues with model estimation, which include the addition of seasonal dummies, specifically day-of-the-week and monthly, and subsections of the original stock returns, an explanation for this will be timely given.

Later, this data was dropped and price data was used in the models since log-returns did not provide an appropriate orthogonalization of residuals. This change will be discussed and explained later on.

## 3.2 Empirical Strategy

### 3.2.1 US Indexes - Cryptocurrencies

The causality requisite could be fulfilled in a Granger sense in Bitcoin and in Ethereum at a 5% significance at lag 2. For Ripple and Monero it is not particularly significant, p-values between 0.10 and 0.20, but this method will still be applied and the results will be evaluated.

There was a already a suspicion that the causality effects could be weaker on Ripple and Monero. Even though they are also cryptocurrency, they have distinctive features from the other two coins.

Ripple has the interest of European institutions and markets, with the possible introduction of a digital euro, there could be a space for it to fit in this system with its banking utility. It would be interesting to do a similar study as this one but towards European markets where it is the candidate's belief that Ripple would have the limelight.

Monero is as close as possible with current technology to a perfectly untraceable means of payment. Not only it is widely used in the black market to pay for illegal goods and services, it also provides a gateway for money laundering, tax evasion and other white collar crimes. It does have indeed some weak causality from the US indexes but there could be higher significance in developing countries' equity markets where a parallel economy is main source of economic growth, as well as fluctuations in prices of goods and services caused not only by the tug of war between supply and demand but introduction of legislation affecting illegal trade. There could also be a relation of causality between the deficit of expected and real tax revenue of so-called developed countries.

Pertaining to the methodology application, it succeed without any issues in the proposed manner in chapter 2. Although, the creation of the dummy variable was made in a non-obvious manner. Since the dummy is considered upon the coefficient itself and not another descriptive variable, it does not take unit value in the application. This variable is created by multiplying the unit value dummy variable by the time series for which it is a dummy for and so the coefficient has analytical value of the variable itself during the market crash period as if another layer of linearity. This allowed for not only being able to use this coefficient in the proposed manner later on, by including it in the variance and variance ratio calculations, but also, in general, corrected the coefficient pertaining to the entire variable and changed the entire estimation of the model not only during the Covid crash.

### 3.2.2 Stocks - US Indexes

After retrieving the data, a multitude of regression models were estimated to create residuals of stock models that were orthogonal with each other. Although the final step has to be an AR based model be to fruitful, the equation 2.14 does not, and so the term $\mu_{s,t}$ proved quite a challenge. Since DJI has the smallest number of stocks in its composition, it was used as a test to the other indexes and a assumption was made: whatever solution is found for this case in particular, then the other indexes shall follow this solution with minor adjustments. The considered models where:

- AR(k)

- AR(k)-GARCH
- SARIMA(k,0,0)(1,0,0)[period]-GARCH
- AR(k)-GARCH with added seasonality on mean
- TBATS
- Fourier Transform
- LSTM
- AR(k)-gjrGARCH
- AR(k)-gjrGARCH with added exogenous
- AR(k)-TGARCH
- AR(k)-TGARCH with added exogenous
- VAR-GARCH model family

The only evaluation metric considered was the reduction of $\lambda_{LM}$, a value of a Lagrange multiplier statistic, that follows a limiting chi-squared distribution. And so, the lower the better. The $\lambda_{LM}$ without any regression has a value of about 100 000 and with an AR(1) with exogenous variables $\lambda_{LM}$ value was at around 91 000. Must be kept in mind that the chi-squared distribution at 0.05 with 30 degrees of freedom has a $\lambda_{LM}$ of 387.6471.

- **AR(k)** This version, with 1 through $k$ lags of lagged exogenous variables, was not able to produce any significant improvements against the original AR(1). $k > 4$ started to have minimal improvements which transpired a different course of action to be taken.

- **AR(k)-GARCH** There were improvements compared to the AR(k) estimation on $\lambda_{LM}$ but these were not enough with a value of around 80 000. It started to have the same issue as the previous AR(k), where increasing $k$ did not yield major improvements. There was also a version with GARCH(k,k) but increased the metric which means this particular version made the orthogonalization even worse.

- **SARIMA(k,0,0)(1,0,0)(period)-GARCH** At this moment, it was realized that there could be some sort of seasonality, but most tests for seasonality rejected this hypothesis, and also, decomposition of some time series examples at different periods had shown very little seasonality. But still, to perform this analysis a functional requisite is automation of the process since even a careful analysis of each stock would not guarantee that their residuals would be orthogonal. And so, an AR(1) model with exogenous variables was used and then the sum of their absolute values into a single time series. From this, it would be possible to see synchronous spikes over multiple periods on all stocks, taking advantage of the superposition of signals. A periodogram was created that indicated a possible seasonality component with a period of 64, but it did not do much in terms of improving the metric.

- **AR(k)-GARCH with added seasonality on mean** From the possibility of existence of seasonal components, day-of-the-week and monthly dummy variables were tested and showed minimal improvement on $\lambda_{LM}$.

- **TBATS and Fourier Transform** These modelling techniques, TBATS (De Livera et al., 2010), do not support exogenous variables but they are good options to understand seasonality, since they are essentially frequency based, unlike the previous ARIMA based models that are based on a linear regression principle. So these models were used to add seasonality terms to the AR(k)-GARCH model but with no improvement in $\lambda_{LM}$.

- **LSTM** Instead of continuing the search in ARIMA-GARCH models, a different approach was taken. Since even seasonality through Fourier terms was not improving the model, there could be something that could point towards seasonality but still not be it. A BDS (Broock et al., 1996) test was used to check for linearity which has shown that most stocks are indeed non-linear, and so LSTM (Hochreiter & Schmidhuber, 1997), a recursive neural network, could be of use to at least withdraw some conclusions. It was possible to run a few models and graphic visualizations of the data of a few stocks against their predicted values and residuals. The usage of this model was cut short due to hardware failure that caused a lack of computational power. It can only be assumed that it had the potential to create orthogonal residuals, but there was an interesting conclusion to withdraw. LSTM failed in the same manner as previous models to sense asymmetry and to properly estimate large shocks in various stocks' time series even if they appear on exogenous variables. As if there is a well behaved linear time series that is being estimated being superimposed with others that are very relevant in the orthogonalization of residuals, and even if a particular stock does not share the same shock as exogenous variables, a "phantom" shock in residuals would appear and it could be an inability to properly estimate coefficients.

- **AR(k)-gjrGARCH** gjrGARCH is a variant of the GARCH model that includes an asymmetry term. It did not yield any improvement on the metric.

- **AR(k)-gjrGARCH with added exogenous** Taking into account the conclusions of the LSTM model, inspiration was taken from the previous utilization in the cryptocurrency case of a dummy variable on the coefficient itself, but this one was tied to standard deviation. So each exogenous variable was divided into subsections of according to their standard deviation multiplied by a factor: from 2 to 3, 3 to 4 and above 4 and the same was made but with negative factors. This was meant to aid the gjrGARCH in the task of modelling asymmetry and separate the time series into the sum of various coefficients on particularly high shocks. There was still one task to be done, this method sometimes created time series with only a few or even one non-zero value that were synchronous with other time series in the same fashion, creating co-linearity and affecting the rank of the exogenous variable matrix. There is also the chance of creating time series with only 0 values. So, a method of reducing variables and ensuring that the matrix has independent columns was needed and implemented, but the amount of variables is still threefold the of the original exogenous, requiring a significant amount of computational power. This utilization proved effective and reducing $\lambda_{LM}$ to 70 000. Must be kept in mind that it still has day-of-the-week and monthly dummy variables but these obviously are not included in the production of subsections, only the original exogenous variables pertaining to stocks with lag 1 are utilized.

- **AR(k)-TGARCH with (and without) added exogenous** A similar model (Zakoian, 1994) to gjrGARCH since it is sensitive to asymmetry, but it did not yield improvements to $\lambda_{LM}$. Aside from the previous issue, it is very expensive computationally reducing even further the possibility of usage for the other indexes, which have a considerably larger pool of variables.

- **VAR-GARCH-DCC model family** An attempt was made to correct the residuals through multivariate GARCH models (DCC, aDCC, CCC, GO, Copula, etc.). The idea was: if it is possible to fix the parameters in the estimation of the joint matrix that control the covariance, it would be possible to create a covariance matrix with only a diagonal or, in even another way, to utilize algebraic properties of matrices and rotate the covariance matrix so that the covariance terms are the maximum and then use variance terms for the variance target process in the estimation, keeping the joint matrix terms free, which could provide orthogonal residuals. Trying to remove the diagonal from the altered $\alpha$ from the estimation and fix it to zero proved to have an exceptional difficulty. Also, all these models have a multi-step procedure. During the application of AR(k)-GARCH models, separating the estimation into two steps proved to be ineffective, and only the simultaneous estimation of both AR and GARCH equation terms showed improvement.

  It is uncertain if the models are indeed incapable or if it is the process of their estimation that brings unfruitful results. The DCC specification has been subject of controversy concerning its consistency and capability to produce meaningful results but it is common to utilize such methods in volatility and conditional correlation estimation but it cannot be used in this work in favor of other methods since it produced a $\lambda_{LM}$ of about 90 000.

A different approach was required. After testing tens of models and hundreds of iterations with very little results, doubt started to emerge from the data itself: Were log returns really the correct choice? What can be done as an alternative? And so an ARIMA(1,1,0) model was created and modeled on the price of the stock. Then, arithmetic returns were created from the residuals and tested against the aforementioned chi-squared statistic. It showed immediate improvement with $\lambda_{LM}$ having a mere value of 1150. After a few iterations, the final model for the orthogonalization of was an ARIMA(2,1,0)-GARCH(1,1) model for each DJI stock with $\lambda_{LM}$ assuming a value of 349.

Then, following the methodology, these residuals were then introduced in the AR(1) specification for the DJI index and its residuals showed ARCH effects through an Lagrange multiplier based test. Could indicate that even though each stock residual is orthogonal some information could have been missing. Also, interestingly, causality in the Granger sense was found significant from DJI into stocks, this could account for the missing ARCH effect since it could be considered as an information shared by all stocks and so removed during the regressions for orthogonalization. The model applied was an AR(1)-GARCH(1,1)

The same procedure was applied to NDX with a minor change to the model being in this case an ARIMA(1,1,0)-GARCH(1,1) being found significantly orthogonal using the same test previously described. It also shows similar characteristics of heteroskedasticity on the AR(1) index model, and so, model applied was as well an AR(1)-GARCH(1,1).

GSPC at this moment had to be forsaken. Having 500 stocks meant at least having to perform 500 ARIMA(1,1,0)-GARCH(1,1) model estimations with about 500 exogenous variables. At a rate of about just over a day per model estimation, it would take around 2 years to perform just one iteration. This is unsustainable with the current resources but it would be interesting to see these results since DJI and NDX showed different scales of magnitude on the conditional correlations. Having another term of comparison would provide greater understanding of both indexes in analysis.

## 3.3 Results

This section will only comprise of interpretation of the results and not the results *per se*. Were created hundreds of graphs and tables which are documented in appendix 1 and 2. However, specific content of these appendices will be called during analysis to facilitate the reader by providing for the insights here inscribed.

### 3.3.1 US Indexes - Cryptocurrencies

Analysing all of the graphs, there is a characteristic that shows all of them. The introduction of the dummy variables showed to greatly increase the variance ratio, which could mean that there is a better estimation of the model by decreasing the unexplained variance by providing another layer of linearity during the market crash. This has also influence over the non-dummy coefficients. What this could imply is that if the market crash is not accounted for in any form, the cryptocurrencies' own volatility is overbearing when calculating the variance ratio and strongly influence the $h$ term on the fraction, even if the coefficient of the non-dummy variables has lower value. This effect showed that it is likely that these coefficients are better estimated with this second model, which is backed by significances at 1% on the dummy index residual, and as such, the market crash is essential when estimating the variance ratio.

On a more detailed level, it can be seen that not all cryptocurrencies and indexes behave in the same fashion. DJI has the lowest variance ratio throughout the graph excepting on the period during the market crash comparing against the market crash spike, while NDX has the highest, showing remarkable activity on late 2019 with a spike that is comparable to the market crash in 2020 and even surpasses it on BTC with the dummy variable specification while on the other indexes can be considered relatively negligible.

All graphs demonstrate comparable values according to model specification, however, XRP is by far the lowest with values up to 0.02 while most other can go up to 0.07. This could be due to the previously mentioned relationship to the European market, also there is an occasional significance of return variables which is different from the rest of the models, but more work should be developed around this topic.

In conclusion, this section validates its premise through the dummy variables. Although the returns are very rarely significant in models, the residual variables are always significant, indicating their relevance towards estimating cryptocurrency models and consequently how the market crash is an important event to consider in future model development.

### 3.3.2 Stocks - US Indexes

The graphs shown are only the conditional correlations and not the variance ratio as the previous case. There was no need to add lags to the index model estimation since the variables themselves are orthogonal and independent to previous values and so to obtain the variance ratios would be to simply squaring the conditional correlations, which would provide ragged graphs with the same information but necessarily less smooth and harder to analyse.

There is a noticeable difference on the magnitude of conditional correlations on both cases. While the maximum on DJI is about 0.05, in NDX is it about 0.5. It is uncertain to why this difference happens, but there could be three reasons: 1. a causality relationship from the index to stocks, which was found extremely significant in DJI, and could render a cyclical effect on the price and the index itself which spills into the stocks that comprise it. During the orthogonalization this information could be lost since it is shared by most of the stocks. The lack of this characteristic could be one reason as to why NDX stocks are much more independent from each other and provide larger independent movements. 2. the indexes themselves are different, while DJI is a price-average index, NDX is market capitalization-average, this could have a non-trivial influence on the price movement of NDX by having supply independence. 3. the models used to orthogonalize the residuals are slightly different, while in DJI an ARIMA(2,1,0)-GARCH(1,1) with lag 1 and lag 2 stock exogenous variables was needed to achieve the significance in the $\lambda_L M$ statistic, in NDX it was ARIMA(1,1,0)-GARCH(1,1) with lag 1 exogenous variables. More explanatory variables could mean that each stock price is much more explained in DJI than in NDX and so a reduction in the magnitude of variance of the stock residuals. Further work could be necessary to fully understand the reason why this happens.

#### 3.3.2.1 DJI

Right away, from analyzing the graphs, it is possible to notice that during the market crash in early 2020, all the conditional correlations are low, and so it is possible to infer that at least any specific stocks do not have influence over the price of DJI but it was a market wide highly correlated price movement.

There are various groups of stocks separated into overall high, medium and low conditional correlations. Examining stocks with high correlations, for example, BA on figure B.5 and INTC on figure B.15, can be seen that these stocks have a higher coefficient (table B.1) and that they are significant, and then the inverse is true on the stocks with low conditional correlations and as expected the medium group has these values in between the other two groups.

As proposed, the conditional correlations were used to estimate as AR(1) model with an exogenous dummy variable. The dummy variables on table B.2 seem to be mostly insignificant on the conditional correlations, variance ratios and orthogonalization residual returns with no relation to the values that characterize the estimation. But on the residuals themselves from orthogonalization and the standard deviation of the stock residuals appear to be mostly significant. So, in a way, it was confirmed that there was an unexpected movement in the price of the stocks during the market crash period, but it did not cause a significant change on the residual returns and so this lack of significance carried on to the index model, although it is possible to visually recognize the period when the crash happened in the graphs.

#### 3.3.2.2 NDX

All the same conclusions gathered in DJI, can be applied to the NDX case except that only the stock's standard deviation was considered somewhat significant. But there is an interesting addition, there are clear volatility clusters in the conditional correlations, indicating periods where groups of stocks moved in tandem but fairly independently. It would be interesting to analyse in depth what happened in late Q1 to early Q2 2019 in these stocks since it is unclear in overall news about the market as to why it happened, also it was mostly a local trend inversion of the price of these stocks, in particular GOOGL with a 5% drop. The second major volatility cluster in the conditional correlation graphs is in late Q3 of 2020. This spike was mostly composed by RGEN, a pharmaceutical company, and could correspond to speculation and reaction to news about the vaccines for the pandemic.

### 3.3.3 Comparative Assessment

It is shown that 2019 and Q3 2020 were relevant moments on the conditional correlations of stocks into indexes, however, these moments coincide relatively well with ones when the variance ratio of indexes into cryptocurrencies were low, which is fairly unexpected. How come some components suffered enough volatility to account for such a large portion of the variance of the index but did not spill into cryptocurrencies? It should be analysed with more detail, with a more localized approach since it probably pertains to a specific moment in time where a generalisation from their assimilation to major trading markets would not apply. What is clear is that there were moments of price rallying in cryptocurrencies, in particular BTC (using this example because this coin's fluctuations dictate empirically the state of the cryptocurrency market) with about a 244% increase in price in the first half of 2019 and 550% in the second half of 2020, and so, coinciding with these moments. Three guesses can be inferred: 1. the model might not properly estimate these moments since the price rallying in BTC seemed relatively impervious to a drop in indexes on early May in 2020, there is the possibility of a direct relationship from stocks to cryptocurrencies, bypassing the indexes 2. there is a possibility that BTC own volatility was completely dominant during this time with little to no relationship with indexes or equity market 3. could be a signal or indicator of some sort for a relationship between both markets unaccounted by the models, with some sort of external event or expectation which influenced the market.

On the last conclusion, figure 3.1 shows that DJI variance (with AR(1)-GARCH(1,1)) has some influence on the price of BTC. Not only the price drops when there is a spike but also seems to indicate a trend reversal on the price. The indexes seem to have a delayed response towards the trend reversing effect of cryptocurrencies, which could indicate that an event, exogenous to the variables, took place. This effect only happened a few times, but certainly is worth to develop work to further understand it

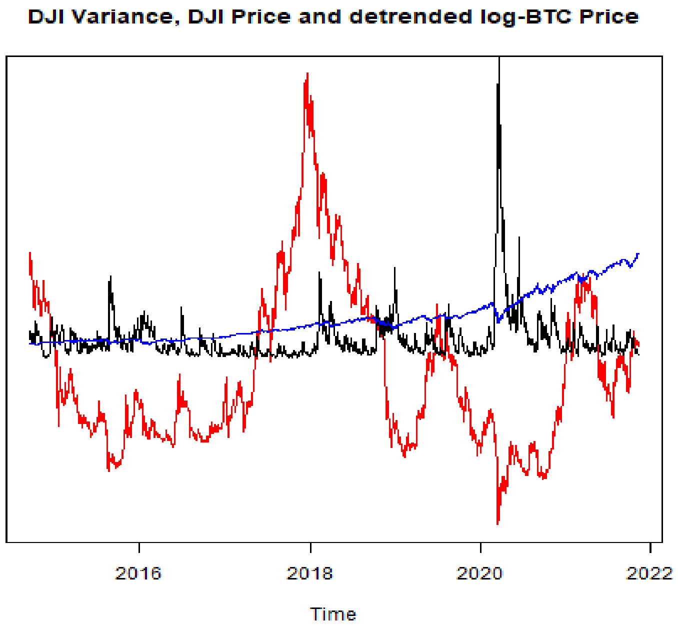


Figure 3.1: DJI Variance (black), DJI Price (blue) and detrended log-BTC Price (red)

since it has the potential to be important on unravelling the functioning of cryptocurrency market cycles, which is of immense value to investors not only in this market but also in equity. Perhaps this effect coupled with the previous two cases can be the foundation, in a purely quantitative and technical analysis sense, for the development of asset portfolios that include cryptocurrency and mutually hedging the volatility throughout all the assets in a hypothetical portfolio with the two previous cases describing correlations and cross-market influence on volatility in the overall market and the mentioned trend reversing effect explaining particular moments of cyclical importance, opening ways for short to mid-term investment strategies and long-term passive investment present-market efficient solutions.

# 4

# Conclusion

It is the belief of the candidate that these results are a near success, falling short in only two aspects of fully achieving the proposed goals: 1. GSPC comprises a far too great number of stocks, rendering computationally heavy the use of overall stocks using this methodology with the available resources. 2. the inability to use log-returns and having to perform regressions on the price itself and then the use of arithmetic returns for further work. However, there is a case to be made here for the usage of arithmetic returns as a substitute of log-returns since they are equivalent at small values.

It was shown that there is a significant difference between the spillovers during the Feb-April 2020 market crash, or Covid crash, and other moments in the cryptocurrency market history before and after this event due to the significance of residual dummy variables included in the various models. These volatility spillovers were quantified using variance ratios and their models were analyzed, probing for insights.

It was shown athat even though it was a traumatic occurrence, the pandemic effect was not relevant in the conditional correlations between stocks and US indexes due in part to the models estimation capability. It was shown that no single stock had a decisive role in the market crash, whichwas a market wide price movement, since this effect was almost completely missing from the index model estimation and posterior calculated conditional correlations, and so, lost during orthogonalization.

It is evident that much of these particular subjects was not covered by this work, including a deeper analysis of the relationship between them. Nevertheless, this study points toward to index volatility, and stock volatility, being relevant in a non-trivial manner to cryptocurrency or even, in fact, behaving in an unexpected fashion. This later conclusion, that appeared almost accidentally suggested by the cases, is in the candidate's belief to be the most interesting aspect of this work and has the potential to change the way the financial sector operates with cryptocurrencies.

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# A

# Appendix 1

This appendix presents tables of the models estimated during the production of the Index - Crypto component of this dissertation. Every pair has three tables, which are discriminated by index model, crypto model and crypto model with dummy variables. And so, this section is comprised of 27 tables (non-repeated index models). There are also two graphs per pair, with and without dummy variables, totaling 24 graphs.

## A.1 DJI

### A.1.1 Index model

Table A.1: Parameter Estimates of the GARCH(1, 1) (DJI)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.084 | 0.015 | 5.462 | 0.00000 |
| ar1 | −0.044 | 0.027 | −1.647 | 0.100 |
| omega | 0.044 | 0.007 | 5.978 | 0 |
| alpha1 | 0.233 | 0.028 | 8.387 | 0 |
| beta1 | 0.732 | 0.026 | 28.046 | 0 |

### A.1.2 BTC

#### A.1.2.1 Crypto model

Table A.2: Parameter Estimates of the GARCH(1, 1) (DJI-BTC)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.245 | 0.096 | 2.542 | 0.011 |
| ar1 | 0.028 | 0.029 | 0.976 | 0.329 |
| $R_{I,t-1}$ | −0.022 | 0.112 | −0.199 | 0.842 |
| $e_{I,t}$ | 0.005 | 0.001 | 4.284 | 0.00002 |
| omega | 1.506 | 0.261 | 5.767 | 0 |
| alpha1 | 0.140 | 0.026 | 5.453 | 0.00000 |
| beta1 | 0.790 | 0.024 | 33.351 | 0 |
| gamma1 | 0.028 | 0.030 | 0.932 | 0.351 |

#### A.1.2.2 Crypto model with dummy variables

Table A.3: Parameter Estimates of the GARCH(1, 1) (DJI-BTC with dummy var.)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.262 | 0.096 | 2.740 | 0.006 |
| ar1 | 0.034 | 0.028 | 1.221 | 0.222 |
| $R_{I,t-1}$ | 0.104 | 0.102 | 1.011 | 0.312 |
| $e_{I,t}$ | 0.003 | 0.001 | 3.416 | 0.001 |
| $R_{I,t-1}D_t$ | 0.027 | 0.344 | 0.079 | 0.937 |
| $e_{I,t}D_t$ | 0.013 | 0.002 | 5.242 | 0.00000 |
| omega | 1.223 | 0.229 | 5.333 | 0.00000 |
| alpha1 | 0.137 | 0.024 | 5.749 | 0 |
| beta1 | 0.817 | 0.023 | 34.905 | 0 |
| gamma1 | −0.007 | 0.024 | −0.282 | 0.778 |

#### A.1.2.3 Variance Ratio Graphs

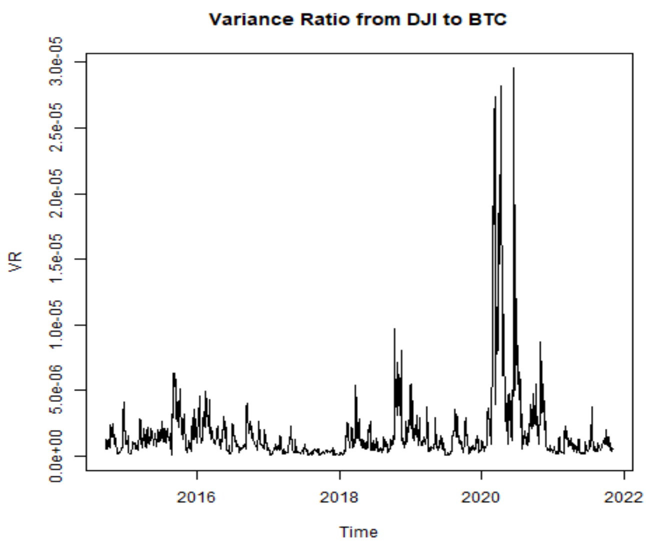


Figure A.1: Variance Ratio of DJI-BTC

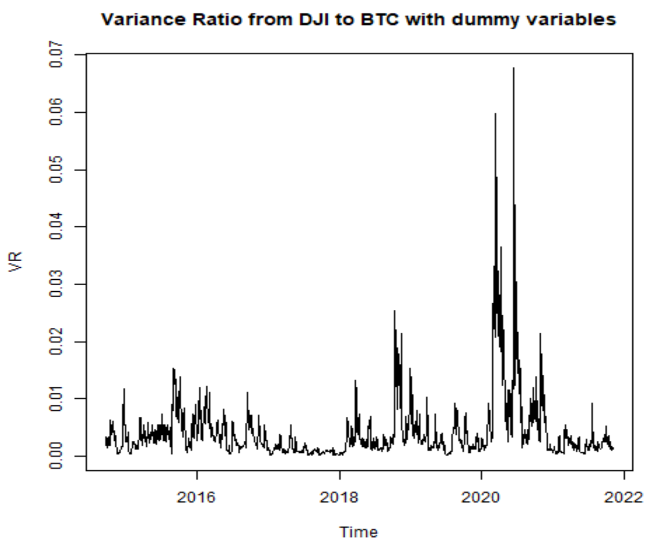


Figure A.2: Variance Ratio of DJI-BTC with dummy variables

### A.1.3 ETH

#### A.1.3.1 Crypto model

Table A.4: Parameter Estimates of the GARCH(1, 1) ((DJI-ETH)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.299 | 0.172 | 1.742 | 0.082 |
| ar1 | 0.029 | 0.033 | 0.897 | 0.370 |
| $R_{I,t-1}$ | 0.145 | 0.181 | 0.802 | 0.423 |
| $e_{I,t}$ | 0.011 | 0.002 | 6.772 | 0 |
| omega | 3.528 | 0.700 | 5.039 | 0.00000 |
| alpha1 | 0.174 | 0.029 | 6.065 | 0 |
| beta1 | 0.800 | 0.026 | 31.035 | 0 |
| gamma1 | −0.072 | 0.028 | −2.546 | 0.011 |

#### A.1.3.2 Crypto model with dummy variables

Table A.5: Parameter Estimates of the GARCH(1, 1) (DJI-ETH with dummy var.)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.308 | 0.171 | 1.804 | 0.071 |
| ar1 | 0.033 | 0.032 | 1.021 | 0.307 |
| $R_{I,t-1}$ | 0.193 | 0.170 | 1.135 | 0.257 |
| $e_{I,t}$ | 0.007 | 0.002 | 4.227 | 0.00002 |
| $R_{I,t-1}D_t$ | 0.174 | 0.452 | 0.384 | 0.701 |
| $e_{I,t}D_t$ | 0.014 | 0.003 | 4.214 | 0.00003 |
| omega | 3.209 | 0.652 | 4.921 | 0.00000 |
| alpha1 | 0.176 | 0.028 | 6.226 | 0 |
| beta1 | 0.805 | 0.025 | 31.942 | 0 |
| gamma1 | −0.078 | 0.027 | −2.888 | 0.004 |

#### A.1.3.3 Variance Ratio Graphs

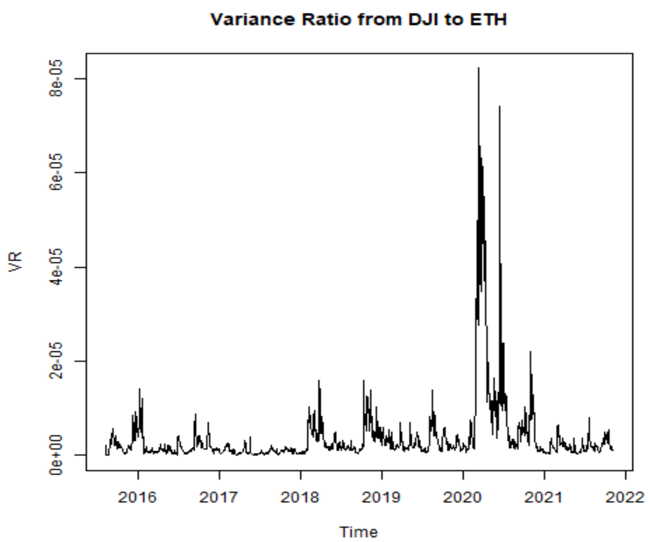


Figure A.3: Variance Ratio of DJI-ETH

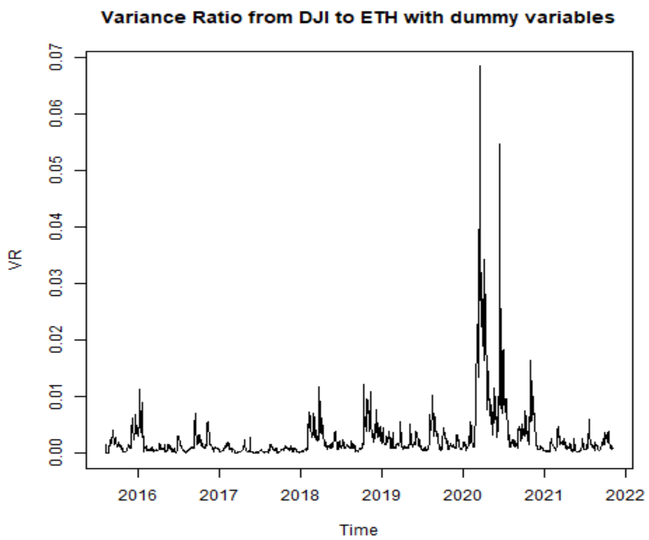


Figure A.4: Variance Ratio of DJI-ETH with dummy variables

### A.1.4 XRP

#### A.1.4.1 Crypto model

Table A.6: Parameter Estimates of the GARCH(1, 1) (DJI-XRP)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | −0.150 | 0.136 | −1.103 | 0.270 |
| ar1 | 0.045 | 0.030 | 1.516 | 0.129 |
| $R_{I,t-1}$ | −0.027 | 0.119 | −0.224 | 0.823 |
| $e_{I,t}$ | 0.009 | 0.001 | 8.697 | 0 |
| omega | 3.160 | 0.434 | 7.278 | 0 |
| alpha1 | 0.352 | 0.045 | 7.743 | 0 |
| beta1 | 0.739 | 0.024 | 30.442 | 0 |
| gamma1 | −0.229 | 0.043 | −5.374 | 0.00000 |

#### A.1.4.2 Crypto model with dummy variables

Table A.7: Parameter Estimates of the GARCH(1, 1) (DJI-XRP with dummy var.)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | −0.135 | 0.137 | −0.981 | 0.326 |
| ar1 | 0.048 | 0.030 | 1.592 | 0.111 |
| $R_{I,t-1}$ | −0.007 | 0.132 | −0.051 | 0.959 |
| $e_{I,t}$ | 0.007 | 0.001 | 5.279 | 0.00000 |
| $R_{I,t-1}D_t$ | 0.245 | 0.288 | 0.850 | 0.395 |
| $e_{I,t}D_t$ | 0.008 | 0.002 | 3.181 | 0.001 |
| omega | 3.181 | 0.431 | 7.388 | 0 |
| alpha1 | 0.352 | 0.047 | 7.553 | 0 |
| beta1 | 0.738 | 0.025 | 29.342 | 0 |
| gamma1 | −0.231 | 0.042 | −5.444 | 0.00000 |

#### A.1.4.3 Variance Ratio Graphs

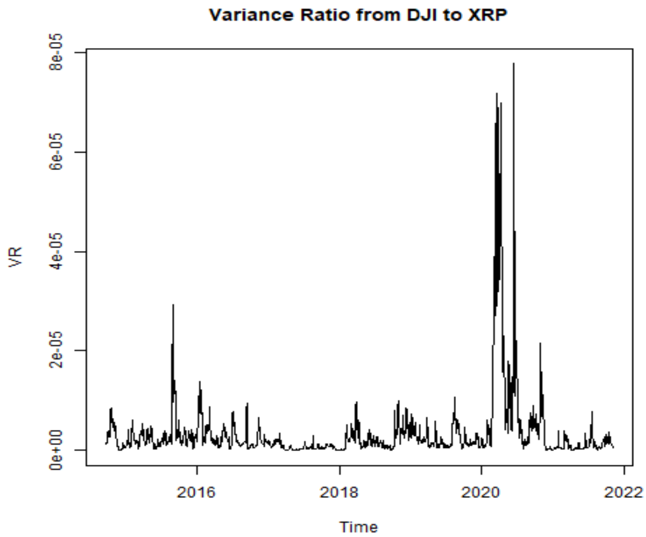


Figure A.5: Variance Ratio of DJI-XRP

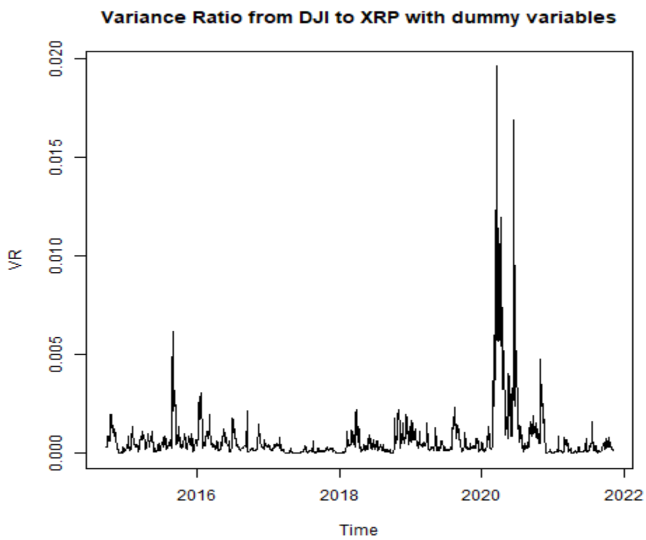


Figure A.6: Variance Ratio of DJI-XRP with dummy variables

### A.1.5 XMR

#### A.1.5.1 Crypto model

Table A.8: Parameter Estimates of the GARCH(1, 1) (DJI-XMR)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.275 | 0.150 | 1.830 | 0.067 |
| ar1 | −0.019 | 0.028 | −0.661 | 0.509 |
| $R_{I,t-1}$ | 0.029 | 0.142 | 0.207 | 0.836 |
| $e_{I,t}$ | 0.008 | 0.001 | 5.474 | 0.00000 |
| omega | 3.209 | 0.723 | 4.438 | 0.00001 |
| alpha1 | 0.175 | 0.025 | 6.910 | 0 |
| beta1 | 0.821 | 0.022 | 36.989 | 0 |
| gamma1 | −0.078 | 0.025 | −3.130 | 0.002 |

#### A.1.5.2 Crypto model with dummy variables

Table A.9: Parameter Estimates of the GARCH(1, 1) (DJI-XMR with dummy var.)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.287 | 0.150 | 1.919 | 0.055 |
| ar1 | −0.013 | 0.028 | −0.463 | 0.643 |
| $R_{I,t-1}$ | 0.069 | 0.151 | 0.457 | 0.648 |
| $e_{I,t}$ | 0.005 | 0.002 | 3.367 | 0.001 |
| $R_{I,t-1}D_t$ | 0.285 | 0.382 | 0.748 | 0.454 |
| $e_{I,t}D_t$ | 0.012 | 0.003 | 3.851 | 0.0001 |
| omega | 2.926 | 0.664 | 4.405 | 0.00001 |
| alpha1 | 0.177 | 0.025 | 7.043 | 0 |
| beta1 | 0.827 | 0.021 | 39.687 | 0 |
| gamma1 | −0.088 | 0.025 | −3.549 | 0.0004 |

