Pedro Jorge Gomes Mota

Licenciatura em Ciências de
Engenharia Electrotécnica e de Computadores

Comparative Analysis of Multicriteria Decision Making Methods

Dissertação para obtenção do Grau de Mestre em Engenharia Electrotécnica e de Computadores

Orientador: Rui Neves-Silva, Prof. Auxiliar da FCT-UNL
Coorientador: Ana Rita Campos, Investigadora da CTS/UNINOVA

FACULDADE DE CIÊNCIAS E TECNOLOGIA
UNIVERSIDADE NOVA DE LISBOA

Setembro 2013

Comparative Analysis of Multicriteria Decision Making Methods for Project Selection

Acknowledgments

I would like to acknowledge Dr. Rui Neves-Silva and Dra. Ana Rita Campos for their assistance during the development process of this dissertation. They were tireless to ensure, that even with full schedules and all the projects in hands, all the help needed was provided. I also have to express my gratitude for the opportunities they gave me, specially the possibility to contact with the project EnPROVE and to participate in the IDT / IIMSS Conference 2013.

I would like to thank all the people that since my first day of kindergarten until my last class in the university, had the difficult but rewarding job of teaching. Special thanks to São, Isabel Fiusa, Isabel, José Teixeira, Manuel Rêgo, Jorge Miranda and Ruy Costa.

Thanks to my friends, colleagues and partners of adventure João Puga Leal, Francisco Parreira do Amaral, Fábio Moreira de Passos and Filipe Martins, that closely shared these six amazing years of my academic and personal life.

I would like to express my deep gratitude to my aunt Nazaré and my cousin Joana that gave me all their support and welcomed me at their house when I moved to college.

I would like to thank my closest family, my father, my mother and my brother João, for all their care, love and dedication. My parents' wise advices guided me through all my academic life, bringing me here confident in my future and grateful for my success. My brother's energy and affection kept me alive and fighting to grant him a better tomorrow.

I have to thank my girlfriend Carolina for all her love, loyalty and tenderness, “What a wonderful life for as long as you've been at my side”. Hope one day she will say “Yes!”.

Finally I would like to dedicate my work to my aunt Cecília, who was and forever will be my second mother. Wherever she is right now, I know for sure she is in everything I do, in the person I have become and close to all that I believe in.

Abstract

The main objective of this dissertation is to perform a Comparative Analysis of different Multicriteria Decision Making Methods applied to real-world problems, in order to produce relevant information to enable the incorporation of those methods on computational platforms. The current document presents a simple case study concerning a decision support application targeted for a real problem regarding retrofitting alternatives of a building with energy efficiency impact. The application process was started with the selection of two Multicriteria Decision Making Methods guided by a preexisting framework, and resulted in the choice of AHP and PROMETHEE II methodologies. These two methods were then combined with three different decision maker profiles (*Conservative, Moderate and Aggressive*) created by means of risk assessment profiling techniques for portfolio allocation. Afterwards, the chosen decision criteria were disposed in a Risk Pyramid according to their inherent level of risk regarding project evaluation. A match was then performed between the decision maker profiles and each criterion, so as to define a proper set of weights for the decision criteria and preference functions, with corresponding preference and indifference thresholds. Finally, three different sets of results (one for each decision maker profile) were produced using appropriate software, and a Sensitivity Analysis was performed over the criteria to understand their influence on the solution. The general conclusion of this Comparative Analysis is that the increase in the preference modelling ability of the methods brings up the least expected alternatives as recommendations for the decision maker. Besides, we have concluded that the decision profiles that allocate bigger weights to the riskiest criteria are the ones that produce the more dispersed set of results within each method application and within each decision maker profile.

Keywords: *Multicriteria Decision Making, Decision Support, Comparative Analysis, Risk Pyramid, AHP, PROMETHEE*

Resumo

O principal objetivo desta dissertação é realizar uma Análise Comparativa de diferentes métodos de suporte à decisão multicritério aplicados a problemas reais, para produzir informações que permitam a incorporação desses métodos em plataformas computacionais. O presente documento exibe um caso de estudo simples de uma aplicação de apoio à decisão direcionada para um problema real, que considera alternativas de renovação de um edifício com impacto na sua eficiência energética. O processo de aplicação teve início com a seleção de dois métodos de decisão multicritério, guiada por uma *framework* pré-existente, e resultou na escolha das metodologias AHP e PROMETHEE II. Estes dois métodos foram então combinados com três perfis diferentes de decisor (Conservador, Moderado e Agressivo) criados por meio de técnicas de análise de avaliação de risco para a alocação de portefólios. Seguidamente, os critérios de decisão escolhidos foram dispostos numa Pirâmide de Risco segundo o seu nível de risco relativamente à avaliação de projeto. Foi então realizada uma correspondência entre os perfis do decisor e cada critério, de modo a definir um conjunto adequado de pesos para os critérios de decisão e funções de preferência, com os respetivos limiares de preferência e indiferença. Finalmente, três conjuntos de resultados (um para cada perfil de tomador de decisão) foram produzidos utilizando *software* adequado, e uma Análise de Sensibilidade foi realizada sobre os critérios, para compreender a sua influência sobre a solução. A conclusão geral da Análise Comparativa é a de que o aumento na capacidade de modelação de preferência nos métodos revela as alternativas menos esperadas como recomendações para o decisor. Além disso, concluímos que os perfis de decisão que alocam maiores pesos para os critérios de maior risco são os que produzem os conjuntos de resultados mais dispersos dentro de cada aplicação do método e dentro de cada perfil de decisor.

Palavras-chave: *Métodos de Apoio à Decisão Multicritério, Suporte à Decisão, Análise Comparativa, Pirâmide de Risco, AHP, PROMETHEE*

Symbols and Notation

Symbol	Description
$\boldsymbol{a_i}$	Alternative i
$\mathbb{A}$	Set of possible actions
$\boldsymbol{A}$	Set of attributes
$\mathbb{A} - \boldsymbol{F}/\boldsymbol{A} - \mathbb{E}$	Classical model to describe a decision making situation
$\mathbf{B}$	Comparison matrix of criteria
$\boldsymbol{b_{ij}}$	Entries of the matrix $\mathbf{B}$
$\mathbf{D}$	Comparison matrix of alternatives
$\boldsymbol{d_j(a,b)}$	Deviation between alternatives a and b for a criterion j
$\boldsymbol{e_{ij}}$	Performance indicator in the performance table
$\mathbb{E}$	Performance Table
$\boldsymbol{F}$	A family of criteria
$\boldsymbol{\phi^-(a)}$	Incoming Flow of alternative $\boldsymbol{a}$
$\boldsymbol{\phi^+(a)}$	Outgoing Flow of alternative $\boldsymbol{a}$
$\boldsymbol{\phi(a)}$	Net Flow of alternative $\boldsymbol{a}$
$\boldsymbol{g_j}$	Criterion j
$\boldsymbol{g_j(a_i)}$	The performance of an alternative a_i under a criterion j
$\{\boldsymbol{g_j}(\cdot), \boldsymbol{P_j(a,b)}\}$	A generalized criterion
$\boldsymbol{I}$	Indifference
$\boldsymbol{l_{ij}}$	Local priority of an alternative a_i under a criterion g_j
$\boldsymbol{Ideal(l_{ij})}$	Local Ideal Mode priority of an alternative a_i under a criterion g_j
$\boldsymbol{\pi(a,b)}$	The preference index
$\boldsymbol{P.\alpha}$	The Choice Problematic
$\boldsymbol{P.\beta}$	The Sorting Problematic
$\boldsymbol{P.\gamma}$	The Ranking Problematic
$\boldsymbol{P.\delta}$	The Description Problematic
$\boldsymbol{p_i}$	Global priority of an alternative a_i
$\boldsymbol{P_j(a,b)}$	A preference function
$\boldsymbol{P}$	Preference
$\boldsymbol{R}$	Incomparability
$\boldsymbol{w_j}$	Weight of the criterion j

Acronyms and abbreviations

Acronym	Description
AHP	Analytic Hierarchy Process
ANP	Analytic Network Process
BOCR	Benefits, Opportunities, Costs and Risks
CAD	Computer Aided Design
CCF	Cumulative Cash Flow
CF	Cash Flows
CI	Consistency Index
CR	Consistency Ratio
CRM	Customer Relationship Management
DCF	Discounted Cash Flow
DM	Decision Maker
DMP	Decision Maker Profiles
DMS	Decision Making Situation
DSS	Decision Support Systems
ELECTRE	Elimination Et Choix Traduisant la Realité
EPDSS	Energy Prediction and Decision Support System
FMADM	Fuzzy Multiple Attribute Decision Making
FMODM	Fuzzy Multiple Objective Decision Making
GAIA	Geometrical Analysis for Interactive Aid
GP	Goal Programming
GUI	Graphical User Interface
HVAC	Heating, Ventilation And Air Conditioning
IRR	Internal Rate of Return
IT	Information Technology
KPI	Key Performance Indicators
LENI	Lighting Energy Numeric Indicator
LIMI	Lighting Maintenance Indicator
LINI	Lighting Initial Investment Indicator
MACBETH	Measuring Attractiveness by a Categorical Based Evaluation Technique
MADM	Multiple Attribute Decision Making
MAUT	Multi-attribute Utility Theory
MAVT/	Multi-attribute Value Theory
MCAP	Multiple Criteria Aggregation Procedure
MCDA	Multicriteria Decision Aid
MCDM	Multicriteria Decision Making
MODM	Multiple Objective Decision Making
MOP	Multi-Objective Programming
MS	Management Science
NAIADE	Novel Approach to Imprecise Assessment and Decision Environments
NPV	Net Present Value
OR	Operations Research
PBP	Discounted Payback Period
PROMETHEE	Preference Ranking Organization Method for Enrichment Evaluations
R&D	Research and Development
SPB	Simple Payback Period
TOPSIS	Technique for Order Preference by Similarity to Ideal Solution
VIKOR	VIseKriterijumska Optimizacija I Kompromisno Resenje

Table of Contents

List of Figures

List of Tables

1. Introduction

The main objective of this dissertation is to perform a comparative analysis of different Multicriteria Decision Making Methods, applied to real-world problems, in order to produce relevant information to enable the incorporation of those methods on computational platforms.

On the present chapter we describe the motivation that guided all our work and the original contributions produced during this process. Also, we present a short summary of each chapter and the organization of the dissertation.

1.1. Motivation

Through the last four decades the concept of decision support has been evolving to keep up with the growing complexity of the decisions taken in the modern world dominated by technology. Nowadays, the application of Decision Support Systems (DSS) starts to be a wider reality due to the advent of technology specially catalyzed by the internet. Even non-traditional areas of the decision support application, such as agriculture and the food sector start to apply DSS to help manage and optimize their outcomes (e.g. pest and diseases control) [1].

Thomas Saaty, a well-known author in the area of decision, draw the attention, in one of his papers to an interesting point about decision, stating that “We are all fundamentally decision makers” and that “Everything we do consciously or unconsciously is the result of some decision” [2]. This statement makes sense not only at a personal level but also when addressing different segments of the industry and other business areas. Companies make decisions constantly and due to the demands of the markets the decisions have increased their complexity, scope and number of actors involved [3]. Bearing in mind the importance of decisions both at a personal and at a business level, we considered the subject of decision support interesting and with a lot of potential to explore. As a consequence of the main objective of our dissertation we intended to produce relevant information for future applications by selecting and comparatively analyzing different applications of decision support methods. The results produced are expected to serve as documentation for future applications and to allow the understanding of the behavior of different decision methods when applied to specific decision making situations.

Moreover, the application of DSS for sustainable energy management is an actual and wide studied topic mainly due to its economic and environmental implications [4]. The development and application of these systems has been an area of work within the university with especial attention to the European projects in course, in particular the one we present as our case study.

The combination of these two situations, represent the motivation that drove the production of this dissertation. The opportunity to apply the concept of decision support to a thematic with such importance as energy management represented a stimulus to the conception of our work.

1.2. Original Contributions

The following points describe the original contributions that resulted from the work performed under the main objective of this dissertation.

1. An **Application of a decision framework to select a decision method** in order to define the appropriate methodologies to solve a real problem, according to the problem characteristics and the preferences of the decision makers involved in the process. This application produced results and information for future work in the area and verified the framework itself.

2. The **Definition of Decision Maker Profiles using risk analysis** to allow the evaluation of different decision methods under the same circumstances. The profiles built upon investment and risk assessment theories guaranteed a stronger and tangible simulation environment to support the criteria weighting and the definition of preference functions.

3. The **Classification of decision criteria according to a Risk Pyramid** guided the weighting process of the decision criteria giving the fundamental guidelines to correlate the decision maker profiles and their behavior towards each criterion.

4. A **Comparative analysis of two widely applied decision methods** resulted in a series of information for future applications. The results obtained in this comparative analysis evidence differences in the methods performances and corresponding outcomes.

1.3. Organization of the Dissertation

Chapter 1, **Introduction**, gives an overview of the motivation that led to the production of this dissertation and its background. It also describes the original contributions achieved and the organization of the document.

Chapter 2, **State of the Art**, introduces fundamental concepts about decision support, describing the different streams of thought, main families of methods and evolution of the discipline. Moreover, it comprises a brief bibliometrics analysis of publications according to the different areas of application and methods used. In the final part, it describes a standard decision process model with all its main elements and phases, and afterwards different methodologies for method selection are discussed.

Chapter 3, **Case study**, presents a real problem based on the selection of retrofitting alternatives of a building with energy efficiency impact, using decision support methods. This chapter contains all the information about the different alternatives and the criteria to use in the decision process.

Chapter 4, **Choosing the appropriate method for the Case Study**, describes the process of method selection based on one of the methodologies found in chapter 2. In addition an explanation of the selected methods, and their associated software tools, is given with simple illustrative examples.

Chapter 5, **Decision Maker Profiles**, comprises the creation of decision maker profiles, based on investment profiles and risk assessment, to test the selected methods from chapter 4. Theses profiles contain information about criteria weights and preference functions defined by the decision makers.

Chapter 6, **Method application results and Sensitivity Analysis,** displays all the results from the methods application regarding the Case Study. Besides, a Sensitivity Analysis is performed to evaluate the influence of each criterion in the final ranking of the alternatives.

Chapter 7, **Comparative Analysis of results,** exploits the results presented in the previous chapter in order to understand the relation between the methods tested. Furthermore, some remarks are drawn around special situations.

Chapter 8, **Conclusions and future work,** summarizes the main aspects of this work, pointing out directions and challenges for future work.

2. State of the Art

This chapter provides an analysis of the Multicriteria Decision Making (MCDM) scenario through time. In the next sections we examine the origins of the discipline, its history and the latest developments. In another section, the most relevant MCDM methods and the different streams of thought related to them are presented. Besides, the structure of the decision process and its elements are described and a general model is displayed. Finally, an overview of different method selection techniques and approaches is performed with special attention to the framework selected for the purpose of this dissertation.

2.1. Origins of MCDM

Multiple Criteria Decision Making is a branch of Operations Research (OR), also called Management Science (MS) or Decision Science, and mentioned sometimes as a sub-field of mathematics. According to Hanne [7], MCDM "deals with (mathematical) theory, methods and methodological issues and case studies (applications) for decision processes where multiple criteria (objectives, goals, attributes) have to be (or should be) considered".

The International Society of MCDM refers in its website that the earliest reference of MCDM is due to the American scientist and politician Benjamin Franklin (1706 – 1790). Franklin had a simple decision method, based on writing in one side of a sheet of paper the arguments in favor and on the other side the ones against the decision. To find how to manage the decision one has to eliminate the pros and cons of equal importance. In the end, the side of the paper with more arguments left is the solution of the problem. Although this is an interesting reference, the MCDM discipline, as we know it nowadays, is an indirect result of a war state and post war situation.

During the Second World War, in order to gain an advantage against the enemies, the nations started to develop and combine different fields of knowledge. These areas suffered a massive expansion and as a consequence new disciplines emerged, e.g. Operations Research. After the World War II, with a prosperous economic and political scenario, OR evolved promptly and extended its applications to other areas than the military, such as industry and logistics. The main objective of OR is to improve the decision making process by providing mathematical tools of analysis, modelling and optimization that aid making better decisions in empirical contexts. As a part of OR, MCDM also results from an interdisciplinary background, combining different areas like engineering, economics, psychology, computer science and of course, mathematics.

MCDM has changed along with OR since the early seventies becoming a very important asset to decision making processes nowadays. In its evolutionary process MCDM has turned from "a conceptual-theoretical enterprise of interests practiced by a limited number of disciplines and individuals to a universally embraced philosophy" [8]. Furthermore MCDM has transformed its paradigm to give voice to the decision maker (DM), we are no longer finding the optimal solution but a solution that satisfies more the DM [9].

2.2. Classifications and Definitions

In the MCDM literature, one can find two main streams of thought sometimes called schools. The first to arise was the French School or also mentioned as the European School, and it is famous for its connection to the outranking methods created and developed by Bernard Roy [10].

In opposition, the American school is associated with Multi-Attribute Value/Utility Theories (MAVT/MAUT) motivated by the work of Keeney and Raiffa and made famous by one of the most studied and used methods worldwide, the Analytic Hierarchy Process (AHP) by T.L. Saaty [11].

Along with these two different approaches also two distinct denominations emerged to define the discipline. The French practitioners dislike the acronym MCDM, as they think that the MCDM "approach is based on a misconception of the decision process and the way a decision analyst or a multicriteria decision method is involved into it" [7]. The word "making" is then replaced by "aid" – Multicriteria Decision Aid (MCDA) – on the tentative to step aside the role of the decision analyst from the one played by the DM.

In some cases this field of studies is also mentioned as Multi-criteria Decision Analysis, a definition which tries to bring both MCDA and MCDM supporters to a consensus or is sometimes adopted by international teams gathering researchers from both schools. Besides these two approaches there are still some major definitions which could be assigned to both MCDM and MCDA and were established to assist a methodical and structured research in the field.

Hwang and Yoon have proposed two main categories for grouping different MCDM problems according to their purposes and available information. The classes defined are Multiple Attribute Decision Making (MADM) and Multiple Objective Decision Making (MODM). The later handles decision problems that consider a continuous decision space, and are usually related to design and planning. On the other hand, MADM problems are assigned to an evaluation component with a discrete decision space and a predetermined set of alternatives/potential actions normally considering information from the DM [11] [12].To better illustrate these classes we will now briefly describe the most common used methods of each class.

In the MODM methods class, we can find the Multi-Objective Programming (MOP) and the Goal Programming (GP) methods: The first method MOP is used in the optimization of mathematical problems where there is a need to simultaneously optimize multiple objective functions; GP is a branch of MOP and represents a generalization of linear programming used to deal with multiple and differing objective measures.

On the other hand, the MADM class presents methods such as Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), AHP and its generalization the Analytic Network Process (ANP), Fuzzy Set Theory, Elimination Et Choix Traduisant la Realité (ELECTRE) and Preference Ranking Organization Methods for Enrichment Evaluations (PROMETHEE): TOPSIS is a compensatory method that uses the notion of geometric distance to evaluate the alternatives of a problem in relation to the ideal solution. AHP is a method based on mathematics and founded on a psychological background. It uses a hierarchy structure and pairwise comparisons to convert human judgment into a set of scores addressing the alternatives of the problem. Besides the original AHP methods, various methods were created based on the original concept. An example is the Analytic Network Process (ANP), which is a generalization of the AHP that allows the interdependency of different levels of the hierarchy forming a network of relations. The Fuzzy Set Theory methods were created to deal with imprecision in defining activities and expressions on the definition of problems. ELECTRE methods define a family with the same name which is closely related to the foundation of the European school of thought. These methods are based on the concept of outranking relations between the alternatives of the problem. PROMETHEE is another family of methods, also settled in the concept of outranking relations. This family was created as a simplest alternative to the use of ELECTRE. PROMETHEE uses preference functions to model the judgments and preferences of the decision makers.

Another classification used for MCDM methods is related to the quality of the available information. The application of MCDM to real world problems faces some issues related with imperfect knowledge from human evaluations, consequence of modelling complex real decision problems. Thus, the information is often catalogued as Crisp, when there is precise data or as Fuzzy, when it is incomplete or vague. In the same way, MCDM methods are subdivided into MADM/MODM if they use crisp information or into FMADM/FMODM (Fuzzy MADM/ Fuzzy MODM) if they use fuzzy knowledge. One of the modelling and solution techniques to solve this kind of problems is Fuzzy Set Theory, which has been on study over the last four decades (for more information on this subject see [13]).

2.3. Bibliometrics of MCDM

During the literature search conducted for this work, two important bibliometric studies emerged and showed some significant conclusions [14] [15]. This type of qualitative analysis, based mathematical and statistical examination of literature shows the development of research in certain areas. Although this practice is commonly used in other areas, there are few studies in the field of MCDM studies.

In 2008, Bragge et al. [14] presented at the International Conference on MCDM in Auckland a work on MCDM / MAUT Bibliometrics. This document represents a key study to understand the exponential growth of this field and how it has influenced other neighboring disciplines. Later, in 2011, Toloie-Eshlaghy and Homayonfar [15] published a review of the literature from 1999 to 2009, which presents the most relevant areas of application for MCDM methods through a comparative analysis, and also shows the most dominant methods within each area.

The main reason for mentioning these studies in the present dissertation remains in the fact that the conclusions presented by them allow us to give form to our objective of showing the scope of MCDM nowadays. Moreover, these two documents are mandatory to any initial research on the area, as they illustrate the multitude of publications accounting them by country, source, year, and research area.

According to Toloie-Eshlaghy and Homayonfar to ease the task of pointing out the relevant topics, the analyzed papers were divided in twelve categories: **Transportation and Logistics, Business and Financial Management, Managerial and Strategic Planning, Project Management and Evaluation, Other topics, Manufacturing and Assembly, Environment Management, Water Management, Energy Management, Agricultural and Forestry Management, Social service and Military Service.** The category with bigger percentage of published papers was the Transportation and Logistics with around 20% of the total 386 application documents.

Among the 628 papers (application and non-application) analyzed on the same study the method with more papers dedicated was AHP (142 papers), followed by TOPSIS (54 papers), MOP (53 papers), GP (37 papers), ANP (37 papers), Fuzzy Set Theory (33 papers) and PROMETHEE (22 papers).

2.4. The Decision Process Model

The MCDM literature is divergent on the right approach to organize and define a decision making process or a Decision Making Situation (DMS). However a famous quote by Albert Einstein is largely used and points out the importance of this step:

> "The formulation of a problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skill"

According to Roy [16] to best analyze and structure a decision making process, three key concepts must be taken into account, as they generally are of utmost importance for its success. Thus, the next three sub-sections present some important aspects related to *alternatives*, or generally potential actions, *criteria* and *problematic*, the main elements of the decision process pointed by the author.

2.4.1. Alternatives or Potential actions

The concepts of alternatives and potential actions come together as they represent the main goal of the decision process, or the possible choices for the DM. Every decision process starts with a problem that needs a solution or a set of solutions that together can solve the initial situation.

Different problems require different modelling approaches, which points out the difference between alternative and potential action. An action is called potential when it is possible to implement or it has something to add to the decision process. On other hand, an alternative results from modelling situations where two potential actions are mutually exclusive, so they are expected to operate separately [16]. Thus, when referring to the best alternative to a problem, one can think of it as the only potential solution to implement from the initial set. It is also essential to mention that a set of potential solutions can change through the decision process as more information is gathered, leaving out some actions.

Let $\mathbb{A}$ be the set of possible actions, when we analyze a discrete decision space, then:

$$\mathbb{A} = \{\boldsymbol{a_1}, \dots, \boldsymbol{a_i}, \dots, \boldsymbol{a_m}\}$$

2.4.2. Family of Criteria and Performance Table

The concept of criteria is connected to both the notions of attribute and objective, as we already observed when describing MADM and MODM. Eldrandaly et al. refer that an attribute measures the system performance regarding an objective, whereas the objective is a statement of the desired situation of the system [17].

A criterion that we denote by $\boldsymbol{g_j}$, represents one of the possible dimensions from which the alternatives or possible actions can be evaluated, according to a defined point of view, in general the DM's angle. The criteria measures how well a potential action is performing towards the goals of the problem.

It is important that the criteria are descriptive of the goals in order to understand the performance of each alternative under those goals. Thus, we denote by $\boldsymbol{g_j}(\boldsymbol{a_i})$ the performance of an alternative $\boldsymbol{a_i}$ regarding a certain criterion. This indicator assesses the level of fulfilment of a certain goal, and also allows the comparison of different alternatives concerning a given criterion.

Depending on the decision method, criteria can be expressed under two data types, qualitative or quantitative: these types can be found either together or separately [18]. To better perform the judgment of alternatives the definition of a scale is needed. The most common options mentioned across the literature are: nominal, ordinal, ratio, absolute, and interval. (see [16], [19] and [20]).

A large number of decision methods use criteria weighting in order to favor a certain aspect of the decision makers' preferences. A well-known example of these methods is the AHP [21].

Choosing the right criteria for the problem situation in hands is very important, as it can shorten the number of alternatives or assure a consistent evaluation of the set of actions (for more details see [12], [16] and [22]). On the definition of the criteria, situations of independence, cooperation, or conflict can happen, thereby it is also relevant to analyze the way criteria interact.

2.4.3. Problematic

This last concept is related to the expected outcome of the decision problem and represents a major role in choosing the right method for the DMS under consideration.

Bernard Roy [23] categorized the decision making situations according to four major problematics, and the way the decision support should be envisaged:

The Description problematic $(\boldsymbol{P.\delta})$ **–** Decision support focuses in providing an appropriate set of actions and a suitable family of criteria, without making any recommendation.

The Choice Problematic $(\boldsymbol{P.\alpha})$ **–** The support intends to narrow down the number of actions to find a single alternative or a possible smaller subset (usually containing the most fulfilling actions to the predefined goals).

The Sorting Problematic $(\boldsymbol{P.\beta})$ **–** In this problematic the support seeks to assign each action a category from a set defined a priori. These categories can be related with the feasibility of the actions and the possibility of their implementation.

The Ranking Problematic $(\boldsymbol{P.\gamma})$ **–** The decision support results in a complete or partial preorder of the set of alternatives, after comparing them with each other.

Although, these are the most common problematics across the literature other categories could be considered (see [24]).

2.4.4. Structure of a Decision Process

The most applied decision methods rely on Multiple Criteria Aggregation Procedure MCAP. This means that they use mathematical and algorithmic procedures, which given a set of alternatives, and considering a certain problematic, lead to a desired solution. Guitouni [25] proposed a general decision process model that focused the DMS on the MCAP concept. We adopted this model as it combines different aspects of the decision support and closely relates to the selection methodology to be presented on the next section, and that will support our work.

It is often mentioned that a universal MCAP does not exist, meaning that a single MCAP is not likely to be used in all DMS. Each MCAP is associated to an approach, the considered possibilities are: *the single synthesizing criterion approach, the outranking synthesizing approach* and *the interactive approach.*

According to Guitouni et al. [9] the multi-attribute utility/value theory considers a set of attributes denoted by $\boldsymbol{A}$, while the outranking methods consider a family of criteria denoted by $\boldsymbol{F}$. This leads to a classical model $\mathbb{A} - \boldsymbol{F}/\boldsymbol{A} - \mathbb{E}$ that can be used to describe any DMS. Although the model is considered incomplete (see [26]) it is representative for the purpose of this study.

The $\mathbb{A} - \boldsymbol{F}/\boldsymbol{A} - \mathbb{E}$ model, regards the set of alternatives $\mathbb{A}$ and the family of criteria/attributes $\boldsymbol{F}/\boldsymbol{A}$, and adds a new concept of Performance Table $\mathbb{E}$, also called Decision Matrix. In this table the rows represent the alternatives, as the columns represent the criteria. A value on the intersection of a certain $\boldsymbol{i^{th}}$ alternative with a $\boldsymbol{j^{th}}$ criterion is the performance indicator $\boldsymbol{g}(\boldsymbol{a})$, denoted $\boldsymbol{e_{ij}}$ on the performance table.

$$\mathbb{A} = \{\boldsymbol{a_1}, \dots, \boldsymbol{a_i}, \dots, \boldsymbol{a_m}\}$$

$$\boldsymbol{F}/\boldsymbol{A} = \{\boldsymbol{g_1}, \dots, \boldsymbol{g_j}, \dots, \boldsymbol{g_n}\}$$

$$\mathbb{E} = \{\boldsymbol{e_{ij}} = \boldsymbol{g_j}(\boldsymbol{a_i})\ \boldsymbol{i} = \boldsymbol{1}, \dots, \boldsymbol{m}; \boldsymbol{j} = \boldsymbol{1}, \dots, \boldsymbol{n}\}$$

The $\mathbb{A} - \boldsymbol{F}/\boldsymbol{A} - \mathbb{E}$ model is included on the first stage of a five step decision-making process seen as recursive and nonlinear, with the decision maker and the decision analyst providing information and changes to the loop. Hence, we consider the following steps of the process represented in Figure 2.1, developed by Guitouni [25] :

I. **Structuration** – the structuring of the DMS (alternatives, criteria and Performance Table)
II. **Preferences Articulation and Modelling** – determination of criteria relative importance, inter-criteria information, value and utility functions, thresholds, etc.
III. **Preferences Aggregation** – establishment of a preference relational system
IV. **Exploitation** (depends on each MCAP)
V. **Recommendation –** the output of the process

This decision model served as structure to deal with the case study problem (to be presented on the next chapter). The model is transversal to all the current document and each one of its phases will be addressed on further chapters.

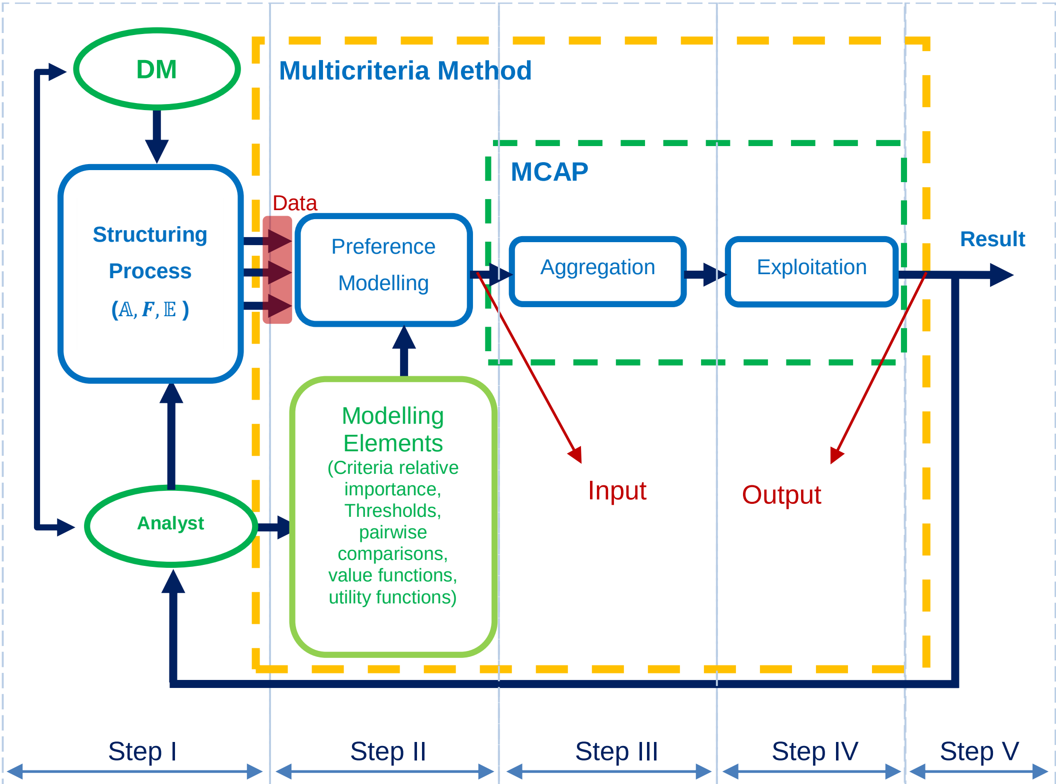


Figure 2.1. Model of a Multicriteria Decision Process (Adapted from [25])

2.5. How to choose a Decision Method

A fundamental step on the application of MCDM is to define or choose the appropriate MCDM method to solve the problem under consideration. Not all the methods are suitable for the same situations, for that reason there is a need to find the right method for a certain situation.

