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*The potential of using different representations is widely recognized, but not much is known about how teachers use them nor about the impact of the technology on such use. The goal of this study is to characterize the teachers' representational fluency when teaching functions at high school level, discussing, at the same time, the impact in the use of representations resulting from the use of technology. Adopting a qualitative approach, I analyze one teacher's practice. The results suggest that algebraic and graphical representations are seen as more important, that tabular representation is assumed as irrelevant and that the access to technology impacts the representations used and how they are used.*

*Keywords: Technology, different representations, functions.*

## Introduction

Work with different representations is recognized by several authors as an important experience to promote the development of links and strengthen the understanding of mathematical concepts (Dreher, Kuntze, & Lerman, 2016). And the technology is pointed as having the potential to easily allow the students to alternate among representations (Cavanagh & Mitchelmore, 2003). Although the potential of using different representations is widely recognized, only a few authors paid attention to how they are used (Dreher et al., 2016; Molenje & Doerr, 2006). And even those who do it, do not focus on the impact of the technology on the teachers' choices in what concerns the use of the different representations. This is the new aspect addressed in this article. The goal is to characterize the teachers' representational fluency when teaching functions at high school level in a context where technology is available, discussing, at the same time, the impact in the use of representations resulting from the availability of technology. With this focus, I specifically intend to answer the following research questions: How does the teacher balance the use of the different representations of functions? How does the teacher articulate the multiple ways of representing a function within and among representations? What is the impact of technology on the teachers' balance and articulation of the different representations?

## Representational fluency

Zbiek, Heid, Blume and Dick (2007) characterize the representational fluency as the ability to move from one representation to another, transferring the knowledge from one representation to another and combining it with the new knowledge available on the new representation. And this includes the transition between representations of different kinds but also transitions between different representations of the same type of representation, as emphasized by Moore et al. (2013) and Even (1998). This means that it is important to consider a transition, for instance, from an algebraic representation of a function to a graphical representation, but that it is also important to consider a transition from a graphical representation of a function to another graphical representation of that same function (for example, when we are using technology and we need to look for a suitable viewing window for the graph).

Kendal and Stacey (2001) use the expression privileging (used originally by Wertsch in 1990) to refer to the teachers' specific way of teaching. And the authors think of it as a reflection of the teachers' beliefs and professional knowledge that characterizes their practice. Privileging includes the teachers' options about what they teach and how they teach it. In what is relevant to this study, privileging includes the teachers' decisions about what representations are (intentionally or unintentionally) used and about how they are used. And how the teachers privilege a specific approach and devalue others can tell us much about their professional knowledge (Dreher et al., 2016; Rocha, 2016). When studying the teacher's representational fluency it is important to know if some representations are preferred over others. I use the word *balance* to refer to how the teachers privilege specifically the use of some representations over others.

Although several authors recognized the relevance of using different representations based on the contribution it can bring to the development of mathematical understanding (Duval, 2006; Zbiek et al., 2007), the way how the teachers balance the representations has not received much attention. Molenje and Doerr (2006) conducted one of the few studies with this focus. Their conclusions suggest that the teachers express some concern about how they balance the different representations. Nevertheless, the use of algebraic and graphical representations is dominant in relation to the numerical representation. And this study also achieved another interesting conclusion: when the teachers actually use the different representations, it is possible to identify a pattern in the way how they do it. According to the authors, some teachers tend to start by an algebraic representation, move to a graphical representation and then to a numeric representation. Other teachers tend to move from the algebraic representation to a numerical representation and only then to a graphical representation. In a previous study conducted by myself on the teachers' use of different representations (Rocha, 2016), I found more diversity on the articulation of representations than the one found by Molenje and Doerr (2006), but I also found some preference for the use of some specific sequences of representations. These options will promote a limited use of representations by the students (because they tend to reproduce their teachers), with consequences for the students' learning and for their flexibility with different representations and with functions in general (Rocha, 2016). As Dreher et al. (2016) emphasize, the use of different representations is not enough to promote learning, it is essential to pay attention to how they are being used. And these findings suggest that besides studying how the teachers balance the different representations, it is also important to understand how the teachers articulate them. I use the word *articulate* to refer to how the teachers privilege a specific pattern when going from one representation to another. The way how the teacher balances and articulates the different representations is assumed as central on the characterization of the teacher's representational fluency. As so, these are the concepts used to analyze the teacher's representational fluency.