#### A.1.5.3 Variance Ratio Graphs

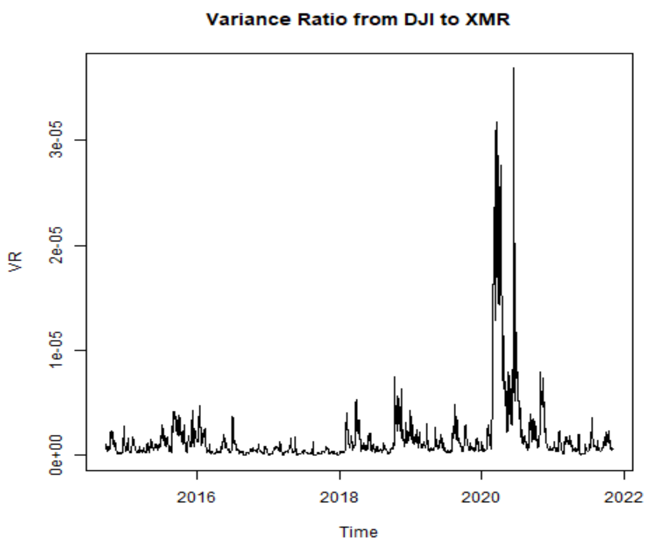


Figure A.7: Variance Ratio of DJI-XMR

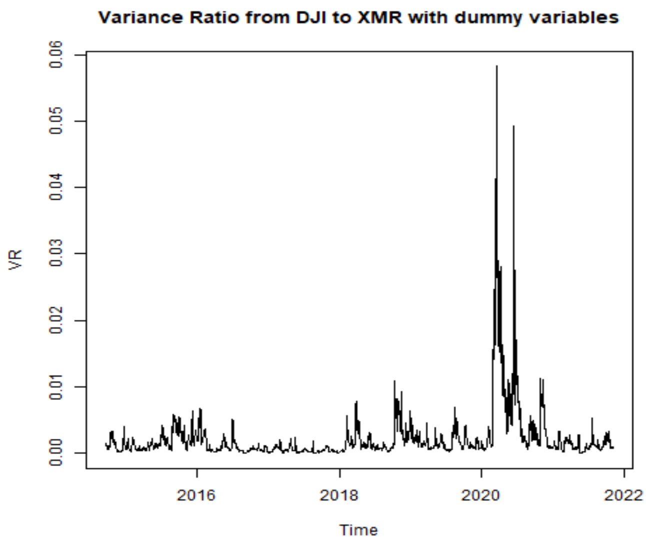


Figure A.8: Variance Ratio of DJI-XMR with dummy variables

## A.2 GSPC

### A.2.1 Index model

Table A.10: Parameter Estimates of the GARCH(1, 1) (GSPC)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.091 | 0.015 | 6.232 | 0 |
| ar1 | −0.083 | 0.027 | −3.072 | 0.002 |
| omega | 0.044 | 0.007 | 6.423 | 0 |
| alpha1 | 0.252 | 0.029 | 8.758 | 0 |
| beta1 | 0.720 | 0.024 | 29.473 | 0 |

### A.2.2 BTC

#### A.2.2.1 Crypto model

Table A.11: Parameter Estimates of the GARCH(1, 1) (GSPC-BTC)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.253 | 0.096 | 2.628 | 0.009 |
| ar1 | 0.027 | 0.029 | 0.921 | 0.357 |
| $R_{I,t-1}$ | −0.080 | 0.108 | −0.738 | 0.460 |
| $e_{I,t}$ | 0.005 | 0.001 | 4.573 | 0.00000 |
| omega | 1.487 | 0.260 | 5.716 | 0 |
| alpha1 | 0.138 | 0.025 | 5.530 | 0.00000 |
| beta1 | 0.792 | 0.024 | 33.613 | 0 |
| gamma1 | 0.028 | 0.029 | 0.967 | 0.333 |

#### A.2.2.2 Crypto model with dummy variables

Table A.12: Parameter Estimates of the GARCH(1, 1) (GSPC-BTC with dummy var.)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.266 | 0.096 | 2.776 | 0.006 |
| ar1 | 0.034 | 0.028 | 1.212 | 0.225 |
| $R_{I,t-1}$ | 0.085 | 0.103 | 0.825 | 0.409 |
| $e_{I,t}$ | 0.004 | 0.001 | 3.747 | 0.0002 |
| $R_{I,t-1}D_t$ | 0.019 | 0.368 | 0.052 | 0.959 |
| $e_{I,t}D_t$ | 0.014 | 0.003 | 5.355 | 0.00000 |
| omega | 1.194 | 0.227 | 5.269 | 0.00000 |
| alpha1 | 0.133 | 0.023 | 5.735 | 0 |
| beta1 | 0.820 | 0.023 | 35.341 | 0 |
| gamma1 | −0.005 | 0.024 | −0.204 | 0.839 |

#### A.2.2.3 Variance Ratio Graphs

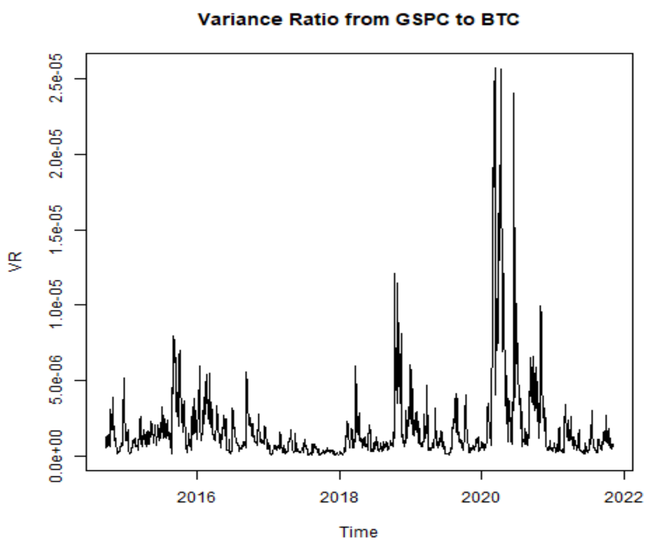


Figure A.9: Variance Ratio of GSPC-BTC

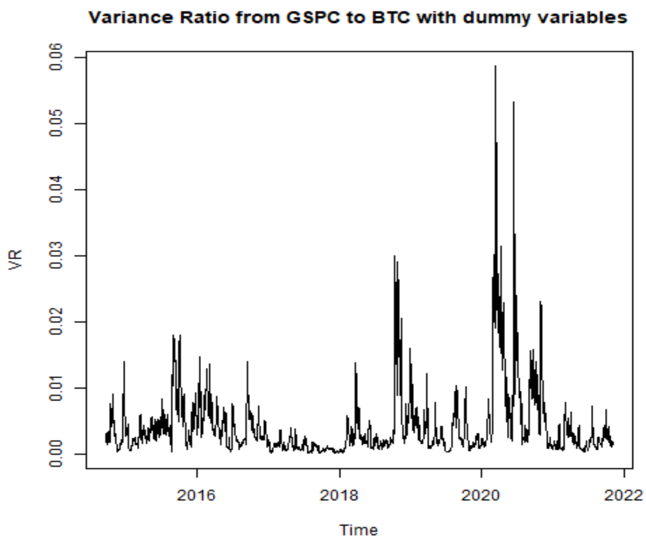


Figure A.10: Variance Ratio of GSPC-BTC with dummy variables

### A.2.3 ETH

#### A.2.3.1 Crypto model

Table A.13: Parameter Estimates of the GARCH(1, 1) (GSPC-ETH)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.301 | 0.171 | 1.763 | 0.078 |
| ar1 | 0.026 | 0.032 | 0.812 | 0.417 |
| $R_{I,t-1}$ | 0.126 | 0.190 | 0.665 | 0.506 |
| $e_{I,t}$ | 0.012 | 0.002 | 7.003 | 0 |
| omega | 3.458 | 0.697 | 4.960 | 0.00000 |
| alpha1 | 0.173 | 0.029 | 6.037 | 0 |
| beta1 | 0.803 | 0.026 | 31.101 | 0 |
| gamma1 | −0.074 | 0.028 | −2.635 | 0.008 |

#### A.2.3.2 Crypto model with dummy variables

Table A.14: Parameter Estimates of the GARCH(1, 1) (GSPC-ETH with dummy var.)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.308 | 0.170 | 1.809 | 0.070 |
| ar1 | 0.031 | 0.032 | 0.966 | 0.334 |
| $R_{I,t-1}$ | 0.185 | 0.174 | 1.067 | 0.286 |
| $e_{I,t}$ | 0.008 | 0.002 | 4.486 | 0.00001 |
| $R_{I,t-1}D_t$ | 0.081 | 0.455 | 0.177 | 0.859 |
| $e_{I,t}D_t$ | 0.015 | 0.004 | 4.176 | 0.00003 |
| omega | 3.142 | 0.645 | 4.870 | 0.00000 |
| alpha1 | 0.174 | 0.028 | 6.183 | 0 |
| beta1 | 0.808 | 0.025 | 32.147 | 0 |
| gamma1 | −0.077 | 0.027 | −2.865 | 0.004 |

#### A.2.3.3 Variance Ratio Graphs

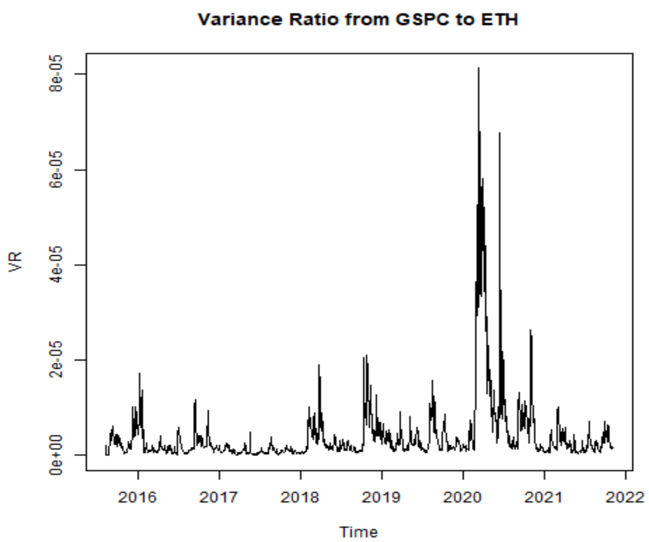


Figure A.11: Variance Ratio of GSPC-ETH

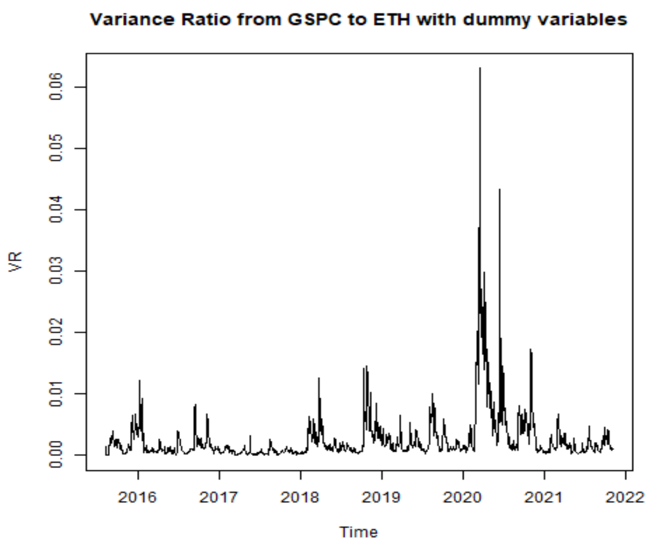


Figure A.12: Variance Ratio of GSPC-ETH with dummy variables

### A.2.4 XRP

#### A.2.4.1 Crypto model

Table A.15: Parameter Estimates of the GARCH(1, 1) (GSPC-XRP)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | −0.144 | 0.136 | −1.063 | 0.288 |
| ar1 | 0.042 | 0.030 | 1.384 | 0.166 |
| $R_{I,t-1}$ | −0.114 | 0.125 | −0.910 | 0.363 |
| $e_{I,t}$ | 0.009 | 0.001 | 8.450 | 0 |
| omega | 3.076 | 0.433 | 7.101 | 0 |
| alpha1 | 0.347 | 0.044 | 7.847 | 0 |
| beta1 | 0.742 | 0.024 | 31.140 | 0 |
| gamma1 | −0.222 | 0.042 | −5.349 | 0.00000 |

#### A.2.4.2 Crypto model with dummy variables

Table A.16: Parameter Estimates of the GARCH(1, 1) (GSPC-XRP with dummy var.)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | −0.127 | 0.137 | −0.926 | 0.354 |
| ar1 | 0.045 | 0.030 | 1.507 | 0.132 |
| $R_{I,t-1}$ | −0.081 | 0.139 | −0.580 | 0.562 |
| $e_{I,t}$ | 0.007 | 0.001 | 5.372 | 0.00000 |
| $R_{I,t-1}D_t$ | 0.273 | 0.301 | 0.907 | 0.364 |
| $e_{I,t}D_t$ | 0.009 | 0.003 | 3.290 | 0.001 |
| omega | 3.108 | 0.427 | 7.274 | 0 |
| alpha1 | 0.345 | 0.046 | 7.585 | 0 |
| beta1 | 0.742 | 0.025 | 29.911 | 0 |
| gamma1 | −0.225 | 0.041 | −5.424 | 0.00000 |

#### A.2.4.3 Variance Ratio Graphs

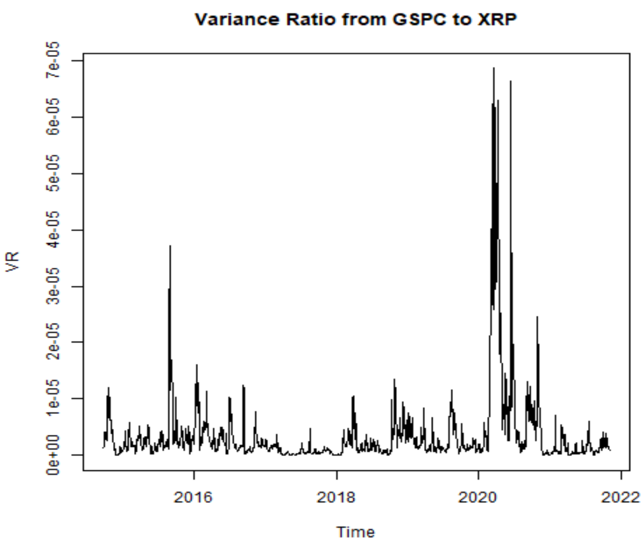


Figure A.13: Variance Ratio of GSPC-XRP

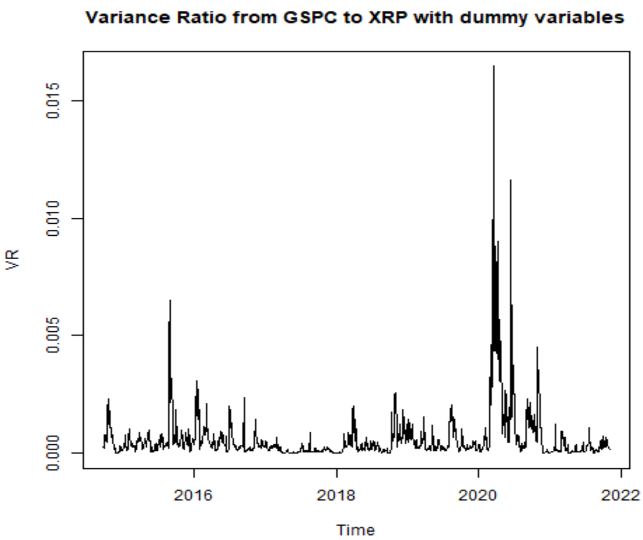


Figure A.14: Variance Ratio of GSPC-XRP with dummy variables

### A.2.5 XMR

#### A.2.5.1 Crypto model

Table A.17: Parameter Estimates of the GARCH(1, 1) (GSPC-XMR)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.275 | 0.150 | 1.837 | 0.066 |
| ar1 | −0.021 | 0.028 | −0.748 | 0.455 |
| $R_{I,t-1}$ | 0.023 | 0.150 | 0.152 | 0.879 |
| $e_{I,t}$ | 0.008 | 0.001 | 5.689 | 0 |
| omega | 3.187 | 0.724 | 4.401 | 0.00001 |
| alpha1 | 0.174 | 0.025 | 6.920 | 0 |
| beta1 | 0.822 | 0.022 | 36.931 | 0 |
| gamma1 | −0.077 | 0.025 | −3.140 | 0.002 |

#### A.2.5.2 Crypto model with dummy variables

Table A.18: Parameter Estimates of the GARCH(1, 1) (GSPC-XMR with dummy var.)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.286 | 0.149 | 1.913 | 0.056 |
| ar1 | −0.015 | 0.028 | −0.517 | 0.605 |
| $R_{I,t-1}$ | 0.085 | 0.158 | 0.538 | 0.590 |
| $e_{I,t}$ | 0.006 | 0.002 | 3.692 | 0.0002 |
| $R_{I,t-1}D_t$ | 0.270 | 0.405 | 0.666 | 0.505 |
| $e_{I,t}D_t$ | 0.013 | 0.003 | 3.895 | 0.0001 |
| omega | 2.874 | 0.656 | 4.381 | 0.00001 |
| alpha1 | 0.175 | 0.025 | 7.057 | 0 |
| beta1 | 0.829 | 0.021 | 40.246 | 0 |
| gamma1 | −0.087 | 0.025 | −3.559 | 0.0004 |

#### A.2.5.3 Variance Ratio Graphs

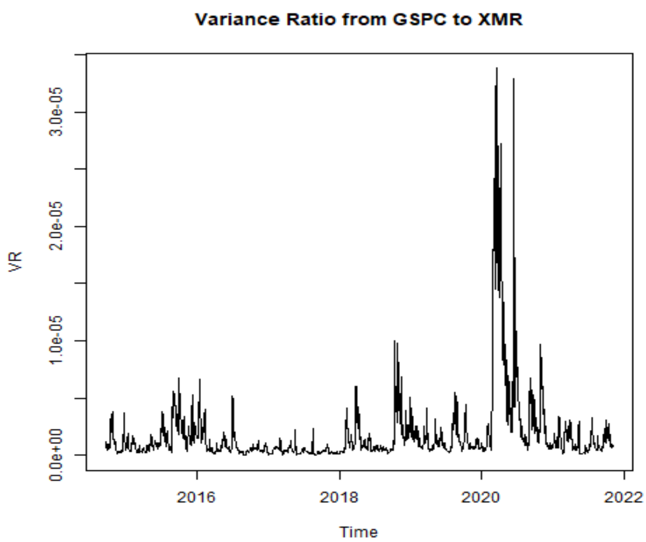


Figure A.15: Variance Ratio of GSPC-XMR

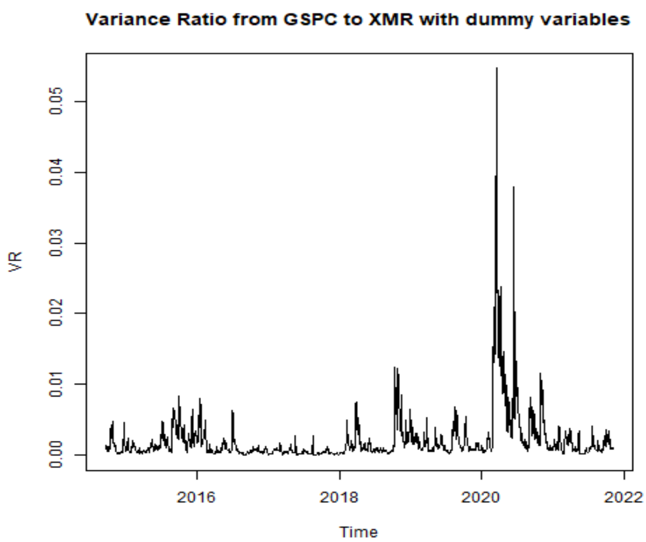


Figure A.16: Variance Ratio of GSPC-XMR with dummy variables

## A.3 NDX

### A.3.1 Index model

Table A.19: Parameter Estimates of the GARCH(1, 1) (NDX)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.109 | 0.020 | 5.400 | 0.00000 |
| ar1 | −0.060 | 0.026 | −2.281 | 0.023 |
| omega | 0.070 | 0.013 | 5.565 | 0.00000 |
| alpha1 | 0.181 | 0.024 | 7.642 | 0 |
| beta1 | 0.777 | 0.025 | 31.287 | 0 |

### A.3.2 BTC

#### A.3.2.1 Crypto model

Table A.20: Parameter Estimates of the GARCH(1, 1) (NDX-BTC)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.230 | 0.097 | 2.388 | 0.017 |
| ar1 | 0.029 | 0.029 | 1.004 | 0.315 |
| $R_{I,t-1}$ | 0.008 | 0.087 | 0.093 | 0.926 |
| $e_{I,t}$ | 0.004 | 0.001 | 5.007 | 0.00000 |
| omega | 1.428 | 0.253 | 5.642 | 0.00000 |
| alpha1 | 0.134 | 0.024 | 5.610 | 0.00000 |
| beta1 | 0.796 | 0.023 | 34.678 | 0 |
| gamma1 | 0.034 | 0.027 | 1.256 | 0.209 |

#### A.3.2.2 Crypto model with dummy variables

Table A.21: Parameter Estimates of the GARCH(1, 1) (NDX-BTC with dummy var.)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.256 | 0.096 | 2.674 | 0.008 |
| ar1 | 0.034 | 0.027 | 1.221 | 0.222 |
| $R_{I,t-1}$ | 0.076 | 0.082 | 0.928 | 0.353 |
| $e_{I,t}$ | 0.004 | 0.001 | 4.391 | 0.00001 |
| $R_{I,t-1}D_t$ | 0.329 | 0.380 | 0.865 | 0.387 |
| $e_{I,t}D_t$ | 0.016 | 0.002 | 6.355 | 0 |
| omega | 1.105 | 0.216 | 5.129 | 0.00000 |
| alpha1 | 0.128 | 0.023 | 5.693 | 0 |
| beta1 | 0.829 | 0.023 | 36.622 | 0 |
| gamma1 | −0.006 | 0.023 | −0.271 | 0.786 |

#### A.3.2.3 Variance Ratio Graphs

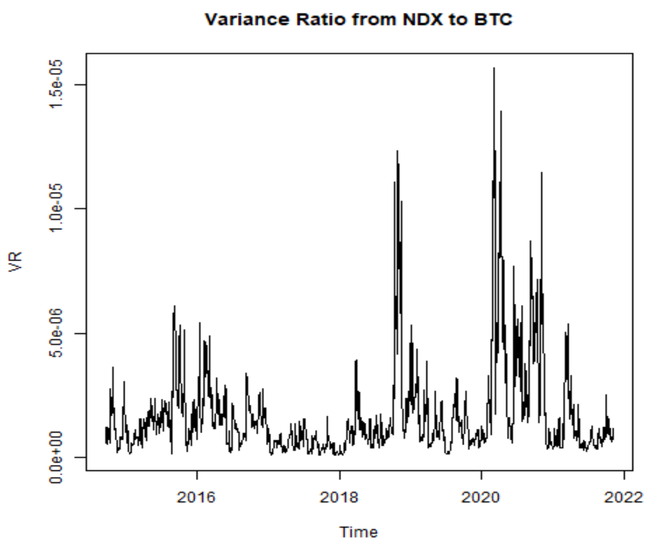


Figure A.17: Variance Ratio of NDX-BTC

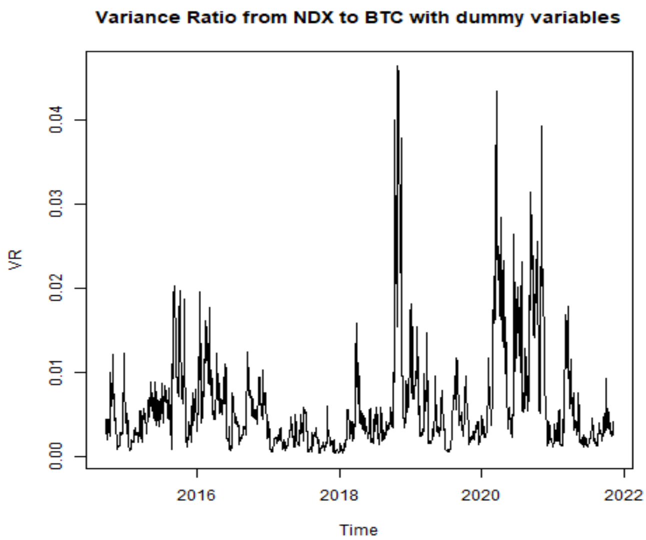


Figure A.18: Variance Ratio of NDX-BTC with dummy variables

### A.3.3 ETH

#### A.3.3.1 Crypto model

Table A.22: Parameter Estimates of the GARCH(1, 1) (NDX-ETH)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.262 | 0.172 | 1.524 | 0.127 |
| ar1 | 0.029 | 0.033 | 0.887 | 0.375 |
| $R_{I,t-1}$ | 0.191 | 0.146 | 1.313 | 0.189 |
| $e_{I,t}$ | 0.009 | 0.002 | 5.778 | 0 |
| omega | 3.593 | 0.721 | 4.986 | 0.00000 |
| alpha1 | 0.177 | 0.030 | 6.003 | 0 |
| beta1 | 0.795 | 0.026 | 30.078 | 0 |
| gamma1 | −0.068 | 0.030 | −2.283 | 0.022 |

#### A.3.3.2 Crypto model with dummy variables

Table A.23: Parameter Estimates of the GARCH(1, 1) (NDX-ETH with dummy var.)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.278 | 0.170 | 1.635 | 0.102 |
| ar1 | 0.031 | 0.032 | 0.958 | 0.338 |
| $R_{I,t-1}$ | 0.184 | 0.136 | 1.349 | 0.177 |
| $e_{I,t}$ | 0.006 | 0.001 | 4.486 | 0.00001 |
| $R_{I,t-1}D_t$ | 0.337 | 0.435 | 0.777 | 0.437 |
| $e_{I,t}D_t$ | 0.017 | 0.003 | 5.114 | 0.00000 |
| omega | 3.011 | 0.636 | 4.731 | 0.00000 |
| alpha1 | 0.169 | 0.028 | 6.015 | 0 |
| beta1 | 0.815 | 0.025 | 32.207 | 0 |
| gamma1 | −0.077 | 0.026 | −2.980 | 0.003 |

#### A.3.3.3 Variance Ratio Graphs

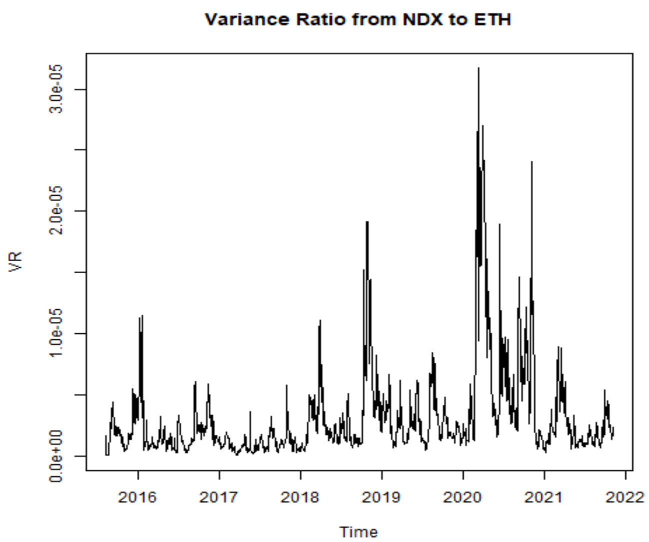


Figure A.19: Variance Ratio of NDX-ETH

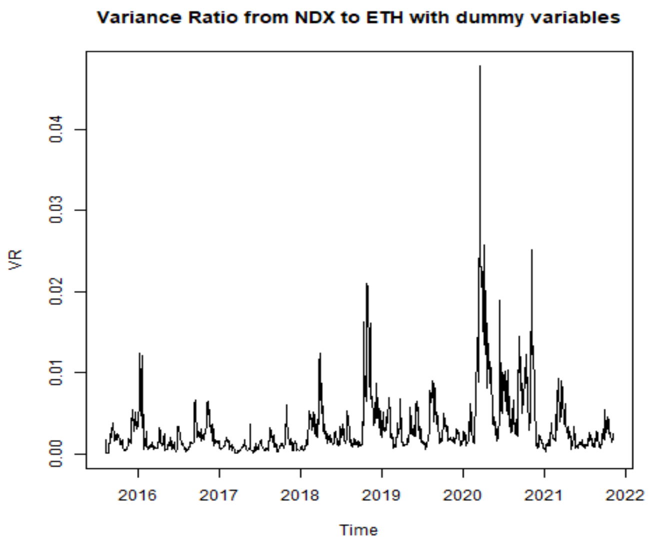


Figure A.20: Variance Ratio of NDX-ETH with dummy variables

### A.3.4 XRP

#### A.3.4.1 Crypto model

Table A.24: Parameter Estimates of the GARCH(1, 1) (NDX-XRP)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | −0.137 | 0.137 | −0.999 | 0.318 |
| ar1 | 0.043 | 0.030 | 1.418 | 0.156 |
| $R_{I,t-1}$ | −0.097 | 0.110 | −0.884 | 0.377 |
| $e_{I,t}$ | 0.007 | 0.001 | 6.894 | 0 |
| omega | 3.177 | 0.448 | 7.085 | 0 |
| alpha1 | 0.332 | 0.043 | 7.787 | 0 |
| beta1 | 0.744 | 0.025 | 30.009 | 0 |
| gamma1 | −0.204 | 0.040 | −5.149 | 0.00000 |

#### A.3.4.2 Crypto model with dummy variables

Table A.25: Parameter Estimates of the GARCH(1, 1) (NDX-XRP with dummy var.)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | −0.103 | 0.138 | −0.745 | 0.456 |
| ar1 | 0.049 | 0.030 | 1.616 | 0.106 |
| $R_{I,t-1}$ | −0.092 | 0.110 | −0.835 | 0.404 |
| $e_{I,t}$ | 0.005 | 0.001 | 4.999 | 0.00000 |
| $R_{I,t-1}D_t$ | 0.509 | 0.302 | 1.686 | 0.092 |
| $e_{I,t}D_t$ | 0.011 | 0.003 | 4.458 | 0.00001 |
| omega | 3.075 | 0.424 | 7.252 | 0 |
| alpha1 | 0.328 | 0.043 | 7.558 | 0 |
| beta1 | 0.751 | 0.025 | 30.378 | 0 |
| gamma1 | −0.215 | 0.039 | −5.480 | 0.00000 |

#### A.3.4.3 Variance Ratio Graphs

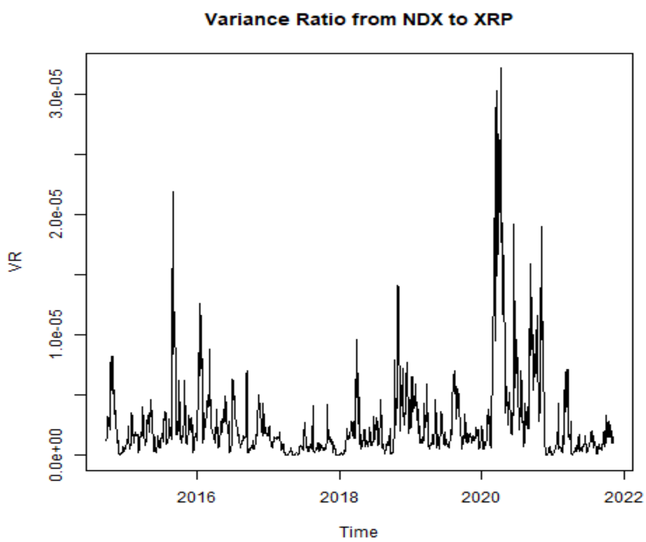


Figure A.21: Variance Ratio of NDX-XRP

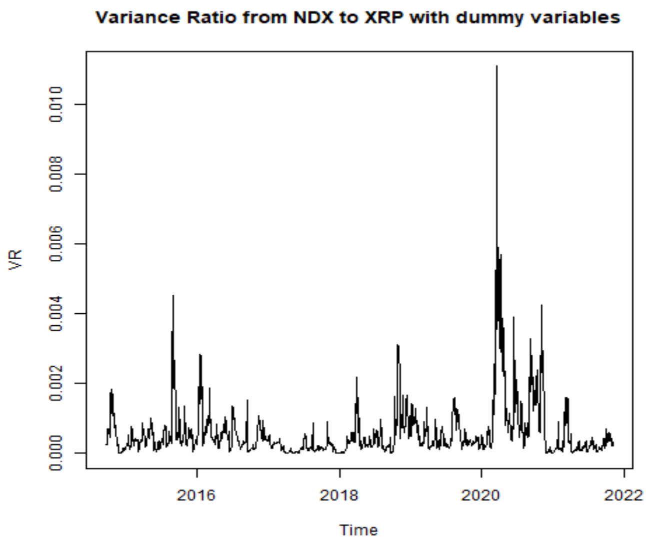


Figure A.22: Variance Ratio of NDX-XRP with dummy variables

### A.3.5 XMR

#### A.3.5.1 Crypto model

Table A.26: Parameter Estimates of the GARCH(1, 1) (NDX-XMR)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.257 | 0.151 | 1.708 | 0.088 |
| ar1 | −0.021 | 0.028 | −0.755 | 0.450 |
| $R_{I,t-1}$ | 0.056 | 0.125 | 0.449 | 0.653 |
| $e_{I,t}$ | 0.006 | 0.001 | 4.889 | 0.00000 |
| omega | 3.325 | 0.751 | 4.426 | 0.00001 |
| alpha1 | 0.173 | 0.025 | 6.849 | 0 |
| beta1 | 0.819 | 0.023 | 35.585 | 0 |
| gamma1 | −0.073 | 0.025 | −2.970 | 0.003 |

#### A.3.5.2 Crypto model with dummy variables

Table A.27: Parameter Estimates of the GARCH(1, 1) (NDX-XMR with dummy var.)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.277 | 0.150 | 1.848 | 0.065 |
| ar1 | −0.014 | 0.028 | −0.488 | 0.625 |
| $R_{I,t-1}$ | 0.084 | 0.127 | 0.656 | 0.512 |
| $e_{I,t}$ | 0.004 | 0.001 | 3.256 | 0.001 |
| $R_{I,t-1}D_t$ | 0.556 | 0.382 | 1.454 | 0.146 |
| $e_{I,t}D_t$ | 0.016 | 0.003 | 5.039 | 0.00000 |
| omega | 2.799 | 0.644 | 4.348 | 0.00001 |
| alpha1 | 0.170 | 0.024 | 7.039 | 0 |
| beta1 | 0.834 | 0.020 | 41.177 | 0 |
| gamma1 | −0.085 | 0.024 | −3.561 | 0.0004 |

#### A.3.5.3 Variance Ratio Graphs

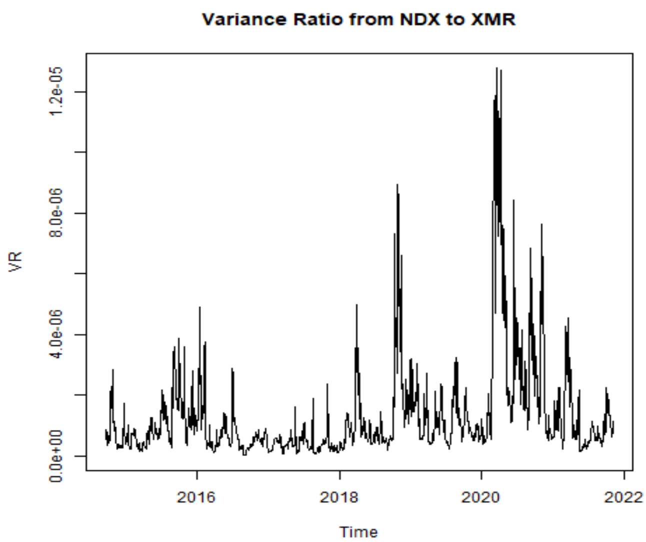


Figure A.23: Variance Ratio of NDX-XMR

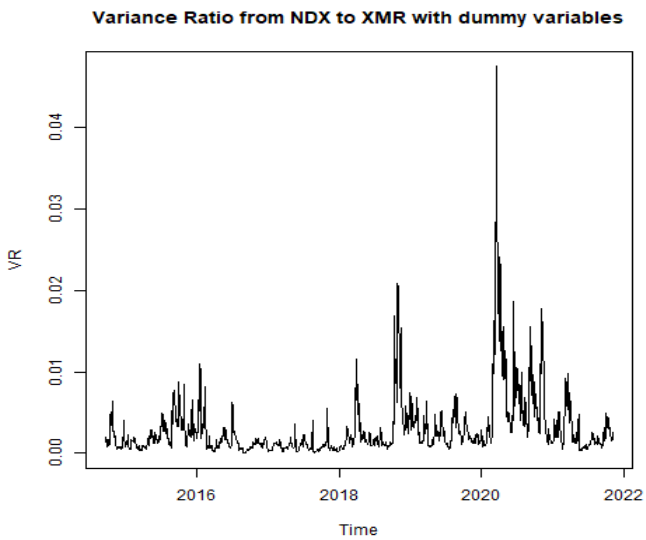


Figure A.24: Variance Ratio of NDX-XMR with dummy variables

# B

# Appendix 2

This annex shows the results from the stocks to US indexes conditional correlations, as well as important models such as the index model using the orthogonal stock residuals and the dummy variable models displaying the significance of the dummy variables during the Covid market crash.

