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Steffen Hoernig

The Strength of the Waterbed Effect Depends on Tariff Type

Steffen Hoernig*

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Abstract

We show that the waterbed effect, i.e. the pass-through of a change in one price of a firm to its other prices, is much stronger if the latter include subscription rather than only usage fees. In particular, in mobile network competition with a fixed number of customers, the waterbed effect is full under two-part tariffs, while it is only partial under linear tariffs.

Keywords: Waterbed effect; two-part tariff; linear tariff; mobile termination; two-sided platforms.

JEL: D43, L13, L51

1 Introduction

The "waterbed effect" describes the interdependence between prices at multiple-good firms and multi-sided platforms. As much as a waterbed rises on one side if it is pressed down on the other, firms may optimally change prices if some other price is forced to a different level, for example through regulatory interventions. The extent of the waterbed effect can be a contentious issue when it would weaken the effectiveness of the regulatory measures. In the debate about the downward regulation of the charges paid by fixed networks to mobile networks for routing calls from the former to their receivers on the latter, the so-called "mobile termination rates", mobile networks have claimed that the result would be higher retail prices for mobile customers, while regulators argued there would be no effect.¹

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¹See Schiff (2008) for an introduction to the waterbed effect and a discussion of these issues.

In this note we show how the waterbed effect depends on the type of tariff that is charged to the unregulated side of the market. On the regulated side, the firm receives a fixed payment per customer of the unregulated side. This payment can be the profits from fixed-to-mobile termination of calls, or advertising, or any other profits that depend on the customer's existence (rather than his usage). We determine the pass-through for two-part tariffs, where customers pay for subscription and usage, and for linear tariffs where they only pay for usage.² We show that the waterbed effect is much stronger under two-part than under linear tariffs; in particular, under the assumption of a fixed number of mobile subscribers we show that under two-part tariffs the waterbed effect is full, while it is only partial with linear tariffs. This implies that downward regulation of some price leads to a stronger rise negative effect on clients of the other services if the latter are charged a multi-part tariff. In particular, this result contracts mobile networks' contention that lower fixed-to-mobile termination rates would disproportionately hurt customers on pre-pay tariffs.

The issue of the strength of the waterbed effect has been studied in both in theoretical and empirical work. Wright (2002) remains the most important theoretical treatment of fixed-to-mobile interconnection. He shows, generically, that if the pass-through of fixed costs to profits is full (partial), networks are indifferent about termination rates (jointly want to set them at the monopoly level). Below we show that these cases arise due to competition in two-part or linear tariffs, respectively.³

Genakos and Valletti (2011a, 2011b) provide an empirical study of the waterbed effect with simultaneous fixed-to-mobile and mobile-to-mobile interconnection. They show that the waterbed effect is significantly stronger for post-pay (two-part) than for pre-pay (linear) tariffs. They ascribe this difference to how the regulation of mobile termination rates affects the interconnection of calls between mobile networks, and therefore indirectly changes how intensively networks compete for subscribers. While their argument is certainly correct, it does not take into account that the actual direct pass-through of fixed-to-mobile termination profits depends on the type of tariffs in the mobile market. In this note, we isolate this factor by considering the two types of termination separately.

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²In the market, these types of contract are normally denoted as "post-pay" or "pre-pay / pay-as-you-go" tariffs.

³Armstrong (2002) discusses a model of perfect competition in two-part tariffs. It exhibits a full waterbed effect due to the type of tariff, not due to the type of competition.

2 Model Setup

The model setup is a generalization of Laffont, Rey and Tirole (1998) to many networks and general (instead of Hotelling) subscription demand. We assume that there are $n \geq 2$ symmetric mobile networks $i = 1, \dots, n$ who compete in tariffs. In the main text we consider linear and two-part tariffs that do not discriminate between calls within the same network (on-net calls) and those to rival networks (off-net calls), while in the appendix we analyze tariffs which price discriminate between these types of calls. Thus for now we assume that network i charges a price p_i for each call minute. In case networks compete in two-part tariffs it also charges a fixed fee F_i .