Many attempts have been made to define a framework that links each DMS to the most suitable decision method. This is an exhaustive, thorough, and nearly impossible procedure that must take into consideration all the decision process dimensions, the DM's role, not to mention the extensive number and variety of methods, and the information available [27].

However it is unquestionable that the selection problem is primal to the success of the process [17], which explains some of the meticulous studies in this area (see [26]).

One of the first methodologies to help in the selection of a method was defined by Hwang and Yoon [28], and it is still in use. They organized some decision methods on a diagram tree according to the available information, then the DM only needs to follow the branches of the tree according to the DMS he is analyzing. In the end of the process the DM will find a proposed decision method or a group of possible methods. This approach provided the decision analysts and the decision makers with a simple tool to make a choice. Nevertheless, it is a restricted approach and leaves out important aspects of the decision process as well as powerful methods, not considered in the definition of the tree.

Later, a study conducted by Kornyshova et al. [20] presented a state of the art of the existing approaches to select MCDM methods. This study considered nine different approaches and compared them with each other regarding their characteristics. In this document the authors pointed out four major facets and their inner features which, according to them, guarantee the characterization of the decision problem in the selection context. Those facets are:

The Problem facet – type of decision problematic, problem scale (workplace, department, enterprise, corporation…).

The Potential Action facet – number of alternatives, ability to consider new alternatives, incompatibility and conflict, organization of the alternatives, nature of the alternatives set (discrete, continuous).

The Criteria facet – data type, measure scale, criteria weighting, criteria interaction.

The Usage facet – tool (Software), Approaches for giving partial and final evaluations, Easiness of use, cost for implementing (purchasing the tool, costs for training), decision maker preferences (DM understanding, skills and habits).

By analyzing these four facets and their elements, one can easily understand the amount of possibilities to define a selection methodology. Moreover, these facets address different aspects of a DMS and also different DM's points of view. For example, it is more likely for a DM without proper training in the field of decision making to rely his choice on the *Usage* facet rather than on more technical facets such as the *Criteria* or the *Potential Action*.

2.5.1. Designated methodology for method selection

In order to capture the essential characteristics of the decision methods, we decided to apply an alternative methodology to the ones already mentioned. The procedure that we applied on our work combines an easy structure and a careful description of different methods, resulting from a comparative study of twenty-nine MCDA discrete methods. Once more, the methodology used was presented by Guitouni and Martel [9], already cited in the previous chapter. Their technique is based on the definition of seven guidelines that help choosing an appropriate decision method. Those guidelines are synthesized below and they will be observed in detail in the next section.

Guideline 1: Determine the stakeholders of the decision process.
Guideline 2: Consider the DM 'cognition' when choosing a certain preference evaluation mode.
Guideline 3: Determine the decision problematic pursued by the DM.
Guideline 4: Choose the MCAP that can handle properly the input information**.**
Guideline 5: Consider the compensation degree of the MCAP method

Guideline 6: Verify the fundamental hypothesis of the method
Guideline 7: Consider the decision support system

These seven guiding principles supported the designing of a typological tree of discrete MCAP. Similarly to what Hwang and Yoon, the DM or the analyst only needs to follow the branches of the tree according to the guidelines and one or several decision methods (MCAP) will be presented as possibilities for the DMS under consideration.

Although it may seem like an analogous technique, the *seven guidelines* approach presents more advantages to the selection process. Beyond the guidelines and the typological tree, Guitouni and Martel presented twenty-nine possible MCAP with detailed information about their characteristics and the way they fulfil the seven guidelines (see [9]). This information makes the selection process easier and less time-consuming, increasing the probability of having less possible methods as an output.

Before exploring the guidelines and typological tree it is important to mention that some limitations come with this strategy as, once more, it does not take into consideration all the possible methods and dimensions of the decision situations. Hence, not always an unequivocal choice is the result of its use. Still it represents a powerful tool for guiding the method selection and can be improved by adding new branches to the tree, new guidelines, and more easily other methods to the list, for example MACBETH [29], VIKOR [30], and ANP [31].

2.5.1.1. The Seven Guidelines

The first guideline (G1) intends to define the proper operational approach, one that will be in line with the perspectives of the stakeholders of the process, or the DM.

Guideline number two (G2) is divided in four different points concerning the preference elucidation modelling. The first point addresses the preference elucidation mode itself, *pairwise comparisons* and *tradeoffs* are two common examples. The second point refers to the moment of preference elucidation, which for the twenty-nine methods studied always happens *a priori*. The global DM preference structure considered is the third point in G2 and it regards the preference structures including for example Preference (P), Indifference (I), and Incomparability $(R) - \{P, I, R\}$. The last point in G2 is the type of ordering of the alternatives, that results from the application of the method, *total preorder*, *partial semiorder*, and *partial interval order* are some of the possibilities.

The guideline G3 intends to determine which kind of decision problematic is perused by the DM, as we already mentioned, *choice*, *ranking*, *description* and *sorting* are the most common possibilities among the methods.

The fourth guideline is related to information. Using G4 allows understanding the kind of information considered (*ordinal*, *cardinal* or *mixed*) and the nature of that information or its determinism.

G5 considers the discrimination power of the criteria (*absolute* or *non-absolute*), the compensation degree of the method and the inter-criteria information.

The guideline G6 regards the hypothesis of the method (e.g. Independence, commensurability, invariance, transitivity, dominance).

The last guideline (G7) refers to the existence of a software or tool to support the method application.

The twenty-nine methods are catalogued according to these seven guidelines and their inner elements, allowing a simple selection among those methods (see Annex A.). The concepts behind the guidelines also appear in the typological tree conditioning the assortment of the branches. The next section presents the different levels of the typological tree in an adaptation of the original.

2.5.1.2. The Typological tree

The typological tree represents a graphical application of the guidelines. Through its use one can solve the selection process in an easier way, checking the different characteristics of the problem against the possible methods.

In the original typological tree [9], the authors present three stages of selection. However, to clearly identify not only the guidelines but also their inner elements we split up one of the stages in two smaller ones.

Every selection stage begins with a question. According to the answer we eliminate a group or groups of methods and we move to the next question, and also the next stage.

The first stage (Figure 2.2) asks the question *"What is the operational approach?".* This question brings the guideline G1 and four possible answers. This first level of the tree allows the removal of a large number of methods, since it requests the DM to choose a family of methods.

Stages two (Figure 2.3) and three (Figure 2.4) are relative to the information involved in the decision process. The guideline that rules both stages is the guideline G4. These two stages are the ones that are presented together in the original typological tree, but since each stage has its own question we decided to describe them separately.

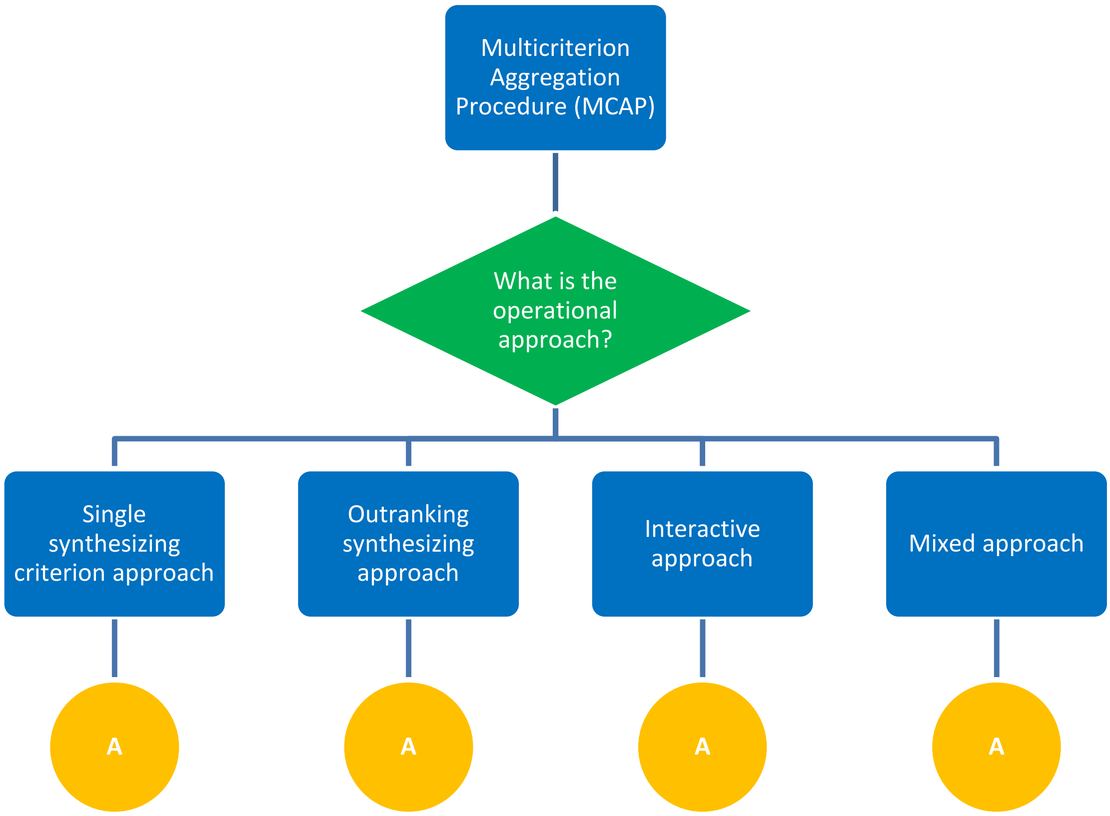


Figure 2.2. First Stage of the Typological Tree

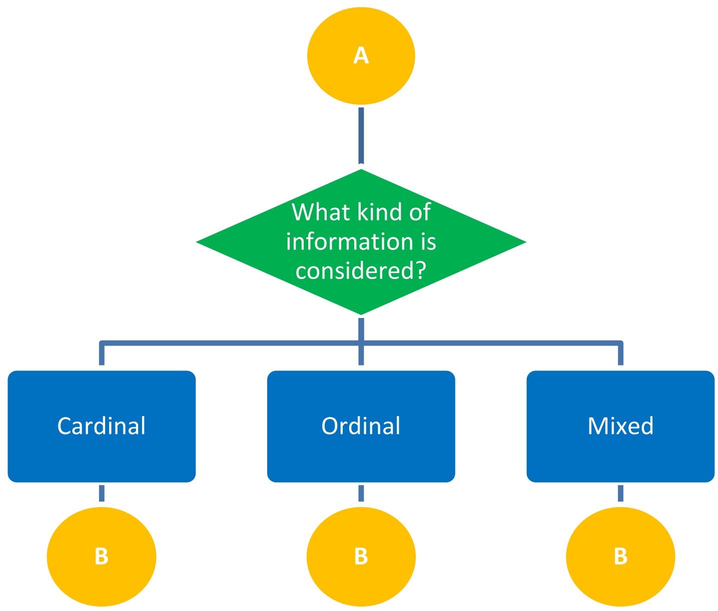


Figure 2.3.Second Stage of the Typological Tree

Stage two asks the question “What kind of information is considered?”. The possible answers are three: ordinal, cardinal and mixed. Once again, and considering that we are selecting a method among one of the families selected in the first stage, we are shortening the number of methods since some of the possibilities cannot deal with both kinds of information.

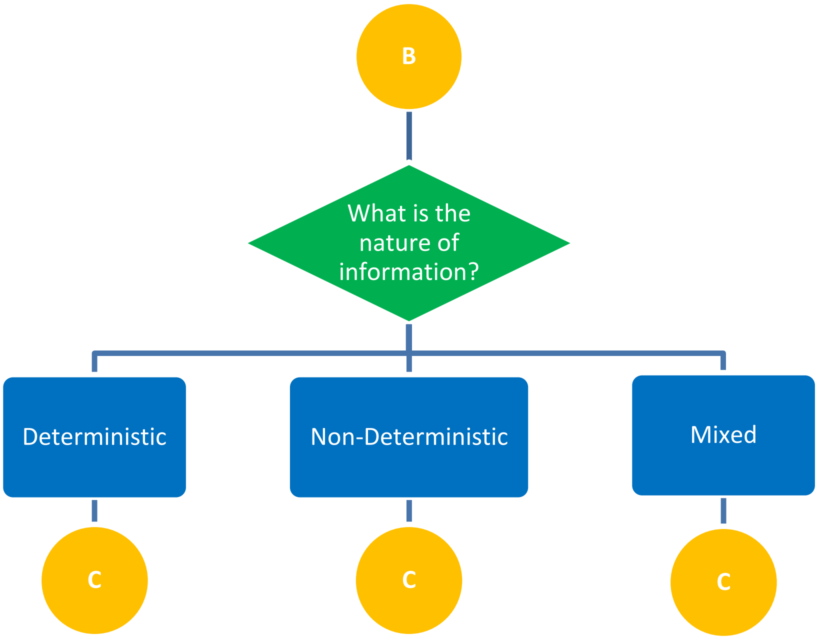


Figure 2.4. Third Stage of the Typological Tree

Similarly to stage two, stage three asks “What is the nature of the information?”, to assess the determinism of the information. With the proper answer one can choose a method that is able to deal with certain, uncertain, fuzzy or other types of information.

Last stage of the typological tree defines the final selection through the question “Which decision problematic is addressed?”. This level relates to the guideline G3 to define a method that suits the DM intentions. As we can see in Figure 2.5 these are the last branches of the tree, and they lead us to a selected MCAP or multiple.

Although the tree is only able to presents these four stages referring to guidelines G1, G3 and G4, it is important not to forget the other four guidelines. The application of G2, G5, G6 and G7 can sometimes guarantee an unequivocal output, and this is why the application of these four remaining guidelines is usually performed after using the typological tree to further refine the results.

Analyzing our designated methodology and all the other studies, we found that a common denominator to all of them is the fact that, even though some of them are very extensive and accurate, none of them is able to encompass all the methods and all the DMS.

This problem could be solved with a standard tabulation for all the methodologies, creating a universal taxonomy for one or several of the available selection techniques. Despite the fact it is an interesting research topic, the method selection issue is outside the boundaries of our work. Thus we will accept the drawback of having multiple possibilities as an output of the method selection technique used, and justify our choices with other arguments.

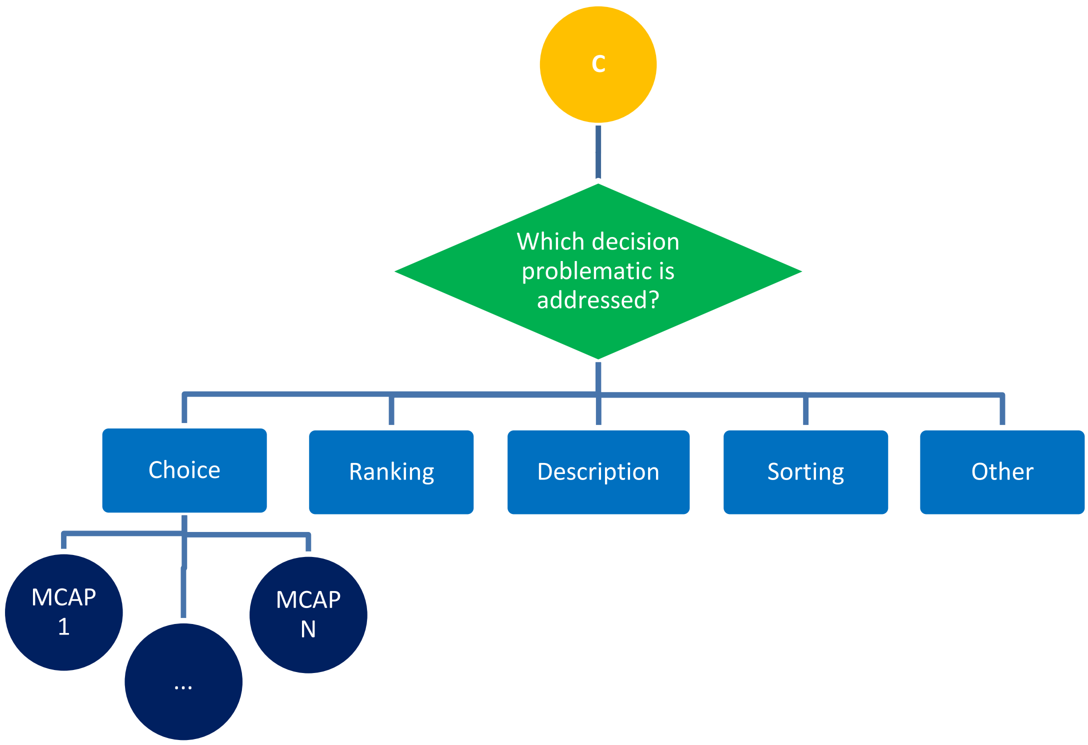


Figure 2.5. Fourth Stage of theTypological Tree

3. Case Study

EnPROVE, Energy consumption prediction with building usage measurements for software-based decision support, is a European project supported by the European Union's Seventh Framework Programme (FP72007-2013) under grant agreement 248061. This project ran between 2010 and 2013 and gathered institutions from Portugal, Spain, Germany, Netherlands, Poland and Ireland.

3.1. EnPROVE project description

Most building owners forgo building renovation and direct their investment to other areas, which may have a bigger impact. In addition, there are so many technologies related to energy efficiency measures, that it becomes an impossible mission to select the most appropriate ones for a specific building.

The EnPROVE project's main objective is to convince, in an objective and accurate way, the investors, either building owners or not, to invest in renovation of existing infrastructures. The recovery of invested capital happens by the reduction of energy consumption and in shorter periods than usually perceived.

EnPROVE developed a method to predict energy consumption of a building once appropriate energy-efficient technologies were employed. This was used to prepare an implementation plan convincing building owners to renovate with energy-efficient solutions. The result was an easy-to-use software decision-support tool, structured to fit on a variety of architectural software programs.

The key hypothesis followed by EnPROVE is that it is possible, from the adequate gathering and assessing of data on how an infrastructure is being used, to build Energy Consumption Models relevant for prediction of alternative scenarios. By relevant prediction, it is meant enabling the assessment of the energy-efficiency impact of several alternative technologies for which available investment resources can be directed and, thus, supporting the decision maker in finding the best set of energy-efficient solutions to be implemented.

EnPROVE assumed that the data gathered on how an infrastructure is used may serve to improve the accuracy on prediction of future energy consumption impact of installing alternative sets of available technologies. This also justifies the necessary renovation investment based on a financial return-on-investment calculation.

In short, EnPROVE monitors the usage of a building, models the building's energy consumption, and uses these two elements to predict energy consumption under alternative scenarios based on available market solutions and provide recommendations for a best solution, taking into consideration the decision-maker's criteria and restrictions.

The concept of the EnPROVE platform is based on analyzing the real use of the building and proposing sets of control technologies that could be installed in the building, predicting the energy consumption.

The EnPROVE platform consists of two major systems [31]:

- The Building Performance and Usage Auditing includes a wireless sensor network deployed in the building to be renovated, connected to local gateways that transmit data to the remote building performance and usage server, which processes this.
- The Energy Prediction and Decision Support System responsible for interacting with the technical consultant to extrapolate the data collected from the building and predict energy consumption for several possible technical solutions, and enable the investor in selecting the best renovation scenario considering tangible (e.g. return on investment) and intangible (e.g. comfort level) criteria.

The EnPROVE platform is used to support a full assessment of a building to be renovated, suggest a set of possible renovation scenarios, and help the decision maker in selecting the most appropriate one.

A wireless sensor network is deployed to the building being assessed, to collect data on occupation, temperature inside and outside, daylight inside and outside, luminance, lighting and HVAC actuation. Sensors are deployed to typical zones, avoiding having to audit the complete building. The audit results are extrapolated to achieve full yearly profiles of a building's use, which together with installation information, comprise the building's baseline scenario, or starting point.

The EnPROVE decision support system suggests a set of renovation scenarios to be applied to the building, which can be compared with the baseline scenario. The investor has to select the most appropriate scenario to be implemented.

3.2. Technical and Financial Analysis

All the scenarios proposed are compared in terms of energy consumption following three Key Performance Indicators (KPI).

The defined KPI for evaluating the different scenarios are the Lighting Energy Numeric Indicator (LENI) the Lighting Initial Investment Indicator (LINI), and the Lighting Maintenance Indicator (LIMI).

LENI accesses the total annual lighting energy required in a building and it is expresses in $[\mathrm{kWh}/(\mathrm{m}^2 * \mathrm{year})]$. On the other hand, LIMI defines the total amount of money spent per year in maintenance, and it is expressed in $[€/(\mathrm{m}^2 * \mathrm{year})]$.

The last indicator is related to the initial investment for each scenario. LINI is presented in $[€/\mathrm{m}^2]$.

After the technical evaluation, a financial analysis is performed to supply another set of indicators. This means that each scenario becomes a possible project to implement, so its financial characteristics need to be assessed in order to understand the project's validity.

The financial perspective proposed by EnPROVE relies on three major indicators: Discounted Payback Period (PBP), Net Present Value (NPV) and Internal Rate of Return (IRR).

These indicators result from the combination of the KPI and the information related to the building. Following the indicators, the EnPROVE platform uses the available data to calculate standard, discounted, and cumulative Cash Flows (CF) for each scenario and then the Initial Investment, and the PBP, the NPV and the IRR.

3.2.1. Cash Flow

Since the aim of EnPROVE is to maximize energy efficiency, the Cash Flow results from an investing activity where the energy savings are considered the project Inflow.

Normally on a project, after the Initial Investment, the cash inflow is expected to be presented by the amount of money coming, for example from revenues. However, in this situation the cash inflow is the difference between what would be spent in a baseline scenario and what is really spent in one of the renovation scenarios provided by the platform. This means that the cash inflow is the amount of money saved after the renovation.

On the other hand, the cash outflow is the initial investment and, after that, the amount of money spent on building maintenance. In that way, the Cash Flow is the difference between Inflows and Outflows for a certain period, which in this case study is a year.

Two other concepts related to Cash Flow (or Net Cash Flow) are important, the first is the Discounted Cash Flow (DCF) and the second is the Cumulative Cash Flow (CCF). The former takes into account the time value of money to represent the present value of future Cash Flows.

The latter is the sum of all Cash Flows (Net or Discounted) since the inception of the project or the company until a certain period, and it allows understanding the long term strength of a project.

To obtain the Discounted Cash Flow:

$$DCF = CF \times \frac{1}{(1+i)^n}$$

Where, $\boldsymbol{i}$ – discount rate

$\boldsymbol{n}$ – period

3.2.2. Payback Period

There are two possible Payback Period indicators, the *Simple Payback Period (SPB)* and the *Discounted Payback Period,* being the latter considered much more accurate to make a decision.

The SPB represents the time the initial investment (outflow) is expected to be recovered from the inflows created by the investment.

Since the SPB does not account for the time value of the money, the PBP approach is followed in order to overcome that drawback.

To determine the PBP, first we should calculate the discounted cash flow and then the accumulated cash flow. Then we follow the formula:

$$Discounted\ Payback\ Period = A + \frac{B}{C}$$

Where, $\boldsymbol{A}$ – Last period with a negative discounted cumulative cash flow

$\boldsymbol{B}$ – Absolute value of discounted cumulative cash flow at the end of period A

$\boldsymbol{C}$ – Discounted cash flow during the period after A

Both Simple and Discounted indicators do not take into account the cash inflows after the Payback Period. This means that other indicators should also be used to evaluate the project.

3.2.3. Net Present Value

NPV is the sum of the present value of the cash flows of a project over its lifetime. It is a reliable indicator to evaluate the profitability of a project, since it accounts for the time value of money, by using discounted cash flows. To determine NPV a discount rate must be considered to discount the net cash flows.

The following formula stands for the NPV:

$$NPV = -\,CF_0 + \sum_{n=1}^{T} \frac{CF_n}{(1+i)^n}$$

Where, $\boldsymbol{CF_0}$ – Initial Investment

$\boldsymbol{CF}$ – Cash Flow

$\boldsymbol{i}$ – Discount rate

$\boldsymbol{T}$ – Project life time

3.2.4. Internal Rate of Return

The IRR is the discount rate that brings the net present value of an investment to zero. In other words, the IRR is the discount rate that equals NPV to the current value of the Initial Investment, or the break even rate. IRR is also a reliable measure for project appraisal. It allows a meaningful comparison with the defined discount rate showing the quality of the investment.

To determine the IRR:

$$NPV = 0 \Leftrightarrow -\,CF_0 + \sum_{n=1}^{T} \frac{CF_n}{(1+irr)^n} = 0$$

Where, $\boldsymbol{NPV}$ – Net Present Value

$\boldsymbol{CF_0}$ – Initial Investment

$\boldsymbol{CF_n}$ – Cash Flow per period

$\boldsymbol{irr}$ – Internal Rate of Return

$\boldsymbol{T}$ – Project life time

3.2.5. Decision Rules

All the financial indicators mentioned above have a decision rule which helps understanding if a certain project, under the evaluation of a particular factor, represents or not a good investment. These rules can be used either to evaluate a single project, or compare between a set of alternatives.

The decision rule for the PBP states that one should invest in the project with the smallest period. For instance, if there is a target PBP, a project with a shorter period, than the target value, is most likely to be accepted.

The decision rules for the NPV and IRR are slightly different, since the bigger the value of the indicator, the more probable is the project to be accepted. Usually a project is accepted if its NPV is positive or zero. However other indicators should be considered if the NPV is null. Moreover, whenever judging different projects, the one with the highest value of NPV should prevail.

Respecting the comparison of multiple projects, with equal initial investments, the IRR rule follows the previous, being the project with the highest value of IRR the right choice. Another important point is that a project with an IRR smaller than the target discount rate should be put aside.

These rules are very significant and can be used to provide a conjoint approach to help choosing the best project. On chapter 5 we will analyze these decision rules under a MCDM approach.

3.3. Test Case: Building in Dublin, Ireland

The first test of EnPROVE was realized in an office building in Ireland. The objective was to renovate only the lighting infrastructure of a portion of a building of 445 m^2. The EnPROVE platform suggested twelve lighting renovation scenarios with energy savings between 300 kWh/year and 6 000 kWh/year, and investment efforts between 60 € and 9 000 € have been selected as input to the decision support process and the review by the investor.

The set of results produced by the EnPROVE platform in the Irish building were the beginning of the comparative analysis proposed in this dissertation. According to what we have described in chapter 2 the structure of a decision process is divided into five different phases. The first one is the *Structuration* and represents the definition of the basic elements of the decision situation: the *criteria*, the *alternatives* and the corresponding *performance table*.

Until now we have mentioned twelve renovation scenarios that henceforth will be mentioned as the alternatives/potential actions of the DMS. These alternatives are displayed in Table 3.1 alongside with the baseline scenario, or the original configuration of the building, before the auditing process.

We have also presented three financial indicators, the *PBP*, the *NPV* and the *IRR*, that in the context of the *Structuration* phase, and by EnPROVE default, we will consider the criteria/attributes of the DMS.

Lastly the performance table is obtained by the processes previously described in this chapter. The KPI (Table 3.2) were used to produce the values of the three decision criteria for each alternative, considering a discount rate of 2% and an energy price of 0,165 $€/\mathrm{kWh}$ (values determined by EnPROVE). The final values of the performance table are exhibited in Table 3.3

Table 3.1. Baseline and renovation scenarios with energy savings information

Scenario	Scenario Description	Energy Savings
A	Baseline	-
B	Scheduling (all zones) - auto 1-KeepLP-KeepCtrl	12,83%
C	Scheduling (all zones) - auto 1 - KeepLP-LocalCtrl	12,83%
D	Scheduling (all zones) - auto 1 - KeepLP-AreaCtrl	12,83%
E	Manual On/Occupancy Off (improving) - auto 1 - KeepLP-LocalCtrl	30,01%
F	Occupancy On/Occupancy Off (improving) - auto 1 - KeepLP-KeepCtrl	10,45%
G	Occupancy On/Occupancy Off (improving) - auto 1 - KeepLP-LocalCtrl	10,45%
H	Daylight Dimming (improving) - auto 1 - KeepLP-LocalCtrl	74,70%
I	Daylight Dimming (improving) - auto 1 - KeepLP-AreaCtrl	74,70%
J	Scheduling (all zones) - auto 1 & Manual On/Occupancy Off (improving) -auto 1-KeepLP-KeepCtrl	38,13%
K	Scheduling (all zones) - auto 1 & Manual On/Occupancy Off (improving) -auto 1-KeepLP-LocalCtrl	38,13%
L	Scheduling (all zones) - auto 1 & Daylight Dimming (improving) -auto 1-KeepLP-LocalCtrl	82,53%
M	Scheduling (all zones) - auto 1 & Daylight Dimming (improving) -auto 1-KeepLP-AreaCtrl	82,53%

Table 3.2. KPI and Equipment life time of each scenario

Scenario	LENI $[\mathbf{kWh}/(\mathbf{m}^2 * \mathbf{year})]$	LINI $[€/\mathbf{m}^2]$	LIMI $[€/(\mathbf{m}^2 * \mathbf{year})]$	Equipment Life time
A	17,23	0	0,36	-
B	15,02	0,17	0,37	20
C	15,02	0,72	0,37	20
D	15,02	5,76	0,37	20
E	12,06	1,28	0,37	20
F	15,43	0,13	0,37	20
G	15,43	0,16	0,37	20
H	4,36	14	0,37	20
I	4,36	18,22	0,37	20
J	10,66	1,45	0,37	20
K	10,66	6,84	0,37	20
L	3,01	14,72	0,37	20
M	3,01	19,88	0,37	20

Table 3.3. Performance Table – Case study

Scenario	IRR	NPV	PBP (years)
B	208,62%	2.504,92 €	1
C	49,24%	2.260,17 €	3
D	2,07%	17,37 €	20
E	65,86%	5.564,76 €	2
F	220,77%	2.030,47 €	1
G	179,37%	2.017,12 €	1
H	14,00%	9.149,01 €	8
I	9,82%	7.271,11 €	10
J	74,07%	7.169,95 €	2
K	14,69%	4.771,40 €	7
L	14,88%	10.449,42 €	7
M	10,01%	8.153,22 €	10

4. Choosing the appropriate decision method for the Case Study

After defining the case study problem with all the fundamental characteristics, and with that defining the DMS, we can bring the methodology presented in chapter 2 and through it select the appropriate decision method to help solve it. As we have already mentioned the chosen methodology applies a set of guidelines and a typological tree to determine the suitable decision method among a list of twenty-nine possibilities (see Annex A.).

In the present chapter we will go through the process of selection step by step, simulating the application of this methodology by a DM or a decision analyst.

The first element that we have considered was the typological tree. Therefore, the guidelines G1, G3 and G4 are the leading components of the selection. The remaining guidelines were applied after removing most methods, guaranteeing a refined output of the process.

The first question of the typological tree placed by guideline G1 is “What is the operational approach?”. This interrogation usually has one single answer depending on the DM preferences. However, the main objective of our work is the performance of a comparative analysis between different MCDM approaches. Examining the four possible operational approaches in the methods catalogue (see Annex A.), we noted that the single synthesizing criterion approach and outranking synthesizing approach were the ones with more available options. Therefore the answers to the first question of the typological tree are the ones highlighted in Figure 4.1 to assure a broader and richer analysis.