## B.1 DJI

Table B.1: Parameter Estimates of the GARCH(1, 1) (DJI-Stocks)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.0001 | 0.0002 | 0.485 | 0.628 |
| ar1 | −0.236 | 0.036 | −6.540 | 0 |
| MMM | 0.00000 | 0.00000 | 1.096 | 0.273 |
| AXP | −0.00000 | 0.00000 | −0.518 | 0.604 |
| AMGN | 0.00000 | 0.00000 | 0.672 | 0.502 |
| AAPL | −0.00001 | 0.00001 | −0.790 | 0.429 |
| BA | −0.00002 | 0.00000 | −3.879 | 0.0001 |
| CAT | −0.00000 | 0.00001 | −0.362 | 0.717 |
| CVX | 0.00001 | 0.00000 | 4.904 | 0.00000 |
| CSCO | −0.00000 | 0.00000 | −0.754 | 0.451 |
| KO | −0.00002 | 0.00002 | −0.844 | 0.399 |
| DOW | −0.00000 | 0.00000 | −7.166 | 0 |
| GS | 0.00001 | 0.00001 | 1.328 | 0.184 |
| HD | 0.00000 | 0.00000 | 1.369 | 0.171 |
| HON | 0.00000 | 0.00000 | 1.016 | 0.310 |
| IBM | −0.00001 | 0.00001 | −1.074 | 0.283 |
| INTC | −0.00003 | 0.00001 | −3.971 | 0.0001 |
| JNJ | 0.00002 | 0.00001 | 2.551 | 0.011 |
| JPM | 0.00000 | 0.00000 | 0.731 | 0.465 |
| MCD | −0.00000 | 0.00000 | −1.047 | 0.295 |
| MRK | −0.00000 | 0.00002 | −0.110 | 0.912 |
| MSFT | 0.00001 | 0.00000 | 1.836 | 0.066 |
| NKE | 0.00002 | 0.00002 | 0.866 | 0.387 |
| PG | 0.00000 | 0.00000 | 0.039 | 0.969 |
| CRM | −0.00001 | 0.00002 | −0.398 | 0.690 |
| TRV | 0.00003 | 0.00001 | 2.775 | 0.006 |
| UNH | −0.00000 | 0.00001 | −0.400 | 0.689 |
| VZ | 0.00001 | 0.00001 | 0.800 | 0.424 |
| V | −0.00000 | 0.00000 | −0.496 | 0.620 |
| WMT | 0.00000 | 0.00001 | 0.423 | 0.673 |
| WBA | −0.00000 | 0.00001 | −0.104 | 0.917 |
| DIS | 0.00003 | 0.00001 | 2.361 | 0.018 |
| omega | 0.00000 | 0.00000 | 0.274 | 0.784 |
| alpha1 | 0.050 | 0.009 | 5.702 | 0 |
| beta1 | 0.900 | 0.021 | 42.988 | 0 |

Table B.2: Significance of dummy variables during the COVID-19 market crash on Conditional Correlation and Variance Ratio, Returns of Price Residuals of Stocks, Stock Price Residuals, Stock Standard Deviation from GARCH models

| Stocks | Cond. Corr | VR | Returns on Residuals | Residuals | Stock std. |
|---|---|---|---|---|---|
| MMM | 0.06 | 0.36 | 0.89 | 0.97 | 0 |
| AXP | 0.6 | 0.93 | 0.84 | 0 | 0.03 |
| AMGN | 0.68 | 0.45 | 0.93 | 0.88 | 0 |
| AAPL | 0.78 | 0.98 | 0.99 | 0.02 | 0.33 |
| BA | 0.99 | 0.78 | 0.72 | 0 | 0 |
| CAT | 0.99 | 0.98 | 0.52 | 0.71 | 0.85 |
| CVX | 0.97 | 0.86 | 0.58 | 0.08 | 0.14 |
| CSCO | 0.06 | 0.1 | 0.85 | 0.14 | 0 |
| KO | 0.98 | 0.83 | 0.61 | 0.01 | 0.13 |
| DOW | 0.87 | 0.91 | 0.86 | 0.11 | 0.91 |
| GS | 0.56 | 0.75 | 0.53 | 0 | 0.52 |
| HD | 0.89 | 0.75 | 0.81 | 0.01 | 0.19 |
| HON | 0.89 | 0.98 | 0.98 | 0.01 | 0.21 |
| IBM | 0.22 | 0.2 | 0.8 | 0.01 | 0 |
| INTC | 0.5 | 0.4 | 0.03 | 0.58 | 0 |
| JNJ | 0.36 | 0.17 | 0.99 | 0.63 | 0 |
| JPM | 0.4 | 0.59 | 0.88 | 0 | 0.05 |
| MCD | 0.82 | 0.98 | 1 | 0.01 | 0.01 |
| MRK | 0.96 | 0.85 | 0.43 | 0.83 | 0.2 |
| MSFT | 0.96 | 0.87 | 0 | 0.07 | 0.25 |
| NKE | 0.53 | 0.58 | 0.99 | 0.06 | 0 |
| PG | 0.98 | 0.94 | 0.9 | 0.2 | 0.29 |
| CRM | 0.72 | 0.65 | 0.86 | 0.07 | 0.19 |
| TRV | 0.44 | 0.23 | 0 | 0.01 | 0 |
| UNH | 0.92 | 0.95 | 0.98 | 0.07 | 0.28 |
| VZ | 0.49 | 0.29 | 0.86 | 0.89 | 0 |
| V | 0.69 | 0.9 | 0.78 | 0.01 | 0.04 |
| WMT | 0.45 | 0.38 | 0.95 | 0.49 | 0 |
| WBA | 0.94 | 0.69 | 0.22 | 0.17 | 0.11 |
| DIS | 0.89 | 0.81 | 0.39 | 0 | 0.13 |

