A Work Project, presented as part of the requirements for the Award of a Master's degree in Economics from the Nova School of Business and Economics.

THE BLUE BOND PROPOSAL IN THE PRESENCE OF SPILLOVER EFFECTS

DANIEL HENRIQUE MARCOS

Work project carried out under the supervision of:

André Castro Silva

02-01-2021

Abstract

I show that a Blue and Red Bonds format of debt mutualisation can be beneficial for the assistance-providing countries and that the benefit from joining such scheme depends positively on the size of a fixed default output cost and negatively on the threshold for the issuance of blue bonds. I assume the presence of spillover effects and compare the resources of Core, the fiscally responsible country in the union, under the Blue Bond scheme against the case of no debt mutualisation and show the difference between both schemes under different output shock realizations. I also present a formula for the expected difference in resources for both formats and solve the model for different values of the most relevant parameters.

Keywords

Interest Rates; Fiscal Policy; Eurobonds; Sovereign Debt

1. Introduction

This paper proves that in a union of countries that mutualise their public debt through a Blue and Red Bonds scheme, the more fiscally responsible countries can benefit from the program and achieve higher utility and resources, even in circumstances in which they have to provide financial support in order to avoid another country defaulting on its blue bonds. I show what the result from participating in the scheme is for these assistance-providing countries for different states of the world and conclude that default costs are the reason why a country can be better off by providing support to another member of the union, as it avoids output costs caused by decreasing general confidence in the economy. When the fixed default output cost is large enough, a country is better off by participating in the program and guaranteeing the distressed country's blue bonds. I also show that Core's incentive to join a Blue and Red Bonds scheme depends negatively on the threshold for the issuance of Blue Bonds, the amount of Periphery's debt that is guaranteed by the union.

The debate around debt mutualisation in the Eurozone has been around for a long time. Eurobonds, sovereign debt bonds issued in Euros jointly by the Eurozone states, constituted a hot topic during the European sovereign debt crisis, as multiple proposals were published, inclusively one by the European Commission, which presented a Green Paper on the feasibility of introducing Stability Bonds in November 2011. Recently, with the emergence of the Covid-19 pandemic and the ensuing economic crisis that affected the entire world economy, Eurobonds came to the public's eye again, this time labelled as Corona bonds, with many proponents both on the media and in the European Council. The idea was initially rejected by Germany, the Netherlands, Austria, Estonia and Finland, which have opposed debt mutualisation schemes in Europe due to the possibility of moral hazard arising and possible higher debt service costs. In July 2020, however, the EU approved a €750 billion economic recovery fund which would be partly financed by common European debt and on the 21st of October the European commission issued inaugural social bonds in the value of €17 billion to be transferred to the beneficiary member states in the form of loans.

Eurobonds have been a controversial topic along the years and the arguments of both their opponents and proponents have been consistent.

The issuance of Eurobonds would imply the creation of a huge market of a safe and liquid asset which could rival in size with the US Treasury bond market. Arguably, the liquidity of such assets would imply lower transaction costs and, consequently, lower yields. Countries with a smaller liquidity in their national bond markets would benefit the most from the liquidity gains in Eurobonds. These countries would be benefiting from smaller yields already, since having their public debt guaranteed by the union would significantly decrease their risk of default, which is the main determinant of yield spreads in the Euro Area (Missale and Favero, 2012).

Moreover, Eurobonds could be an important tool to stop debt crisis in the Euro Area such as the European Sovereign debt crisis, by mitigating sudden shifts in market sentiment and stopping contagion between member states. If fiscally vulnerable countries had at least a portion of their public debt guaranteed by the union, financial markets would not react so irrationally in some situations and self-fulfilling crisis could be avoided.

The opponents of Eurobonds, however, fear that a moral hazard issue would arise if the Euro Area countries mutualised their debt. Since each country's public debt would be guaranteed in some way by the rest of the union, an incentive would be created to issue too much debt, while if no debt was mutualised interest rates would respond and prevent too much debt issuance. Without any mechanism aiming to tackle this problem, countries would know they could overaccumulate public debt without consequence, since the rest of the union would always intervene in case of possible default.

Furthermore, the highest rated countries with a sound fiscal position, such as Germany and other northern European countries, believe their debt servicing costs would increase in an Eurobonds scheme, at least initially, as they would be guaranteeing the debt of countries with lower ratings.

The format of Eurobonds I explore in this paper is the Blue Bond format, which was first proposed by (Von Weizsäcker and Delpla, 2011). In this format of debt mutualisation, countries can issue two types of bonds: blue and red. Blue bonds can only be issued up to a certain % of each country's GDP (60% in the initial proposal). The senior Blue bonds tranche is pooled among all participating countries and is secured collectively by them. The junior red tranche, from 60% of GDP beyond, is issued by each country and is not guaranteed by the rest of the union. The intention behind the division of blue and red bonds was to enforce fiscal discipline by making borrowing more expensive at the margin and provide incentives for countries to adopt a sound fiscal stance. According to the proposal, red bonds should be largely kept out of the banking system, but I relax that assumption in this paper.

Although this proposal was deemed as suboptimal in (Beetsma and Mavromatis, 2014), as it did not provide an incentive for less disciplined countries to lower their level of public debt, I analyse whether or not it can still leave all the union better off against the case of not mutualizing any debt. Since default on the blue portion of a country's debt is avoided under this scheme, there is a possibility that avoiding a fixed output default cost and the otherwise ensuing spillover effects in the banking system outweighs the losses faced by fiscally responsible countries if they provide financial support. In Chapter 2, I go through the most relevant literature for the context of this paper. In chapters 3 and 4, I shall lay down the details of the model and proceed to analyse the implications of joining the Blue and Red Bonds scheme for Core, the fiscally responsible country, under different shock realizations and parameter values.