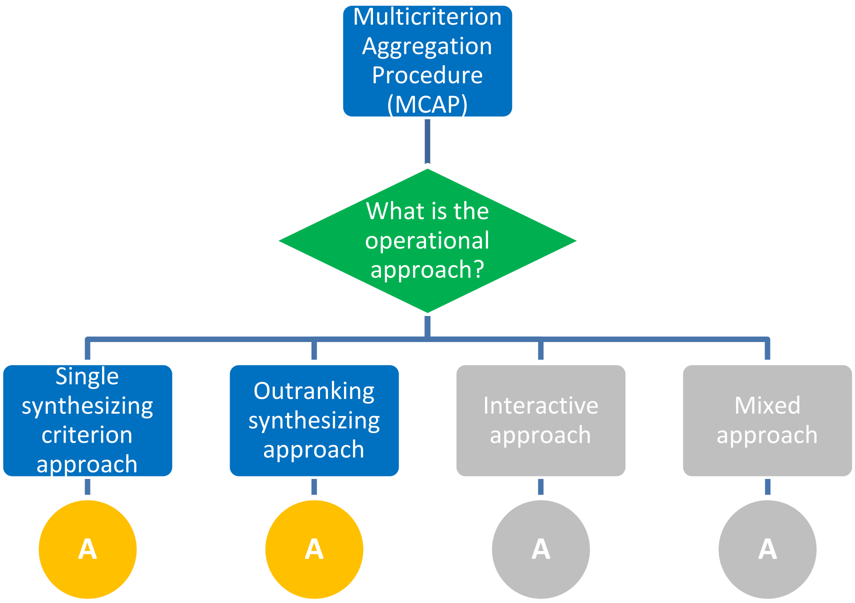


Figure 4.1. First stage of the typological tree - Method selection

The next questions on the tree are relative to the information involved in the DMS. The guideline G4 embodies those two questions, “What kind of information is considered?” and “What is the nature of information?”. According to the data of the Case Study the answers are easily obtained, since we are dealing with cardinal and deterministic information (Figure 4.2 and Figure 4.3).

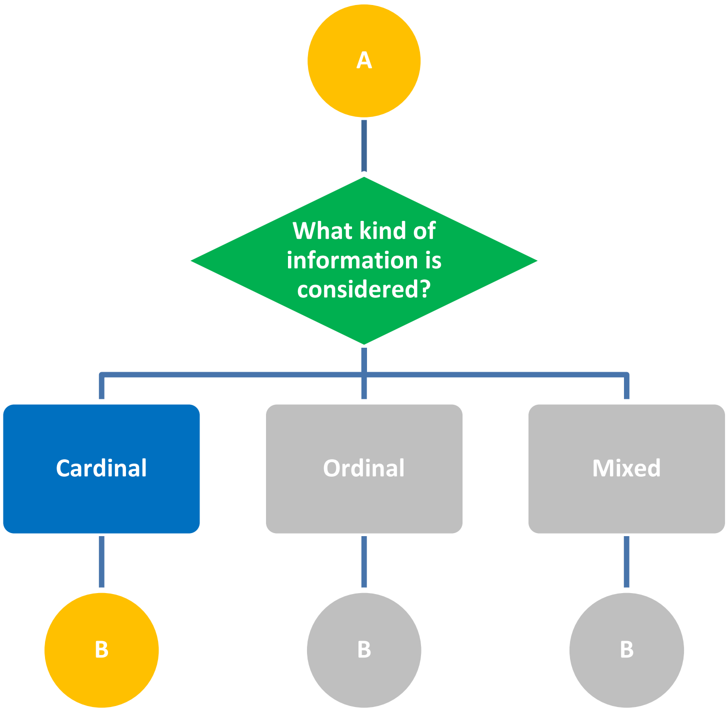


Figure 4.2. Second stage of the typological tree – Method selection

By the end of this stage, we are still considering six methods from the single synthesizing criterion approach and nine from the outranking synthesizing approach. This means that at this point of the selection process we have already eliminated fourteen methods from the original twenty-nine.

The final step of the typological tree represents the guideline G3 and the question “Which decision problematic is addressed?”. The obvious answer is ranking, since the objective of our DMS is to find the best renovation scenario and understand the order of the following possibilities. At this point we present the possibilities found by using the typological tree for both approaches, single synthesizing criterion (Figure 4.4) and outranking synthesizing (Figure 4.5).

The application of the typological tree resulted in the elimination of twenty-one methods, leaving two methods from the *single synthesizing criterion approach* and six from the *outranking synthesizing approach*.

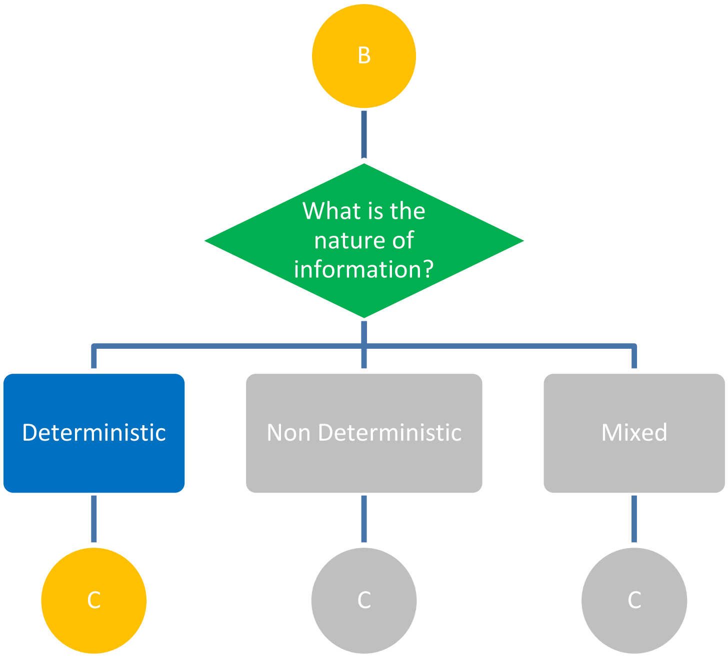


Figure 4.3. Third stage of the typological tree - Method selection

Since we could not find the two unequivocal outputs we were looking for with the use of the first element of the methodology, the application of the four remaining guidelines is necessary.

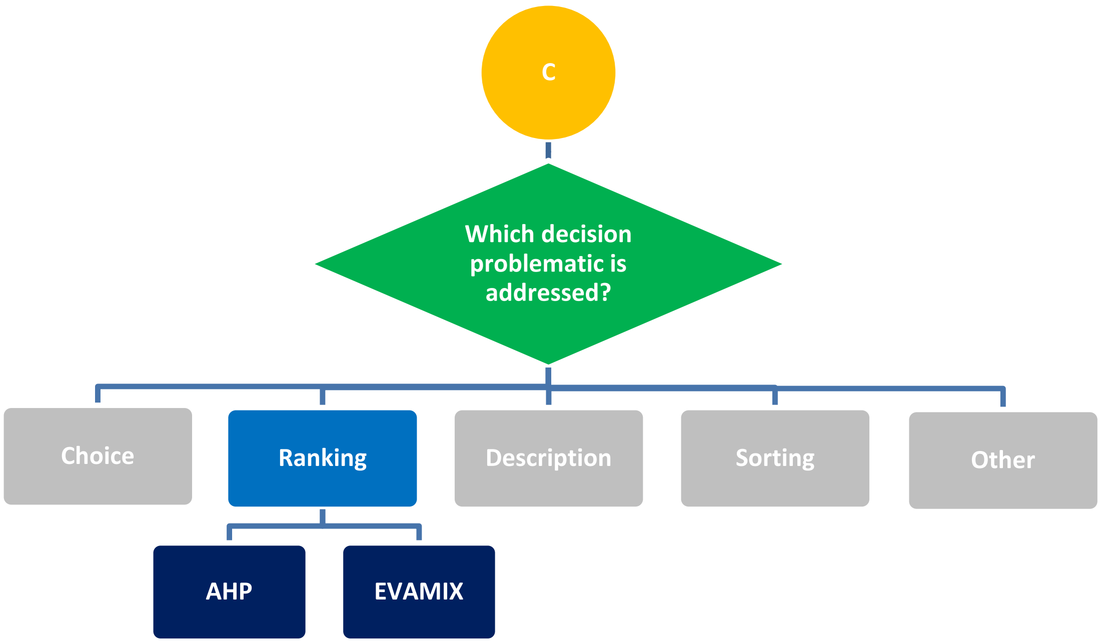


Figure 4.4. Fourth stage of the typological tree - Method Selection (1)

We will start by considering the last guideline G7, since it is the easiest one to apply. This guideline evaluates the existence of a support system, more precisely a software tool. It represents an important subject for our investigation, since the use of a support system to run the gathered data through the decision methods, will streamline the process of obtaining and analyzing the results.

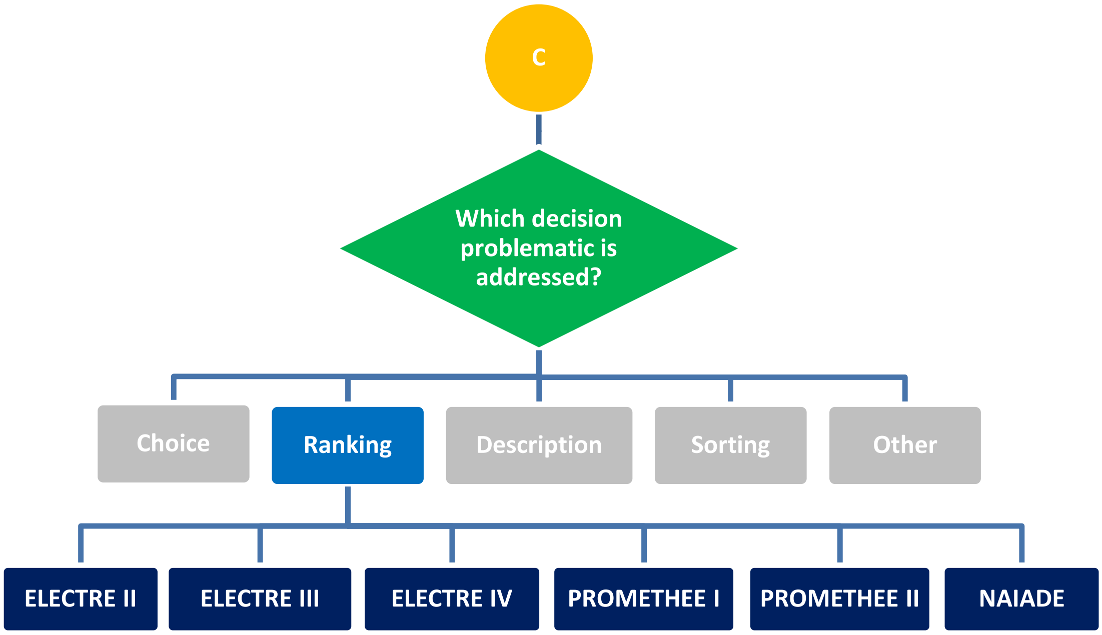


Figure 4.5. Fourth stage of the typological tree - Method Selection (2)

According to the catalogue created for the methodology in use (see Annex A.), the guideline G7 allows to remove two methods, one from each approach. This situation highlights the first unequivocal solution, corresponding to the single synthesizing criterion approach – the *Analytic Hierarchy Process (AHP)* method. G7 also takes out one method from the outranking family, leaving five potentials solutions. In that way another guideline must be used to conclude the selection or to reduce the number of methods.

The next chosen guideline will be G2, which is divided in four topics. G2 refers to the DM cognition towards the preference elucidation modelling. The first of its composing topics that we will consider refers to *ordering of the alternatives*, resulting from the method application. This is probably the most important topic of G2 for our concerns, as we are considering a *ranking* problematic, where the type of resulting order is very important.

The first output that we achieved – *AHP* method – exhibits a *total preorder* when considering the G2 order topic. This happened due to the selection process, but it is in line with what we are looking for. The best renovation scenario can only be found with a *total preorder* of the alternatives. Considering the five remaining *outranking* methods, there are only two that satisfy the order condition we mentioned – *PROMETHEE II* and *NAIADE*.

Finally it is possible to choose between these two methods using the other topics of G2 and also G5 and G6. We will combine aspects of all the guidelines in order to select between the two methods. Moreover, we will simultaneously explore these guidelines on the AHP method so as to choose a method with as similar characteristics as possible, allowing a more significant comparison.

According to the corresponding topic of G2, PROMETHEE has a *preference structure* more similar to that of AHP - both methods are based on a *Preference* and *Indifference* structure. Next guideline to be observed will be G5, the one related to the compensation degree of the MCAP. G5 is divided into three topics, *discrimination power of the criteria, compensation* and *information inter-criteria.* Once again, and although they do not match perfectly, the method that has more in common with AHP is PROMETHEE. Lastly, following the G6 guideline one can select the method by considering its hypothesis. Both PROMETHEE and NAIADE use leaving and entering flows. Nonetheless NAIADE also considers fuzzy arithmetic, an aspect that makes it more difficult to apply. Thereby, bearing in mind these three last guidelines and the similarities with AHP, we selected PROMETHEE over NAIADE.

4.1. Selected methods for the comparative analysis

The result of the selection process brought up as solutions the two methods mentioned above*: Analytic Hierarchy Process (AHP)* and *Preference Ranking Organization Method for Enrichment Evaluations II (PROMETHEE II)*. In the present section a brief explanation of these two methods will be performed in order to explain their structures of application.

4.1.1. Analytic Hierarchy Process

The Analytic Hierarchy Process (AHP) was first introduced by Thomas L. Saaty in 1977 as "a theory of relative measurement on absolute scales of both tangible and intangible criteria based both on the judgment of knowledgeable and expert people and on existing measurements and statistics needed to make a decision" [16].

This method can be categorized under the group of the single synthesizing criterion approaches, where methods like MAUT, MAVT and TOPSIS [28] can be found.

AHP has been evolving since the 1980's by the hand of its creator [21] but also through the work of a wide scientific community that gathered around the potential of the application of this method in different areas (see [40]).

A major reference in the history of the method is the creation of the Analytic Network Process (ANP) [41], a generalization of AHP based on feedback and dependence that allowed the original concept to reach new areas and take part in new applications. An interesting example is the use of AHP/ANP in BOCR analysis - Benefits, Opportunities, Costs and Risks (see [42] and [43]).

The method is largely mentioned as the best known and most used decision method worldwide, which means that every year more and more enthusiasts, practitioners and researchers dedicate their time to improve the method and its applications. However, this also means that the method is constantly under observation and is subject to a lot of criticism from the scientific community, especially from supporters of other methodologies. Among all the situations pointed out as drawbacks of the AHP method, the rank reversal problem collects most of the attentions, thus it will be under our observation further in this chapter.

In the next sections we analyze some core aspects of the AHP method that will be essential on the decision process of the study case. In the first section we present some fundamental characteristics of the method and in the second section we describe the structure of AHP application. In the third section we explain rank reversal and we show the different approaches used to deal with this issue. Finally in the last section we briefly describe a software tool based on AHP and commonly used to process the data, obtain the ranking of the alternatives and perform the sensitivity analysis. We also provide an illustrative example using the software tool.

4.1.1.1. AHP theory fundamentals

The best way to understand AHP is to start by analyzing its roots. The method has three primal facets that inspired its designation (see [44] and [45]). Those facets are:

Analytic Facet - The method approaches every problem by separating and identifying its core elements. This analysis allows the decision maker to understand the different dimensions of the decision situation and to easily evaluate them. Analysis is the opposite of synthesis and this means that this facet also has a connection to the synthesis ability of the method. AHP is well known as the best method to facilitate the synthesis of complex factors in a decision.

Hierarchical Facet - It is a natural human response divide an intricate problem into multiple smaller and less complex problems. Saaty captured this natural reaction and included it in the method as a way to structure the decision situation and easily describe and solve it.

Procedural Facet - The application of AHP is based on different steps that allow the decision maker to progressively reach a result, in this case a solution for the problem considered. These steps will be discussed below.

Beyond those three dimensions of AHP we can find other important features that result from the evolution of the methodology and represent now its basic structure. Saaty stated that the method is based on seven pillars [46]:

1. Ratio scales, proportionality, and normalized ratio scales
2. Reciprocal paired comparisons
3. Sensitivity of the principal right eigenvector
4. Homogeneity and clustering
5. Synthesis that can be extended to dependence and feedback
6. Rank preservation and reversal
7. Group judgments

Some of these aspects will be under observation along this chapter, with special attention to rank preservation and reversal.

4.1.1.2. AHP structure

In the previous section we mentioned the three facets that are in the origin of AHP. Those facets can be easily identified in the application structure of the method. Saaty proposed that to apply the AHP four major steps must be followed [3]:

Step 1 - Define the problem to determine the type of knowledge sought.

In this first step it is expected to understand the dimension of the problem, the stakeholders and what kind of solution they are looking for. The method has been used for wide range of applications but mainly to solve choice problematic problems, ranking and resource allocation situations, benchmarking of processes or systems and quality management.

Step 2 - Define a hierarchical structure for the problem by identifying its core elements: goal, attributes/criteria, sub-criteria and alternatives.

The AHP method is based on a hierarchical system, composed by different levels. For AHP every decision problem has a hierarchical structure that starts with the goal of the problem as the first level, e.g. “buying a house”, “choosing a location for a new facility”.

The next level of the hierarchy includes the attributes or criteria used to evaluate the alternatives or possible actions. For a problem such as “buying a house”, criteria like price, neighborhood safety or age of the house could be considered.

The lower levels of the structure can be associated to sub-criteria related to the criteria in the level above. However, the lowest level of all hierarchies always represents the alternatives of the problem, which in the simplest structure are placed in the base, as we can see in Figure 4.6.

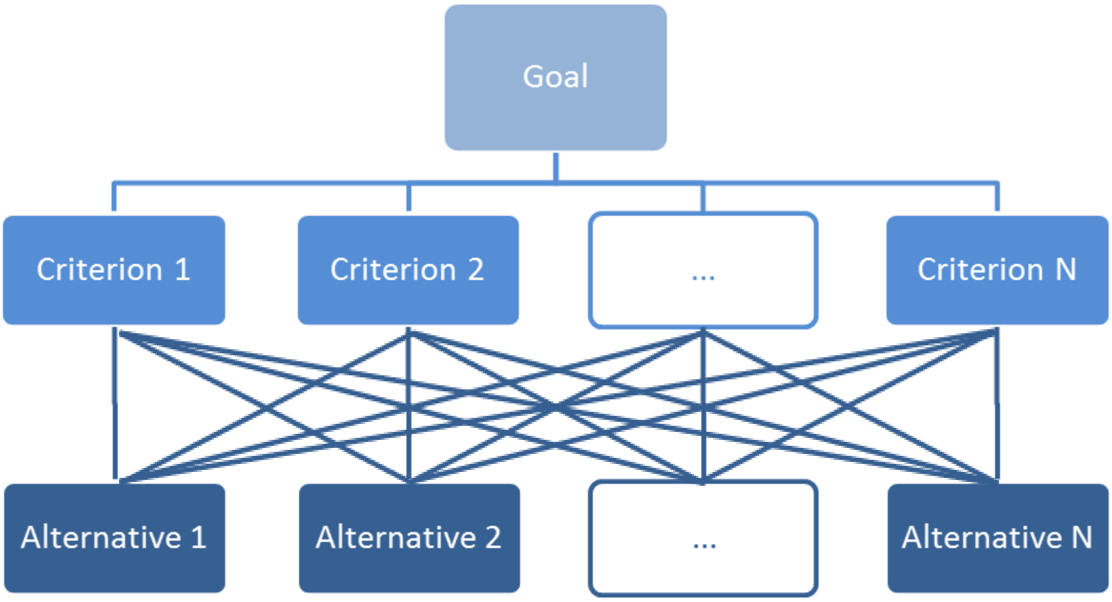


Figure 4.6. AHP Hierarchy of a problem (simplest form)

Step 3 - Compare by pairwise comparisons elements of a level with respect to the one in a level above.

In this third step we are defining the weights of each criterion and the priority of the alternatives considering that criterion. The procedure to achieve the weights and priorities is based on pairwise comparisons, which represent one of the seven pillars of the method.

In AHP the pairwise comparisons can be performed by three different judgment elicitation modes: verbal, numerical and graphical. These modes have different associated scales that allow to determine the importance or dominance of an element over another.

The fundamental scale of absolute numbers is the original AHP scale, also known as 1-9 verbal scale (Table 4.1).

At the end of step three it is expected that a set of comparison matrices is produced. These comparisons matrices refer to the evaluation of the criteria under the goal of the problem, the sub-criteria under each criterion and the alternatives under each criterion or the sub-criterion. The standard comparison matrices are represented in Table 4.2 and Table 4.3.

Table 4.1. AHP verbal scale (Source: [3])

Intensity of Importance	Definition	Explanation
1	Equal Importance	Two activities contribute equally to the objective
2	Weak or slight	
3	Moderate importance	Experience and judgment slightly favor one activity over another
4	Moderate plus	
5	Strong importance	Experience and judgment strongly favor one activity over another
6	Strong plus	
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice
8	Very, very strong	
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation

Table 4.2. Criteria pairwise comparison table

Goal	*Criterion 1*	*…*	*Criterion N*
Criterion 1			
…			
Criterion N			

From Table 4.2 we define the first comparison matrix, which has a NxN structure and it is denoted as $\mathbf{B} = (\boldsymbol{b_{ij}})\ (i, j = 1, 2, \dots, N)$. This matrix results from the pairwise comparative judgment criteria like (g_i, g_j).

The entries b_{ij} of the matrix follow two rules [4]:

Rule 1 – If $b_{ij} = \alpha$, then $b_{ji} = {}^{1}/_{\alpha}$, $\alpha \neq 0$

Rule 2 – If g_i has equal relative importance as g_j, then $b_{ij} = 1$, $b_{ji} = 1$, and $b_{ii} = 1$, for all i

Considering these rules, the reciprocal matrix $\mathbf{B}$ is represented as:

$$\mathbf{B} = \begin{bmatrix} 1 & b_{12} & \cdots & b_{1N} \\ {}^{1}/_{b_{12}} & 1 & \cdots & b_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ {}^{1}/_{b_{1N}} & {}^{1}/_{b_{2N}} & \cdots & 1 \end{bmatrix}$$

In the context of our dissertation the weights of the criteria will be directly assigned. This means that the weights of the criteria $(w_i, w_j, \dots, w_N)$ are known and then $b_{ij} = {}^{w_i}/_{w_j}\ (i, j = 1, \dots, N)$.

Table 4.3. Alternatives pairwise comparison table

Criterion N	Alternative 1	...	Alternative M
Alternative 1			
...			
Alternative M			

For Table 4.3 the comparison matrix obtained is called $\mathbf{D}$ and it is related to priority of the alternatives $(\boldsymbol{a_1}, \boldsymbol{a_1}, \dots \boldsymbol{a_M})$ under a certain criterion g_j, in other words the local priority of the alternatives. The rules for obtaining the matrix $\mathbf{D}$ are similar to the ones presented for matrix $\mathbf{B}$.

Matrix $\mathbf{D}$ has a MxM structure and it is denoted as $\mathbf{D}_{\boldsymbol{j}} = (d_{mn})$ $(m, n = 1, 2, \dots, M)$ $(j = 1, 2, \dots, N)$. This matrix results from the pairwise comparative judgment criteria like (g_i, g_j).

The entries d_{mn} are defined by the same type of rules [4]:

Rule 1 – If $d_{mn} = \alpha$, then $d_{nm} = {}^{1}/_{\alpha}$, $\alpha \neq 0$

Rule 2 – If a_m has equal relative importance as a_n, then $d_{mn} = 1$, $d_{nm} = 1$, and $d_{mm} = 1$, for all m

Considering these rules, the reciprocal matrix $\mathbf{B}$ is represented as:

$$\mathbf{D}_{\boldsymbol{j}} = \begin{bmatrix} 1 & d_{12} & \cdots & d_{1M} \\ {}^{1}/_{d_{12}} & 1 & \cdots & d_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ {}^{1}/_{d_{1M}} & {}^{1}/_{d_{2M}} & \cdots & 1 \end{bmatrix} (j = 1, 2, \dots, N)$$

Step 4 - Define the final/global priorities of the alternatives by combining the weights of the criteria and the priorities for each element under those criteria.

In this final step, different methods can be considered to find the global priorities of the alternatives, using both matrixes $\mathbf{B}$ and $\mathbf{D}$. Among those methods, such as the geometric mean method or the lambda-max method, the eigenvalue method is the most used [11].

A fifth step can be included in order to evaluate the consistency of the paired judgments provided. This step uses the consistency index (CI) and the consistency ratio (CR) to determine if the judgments on the matrices are inconsistent or not. Since this step can be set aside in the context of our work, for more information on the topic we recommend the additional reference [16].

4.1.1.3. Rank reversal

The rank reversal is a transversal issue in the field of MCDM, happening in different methods from different methodological approaches. However, the development of AHP drew a lot of attention to this ranking problem.

Rank reversal happens when the order previously determined among the old alternatives suffers a change with the addition or deletion of alternatives. This happens when alternatives are dependent among themselves [47].

The reason researchers give so much importance to rank reversal is based on the fact that the axioms where utility theory and multiattribute utility theory were founded, mention the following:

> *"Adding new acts (alternatives) to a decision problem under uncertainty, each of which is weakly dominated (preferred) by or is equivalent to some old act, has no effect on the optimality or non-optimality of an old act." Luce and Raiffa [48]*
>
> *"If an act is non-optimal for a decision problem under uncertainty, it cannot be made optimal by adding new acts to the problem." Luce and Raiffa [48]*

These arguments resulted in the creation of different approaches to deal with rank reversal [49], since, there are some situations where rank reversal should not exist and others where it is valid and can occur.

As a consequence of the criticism directed to the method, the original AHP model received an extension to allow both rank preservation and reversal. It now incorporates two synthesis modes one that allows rank reversal (Distributive Mode) and another that preserves the ranking of the alternatives (Ideal Mode). In the next subsections we summarize these two modes and we also provide some guidelines to understand under which circumstances one is chosen over the other.

Distributive Mode

The Distributive Mode is a synthesis used to deal with closed systems. In a closed system the resources are limited, usually it is said that in a closed system, scarcity is germane. Examples of this kind of systems are the distribution of votes on a presidential election or the allocation of corporation's R&D budget [44].

For the purpose of the AHP a closed system means that the alternative scores under each criterion are normalized to sum to one. The alternatives are dependent, and if we reduce the performance score of a certain alternative the preference for any other increases. The same happens if an alternative is removed, this resumes the issue of rank reversal [50].

In this synthesis the global priority of an alternative a_i is obtained as follows:

$$p_i = \sum_j w_j * l_{ij}$$

Where: p_i is the global priority of the alternative a_i

w_j is the weight of criterion g_j

l_{ij} is the local priority of the alternative a_i under the criterion g_j

Ideal Mode

The Ideal Mode deals with open systems, where scarcity is not germane, meaning that they allow the addition or removal of resources.

In this synthesis, for each criterion the best performing alternative is considered the ideal alternative or the benchmark. On that criterion, the local priority of this ideal alternative is equal to one, and the local priority of other alternatives is a fraction of the benchmark value [44]. In this mode "the preference for any given alternative is independent of the performance of other alternatives, except for the alternative selected as benchmark" [50]. In the Ideal mode rank is preserved.

In the Ideal synthesis the way we obtain the global priority of an alternative a_i is similar to the previous mode, but we have to consider the step relative to the benchmarking of the alternatives. In that way the local priority of an alternative a_i under a criterion g_j, for the ideal mode is obtained as presented below:

$$Ideal(l_{ij}) = {l_{ij}}\Big/\left(\max\{l_{1j}\,,\ l_{2j}\,, \dots,\ l_{Nj}\}\right)$$

Where, l_{ij} is the local priority of the alternative a_i under the criterion g_j

The global priority of an alternative a_i in the Ideal mode is given by:

$$p_i = \sum_j w_j * Ideal(l_{ij})$$

Where, p_i is the global priority of the alternative a_i

w_j is the weight of criterion g_j

$Ideal(l_{ij})$ is the Ideal mode local priority of the alternative a_i under the criterion g_j

4.1.1.4. Guidelines to choose the synthesis mode

Some guidelines were proposed to choose the appropriate mode for a given problem [50].

Distributive Synthesis Mode:

- Used when the DM is concerned with the extent to which each alternative dominates all other under the criterion.
- The DM indicates that the preference for a top ranked alternative under a given criterion would improve if the performance of any lower ranked alternative was adjusted downward.

Ideal Synthesis Mode:

- Used when the DM is concerned with how well each alternative performs relative to a fixed benchmark.
- Following the guidelines above, the DM chooses which situations are more suitable for his decision situation, and according to his choices the mode is determined.

4.1.1.5. Expert Choice - Comparion^{TM} Suite

The *Expert Choice* software is a worldwide used tool for decision making and it is based on the AHP methodology. This software is largely used by organizations, academic institutions and industry, as it provides a reliable tool, easy to use and understand. It has been evolving through the last years along with the development of AHP and it incorporates the different modes and possibilities of the method as whole. The *Expert Choice* is a paid software. Nevertheless, it has a web-based application that can be used for free, the *Comparion^{TM} Suite*. This tool will be used in the proposed case study, since it assists the result analysis allowing sensitivity evaluation.

4.1.2. Preference Ranking Organization Methods for Enrichment Evaluations

PROMETHEE represents a family of outranking methods proposed by J.P. Brans in 1982 [33].These methods are widely used by decision makers and analysts all over the world, and they also play a major role in academic research for improving decision making on different areas [34].

This family was shaped in order to establish a new group of outranking methods, as easy as possible to be understood and used by the DM. PROMETHEE was created, after the original outranking family, the ELECTRE and it is also based on the concept of dominance order (see [16] [35]).

The first outranking family, the ELECTRE, stands on an extensive group of parameters to be set by the DM and the analyst. The drawback of the use of ELECTRE methods resides on the nature of the required parameters. Although some of them have a real economic meaning, others have a technical character more difficult to understand (e.g. discordance and discrimination thresholds) [35]. In opposition, PROMETHEE relies on extensions of the notion of criterion, which are presented to the DM as different preference functions with few but meaningful parameters (maximum two).

Following the footsteps of ELECTRE, the PROMETHEE family presents different methods suitable for different decision situations. PROMETHEE started its evolution with PROMETHEE I and PROMETHEE II in 1982 [33]. Those methods were immediately used in different real problems which opened the way for the development of the first two methods ([33], [36] and [37]) and the creation of PROMETHEE III (interval order) and PROMETHEE IV (for a continuous set of alternatives) a few years later ([38] [39]).

The creators of the first four PROMETHEE methods also presented a visual interactive module called GAIA [38], a method supported on the ideas of the previous four, but standing on graphical representation.

Later on 1992 and 1994, through a series of modifications, they proposed PROMETHEE V (with segmentation constraints) and PROMETHEE VI (representation of the human brain) [16].

Although all the methods have been used and studied with incredible success in a wide set of applications and areas, for the purpose of the present document, only PROMETHEE I (partial ranking) and PROMETHEE II (complete ranking), methods are analyzed in detail.

In addition, we also consider the academic free software *Visual PROMETHEE* to support the application of the methods. This is a powerful tool that includes all the variants of PROMETHEE already mentioned. A brief use of its potential is presented in the example on the last section of this chapter.

4.1.2.1. Principles of the PROMETHEE methods

PROMETHEE deals with multicriteria problems expressed as follows:

$$max\{g_1(a), g_2(a), \dots, g_j(a), \dots, g_k(a) | a \in A\}$$

$$min\{g_1(a), g_2(a), \dots, g_j(a), \dots, g_k(a) | a \in A\}$$

In the equations, A is the finite set of possible alternatives or actions $\{a_1, a_2, \dots, a_i, \dots, a_n\}$ and $\{g_1(a), g_2(a), \dots, g_j(a), \dots, g_k(a)\}$ is the set of criteria.

The information gathered from a problem like the one presented above is grouped on an Evaluation Table such as:

$$
\begin{array}{c|cccccc}
a & g_1(\cdot) & g_2(\cdot) & \cdots & g_j(\cdot) & \cdots & g_k(\cdot) \\
\hline
a_1 & g_1(a_1) & g_2(a_1) & \cdots & g_j(a_1) & \cdots & g_k(a_1) \\
a_2 & g_1(a_2) & g_2(a_2) & \dots & g_j(a_2) & \dots & g_k(a_2) \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
a_i & g_1(a_i) & g_2(a_i) & \cdots & g_j(a_i) & \cdots & g_k(a_i) \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
a_n & g_1(a_n) & g_2(a_n) & \cdots & g_j(a_n) & \cdots & g_k(a_n)
\end{array}
$$

Moreover, the PROMETHEE methods are based on three main steps, which are examined in the following sections.