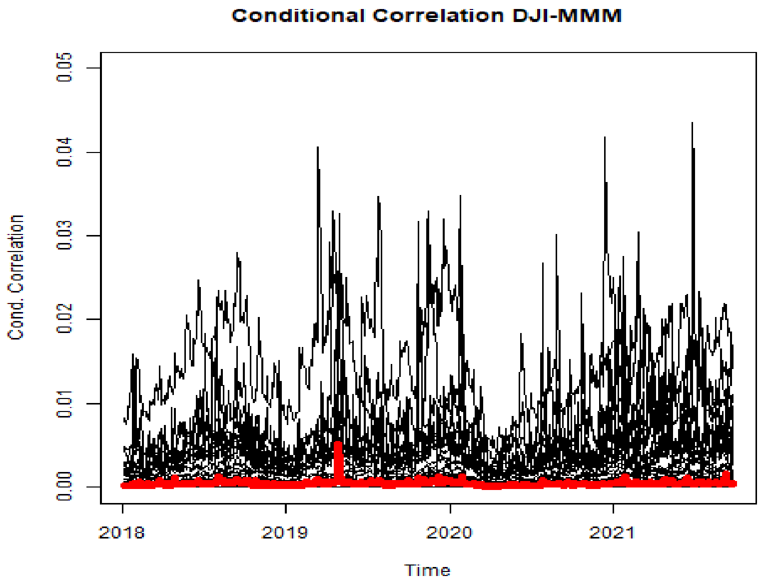


Figure B.1: Conditional Correlation DJI-MMM

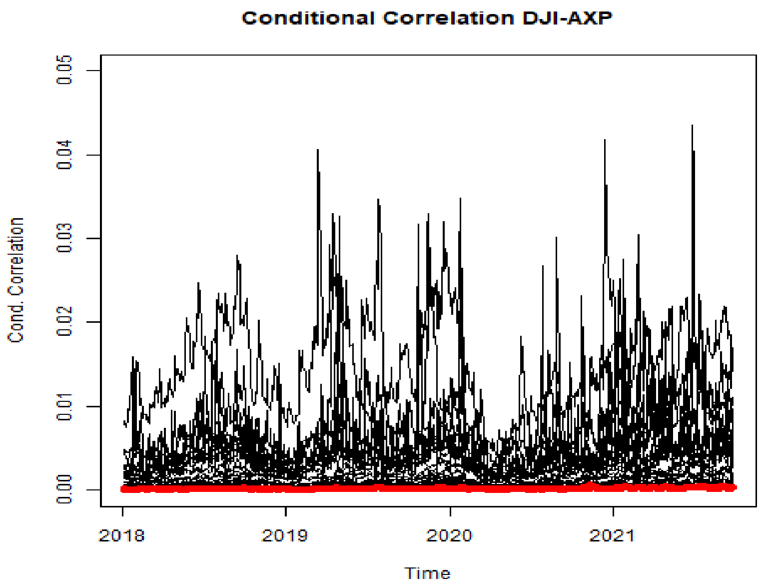


Figure B.2: Conditional Correlation DJI-AXP

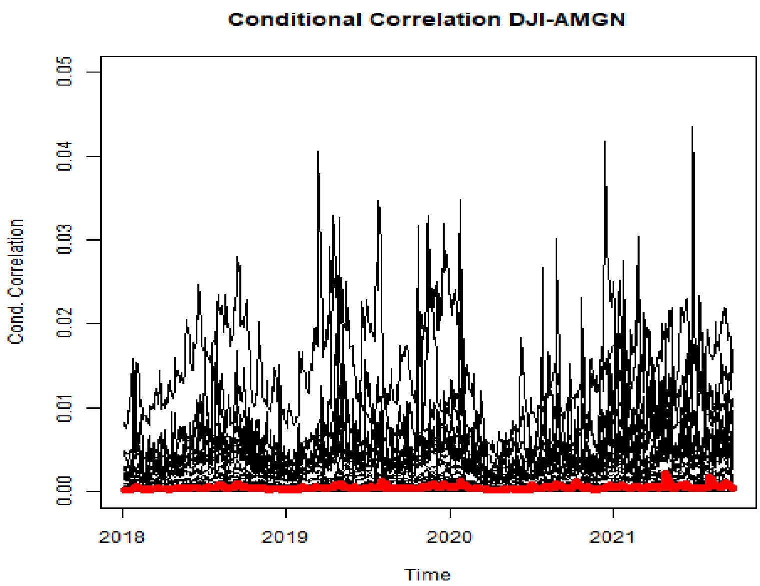


Figure B.3: Conditional Correlation DJI-AMGN

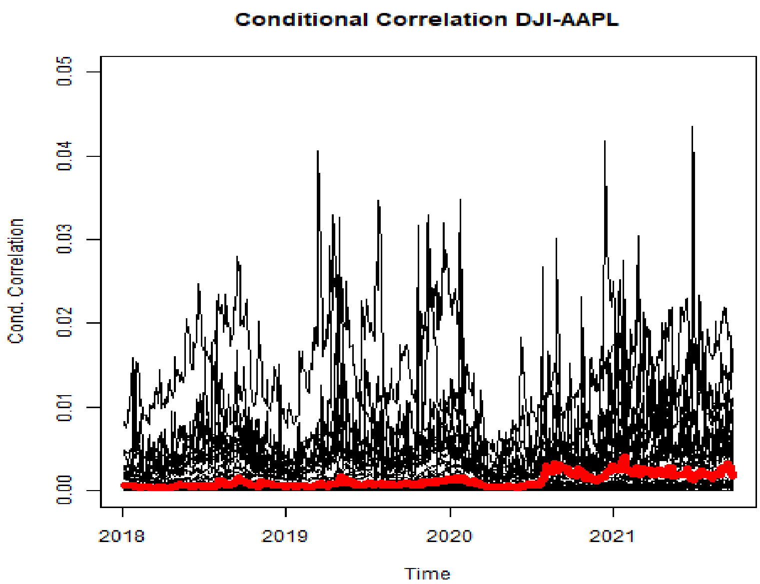


Figure B.4: Conditional Correlation DJI-AAPL

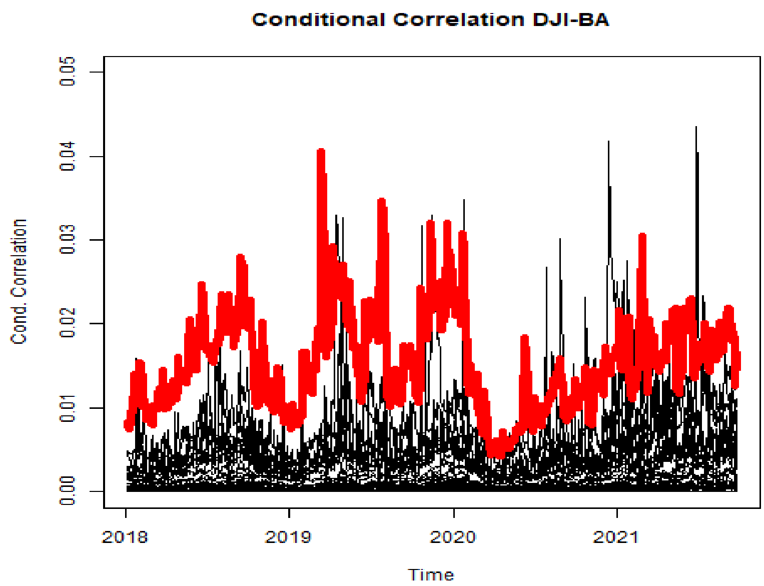


Figure B.5: Conditional Correlation DJI-BA

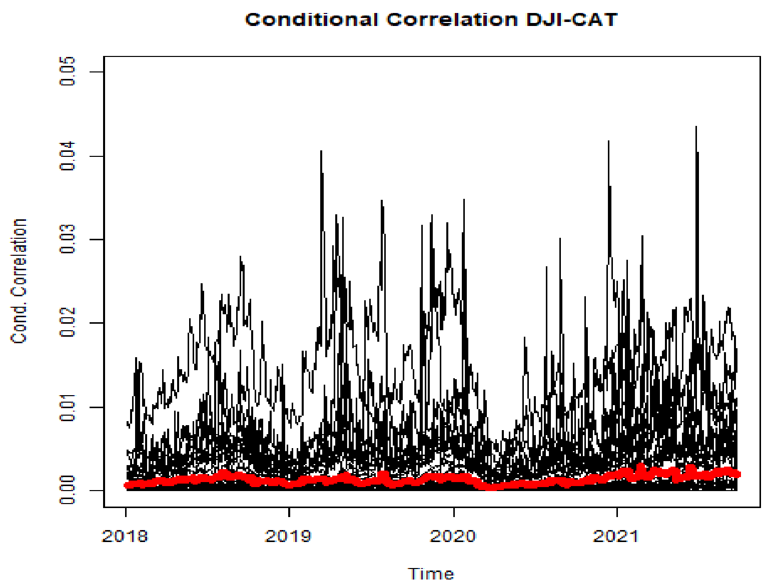


Figure B.6: Conditional Correlation DJI-CAT

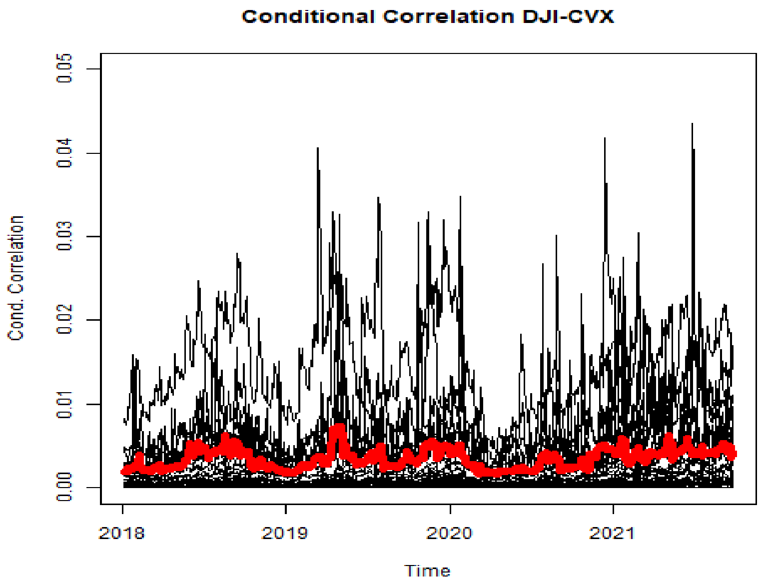


Figure B.7: Conditional Correlation DJI-CVX

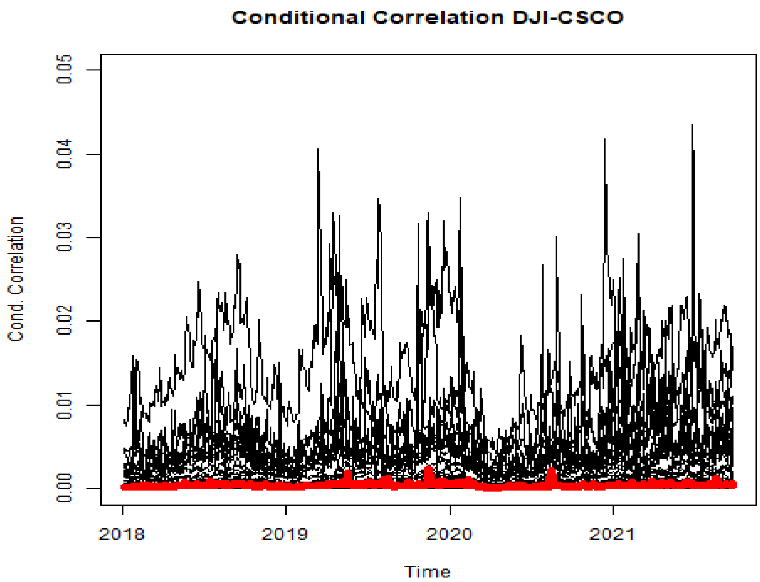


Figure B.8: Conditional Correlation DJI-CSCO

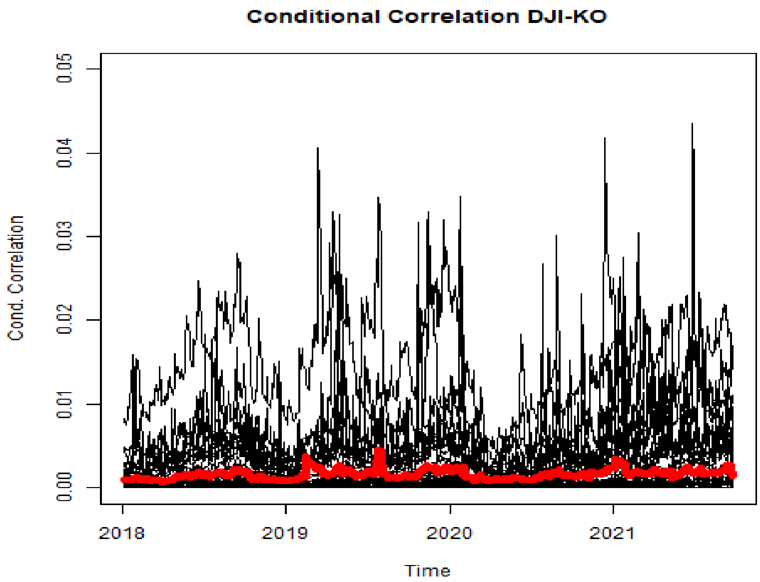


Figure B.9: Conditional Correlation DJI-KO

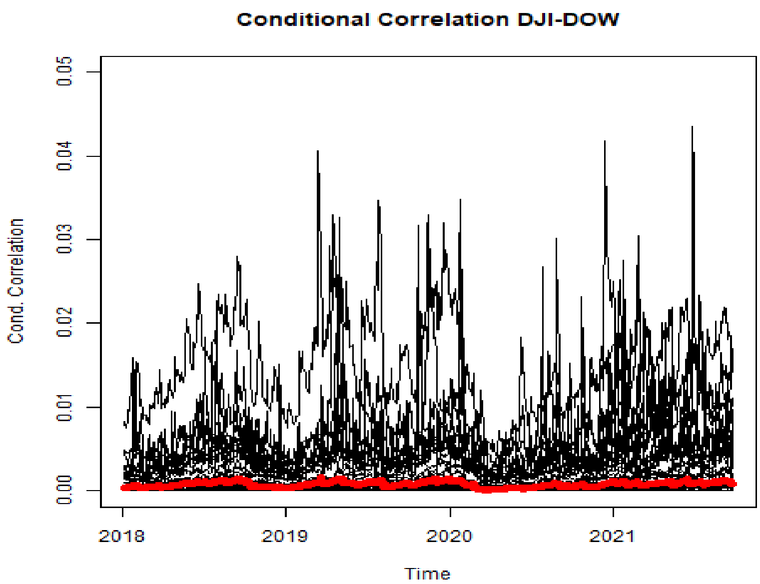


Figure B.10: Conditional Correlation DJI-DOW

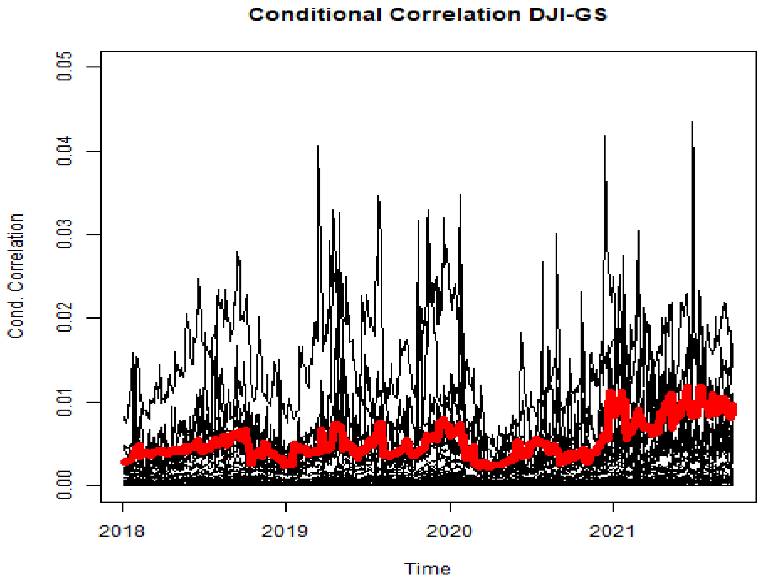


Figure B.11: Conditional Correlation DJI-GS

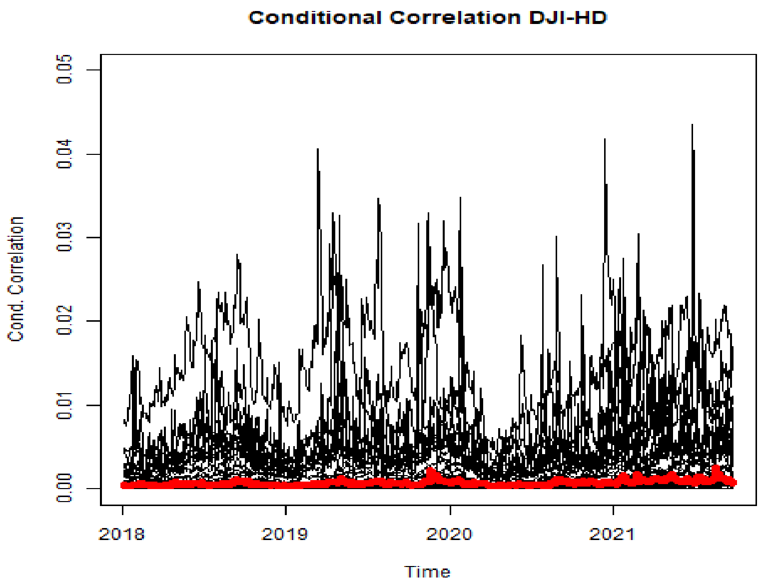


Figure B.12: Conditional Correlation DJI-HD

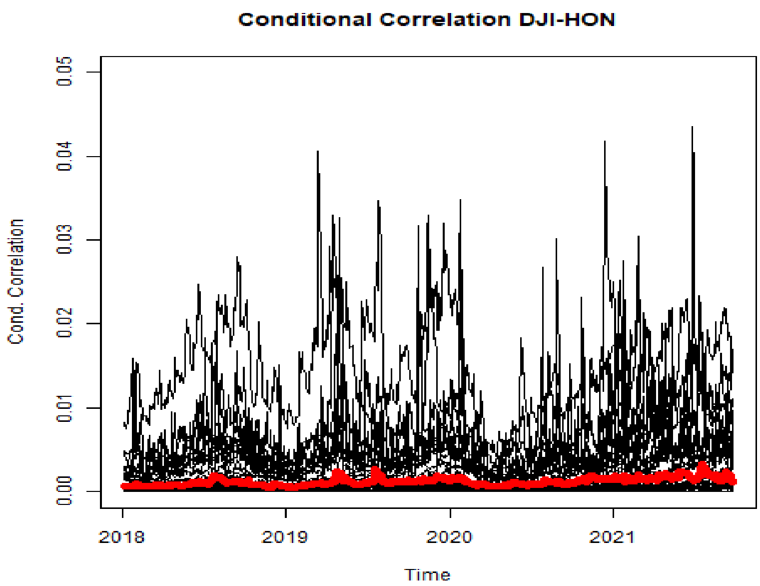


Figure B.13: Conditional Correlation DJI-HON

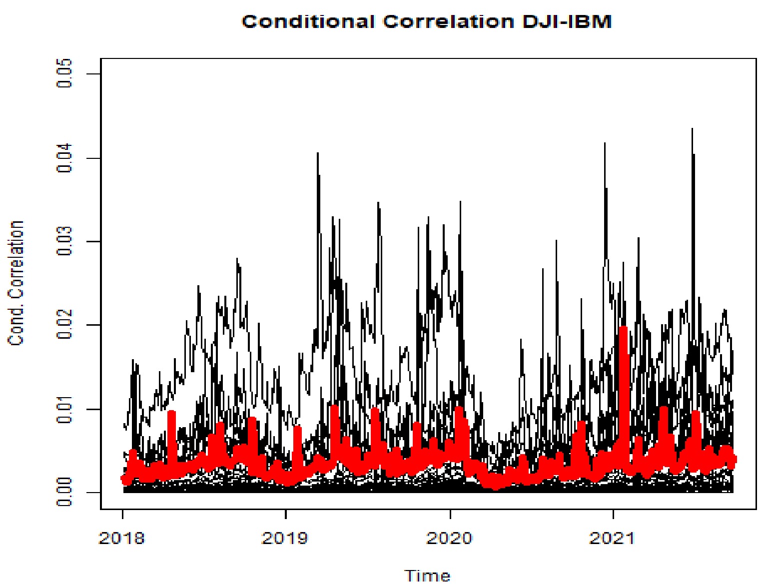


Figure B.14: Conditional Correlation DJI-IBM

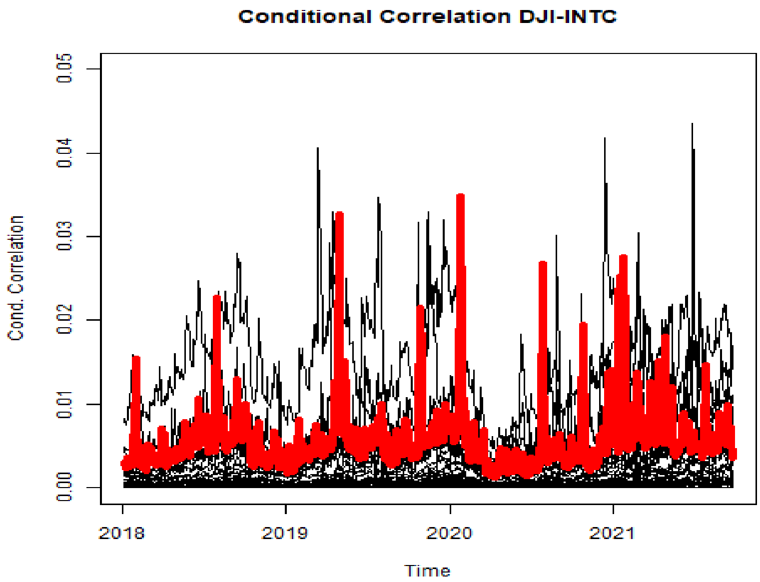


Figure B.15: Conditional Correlation DJI-INTC

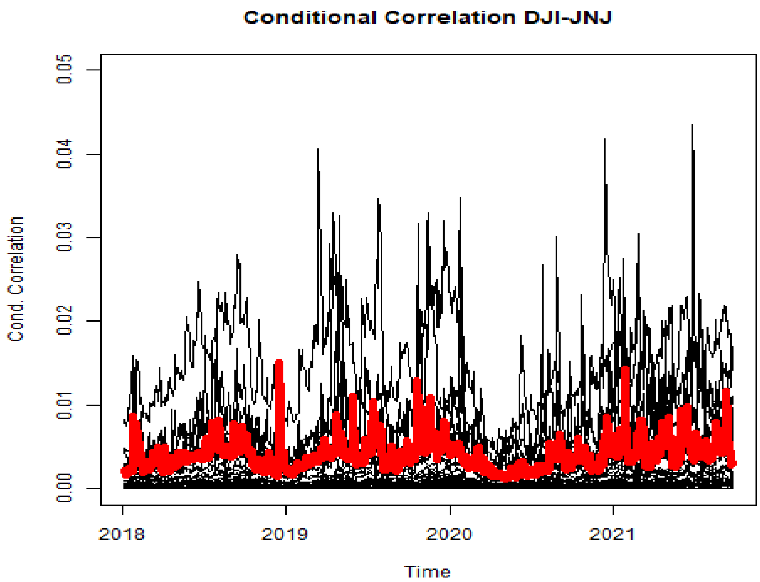


Figure B.16: Conditional Correlation DJI-JNJ

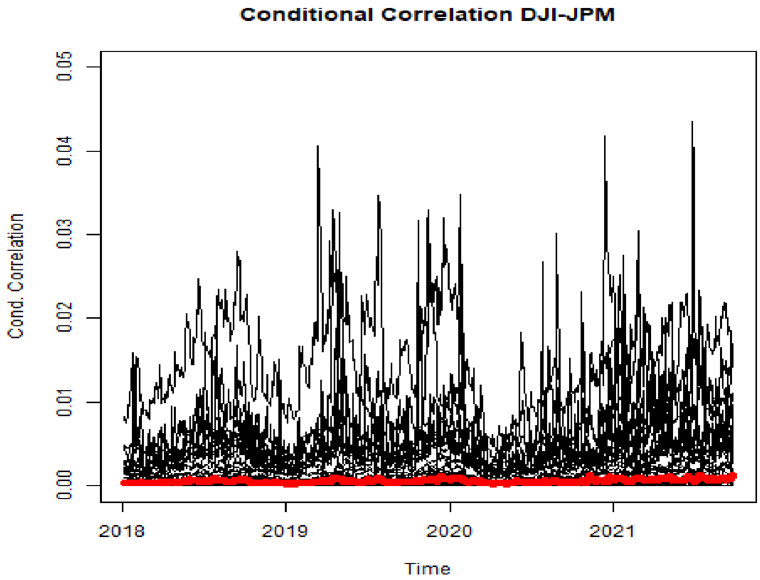


Figure B.17: Conditional Correlation DJI-JPM

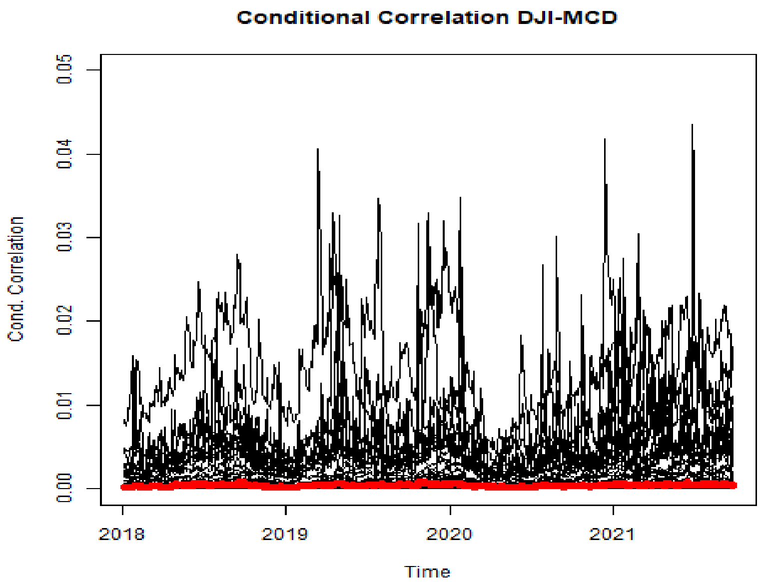


Figure B.18: Conditional Correlation DJI-MCD

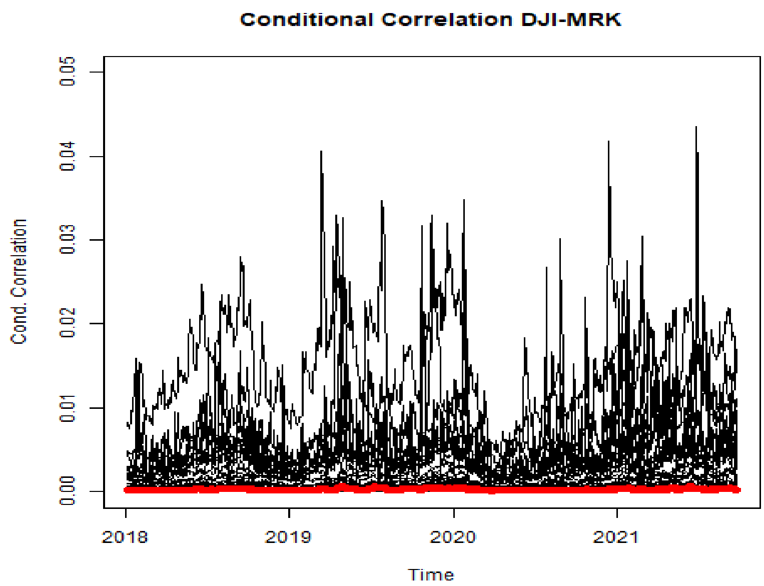


Figure B.19: Conditional Correlation DJI-MRK

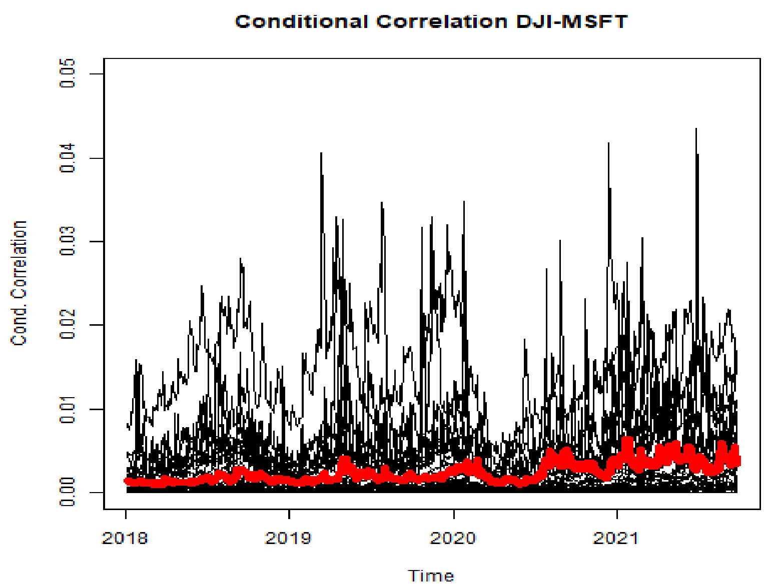


Figure B.20: Conditional Correlation DJI-MSFT

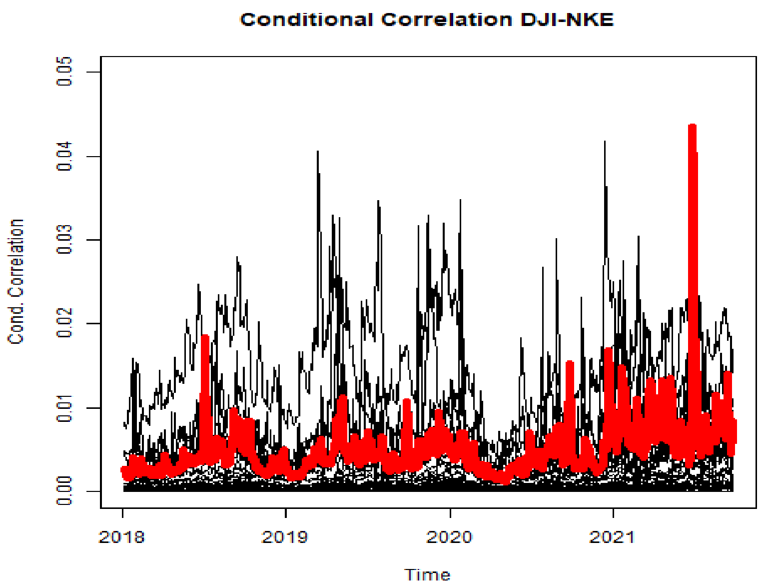


Figure B.21: Conditional Correlation DJI-NKE

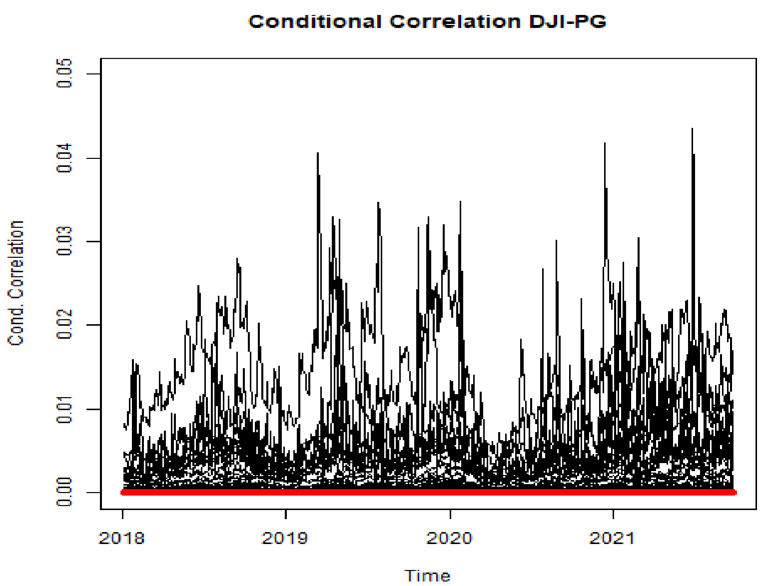


Figure B.22: Conditional Correlation DJI-PG

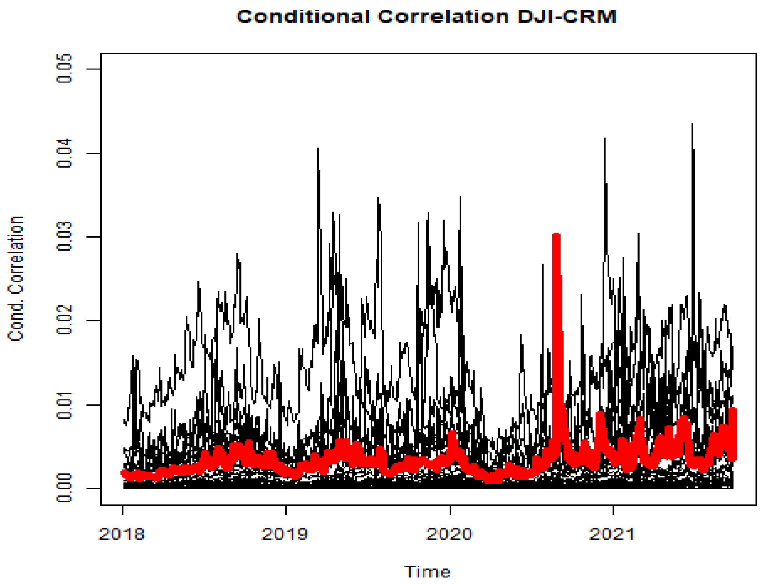


Figure B.23: Conditional Correlation DJI-CRM

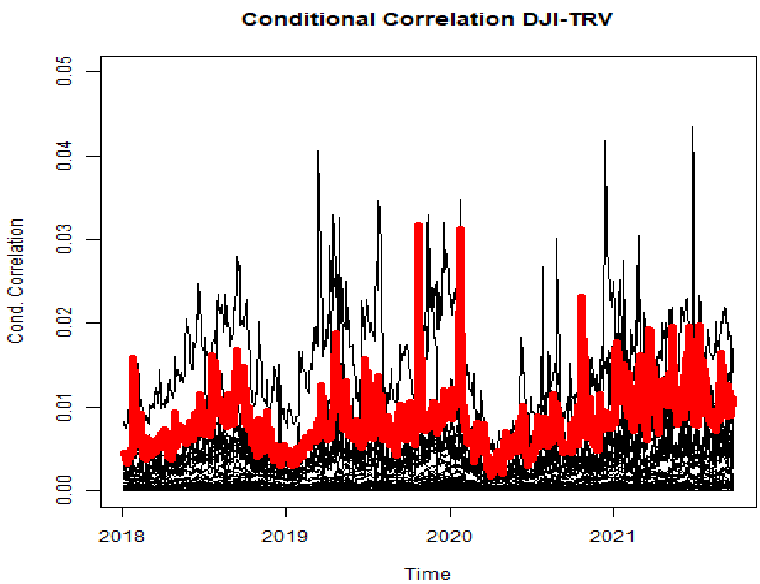


Figure B.24: Conditional Correlation DJI-TRV

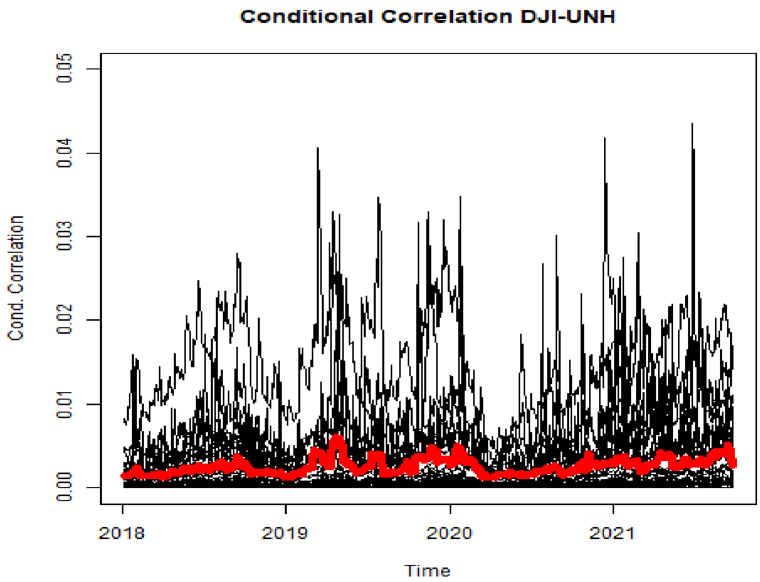


Figure B.25: Conditional Correlation DJI-UNH

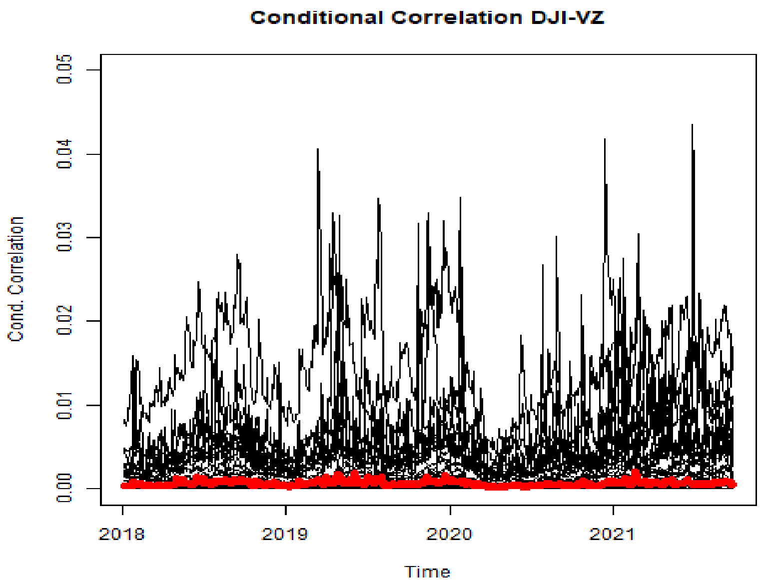


Figure B.26: Conditional Correlation DJI-VZ

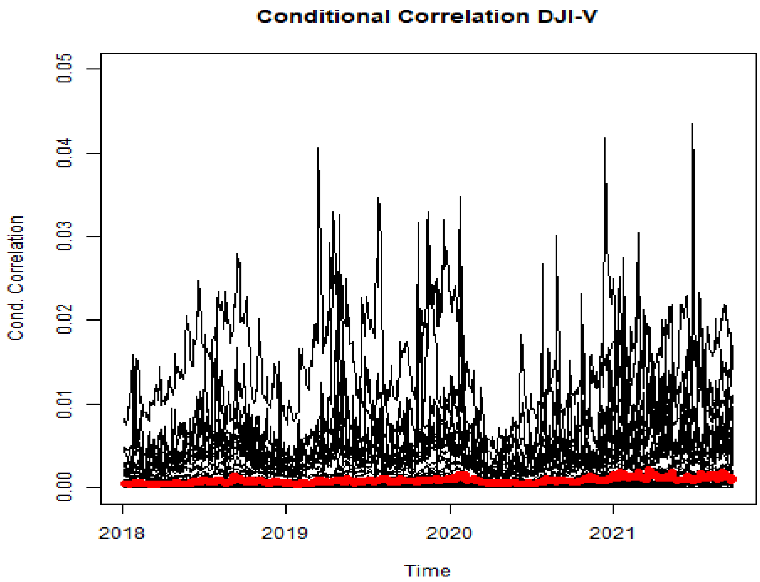


Figure B.27: Conditional Correlation DJI-V

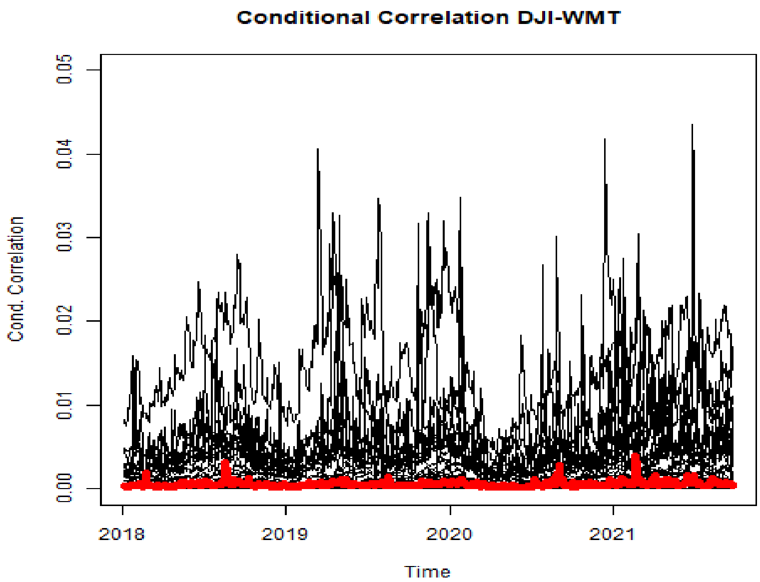


Figure B.28: Conditional Correlation DJI-WMT

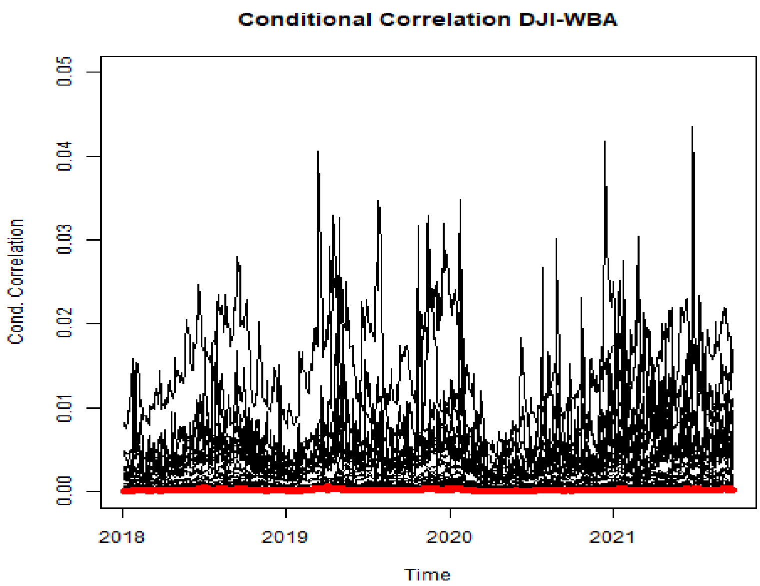


Figure B.29: Conditional Correlation DJI-WBA

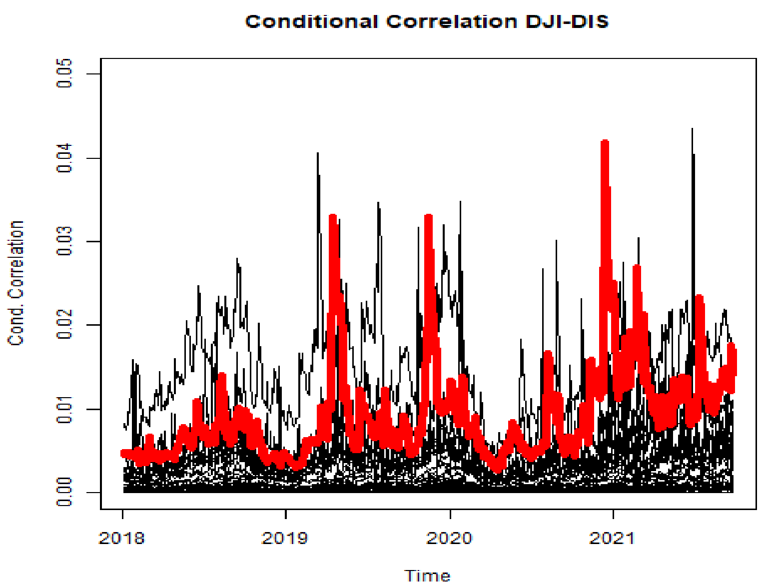


Figure B.30: Conditional Correlation DJI-DIS

## B.2 NDX

Table B.3: Parameter Estimates of the GARCH(1, 1) (NDX-Stocks)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| mu | 0.001 | 0.0004 | 2.587 | 0.010 |
| ar1 | −0.297 | 0.039 | −7.587 | 0 |
| AAPL | 0.00001 | 0.00001 | 0.481 | 0.631 |
| MSFT | −0.00000 | 0.00000 | −3.360 | 0.001 |
| AMZN | −0.00000 | 0.00000 | −5.798 | 0 |
| TSLA | 0.00003 | 0.00001 | 1.941 | 0.052 |
| GOOG | −0.00002 | 0.00001 | −1.481 | 0.138 |
| FB | −0.00003 | 0.00002 | −1.748 | 0.080 |
| GOOGL | 0.00004 | 0.00003 | 1.046 | 0.296 |
| NVDA | −0.00000 | 0.00000 | −1.825 | 0.068 |
| PYPL | −0.00003 | 0.0001 | −0.470 | 0.638 |
| ADBE | 0.00001 | 0.00001 | 1.184 | 0.236 |
| NFLX | 0.00001 | 0.00001 | 1.339 | 0.180 |
| CMCSA | −0 | 0.00000 | −0.012 | 0.991 |
| CSCO | −0.00001 | 0.00001 | −1.160 | 0.246 |
| INTC | −0.0001 | 0.0001 | −1.331 | 0.183 |
| PEP | 0.00001 | 0.00001 | 2.123 | 0.034 |
| AVGO | −0.00000 | 0.00001 | −0.624 | 0.533 |
| COST | −0.00001 | 0.00000 | −3.266 | 0.001 |
| TXN | −0.0001 | 0.00002 | −5.900 | 0 |
| TMUS | 0.00000 | 0.00000 | 0.857 | 0.392 |
| MRNA | −0.00001 | 0.00001 | −1.127 | 0.260 |
| INTU | 0.00000 | 0.00000 | 1.877 | 0.060 |
| HON | 0.00001 | 0.00002 | 0.401 | 0.688 |
| QCOM | −0.00005 | 0.00003 | −1.694 | 0.090 |
| CHTR | −0.00000 | 0.00000 | −0.851 | 0.395 |
| SBUX | −0.00000 | 0.00001 | −0.317 | 0.751 |
| AMD | −0.00001 | 0.00001 | −1.078 | 0.281 |
| AMGN | 0.00001 | 0.00001 | 1.001 | 0.317 |
| ISRG | −0.00001 | 0.00000 | −2.427 | 0.015 |
| AMAT | −0.00000 | 0.00000 | −0.558 | 0.577 |
| BKNG | 0.00001 | 0.00002 | 0.593 | 0.553 |
| ADI | −0.00000 | 0.00002 | −0.222 | 0.824 |
| GILD | 0.00000 | 0.00001 | 0.658 | 0.510 |
| ADP | 0.00002 | 0.00001 | 1.548 | 0.122 |
| MELI | 0.00000 | 0.00002 | 0.012 | 0.990 |
| MDLZ | −0.00001 | 0.00003 | −0.368 | 0.713 |
| LRCX | 0.00001 | 0.00001 | 1.334 | 0.182 |
| MU | 0.00000 | 0.00004 | 0.133 | 0.894 |
| FISV | −0.00001 | 0.00001 | −0.805 | 0.421 |
| CSX | 0.00000 | 0.00000 | 1.084 | 0.278 |
| REGN | 0.00005 | 0.00001 | 3.382 | 0.001 |
| ILMN | 0.00000 | 0.00000 | 0.086 | 0.932 |
| ZM | −0.00000 | 0.00001 | −0.294 | 0.769 |
| ADSK | 0.0001 | 0.00004 | 1.435 | 0.151 |
| ASML | −0.0001 | 0.00002 | −2.535 | 0.011 |
| ATVI | 0.00001 | 0.00001 | 1.015 | 0.310 |
| JD | 0.0001 | 0.00003 | 3.756 | 0.0002 |
| TEAM | 0.00003 | 0.00003 | 1.181 | 0.238 |

Table B.4: Parameter Estimates of the GARCH(1, 1) (NDX-Stocks) (cont. 1)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| IDXX | −0.00001 | 0.00001 | −1.003 | 0.316 |
| DXCM | −0.00000 | 0.00000 | −0.043 | 0.966 |
| ALGN | 0.00001 | 0.00000 | 1.487 | 0.137 |
| NXPI | 0.00000 | 0.00000 | 0.254 | 0.799 |
| KLAC | 0.00000 | 0.00000 | 0.153 | 0.878 |
| LULU | −0.00000 | 0.00000 | −0.031 | 0.975 |
| DOCU | −0.00001 | 0.00001 | −1.342 | 0.180 |
| MRVL | −0.00002 | 0.00003 | −0.677 | 0.498 |
| CRWD | −0 | 0.00000 | −0.0001 | 1.000 |
| KDP | −0.00001 | 0.00001 | −1.700 | 0.089 |
| MAR | −0.00003 | 0.00002 | −1.296 | 0.195 |
| WDAY | 0.0001 | 0.00004 | 1.566 | 0.117 |
| EXC | −0.00000 | 0.00000 | −0.172 | 0.863 |
| VRTX | 0.0001 | 0.00003 | 3.210 | 0.001 |
| MNST | −0.00000 | 0.00001 | −0.369 | 0.712 |
| SNPS | 0.00000 | 0.00000 | 0.931 | 0.352 |
| EBAY | −0.00001 | 0.00002 | −0.713 | 0.476 |
| KHC | 0.00002 | 0.00001 | 1.858 | 0.063 |
| MTCH | 0.00000 | 0.00002 | 0.031 | 0.975 |
| BIIB | 0.00000 | 0.00003 | 0.045 | 0.964 |
| ORLY | 0.00000 | 0.00000 | 1.340 | 0.180 |
| MCHP | 0.00002 | 0.00001 | 1.604 | 0.109 |
| CDNS | 0.00004 | 0.00001 | 4.894 | 0.00000 |
| WBA | −0.00000 | 0.00000 | −1.940 | 0.052 |
| AEP | 0.00000 | 0.00000 | 1.336 | 0.181 |
| EA | −0.00000 | 0.00001 | −0.300 | 0.764 |
| PAYX | 0.00003 | 0.00003 | 1.167 | 0.243 |
| CTAS | −0.00002 | 0.00001 | −1.624 | 0.104 |
| CTSH | −0.00002 | 0.00003 | −0.705 | 0.481 |
| ROST | −0.00000 | 0.00001 | −0.435 | 0.664 |
| BIDU | −0.00004 | 0.00002 | −2.071 | 0.038 |
| XLNX | 0.00000 | 0.00001 | 0.727 | 0.467 |
| PDD | 0.00001 | 0.00002 | 0.371 | 0.711 |
| KTA | −0.00000 | 0.00000 | −1.688 | 0.091 |
| XEL | 0.00001 | 0.00002 | 0.527 | 0.598 |
| CPRT | −0.00000 | 0.00002 | −0.084 | 0.933 |
| VRSK | 0.00000 | 0.00000 | 4.005 | 0.0001 |
| SGEN | 0.00000 | 0.00000 | 0.420 | 0.674 |
| ANSS | 0.00001 | 0.00002 | 0.930 | 0.352 |
| FAST | −0.0001 | 0.00003 | −1.952 | 0.051 |
| PCAR | 0.0001 | 0.00005 | 1.503 | 0.133 |
| SWKS | −0.00000 | 0.00001 | −0.353 | 0.724 |
| NTES | −0.00001 | 0.00002 | −0.343 | 0.732 |
| CDW | −0.00000 | 0.00002 | −0.157 | 0.875 |
| SIRI | 0.00001 | 0.00002 | 0.350 | 0.726 |
| SPLK | −0.00001 | 0.00002 | −0.552 | 0.581 |
| PTON | 0.00002 | 0.00003 | 0.520 | 0.603 |

Table B.5: Parameter Estimates of the GARCH(1, 1) (NDX-Stocks) (cont. 2)

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| VRSN | 0.00000 | 0.00000 | 0.132 | 0.895 |
| DLTR | 0.00002 | 0.00003 | 0.770 | 0.441 |
| CERN | 0.00000 | 0.00001 | 0.066 | 0.948 |
| TCOM | −0.00000 | 0.00000 | −0.401 | 0.688 |
| INCY | 0.00000 | 0.00001 | 0.592 | 0.554 |
| CHKP | 0.00002 | 0.00002 | 1.210 | 0.226 |
| FOXA | −0 | 0.00000 | −0.001 | 1.000 |
| FOX | −0.00000 | 0.00000 | −0.592 | 0.554 |
| omega | 0.00000 | 0.00000 | 1.423 | 0.155 |
| alpha1 | 0.050 | 0.002 | 30.110 | 0 |
| beta1 | 0.900 | 0.001 | 1,095.711 | 0 |

Table B.6: Significance of dummy variables during the COVID-19 market crash on Conditional Correlation and Variance Ratio, Returns of Price Residuals of Stocks, Stock Price Residuals, Stock Standard Deviation from GARCH models

| Stocks | Cond. Corr | VR | Returns on Residuals | Residuals | Stock std. |
|---|---|---|---|---|---|
| AAPL | 0.782 | 0.816 | 0.955 | 0.529 | 0.313 |
| MSFT | 0.845 | 0.600 | 0.995 | 0.637 | 0.129 |
| AMZN | 0.601 | 0.668 | 0.915 | 0.754 | 0.775 |
| TSLA | 0.935 | 0.831 | 0.722 | 0.991 | 0.912 |
| GOOG | 0.331 | 0.513 | 0.395 | 0.499 | 0.186 |
| FB | 0.224 | 0.360 | 0.781 | 0.515 | 0.608 |
| GOOGL | 0.036 | 0.207 | 0.747 | 0.721 | 0.978 |
| NVDA | 0.823 | 0.879 | 0.895 | 0.953 | 0.181 |
| PYPL | 0.486 | 0.875 | 0.348 | 0.761 | 0.907 |
| ADBE | 0.978 | 0.844 | 0.757 | 0.678 | 0.082 |
| NFLX | 0.419 | 0.505 | 0.506 | 0.790 | 0.593 |
| CMCSA | 0.992 | 0.999 | 0.853 | 0.517 | 0.001 |
| CSCO | 0.438 | 0.501 | 0.812 | 0.540 | 0.065 |
| INTC | 0.748 | 0.581 | 0.888 | 0.441 | 0.0000 |
| PEP | 0.032 | 0.097 | 0.863 | 0.646 | 0.013 |
| AVGO | 0.845 | 0.903 | 0.762 | 0.526 | 0.318 |
| COST | 0.343 | 0.848 | 0.789 | 0.694 | 0.076 |
| TXN | 0.713 | 0.734 | 0.660 | 0.459 | 0.300 |
| TMUS | 0.856 | 0.931 | 0.820 | 0.995 | 0.808 |
| MRNA | 0.092 | 0.082 | 0.604 | 0.899 | 0.016 |
| INTU | 0.830 | 0.802 | 0.871 | 0.687 | 0.289 |
| HON | 0.664 | 0.904 | 0.781 | 0.815 | 0.088 |
| QCOM | 0.602 | 0.608 | 0.343 | 0.650 | 0.367 |
| CHTR | 0.311 | 0.396 | 0.0000 | 0.681 | 0.513 |
| SBUX | 0.409 | 0.662 | 0.959 | 0.716 | 0.370 |
| AMD | 0.244 | 0.561 | 0.921 | 0.820 | 0.019 |
| AMGN | 0.409 | 0.534 | 0.991 | 0.678 | 0.731 |
| ISRG | 0.832 | 0.732 | 0.822 | 0.728 | 0.370 |
| AMAT | 0.710 | 0.963 | 0.780 | 0.647 | 0.291 |
| BKNG | 0.286 | 0.469 | 0.834 | 0.639 | 0.001 |
| ADI | 0.771 | 0.619 | 0.907 | 0.467 | 0.037 |
| GILD | 0.642 | 0.864 | 0.806 | 0.797 | 0.860 |
| ADP | 0.432 | 0.495 | 0.622 | 0.863 | 0.926 |
| MELI | 0.208 | 0.073 | 0.733 | 0.474 | 0.586 |
| MDLZ | 0.743 | 0.785 | 0.569 | 0.949 | 0.739 |
| LRCX | 0.302 | 0.715 | 0.542 | 0.626 | 0.0000 |
| MU | 0.950 | 0.887 | 0.678 | 0.396 | 0.041 |
| FISV | 0.633 | 0.461 | 0.767 | 0.825 | 0.003 |
| CSX | 0.746 | 0.802 | 0.895 | 0.641 | 0.085 |
| REGN | 0.531 | 0.476 | 0.863 | 0.457 | 0.004 |
| ILMN | 0.802 | 0.597 | 0.845 | 0.941 | 0.128 |
| ZM | 0.647 | 0.906 | 0.621 | 0.657 | 0.590 |
| ADSK | 0.880 | 0.690 | 0.947 | 0.833 | 0.468 |
| ASML | 0.173 | 0.867 | 0.933 | 0.596 | 0.0001 |
| ATVI | 0.768 | 0.522 | 0.897 | 0.952 | 0.198 |
| JD | 0.824 | 0.988 | 0.0001 | 0.788 | 0.955 |
| TEAM | 0.634 | 0.945 | 0.863 | 0.774 | 0.820 |

Table B.7: Significance of dummy variables during the COVID-19 market crash on Conditional Correlation and Variance Ratio, Returns of Price Residuals of Stocks, Stock Price Residuals, Stock Standard Deviation from GARCH models (cont. 1)

| Stocks | Cond. Corr | VR | Returns on Residuals | Residuals | Stock std. |
|---|---|---|---|---|---|
| IDXX | 0.582 | 0.539 | 0.886 | 0.333 | 0.301 |
| DXCM | 0.858 | 0.485 | 0.846 | 0.607 | 0.766 |
| ALGN | 0.873 | 0.930 | 0.845 | 0.879 | 0.680 |
| NXPI | 0.772 | 0.482 | 0.890 | 0.471 | 0.034 |
| KLAC | 0.758 | 0.588 | 0.848 | 0.656 | 0.231 |
| LULU | 0.972 | 0.736 | 0.886 | 0.761 | 0.703 |
| DOCU | 0.359 | 0.492 | 0.641 | 0.762 | 0.033 |
| MRVL | 0.865 | 0.719 | 0.687 | 0.617 | 0.769 |
| CRWD | 1.000 | 1.000 | 0.845 | 0.968 | 0.924 |
| KDP | 0.214 | 0.485 | 0.978 | 0.768 | 0.0001 |
| MAR | 0.046 | 0.143 | 0.762 | 0.570 | 0.0000 |
| WDAY | 0.548 | 0.975 | 0.857 | 0.678 | 0.252 |
| EXC | 0.633 | 0.453 | 0.901 | 0.563 | 0.009 |
| VRTX | 0.672 | 0.725 | 0.805 | 0.971 | 0.015 |
| MNST | 0.941 | 0.702 | 0.854 | 0.744 | 0.018 |
| SNPS | 0.796 | 0.664 | 0.785 | 0.654 | 0.207 |
| EBAY | 0.874 | 0.620 | 0.955 | 0.819 | 0.191 |
| KHC | 0.513 | 0.360 | 0.689 | 0.356 | 0.108 |
| MTCH | 0.830 | 0.753 | 0.580 | 0.573 | 0.288 |
| BIIB | 0.894 | 0.617 | 0.819 | 0.722 | 0.870 |
| ORLY | 0.717 | 0.836 | 0.917 | 0.782 | 0.046 |
| MCHP | 0.501 | 0.580 | 0.963 | 0.965 | 0.026 |
| CDNS | 0.604 | 0.625 | 0.852 | 0.579 | 0.163 |
| WBA | 0.661 | 0.367 | 0.850 | 0.724 | 0.002 |
| AEP | 0.196 | 0.155 | 0.848 | 0.424 | 0.0000 |
| EA | 0.867 | 0.842 | 0.778 | 0.645 | 0.416 |
| PAYX | 0.684 | 0.836 | 0.938 | 0.749 | 0.019 |
| CTAS | 0.245 | 0.455 | 0.984 | 0.492 | 0.573 |
| CTSH | 0.848 | 0.867 | 0.604 | 0.884 | 0.010 |
| ROST | 0.203 | 0.327 | 0.815 | 0.738 | 0.278 |
| BIDU | 0.163 | 0.534 | 0.529 | 0.528 | 0.949 |
| XLNX | 0.875 | 0.455 | 0.965 | 0.774 | 0.028 |
| PDD | 0.474 | 0.455 | 0.813 | 0.931 | 0.669 |
| OKTA | 0.502 | 0.494 | 0.946 | 0.953 | 0.617 |
| XEL | 0.754 | 0.619 | 0.837 | 0.714 | 0.852 |
| CPRT | 0.740 | 0.412 | 0.909 | 0.866 | 0.792 |
| VRSK | 0.556 | 0.643 | 0.792 | 0.965 | 0.781 |
| SGEN | 0.757 | 0.732 | 0.827 | 0.470 | 0.708 |
| ANSS | 0.240 | 0.453 | 0.901 | 0.637 | 0.010 |
| FAST | 0.702 | 0.646 | 0.831 | 0.799 | 0.745 |
| PCAR | 0.422 | 0.466 | 0.813 | 0.395 | 0.037 |
| SWKS | 0.577 | 0.425 | 0.895 | 0.473 | 0.798 |
| NTES | 0.422 | 0.406 | 0.910 | 0.450 | 0.514 |
| CDW | 0.873 | 0.835 | 0.701 | 0.942 | 0.843 |
| SIRI | 0.876 | 0.939 | 0.916 | 0.832 | 0.008 |
| SPLK | 0.445 | 0.567 | 0.843 | 0.713 | 0.001 |
| PTON | 0.269 | 0.912 | 0.996 | 0.814 | 0.286 |

Table B.8: Significance of dummy variables during the COVID-19 market crash on Conditional Correlation and Variance Ratio, Returns of Price Residuals of Stocks, Stock Price Residuals, Stock Standard Deviation from GARCH models (cont. 2)

| Stocks | Cond. Corr | VR | Returns on Residuals | Residuals | Stock std. |
|---|---|---|---|---|---|
| VRSN | 0.543 | 0.477 | 0.876 | 0.516 | 0.156 |
| DLTR | 0.600 | 0.575 | 0.878 | 0.620 | 0.224 |
| CERN | 0.307 | 0.517 | 0.916 | 0.845 | 0.001 |
| TCOM | 0.494 | 0.606 | 0.694 | 0.662 | 0.567 |
| INCY | 0.999 | 0.809 | 0.811 | 0.846 | 0.988 |
| CHKP | 0.463 | 0.551 | 0.965 | 0.449 | 0.976 |
| FOXA | 1.000 | 1.000 | 0.844 | 0.794 | 0.927 |
| FOX | 0.711 | 0.472 | 0.927 | 0.509 | 0.420 |