2. Literature Review

The concept of Eurobonds and its different formats have been subject to analysis by several authors. (Claessens et al., 2012) reviewed in detail 5 different proposals for common euro debt from the perspective of debtors, creditors and how the transition from national issuances to Eurobonds would play out. They also analysed the legal and institutional challenges that the adoption of Eurobonds would impose, especially how Eurobonds could clash with Article 125 of the European treaty, which declares that no member state can assume the debt of another member state.

The idea of conditional Eurobonds was defended in (Muellbauer,2013). In this paper, the author proposes all Euro Area sovereign borrowing to be undertaken in a format of jointly guaranteed Eurobonds, which aims to combat the classical moral hazard problem by obliging each country to pay a risk premium to a joint debt management agency. This risk premium would be dependent on competitiveness, public and private debt to GDP ratio and the fall-out from housing market crises, as they were proven to be the most important economic fundamentals in determining credit risk.

(Gilbert et al.,2013) argue for a complete centralization of debt issuance with an Eurobonds format under which European member states lose the ability to raise debt individually and an independent budgetary authority provides countries with the necessary Eurobond funding, contingent on the compliance with the fiscal policies defined in the Maastricht Treaty or on the implementation of an adjustment programme. In addition, they propose different features they deemed fundamental to improve the existing Eurobond proposals and that are present in their own proposal: Complete centralizations of debt issuance, banning countries from accessing money and capital markets on their own initiative and having an independent budgetary authority controlling the access to Eurobonds

(Baglioni and Cherubini, 2011) devised a structural model of sovereign credit risk where interest rate spreads depended on the government's budget balance, debt-to-GDP ratio and on the country's expected GDP growth. They applied this model to a scheme in which Eurobonds could be issued up to a limit of 40% of GDP and there were sufficient guarantees for the interest payments on these securities to be equal to the risk-free interest rate, and found Eurobonds to be able to reduce debt servicing costs for some countries in the Euro Area without increasing those of others. The increase in interest rates of the national junior tranches of debt was also found to be a strong mechanism to fight moral hazard, as it makes the marginal cost of public debt higher and creates an incentive for countries not to issue too much debt.

Another formal model that represented an important contribution to the subject was provided in (Beetsma and Mavromatis, 2014). The authors built a two-period model with 2 countries, one of them with a political distortion, and used it to study the debt accumulation incentives for that country under different Eurobonds schemes, as well as how structural reform could be boosted through a well devised Eurobonds scheme. They concluded that a limited debt repayment guarantee provided by Core, the fiscally responsible country, was able to induce Periphery, the country with a political distortion, to reduce its public debt, unlike the case of an unlimited debt repayment guarantee. Their most important conclusion for the scope of this paper, however, was that a Blue and Red Bonds scheme was incapable of inducing Periphery, the politically distorted country, to reduce its debt level under any circumstances. I shall build on their work by analysing what this format would entail for Core in the presence of default costs.

3. The Model:

I use the model employed in *An Analysis of Eurobonds (Beetma and Mavromatis, 2014).* In this two-period model, there are two different countries, named Core and Periphery, which can choose to enter a debt mutualisation scheme. Periphery suffers from a political distortion that leads to a higher debt level than the socially desirable. Its debt is sold in the international markets to risk-neutral investors, which means the interest rate of such debt is the lowest one that ensures investors are repaid in expected terms.

3.1. Core

Core's economy features a large number of competitive risk-neutral banks. The country is financially connected to Periphery via the credit these banks provide to Periphery's economy. In the first period, Core's banks invest in a portion $\boldsymbol{\delta} \geq \mathbf{0}$ of Periphery's debt. In the second period, Core's banks are repaid, partially or totally, an amount $\mathbf{z}$, which makes the total payment of Periphery to Core's banks equal to $\boldsymbol{\delta}\mathbf{z} \geq \mathbf{0}$. The amount of resources available for provision from Core's banks to the country's firms in period 2 is given by $\boldsymbol{q} - \boldsymbol{\delta b} + \boldsymbol{\delta z}$, where $\boldsymbol{\delta b}$ is the amount of Periphery's debt purchased in the 1st period by Core's banks and $\boldsymbol{q} > \mathbf{0}$ is a constant large enough such that Core is always able to incur any cost that may arise from a Periphery's default or to provide the necessary rescue funds to avoid default in Periphery. Core's resources in the second period are determined by a production function of the type $\boldsymbol{y} = \boldsymbol{Ak}$, where $\boldsymbol{A} \geq \mathbf{1}$ is a capital productivity constant and $\boldsymbol{k}$ is the amount of capital employed in the production process. Capital fully depreciates in each period and therefore Core's firms must finance themselves by borrowing from the country's banks at risk-free rate. There are two channels through which a Periphery's partial or total default affects Core's second period resources: When Periphery defaults, Core's banks are not repaid in full, which lowers $\boldsymbol{z}$, and therefore also lowers the banks' credit provision. The larger the amount Periphery defaults on, the lower $\boldsymbol{z}$ is and the lower Core's resources will be. Furthermore, Core suffers a fixed default output cost $\boldsymbol{\psi} \geq \mathbf{0}$, which is the same independently of the size of Periphery's default.