## Methodology and study context

Given the nature of the problem under study and in line with the ideas advocated by Yin (2003), the study adopts a qualitative and interpretative methodological approach, undertaking one teacher case study (in the part of the study presented here). Data were collected by semi-structured interviews, class observation, and documental data gathering. The teacher's lessons with one of her classes were followed while she taught functions. It was performed an interview after each lesson, with the purpose of knowing the analysis that the teacher did of the events. A test, with questions about changing between representations, was applied to the students by the researcher at the end of the study. The results of this test and some selected answers given by the students were used on a final interview to the teacher as a starting point to discuss her use of representations in her lessons. It was only at this point that the teacher became aware of the specific focus of the study on the use of representations. During the study, 12 lessons of 90 minutes at 10*th* grade (age 16) were observed. All interviews and observed lessons were audio-recorded and later transcribed. Data analysis was mainly descriptive and interpretative in nature, and guided by the goals of the study. The tasks proposed by the teacher were assumed as the unit of analysis. For each task the representations adopted by the teacher and the articulation between them were identified. Each type of articulation identified was then analyzed with the intention of characterizing it and understand the balance ascribed by the teacher to each representation. The representations taken into account were the ones usually available on the technologies and used on the study of functions: algebraic, graphical, numerical and tabular. Due to the goals of the study, when analyzing a task what was taken into account was the teacher's point of view, ie, the representations she chooses to use or suggests to the students.

The teacher participating in the study has a professional experience of about 20 years and a positive attitude towards the use of technology to teach mathematics. She likes to use computers on her practice, but when teaching functions she prefers to use graphing calculators and avoid the trouble of moving to a different classroom. As so, the graphing calculator was the only technology she used during this study, and this is why this is the only technology mentioned in this article. All the 25 students in this class had their own graphing calculator, which they could use at all times.

## Results and discussion

An analysis of the tasks proposed by the teacher allowed the identification of nine different types of articulation between representations. Table 1 presents the sequences of representations used, a brief description of the strategy used to solve the task and an example of a possible task (please take into account that the examples presented were simplified due to space constrains). The sequence algebraic $\rightarrow$ graphical $\rightarrow$ numerical was used often on tasks with a real context and asking something about it (for instance, corresponding to the zeros or maximum of the function, or to the values for which the function is increasing or assumes positive values). It was also used on strictly mathematical questions, namely to solve inequalities. The sequence algebraic $\rightarrow$ graphical was used when the goal was the graph of the function. And when the standard window did not offer a good viewing of the graph, the sequence became algebraic $\rightarrow$ graphical $\rightarrow$ graphical, including a change among graphical representations, ie, using different representations of the same type of representation. The sequence algebraic $\leftrightarrow$ graphical was used on exploratory tasks where the teacher intended the students to relate these two representations, expanding their mathematical knowledge, or to use previous knowledge to guide their successive attempts to solve the task. All these four sequences required the use of technology. And in all the other five sequences the technology was not used. However, some of these sequences were not as used as others. That is the case of the sequence graphical → algebraic that was only used once, and of the sequence graphical → numerical that was only used on an introductory lesson to the theme. Besides that, it is possible to notice that certain sequences were never used and that the tabular representation was never used as well.