The marginal on-net cost of a call is $c > 0$ and the cost of terminating a call is $c_0 > 0$. Networks charge each other the access charge a per incoming call minute. Thus the marginal cost of an off-net call is $c+m$, where $m = a - c_0$ is the termination margin. There is a monthly fixed cost f per customer, and networks receive further monthly profits of Q per customer that do not originate from payments for retail services offered to them. Our focus will be on how equilibrium profits depend on Q .

From making a call of length q , a consumer obtains utility $u(q)$, where $u(0) = 0$, $u' > 0$ and $u'' < 0$. For call price p , the indirect utility is $v(p) = \max_q u(q) - pq$, call demand is $q(p) = -v'(p)$ with elasticity $\eta(p) = -pq'(p)/q(p)$. Receiving a call of length q yields utility $\beta u(q)$, where $\beta \geq 0$ indicates the strength of the call externality. Letting $v_i = v(p_i)$ and assuming a balanced calling pattern (i.e. subscribers call any other subscriber with the same probability) the surplus of a consumer on network i is given by

$$w_i = v_i + \beta \sum_{j=1}^n \alpha_j u_j - F_i,$$

where F_i is zero for a linear tariff. The market share of network $i = 1, \dots, n$ is assumed to be

$$\alpha_i = A(w_i - w_1, \dots, w_i - w_n),$$

where $A_n : \mathbb{R}^n \rightarrow \mathbb{R}$ is strictly increasing and symmetric in its arguments, with $0 \leq \alpha_i \leq 1$, $\sum_{i=1}^n \alpha_i = 1$, from which follows that $A(0, \dots, 0) = 1/n$. Let $\sigma = dA(x, 0, \dots, 0)/dx|_{x=0}$.⁴

Denote the profits from a pair of originated and terminated calls between networks i and j as $P_{ij} = (p_i - c - m)q_i + mq_j$, $i, j = 1, \dots, n$ (access payments

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⁴This demand specification encapsulates both the generalized Hotelling model of Hoernig (2014) and the logit model $\alpha_i = \exp(w_i) / \sum_{j=1}^n \exp(w_j)$. We can allow for the more general specification $\alpha_i = D_i(w)$, but in this case σ is no longer constant. Expression (3) remains the same, but is harder to sign.

cancel for on-net calls). Network i 's profits are

$$\pi_i = \alpha_i \left(\sum_{j=1}^n \alpha_j P_{ij} + F_i - f + Q \right).$$

3 Equilibrium Profits and the Waterbed Effect

We will now derive equilibrium profits and determine their dependence on profits Q , for both linear and two-part tariffs. As for the latter, network i 's first-order condition for a profit maximum is

$$0 = \frac{\partial \pi_i}{\partial F_i} = \frac{\pi_i}{\alpha_i} \frac{\partial \alpha_i}{\partial F_i} + \alpha_i \left(\sum_{j=1}^n \frac{\partial \alpha_j}{\partial F_i} P_{ij} + 1 \right).$$

In a symmetric Nash equilibrium we have $\alpha_i = 1/n$, $\partial \alpha_i / \partial F_i = -(n-1)\sigma$, and for all $j \neq i$, $\partial \alpha_j / \partial F_i = \sigma$ and $P_{ij} = P_{ii}$. Solving the first-order condition for π_i we obtain

$$\pi_i = \frac{1}{(n-1)n^2\sigma}. \quad (1)$$

These profits do not depend on Q , i.e. we have a full waterbed effect. As for linear tariffs, consider the first-order condition for maximizing profits with respect to the call price p_i :

$$0 = \frac{\partial \pi_i}{\partial p_i} = \frac{\pi_i}{\alpha_i} \frac{\partial \alpha_i}{\partial p_i} + \alpha_i \left(\sum_{j=1}^n \frac{\partial \alpha_j}{\partial p_i} P_{ij} + \sum_{j=1}^n \alpha_j \frac{\partial P_{ij}}{\partial p_i} \right).$$