3.2. Periphery

Periphery's social welfare function is given by:

$$U_S(f_1, f_2, g_1, g_2) = u(f_1 + g_1) + E[f_2 + g_2],$$

where $f_t \geq 0$ and $g_t \geq 0$ are two different public goods and $E[\cdot]$ denotes the expectations operator taken at the start of period 1. As the function portrays, society as a whole is indifferent between both goods and is also indifferent between both periods. The utility function is twice continuously differentiable, with $u'(x) > 0$, $u''(x) \leq 0$ and $u'(x), u''(x) \rightarrow 0$ as $x \rightarrow \infty$. Furthermore, for the sake of convenience, it is also assumed that $u(0) = 0$ and $u'(1) = 1$.

The political distortion present in this country's economy comes in the form of political parties that value public goods differently and, unsure about whether or not they will be running the country in the second period, try to maximize their own utility by increasing consumption of the public good they value the most in the first period, by issuing more debt than society at large (whose optimal debt level is 0, under a positive interest rate) would desire. This distortion is modelled by having two political parties, **F** and **G**, whose utility functions are the following:

$$U_F(f_1, f_2) = u(f_1) + E[f_2],$$

$$U_G(g_1, g_2) = u(g_1) + E[g_2].$$

We assume that party F is in office in period 1 and that the probability of re-election is exogenous and denoted by p, where $0 < p \leq 1$.

Core's resources for both periods are given by:

$$h_1 = 1 + b,$$

$$h_2 = 1 + \varepsilon - b(1 + r),$$

where b denotes the amount of debt that is issued and r is the interest rate on said debt. ε is a uniformly distributed random shock with mean $E[\varepsilon] = 0$. Its value ranges from $[\varepsilon L, \varepsilon H[$, where $\varepsilon L = -\varepsilon H$. Its density function is given by $g(\varepsilon) = \frac{1}{\varepsilon H - \varepsilon L} = \frac{1}{2\varepsilon H}$.

Moreover, there is a lower limit for second period resources denoted by ρ_L, where $0 < \rho_L < 1$ which the country values above its debt obligations. The country will honour its debt servicing as far as second period resources remain equal or larger than ρ_L.

Furthermore, for reasons that are explained in (Beetma and Mavromatis, 2014), the following assumptions are made:

- $1 + \varepsilon L = \rho_L$
- p **is sufficiently high such that** $b^{NG} \leq \varepsilon H$

Since Periphery's second period resources have a random component in ε, whether or not the country defaults on its debt will depend on the size of that random shock.

4. Blue and Red Bonds

4.1. Introduction

In (Beetma and Mavromatis, 2014), it was proven that no matter what the threshold $\tilde{b}$ for the amount of blue bonds that Periphery can issue, the blue and red bonds format of debt mutualisation is not able to induce Periphery to reduce its debt. Under a blue and red bonds scheme, Periphery's debt will be equal or larger than in the absence of a debt mutualisation scheme: If $\tilde{b} \leq b^{NG}$, where b^{NG} is Periphery's chosen debt level in the case of no debt mutualisation and $\tilde{b}$ is the maximum amount of blue bonds allowed, $b = b^s + b^n = b^{NG}$, where $\boldsymbol{b^s}$ and $\boldsymbol{b^n}$ are the blue and red bonds' portion of Core's debt, respectively. If $\widetilde{\boldsymbol{b}} > \boldsymbol{b^{NG}}$, $\boldsymbol{b^n} = \boldsymbol{0}$ and Periphery sets its debt level at $\boldsymbol{b} = \boldsymbol{b^s} = \widetilde{\boldsymbol{b}}$ or at $\boldsymbol{b} = \boldsymbol{b^s} = \boldsymbol{b^G} > \boldsymbol{b^{NG}}$. In both cases, Periphery sets the amount of blue debt at $\widetilde{\boldsymbol{b}}$, since it pays no interest rate on blue bonds and thus the country issues all the blue debt that it is allowed to.

In this paper, I will focus on the cases where $\widetilde{\boldsymbol{b}} \leq \boldsymbol{b^{NG}}$. In these cases, the debt level of Periphery is the same as if there was no debt mutualisation. However, Periphery's society utility increases, as the country's debt-servicing costs are lower under this scheme, which will lead to larger resources in the second period. This utility gain is achieved at the expense of Core, which must act if Periphery is not able to honour its blue debt commitments. If Periphery's shock realization $\boldsymbol{\varepsilon}$ in the second period is not large enough to repay the blue debt that it issued in the first period, Core must come to rescue by providing financial support and covering the amount Periphery is defaulting on. However, this does not mean that, even with its limitations, a Blue and Red Bonds cannot be beneficial for Core. It is easy to see how, for some values of $\boldsymbol{\delta}$, the fraction of Periphery's bonds that Core's banks buy in the first period, Core can achieve a higher utility by bailing Periphery out and saving its banks from suffering impairment losses. Furthermore, by including a fixed output cost $\boldsymbol{\psi}$ for Core when Periphery defaults on its debt, the range of situations for which the Blue and Red Bonds format becomes beneficial for Core increases. In chapter **2.2**, I will figure out under what situations the Blue and Red Bonds format increases utility for Core. It is worth noting that under the Blue and Red Bonds scheme, the portion $\boldsymbol{\delta}$ of Periphery's blue and red bonds that Core buys will be assumed to be the same. However, this would not have to be necessarily the case, and the results would change if, for example, there was some limit for the amount of Red Bonds that Core's banks could buy.

4.2. Core's Resources

In order to know whether or not it is favourable for Core to opt in the Blue and Red Bonds agreement instead of not being part of any debt mutualisation agreement, we must compare the different possible outcomes for both schemes.

Under the Blue and Red Bonds scheme, Core guarantees the blue bonds portion of Periphery's debt, $\boldsymbol{b^s = \tilde{b}}$, and is called to help whenever Periphery's resources are not sufficient to repay those bonds, partially or totally. Those situations correspond to the shocks $\boldsymbol{\varepsilon L \leq \varepsilon < \varepsilon L + \tilde{b}}$. Core's resources under three different states of the world are exposed in **Table 1**.