4.1.2.2. Extension of the notion of criterion

The first step of these methods is the extension of the notion of criterion. A generalized criterion $\{\boldsymbol{g_j}(\cdot), \boldsymbol{P_j}(\boldsymbol{a}, \boldsymbol{b})\}$ is related to each criterion $\boldsymbol{g_j}$ by means of a preference function. The function accesses the preference of a DM for an action $\boldsymbol{a}$ regarding an action $\boldsymbol{b}$**,** and has a value between 0 and 1. Values closer to 0, show greater indifference from the DM. On the other side, values closer to 1 represent greater preference, and functions with value equal to 1, represent strict preference. Thus, for each criterion the decision maker defines a preference function:

$$P_j(a,b) = H_j[d_j(a,b)] \; \forall a,b \; \in A$$

Where:

$$d_j(a,b) = g_j(a) - g_j(b)$$

Pairwise comparisons define the preference structure of PROMETHEE [16]. In this last equation $d_j(a,b)$ represents the deviation between a and b for a criterion j. It also indicates the areas of indifference on the neighborhood of $g_j(b)$ [35].

As mentioned above:

$$0 \leq P_j(a,b) \leq 1$$

For a criterion to be maximized, the previous function characterizes the preference of a over b. Its graphical representation is showed in Figure 4.7.

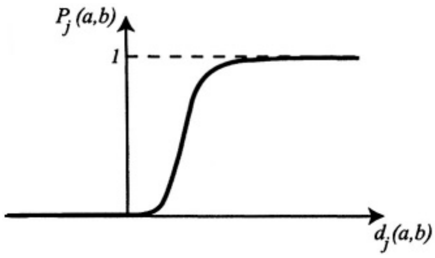


Figure 4.7. Representation of the preference function

From this function the following property is observed:

$$P_j(a,b) > 0 \Rightarrow P_j(b,a) = 0$$

On the other hand for a criterion to be minimized the preference function needs to be reversed or given by:

$$P_j(a,b) = H_j\left[-d_j(a,b)\right] \forall a,b \in A$$

The authors of PROMETHEE proposed six possible types of generalized criteria in the form of preference functions. Some of these functions require one or two parameters to be fixed by the decision maker. The possible parameters are the following:

- $\boldsymbol{q}$ – **Indifference Threshold –** If the value of the distance d is below this threshold the DM considers two alternatives indifferent;
- $\boldsymbol{p}$ – **Strict Preference Threshold –** If the value of the distance d is above this threshold the DM considers strict preference
- $\boldsymbol{s}$ – **A value between $\boldsymbol{q}$ and $\boldsymbol{p}$ –** defines the inflection point of the preference function.

The six possible types of generalized criteria are shown in Table 4.4. The first column contains the type and the description of each criterion, the second column has an analytic definition $H(d)$ based on the distance $d(a,b)$, the third shows the shape of the function and the last column presents which parameters should be fixed for each type.

4.1.2.3. Valued Outranking Relation

The second step of the PROMETHEE deals with the outranking relation, the proposed approach is considered easier to understand and much less sensitive to small modifications, compared with other outranking relations such as the one used in ELECTRE [35]. The relation is based on the concepts of *Preference Index* and *Valued Outranking Graph*.

Preference Index

A Preference Index is defined for each couple of actions a and b from the set of alternatives considered. This index is a measure of the preference of the DM for an action over another, for all the criteria. It has a value between 0 and 1. This means that for values closer to 1, the greater the preference.

The preference index $\pi(a,b)$, when all the weights of the criteria are the same is given by:

$$\boldsymbol{\pi(a,b)} = \frac{\mathbf{1}}{\boldsymbol{n}}\sum_{\boldsymbol{j=1}}^{\boldsymbol{n}} P_j(a,b)$$

The PROMETHEE does not include an approach to weight the criteria. However, using the weighting approach of another method (e.g. AHP), it is possible to calculate the weighted preference index with the following equation:

$$\boldsymbol{\pi(a,b)} = \sum_{\boldsymbol{j=1}}^{\boldsymbol{n}} w_j P_j(a,b)$$

Some important properties hold for all $(a,b) \in A$ (see [16]):

$$\begin{cases} \boldsymbol{\pi(a,a) = 0} \\ \boldsymbol{0 \leq \pi(a,b) \leq 1} \\ \boldsymbol{0 \leq \pi(b,a) \leq 1} \\ \boldsymbol{0 \leq \pi(a,b) + \pi(b,a) \leq 1} \end{cases}$$

Valued Outranking Graph

The Valued Outranking Graph is defined through a set of nodes, one for each action or alternative. Furthermore, between each two actions two arcs are outlined with the values of the Preference Indexes for those actions. Thereby, for actions $a, b \in A$ the arcs (ab) and (ba)have the values $\boldsymbol{\pi(a,b)}$ and $\boldsymbol{\pi(b,a)}$ respectively. Figure 4.8 is an example of a graph with three possible actions.

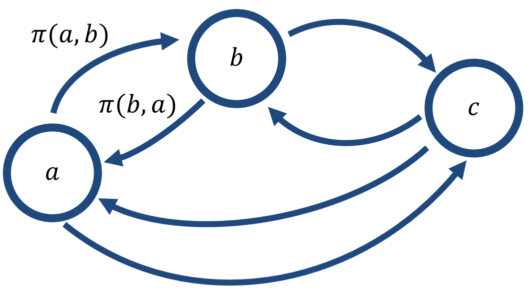


Figure 4.8. Valued Outranking Graph

4.1.2.4. Exploitation of Outranking Relation

After the definition of the Valued Outranking Graph, everything is set for the last step, the *Exploitation of the Outranking Relation*.

The Graph provides meaningful information, with easy interpretation for the DM. From the data gathered on the first and second steps it is now possible to solve the decision problem.

The Graph shows the existence of $(n-1)$ arcs leaving and $(n-1)$ arcs entering each alternative $a \in A$. This defines the Outgoing and Incoming Flows or the also called Positive and Negative Outranking Flows:

- **Outgoing Flow:**

$$\phi^{+}(a) = \frac{1}{n-1}\sum_{x \in A} \boldsymbol{\pi}(\boldsymbol{a}, \boldsymbol{x})$$

- **Incoming Flow:**

$$\phi^{-}(a) = \frac{1}{n-1}\sum_{x \in A} \boldsymbol{\pi}(\boldsymbol{x}, \boldsymbol{a})$$

The Positive Outranking Flow defines how much an alternative outranks all the others. The higher the flow value, the better the alternative (the more an action dominates the others). On the other side, the Negative Outranking Flow expresses how much an alternative is outranked by all the others. Thus, the lower the flow value, the better the alternative (the less an action is dominated).

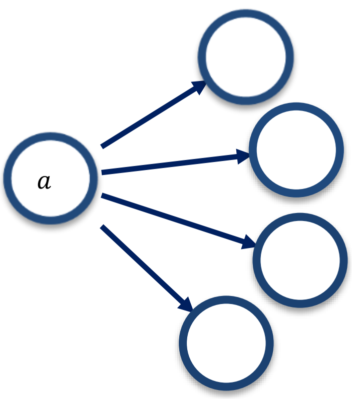


Figure 4.9. Outgoing Flow

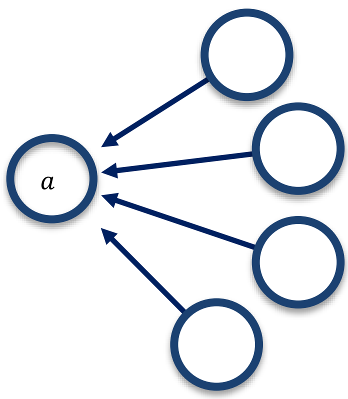


Figure 4.10. Incoming Flow

4.1.2.5. PROMETHEE I

The PROMETHEE I provides a partial preorder or ranking $(P^I,\ I^I, R)$ of the alternatives. To better understand this relation the two total preorders $(P^+,\ I^+)$ and $(P^-,\ I^-)$, induced by the positive and negative flows, are defined as follows:

$$\begin{aligned} &aP^+b \quad iff\ \phi^+(a) > \phi^+(b) \\ &aP^-b \quad iff\ \phi^-(a) < \phi^-(b) \end{aligned}$$

$$\begin{aligned} &aI^+b \quad iff\ \phi^+(a) = \phi^+(b) \\ &aI^-b \quad iff\ \phi^-(a) = \phi^-(b) \end{aligned}$$

The intersection of the previous total preorders satisfies the principals below, and expresses the PROMETHEE I partial preorder:

$$aP^Ib\ (a\ outranks\ b), if\ \begin{cases} aP^+b\ and\ aP^-b, \\ aP^+b\ and\ aI^-b, \\ aI^+b\ and\ aP^-b \end{cases}$$

$$aI^Ib\ (a\ is\ indifferent\ to\ b), if\ aI^+b\ and\ aI^-b$$

$$aRb\ (a\ and\ b\ are\ incomparable), otherwise$$

The incomparability of two actions is here considered for situations where the information, expressed by the positive and negative flow, does not allow a consistent evaluation.

4.1.2.6. PROMETHEE II

Some situations require a complete preorder or ranking of the alternatives. This is the case of PROMETHEE II, which consists on a complete ranking $(P^{II},\ I^{II})$ (without incomparable actions).

A new concept of Net Flow is then defined for each alternative:

$$\phi(a) = \phi^+(a) - \phi^-(a)$$

$$-1 \le \phi(a) \le 1$$

The Net Flow allows a complete ranking of the alternatives by balancing the positive and negative flows. However, it is responsible for some losses of information making PROMETHEE I more realistic than PROMETHEE II. Nevertheless, it is responsible for establishing the following useful relation between alternatives:

$$aP^{II}b\ (a\ outranks\ b), iff\ \phi(a) > \phi(b)$$

$$aI^{I}b\ (a\ is\ indifferent\ to\ b), iff\ \phi(a) = \phi(b)$$

4.1.2.7. Rank Reversal

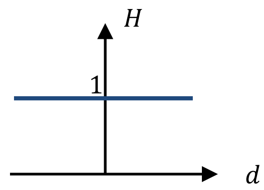
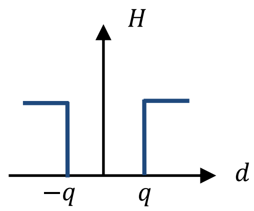
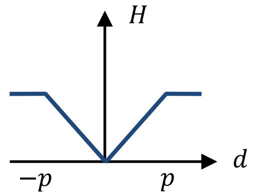
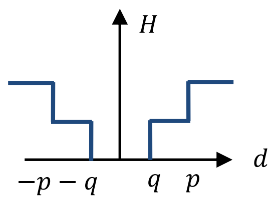
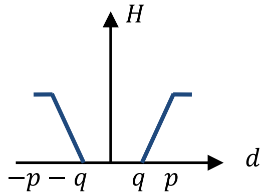
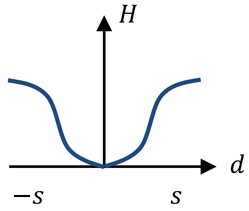
Similarly to what happens in AHP, the rank reversal issue is also presented on PROMETHEE. This singularity is closely related to all the pairwise-comparison based methods and besides the ones under analysis we can mention others like MACBETH and ELECTRE.
In the PROMETHEE case, rank reversal is limited since it mostly occurs when the flows of two alternatives are close to each other. This is generally a consequence of wrong preference modelling, which is related to the choice of the preference functions.

4.1.2.8. Visual PROMETHEE

Visual PROMETHEE is one of the most used outranking based pieces of software. This tool resulted from the evolution of the well-known *PROMCALC* and *Decision Lab* software applications developed in the 1980’s and 1990’s, respectively.

An academic version of *Visual PROMETHEE* is available for non-profit applications. We decided to apply this tool to our work in order to achieve faster and more reliable results. This software can easily solve the extensive calculations involved in a selection project such as the one under study. The *Visual PROMETHEE* also allows to test the final results of the process and perform sensitivity analysis.

Table 4.4. Generalized criteria - The most common types (Source: [37])

Types of Criteria	Analytical Definition	Shape	Parameters
Type I: *Usual Criterion*	$H(d) = \begin{cases} 0, & d = 0 \\ 1, & \lvert d \rvert > 0 \end{cases}$		-
Type II: *Quasi-Criterion or* *U-Shape Criterion*	$H(d) = \begin{cases} 0, & \lvert d \rvert \leq q \\ 1, & otherwise \end{cases}$		$\boldsymbol{q}$
Type III: *V-Shape Criterion*	$H(d) = \begin{cases} \frac{\lvert d \rvert}{p}, & \lvert d \rvert \leq p \\ 1, & \lvert d \rvert > p \end{cases}$		$\boldsymbol{p}$
Type IV: *Level-Criterion*	$H(d) = \begin{cases} 0, & \lvert d \rvert \leq q \\ \frac{1}{2}, & q < \lvert d \rvert \leq p \\ 1, & otherwise \end{cases}$		$\boldsymbol{q, p}$
Type V: *Linear Criterion or* *V-Shape with* *Indifference Area* *Criterion*	$H(d) = \begin{cases} 0, & \lvert d \rvert \leq q \\ \frac{\lvert d \rvert - q}{p - q}, & q < \lvert d \rvert \leq p \\ 1, & otherwise \end{cases}$		$\boldsymbol{q, p}$
Type VI: *Gaussian Criterion*	$H(d) = 1 - exp\left\{\frac{d^2}{2s^2}\right\}$		$\boldsymbol{s}$

5. Decision Maker Profiles

On the previous chapters we have presented all the important data regarding the case study problem, such as the set of decision criteria and the set of alternatives. We have also described the characteristics and the hypothesis of the chosen decision methods (AHP and PROMETHEE) used to deal with the case study. However, both decision methods request more input information: (1) the weights of the decision criteria, (2) the preference functions for each criterion and (3) the indifference and preference thresholds of those preference functions. In the considered case study such kind of inputs are the result of human interaction with the decision process, meaning that this is where the DM/analyst plays his role. This is also the phase of the decision process model called *Preference Modelling*.

For the purpose of the present dissertation, the information gathered from the EnPROVE project only includes technical data, without referring the preferences, profile or inputs of the DM. Although it could have been seen as a drawback, this situation was considered an opportunity since it allowed the creation of different decision maker profiles that provided interesting results.

In the present chapter, we present the process of defining the DM profiles and the consequent inputs they generate for the application of both decision methods. As a result of the profiling process three decision groups will be created.

5.1. Defining the profiles

In order to analyze different behaviors and understand how different attitudes towards the decision situation can influence the outcome of that decision, we have created three Decision Maker Profiles (DMP): *Conservative, Moderate and Aggressive*.

The DMP were based on standard investing styles generally used to define investment portfolios [51] [52]. These styles are a measure of risk tolerance, investment time horizon, personal investment goals, experience, and other factors. Fundamentally they represent the personality of the investor and the financial environment where he is inserted.

Among all the factors used to define investment profiles, risk is a key element when dealing with investments and project selection. The degree of risk for a certain investment is proportional to its potential of return. Additionally, the investment time horizon normally dictates the degree of risk an investor is willing to take.

In order to help different investors allocate their money, according to their personal characteristics, a common tool, called the Risk Pyramid or Investment Pyramid, is generally used [53].

In this methodology both risk and potential return on investment grow from the base to the top levels of the pyramid. Figure 5.1 is an example of a Risk Pyramid. This multi-levelled structure presents the low risk investments on the base level showing safety and stability, with *Principal Preservation*. As we climb to the top, towards *Speculation* level, we notice that each level is narrower and more unstable than the previous, meaning that the risk increases, but so does the potential return on investment.

Liquidity is also an important feature of the pyramid structure. It represents how easily and fast an asset or investment can be converted to cash. The base levels of the pyramid are typically the ones with the highest liquidity.

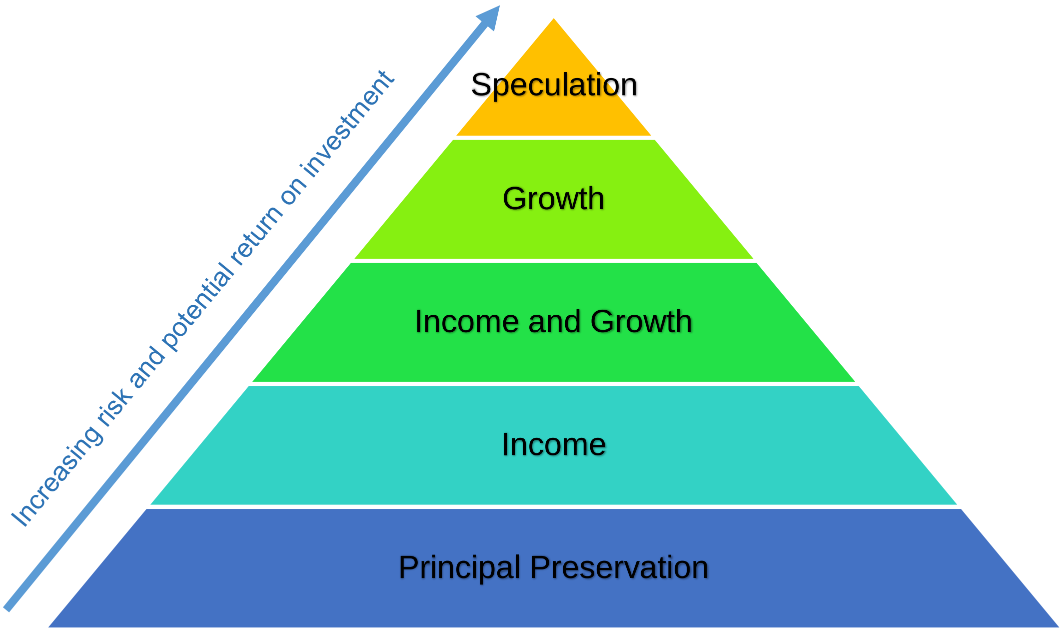


Figure 5.1. Risk Pyramid

According to their personalities the investors will allocate their investments towards the different levels of the pyramid. A risk averse investor would probably invest on the base levels of the pyramid like *Income* and *Principal Preservation,* which would grant him a low potential return on investment but a high level of security and stability.

Grounded on the concept of the Risk Pyramid and the standard investment profiles we developed the three DMP that we now present. Furthermore, the profiles were created having in mind the characteristics of the case study to establish proper decision scenarios.

5.1.1. Conservative Decision Maker

The Conservative DM was characterized as the one with the shortest investment time horizon (less than 5 years). This DM privileges the safety, principal preservation and a high level of liquidity. He has a risk averse personality and consequently his investments have low potential return. For him this kind of projects is a one-time only investment.

5.1.2. Moderate Decision Maker

The Moderate DM represents a position between the Conservative and the Aggressive personalities. His time horizon is longer than the previous DMP, going from 5 to 15 years. He has a balanced approach to the different levels of the pyramid, which grants him medium risk level and also medium potential return. He is willing to take short-term losses for long-term returns.

5.1.3. Aggressive Decision Maker

The Aggressive DM was described as the investor that is willing to take the highest risk, but that is also expecting the highest possible return. His time horizon is the longest of the three DMP – over 15 years. This allows him to balance the possible losses over time. This DMP deals with more than one of this type of projects simultaneously. For an Aggressive DM liquidity is not a concern and that is the reason he usually does not invest in the base levels of the pyramid.

5.2. Weighting the decision criteria

After defining the decision maker profiles the next step was to use them to create the input information necessary to the application of the decision methods. We started by weighting the different criteria according to each profile but first we needed to understand how the different criteria or the financial indicators fit to the approach of the Risk Pyramid and to the personality of the three decision makers that we have created.

On chapter 3, we presented the set of decision criteria, which is based on conventional financial indicators used to evaluate projects and investments (PBP, NPV and IRR). These indicators belong to the EnPROVE platform structure and that is why we considered them instead of other possibilities. The reason they were used in the project relies on the fact that they access different dimensions of investment and also different perspectives of the decision maker. To better understand how they can be used together and how they integrate our current approach we will observe these indicators in detail. A relation between the indicators and their respective levels of risk was established.

5.2.1. Discounted Payback Period

The discounted payback period, or PBP can be considered a preferential indicator to address decisions where low risk is expected. This is explained by the fact that choosing a project based on this indicator results in a decision mainly focused on recovering the investment instead of a decision dedicated to increase the investors wealth. PBP highlights the return of capital rather than the return on capital, an aspect that grants the investors that use it a low risk exposure [54].

Moreover, we can still relate PBP with another aspect of the Risk Pyramid. The payback indicator points towards liquidity since it favors short-term projects that quickly free up cash for other investments [55].

A commonly identified drawback of PBP is that it ignores the cash flows after the cutoff period. However this happens in order to avoid their uncertainty and for some investors it represents a protection against additional risk [56].

As we can see using the PBP indicator privileges low risk exposure and a high level of liquidity, consequently it also leads to a low potential return on investment.

5.2.2. Net Present Value

NPV is one of the most used indicators for project evaluation and to access profitability of investments [55].

Unlike the PBP, the NPV indicator has associated risk due to the estimation of the discount rate. A poor estimation can compromise the evaluation, since the estimated rate will influence the future cash flows.

As we have mentioned on a previous chapter, NPV can be used to analyze a single project or select among a set of possible solutions.

Following our current approach, one can consider NPV a medium risk and a medium potential return on investment indicator.

5.2.3. Internal Rate of Return

Despite its downsides, the IRR indicator is normally seen as the best alternative to NPV. This indicator as it name denotes refers only to internal factors of the project and its cash flows, meaning that the rates practiced in the markets do not affect the value of the IRR [55].

One of the assumptions made about IRR is that the cash flows are reinvested at a rate equal to the indicator value [54].This is less likely to happen for higher IRR and in the context of small companies, which rises a level of uncertainty and risk in using this measure [57].

A list and description of all the IRR application problems can be found in [56]. For the purpose of our work we only focus our attention in the *mutually exclusive projects* problem. Defining two projects, for example A and B, as mutually exclusive means that if we implement project A, we cannot implement project B, and vice-versa. In this situation the application of the IRR to choose the projects may lead to incorrect decisions if the considered projects have different initial investments. In the considered case study, the different available alternatives are mutually exclusive projects with different initial investments. Since the IRR indicator belongs to the original EnPROVE platform features, we decided to consider it as the riskiest of all the indicators, not only because of the mutually exclusive projects problem but also because of all the other characteristics of the measure.

5.3. Criteria Pyramid

After analyzing all the criteria of the decision problem, we combined their characteristics with the Risk Pyramid methodology. The indicators were assigned to different levels of a pyramid structure according to their risk exposure, potential return on investment and level of liquidity. Figure 5.2 shows the result of the combined approach, the Criteria Pyramid.

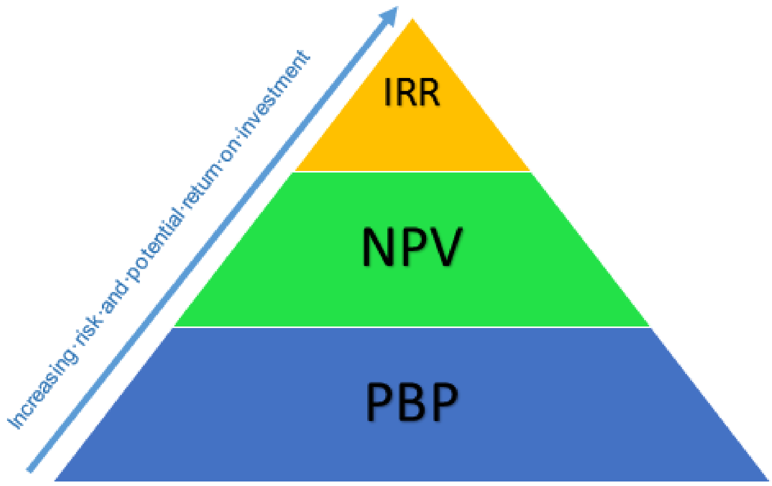


Figure 5.2. Criteria Risk Pyramid

In the beginning of the present chapter we stated that to apply the AHP and PROMETHEE methods to the decision problem we needed input information from DM. After defining the Decision Maker Profiles and the Criteria Risk Pyramid we were able to create that input information based on the DMP and the Criteria Pyramid.

5.4. Decision Groups

Henceforward we will consider three decision groups, one for each DMP. These groups contain the criteria weights that will be used in both decision methods. They also describe the PROMETHEE preference functions and the respective thresholds that once more, depend on the different personalities of the decision makers.

The definition of the preference functions for the PROMETHEE method was based on the guidelines proposed on the VISUAL PROMETHEE software manual [58]. According to our type of criteria the best preference functions are type III (V-shape function) and type V (Linear function), since they are the most suitable for quantitative criteria. The only difference between this two type of functions remains in the introduction of an indifference threshold.

In all the three groups the same type of functions were assigned to the three criteria, the PBP criterion uses a type III preference function and the other two criteria, NPV and IRR, use type V preference functions.

The reason we chose not to introduce an indifference threshold for the preference function of PBP criterion is based on the fact that it seemed reasonable to consider that when dealing with time, in this case expressed in years, the DM will always express some kind of preference when evaluating even the slightest difference between two projects. For the other two criteria the indifference threshold allowed to neglect minor differences between the projects that would not be significant for the DM.

5.4.1. Conservative Decision Group

According to the Conservative DM's personality, the PBP indicator represents the principal criterion and the one that receives the biggest priority in the decision. The two other criteria obtain less weight as their risk factors increase (Figure 5.3).

The Conservative DM was characterized as a very cautious person when dealing with the definition of the preference functions thresholds.

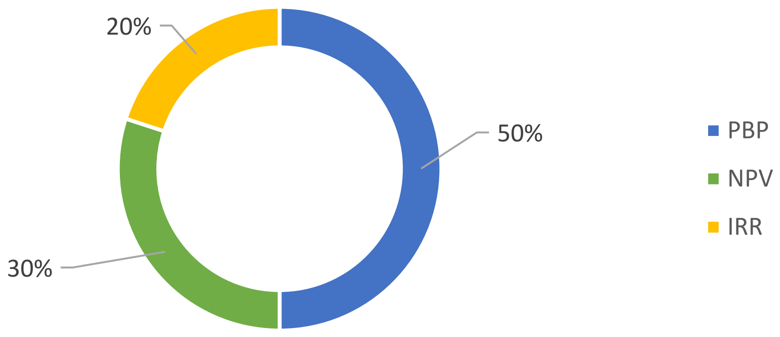


Figure 5.3. Conservative DM Criteria Weights

His main objective is to recoup his investment with the less risk possible, so the preference threshold for the PBP preference functions was set to the minimum value possible as we can see below:

$$HC_{PBP}(d) = \begin{cases} \frac{|d|}{1}, & |d| \leq 1 \\ 1, & |d| > 1 \end{cases}$$

For the NPV and IRR criteria he allows himself for some indifference but his preference thresholds are set to very low values as shown by the following functions:

$$HC_{NPV}(d) = \begin{cases} 0, & |d| \leq 100 \\ \frac{|d| - 100}{500 - 100}, & 100 < |d| \leq 500 \\ 1, & otherwise \end{cases}$$

$$HC_{IRR}(d) = \begin{cases} 0, & |d| \leq 2 \\ \frac{|d| - 2}{10 - 2}, & 2 < |d| \leq 10 \\ 1, & otherwise \end{cases}$$

5.4.2. Moderate Decision Group

We have already stated that a Moderate DM balances the different levels of the pyramid in order to obtain a compromise between risk and return. According to that search for equilibrium all the criteria were weighted the same way, as shown in the Figure 5.4.

Having in mind the characteristics of the moderate DM his criteria preference functions were set to reflect his need to find balance between all the attributes used to evaluate the project.

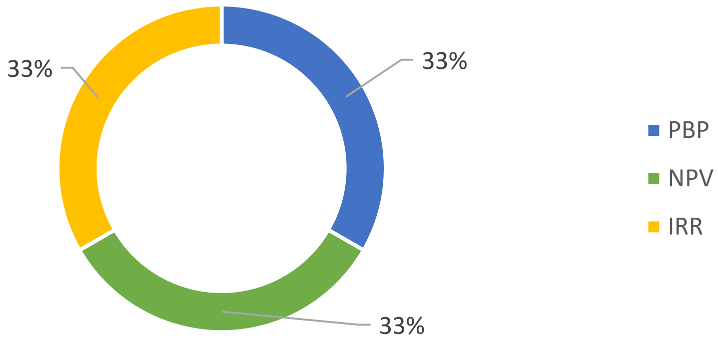


Figure 5.4. Moderate DM Criteria Weights

Therefore the PBP preference threshold was set to match his investment time horizon.

$$HM_{PBP}(d) = \begin{cases} \frac{|d|}{5}, & |d| \leq 5 \\ 1, & |d| > 5 \end{cases}$$

Analyzing the performance of the different alternatives according to the NPV and IRR criteria, the preference and indifference thresholds were set to echo once more the compromise between risk and return sought by this DM.

$$HM_{NPV}(d) = \begin{cases} 0, & |d| \leq 500 \\ \frac{|d| - 500}{2000 - 500}, & 500 < |d| \leq 2000 \\ 1, & otherwise \end{cases}$$

$$HM_{IRR}(d) = \begin{cases} 0, & |d| \leq 10 \\ \frac{|d| - 10}{30 - 10}, & 10 < |d| \leq 30 \\ 1, & otherwise \end{cases}$$

5.4.3. Aggressive Decision Group

This kind of investor largely favors the IRR indicator to ensure the growth and the profitability of his investments despite the knowledge of its drawbacks. As we have already mentioned liquidity is not a concern for an Aggressive investor and that is the reason why the PBP criterion receives the smallest priority of all the attributes considered (Figure 5.5).

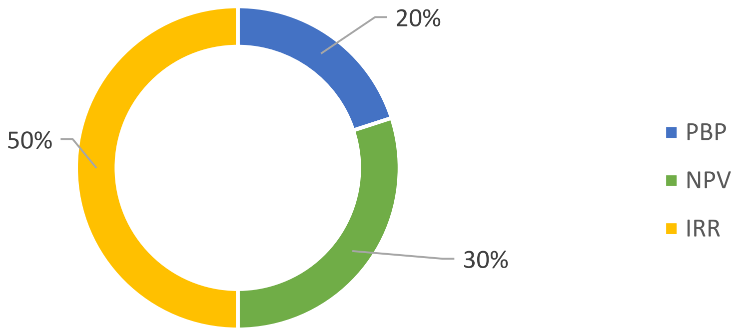


Figure 5.5. Aggressive DM Criteria Weights

The highest values for both indifference and preference thresholds were defined by this DM. His aggressive style of investment, his time horizon and the fact that he is involved in other projects allows him to consider the following preference functions:

$$HA_{PBP}(d) = \begin{cases} \frac{|d|}{10}, & |d| \leq 10 \\ 1, & |d| > 10 \end{cases}$$

$$HA_{NPV}(d) = \begin{cases} 0, & |d| \leq 2000 \\ \frac{|d| - 2000}{5000 - 2000}, & 2000 < |d| \leq 5000 \\ 1, & otherwise \end{cases}$$

$$HA_{IRR}(d) = \begin{cases} 0, & |d| \leq 30 \\ \frac{|d| - 30}{70 - 30}, & 30 < |d| \leq 70 \\ 1, & otherwise \end{cases}$$

The definition of the three decision groups, with all the necessary input information for the application of the decision methods, closes this chapter. Until this stage we gathered all the material and all the data to finally find the solutions to the study case. The next step in our work was the application of the decision methods PROMETHEE and AHP and the respective sensitivity analysis.

6. Method application results and Sensitivity Analysis

The third and fourth steps of the decision process model considered in chapter 2 are the aggregation and the exploitation of the MCAP. Both steps are inner procedures of each method, and vary according to the way the method is structured. These two phases come before giving a final recommendation to the DM. In the present chapter we will show the results of the MCAP aggregation and exploitation phases for each method (AHP and PROMETHEE) and for each DM profile that we have created. Moreover, we perform a Sensitivity Analysis based on the tools provided by the software packages used.