| Representation | Teacher's approach | Example of task |
|---|---|---|
| algebraic → graphical | Insert the function on the calculator and press graph | Draw the graph of $f(x) = -x^2 + 3x + 5$ |
| algebraic → graphical → graphical | Insert the function on the calculator and press graph<br>Use zoom / change the window | Draw the graph of $f(x) = -x^2 + 100$ |
| algebraic → graphical → numerical | Insert the function on the calculator and press graph. Use menu *calc*<br>In some cases (see 2nd example) it is possible an approach without using the calculator | Knowing that the height of a remote controlled airplane is given by $f(x)=-x^2+4x$ find the maximum height reached by the airplane.<br>Solve the inequality: $x^2+4x+3>0$ |
| graphical → algebraic | Use paper and pencil and visually get information from the graph (calculator used at most to check) | Look at the graph of the function and find its algebraic expression |
| algebraic ↔ graphical | Insert one (or more) function on the calculator, see the graph, change the function and move back and forward between the representations trying to come to a conclusion | Study the family of functions $y=ax^2$, $a\in\Re\backslash\{0\}$<br>Slalom: Find a polynomial function to represent the course of a skier when he goes through both doors without touching the flags at (1, 4), (2, 4), (5, 4), (6, 4) |
| algebraic → numerical | Use paper and pencil | Find the zeros of $f(x)=2x^2-8x+6$<br>Solve the inequality: $2x^2+12x+10<0$ |
| algebraic → algebraic | Use paper and pencil | Write the polynomial function $f(x)= x^3-x$ as a product of factors |
| numerical → algebraic | Use paper and pencil | Write the expression of the 2nd degree polynomial function with a zero for x=1 and x=-2 and going through the point (0, 4) |
| graphical → numerical | Direct reading of the graph (use paper and pencil) | What is the maximum value reached by the function on the graph? |

**Table 1: Sequence of representations used by the teacher**

The teacher recognizes the importance of using different representations to promote a deeper understanding of the concepts and emphasizes in particular the contributions of graphic representation to the understanding of algebraic representation:

> T: I think that using graphs and algebraic expressions is very important. Some students have some difficulty in working analytically. They get lost and at a certain point they no longer understand what they are doing. For instance, they don't know why they equal the expression to zero when they are looking for the zeros of the function... and why they can't just replace the x by 0. Having the graph helps to understand... we want the value of x when y is 0. (...) I think the graphical representation is easier for them and if you work with it and with the algebraic representation you are helping them.

And she assumes that the technology allows the students to quickly access a wide variety of graphs and that it turns the work around functions much easier. It becomes possible to link different aspects about functions in a way that was not possible before:

> T: Technology changes everything! The technology allows us to move instantly from the algebraic expression to the graph. And that turns possible to develop deeper relations among these representations. We can see on the graph the impact of changing a parameter on the function. This is not possible without technology.

The results achieved by the students on the test on representations suggest that they are effectively familiar with the relationship between algebraic and graphical representations. Nevertheless, the results also show that the students have difficulties in getting information from a table. 48% of the students failed to identify, based on a table including the relevant information of a function *f*, an interval for x where the values of *f* were increasing; but 80% managed to answer the same question based on a graph. 88% of the students were unable to find the expression of a function using the information on a table of values; but all recognized the quadratic function when the information was provided by a graph and 92% were able to find the algebraic expression. When confronted with these results, the teacher recognizes she does not use the tabular representation, assuming that the students' lack of familiarity with the representation might be the source for the difficulties of some students. But somehow she seems surprised and expresses her belief about the similarity between the numerical and tabular representation. A similarity that is actually recognized by several authors, such as Goos and Benninson (2008). In the teacher's own words:

> T: I have to admit that nowadays I don't use tables so much. When I was a student we didn't have technology and we use a table to register some values of the function and then mark them on a graph or whatever… With technology we don't need that and… I know the calculator provides a table but we can get everything from the graph using the menu *calc*, so there is no need… for the table. It's the same. But maybe not for them. I think they get confused and didn't know in what column they should read the values. Maybe I should use the table. I don't know… I thought it was the same.

About privileging paper and pencil or technology when moving from one representation to another, the teacher says that she tries to balance the two options:

T: I try to do everything with and without technology because our syllabus requires that. (…) There are some exercises that they don't know how to do without the technology. For instance, find the maximum of a function. And there are also others that they cannot do on the calculator. For instance, factor a polynomial function… But we solve inequalities with and without technology. I think we achieve a deeper understanding when we can do something in two different ways.