Table 1

	Core's Period 2 resources
Shock (ε)	**Blue and Red Bonds**
$[\varepsilon L, \varepsilon L + \tilde{b}[$	$A[q - \delta b + \delta\tilde{b}]$ - $(\varepsilon L + \tilde{b} - \varepsilon) - \psi$
$[\varepsilon L + \tilde{b}, \varepsilon L + \tilde{b} + b^n(1 + r^n)[$	$A[q - \delta b + \delta\tilde{b} + \delta(\varepsilon - \varepsilon L - \tilde{b})]$ - ψ
$[\varepsilon L + \tilde{b} + b^n(1 + r^n), \varepsilon H]$	$A[q - \delta b + \delta\tilde{b} + \delta b^n(1 + r^n))$

In the first range of shocks in the table, $\boldsymbol{[\varepsilon L, \varepsilon L + \tilde{b}[}$, Periphery defaults fully on its red bonds and fully or partially on its blue bonds. In this scenario, core's production function includes the blue bonds that were repaid to the banks but does not include the repayment of red bonds, since those are not guaranteed by Core. The second term, $\boldsymbol{(\varepsilon L + \tilde{b} - \varepsilon)}$ corresponds to the amount of resources Core provides to avert Periphery's default on blue bonds. Finally, there is a default output cost $\boldsymbol{\psi}$ imposed by Periphery's default on the red bonds.

In the second range of shocks in the table, $[\boldsymbol{\varepsilon L}+\widetilde{\boldsymbol{b}}, \boldsymbol{\varepsilon L}+\widetilde{\boldsymbol{b}}+\boldsymbol{b}^{\boldsymbol{n}}(\mathbf{1}+\boldsymbol{r}^{\boldsymbol{n}})[$, Periphery is able to repay the blue bonds portion of its debt but defaults partially or fully on its red bonds. When this situation occurs, Core's production function includes the total repayment of blue bonds by core plus the repayment of $\boldsymbol{\delta}(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon L}-\widetilde{\boldsymbol{b}})$, which is the available amount of resources Periphery has to repay its red bonds' obligations. Under this situation, there is no financial support from Core and the country suffers an output default cost again.

Finally, there is the range of shocks $[\boldsymbol{\varepsilon L}+\widetilde{\boldsymbol{b}}+\boldsymbol{b}^{\boldsymbol{n}}(\mathbf{1}+\boldsymbol{r}^{\boldsymbol{n}}), \boldsymbol{\varepsilon H}]$ in which Periphery is able to pay all its debt service costs by repaying both the blue bonds and the red bonds plus their respective interest. Hence, Core's resources are given by the production function where the total repayment of debt by Periphery is included and, logically, there is no output default cost.

In the absence of a debt mutualisation scheme such as the Blue and Red Bonds scheme, Core's resources are described by characterising two different ranges of shocks, which are presented in **Table 2**.

Table 2

	Core's Period 2 resources
Shock (ε)	**No Debt Mutualisation**
$[\varepsilon L, \varepsilon L+b^{NG}(1+r^{NG})[$	$A[q-\delta b+\delta(\varepsilon-\varepsilon L)]$ - ψ
$[\varepsilon L+b^{NG}(1+r^{NG}), \varepsilon H]$	$A[q-\delta b+\delta b(1+r^{NG})]$

Under the first range of shocks, $[\boldsymbol{\varepsilon L}, \boldsymbol{\varepsilon L}+\boldsymbol{b}^{\boldsymbol{NG}}(\mathbf{1}+\boldsymbol{r}^{\boldsymbol{NG}})[$, Periphery defaults partially or fully on its debt and Core's resources are given by the production function which includes the amount of resources available, $(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon L})$, that Periphery employs in its debt-servicing payments. The default cost is present again.

Under the second range of shocks, $[\boldsymbol{\varepsilon L} + \boldsymbol{b^{NG}}(\mathbf{1} + \boldsymbol{r^{NG}}), \boldsymbol{\varepsilon H}]$, Core's resources correspond to the production function which includes the total repayment of Periphery's debt-servicing costs and no output default cost.

Considering the fact that under $\boldsymbol{b^s} + \boldsymbol{b^n} = \boldsymbol{b^{NG}}$ we know that $\boldsymbol{b^{NG}}(\mathbf{1} + \boldsymbol{r^{NG}}) > \widetilde{\boldsymbol{b}} + \boldsymbol{b^n}(\mathbf{1} + \boldsymbol{r^n})$, we can compare core's period 2 resources under all the ranges of shocks that affect both the Blue and Red Bonds and the No Debt Mutualisation scenarios, which are exposed in Table 3.

Table 3

	Core's Period 2 resources	
Shock (ε)	**Blue and Red Bonds**	**No Debt Mutualisation**
$[\varepsilon L, \varepsilon L + \tilde{b}[$	$A[q - \delta b + \delta\tilde{b}]$ - $(\varepsilon L + \tilde{b} - \varepsilon) - \psi$	$A[q - \delta b + \delta(\varepsilon - \varepsilon L)] - \psi$
$[\varepsilon L + \tilde{b}, \varepsilon L + \tilde{b} + b^n(1 + r^n)[$	$A[q - \delta b + \delta\tilde{b} + \delta(\varepsilon - \varepsilon L - \tilde{b})]$ - ψ	$A[q - \delta b + \delta(\varepsilon - \varepsilon L)] - \psi$
$[\varepsilon L + \tilde{b} + b^n(1 + r^n), \varepsilon L + b^{NG}(1 + r^{NG})[$	$A[q - \delta b + \delta\tilde{b} + \delta b^n(1 + r^n)$	$A[q - \delta b + \delta(\varepsilon - \varepsilon L)] - \psi$
$[\varepsilon L + b^{NG}(1 + r^{NG}), \varepsilon H]$	$A[q - \delta b + \delta\tilde{b} + \delta b^n(1 + r^n)$	$A[q - \delta b + \delta b(1 + r^{NG})]$