6.1. Method application results

The following results are presented for each DM and consist in the final ranking of the alternatives obtained by using AHP and PROMETHEE. According to the method, and mainly due to the differences in the software applied, different kinds of graphical representations are used.

6.1.1. AHP application results

Under the AHP usage we will start by presenting the resulting ranking of each alternative towards each criterion separately, for all the three DM profiles. This will allow to understand the dominant alternatives for each attribute, and how they will affect the final ranking after considering the corresponding sets of criteria weights given by each DM. Figure 6.1 shows the ranking for PBP with the alternatives B, F and G assuming top positions with the equal scores.

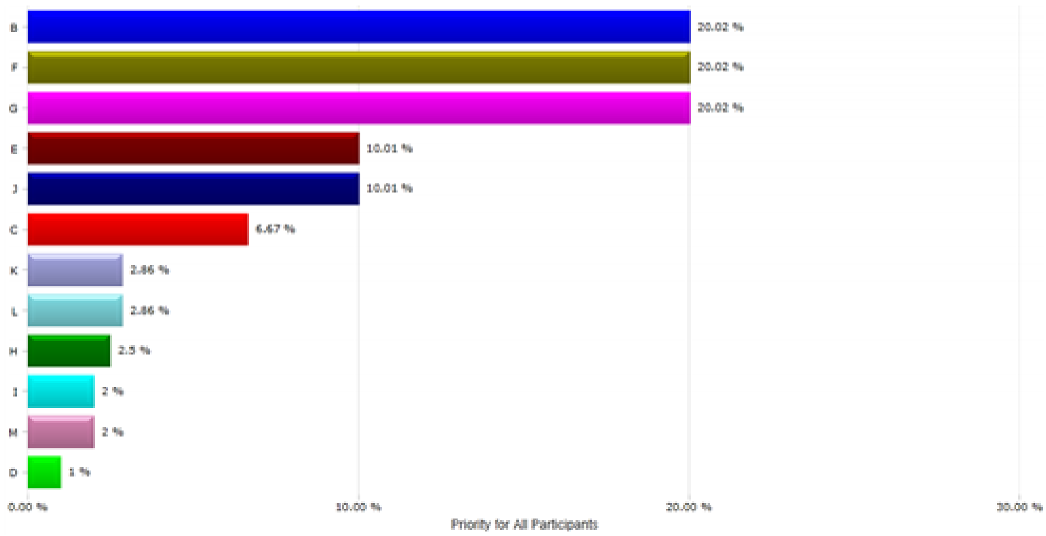


Figure 6.1. Ranking of the alternatives for the PBP criterion

On the other hand, Figure 6.2 presents different dominant alternatives for NPV, with L, H and M assuming the three leading positions with considerable differences between them.

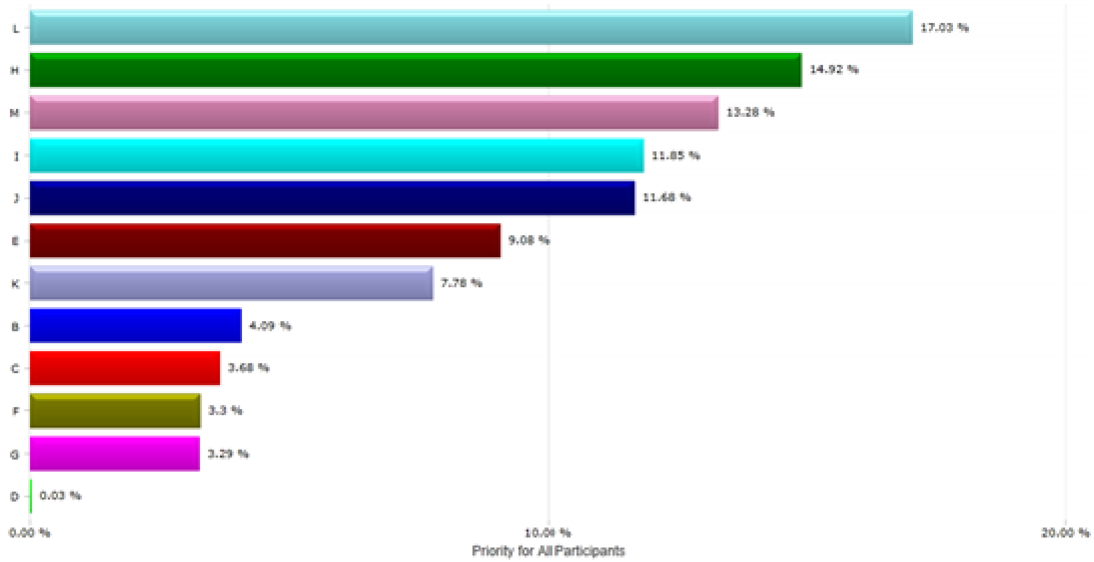


Figure 6.2. Ranking of the alternatives for the NPV criterion

Finally in Figure 6.3 the alternatives F, B and G take the three head positions, also with different score values.

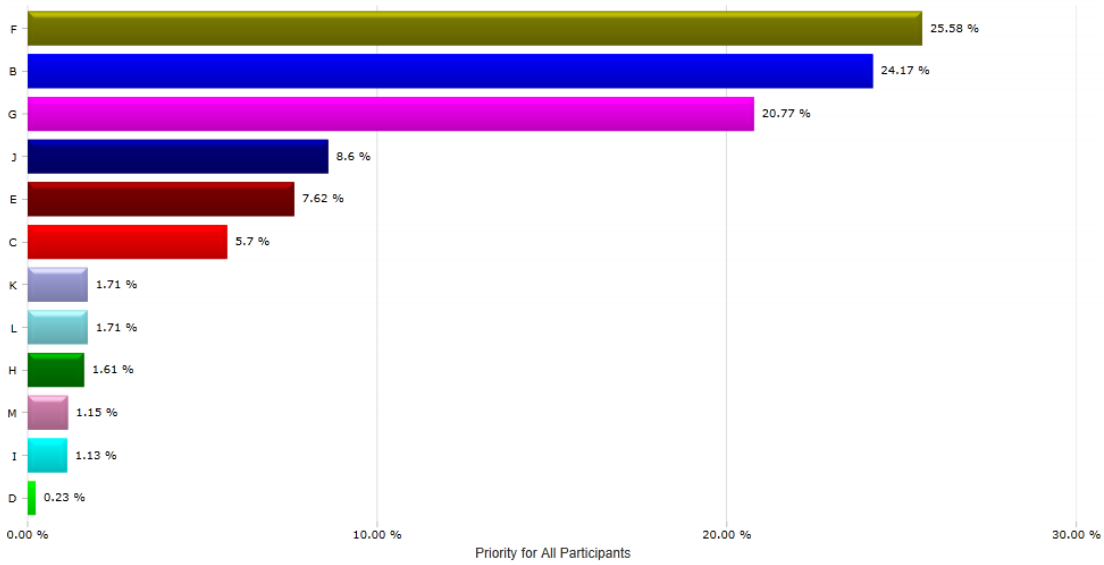


Figure 6.3. Ranking of the alternatives for the IRR criterion

The next step on the AHP application is the addition of the three different sets of weights defined for each DM. Below we display the figures regarding the final results for each DM profile, which by the end of this chapter will be compared against each other and the results from the PROMETHEE II method.

Starting with the Conservative DM, the final ranking of alternatives is displayed in Figure 6.4, which shows the alternative B surpassing all other alternatives.

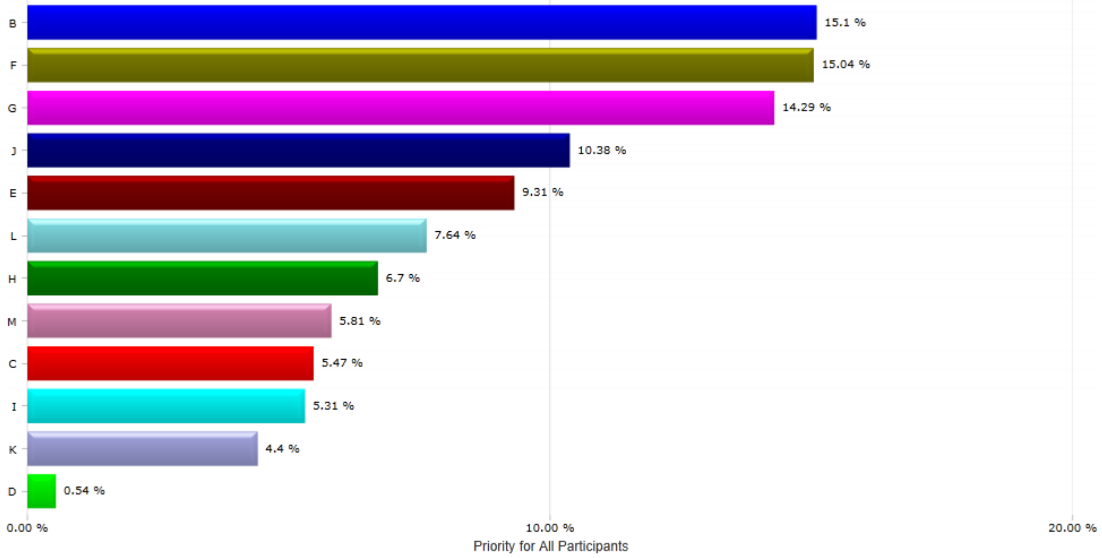


Figure 6.4. Final ranking of the alternatives for the Conservative DM

Considering the Moderate DM, Figure 6.5 displays the final ranking of alternatives, with alternative F taking the first place and B the second, with a small percentage difference.

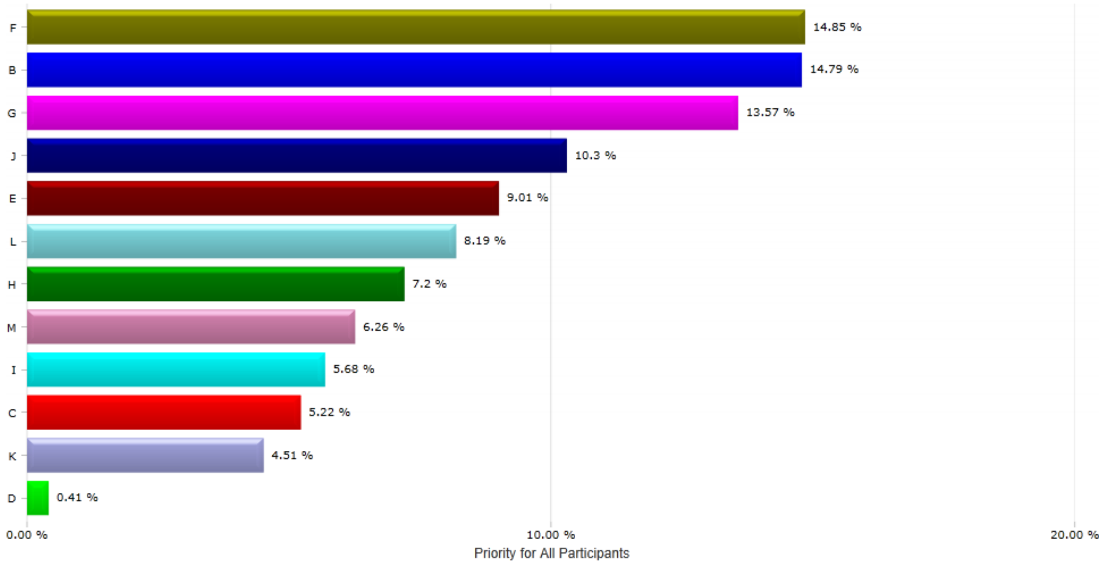


Figure 6.5. Final ranking of the alternatives for the Moderate DM

Finally the results for the Aggressive DM are shown in Figure 6.6. Following the Moderate DM, the top two alternatives are F and B, once again separated by a small distance.

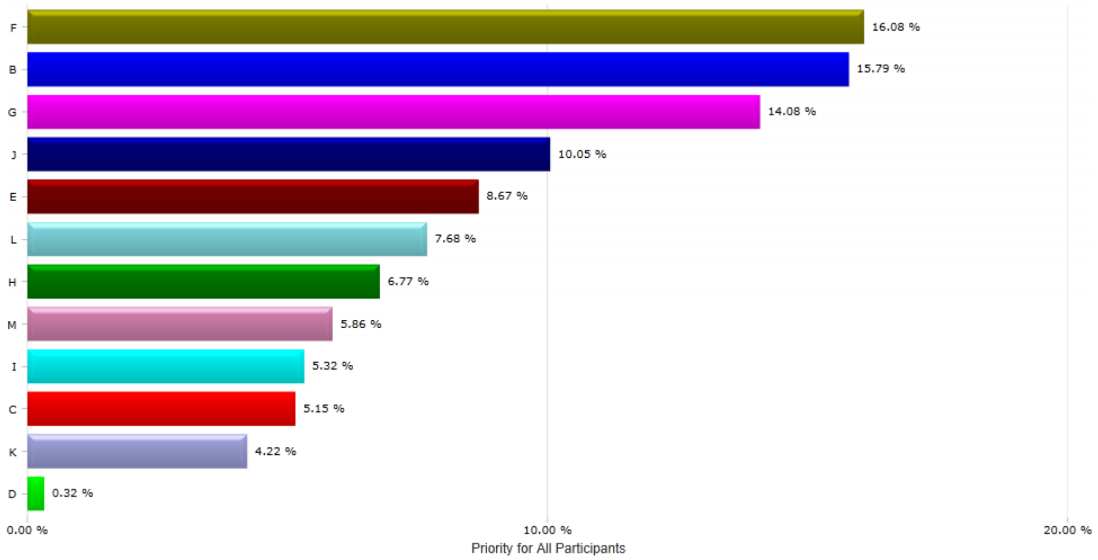


Figure 6.6. Final ranking of the alternatives for the Aggressive DM

6.1.2. PROMETHEE II application results

The application of PROMETHEE II was performed considering two different ways. The reason for this segmentation relies on the possibility to analyze the influence of the preference functions in the final ranking of the alternatives within the PROMETHEE II application.

The first way analysis the different alternatives considering that for each decision criterion no preference functions are defined by the DM. This means that *Usual Criterion (Type I)* functions are used, an approximation to what happens in AHP. In contrast, the second way includes the preference functions defined on chapter 5, according to each DM profile.

Similarly to the AHP results analysis we will present the results for each DM profile obtained through the *Visual PROMETHEE* software. These results comprise a table with the final ranking and the flows for each alternative and a PROMETHEE I flow chart that helps to clarify situations where incomparability can be found on the top alternatives. The PROMETHEE I charts can be found on the Annex B.

6.1.2.1. PROMETHEE II – Without DM preference functions

The Table 6.1 shows the results for the Conservative DM, with alternative B occupying the first position followed by alternatives F and J. Analyzing the Figure B.1 it is possible to confirm that no incomparability relations are found in the top alternatives.

Table 6.1. Final ranking of the alternatives and PROMETHEE flows for the Conservative DM without DM preference functions

Ranking	Alternatives	Phi	Phi+	Phi-
1	B	0,4909	0,7000	0,2091
2	F	0,4182	0,6636	0,2455
3	J	0,3545	0,6545	0,3000
4	G	0,2909	0,6000	0,3091
5	E	0,2636	0,6091	0,3455
6	L	0,1727	0,5545	0,3818
7	H	-0,0727	0,4636	0,5364
8	C	-0,0727	0,4636	0,5364
9	K	-0,1545	0,3909	0,5455
10	M	-0,3182	0,3091	0,6273
11	I	-0,3727	0,2818	0,6545
12	D	-1,0000	0,0000	1,0000

Taking into account the results of the Moderate DM, Table 6.2 indicates that once again alternatives B, F and J take the first three positions, respectively. Similarly to the Conservative DM no incomparability relations are found among the top alternatives in Figure B.2.

Table 6.2. Final ranking of the alternatives and PROMETHEE flows for the Moderate DM – without DM preference functions

Ranking	Alternatives	Phi	Phi+	Phi-
1	B	0,4545	0,6970	0,2424
2	F	0,3939	0,6667	0,2727
3	J	0,3636	0,6667	0,3030
4	E	0,2424	0,6061	0,3636
5	G	0,2121	0,5758	0,3636
6	L	0,2121	0,5758	0,3636
7	H	-0,0303	0,4848	0,5152
8	C	-0,0909	0,4545	0,5455
9	K	-0,1515	0,3939	0,5455
10	M	-0,2727	0,3333	0,6061
11	I	-0,3333	0,3030	0,6364
12	D	-1,0000	0,0000	1,0000

Finally the results for the Aggressive DM are shown in Table 6.3, and for the third time the top three alternatives are B, F and J, without incomparability relations exhibited in Figure B.3.

Table 6.3. Final ranking of the alternatives and PROMETHEE flows for the Aggressive DM – without DM preference functions

Ranking	Alternatives	Phi	Phi+	Phi-
1	B	0,4909	0,7273	0,2364
2	F	0,4727	0,7182	0,2455
3	J	0,3818	0,6818	0,3000
4	E	0,2364	0,6091	0,3727
5	G	0,2364	0,6000	0,3636
6	L	0,1727	0,5545	0,3818
7	H	-0,0727	0,4636	0,5364
8	C	-0,0727	0,4636	0,5364
9	K	-0,1545	0,3909	0,5455
10	M	-0,3182	0,3091	0,6273
11	I	-0,3727	0,2818	0,6545
12	D	-1,0000	0,0000	1,0000

6.1.2.2. PROMETHEE II – With DM preference functions

The results presented below differ from the previous ones due to the introduction of the preference functions defined by the DM.

Table 6.4 shows the results for the Conservative DM, with alternatives B, F and J occupying the top positions. Although, Figure B.4 shows an incomparability within this group, it does not affect the choice of the dominant alternative, as it is settle between alternatives F and J.

The ranking of the alternatives produced for the Moderate DM are shown in Table 6.5. The alternative J is for the first time the dominant alternative followed by alternatives F and B. The analysis of PROMETHEE I flow chart (Figure B.5) confirms that no incomparability relations affect the ranking of the top alternative J.

Table 6.4. Final ranking of the alternatives and PROMETHEE flows for the Conservative DM with DM preference functions

Ranking	Alternatives	Phi	Phi+	Phi-
1	B	0,4709	0,6800	0,2091
2	F	0,4111	0,6364	0,2253
3	J	0,3772	0,6500	0,2728
4	G	0,3365	0,6000	0,2635
5	E	0,2682	0,6091	0,3409
6	L	0,1318	0,5136	0,3818
7	H	-0,0636	0,4364	0,5000
8	C	-0,0913	0,4277	0,5190
9	K	-0,1955	0,3500	0,5455
10	M	-0,2864	0,3045	0,5909
11	I	-0,3681	0,2501	0,6182
12	D	-0,9909	0,0000	0,9909

Table 6.5. Final ranking of the alternatives and PROMETHEE flows for the Moderate DM with DM preference functions

Ranking	Alternatives	Phi	Phi+	Phi-
1	J	0,4175	0,5966	0,1790
2	F	0,3303	0,5424	0,2121
3	B	0,3242	0,5394	0,2152
4	E	0,2910	0,5377	0,2467
5	G	0,2364	0,5091	0,2727
6	L	0,0389	0,3965	0,3576
7	C	-0,0030	0,3818	0,3848
8	H	-0,0545	0,3374	0,3919
9	M	-0,1925	0,2599	0,4524
10	I	-0,2415	0,2365	0,4780
11	K	-0,2862	0,2288	0,5150
12	D	-0,8606	0,0000	0,8606

To finish, the results relative to the Aggressive DM (Table 6.6) point out alternative B as the dominant one, followed by alternative F. For the first time an incomparability relation is established between this two top alternatives, showing that in a PROMETHEE I context both will be consider as top ranking solutions (see Figure B.6 and Figure B.7)

Table 6.6. Final ranking of the alternatives and PROMETHEE flows for the Aggressive DM with DM preference functions

Ranking	Alternatives	Phi	Phi+	Phi-
1	B	0,3630	0,5063	0,1433
2	F	0,3585	0,5156	0,1570
3	G	0,3309	0,5018	0,1709
4	J	0,2836	0,4370	0,1534
5	E	0,1376	0,3254	0,1878
6	L	-0,0101	0,2458	0,2559
7	H	-0,0760	0,1978	0,2739
8	M	-0,1505	0,1725	0,3229
9	I	-0,1740	0,1570	0,3309
10	C	-0,1789	0,1240	0,3030
11	K	-0,2489	0,0766	0,3255
12	D	-0,6352	0,0000	0,6352

6.2. Sensitivity Analysis

This kind of analysis seeks to determine the impact caused by modifications of independent system variables over the outcome of the system. Considering our case study, the Sensitivity Analysis aims to evaluate the influence of the weights of the criteria in the final ranking of the alternatives.

We constructed the Sensitivity Analysis through positive and negative variations of the weights of the three decision criteria regarding the proportion established by each DM. The boundaries defined to perform this analysis are values ranging from -10% to +10% around the value of the weight of the criteria originally defined. The variations are performed for each criterion separately, with the value of the other two criteria following the original proportion implemented.

To ensure the conformity between the deviations in each criterion and the original weighting proportions defined by the DM we also evaluated each variation to understand the validity of the corresponding Sensitivity Analysis situation. All the situation where the proportion was disrespected, as a result of the changes performed by the Sensitivity Analysis, were excluded from the set of results and considered invalid.

6.2.1. AHP Sensitivity Analysis

For the AHP analysis we used the Performance Sensitivity tool, included in the Expert Choice software, which presents the final ranking of the alternatives and the ranking of the alternative for each criterion.

For each DM we present the results for the original ranking of the alternatives and alongside the results for the positive and negative valid variations of each criterion. All the Sensitivity Analysis results can be found in the Annex C.

Figure C.1 shows the original final ranking for the Conservative DM. Figure C.2 and Figure C.3 are relative to the positive and negative variations of the PBP criterion. Additionally Figure C.4 presents the final ranking for +10% change of the NPV and in the same way Figure C.5 displays the results for -10% deviation of the original IRR weight. The situations addressing -10% NPV and -10% IRR where considered invalid since they do not respect the proportion defined by the Conservative DM. The original set of weights was established in such a way that the weight of the NPV criterion should never be under the value of the IRR.

Relatively to the Moderate DM, the original ranking of the alternatives is depicted in Figure C.6, with the variations for the PBP criterion shown in Figure C.7 and Figure C.8. The changes to the NPV criterion can be seen in Figure C.9 and Figure C.10, and the ones performed to the IRR criterion in Figure C.11 and Figure C.12. For this DM all the situations were considered valid for the reason that every time a criterion was changed the other two kept equal percentage values, as expected.

Finally the Sensitivity Analysis results for the Aggressive DM are compared with Figure C.13, the original ranking of the alternatives. The PBP variation is presented on Figure C.14, the NPV on Figure C.15, and the IRR on Figure C.16 and Figure C.17. The situations regarding +10% PBP and -10% NPV were considered invalid as the characteristics of the Aggressive profile prevent that the value of PBP criterion becomes bigger than the value of the NPV.

6.2.2. PROMETHEE II Sensitivity Analysis

The PROMETHEE II Sensitivity Analysis was performed using the *Walking Weights* tool available on the *Visual PROMETHEE* software. The analysis was performed for both PROMETHEE II applications (with and without DM preference functions). The results for both situations can be found in Annex C. with a similar structure to what was presented for the AHP results: Firstly the results regarding the application without DM preference functions ordered by DM - Conservative, Moderate and Aggressive - and then the results regarding the application with DM preference functions. Under the domain of each DM profile the results for the positive and negative variations of each of the three decision criteria – PBP, NPV and IRR.

7. Comparative Analysis of results

The final step in a decision process is the *recommendation*, when a solution is proposed to the DM. This step embodies an important phase in the process, since the acceptance of the alternative by the DM may or may not restart all the process in order to redefined weights, preference functions or even the selected method (see Chapter 2).

In this chapter, we present a compilation of all the results obtained from the *exploitation phase* and from the corresponding Sensitivity Analysis. Those results are grouped by DM and are relative to the three method applications explored: AHP and PROMETHEE II with and without DM preference functions.

Our Comparative Analysis was focused on the similarities between each one of the three applications, but also in the changes to the final solution triggered by the Sensitivity Analysis. In addition, the situations where rank reversal occurs in the PROMETHEE applications will be analyzed to understand the validity of the preference functions defined.

Conservative DM results

To start with the Conservative DM, Table 7.1 contains all the final recommendations for each one of the three applications. The first line of the table presents the solutions obtained considering the original set of weights. All the three recommendations point to alternative B as the best renovation scenario for the building retrofit. Whenever a solution is transversal to all the three application the values on the corresponding line are presented in green, a situation that is repeated in the lines relative to the results of +10% and -10% of the PBP Sensitivity Analysis and the -10% IRR.

Table 7.1. Final Recommendations of all the applications for the Conservative DM

Sensitivity Analysis	AHP	PROMETHEE II Without DM preference functions	PROMETHEE II With DM preference functions
Original	B	B	B
+10% PBP	B	B	B
-10% PBP	B	B	B
+10% NPV	B	B	J
-10% IRR	B	B	B

The only difference observed between the three applications was found in the +10% NPV Sensitivity Analysis results, where the influence of the preference functions brought the alternative J to the top.

After all the conclusions concerning the recommendations for the Conservative DM, it is possible to state that within all the final solutions proposed for the original weighting of criteria and the

Sensitivity Analysis, only the situation regarding the +10% NPV variations presents a different top alternative when we apply the PROMETHEE II with DM preference functions. In all the other rows of the table all the solutions point out to alternative B as the best renovation scenario.

To verify this particularity and understand why this solution is different from the others we can examine the PROMETHEE I flow chart for the +10% NPV situation (Figure 7.1). The chart shows that the top alternatives J and B are incomparable. This allows us to state that alternative B is under the mentioned circumstances the most probable choice for the Conservative DM.

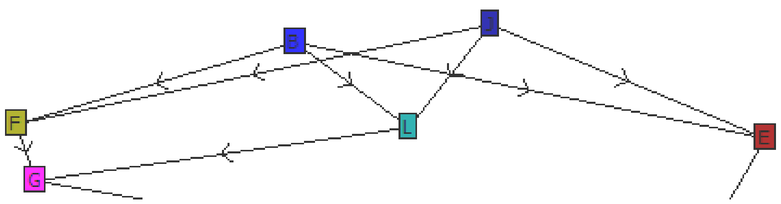


Figure 7.1. Top alternatives - PROMETHEE I flow chart - Conservative DM with DM preference functions (+10%NPV)

This solution is in line with the characteristics of this DM profile since it presents the shortest PBP (1 year), for a low initial investment (75.65€) and low risk exposure. Furthermore, alternative B only grants 12.83% of energy savings, a low value that was expected for a Conservative DM.

Moderate DM results

The Comparative Analysis of the recommendations proposed for the Moderate DM follows the same structure presented before. However, the results presented in Table 7.2 are totally different from the results of the previous one. Instead of having the same solution for the three applications, the pattern that can be identified shows that for four different situations (original, +10% PBP, -10% PBP and -10% NPV) all the three outcomes of the methods diverge (as we can see with the rows with alternatives marked in red). These results are a consequence of both the different structure of the methods used, but also the introduction of the preference functions, all combined with a balanced set of weights – inherent to the profile of the Moderate DM.

Moreover, it is interesting to analyze the results in terms of the alternatives found within each application. The AHP application column in the table presents five out of seven recommendations pointing out alternative F as the best solution and the other two indicating alternative B. This is in line with the Gradient Analysis results and the ranking of alternatives for each criterion, presented in the previous chapter, and shows that in the context of the AHP and the Moderate DM way of thinking, the alternative B surpasses F if the NPV weight increases and the IRR decreases.

Table 7.2. Final Recommendations of all the applications for the Moderate DM

Sensitivity Analysis	AHP	PROMETHEE II Without DM preference functions	PROMETHEE II With DM preference functions
Original	F	B	J
+10% PBP	F	B	J
-10% PBP	F	B	J
+10% NPV	B	J	J
-10% NPV	F	B	J
+10% IRR	F	B	F
-10% IRR	B	B	J

In the second column relative to the PROMETHEE II without DM preference functions only the +10% NPV situation presents alternative J as the top alternative instead of B. This is a consequence of the preference functions and the influence of the NPV in the alternatives, but once again we can go deeper in the analysis and observe the PROMETHEE I flow chart to realize that alternatives J and B have an incomparability relation (Figure 7.2).

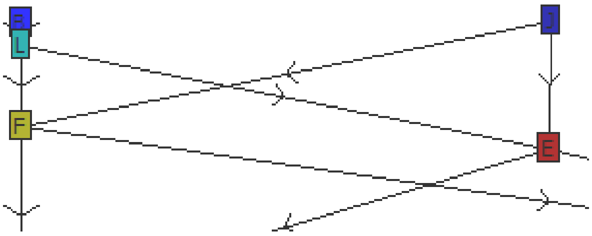


Figure 7.2. Top Alternatives – PROMETHEE I flow chart- Moderate DM without DM preference functions (+10% NPV)

Another pair of incomparable alternatives shown by the PROMETHEE I flow chart can be found in the +10% IRR situation (Figure 7.3), the only one in the column of the PROMETHEE II with DM preference functions application that does not point alternative J as the solution.

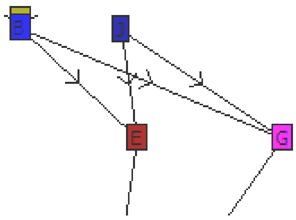


Figure 7.3. Top Alternatives – PROMETHEE I flow chart- Moderate DM with DM preference functions (+10% IRR)

Summing up all the previous conclusions, it is possible to state that each application is closely related to one alternative and that all the three are different from each other, F for AHP, B for PROMETHEE II without DM preference functions, and J for the remaining application.

At a first and inaccurate observation of the set of alternatives, F is the best solution since it is the dominant alternative in two different criteria (PBP and IRR). Nevertheless, alternative B is also the dominant alternative for the PBP criterion, surpasses F under the NPV, and its IRR performance differs in a small percentage from alternative F. Considering alternative J, it does not take any top position regarding the criteria but it is a balanced alternative that may get the DM's attention. Bearing in mind these particularities, it is plausible to affirm that, going from the application on the column in the left to the one in the right, the way the methods capture the DM preferences is increasing its precision. The solution goes from a simple sum of independent parts to a well-modulated choice based on the notion of outranking and considering preference and indifference, two fundamental concepts that influence the gap between alternatives. This can be observed when we translate the alternatives into gains for the DM. According to the profile of the Moderate DM both alternatives F and B are valid, but alternative J is the one that better embodies the balanced characteristics of this DM. The alternative has a PBP of two years and the sixth lowest initial investment (645.25€), but results in 38.13% of energy savings. The results are according to what was predicted since the gains are superior as well as the risk tolerance.

Aggressive DM results

The last set of results to be analyzed is relative to the Aggressive DM (Table 7.3). From all the three tables this one shows the most uneven outcomes comparing the applications against each other.

Table 7.3. Final Recommendations of all the applications for the Aggressive DM

Sensitivity Analysis	AHP	PROMETHEE II Without DM preference functions	PROMETHEE II With DM preference functions
Original	F	B	B
-10% PBP	F	B	B
+10% NPV	F	B	J
+10% IRR	F	F	F
-10% IRR	F	B	J

The AHP is the only column that presents the same alternative for all the five possibilities. On the column relative to the second application, among all the five situations only the +10% IRR variation does not point out alternative B as the top one. This is only a matter of influence of the weights of the criteria, as the alternative F becomes the solution with only a 4% positive variation of the IRR criterion.

Finally the results for the application on the column of the right present the most significant dispersion, as two situations point alternative B as the solution, the other two point alternative J and the remaining one points out alternative F.

To explain these results we recall what was referred in the previous chapter regarding PROMETHEE I. It was observed that the only top original solutions presenting an incomparability relation were the ones obtained for the weighting of the Aggressive DM in the PROMETHEE II application with DM preference functions (Figure 7.4).