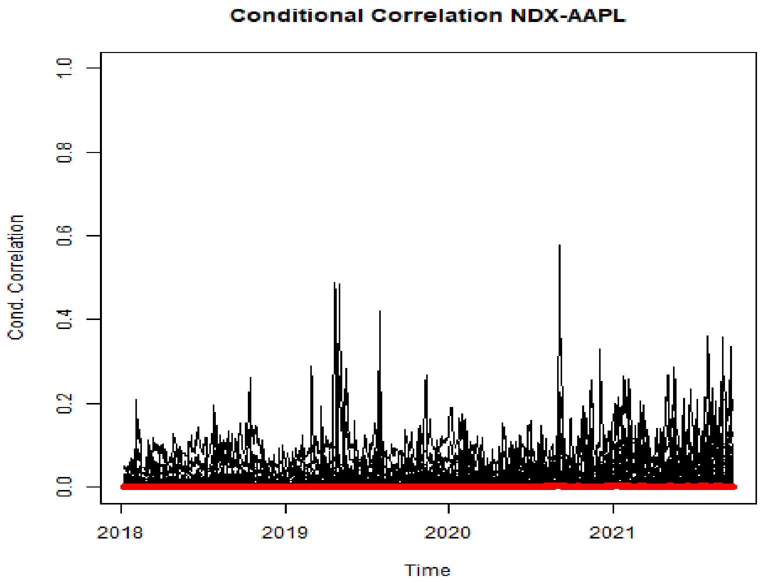


Figure B.31: Conditional Correlation NDX-AAPL

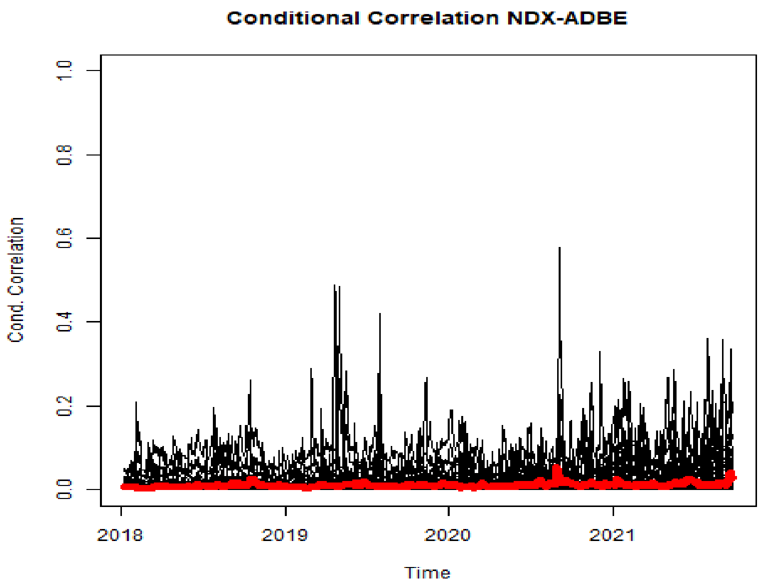


Figure B.32: Conditional Correlation NDX-ADBE

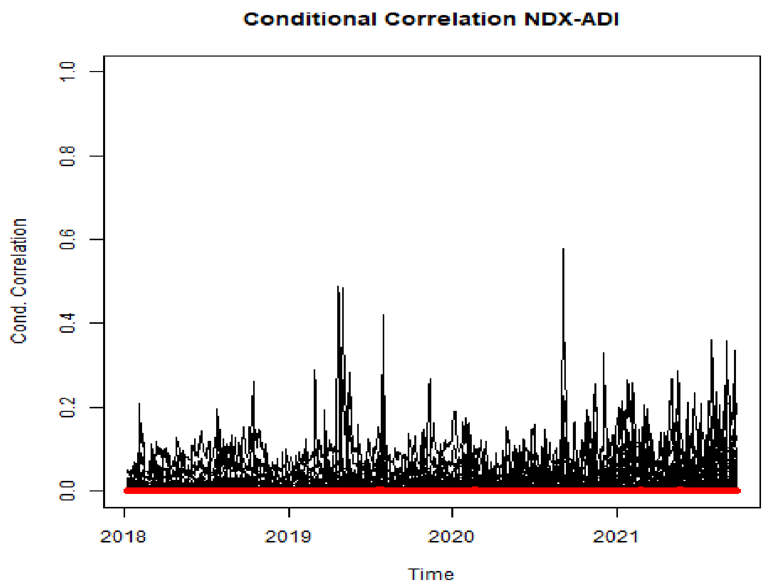


Figure B.33: Conditional Correlation NDX-ADI

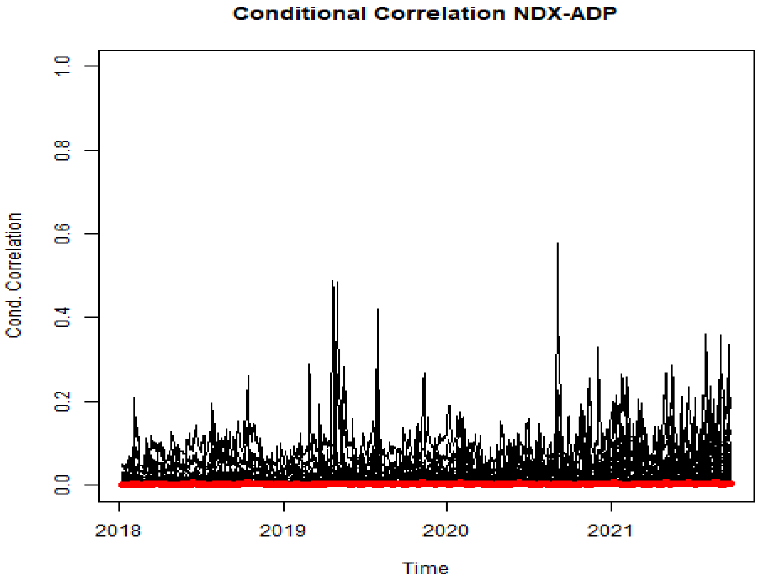


Figure B.34: Conditional Correlation NDX-ADP

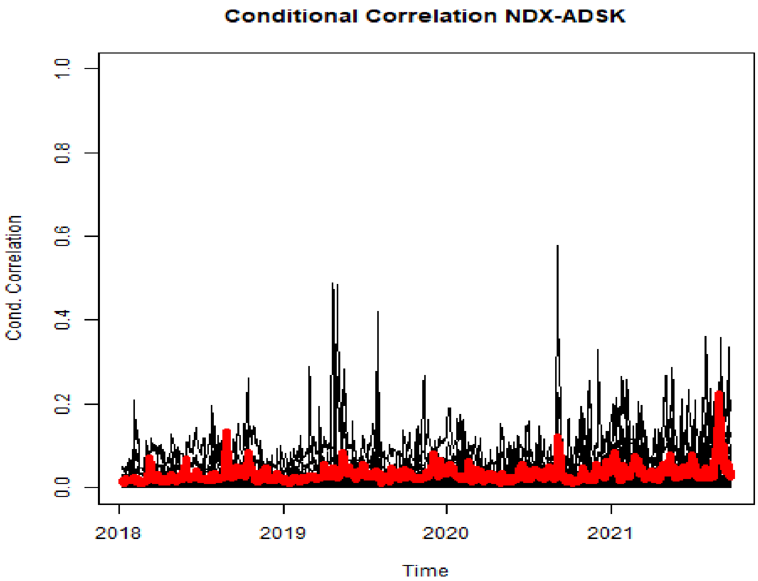


Figure B.35: Conditional Correlation NDX-ADSK

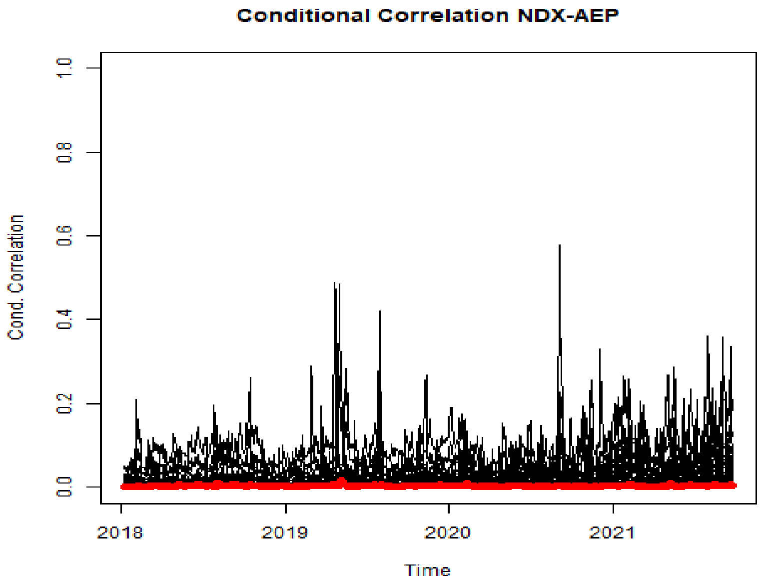


Figure B.36: Conditional Correlation NDX-AEP

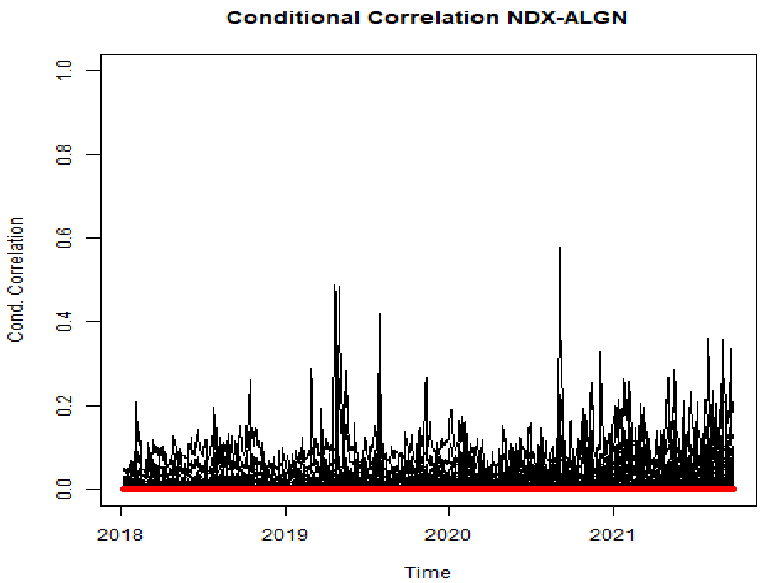


Figure B.37: Conditional Correlation NDX-ALGN

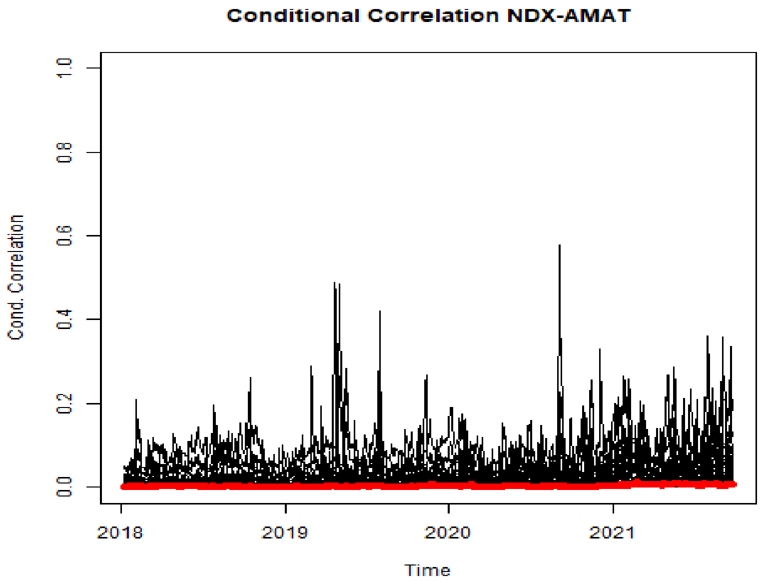


Figure B.38: Conditional Correlation NDX-AMAT

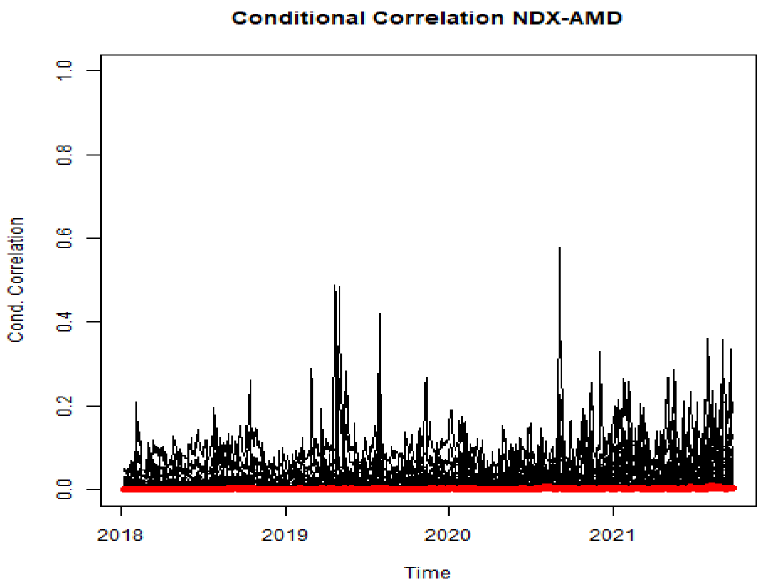


Figure B.39: Conditional Correlation NDX-AMD

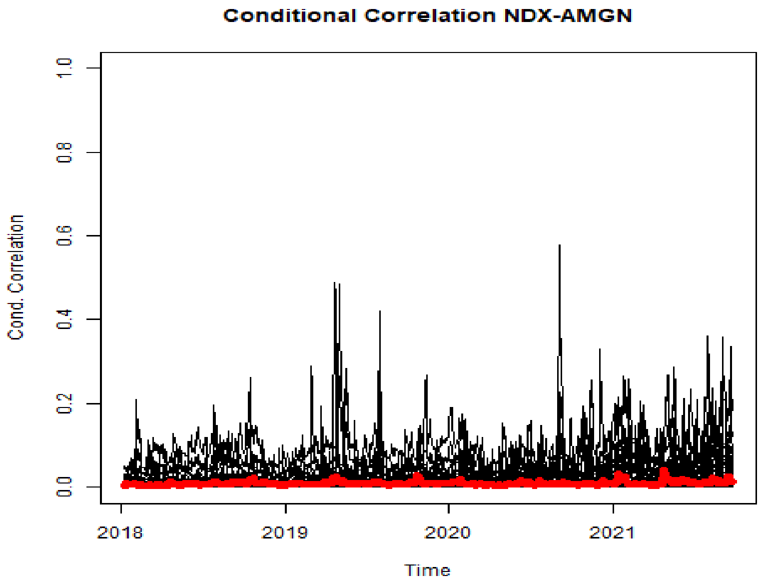


Figure B.40: Conditional Correlation NDX-AMGN

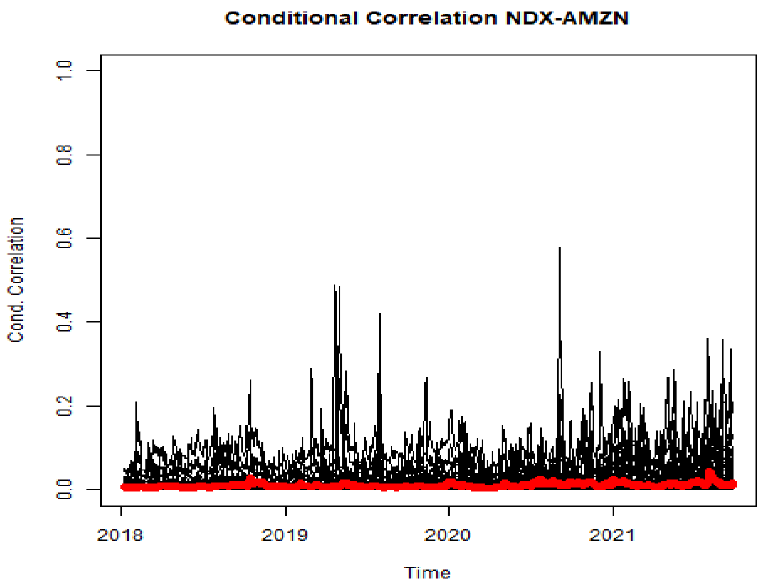


Figure B.41: Conditional Correlation NDX-AMZN

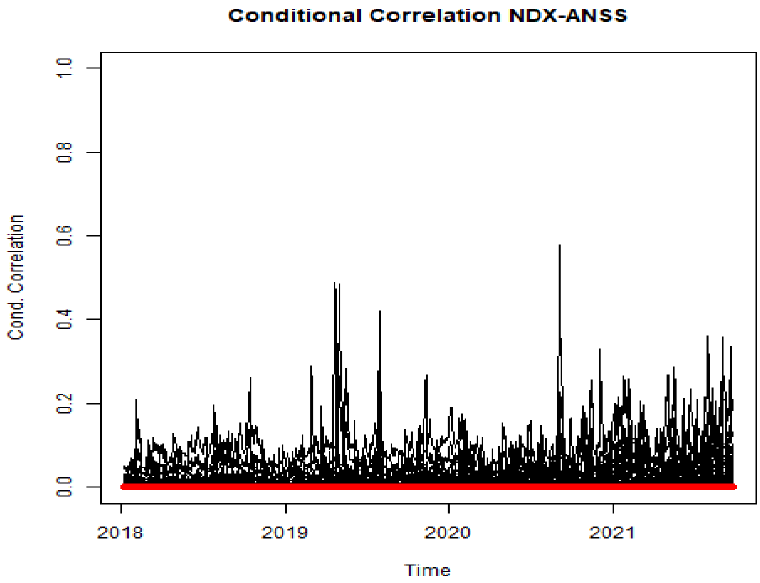


Figure B.42: Conditional Correlation NDX-ANSS

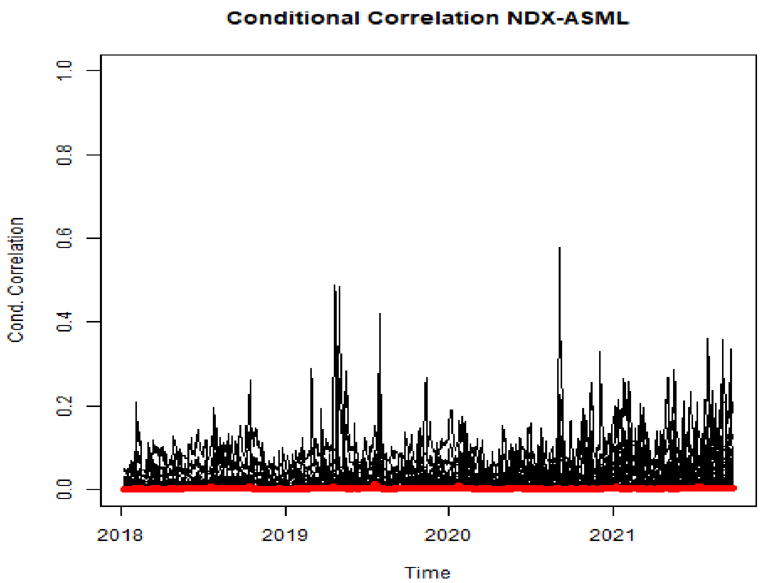


Figure B.43: Conditional Correlation NDX-ASML

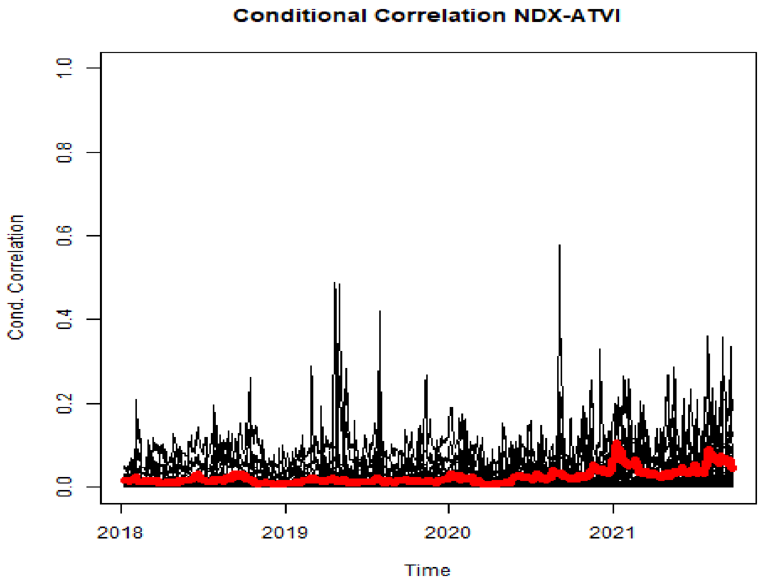


Figure B.44: Conditional Correlation NDX-ATVI

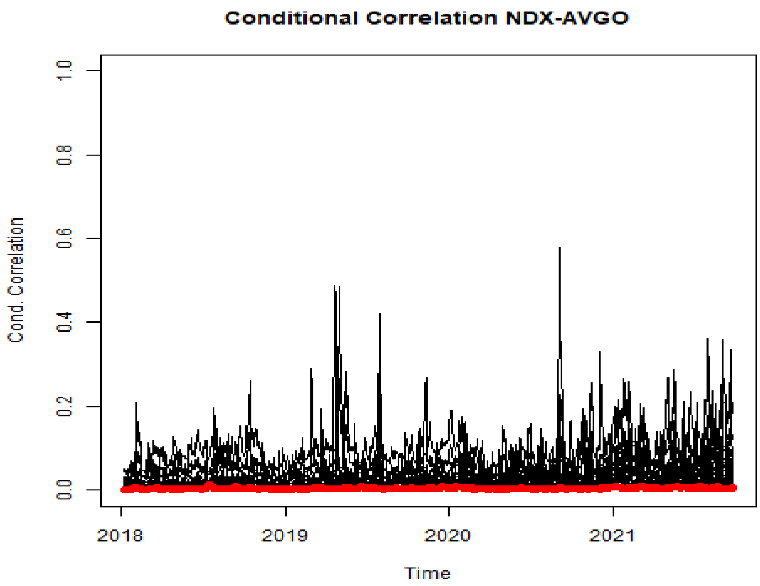


Figure B.45: Conditional Correlation NDX-AVGO

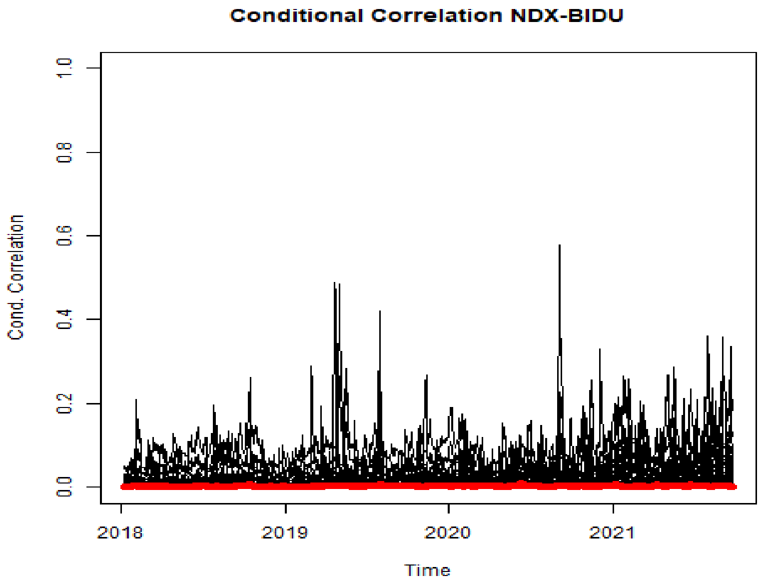


Figure B.46: Conditional Correlation NDX-BIDU

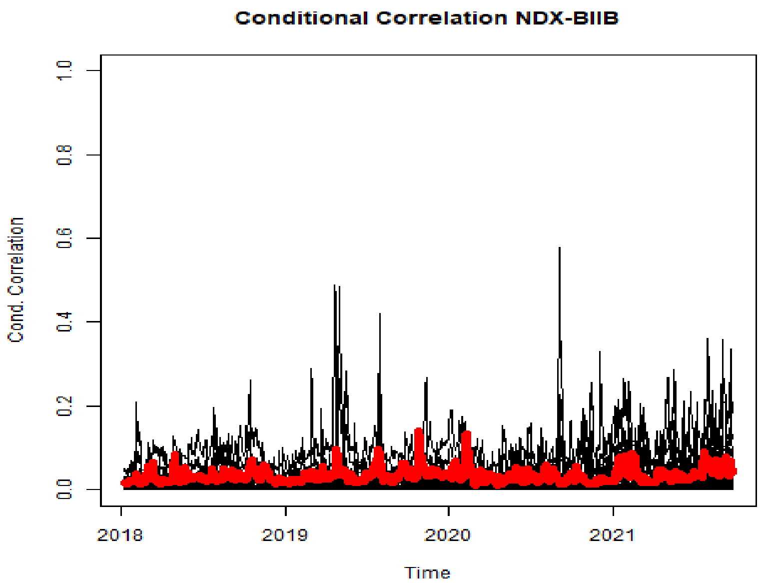


Figure B.47: Conditional Correlation NDX-BIIB

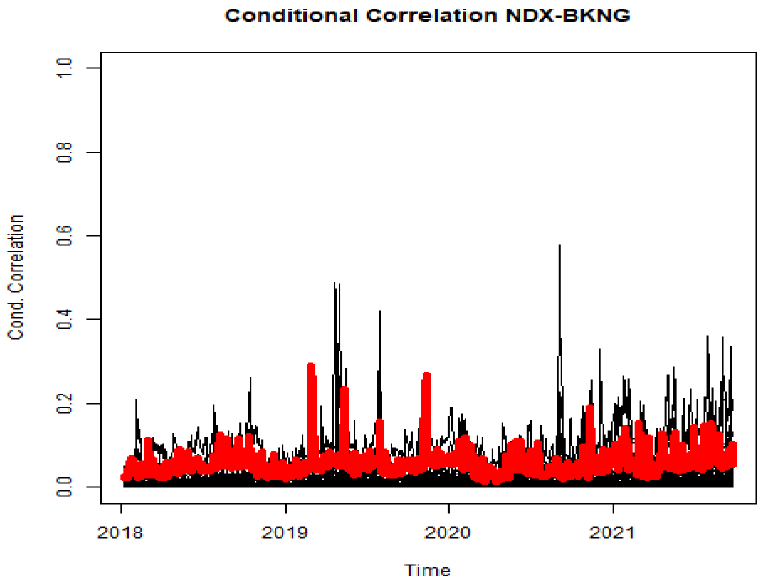


Figure B.48: Conditional Correlation NDX-BKNG

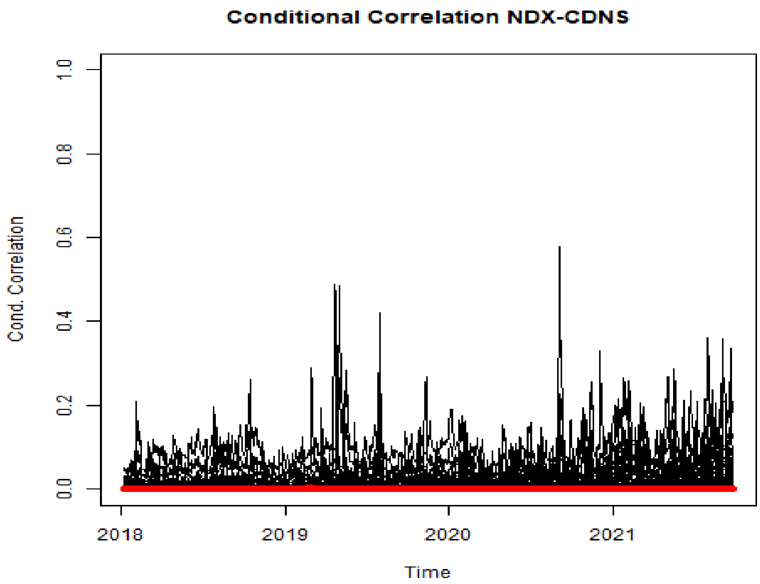


Figure B.49: Conditional Correlation NDX-CDNS

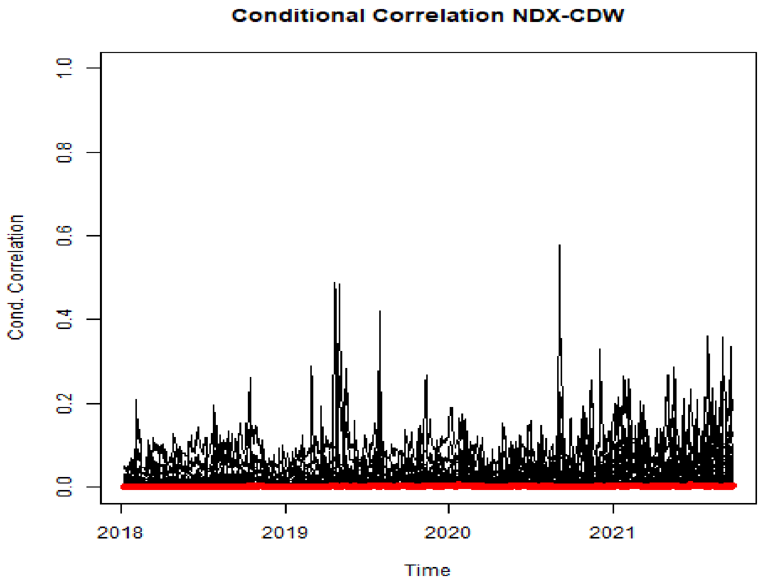


Figure B.50: Conditional Correlation NDX-CDW

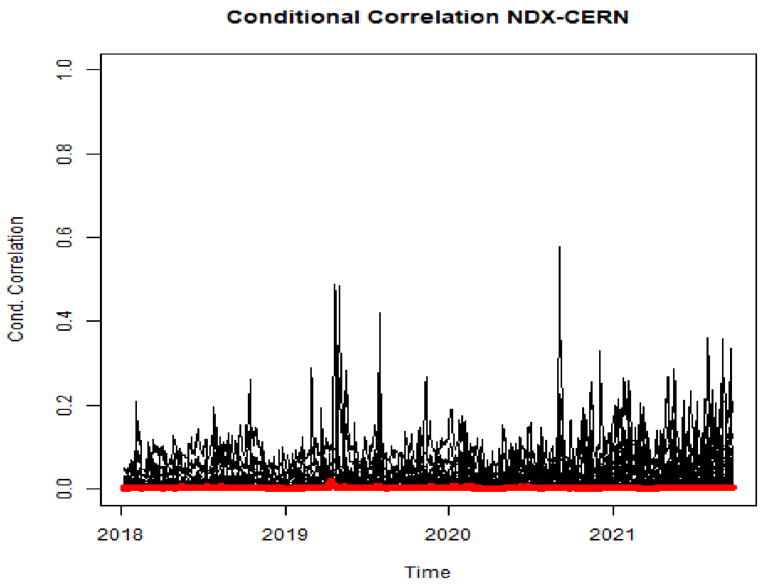


Figure B.51: Conditional Correlation NDX-CERN

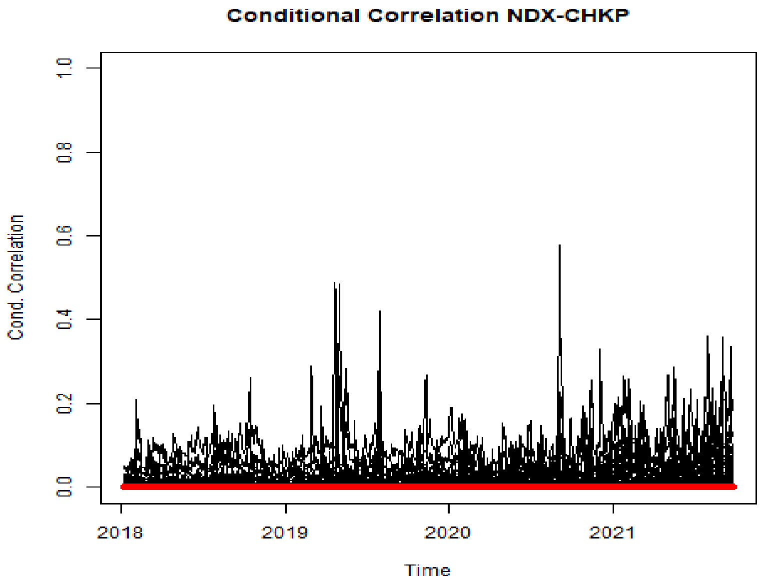


Figure B.52: Conditional Correlation NDX-CHKP

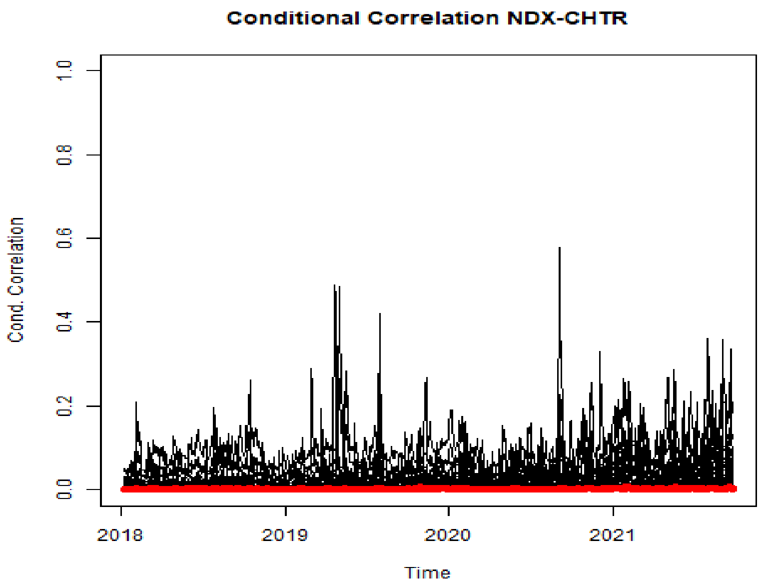


Figure B.53: Conditional Correlation NDX-CHTR

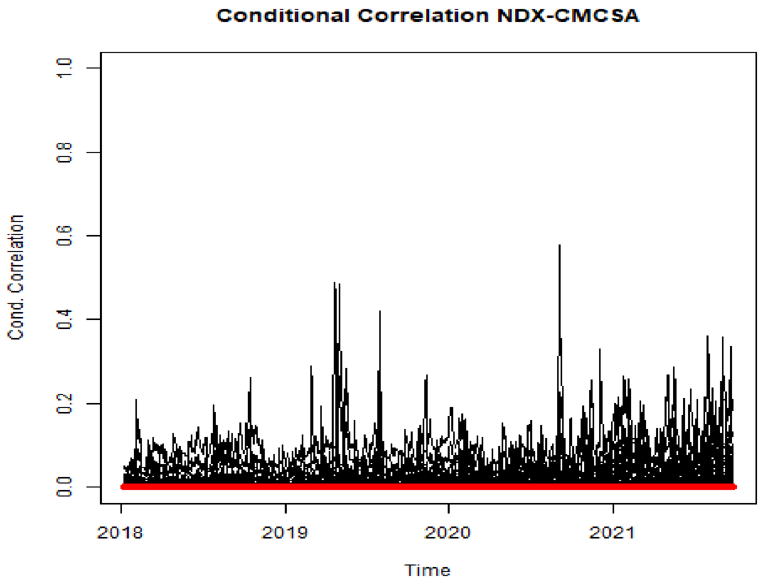


Figure B.54: Conditional Correlation NDX-CMCSA

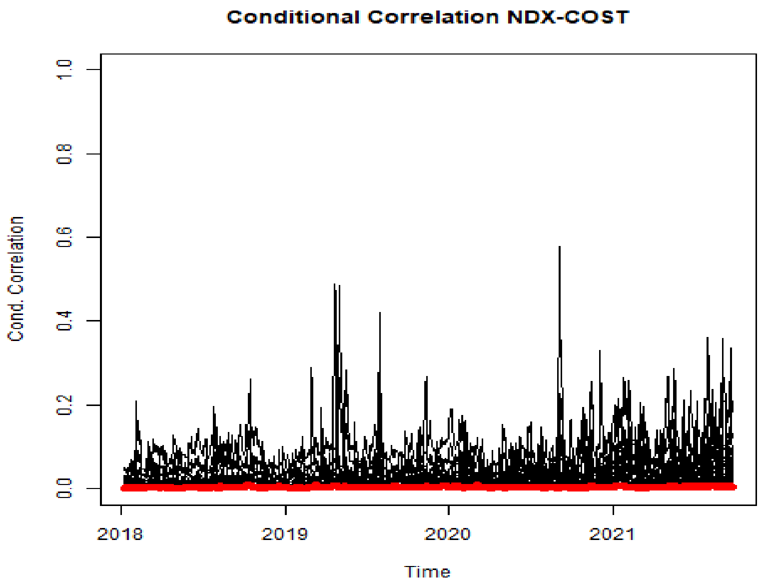


Figure B.55: Conditional Correlation NDX-COST

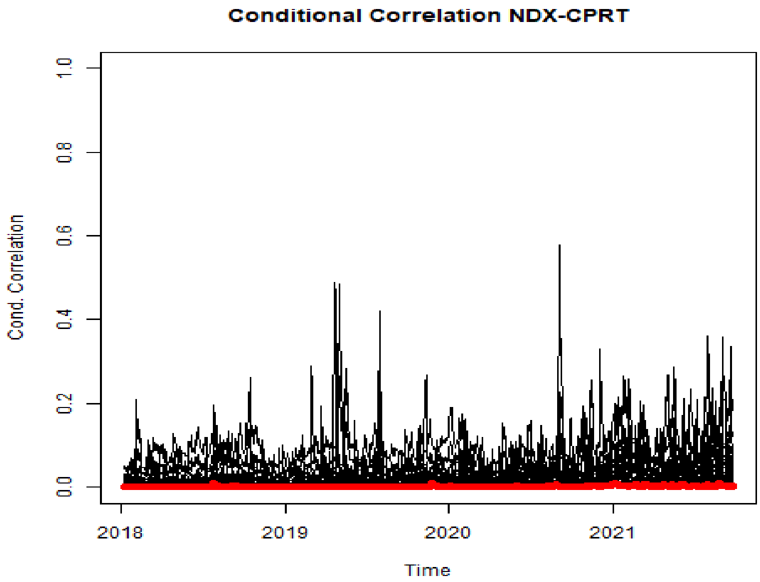


Figure B.56: Conditional Correlation NDX-CPRT

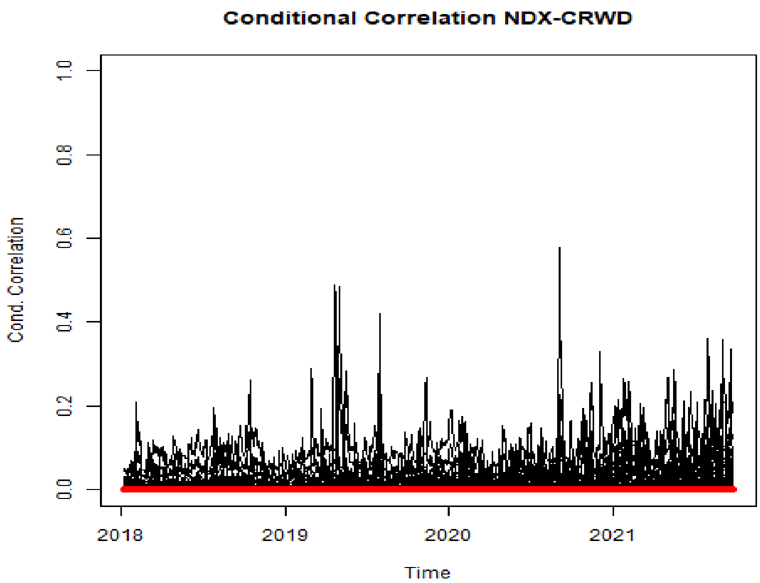