In a symmetric Nash equilibrium, we have $p_i = p^*$ and $q_i = q^*$ for all $i = 1, \dots, n$, and thus $\partial \alpha_i / \partial p_i = -(n-1)\sigma q^*$ and $\partial \alpha_j / \partial p_i = \sigma q^*$, with

$$\pi_i = \frac{1 - \eta^* L^*}{(n-1)n^2\sigma}, \quad (2)$$

where $L^* = (p^* - c - (n-1)m/n)/p^*$ is the Lerner index for the equilibrium call price and η^* the corresponding price elasticity of demand. Combining both expressions for profits shows that even in our more general framework under two-part tariffs the call price continues equal to average cost, i.e. $L^* = 0$ or $p^* = c + (n-1)m/n$, i.e. does not depend on Q at all. On the other hand, we now need to determine $\partial p^* / \partial Q$ for linear tariffs, for which we

combine (2) with the symmetric equilibrium profits $\pi_i = (P^* - f + Q)/n$, $P^* = (p^* - c)q^*$, to obtain

$$\frac{dp^*}{dQ} = -\frac{(n-1)n}{(n-1)n(P^*)' + (\eta^*L^*/\sigma)'},$$

where apostrophes denote derivatives with respect to p^* . Since p^* is below the monopoly price $(P^*)'$ is strictly positive, and the denominator is positive unless the demand elasticity decreases very strongly as the call price increases. The following assumption, common in the economic literature, provides a simple sufficient condition for $(\eta^*L^*)' > 0$.

Assumption 1: The price elasticity of demand $\eta(\cdot)$ is non-decreasing.

Under this assumption, we conclude that under linear tariffs higher Q feeds through to lower call prices, $dp^*/dQ < 0$. Finally, we obtain

$$\frac{d(n\pi_i)}{dQ} = \frac{(\eta^*L^*/\sigma)'}{(n-1)n(P^*)' + (\eta^*L^*/\sigma)'}, \quad (3)$$

which implies that only by chance the waterbed effect is full ($d(n\pi_i)/dQ = 0$). Under Assumption 1, we obtain $0 < d(n\pi_i)/dQ < 1$, i.e. higher Q is translated into higher industry profits, but only partially so.⁵

Summing up:

Proposition 1 *In symmetric equilibrium, the waterbed effect is*

1. *full under two-part tariffs;*
2. *partial under linear tariffs.*

As shown in the Appendix, much the same results hold if networks price discriminate between on- and off-net calls, as originally discussed in Hoernig (2010).

Two conclusions follow from these results: First, in general terms the exact structure of tariffs on one side of a market dictates how price changes on some other side are transmitted, even though different groups of customers are involved. Thus the design of regulation must take types of tariffs in unregulated market segments into account. Second, for the specific case of regulation of fixed-to-mobile termination charges, our results show that concerns about reduced consumer welfare due to the waterbed effect are less justified for consumers on pre-pay (linear) tariffs than those on post-pay (two-part) tariffs, contrary to what networks have often publicly claimed.

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⁵If Assumption 1 were to be strongly violated then industry profit would even decrease in Q . Firms' lobbying for higher Q shows that this case is merely a theoretical curiosity.

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4 Appendix: Destination-Based Price Discrimination

As in Hoernig (2010), we assume that network i charges a per-minute price p_i for calls within the same network (on-net calls) and a per-minute price $\hat{p}_i$ for calls to the other mobile network (off-net calls). Thus either networks charge multi-part tariffs $(F_i, p_i, \hat{p}_i)$ or linear tariffs $(p_i, \hat{p}_i)$. Letting $v_i = v(p_i)$, $\hat{v}_i = v(\hat{p}_i)$, etc., and assuming a balanced calling pattern (i.e. subscribers call

any other subscriber with the same probability) the surplus of a consumer on network i is given by

$$w_i = \alpha_i (v_i + \beta u_i) + \sum_{j \neq i} \alpha_j (\hat{v}_i + \beta \hat{u}_j) - F_i = \sum_{j=1}^n \alpha_j h_{ij} - F_i,$$

where $h_{ii} = v_i + \beta u_i$ and $h_{ij} = \hat{v}_i + \beta \hat{u}_j$ for $j \neq i$, and F_i is equal to zero under a linear tariff.