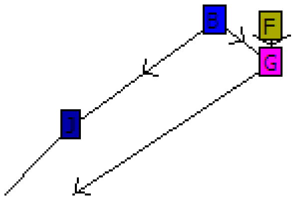


Figure 7.4. Top Alternatives – PROMETHEE I flow chart - Aggressive DM with DM preference functions (original)

Moreover, two other incomparability relations can be identified between alternatives B and F when referring to the -%10 PBP situation (Figure 7.5) and the +10% IRR (Figure 7.6)

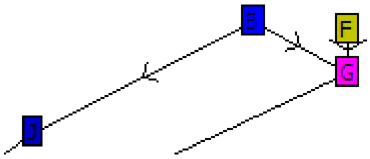


Figure 7.5.Top Alternatives – PROMETHEE I flow chart - Aggressive DM with DM preference functions (-10% PBP)

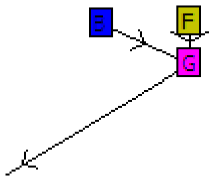


Figure 7.6. Top Alternatives – PROMETHEE I flow chart - Aggressive DM with DM preference functions (+10% IRR)

Lastly, the situations distinguishing the alternative J as the best solution (+10%NPV and -10% IRR) are a consequence of the impact of the NPV criterion, which has previously made the alternative J the most recommended for the PROMETHEE II with DM preference functions application in the Moderate DM's list of results.

An additional point should be mentioned about the results for the Aggressive DM, since it was the only DM to present a rank reversal situation. When analyzing the original situation in the PROMETHEE II application without DM preference functions it was noticed that by removing alternative C from the set of alternatives the alternative on the top, in this case B, was replaced by alternative F. This situation brings additional instability to this set of results that already showed the least predictable outcomes.

Since it was impossible to find a single alternative common to all the application or any kind of pattern to associate a single alternative to each application we observed closely the outcomes to understand in which way they fit the characteristics of the Aggressive DM. All the three possible outcomes, alternatives B, F and J, are valid. However, they do not show how risk tolerant is the Aggressive DM and how much return this kind of DM expects The reason for this detachment stands in the choice of the IRR criterion to evaluate projects with different initial investments. It would be reasonable to accept that an Aggressive DM chooses alternatives like H or L with higher PBP (7/8 years) and higher initial investments but energy savings around 80%.

After performing the Comparative Analysis for each DM and method application individually, it was possible to sum up the most significant conclusions. We have observed an evident influence of the weighing of the criteria in the outcomes of the methods. This was clearly noticed in the AHP application where the alternatives presented were essentially the ones with the dominant performances in the privileged criteria, or a combination of both when the set of weights is balanced.

We also perceived the effect of introducing preference functions and dependent relations between the alternatives. It was easily identified by noting that in the three tables of results, the one relative to the Moderate DM (balanced weights) showed a particular pattern. For each method application in this table a different alternative was recommended for the great majority of the situations within that application. Additionally, and as we mentioned before, the alternative that better suited the characteristics of this profile was the one resulting from the application where the preference of DM was better modulated, in other words in the PROMETHEE II with DM preference functions.

Another conclusion that can be verified with the comparative analysis is relative to the hypothesis of the methods and the preference modelling. The fact that we allow the existence of Incomparability in the PROMETHEE II method to achieve a total ranking of the alternatives showed that in some cases the expected solution is masked by the concept of the Net Flow and another close alternative is displayed, when in reality those alternatives are incomparable.

Finally it was noted that, according to the profiles of the DM that were created, the Aggressive DM is the only one for which the list of recommended solutions does not follow the characteristics of the profile. It was expected that the alternatives selected for this DM would reflect his high risk tolerance so as to perceive the higher returns that this DM looks for. It was also expected that the alternatives presented would characterized with higher PBP values, since the investment time horizon for this DM was the longest of all three.

8. Conclusion and Future Work

The main objective of this dissertation was to perform a Comparative Analysis of Multicriteria Decision Making methods, in order to obtain information for future applications.

Under our main goal we explored the different facets of decision support, going from a general definition of a DMS, to the application of decision methods to a real problem obtaining recommendations to solve it.

During the process we developed some original contributions that supported our work and can hereafter integrate other solutions, systems and processes: **The Application of a Decision Framework to Select a Decision Method** led to an organized and grounded choice of two decision methods to find solutions for the DMS considered. Consequence of the previous contribution, was the main objective of our work itself. The performance of the **Comparative Analysis** of two decision methods gave rise to a set of results establishing relations between those methodologies, the DM involved, and the problem addressed. As a final point, the remaining contributions are attached to the necessity to relate the role of the DM with the choice and weighting of the decision criteria. Therefore we produced a **Definition of Decision Maker Profiles Using Risk Analysis** and a **Classification of the Decision Criteria According to a Risk Pyramid,** which allowed us to settle the required connection between DM and criteria.

The backbone of our work was the structure of the decision process model and its five phases. For each phase we have analyzed and defined the corresponding inner elements, which allowed us to go forward to the next phase.

In the *Structuring Process* phase we have presented the case study, so as to display all the necessary elements to give form to our decision. Thus, we showed the selected criteria and how they were obtained from the variables of the problem, and we also presented the renovation scenarios, which played the role of the alternatives.

The next three phases (*Preference Modelling, Aggregation* and *Exploitation*) were closely related to the MCDM methods selected. Before exploring each one of these phases we started by choosing the decision methods to integrate our comparative analysis. We used a preexisting framework, and from a set of catalogued methods, and following the guidelines proposed, according to our problem, we selected two methods from two different approaches: AHP and PROMETHEE.

It is possible to state that the applied framework is a valid approach to select the methods according to the DMS characteristics, since both methods AHP and PROMETHEE fulfilled the necessities of the decision problem, producing results which are also in line with the DM's preferences. The framework is also valid as an easy and organized approach to the selection process, which is mainly confirmed by the simplicity of the directions taken by following the guidelines proposed.

After describing those methods, we started the second phase of the decision process. This phase seeks to define the elements that for each method define the preference modelling structure (e.g. preference functions, thresholds). Since this phase required the intervention of a DM, we created three different DM profiles. Those profiles were developed based on investment techniques and risk assessment. Moreover, we established a relation between the profiles and the criteria following a Risk Pyramid approach. The result was the definition of three decision groups (one for each DM) comprising the preference functions and weights of the criteria.

Subsequently, we achieved phases three and four, which were considered together. The *Aggregation* and *Exploitation* phases referred to the presentation of the results and the respective Sensitivity Analysis.

Lastly the *Recommendation* phase was the one that embodied our Comparative Analysis. In this phase we have observed the behavior of the methods for each profile of DM. We have detected the influence of the modelling capacities of each method in the outcome of the process. Moreover, we noticed that when we allow a higher degree of preference modelling, the resulting alternatives become more close to the DM's features.

From the Comparative Analysis it was also possible to sustain that both methods present positive aspects that may improve the decision process. AHP presented stable and easy to achieve results with the benchmark approach of the alternatives. The method showed consistent recommendations throughout all the Sensitivity Analysis situations, which suggests that AHP is a solid and simple methodology in the context of this situation. The PROMETHEE II method brought to this DMS another dimension of preference modelling that lacks on AHP. By using preference functions the method allowed a better definition of the notions of preference and indifference letting the solutions of the problem to become even closer to the DM's characteristics. However, this advantage of the method leads to a set of results, less consistent than the one produced by AHP, which presents different solutions for the Sensitivity Analysis situations.

As we have already mentioned, both methods offer advantages to the decision process. In that way, we suggest for future work a combined application of these methodologies in order to explore their benefits and therefore produce synergies.

As a final observation to the work produced, we consider that this study can be improved in order to enhance the way the solutions and recommendations obtained suit the preferences of the DM. On a broader approach to this DMS, we think that an interesting direction to take would be the introduction of other types of decision criteria, especially intangible criteria to evaluate other aspects (e.g. comfort, productivity).

The way decision is envisioned changes from place to place, from person to person. However, there is a common aim to all of them, which is to provide the tools and the help to assure that all decisions taken are founded on strong theories and supported by reliable methods. It is a necessity that the different areas of decision support come together. The fact that the theoretical side of the area was so distant from the practical one was a reason that delayed the wide use of these resources. However the improvement of technology is quickly changing this paradigm.

The results and conclusions of this dissertation intended to fill in the gaps of the decision support field. All the aspects studied and presented can be further explored and improved, using the available technologies, such as the internet, cloud computing and mobile devices.

The findings of our work can be part of future decision support applications and systems and potentiate their benefits and capacities. It is reasonable the perspective of a mobile application based on the combination of methods studied in this dissertation, conceived to support daily personal decisions (e.g. choosing a car, decide which house to buy, select a university to enroll in). It is also a possibility the integration of the framework for selecting a decision method on a Cloud-based DSS addressing multiple decision situations and where the application of different methods can assure more consistent results. We can also conceive a web-based decision support application making use of the Criteria Pyramid concept to help all kinds of decision makers to set their profiles and weight the chosen criteria for a certain DMS.

The possibilities are almost unlimited mostly due to the way technology potentiates the decision support area. In such way, we hope that this dissertation will become part of the evolution of the field, as a source of knowledge and information for future applications.

References

[1] R. Paolo, K. T. Z. Benno, K. Barbara, T. Beate and J. Jeanette, Decision Support Systems in Agriculture: Administration of Meteorological Data, Use of Geographic Information Systems (GIS) and Validation Methods in Crop Protection Warning Service, 2011.

[2] T. Saaty, "Decision making with the analytic hierarchy process," *Int. J. Services Sciences,* vol. 1, no. 1, pp. 83-98, 2008.

[3] A. R. Campos, "Intelligent Decision Support Systems For Collaboration In Industrial Plants," Lisboa, 2010.

[4] S. A. Conrad, J. Geisenhoff, T. Brueck, M. Volna, P. Brink, M. Hall, S. Cook, S. Kenway, W. E. R. Foundation, W. R. Foundation, I. EMA, S. F. University and CSIRO, Decision support system for sustainable energy management. Denver, Colo: Water Research Foundation., Denver, Colo: Water Research Foundation, 2011.

[5] T. Hanne, "On the Classification of MCDM Literature," *Methods of multicriteria decision theory. Proceedings of the 5th Workshop of the DGOR-Working Group. Multicriteria Optimization and Decision Theory,* pp. 113-120, 1995.

[6] Y. Shi, S. Wang, G. Kou and J. Wallenius, "New State of MCDM in the 21st Century," in *Selected Papers of the 20th International Conference on Multiple Criteria Decision Making 2009*, Chengdu, China, 2009.

[7] A. Guitouni and J.-M. Martel, "Tentative guidelines to help choosing an appropriate MCDA method," *European Journal of Operational Research, 109, no. 2,* pp. 501-521, 1998.

[8] F. A. Lootsma, " The French and the American school in multi-criteria decision analysis.," *RAIRO. Recherche opérationnelle,* vol. 24, no. 3, pp. 263-285, 1990.

[9] E. K. Zavadskas and Z. Turskis, "Multiple criteria decision making (MCDM) methods in economics: an overview," in *Technological and Economic Development of Economy*, Taylor & Francis, 2011, pp. 397-427.

[10] G.-H. Tzeng and J.-J. Huang, Multiple attribute decision making : methods and applications, Boca Raton: CRC Press, 2011.

[11] E. Triantaphyllou, B. Shu, S. Nieto Sanchez and T. Ray, "Multi-Criteria Decision Making: An Operations Research Approach," in *Encyclopedia of Electrical and Electronics Engineering*, vol. 15, New York, NY, John Wiley & Sons, 1999, pp. 175-186.

[12] C. Kahraman, Fuzzy multi-criteria decision making : theory and applications with recent developments, New York: Springer, 2008.

[13] J. Bragge, P. Korhonen, H. Wallenius and J. Wallenius, "Bibliometric Analysis of Multiple Criteria Decision Making/Multiattribute Utility Theory," in *Lecture Notes in Economics and Mathematical Systems*, vol. 634, Berlin, New York,, Springer-Verlag, 2010, pp. 259-268.

[14] A. Toloie-Eshlaghy and M. Homayonfar, "MCDM Methodologies and Applications: A Literature Review from 1999 to 2009," *Research Journal of International Studies,* vol. 21, pp. 86-137, 2011.

[15] J. Figueira, S. Greco and M. Ehrgott, Multiple criteria decision analysis : state of the art surveys, New York: Springer, 2005.

[16] K. Eldrandaly, A. H. Ahmed and N. A. Aziz, "An Expert System for Choosing the Suitable MCDM Method for Solving a Spatial Decision Problem," in *9th International Conference on Production Engineering, Design and Control*, Alexandria-Egypt, 2009.

[17] G. Munda, Multicriteria evaluation in a fuzzy environment : theory and applications in ecological economics, Heidelberg, Germany: Physica-Verlag, 1995.

[18] S. Smelev, "Economic Valuation and Decision Making: MCDA as a Tool for the Future," in *Ecological Economics: Sustainability in Practice,*, Springer Science, 2011, pp. 57-74.

[19] E. Kornyshova and S. Camille, "MCDM Techniques Selection Approaches: State of the Art," in *Proceedings of IEEE Symposium on Computational Intelligence in Multicriteria Decision Making*, Honolulu, HI, USA, 2007.

[20] T. L. Saaty, The Analytic Hierarchy Process, Planning, Piority Setting, Resource Allocation, New York, NY: McGraw-Hill, 1980.

[21] D. Baker, D. Bridges, R. Hunter, G. Johnson, J. Krupa, J. Murphy and K. Sorenson, Guidebook to Decision-Making Methods, USA: Department of Energy, 2002.

[22] B. Roy, Methodologie multicritere d'aide a la decision, Paris: Économica, 1985.

[23] C. Bana e Costa, décision, Les problématiques dans le cadre de l'activité d'aide à la, vol. 80, Paris: LAMSADE, Université de Paris Dauphine, 1993.

[24] A. Guitouni, "L'ingénierie du choix d'une procédure d'agrégation multicritère," Québec, 1998.

[25] A. Guitouni, J.-M. Martel and P. Vincke, "A Framework to Choose a Discrete Multicriterion Aggregation Procedure," 1999.

[26] V. M. Ozernoy, "Choosing the best multiple criteria decision-making method," *INFOR,* vol. 30, pp. 159-171, 1992.

[27] C. L. Hwang and K. Yoon, Multiple attribute decision making : methods and applications : a state-of-the-art survey, vol. 186, New York: Springer, 1981.

[28] C. A. Bana e Costa and J. C. Vansnick, "MACBETH: An Interactive Path Towards the Construction of Cardinal Value Functions," in *International Transactions in Operational Research*, vol. 1(4), 1994, pp. 489-500.

[29] S. Opricovic, "Multiple criteria optimization of civil engineering systems," Faculty of Civil Engineering, Belgrade, 1998.

[30] T. L. Saaty, Decision making with dependence and feedback : the analytic network process, Pittsburgh, PA: RWS Publications, 1996.

[31] A. Ishizaka and A. Labib, "Review of the main developments in the analytic hierarchy process," *Expert Systems with Applications,* vol. 38, no. 11, pp. 14336-14345, 2011.

[32] T. L. Saaty, The analytic network process: decision making with dependence and feedback, RWS Publ., 2001.

[33] D. J. D. Wijnmalen, "Analysis of benefits, opportunities, costs, and risks (BOCR) with the AHP–ANP: A critical validation," *Mathematical and Computer Modelling,* vol. 46, no. 7–8, pp. 892-905, 2007.

[34] T. Xuguang, M. Ke, G. Wei and H. Tian, "An Application of ANP with Benefits, Opportunities, Costs and Risks in Supplier Selection: A Case Study in a Diesel Engine Manufacturing Firm," in *Automation and Logistics, 2007 IEEE International Conference on*, Jinan, 2007.

[35] E. H. Forman and S. I. Gass, "The analytic hierarchy process—an exposition," *Operations research,* vol. 49, no. 4, pp. 469-486, 2001.

[36] N. Bhushan and K. Rai, "The Analytic Hierarchy Process," in *Strategic Decision Making: Applying the Analytic Hierarchy Process*, Springer, 2004, pp. 11-21.

[37] T. L. Saaty and L. G. Vargas, "The seven pillars of the analytic hierarchy process," in *Models, methods, concepts & applications of the analytic hierarchy process*, Springer, 2001, pp. 27-46.

[38] T. L. Saaty and M. Sagir, "An essay on rank preservation and reversal," *Mathematical and Computer Modelling,* vol. 49, no. 5, pp. 1230-1243, 2009.

[39] R. Luce and H. Raiffa, Games and Decisions: Introduction and Critical Survey, Dover Publications, 1957.

[40] V. Belton and T. Gear, "On a short-coming of Saaty's method of analytic hierarchies," *Omega,* vol. 11, no. 3, pp. 228-230, 1983.

[41] I. Millet and T. L. Saaty, "On the relativity of relative measures–accommodating both rank preservation and rank reversals in the AHP," *European Journal of Operational Research,* vol. 121, no. 1, pp. 205-212, 2000.

[42] J. Brans, "L'ingénièrie de la décision; Elaboration d'instruments d'aide à la décision. La méthode PROMETHEE," in *R. Nadeau and M. Landry*, Presses de l'Université Laval ed., I. e. P. d. L'aide à la décision:Nature, Ed., Québec, Canada, L'aide à la décision:Nature, Instruments et Perspectives d'Avenir, 1982, p. 183–213.

[43] M. Behzadian, R. B. Kazemzadeh, A. Albadvi and M. Aghdasi, "PROMETHEE: A comprehensive literature review on methodologies and applications," *European Journal of Operational Research,* vol. 200, no. 1, pp. 198-215, 2010.

[44] J. Brans and P. Vincke, "A preference ranking organization method. The PROMETHEE method for MCDM," *Management Science,* vol. 31, no. 6, pp. 647-656, 1985.

[45] J. Brans, B. MAreschal and P. Vincke, "PROMETHEE: A new family of outranking methods in MCDM," in *Operational Research'84*, North Holland, 1984a, pp. 477-490.

[46] J. Brans, B. Mareschal and P. Vincke, "PROMETHEE: A new family of outranking methods in multi-criteria analysis," in *Operational Research '84*, North-Holland, Amsterdam, Elsevier Science Publishers, 1984b, pp. 408-421.

[47] B. Mareschal and J. P. Brans, "The PROMETHEE-GAIA decision support system for multicriteria investigations," ULB-Universite Libre de Bruxelles, 1994.

[48] J. P. Brans, P. Vincke and B. Mareschal, "How to select and how to rank projects: The Promethee method," *European Journal of Operational Research,* vol. 24, no. 2, pp. 228-238, 1986.

[49] Lightbulb Press, Inc., "Key Investment Concepts," 2007.

[50] M. Keršys, "Business Financing Models and Instruments," Vytautas Magnus University, Kaunas, Lithuania, 2010.

[51] C. M. Brown, "Risky climb," *Black Enterprise,* vol. 31, no. 7, p. 52, February 2001.

[52] T. P. Au, A Modern Approach to Graham and Dodd Investing, Hoboken, New Jersey: John Wiley & Sons, Inc., 2004.

[53] S. Ross, R. Westerfield and B. Jordan, Fundamentals of Corporate Finance: Alternate Edition, The McGraw−Hill Companies, Inc., 2003.

[54] R. A. Brealey and S. C. Myers, Principles of Corporate Finance, The McGraw−Hill , 2003.

[55] D. S. Remer and A. P. Nieto, "A compendium and comparison of 25 project evaluation techniques. Part 1: Net present value and rate of return methods," *International Journal of Production Economics,* vol. 42, no. 1, pp. 79-96, 1995.

[56] B. Mareschal, "http://www.promethee-gaia.net/," 23 May 2013. [Online]. Available: http://www.promethee-gaia.net/vpa.html. [Accessed 16 June 2013].

[57] A. Bufardi, R. Gheorghe and P. Xirouchakis, "Fuzzy Outranking Methods: Recent Developments," in *Fuzzy Multi-Criteria Decision Making*, Springer US, 2008, pp. 119-157.

[58] R. Khalid, H. Mohd and H. Ibrahim, "Customer Relationship Management (CRM) Processes from Theory to Practice: The Pre-implementation Plan of CRM System," *International Journal of e-Education, e-Business, e-Management and e-Learning,* vol. 1, no. 1, pp. 22-27, 2011.

[59] R. Buyya, C. Yeoa, S. Venugopala, J. Broberg and B. I., "Cloud computing and emerging IT platforms: Vision, hype, and reality for delivering computing as the 5th utility," *Future Generation Computer Systems,* 2009.

[60] J. P. Shim, M. Warkentin, J. F. Courtney, D. J. Power, R. Sharda and C. Carlsson, "Past, present, and future of decision support technology," *Decision Support Systems,* vol. 33, no. 2, pp. 111-126, 2002.

[61] S. Newman, T. Lync and A. A. Plummer, "Success and failure of decision support systems: Learning as we go," *Journal of Animal Science,* vol. 77, pp. 1-12, 2000.

Annex A. Catalogue of methods – Guitouni and Martel

A.1. Catalogue of methods - part 1 (Source: Tentative guidelines to help choosing an appropriate MCDA method [9])

Method	Guideline									
	G2				G3	G4				
	Pref. eluc. mode	Moment	Pref. struct.	Order	Dec. probl.	Kind of information			Information features	
						Ord.	Card.	Mix.	Deter.	Non deter.
Elementary methods										
Weighted sum	Straightforward (*direct rating*)	a priori	{P, I}	Total preorder	α		✓		✓	
Lexicographic method	Straightforward (*direct rating*)	a priori	{P, I}	Total preorder	α	✓	✓	✓	✓	
Conjunctive method	Straightforward (*direct rating*)	a priori	{P, I}	Filtration	n/a [a]	✓	✓	✓	✓	
Disjunctive method	Straightforward (*direct rating*)	a priori	{P, I}	Filtration	n/a	✓	✓	✓	✓	
Maximin method	Straightforward (*direct rating*)	a priori	{P, I}	Total preorder	α	✓	✓		✓	
Single synthesizing criterion										
Fuzzy weighted sum	Straightforward (*direct rating*)	a priori	{P, Q, I}	Semiorder	α	✓	✓	✓		✓
TOPSIS	Straightforward (*direct rating*)	a priori	{P, I}	Total preorder	α		✓		✓	
MAVT	Tradeoffs	a priori	{P, I}	Total preorder	α		✓		✓	
UTA	Tradeoffs	a priori	{P, I}	Total preorder	α	✓			✓	
SMART	Tradeoffs & rating	a priori	{P, I}	Total preorder	α		✓		✓	
MAUT	Tradeoffs & lotteries	a priori	{P, I}	Total preorder	α		✓			✓
AHP	Pairwise comparison	a priori	{P, I}	Total preorder	α, γ		✓		✓	✓
EVAMIX	Straightforward (*direct rating*)	a priori	{P, I}	Total preorder	α, γ	✓	✓	✓	✓	
Fuzzy maximin	Straightforward (*direct rating*)	a priori	{P, Q, I}	Semiorder	α	✓	✓		✓	✓

[a] n/a: Not applicable.

A.2. Catalogue of methods - part 2 (Source: Tentative guidelines to help choosing an appropriate MCDA method [9])

Method	Guideline									
	G2				G3	G4				
	Pref. eluc. mode	Moment	Pref. struct.	Order	Dec. probl.	Kind of information			Information features	
						Ord.	Card.	Mix.	Deter.	Non deter.
Outranking methods										
ELECTRE I	Pairwise comparison	a priori	{**S**, **R**}	Core	α	✓	✓	✓	✓	
ELECTRE II	Pairwise comparison	a priori	{$\mathbf{S}^F$, $\mathbf{S}_f$, **R**}	Partial semiorder	γ	✓	✓	✓	✓	
ELECTRE III	Pairwise comparison	a priori	Valued {**S**, **R**}	Partial semiorder	γ	✓	✓	✓	✓	
ELECTRE IV	Pairwise comparison	a priori	{$\mathbf{S}^1$, $\mathbf{S}^2$, $\mathbf{S}^3$, $\mathbf{S}^4$, $\mathbf{S}^5$, **R**}	Partial preorder	γ	✓	✓	✓	✓	
ELECTRE IS	Pairwise comparison	a priori	{**S**, **R**}	Partial semiorder	α	✓	✓	✓	✓	
ELECTRE TRI	Pairwise comparison	a priori	{**S**, **R**}	Partial interval order	β	✓	✓	✓	✓	
PROMETHEE I	Pairwise comparison	a priori	Valued {**P**, **I**, **R**}	Partial semiorder	γ	✓	✓	✓	✓	
PROMETHEE II	Pairwise comparison	a priori	Valued {**P**, **I**}	Total preorder	γ	✓	✓	✓	✓	
MELCHIOR	Pairwise comparison	a priori	Valued {**S**, **R**}	Partial semiorder	γ	✓			✓	
ORESTE	Pairwise comparison	a priori	Valued {**P**, **I**, **R**}	Partial semiorder	γ	✓			✓	
REGIME	Pairwise comparison	a priori	{**S**, **R**}	Partial semiorder	γ	✓			✓	
NAIADE	Pairwise comparison	a priori	{**S**, **R**}	Total or partial semiorder	γ	✓	✓	✓	✓	✓
Mixed methods										
QUALIFLEX	Pairwise comparison	a priori	{S, R}	Total semiorder	γ	✓			✓	
Fuzzy conjunctive/disjunctive method	Straightforward	a priori	{P, Q, I}	n/a [a]	α, β			✓		✓
Martel and Zaras method	Pairwise comparison	a priori	{S, R}	Partial semiorder	γ	✓	✓	✓		✓

[a] n/a: Not applicable.

A.3. Catalogue of methods - part 3 (Source: Tentative guidelines to help choosing an appropriate MCDA method [9])

Method	Guideline					
	G5			G6		G7
	Discrimination power of the criteria	Compensation	Information Inter-criteria	Hypothesis [a]	MCAP treatment	Software package
Elementary methods						
Weighted sum	Absolute	Totally	Total and explicit importance coeff.	ind., com., inv., tran., dom.	Algebraic sum	
Lexicographic method	Absolute	Non	n/a [b]	ind., inv., tran., dom.	Cutting planes	
Conjunctive method	Absolute	Non	n/a	ind., inv., tran., dom.	Thresholds	
Disjunctive method	Absolute	Non	n/a	ind., inv., tran., dom.	Thresholds	
Maximin method	Absolute	Non	n/a	ind., inv., tran., dom.	Max and min operators	
Single synthesizing criterion						
Fuzzy weighted sum	non-absolute	Totally	Total and explicit	ind., com., inv., tran., dom.	α-cut and fuzzy arithm.	
TOPSIS	Absolute	Totally	Total and explicit	ind., com., inv., tran., dom.	Euclidien distances	
MAVT	Absolute	Partially	Total and explicit	ind., inv., tran., dom.	Value aggregation (sum or mult)	✓
UTA	Absolute	Partially	Indirect	ind., inv., tran., dom.	Value aggregation (sum)	✓
SMART	Absolute	Partially	Total and explicit	ind., com., inv., tran., dom.	Value aggregation (sum)	✓
MAUT	Absolute	Partially	Total and explicit	ind., inv., tran., dom.	Utility aggregation (sum or mult)	✓
AHP	Absolute	Partially	Total and explicit	inner and outer ind., inv., dom.	Eigenvector method	✓
EVAMIX	Absolute	Partially	Total and explicit	ind., com., inv., tran., dom.	Algebraic sum	
Fuzzy maximin	Non-absolute	Non	n/a	ind., com., inv., dom.	Max and min operators	

[a] ind.: Independence, com.: commensurability, inv.: invariance, tran.: transitivity, dom.: dominance.
[b] n/a: Not applicable.

A.4. Catalogue of methods - part 4 (Source: Tentative guidelines to help choosing an appropriate MCDA method [9])

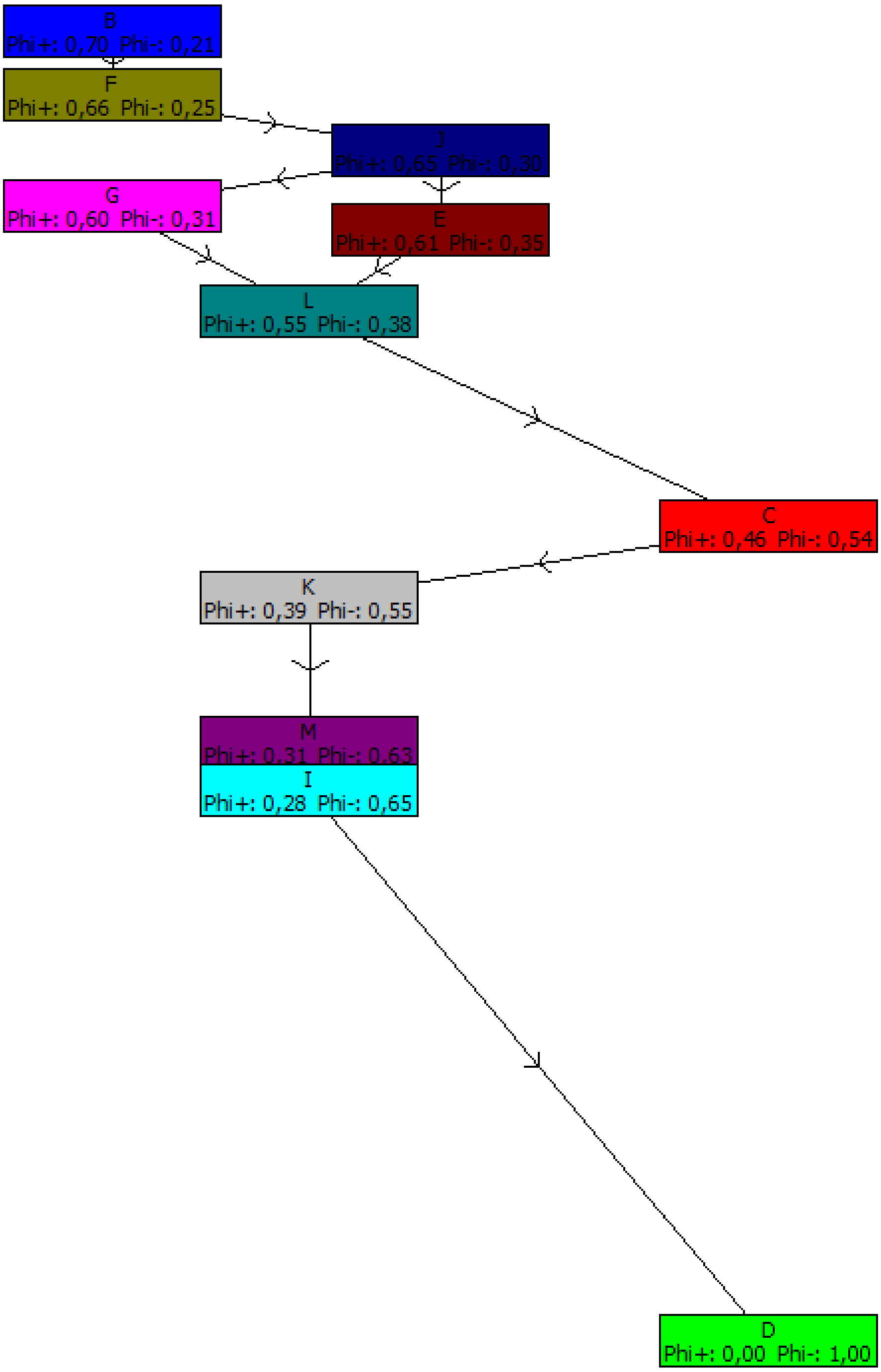
Method	Guideline					
	G5			G6		G7
	Discrimination power of the criteria	Compensation	Information Inter-criteria	Hypothesis [a]	MCAP treatment	Software package
Outranking methods						
ELECTRE I	Absolute	Partially	Total and explicit	ind., inv., coal.	Graph theory (core)	✓
ELECTRE II	Absolute	Partially	Total and explicit	ind., inv., coal.	Graph theory (distillation)	
ELECTRE III	Non absolute	Partially	Total and explicit	ind., inv., coal.	Graph theory (distillation)	✓
ELECTRE IV	Non absolute	Partially	n/a [b]	ind., inv., coal.	Graph theory (distillation)	✓
ELECTRE IS	Non absolute	Partially	Total and explicit	ind., inv., coal.	Graph theory (core)	✓
ELECTRE TRI	Non absolute	Partially	Total and explicit	ind., inv., coal.	Disjunctive and conjunctive	✓
PROMETHEE I	Non absolute	Partially	Total and explicit	ind., inv., coal.	Leaving and entering flows	✓
PROMETHEE II	Non absolute	Partially	Total and explicit	ind., inv., coal.	Leaving and entering flows	✓
MELCHIOR	Non absolute	Partially	Total order	ind., inv.	Graph theory (distillation)	
ORESTE	Absolute	Partially	Total preorder	ind., inv., coal.	Graph theory	✓
REGIME	Absolute	Partially	Total order	ind., inv.	Graph theory	✓
NAIADE	Non absolute	Partially	n/a	ind., inv.	Fuzzy arithm and leaving and entering flows	✓
Mixed methods						
QUALIFLEX	Absolute	Partially	Total or partial and explicit	ind., inv.	Concordance analysis	✓
Fuzzy conjunctive/ disjunctive method	Absolute	No	n/a	ind., inv., tran., dom.	Possibility and necessity measures	
Martel and Zaras method	Non absolute	Partially	Total and explicit	ind., inv., coal.	Graph theory	

[a] ind.: Independence, com.: commensurability, inv.: invariance, tran.: transitivity, dom.: dominance, coal.: coalition (social choice theory).
[b] n/a: Not applicable.