Then, I subtract core's period 2 resources in the no debt mutualisation scenario from the Blue and Red Bonds' one for all the ranges of shocks and present the results of these subtractions in **Table 4**. These results show what is the difference from opting to join the blue and red bonds scheme instead of not mutualising any debt. When they are larger (smaller) than zero, the Blue and Red Bonds program is beneficial (disadvantageous) for Core. When they are equal to zero, Core would have been indifferent between the two schemes.

Table 4

Shock (ε)	Core's Period 2 net result from opting in the Blue and Red Bonds Scheme
$[\varepsilon L, \varepsilon L + \tilde{b}[$	$A\delta\left(\tilde{b} - \varepsilon + \varepsilon L\right) - \varepsilon L - \tilde{b} + \varepsilon$
$[\varepsilon L + \tilde{b}, \varepsilon L + \tilde{b} + b^n(1+r^n)[$	0
$[\varepsilon L + \tilde{b} + b^n(1+r^n),$ $\varepsilon L + b^{NG}(1+r^{NG})[$	$A\delta[\tilde{b} + b^n(1+r^n) - \varepsilon + \varepsilon L] + \psi$
$[\varepsilon L + b^{NG}(1+r^{NG}), \varepsilon H]$	$A\delta(b^n r^n - b r^{NG})$

Under the range of shocks $[\boldsymbol{\varepsilon L}, \boldsymbol{\varepsilon L} + \widetilde{\boldsymbol{b}}[$, Core offers financial support and covers the amount of mutualised debt (blue bonds) that Periphery would default on, given by $(\widetilde{\boldsymbol{b}} - \boldsymbol{\varepsilon} + \boldsymbol{\varepsilon L})$. Consequently, it averts the losses its banks would face in case of a blue bonds default: $\boldsymbol{\delta}(\widetilde{\boldsymbol{b}} - \boldsymbol{\varepsilon} + \boldsymbol{\varepsilon L})$. The size of $\boldsymbol{\Psi}$ is irrelevant in these situations as default occurs in both schemes, although the default is smaller in the case of the Blue and Red Bonds scheme, as Periphery only ends up defaulting on its red debt. This net result is only positive when $\boldsymbol{A\delta} > \mathbf{1}$.

When the shock ranges between $[\boldsymbol{\varepsilon L} + \widetilde{\boldsymbol{b}}, \boldsymbol{\varepsilon L} + \widetilde{\boldsymbol{b}} + \boldsymbol{b^n}(\mathbf{1} + \boldsymbol{r^n})[$, the difference between both schemes for Core's period 2 resources is $\mathbf{0}$, which makes perfect sense: For this range of shocks, Core does not need to provide Periphery any financial support, since the latter is always able to repay the blue bonds portion of its debt. So, under both circumstances, Core's banks are repaid the same amount $\boldsymbol{\delta}\widetilde{\boldsymbol{b}} + \boldsymbol{\delta}\left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon L} - \widetilde{\boldsymbol{b}}\right) = \boldsymbol{\delta}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon L})$. Once again, default happens in both schemes and so, Core's resources will be the same regardless of what scheme is in place.

For the third range of shocks, $[\boldsymbol{\varepsilon L} + \widetilde{\boldsymbol{b}} + \boldsymbol{b^n}(\mathbf{1} + \boldsymbol{r^n}), \boldsymbol{\varepsilon L} + \boldsymbol{b^{NG}}(\mathbf{1} + \boldsymbol{r^{NG}})[$, $\boldsymbol{\psi}$ comes into play, since this range of shocks corresponds to a situation under which Periphery would only default on part of its debt in the No Debt Mutualisation scenario. Besides this output default cost, the difference between both schemes is defined by all the blue and red bonds' debt-service costs that are honoured in one scheme minus the amount $(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon L})$ that Periphery would manage to repay under the other scheme. Since $(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon L}) \geq \widetilde{\boldsymbol{b}} + \boldsymbol{b^n}(\mathbf{1} + \boldsymbol{r^n}),$ the net result will only be positive for situations in which $\boldsymbol{\psi} > |\boldsymbol{A\delta}[\widetilde{\boldsymbol{b}} + \boldsymbol{b^n}(\mathbf{1} + \boldsymbol{r^n}) - \boldsymbol{\varepsilon} + \boldsymbol{\varepsilon L}]|$.

At last, when the shock is such that Core does not default in any of the schemes ($[\boldsymbol{\varepsilon L} + \boldsymbol{b^{NG}}(\mathbf{1} + \boldsymbol{r^{NG}}), \boldsymbol{\varepsilon H}]$), what determines core's period 2 net result from opting in the Blue and Red Bonds scheme is the interest repayments to its banks by Periphery: The interest repayments on the red bonds they bought $(\boldsymbol{\delta b^n r^n})$ minus the interest repayments on the totality of Periphery's debt in the No Debt Mutualisation scenario $(\boldsymbol{\delta b r^{NG}})$. Since we know that $\boldsymbol{b^{NG}}(\mathbf{1} + \boldsymbol{r^{NG}}) > \widetilde{\boldsymbol{b}} + \boldsymbol{b^n}(\mathbf{1} + \boldsymbol{r^n})$, it follows that $\boldsymbol{b^{NG} r^{NG}} > \boldsymbol{b^n r^n}$. Therefore, and considering the fact that $\boldsymbol{A} \geq \mathbf{1}$, the net result from participating in the Blue and Red Bonds program will always be negative for Core in this range of shocks.