Figure B.57: Conditional Correlation NDX-CRWD

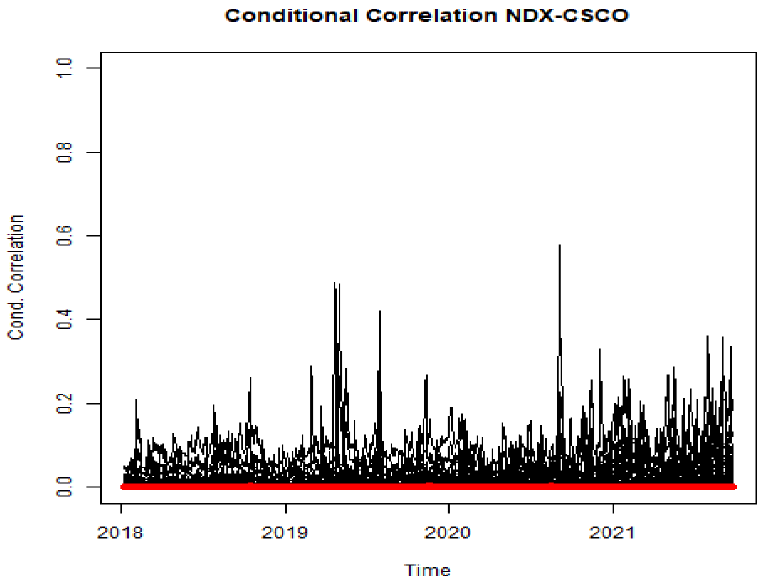


Figure B.58: Conditional Correlation NDX-CSCO

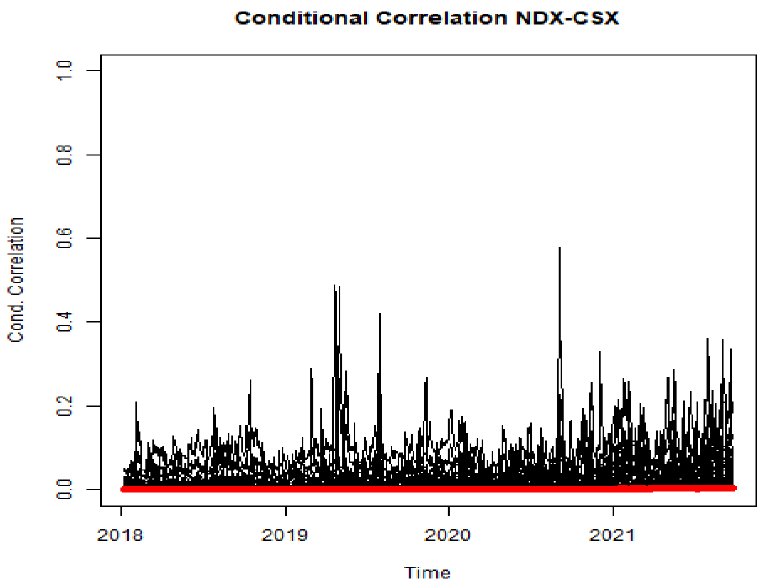


Figure B.59: Conditional Correlation NDX-CSX

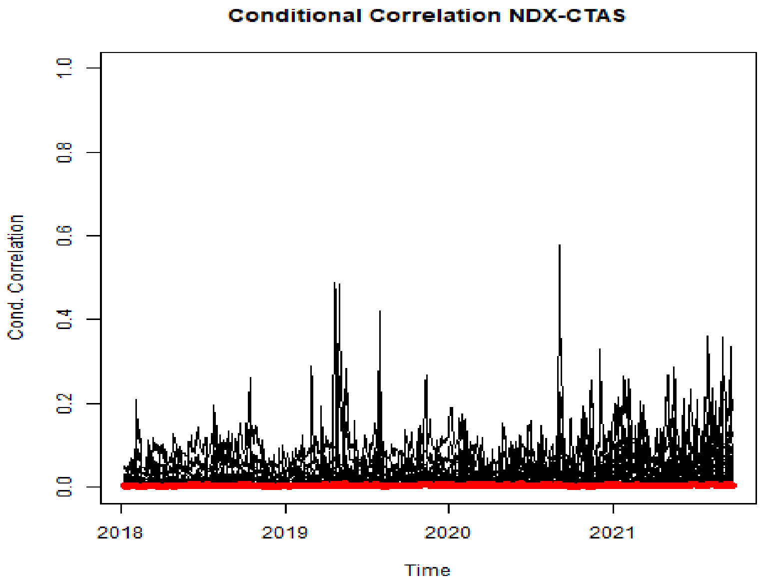


Figure B.60: Conditional Correlation NDX-CTAS

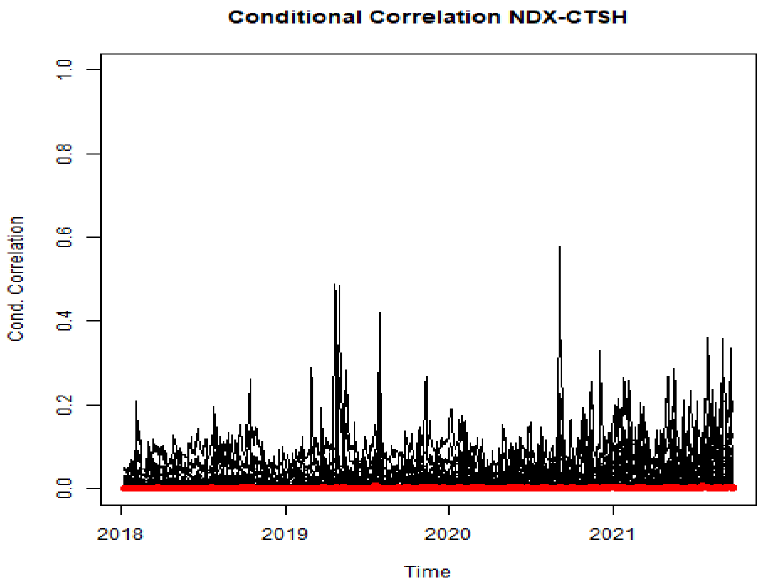


Figure B.61: Conditional Correlation NDX-CTSH

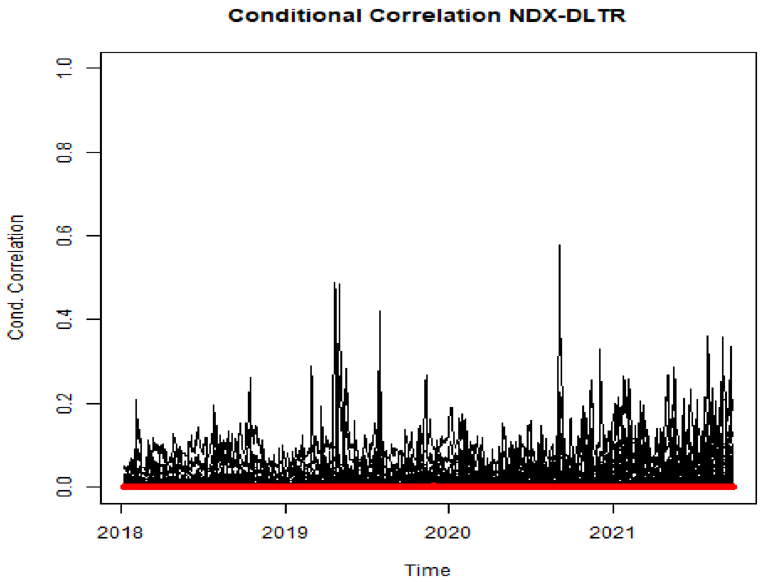


Figure B.62: Conditional Correlation NDX-DLTR

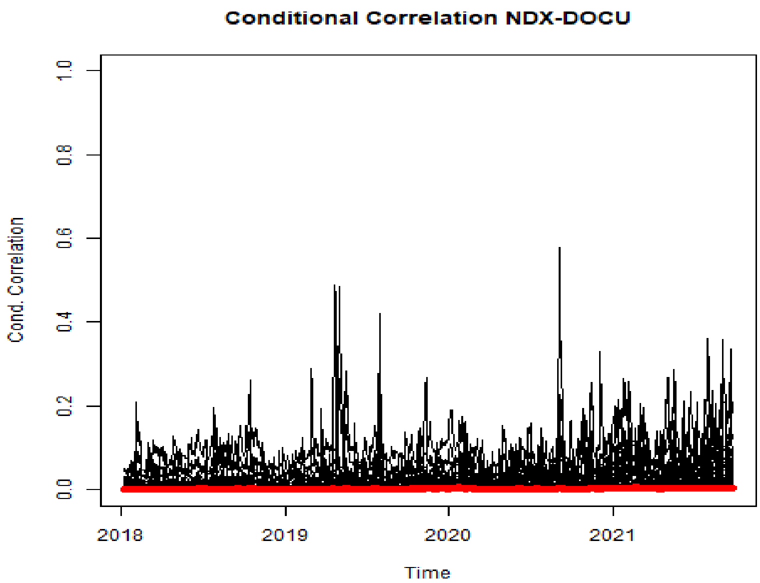


Figure B.63: Conditional Correlation NDX-DOCU

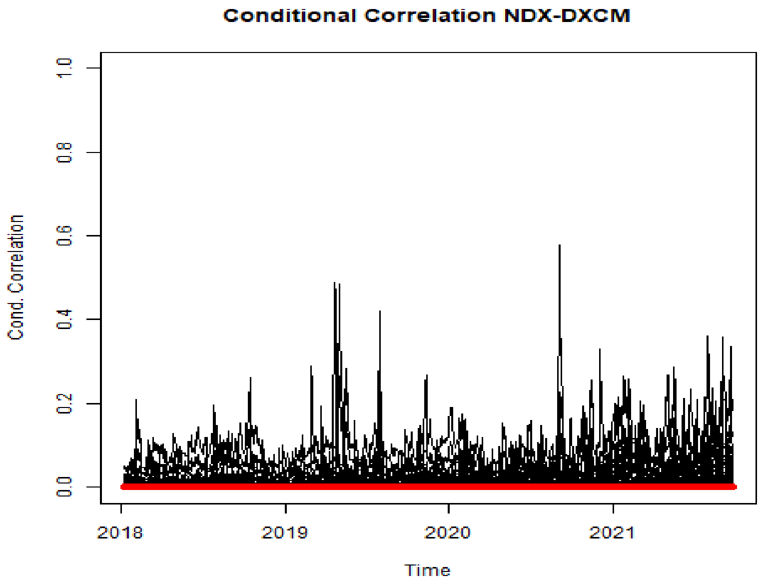


Figure B.64: Conditional Correlation NDX-DXCM

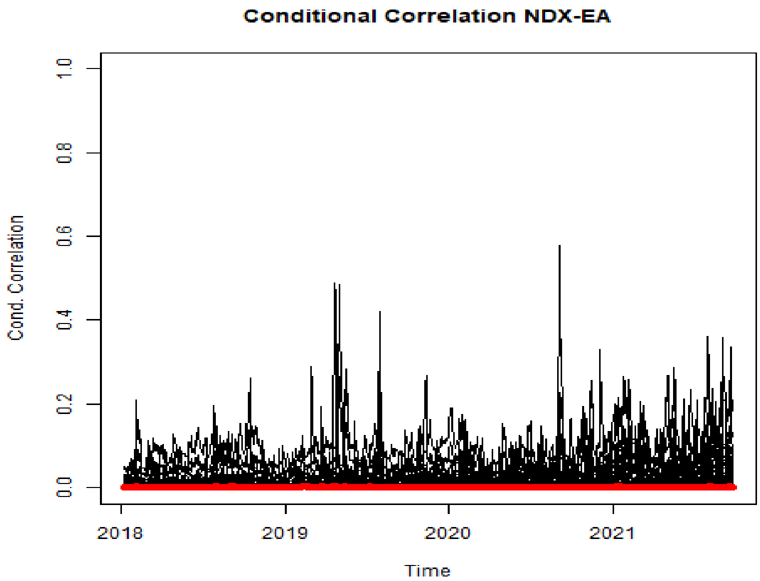


Figure B.65: Conditional Correlation NDX-EA

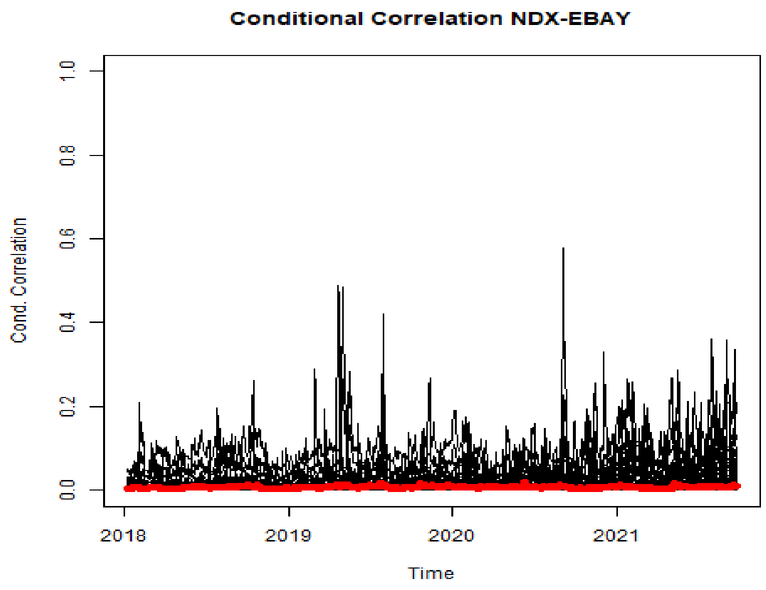


Figure B.66: Conditional Correlation NDX-EBAY

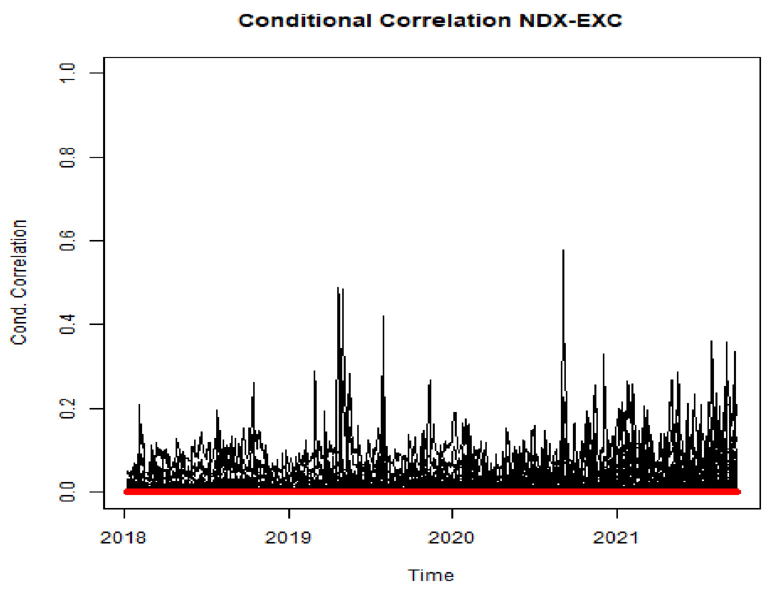


Figure B.67: Conditional Correlation NDX-EXC

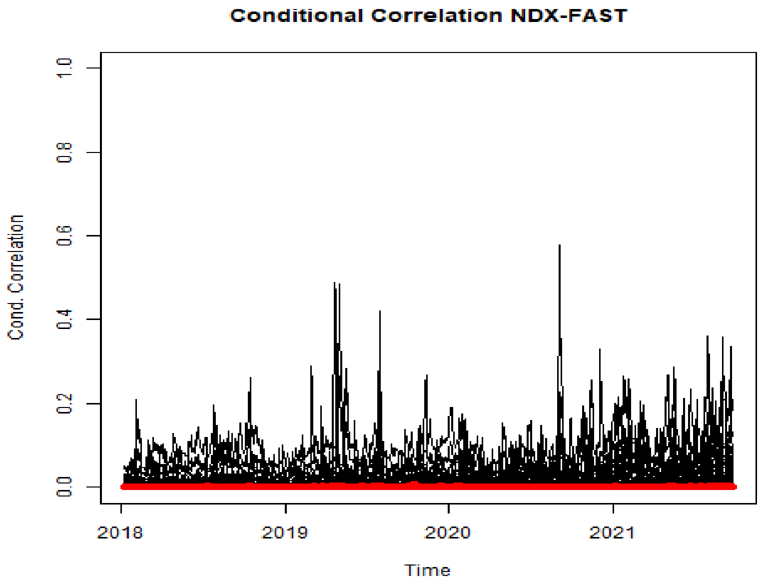


Figure B.68: Conditional Correlation NDX-FAST

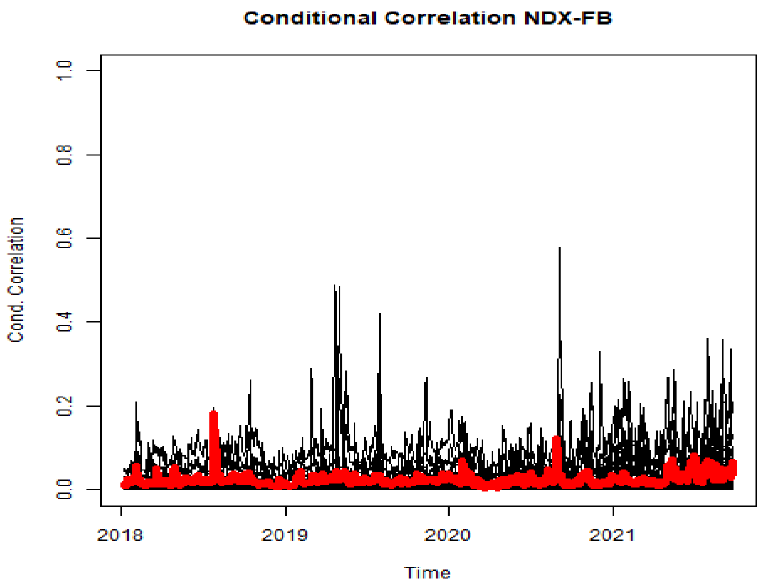


Figure B.69: Conditional Correlation NDX-FB

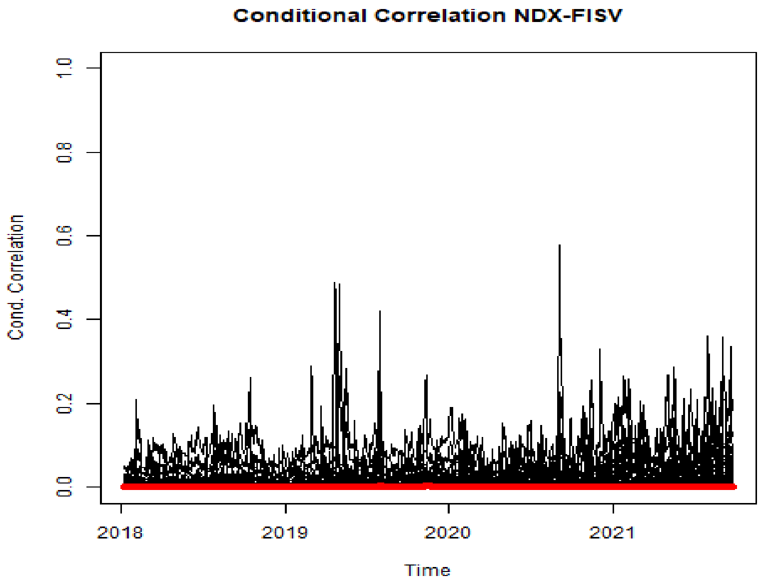


Figure B.70: Conditional Correlation NDX-FISV

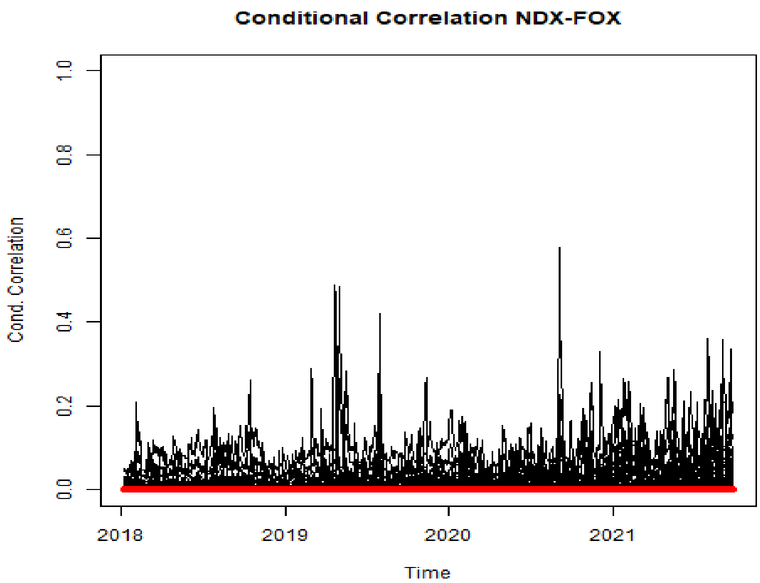


Figure B.71: Conditional Correlation NDX-FOX

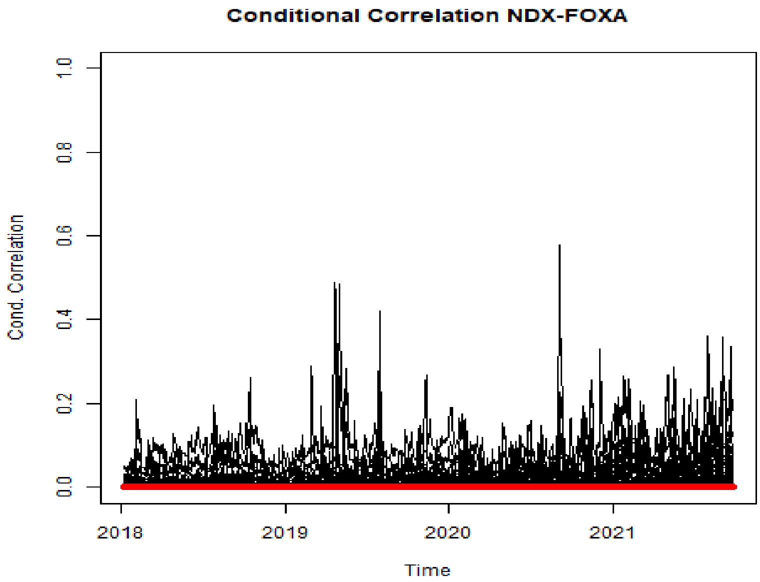


Figure B.72: Conditional Correlation NDX-FOXA

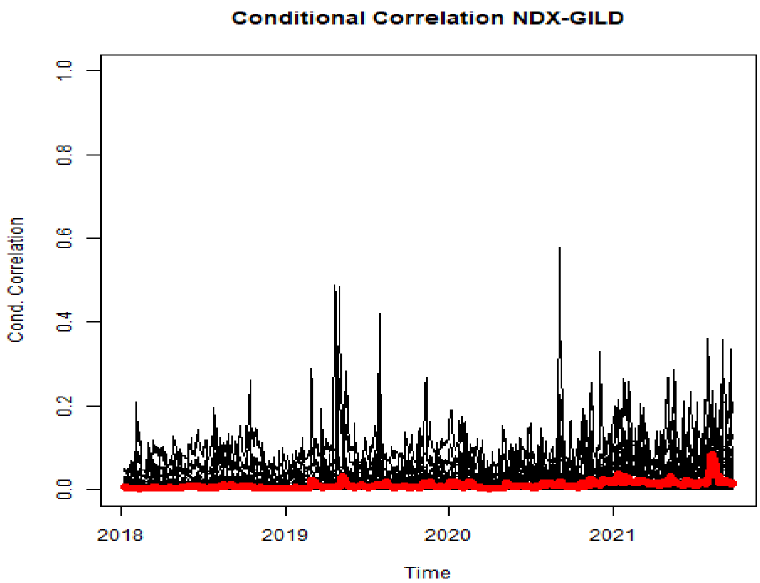


Figure B.73: Conditional Correlation NDX-GILD

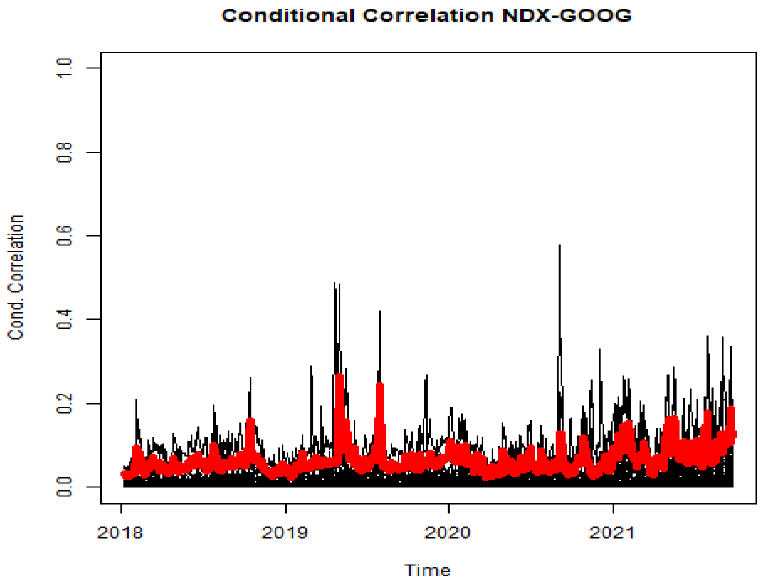


Figure B.74: Conditional Correlation NDX-GOOG

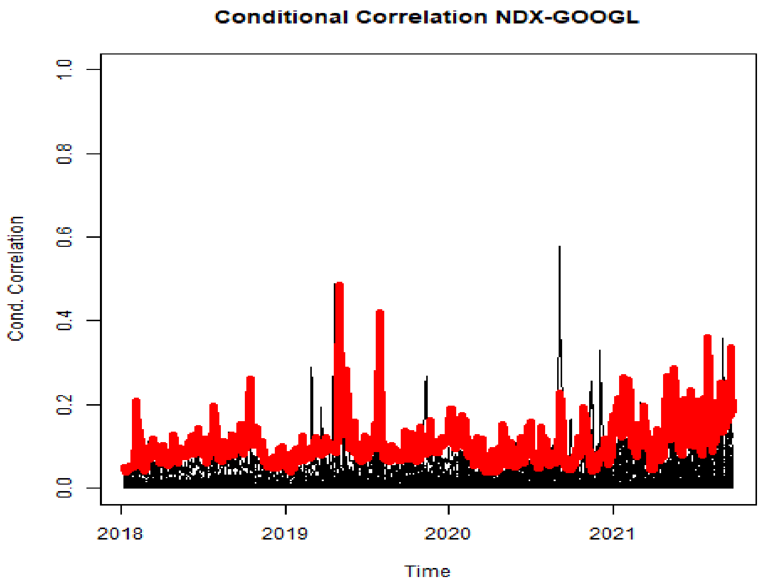


Figure B.75: Conditional Correlation NDX-GOOGL

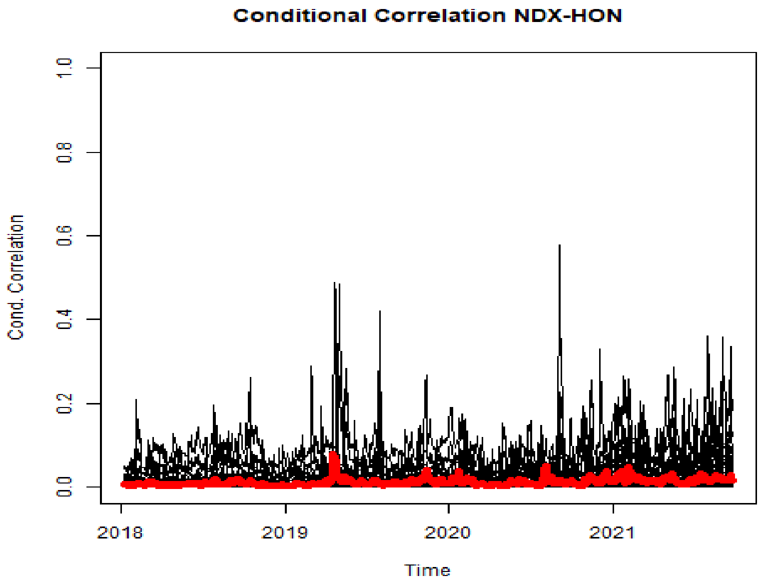


Figure B.76: Conditional Correlation NDX-HON

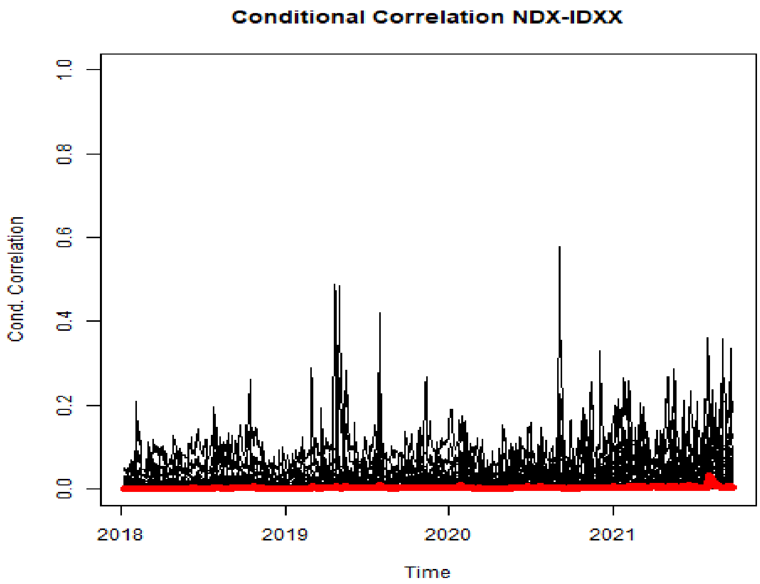


Figure B.77: Conditional Correlation NDX-IDXX

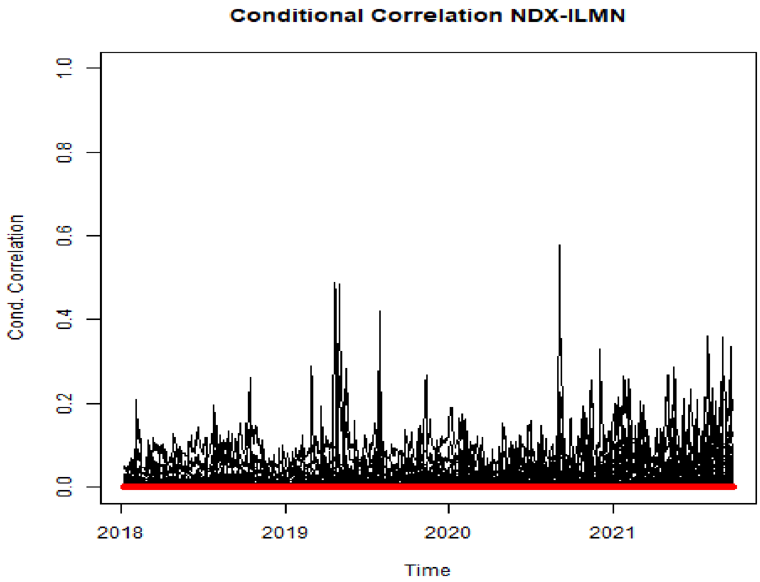


Figure B.78: Conditional Correlation NDX-ILMN

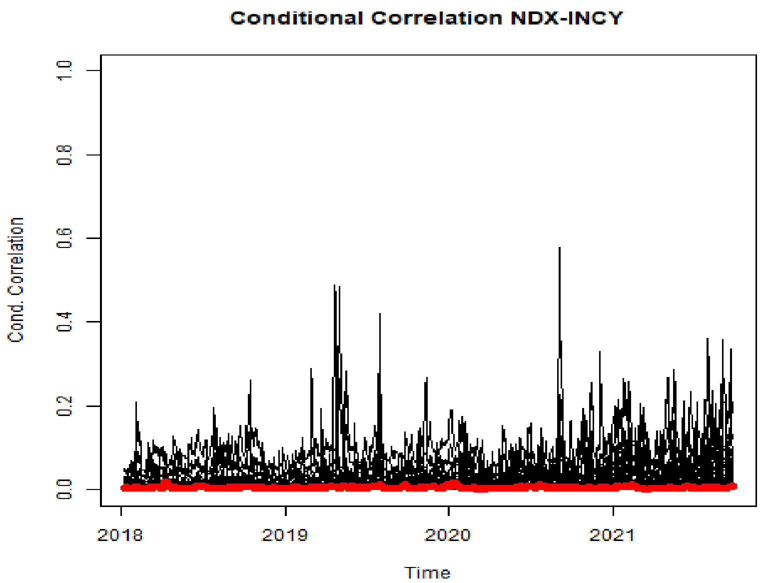


Figure B.79: Conditional Correlation NDX-INCY

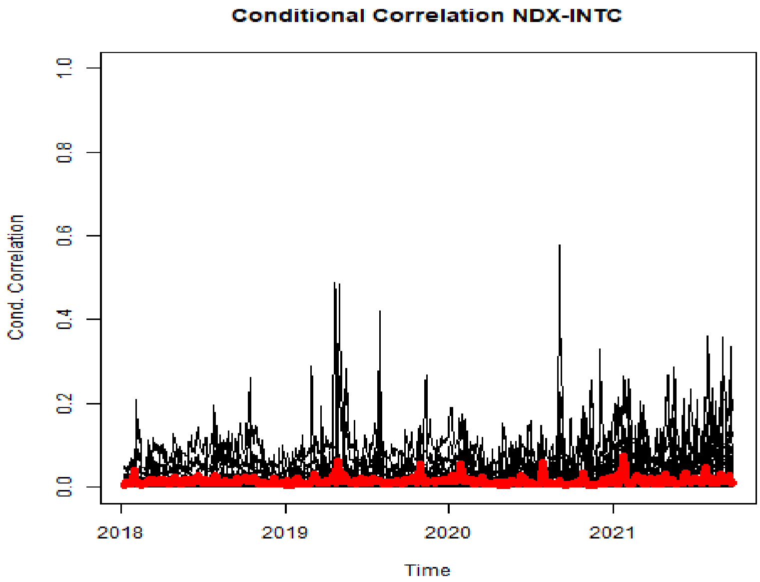


Figure B.80: Conditional Correlation NDX-INTC

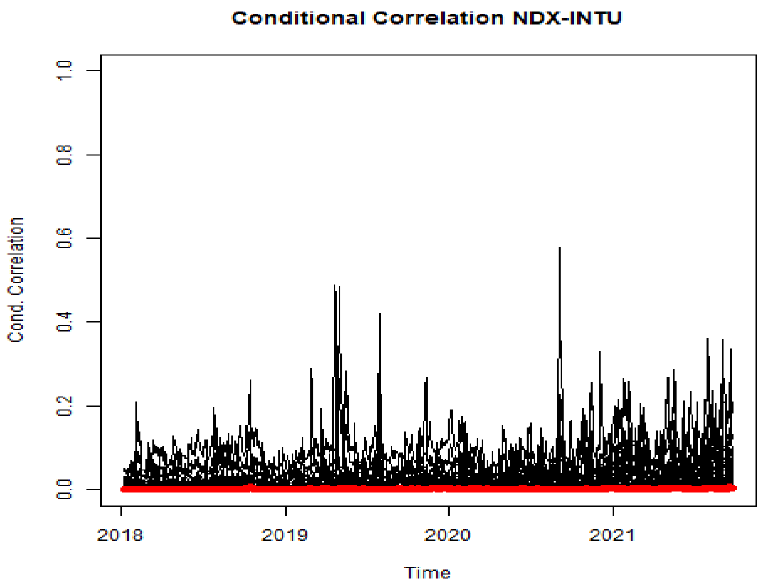


Figure B.81: Conditional Correlation NDX-INTU

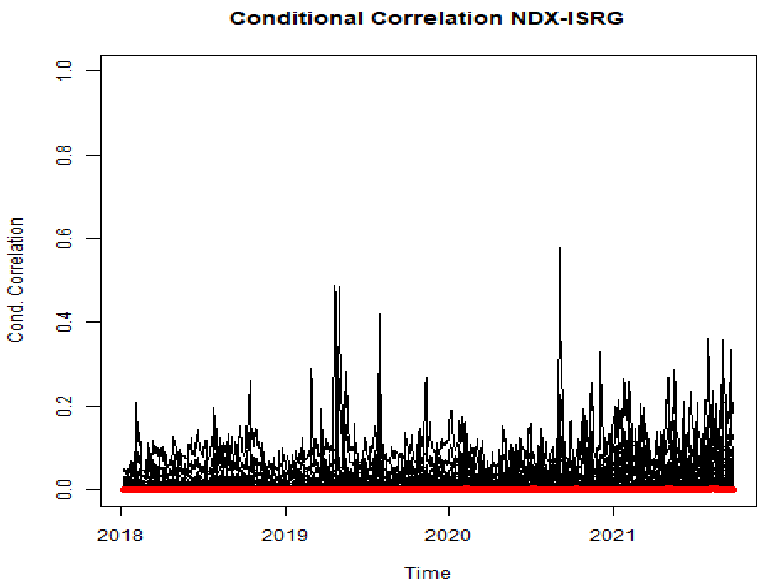


Figure B.82: Conditional Correlation NDX-ISRG

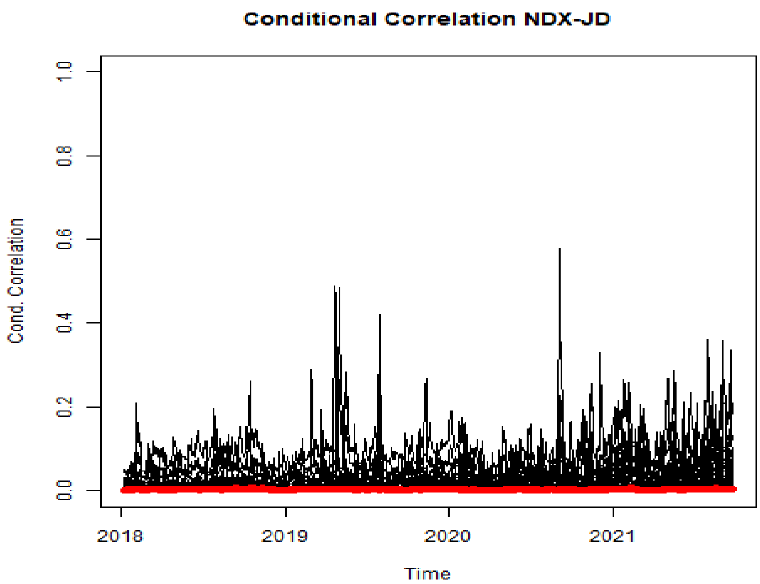