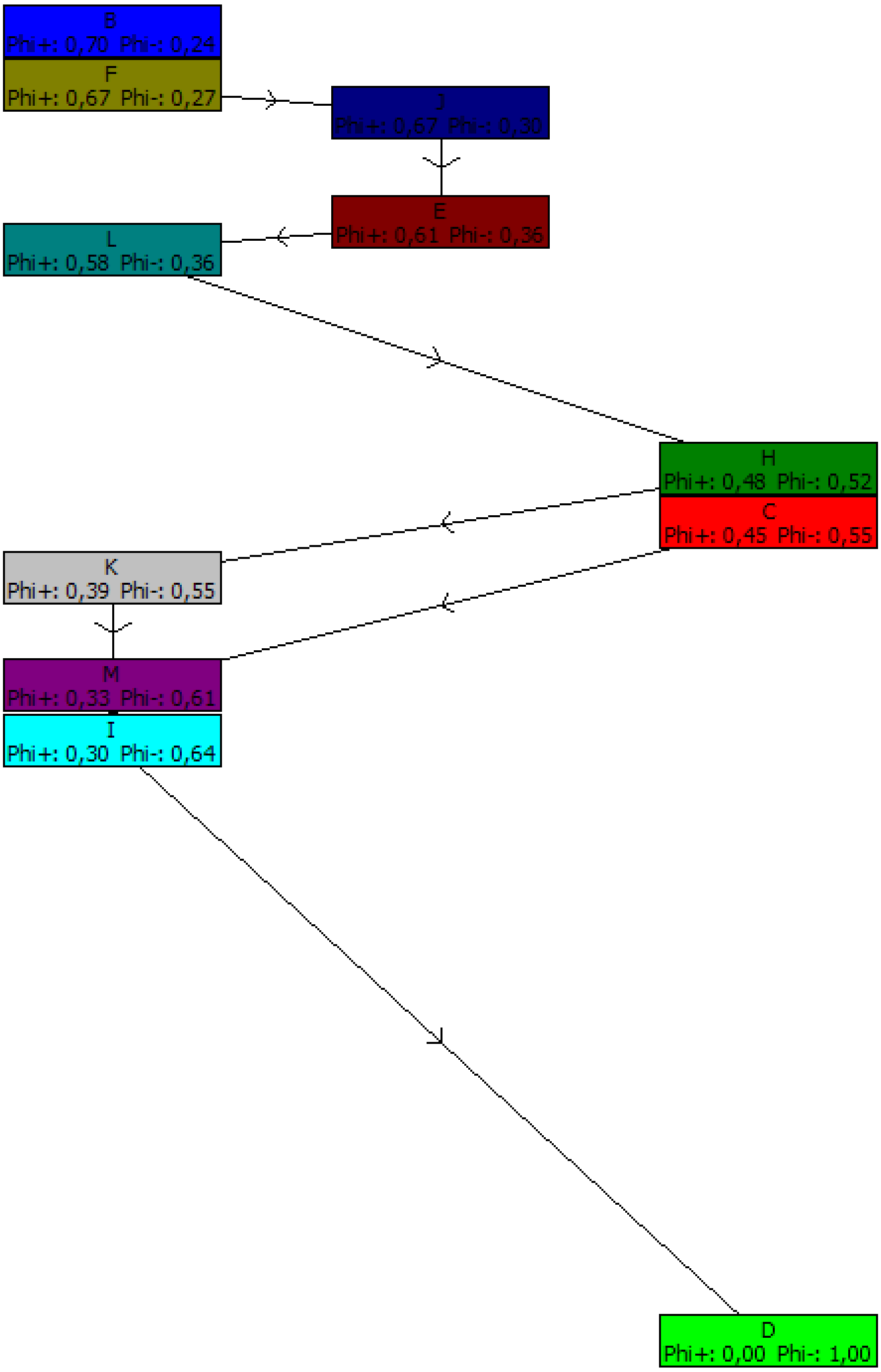
Annex B. PROMETHEE I – Flow charts

The following figures present PROMETHEE I flow charts that allow to understand the existing relations between the alternatives before the application of the Net Flow concept. These flow charts allow to understand the occurrence of incomparability relations within the set of alternatives.

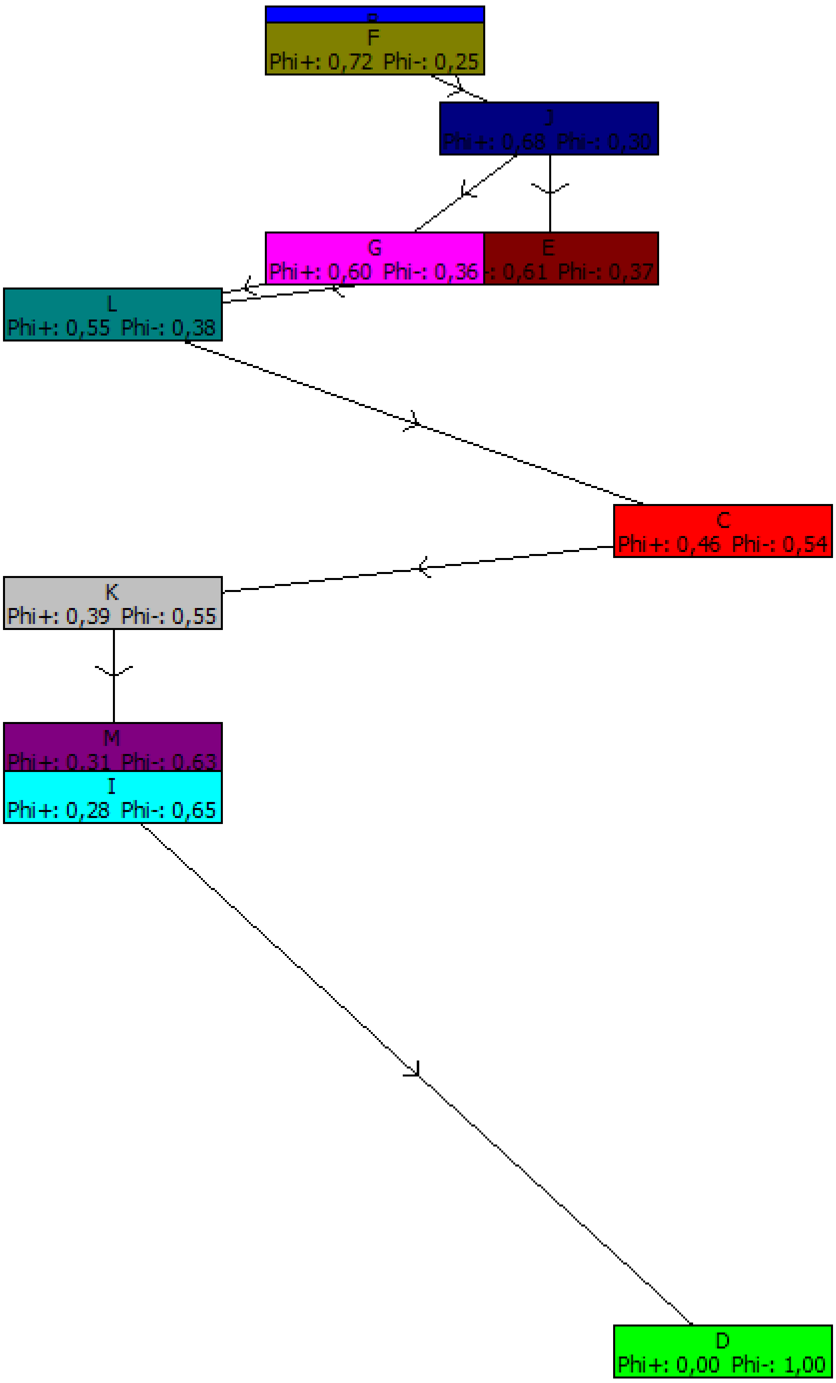
B.1. Conservative DM - without DM preference functions



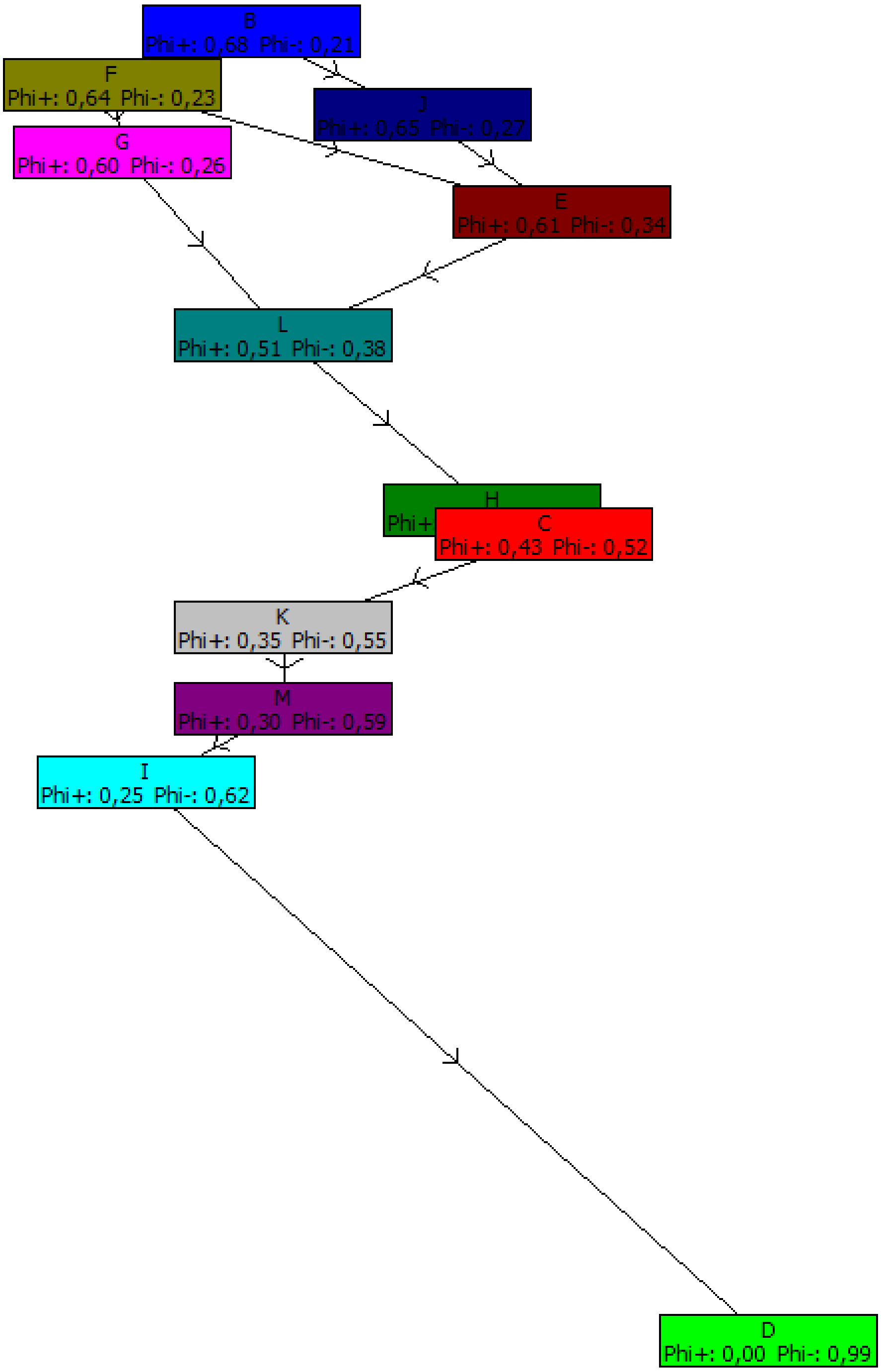
B.2. Moderate DM - without DM preference functions



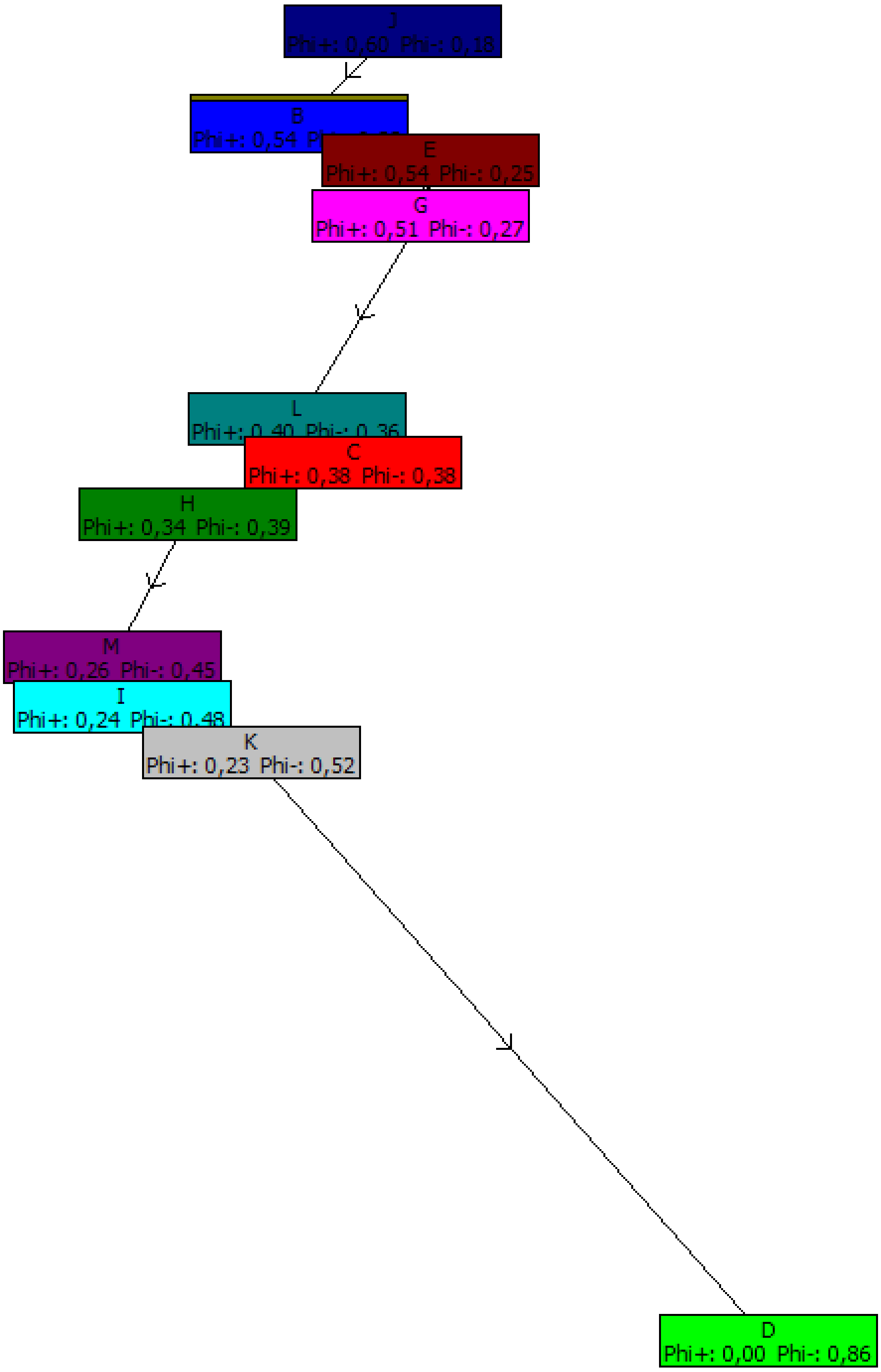
B.3. Aggressive DM - without DM preference functions



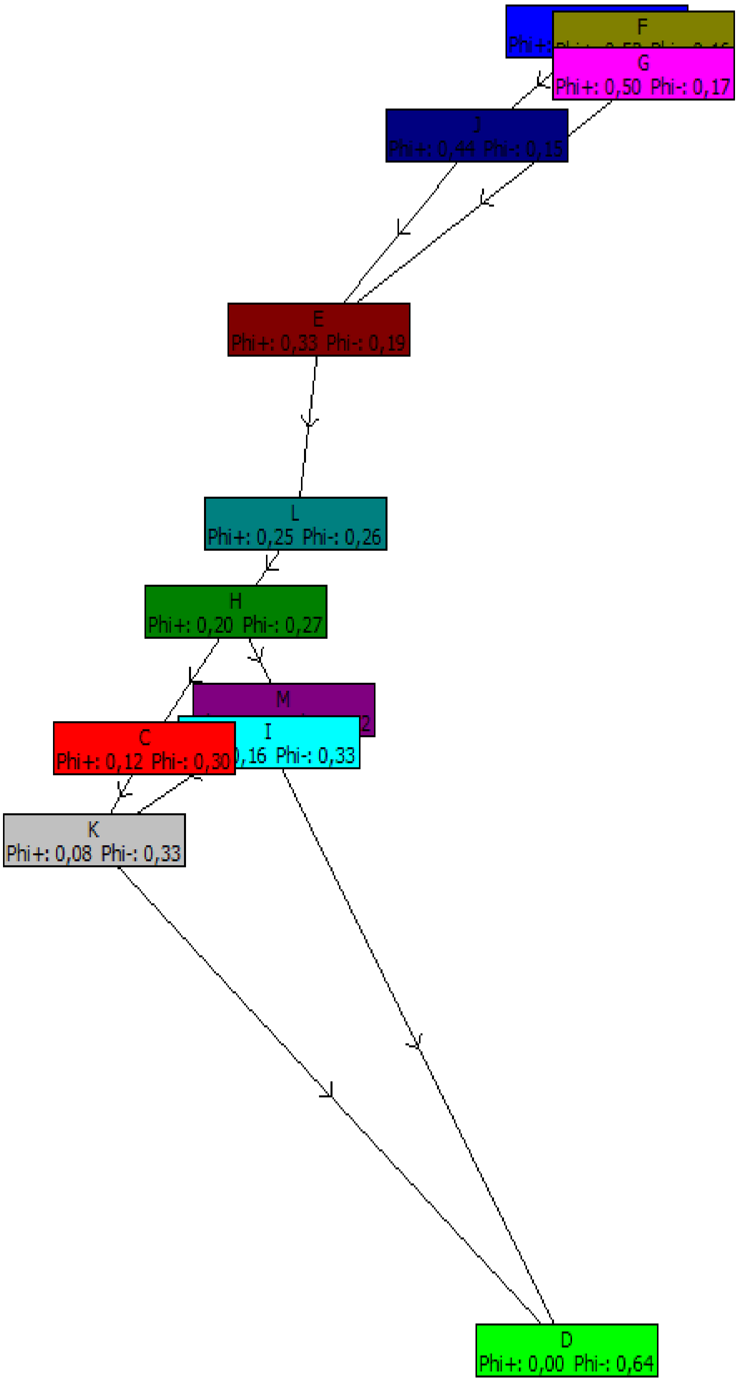
B.4. Conservative DM - with DM preference functions



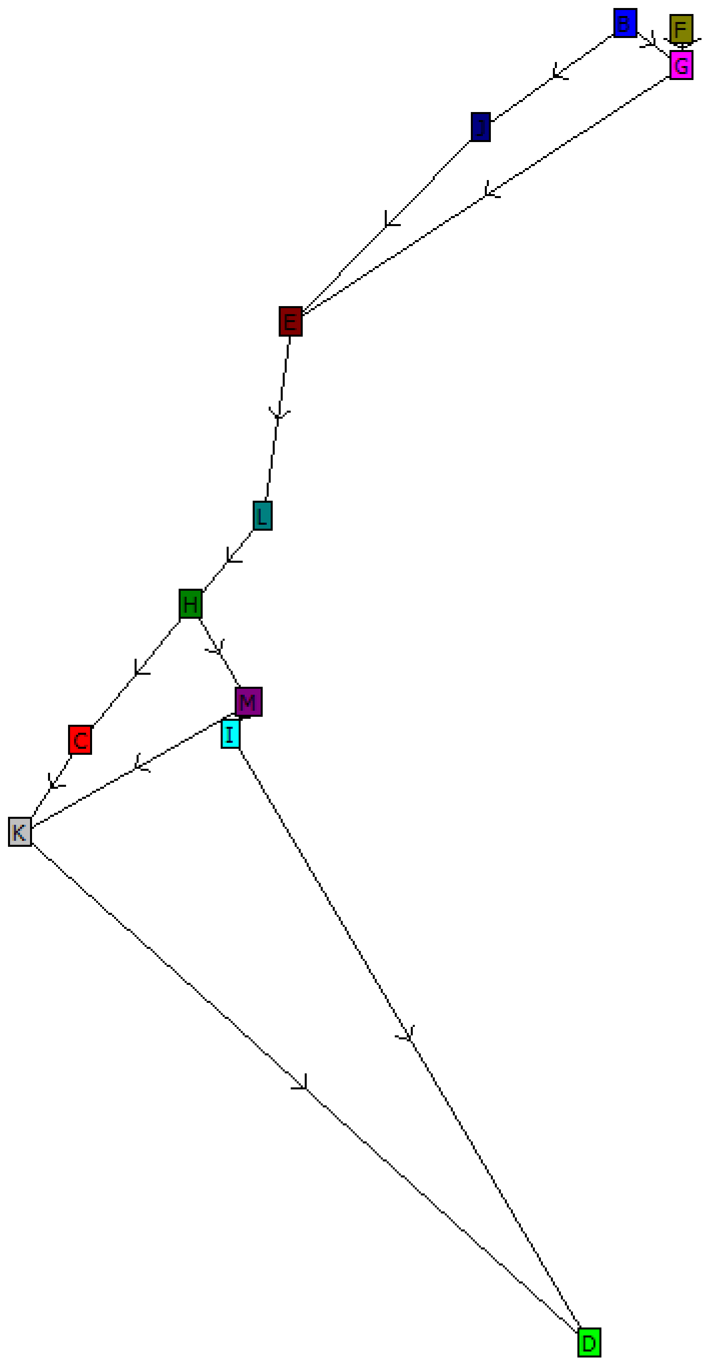
B.5. Moderate DM - with DM preference functions



B.6. Aggressive DM - with DM preference functions (view A)



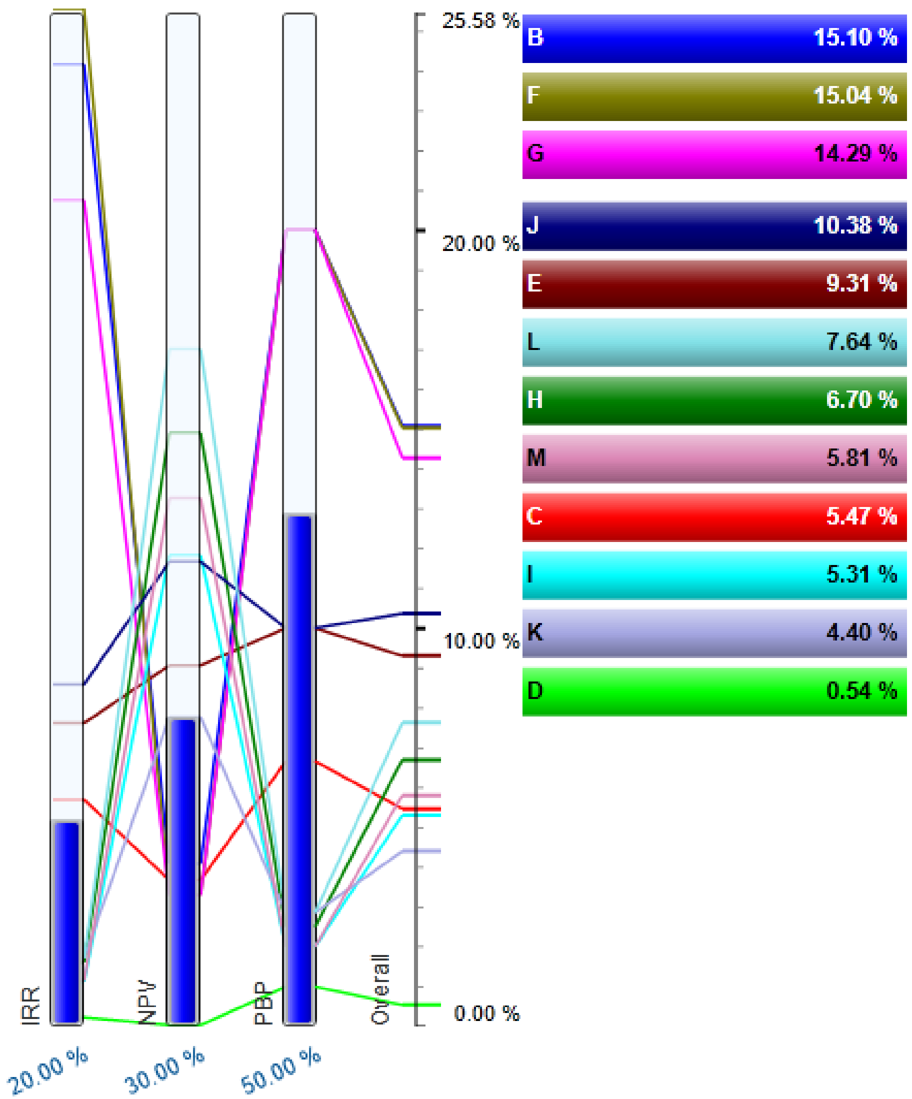
B.7. Aggressive DM - with DM preference functions (view B)



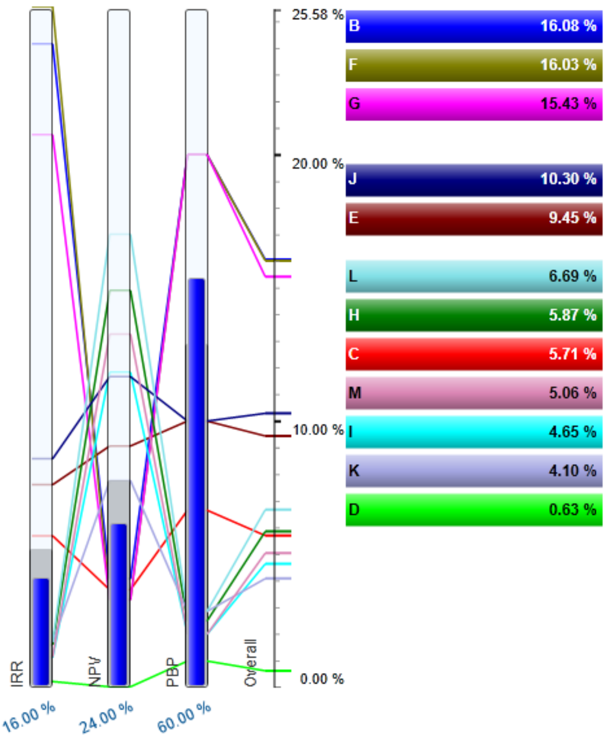
Annex C. Sensitivity Analysis AHP and PROMETHEE II

In this Annex we present the figures displaying the Sensitivity Analysis for both AHP and PROMETHEE II applications. The results are grouped by DM and then by criterion.