However important knowing the different possibilities for the net benefit of participating in the Blue and Red Bonds program may be, Core cannot choose to participate or not in just some of these situations. It must weigh the probability of each of those and analyse whether or not participating in the program will grant the country a higher expected utility.

Let **D** denote Core's period 2 net result from opting in the Blue and Red Bonds proposal:

$$E[D] = \int_{\varepsilon L}^{\varepsilon L + \tilde{b}} \left[A\delta\left(\tilde{b} - \varepsilon + \varepsilon L\right) - \varepsilon L - \tilde{b} + \varepsilon\right] g(\varepsilon) d\varepsilon$$

$$+ \int_{\varepsilon L + \tilde{b} + b^n(1+r^n)}^{\varepsilon L + b^{NG}(1+r^{NG})} \left[A\delta\left[\tilde{b} + b^n(1 + r^n) - \varepsilon + \varepsilon L\right] + \psi\right] g(\varepsilon) d\varepsilon$$

$$+ \int_{\varepsilon L + b^{NG}(1+r^{NG})}^{\varepsilon H} A\delta\left(b^n r^n - b^{NG} r^{NG}\right) g(\varepsilon) d\varepsilon$$

As long as $\boldsymbol{E}[\boldsymbol{D}] > \mathbf{0}$, Core's period 2 resources will be, on average, higher under the Blue and Red Bonds scheme and Core's utility will be higher for all utility functions with $\boldsymbol{u}'(\cdot) > \mathbf{0}$ and $\boldsymbol{u}''(\cdot) \geq \mathbf{0}$.

If, instead, Core's utility function has $\boldsymbol{u''(\cdot) < 0}$, a more realistic assumption, $\boldsymbol{E[D] > 0}$ is not enough to assure Core's expected utility will be higher under the Blue and Red Bonds scheme. For that utility function, the expected utility of both schemes must be compared. In this paper, I shall focus only on the analysis of $\boldsymbol{E[D]}$.

4.3. Parametrization

I solve the model in order to assess the value of $\boldsymbol{E[D]}$ for different values of the most relevant parameters and to show graphically how these parameters affect Core's period 2 average resources. **Table 5** shows the parameter values when each parameter's influence on $\boldsymbol{E[D]}$ is not being subject to analysis.

Table 5

Parameter	Value
ψ	0.05
δ	0.2
εL	-0.4
εH	0.4
p	0.85
$\tilde{b}$	0.2
A	1.5

Furthermore, I assume Periphery's political party F's utility function to be $\boldsymbol{u(f_n) = 2\sqrt{f_n}}$, which complies with all the assumptions laid down in Chapter 3.2. With this utility function, $\boldsymbol{p \geq \sqrt{\frac{1}{\varepsilon H+1}}}$ must always hold since $\boldsymbol{p}$ has to be large enough such that $\boldsymbol{b^{NG} \leq \varepsilon H}$.

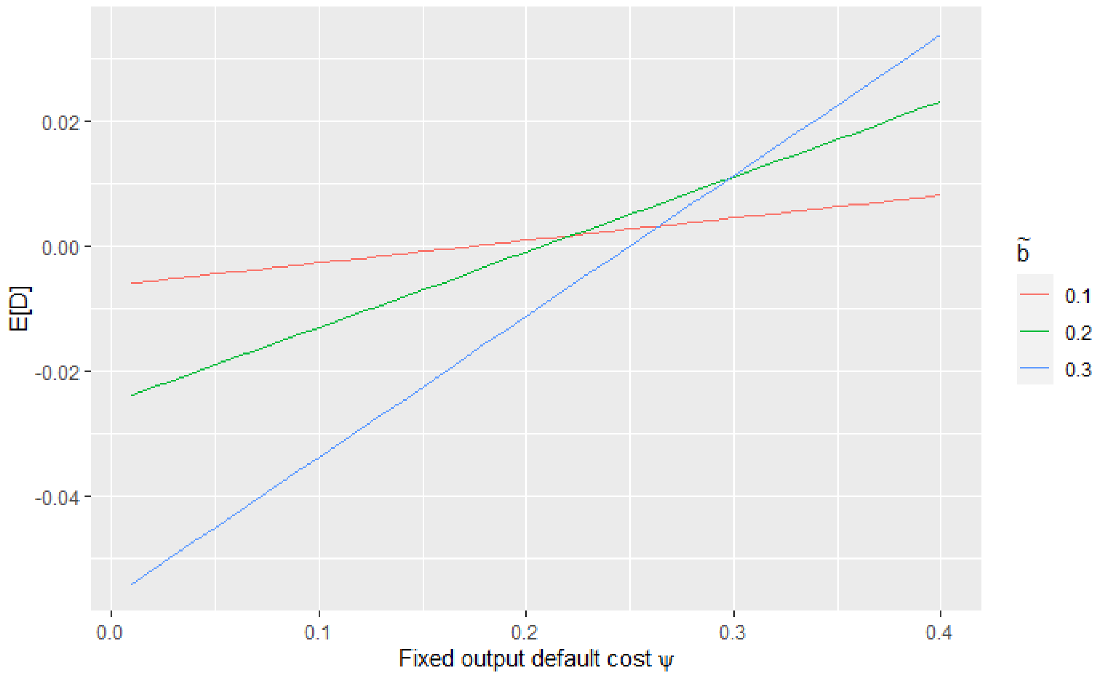
For the purpose of this analysis, the absolute value of the parameters is not what matters the most, but to analyse the different dynamics of the model.