Denote the profits from one on-net call as $P_{ii} = (p_{ii} - c)q_{ii}$ and those of a pair of outgoing and incoming off-net calls as $P_{ij} = (\hat{p}_i - c - m)\hat{q}_i + m\hat{q}_j$, $j \neq i$. Network i 's profits are

$$\pi_i = \alpha_i \left(\sum_{j=1}^n \alpha_j P_{ij} + F_i - f + Q \right).$$

Letting h be the $n \times n$ -matrix of h_{ij} , and w and F the $n \times 1$ -vectors of w_i and F_i , we can write $w = h\alpha - F$. Write market shares as $\alpha_i = D_i(w)$ and $\alpha = D(h\alpha - F)$ for a function $D : \mathbb{R}^n \rightarrow \mathbb{R}^n$ with Jacobian W , then we obtain the market share derivatives

$$\begin{aligned} \frac{d\alpha}{dF} &= W \left(h \frac{d\alpha}{dF} - I \right) \iff \frac{d\alpha}{dF} = -G \\ \frac{d\alpha}{dp} &= W \left(h \frac{d\alpha}{dp} + \frac{dh}{dp} \alpha \right) \iff \frac{d\alpha}{dp} = G \frac{dh}{dp} \alpha, \end{aligned}$$

where I is the identity matrix and $G = (I - Wh)^{-1}W$, with elements G_{ij} , $i, j = 1, \dots, n$. For the derivatives with respect to fixed fees, we obtain

$$\frac{d\alpha_j}{dF_i} = -G_{ji}.$$

As for call prices, note first that dh/dp_i is an $n \times n$ -matrix with entry $dh_{ii}/p_i = -q_i + \beta u'(p_i)q'_i = -q_i(1 + \beta\eta_i)$ at position (i, i) and zeros otherwise; similarly, for $j \neq i$ the matrix $dh/d\hat{p}_i$ has entries $dh_{ij}/d\hat{p}_i = -\hat{q}_i$ and $dh_{ji}/d\hat{p}_i = -\hat{q}_i\beta\hat{\eta}_i$, and is otherwise equal to zero. As a result, we have, for $j = 1, \dots, n$,

$$\frac{d\alpha_j}{dp_i} = -q_i(1 + \beta\eta_i)\alpha_i G_{ji}, \quad \frac{d\alpha_j}{d\hat{p}_i} = -\hat{q}_i \left(G_{ji}(1 - \alpha_i) + \beta\hat{\eta}_i\alpha_i \sum_{k \neq i} G_{jk} \right).$$

Since market shares sum to 1, we have $W_{ii} + \sum_{j \neq i} W_{ij} = 0$ for all i . This implies $\sum_{k \neq i} G_{jk} = -G_{ji}$,⁶ and thus

$$\frac{d\alpha_j}{d\hat{p}_i} = \hat{q}_i G_{ji} ((1 + \beta\hat{\eta}_i)\alpha_i - 1).$$

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⁶If E is the $n \times 1$ -vector of ones, then $W'E = 0$ implies $G'E = 0$.

In a symmetric equilibrium, $W_{ij} = -\sigma$ for all i and $j \neq i$, and thus $W_{ii} = (n-1)\sigma$. Furthermore, in symmetric equilibrium all $h_{ii} \equiv h_{on}$ are identical, and so are all $h_{ij} \equiv h_{of}$, for $j \neq i$. After some computations, we find

$$G_{ii} = G_{on} \equiv \frac{(n-1)\sigma}{1 - \sigma n(h_{on} - h_{of})}, \quad G_{ij} = G_{of} \equiv -\frac{\sigma}{1 - \sigma n(h_{on} - h_{of})}, \quad \forall j \neq i,$$

First we determine the equilibrium profits under multi-part tariffs, following Hoernig (2014): The first-order condition for profit-maximization with respect to fixed fees is

$$0 = \frac{\partial \pi_i}{\partial F_i} = \frac{d\alpha_i}{dF_i} \frac{\pi_i}{\alpha_i} + \alpha_i \left(\sum_{j=1}^n \frac{d\alpha_j}{dF_i} P_{ij} + 1 \right),$$

which can be solved for the symmetric equilibrium profits ($\alpha_i = 1/n$, $P_{ii} = P_{on}^{mp}$, $P_{ij} = P_{of}^{mp}$),

$$\begin{aligned} \pi_i^{mp} &= -\alpha_i^2 \left(\sum_{j=1}^n \frac{G_{ji}}{G_{ii}} P_{ij} - \frac{1}{G_{ii}} \right) \\ &= \frac{1}{n^2} \left(\frac{1}{(n-1)\sigma} + \frac{n(h_{of} - h_{on})}{n-1} + P_{of}^{mp} - P_{on}^{mp} \right). \end{aligned}$$

As is known (e.g. Hoernig 2014),⁷ under multi-part tariffs with price discrimination between on- and off-net calls the equilibrium call prices are $p^{mp} = c/(1 + \beta)$ and $\hat{p}^{mp} = (c + m)/(1 - \beta/(n-1))$. Call prices and h_{on} , h_{of} , P_{of}^{mp} and P_{on}^{mp} do not depend on Q . As a result, equilibrium profits under multi-part tariffs are independent of Q , and the waterbed effect is full.