Nevertheless, some sequences are always done using (or not) technology (see Figure 1). In some cases there is no surprise on that. For instance, the change from one graphical representation to another is usually due to an unsuitable choice of the viewing window. So it is to expect that it happens always when the technology is being used. But there is no reason to never use the technology when moving from a graphical representation to an algebraic one (and this is the option of this teacher). This suggests that the teacher is not fully aware of her options in what concerns the use of the different representations.

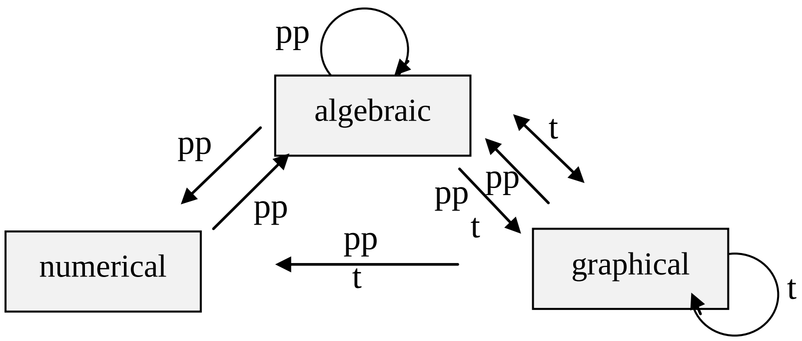


**Figure 1: Representations used by the teacher and their sequence of use (pp - paper and pencil, t - technology)**

## Conclusion

Algebraic and graphical representations seem to be assumed as more important than other representations. A conclusion in line with the results achieved by Molenje and Doerr (2006), who also concluded that these representations are used more often than others, meaning that representations are not equally balanced by the teacher. This can be related to the characteristics of the representations. As stated by Friedlander and Tabach (2001), algebraic representation is concise, comprehensive and effective, being also the most valued representation in Mathematics. But students tend to find it difficult (Quesada & Dunlap, 2008). The graphical representation provides a visual representation and an intuitive approach to the concepts, turning learning more intuitive (Friedlander & Tabach, 2001). It is thus understandable that these two representations are the most used. After all, the graphical representation helps the students understand the most valued representation: the algebraic one. And this is the central point. The representations beyond the algebraic one are mainly used to promote understanding over the most important representation and not so much to promote a global understanding of functions, as advocated by Dreher et al. (2016) and several other authors. The numerical representation, whose relevance is assumed to be minor due to its focus in particular cases (Friedlander & Tabach, 2001), is also used, although not so often. However, that is not the case of the tabular representation that is never used. The reason for the lack of use of this representation has to do with the fact that numerical and tabular representations provide similar information and, as a consequence, tend to be assumed as identical, a result also identified by Rocha (2016).

Although the teacher enrolled in this study articulates the different representations in a flexible way, it is possible to identify a preference for certain sequences and to the use (or not) of technology in each one. It is also possible to notice that the algebraic and the graphical representation are the ones who are articulated in a more diversified way, a circumstance that is certainly related to the fact that these two representations are the ones used more often. These are also the two representations that seem to suffer a deeper impact from the use of technology. The ease of access increases the moments where the graphical representation is used and consequently impacts the balance and the articulation the teacher does of the representations. Simultaneously, the ease of access to numeric values straight from the graphic (when technology is being used), relegates the tabular representation for a situation where its value is no longer recognized. The technology changes the teacher's use of the tabular representation. At the same time, it prevents the teacher from realizing, that from a mathematical point of view, there is a difference between this representation and the numerical one. A difference that somehow is emphasized (or even created) by the technology where is often possible to get a numerical representation based on some automatic process (eg., using technology someone asking for a zero can get an answer even if he has no idea about what that is).

The presence of technology impacts the way how the teacher articulates and balances representations, turning the work around some representations more usual than around others and allowing some different sequences on the use of representations, this implies that technology modifies the teacher's representational fluency. And this fluency is known to be closely related to the teachers' professional knowledge (Dreher et al., 2016; Rocha, 2013). As so, an analysis of the teachers' representational fluency could give important information about their knowledge. And it would be interesting to analyze if different types of representational fluency are related to different levels of teachers' knowledge and, of course, practices.

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