4.4. Results

4.4.1. Fixed Output Default Cost (Ψ)

I start by analysing the impact of the fixed output default cost $\boldsymbol{\Psi}$ on $\boldsymbol{E}[\boldsymbol{D}]$, which is exposed in **Figure 1** for three different values of $\tilde{\boldsymbol{b}}$.

Figure 1



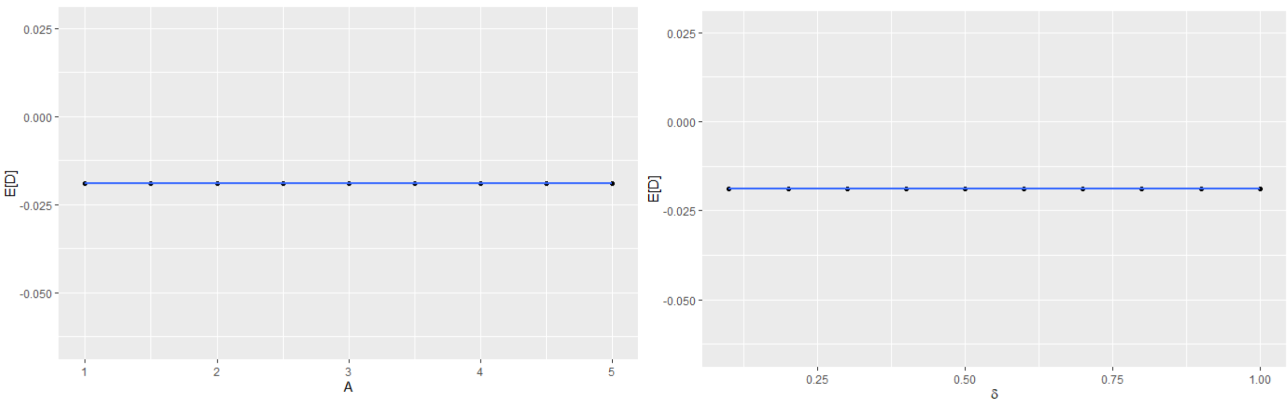
As the figure depicts, the effect of $\boldsymbol{\Psi}$ on the expected net benefit from joining the Blue and Red Bonds Scheme is positive and linear regardless of the value of $\tilde{\boldsymbol{b}}$. That was to be expected since $\boldsymbol{\Psi}$ is a single exogeneous variable present in the second term of the $\boldsymbol{E}[\boldsymbol{D}]$.

Figure 1 also shows that the effect of $\boldsymbol{\Psi}$ on $\boldsymbol{E}[\boldsymbol{D}]$ is stronger for higher values of $\tilde{\boldsymbol{b}}$, since the slope of the lines increases the larger $\tilde{\boldsymbol{b}}$ is. That is explained by the fact that when the limit $\tilde{\boldsymbol{b}}$ for the amount of blue bonds that Periphery can issue increases, the range of shocks under which Periphery would only default in the No Debt Mutualisation scenario also increases, and that is the range of shocks where $\boldsymbol{\Psi}$ comes into play. However, despite the positive relation between $\tilde{\boldsymbol{b}}$ and $\boldsymbol{\Psi}$'s effect on $\boldsymbol{E}[\boldsymbol{D}]$, a larger output default shock $\boldsymbol{\Psi}$ is needed for $\boldsymbol{E}[\boldsymbol{D}]$ to be positive when $\widetilde{\boldsymbol{b}}$ increases. That happens because $\widetilde{\boldsymbol{b}}$ has a negative effect on $\boldsymbol{E}[\boldsymbol{D}]$, as I show later on.

4.4.2. $\boldsymbol{A}$ and $\boldsymbol{\delta}$

The capital productivity constant $\boldsymbol{A}$ and the share of Periphery's debt Core's banks invest in ($\boldsymbol{\delta}$) have both no impact over the attractiveness of the Blue and Red Bonds scheme for Core, as **Figure 2** shows. That happens because these two parameters play the same role in all three different states of the world, and thus their values are irrelevant for the final result of $\boldsymbol{E}[\boldsymbol{D}]$.