As for linear tariffs, the first-order condition for the profit-maximizing on-net price at the symmetric equilibrium is

$$0 = \frac{\partial \pi_i}{\partial p_i} = \frac{d\alpha_i}{dp_i} \frac{\pi_i}{\alpha_i} + \alpha_i \left(\sum_{j=1}^n \frac{d\alpha_j}{dp_i} P_{ij} + \alpha_i \frac{dP_{ii}}{dp_i} \right),$$

with profits under linear tariffs of

$$\begin{aligned} \pi_{on}^{lt} &= -\alpha_i^2 \left(\sum_{j=1}^n \frac{G_{ji}}{G_{ii}} P_{ij} - \frac{1}{(1 + \beta\eta_i) G_{ii}} \left(1 - \frac{p_i - c}{p_i} \eta_i \right) \right) \\ &= \frac{1}{n^2} \left(P_{of} - P_{on} + \frac{1 - \sigma n(h_{on} - h_{of})}{(n-1)\sigma} \frac{1 - \eta L_{on}}{1 + \beta\eta} \right) \end{aligned}$$

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⁷This result can be derived from using the above first-order condition with respect to the fixed fee together with those for call prices discussed below.

and on-net Lerner index $L_{on} = (p - c) / p$. Equally, by using the first-order condition for the profit-maximizing off-net price we obtain

$$0 = \frac{\partial \pi_i}{\partial \hat{p}_i} = \frac{d\alpha_i}{d\hat{p}_i} \frac{\pi_i}{\alpha_i} + \alpha_i \left(\sum_{j=1}^n \frac{d\alpha_j}{d\hat{p}_i} P_{ij} + \sum_{j \neq i} \alpha_j \frac{dP_{ij}}{d\hat{p}_i} \right),$$

or, with $L_{of} = (\hat{p} - c - m) / \hat{p}$,

$$\begin{aligned} \pi_{of}^{lt} &= -\alpha_i^2 \left(\sum_{j=1}^n \frac{G_{ji}}{G_{ii}} P_{ij} + \sum_{j \neq i} \frac{\alpha_j}{((1 + \beta \hat{\eta}_i) \alpha_i - 1) G_{ii}} \left(1 - \frac{\hat{p}_i - c - m}{\hat{p}_i} \hat{\eta}_i \right) \right) \\ &= \frac{1}{n^2} \left(P_{of} - P_{on} + \frac{1 - \sigma n (h_{on} - h_{of})}{\sigma} \frac{1 - \hat{\eta} L_{of}}{n - 1 - \beta \hat{\eta}} \right). \end{aligned}$$

Equating π_{on}^{lt} to π_{of}^{lt} , we obtain

$$\frac{1 - \eta L_{on}}{1 + \beta \eta} = \frac{(n - 1) (1 - \hat{\eta} L_{of})}{n - 1 - \beta \hat{\eta}}.$$

This result implies that the Lerner indices tend L_{on} and L_{of} tend to move in lockstep, that is, if higher Q leads to a lower on-net price then the off-net price will decrease as well. While the exact comparative statics are too involved to be discussed here, this implies that changes in Q are not compensated by opposing shifts in on- and off-net call prices.

If call externalities and access margins are small ($\beta, m \approx 0$), then the equilibrium condition implies $\hat{p} \approx p$, and similar computations as in the main text lead to

$$\frac{d(n\pi^{lt})}{dQ} \approx \frac{(\eta L_{on}/\sigma)'}{(n - 1) n P'_{on} + (\eta L_{on}/\sigma)'},$$

i.e. the above result for the waterbed effect under linear tariffs continues to hold approximately even under discrimination between on- and off-net prices.