Figure B.83: Conditional Correlation NDX-JD

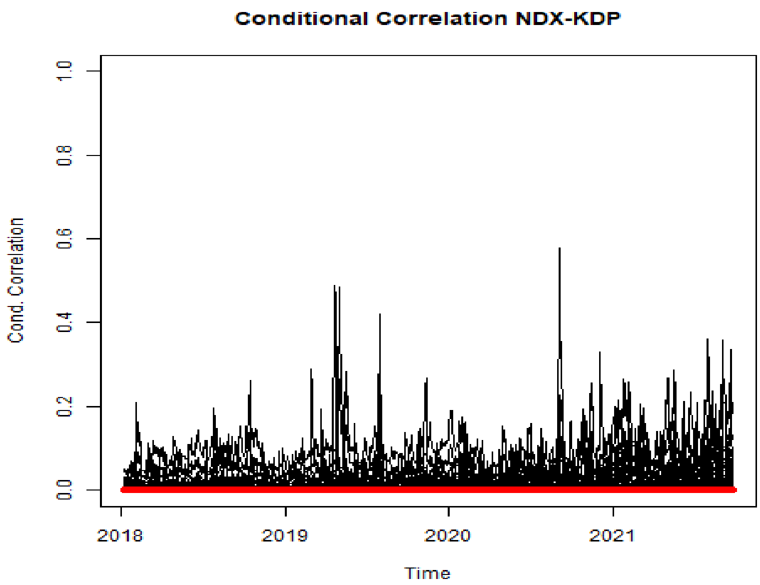


Figure B.84: Conditional Correlation NDX-KDP

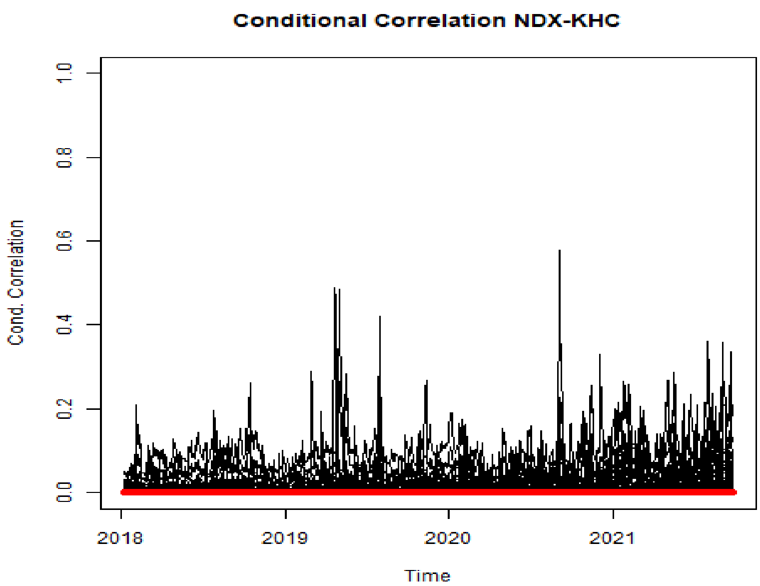


Figure B.85: Conditional Correlation NDX-KHC

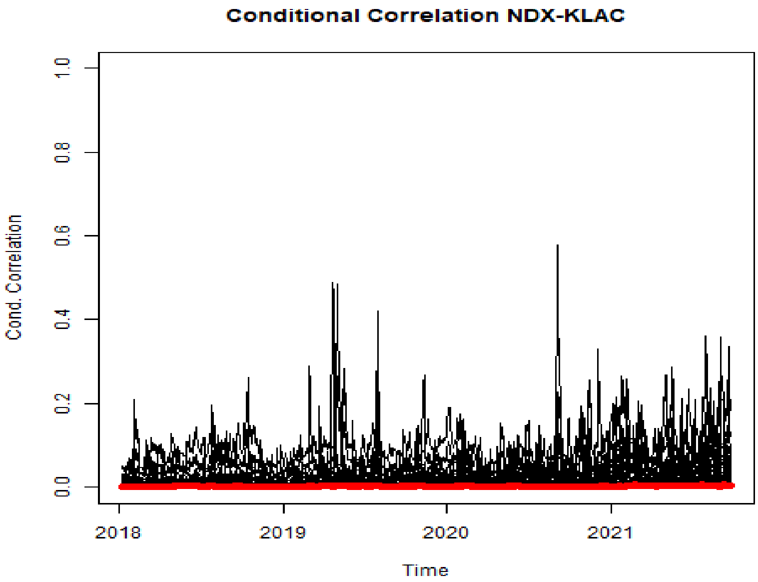


Figure B.86: Conditional Correlation NDX-KLAC

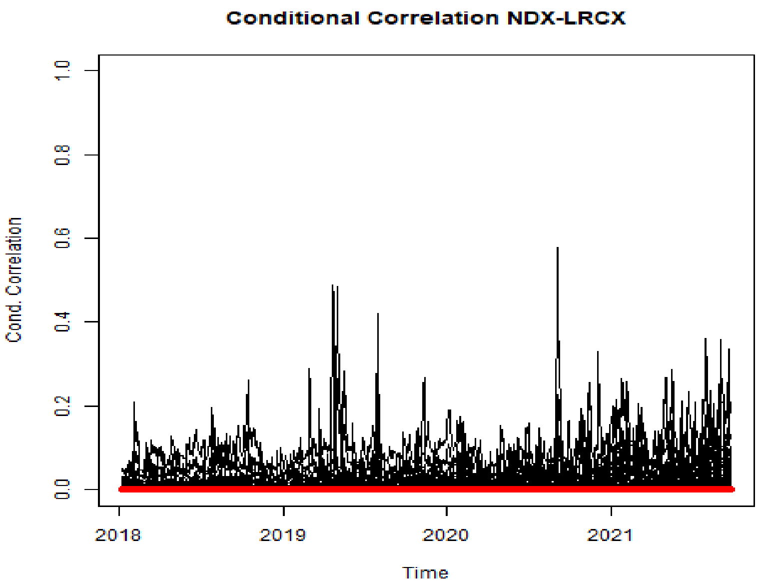


Figure B.87: Conditional Correlation NDX-LRCX

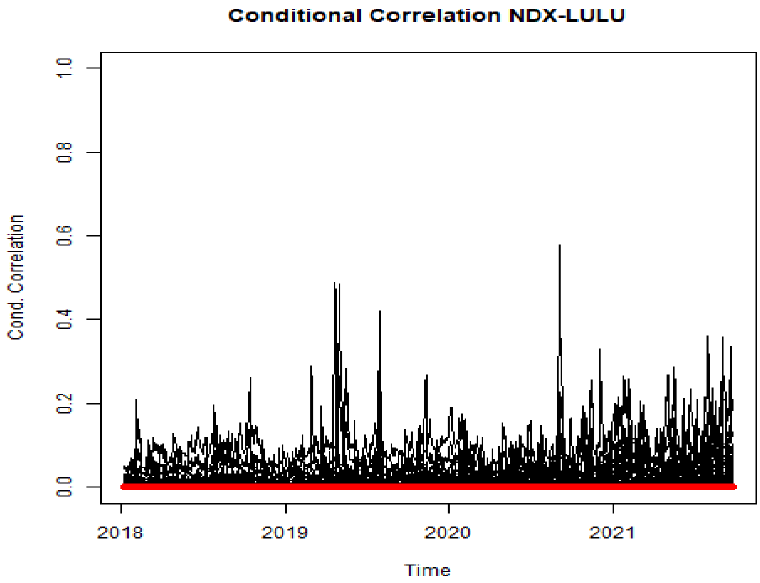


Figure B.88: Conditional Correlation NDX-LULU

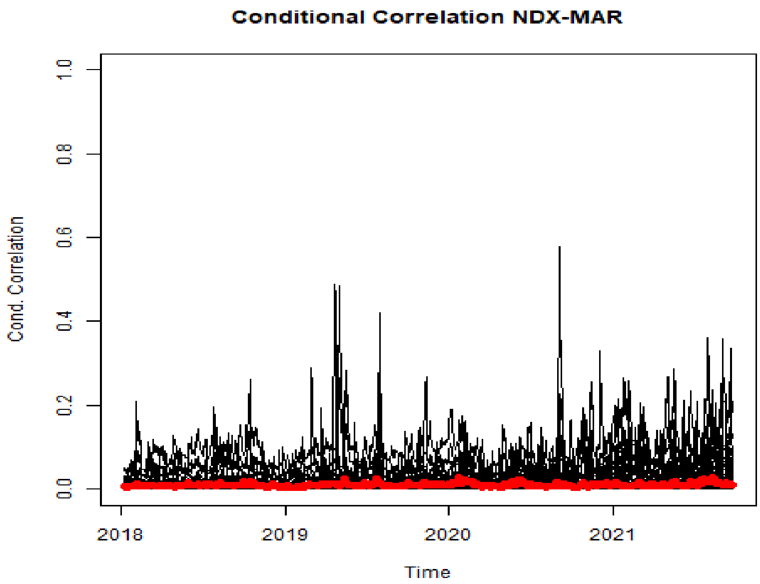


Figure B.89: Conditional Correlation NDX-MAR

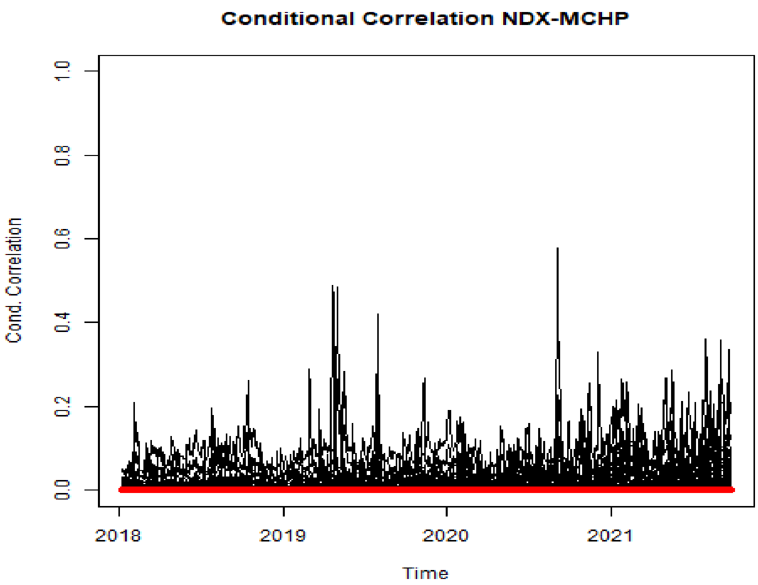


Figure B.90: Conditional Correlation NDX-MCHP

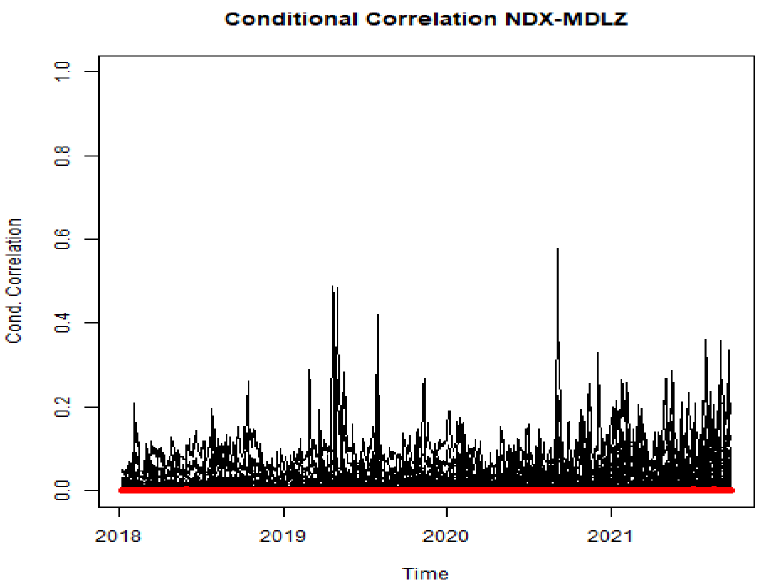


Figure B.91: Conditional Correlation NDX-MDLZ

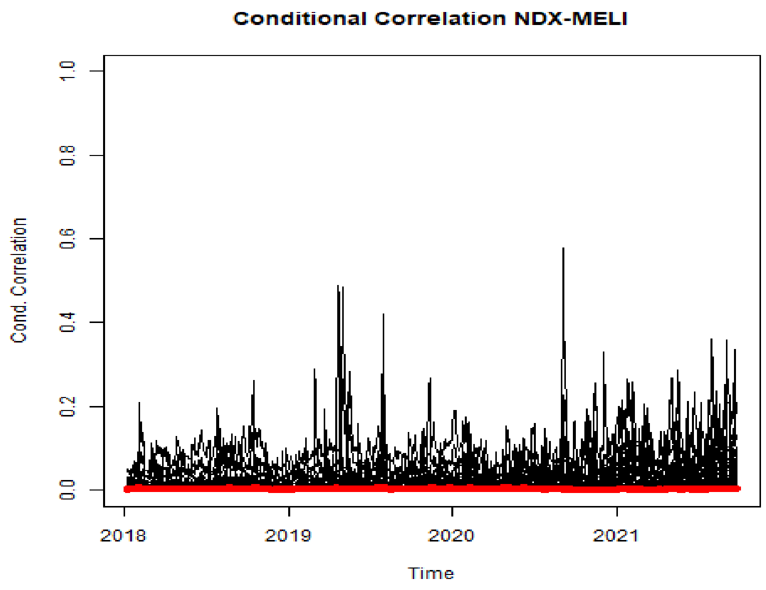


Figure B.92: Conditional Correlation NDX-MELI

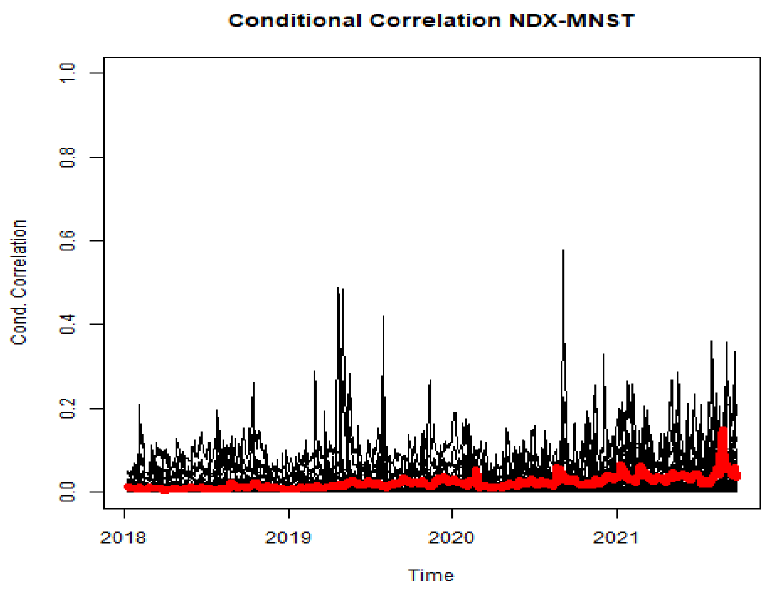


Figure B.93: Conditional Correlation NDX-MNST

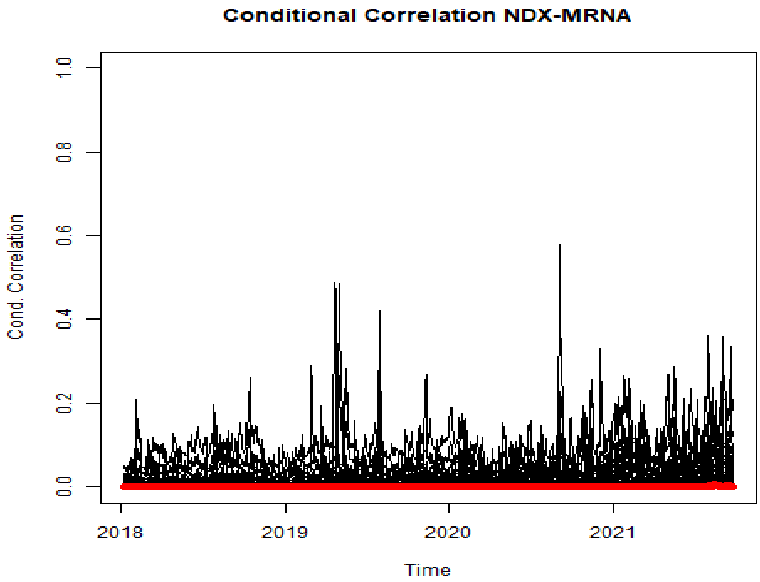


Figure B.94: Conditional Correlation NDX-MRNA

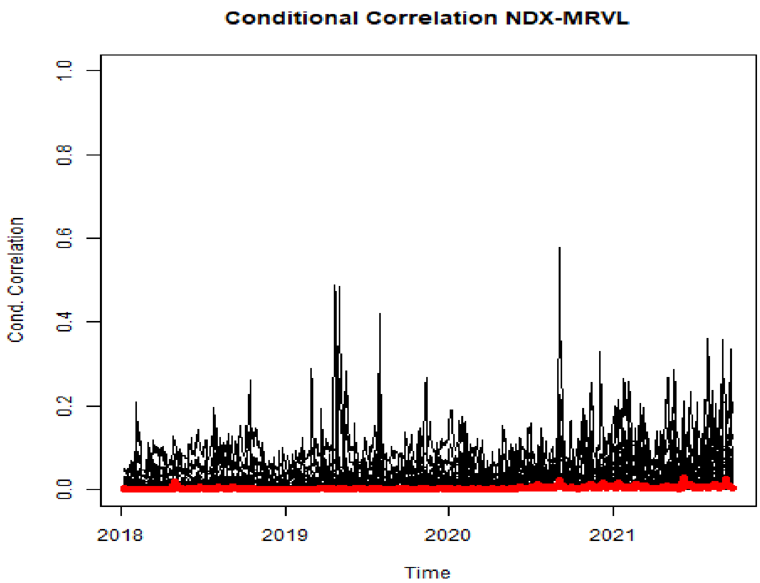


Figure B.95: Conditional Correlation NDX-MRVL

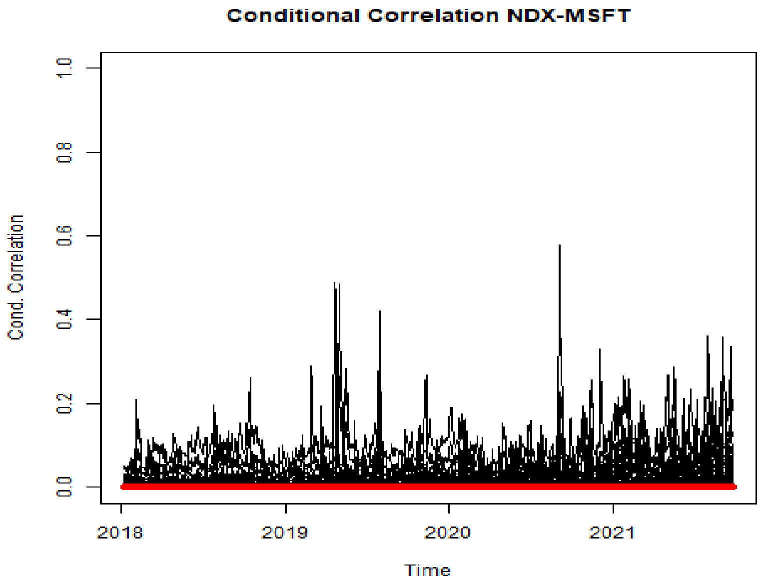


Figure B.96: Conditional Correlation NDX-MSFT

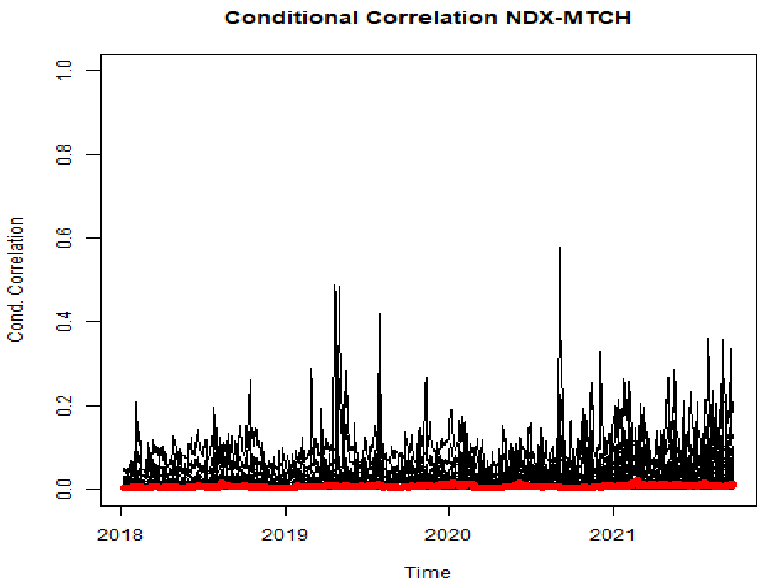


Figure B.97: Conditional Correlation NDX-MTCH

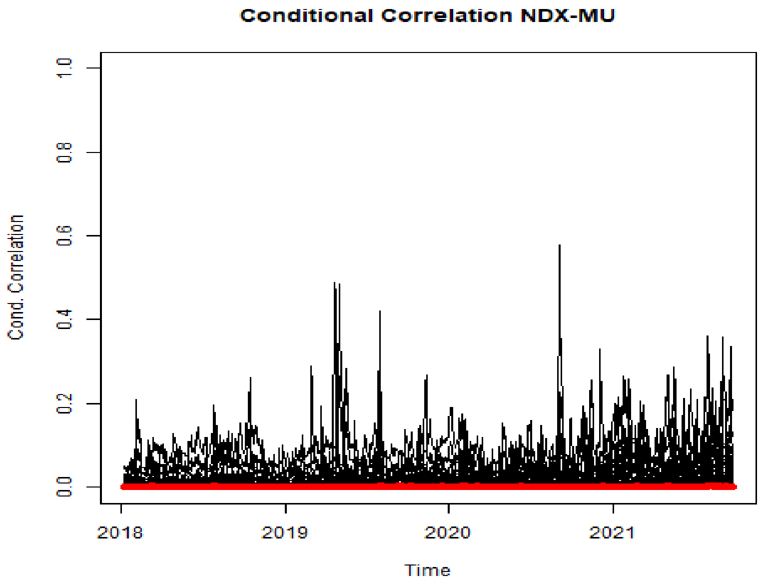


Figure B.98: Conditional Correlation NDX-MU

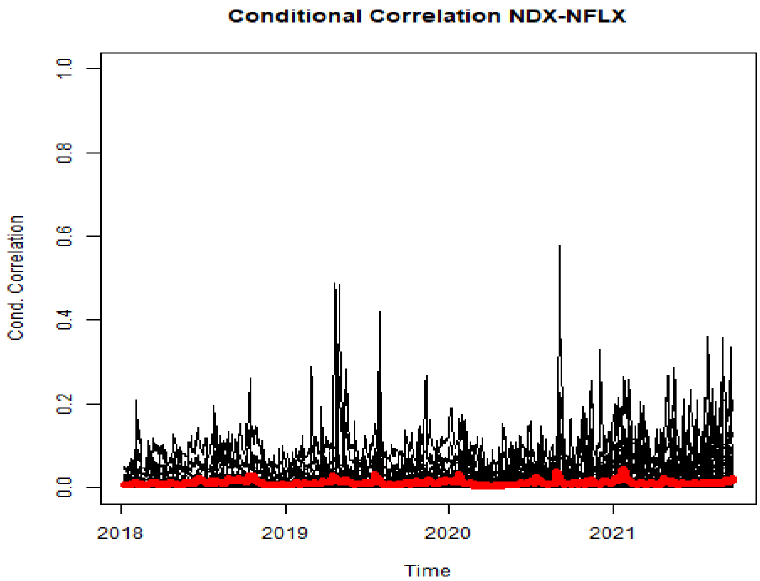


Figure B.99: Conditional Correlation NDX-NFLX

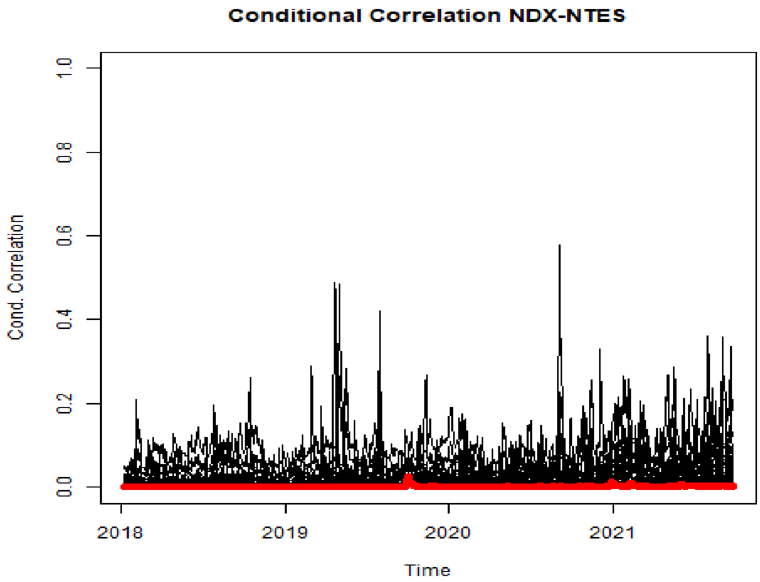


Figure B.100: Conditional Correlation NDX-NTES

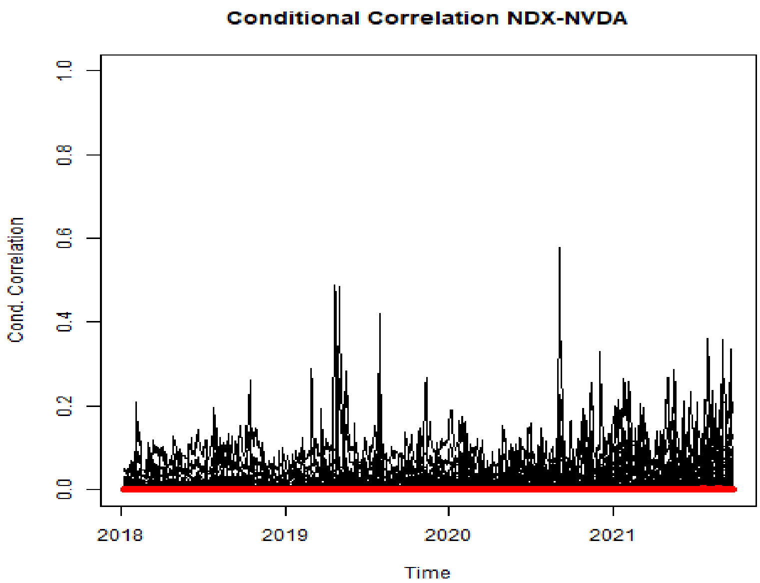


Figure B.101: Conditional Correlation NDX-NVDA

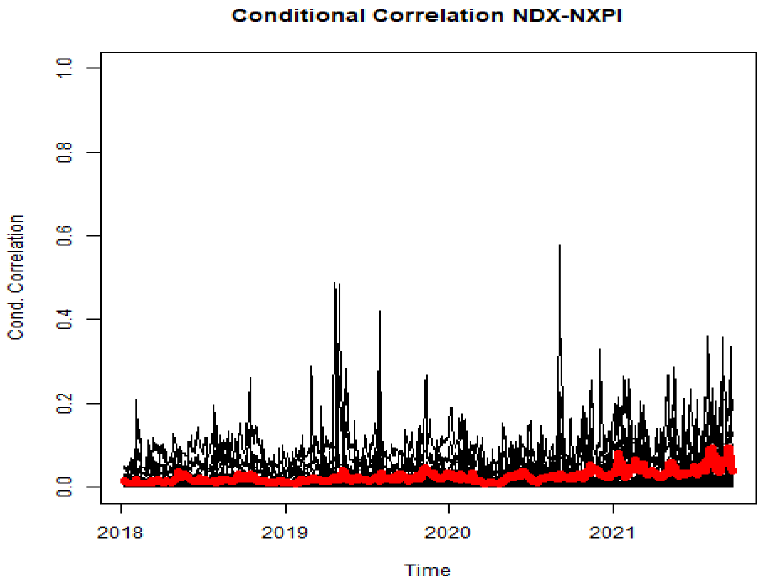


Figure B.102: Conditional Correlation NDX-NXPI

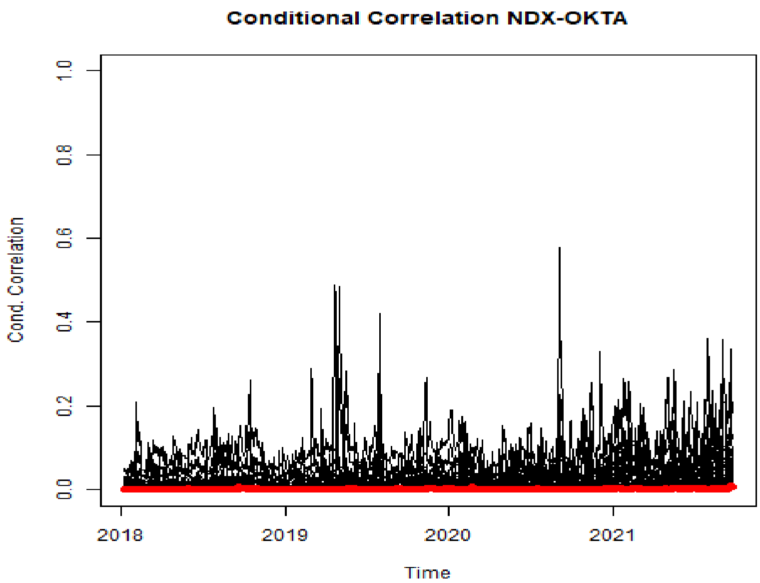


Figure B.103: Conditional Correlation NDX-OKTA

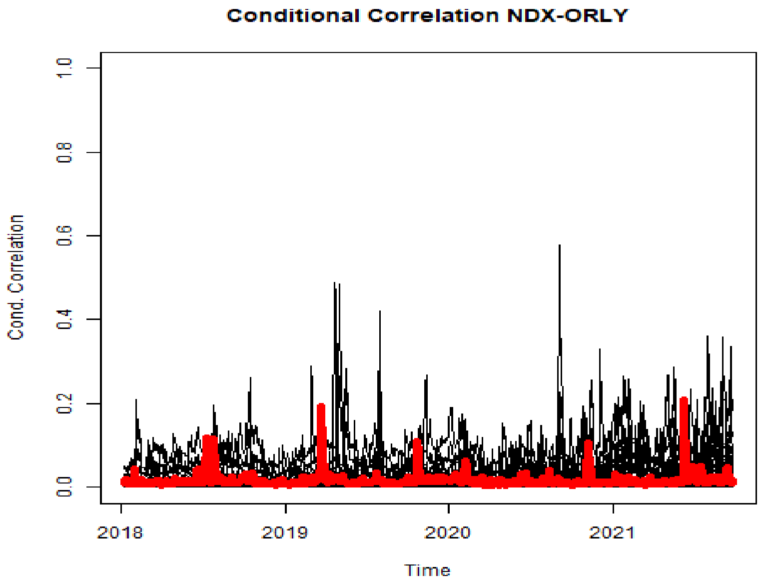


Figure B.104: Conditional Correlation NDX-ORLY

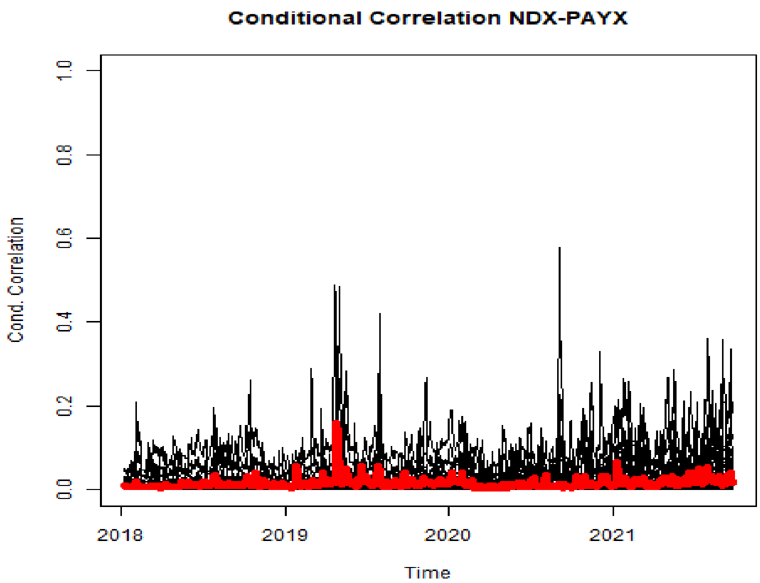


Figure B.105: Conditional Correlation NDX-PAYX

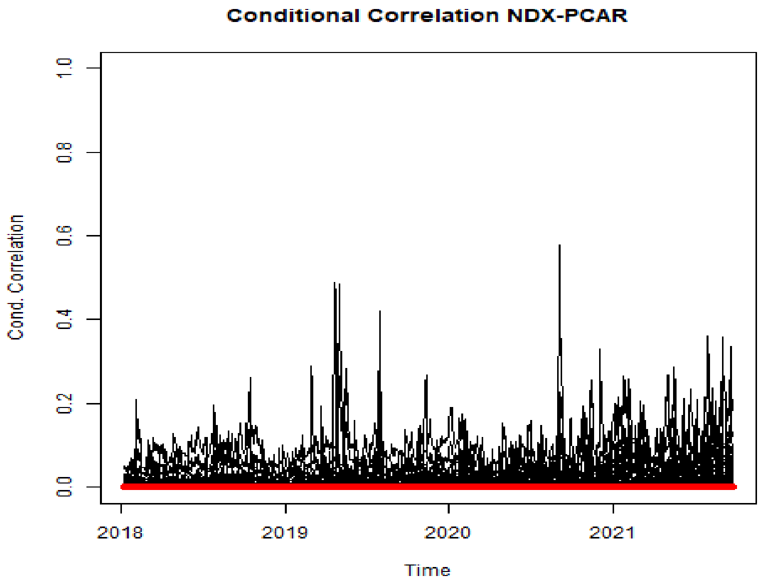


Figure B.106: Conditional Correlation NDX-PCAR

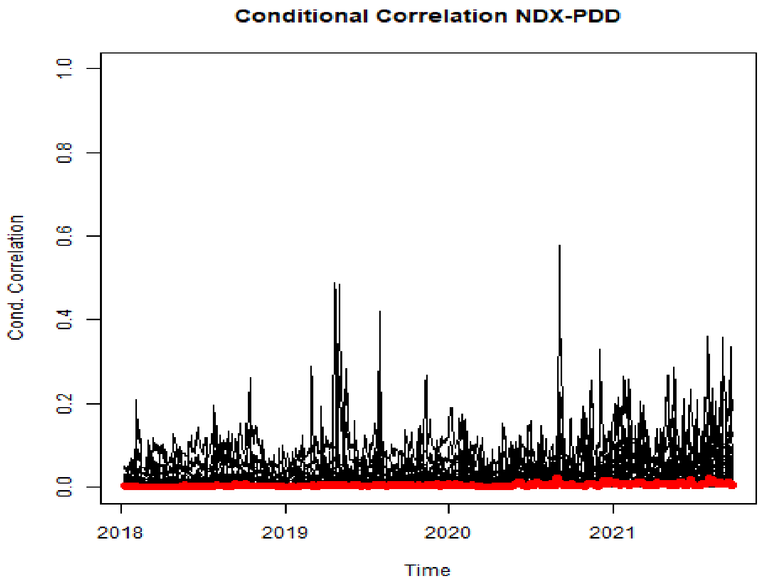


Figure B.107: Conditional Correlation NDX-PDD

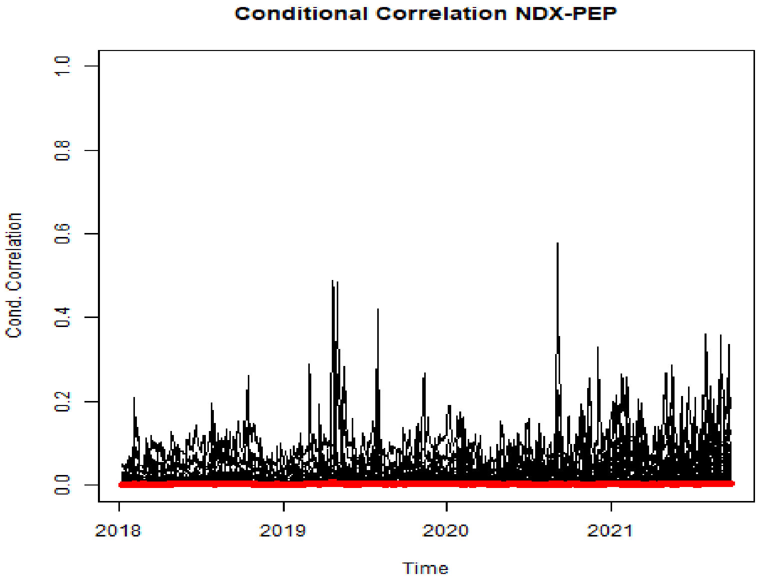


Figure B.108: Conditional Correlation NDX-PEP

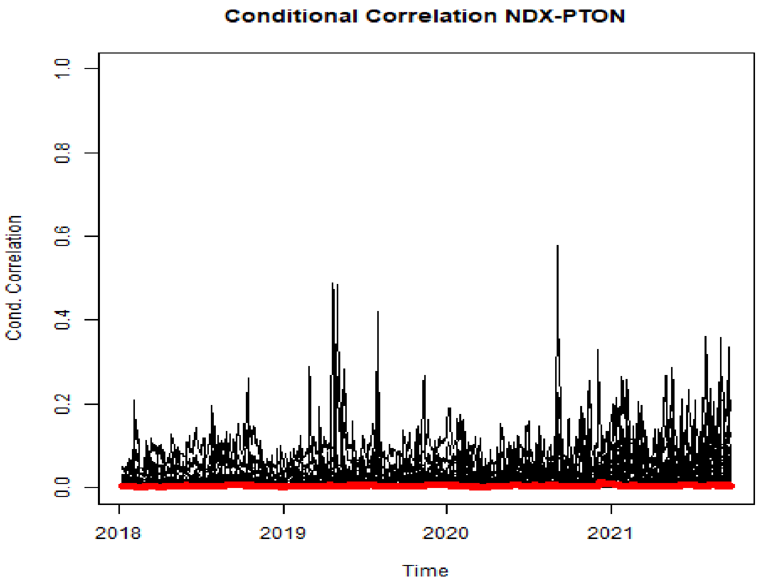


Figure B.109: Conditional Correlation NDX-PTON