Figure 2

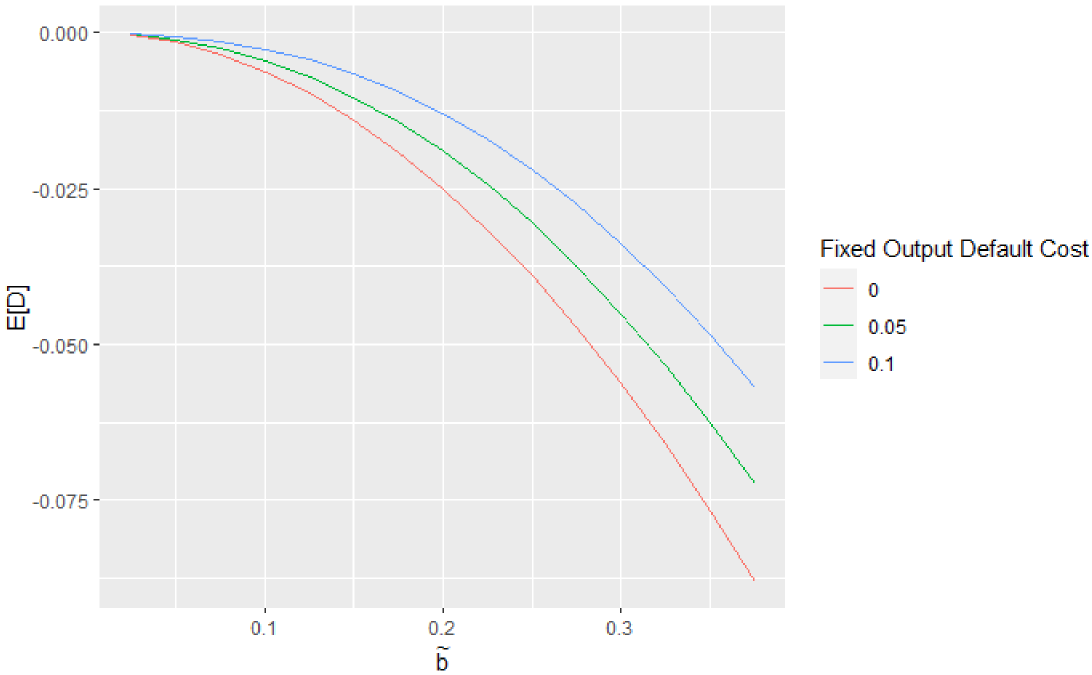


4.4.3. $\widetilde{\boldsymbol{b}}$

Finally, I analyse the impact of the Blue Bonds issuance limit $\widetilde{\boldsymbol{b}}$ on $\boldsymbol{E}[\boldsymbol{D}]$, depicted in **Figure 3**. The figure shows the impact of $\widetilde{\boldsymbol{b}}$ on $\boldsymbol{E}[\boldsymbol{D}]$ for three different values of $\boldsymbol{\Psi}$: 0.05, 0.1 and 0.15. That impact is non-linear for the three curves, which have always a negative slope. That happens because $\widetilde{\boldsymbol{b}}$ has different effects on $\boldsymbol{E}[\boldsymbol{D}]$: On one hand, when $\widetilde{\boldsymbol{b}}$ increases, $\boldsymbol{\varepsilon L} + \widetilde{\boldsymbol{b}} + \boldsymbol{b^n}(\boldsymbol{1} + \boldsymbol{r^n})$ decreases, which in turn increases the range of shocks under which $\boldsymbol{\Psi}$ comes into play, affecting $\boldsymbol{E}[\boldsymbol{D}]$ positively.

The interest paid on red bonds $\mathbf{b^n r^n}$ also decreases when $\widetilde{\boldsymbol{b}}$ increases, affecting $\boldsymbol{E}[\boldsymbol{D}]$ positively as well. On the other hand, the amount $(\widetilde{\boldsymbol{b}} - \boldsymbol{\varepsilon} + \boldsymbol{\varepsilon L})$ Core has to cover when Periphery defaults on its blue bonds increases with $\widetilde{\boldsymbol{b}}$, which in this case affects $\boldsymbol{E}[\boldsymbol{D}]$ negatively, since $\boldsymbol{A\delta} < \mathbf{1}$. Besides that, the gap between $(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon L})$ and $\boldsymbol{b^s} + \boldsymbol{b^n}(\mathbf{1} + \boldsymbol{r^n})$ gets larger, further affecting $\boldsymbol{E}[\boldsymbol{D}]$ negatively.

Figure 3



Throughout the entirety of the curves, the slope is negative because the negative effect of $\widetilde{\boldsymbol{b}}$ on $\boldsymbol{E}[\boldsymbol{D}]$ outweighs the positive one and it becomes stronger along the curves, which become steeper along $\widetilde{\boldsymbol{b}}$'s domain ($\widetilde{\boldsymbol{b}} \leq 0.38$). The smaller $\boldsymbol{\Psi}$ is, the steeper the curves are, because the effect of the avoided output default cost when shocks are such that Periphery would only default if no debt was mutualised becomes weaker.

5. Conclusion

In this paper, I compare Core's second period resources under a Blue and Red Bonds debt mutualisation scheme and under a case of no debt mutualisation. Although not able to induce Periphery to reduce its debt, a Blue and Red Bonds format of debt mutualisation can be beneficial for Core when the fixed output cost that happens when Periphery defaults on some of its debt is high enough. I find that the expected amount of resources Core has in period 2 depends negatively on the Blue Bonds threshold, positively on the fixed output default cost and is not affected by the capital productivity and the share of Periphery's debt owned by its banks.

Moreover, I find that a smaller fixed output default cost exacerbates the negative effect of the Blue Bonds threshold on Core's expected resources and that a lower Blue Bonds threshold mitigates the fixed output default cost's positive effect on Core's expected resources.

6. References

Baglioni, Angelo S.; Cherubini, Umberto. 2011. A theory of eurobonds. SSRN Electronic Journal. 10.2139/ssrn.1928573.

Beetsma, Roel; Mavromatis, Kostas. 2014. An analysis of eurobonds. *Journal of international money and finance*, 45: 91-111.

Claessens, Stijn; Mody, Ashoka; Vallée, Shahin. 2012. Paths to Eurobonds. IMF Working Paper No. 12/172

Delpla, Jaques; Von Weizsäcker, Jakob. 2011. *Eurobonds: The blue bond concept and its implications*. Bruegel Policy Contribution No. 2011/02

Gilbert, Niels D.; Hessel, Jeroen; Verkaart, Silvie. 2013. Towards a Stable Monetary Union: What Role for Eurobonds?. De Nederlandsche Bank Working Paper No. 379

Missale, Alessandro; Favero, Carlo A. 2012. Sovereign spreads in the Euro area: Which prospects for a Eurobond?. Econ. Policy 27 (70), 231–273.

Muellbauer, John. 2013. Conditional eurobonds and the eurozone sovereign debt crisis. *Oxford Review of Economic Policy* 29.3: 610-645.