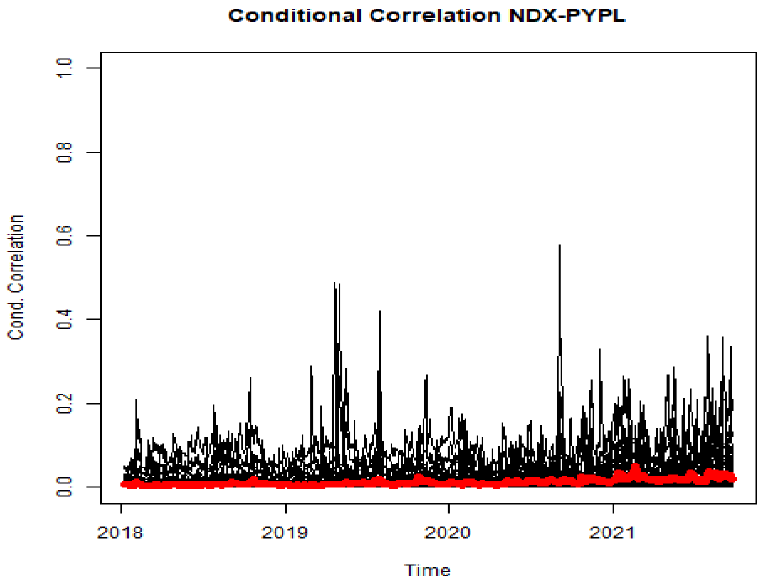


Figure B.110: Conditional Correlation NDX-PYPL

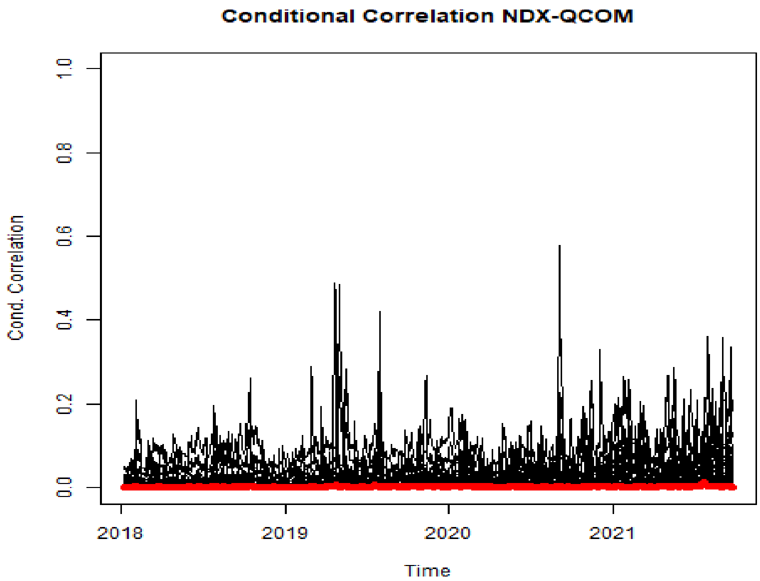


Figure B.111: Conditional Correlation NDX-QCOM

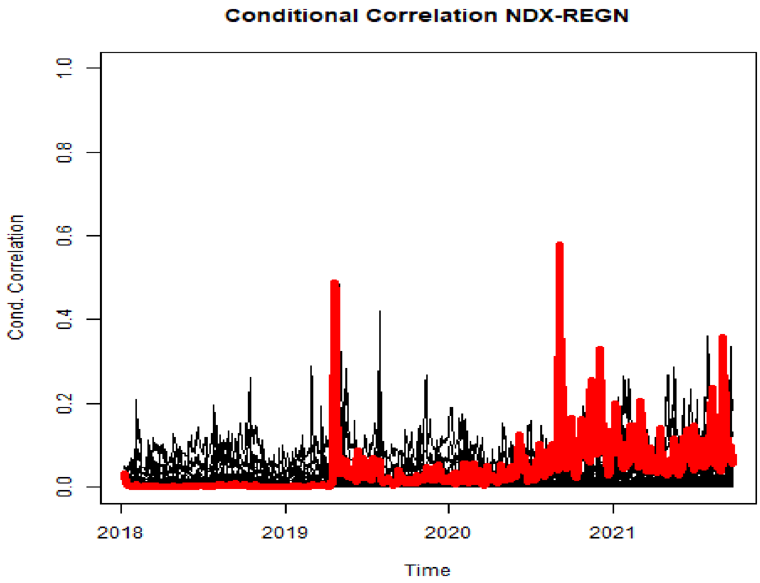


Figure B.112: Conditional Correlation NDX-REGN

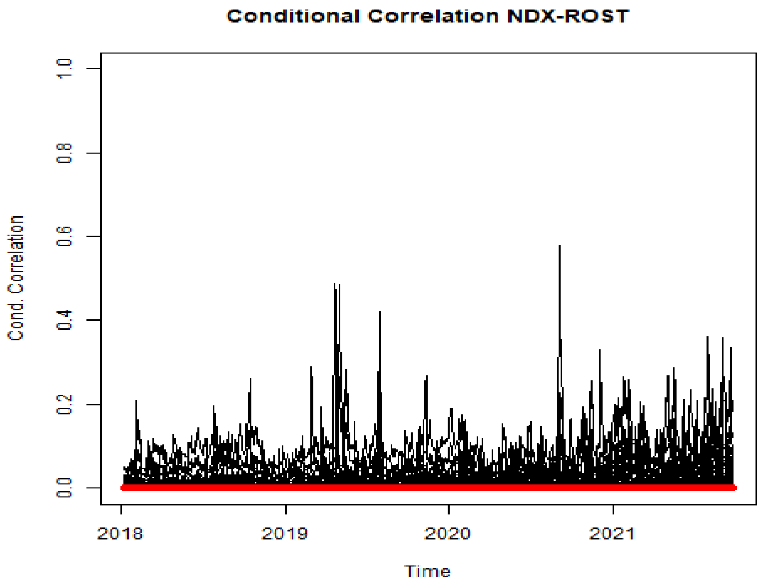


Figure B.113: Conditional Correlation NDX-ROST

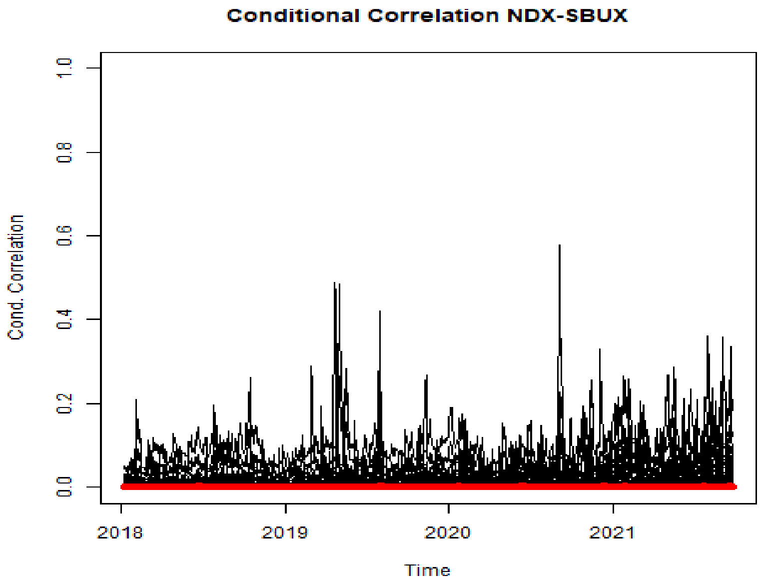


Figure B.114: Conditional Correlation NDX-SBUX

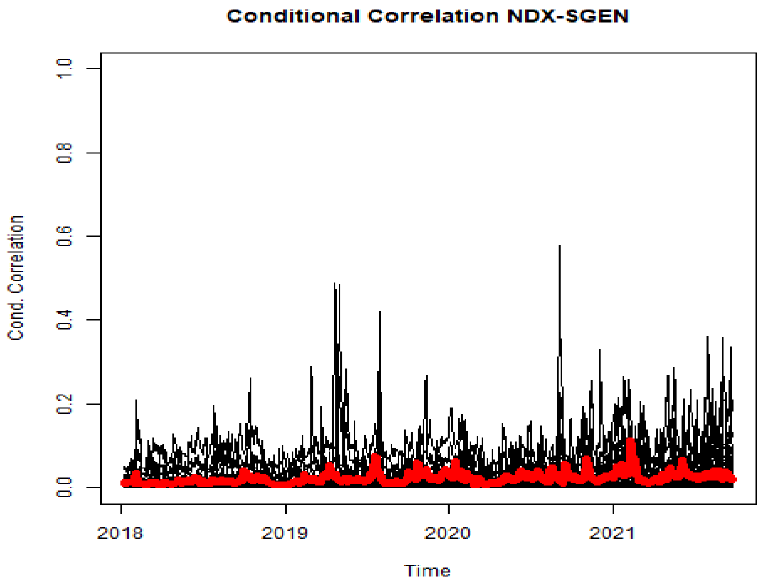


Figure B.115: Conditional Correlation NDX-SGEN

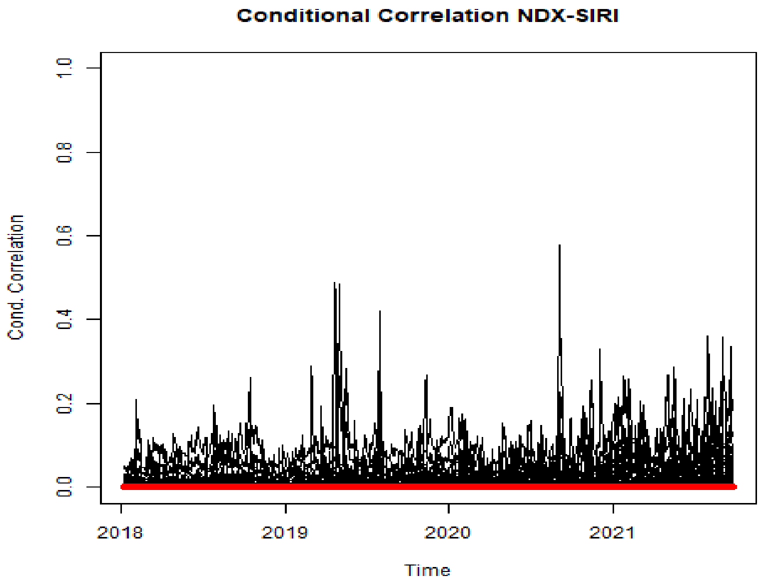


Figure B.116: Conditional Correlation NDX-SIRI

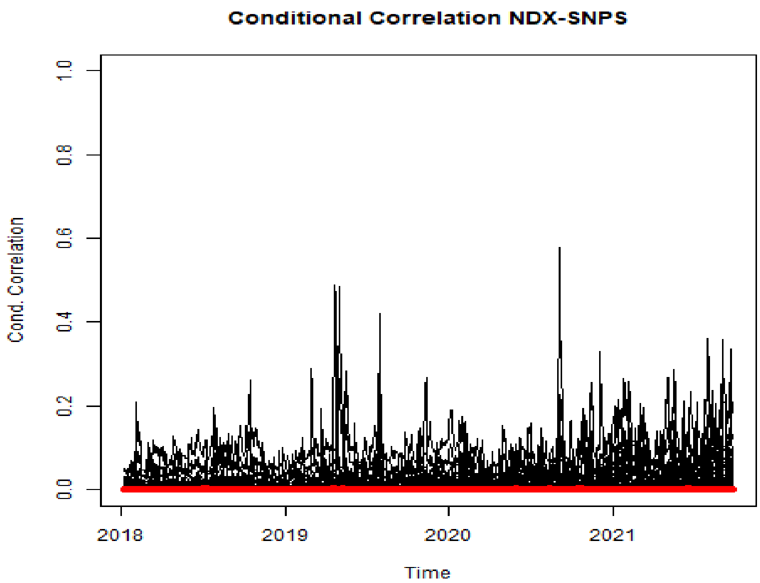


Figure B.117: Conditional Correlation NDX-SNPS

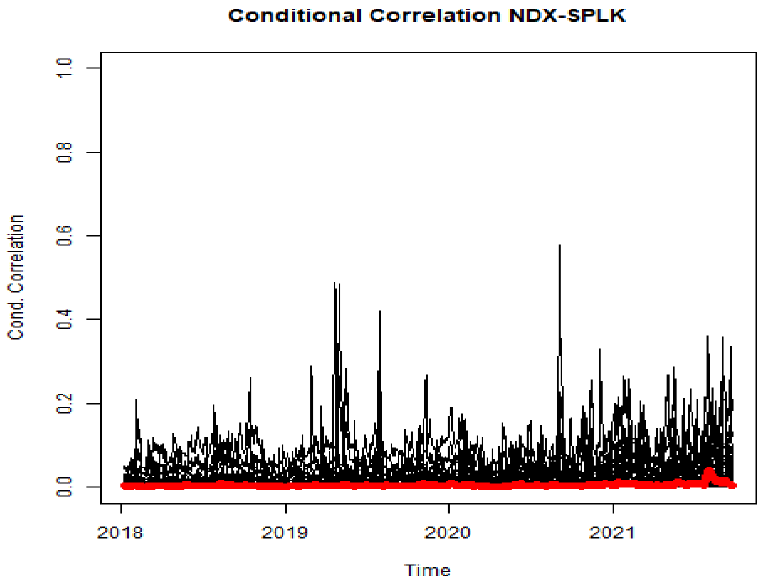


Figure B.118: Conditional Correlation NDX-SPLK

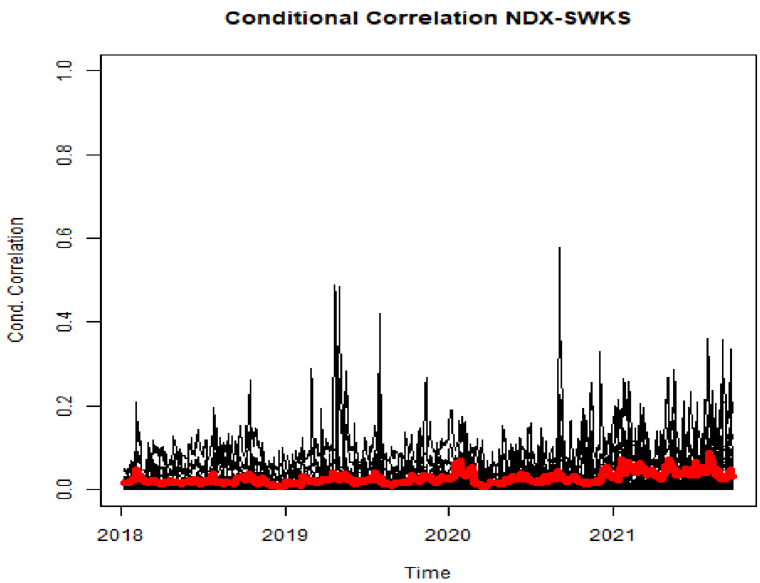


Figure B.119: Conditional Correlation NDX-SWKS

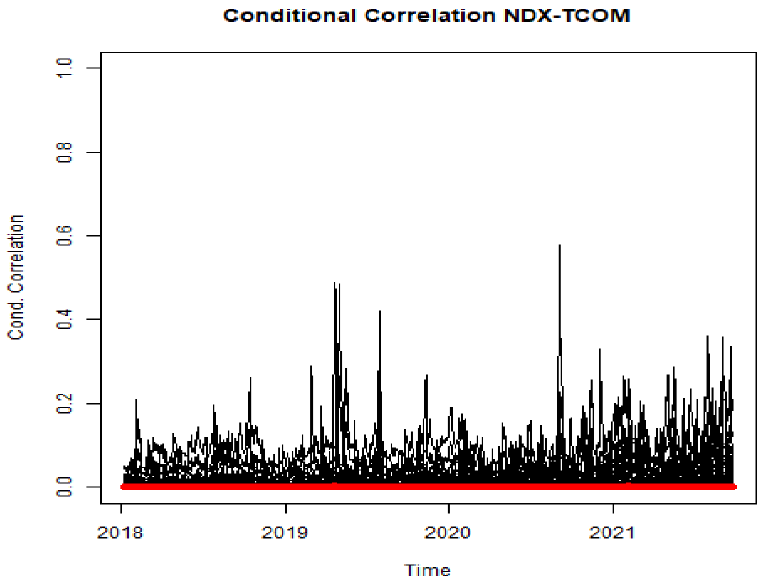


Figure B.120: Conditional Correlation NDX-TCOM

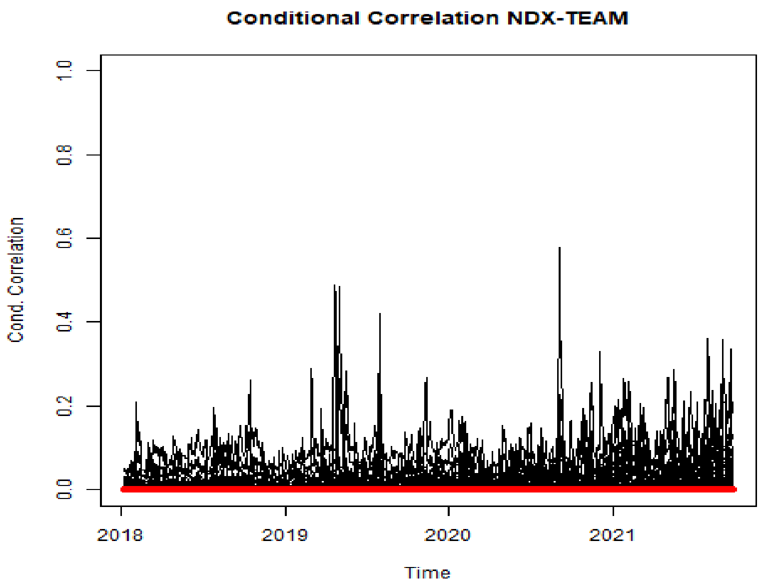


Figure B.121: Conditional Correlation NDX-TEAM

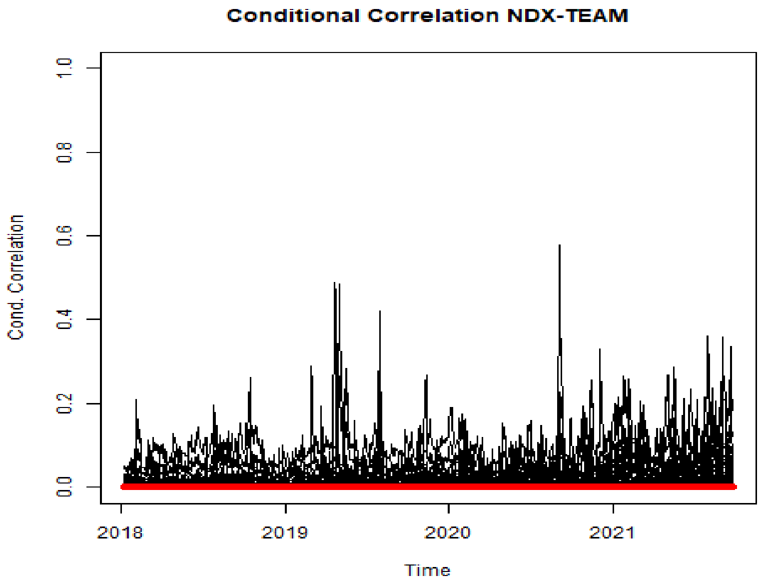


Figure B.122: Conditional Correlation NDX-TEAM

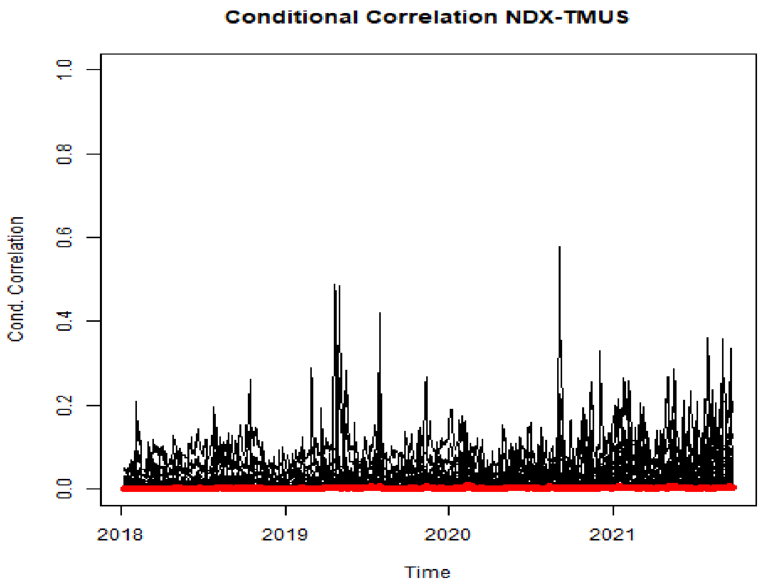


Figure B.123: Conditional Correlation NDX-TMUS

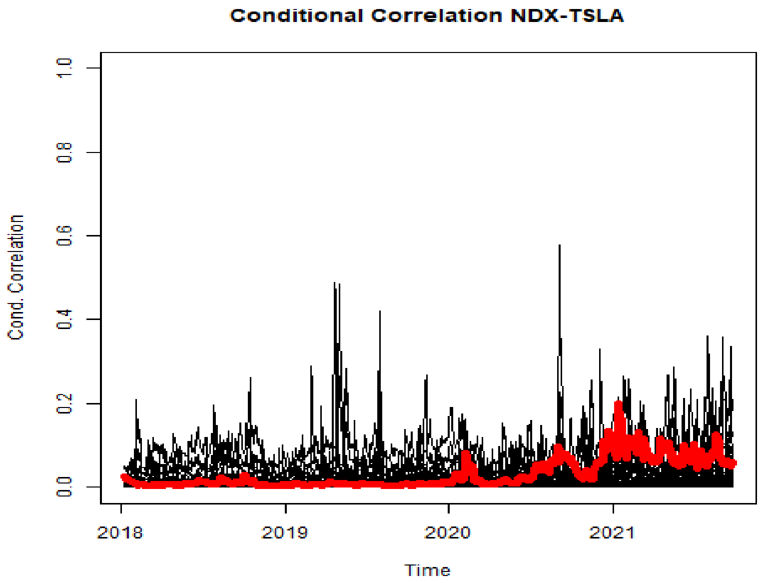


Figure B.124: Conditional Correlation NDX-TSLA

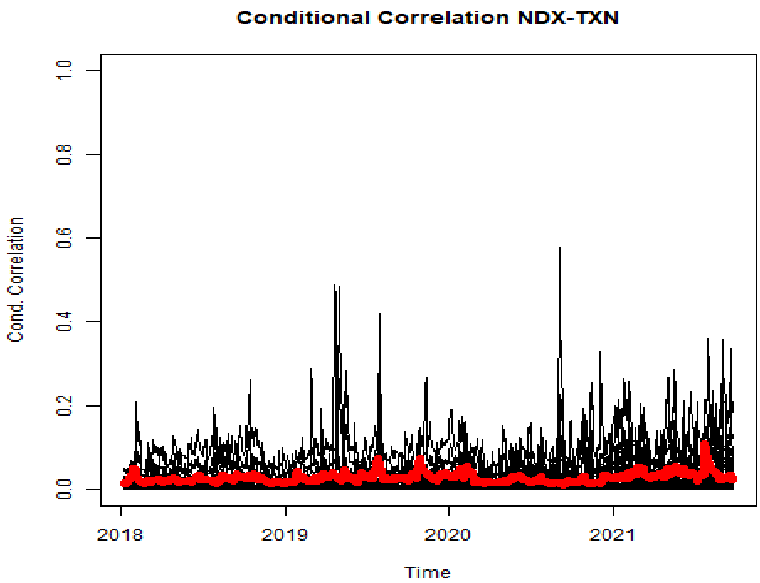


Figure B.125: Conditional Correlation NDX-TXN

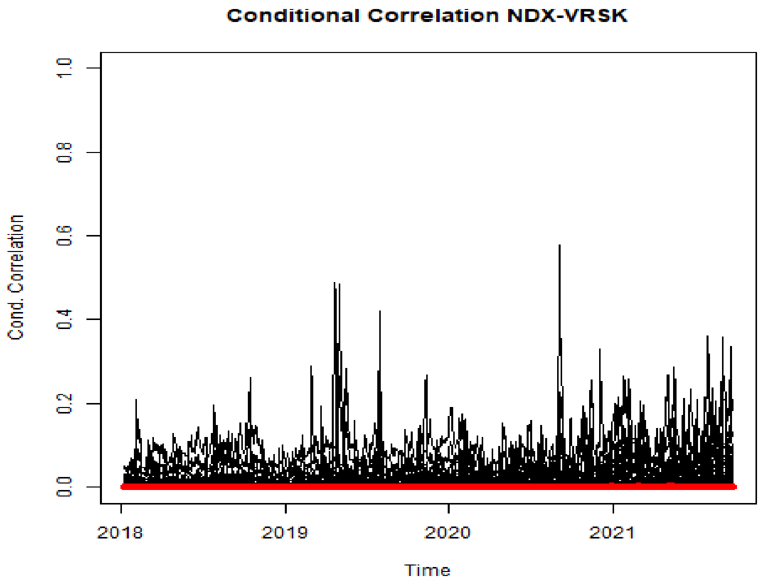


Figure B.126: Conditional Correlation NDX-VRSK

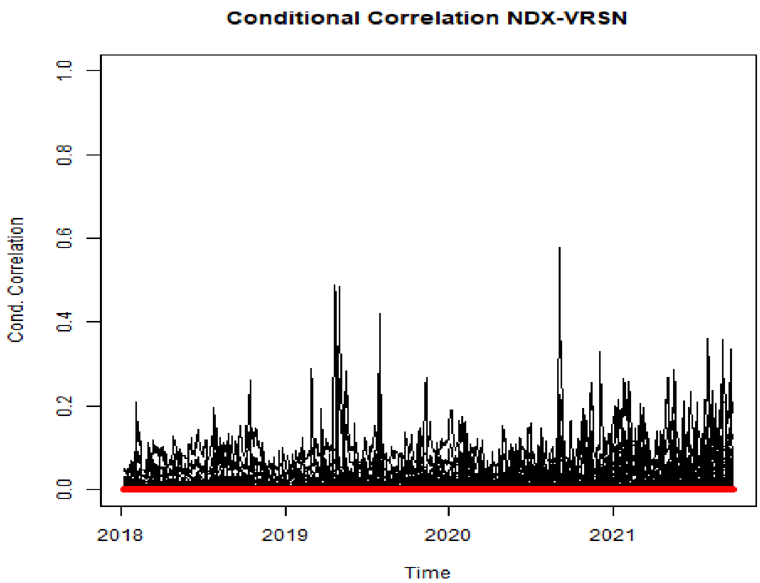


Figure B.127: Conditional Correlation NDX-VRSN

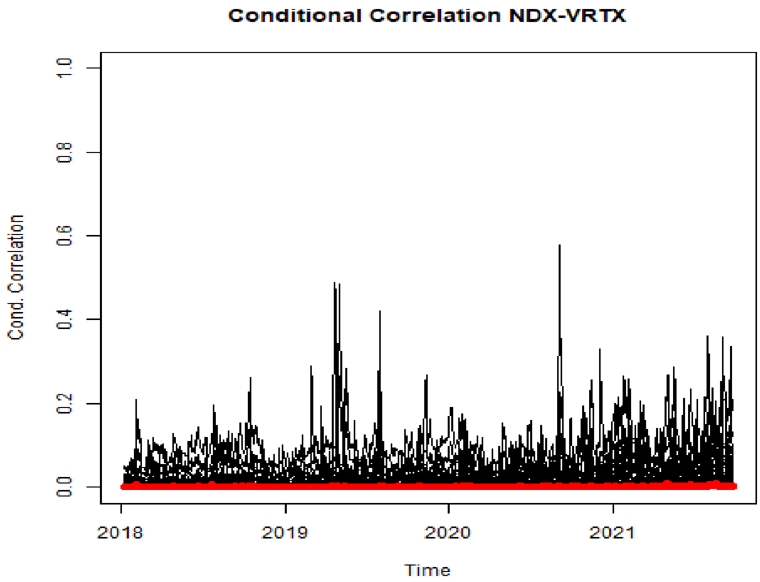


Figure B.128: Conditional Correlation NDX-VRTX

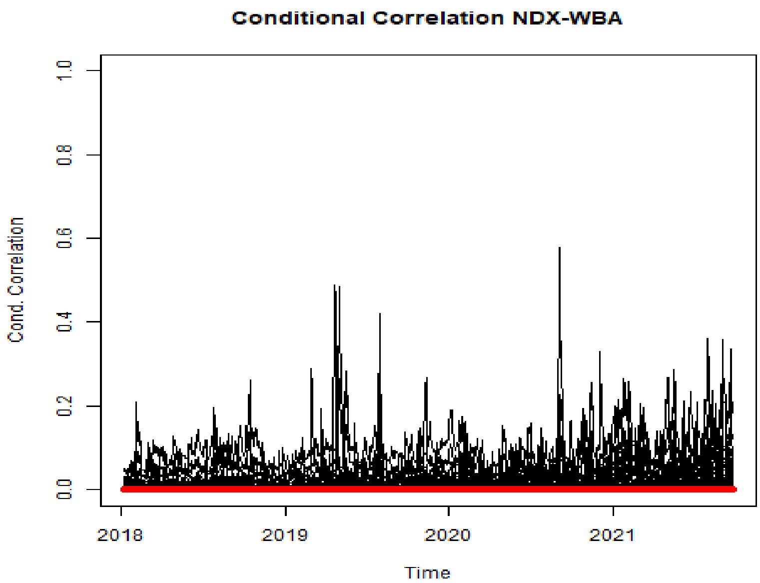


Figure B.129: Conditional Correlation NDX-WBA

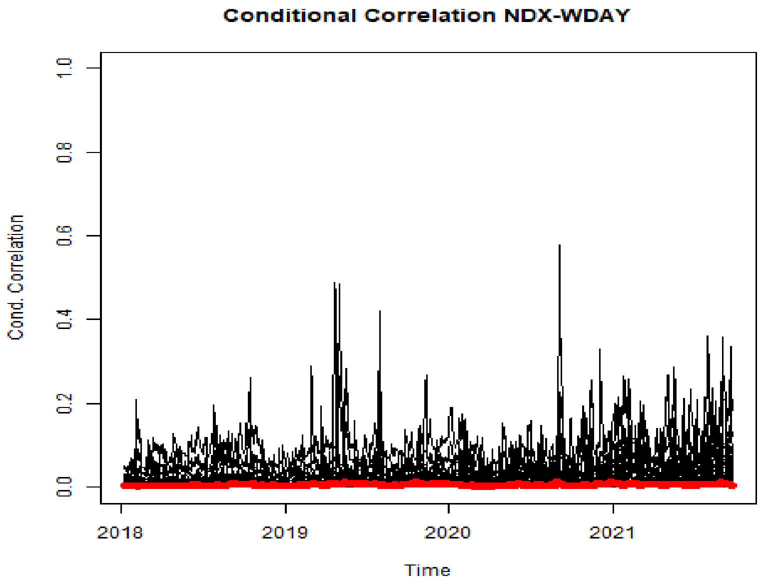


Figure B.130: Conditional Correlation NDX-WDAY

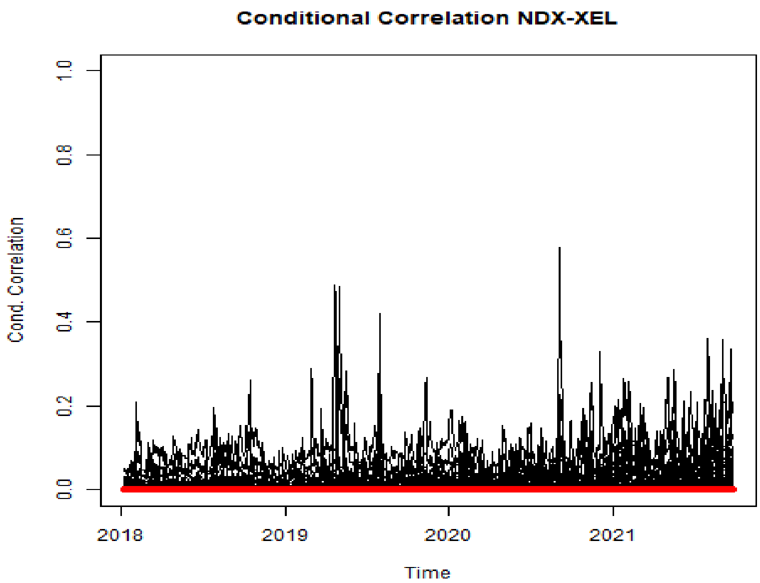


Figure B.131: Conditional Correlation NDX-XEL

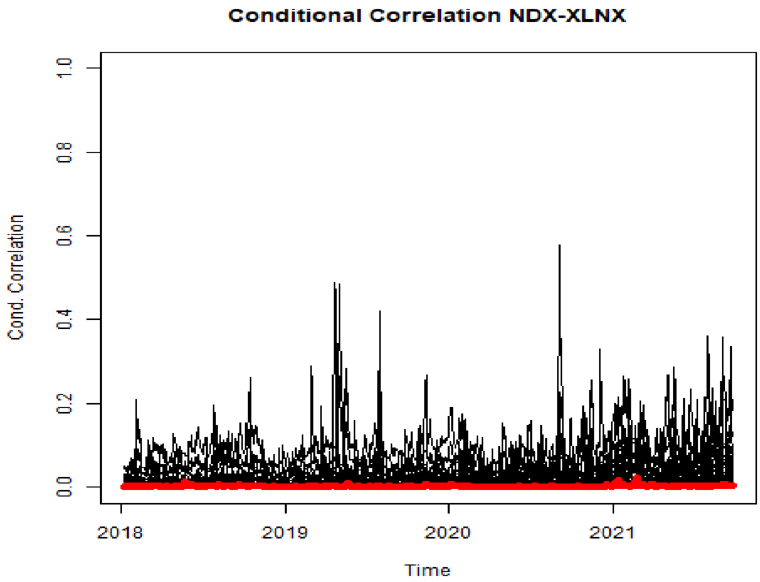


Figure B.132: Conditional Correlation NDX-XLNX

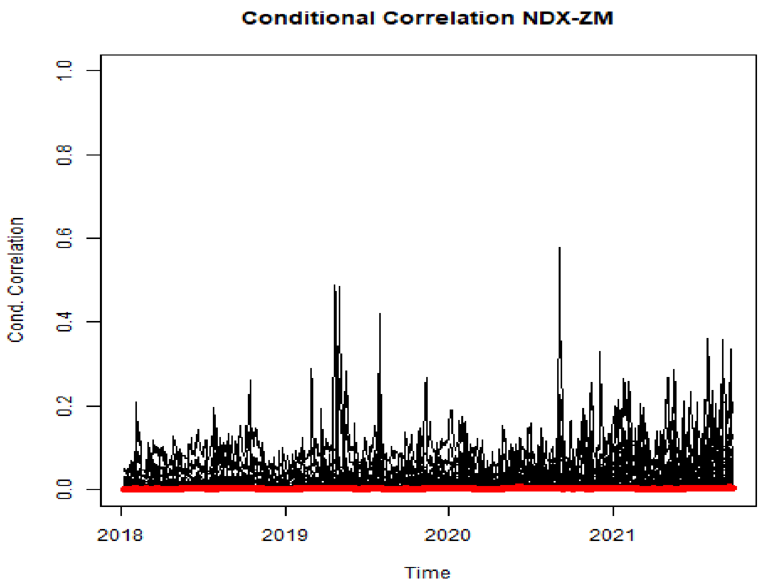


Figure B.133: Conditional Correlation NDX-ZM

2021

Covid Crash: A Study of Volatility Spillovers from Stocks to US Indexes and from US Indexes to Cryptocurrencies

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