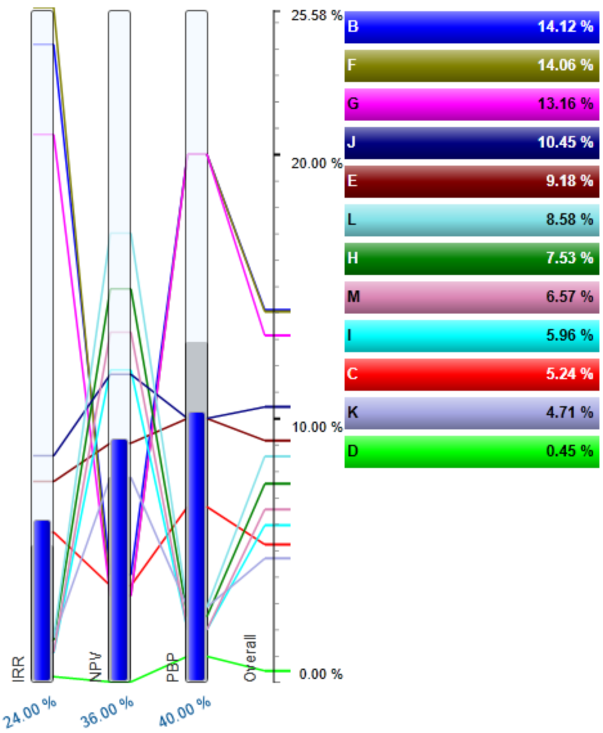
C.1. AHP final ranking of the alternatives for the Conservative DM (original)



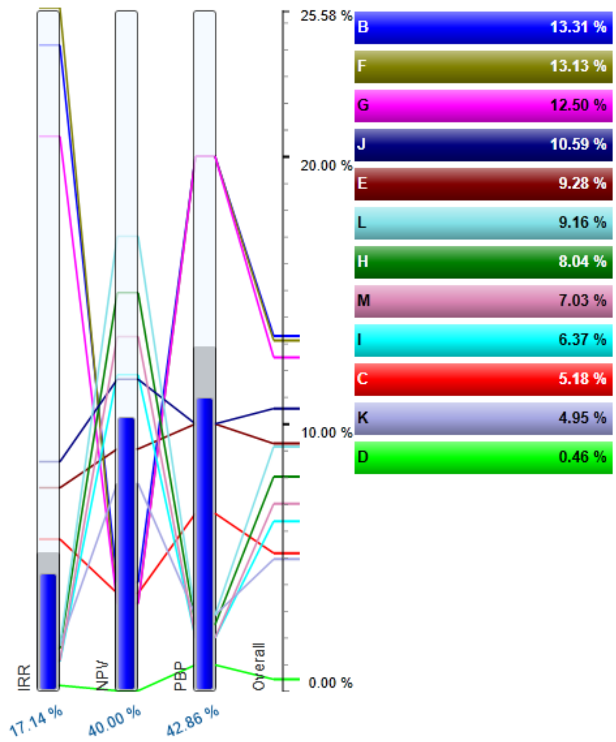
C.2. AHP final ranking of the alternatives for the Conservative DM (+10% PBP)



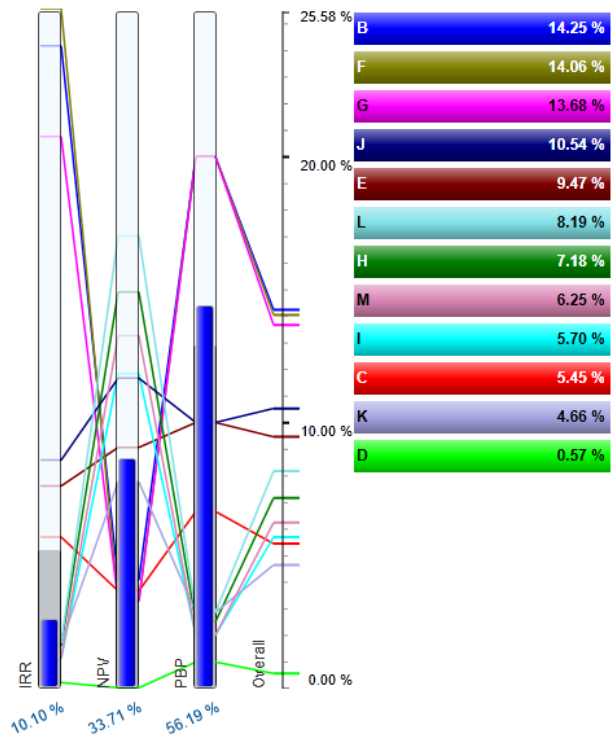
C.3. AHP final ranking of the alternatives for the Conservative DM (-10% PBP)



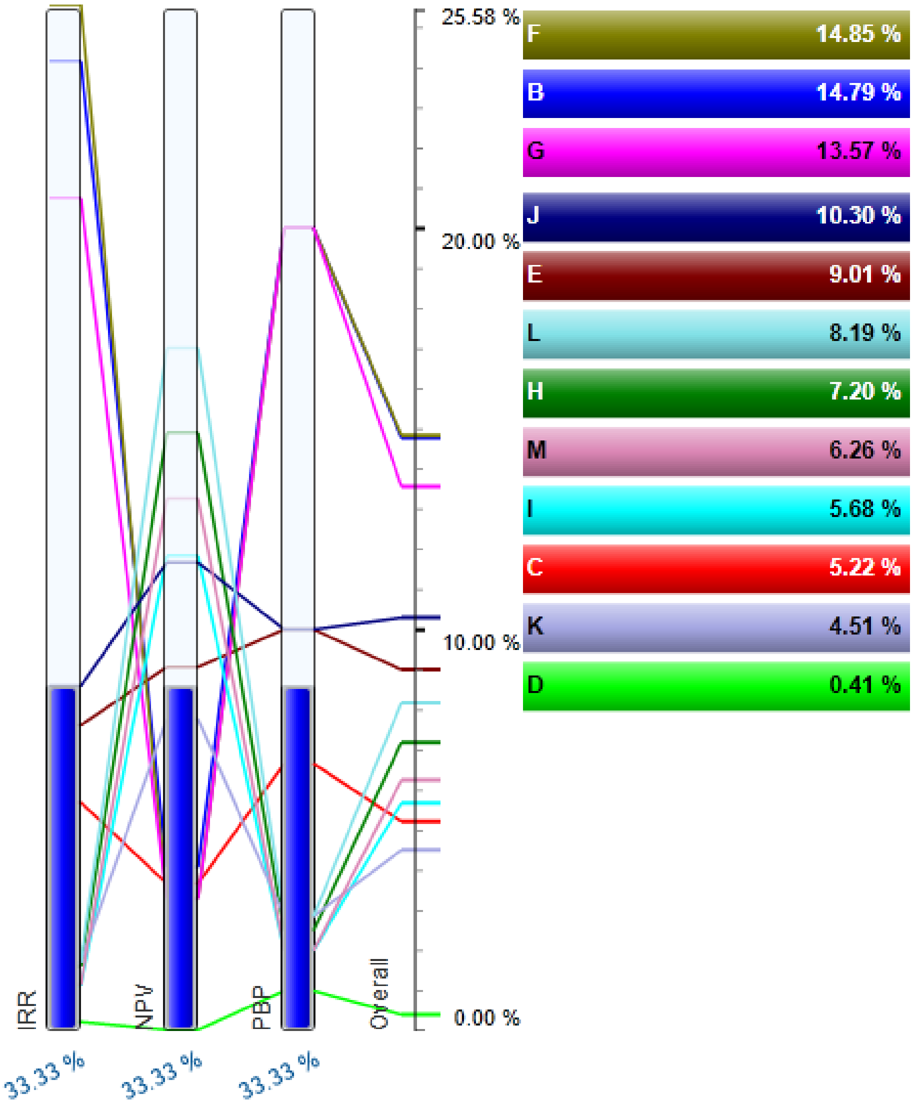
C.4. AHP final ranking of the alternatives for the Conservative DM (+10% NPV)



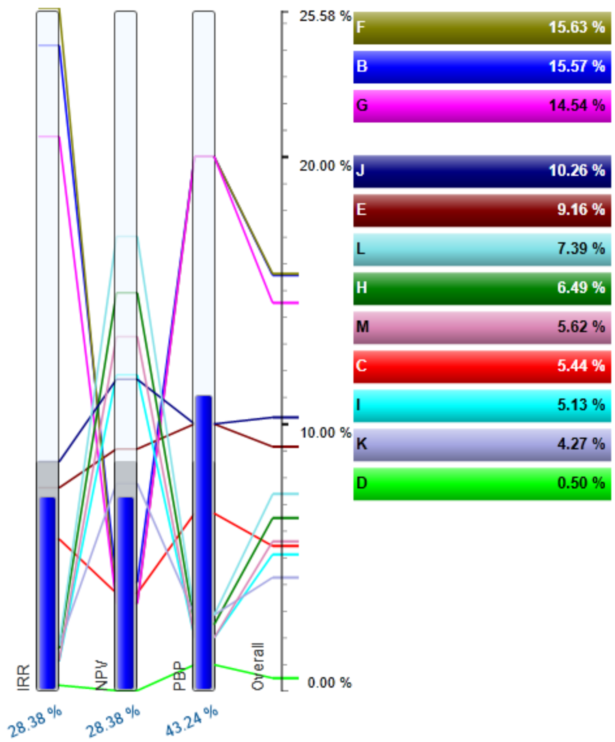
C.5. AHP final ranking of the alternatives for the Conservative DM (-10%IRR)



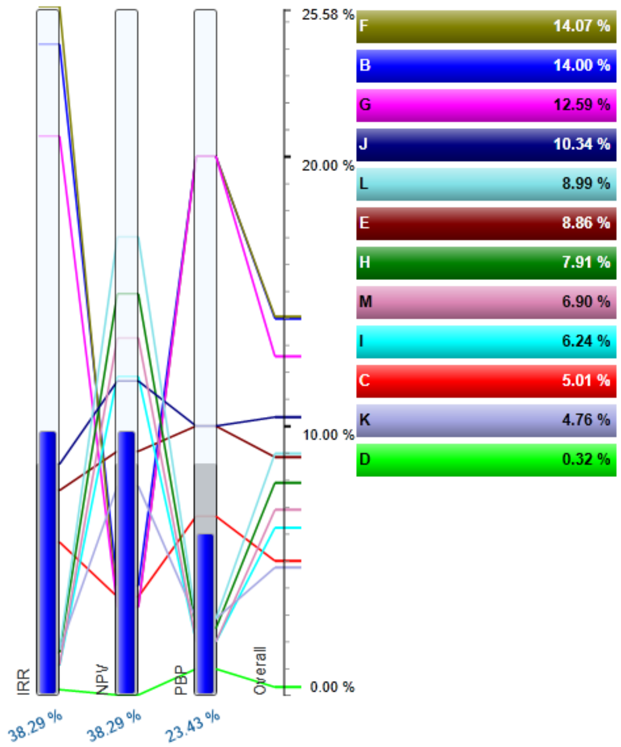
C.6. AHP final ranking of the alternatives for the Moderate DM (original)



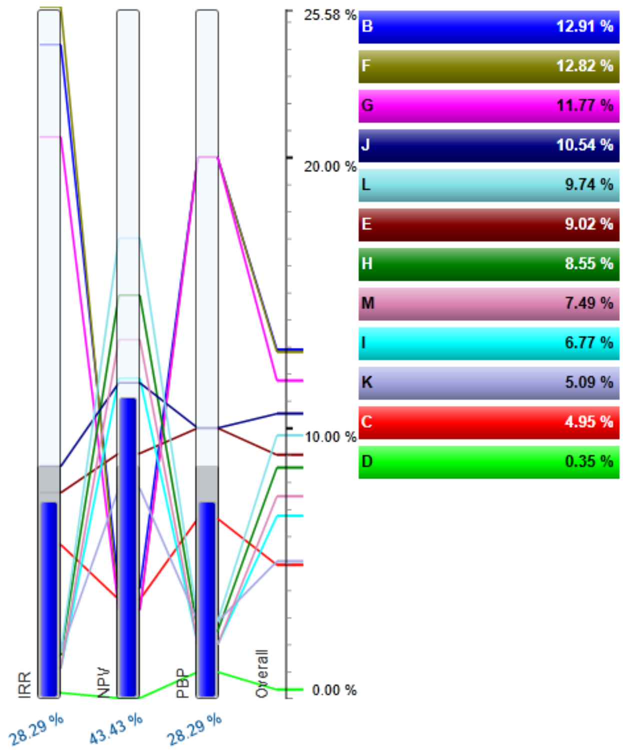
C.7. AHP final ranking of the alternatives for the Moderate DM (+10% PBP)



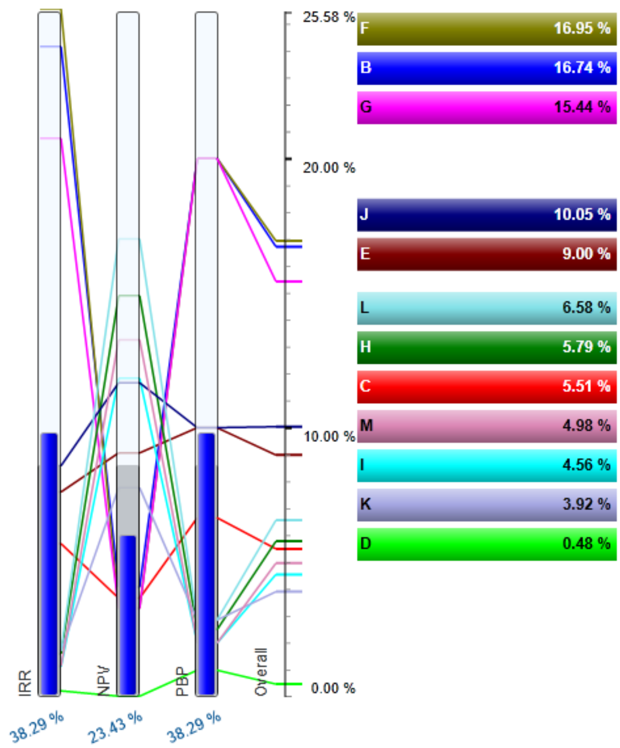
C.8. AHP final ranking of the alternatives for the Moderate DM (-10% PBP)



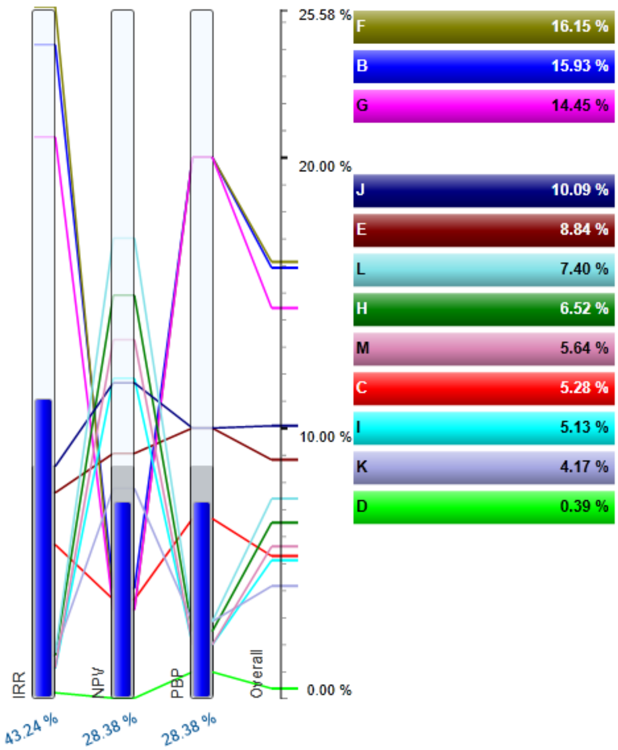
C.9. AHP final ranking of the alternatives for the Moderate DM (+10% NPV)



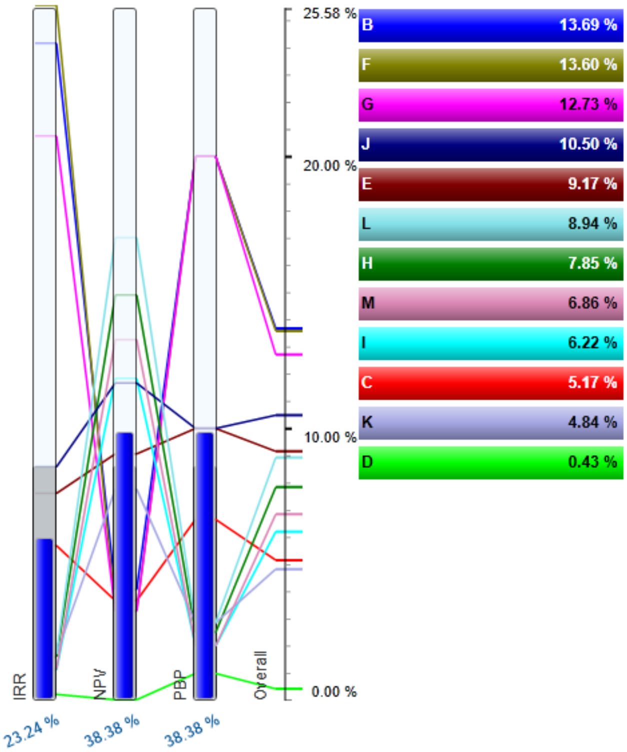
C.10. AHP final ranking of the alternatives for the Moderate DM (-10% NPV)



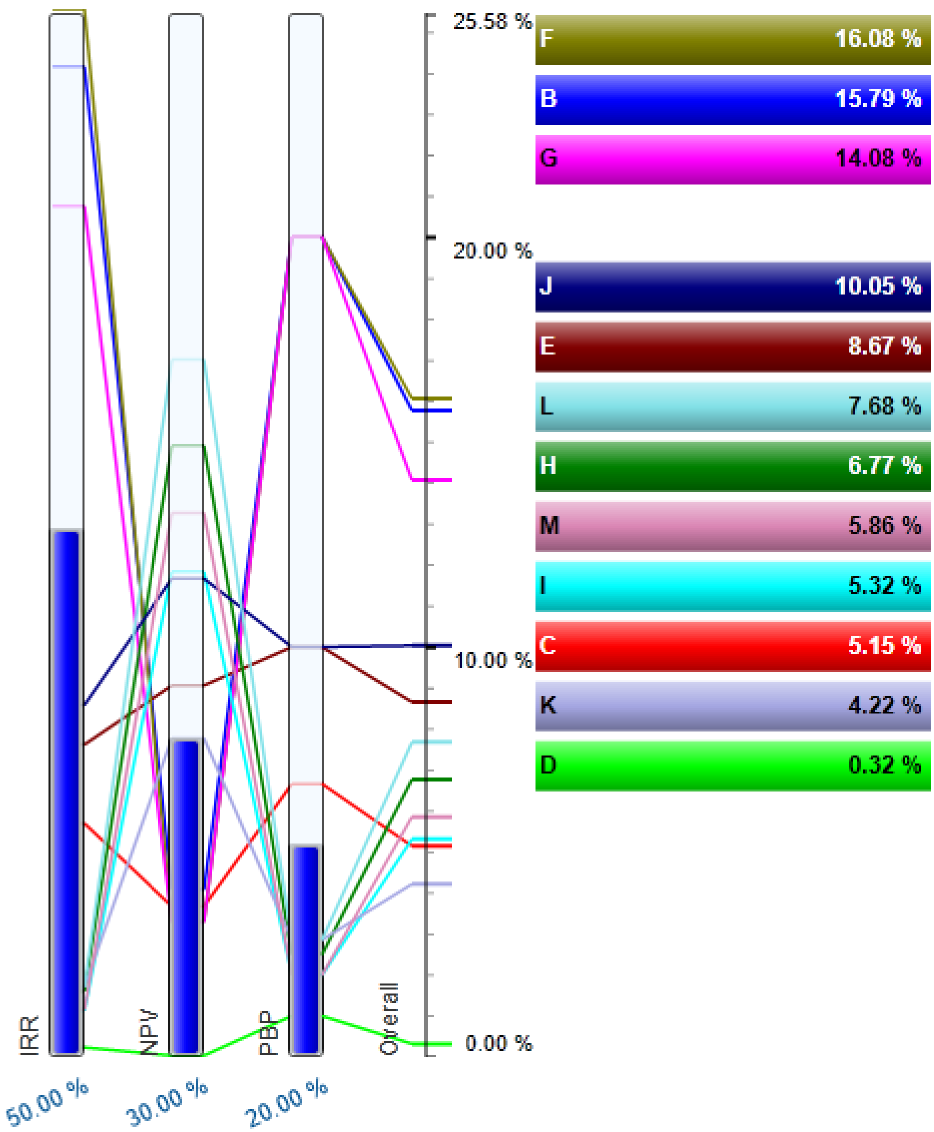
C.11. AHP final ranking of the alternatives for the Moderate DM (+10% IRR)



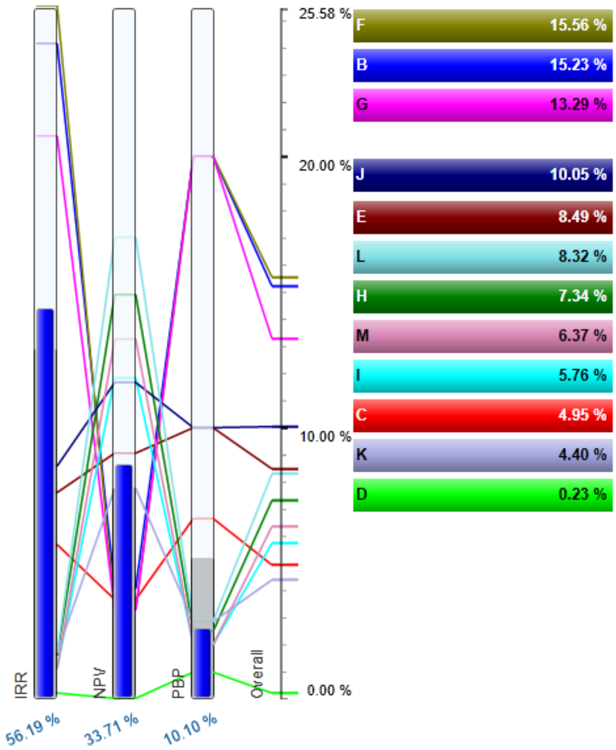
C.12. AHP final ranking of the alternatives for the Moderate DM (-10% IRR)



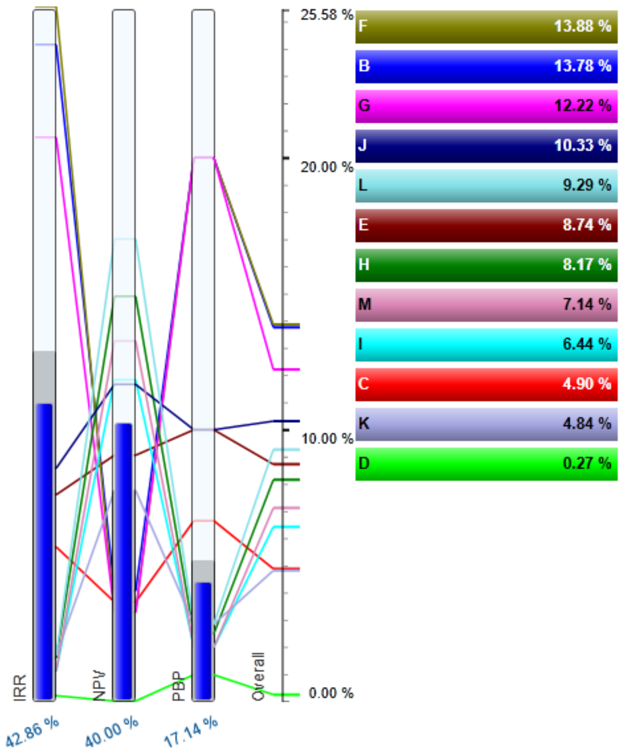
C.13. AHP final ranking of the alternatives for the Aggressive DM (original)



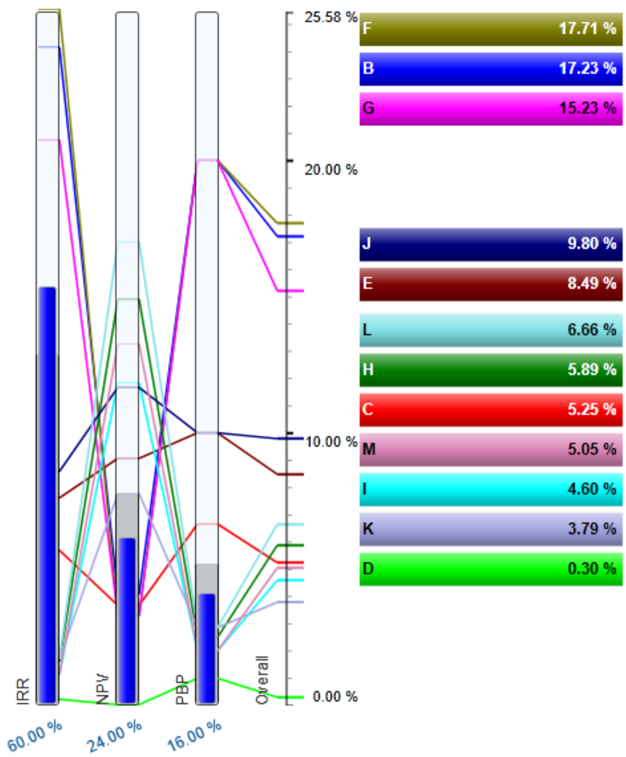
C.14. AHP final ranking of the alternatives for the Aggressive DM (-10% PBP)



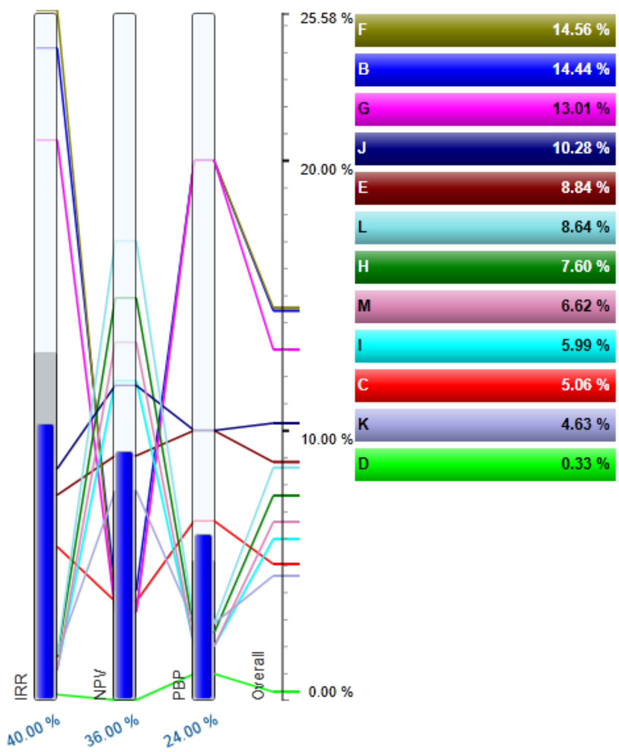
C.15. AHP final ranking of the alternatives for the Aggressive DM (+10% NPV)



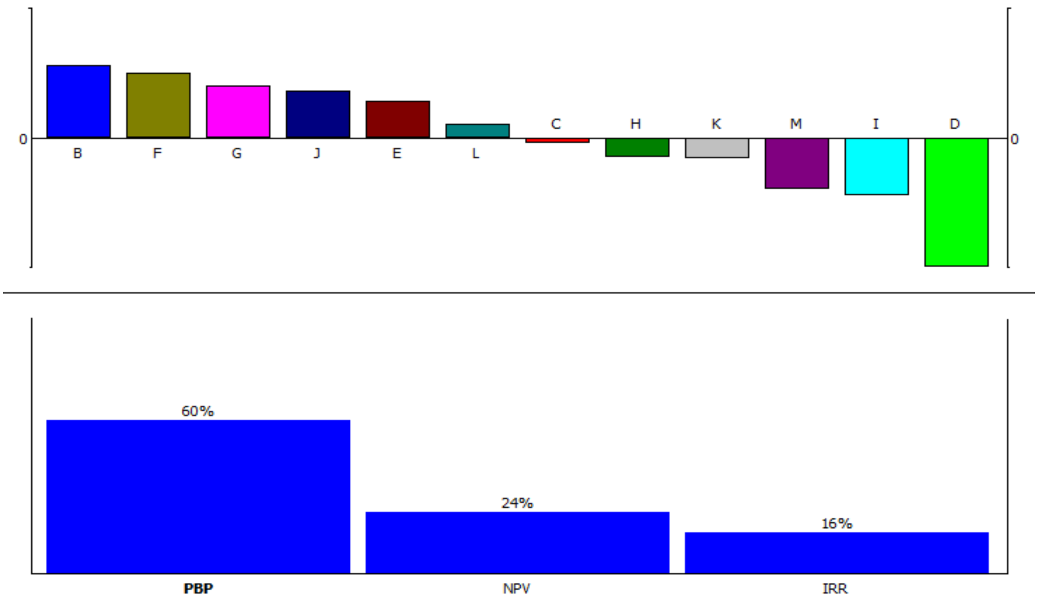
C.16. AHP final ranking of the alternatives for the Aggressive DM (+10% IRR)



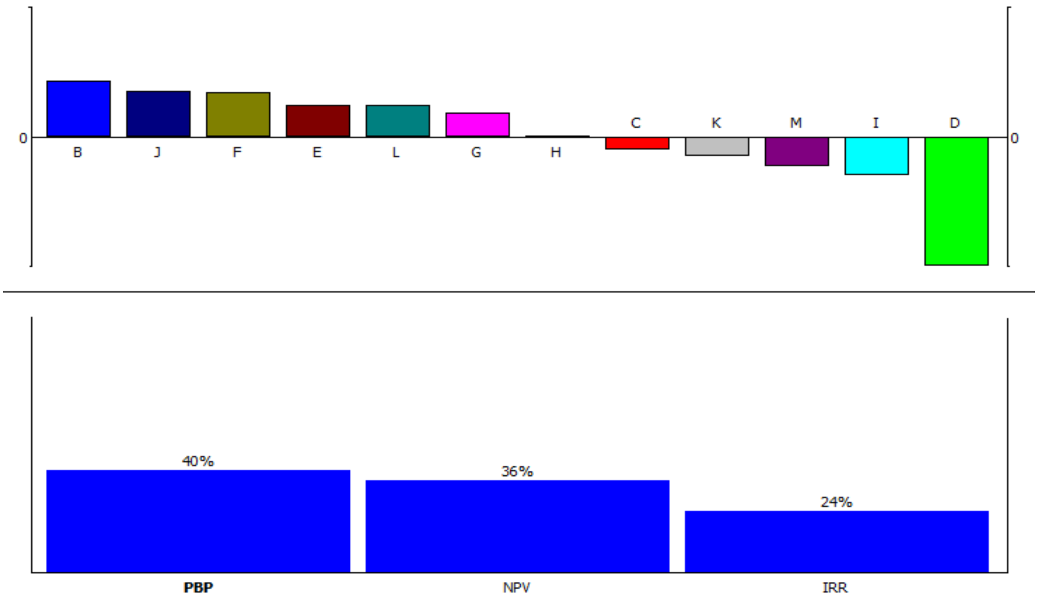
C.17. AHP final ranking of the alternatives for the Aggressive DM (-10% IRR)



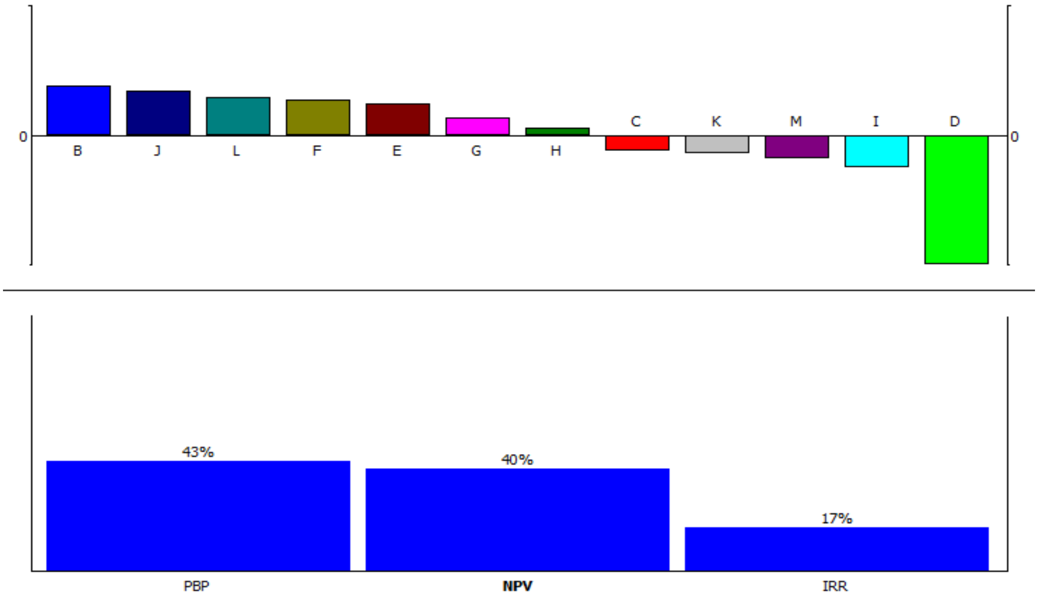
C.18. PROMETHEE final ranking of the alternatives for the Conservative DM – without DM preference functions (+10% PBP)



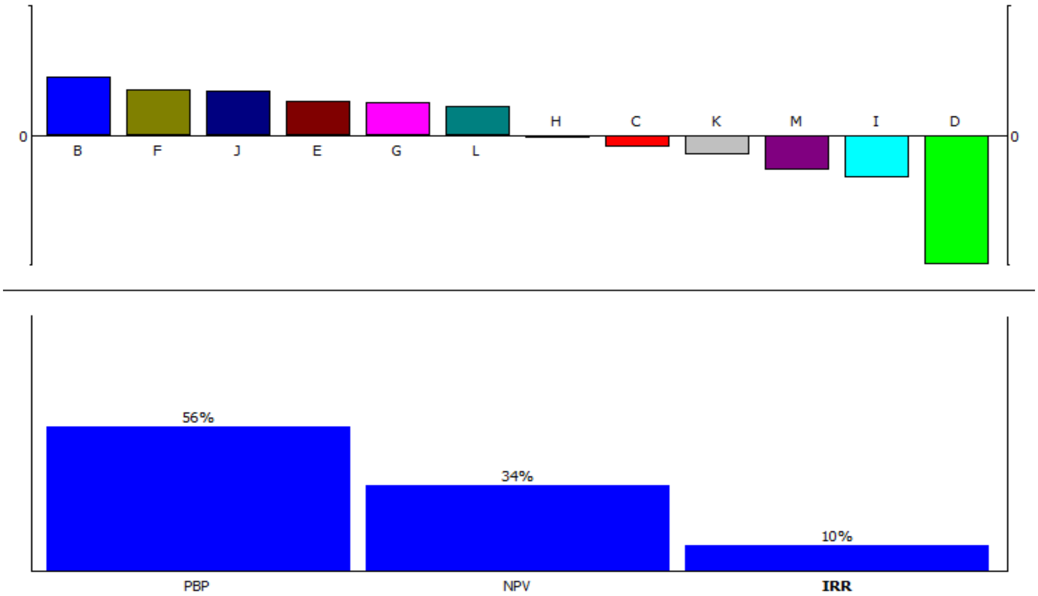
C.19. PROMETHEE final ranking of the alternatives for the Conservative DM – without DM preference functions (-10% PBP)



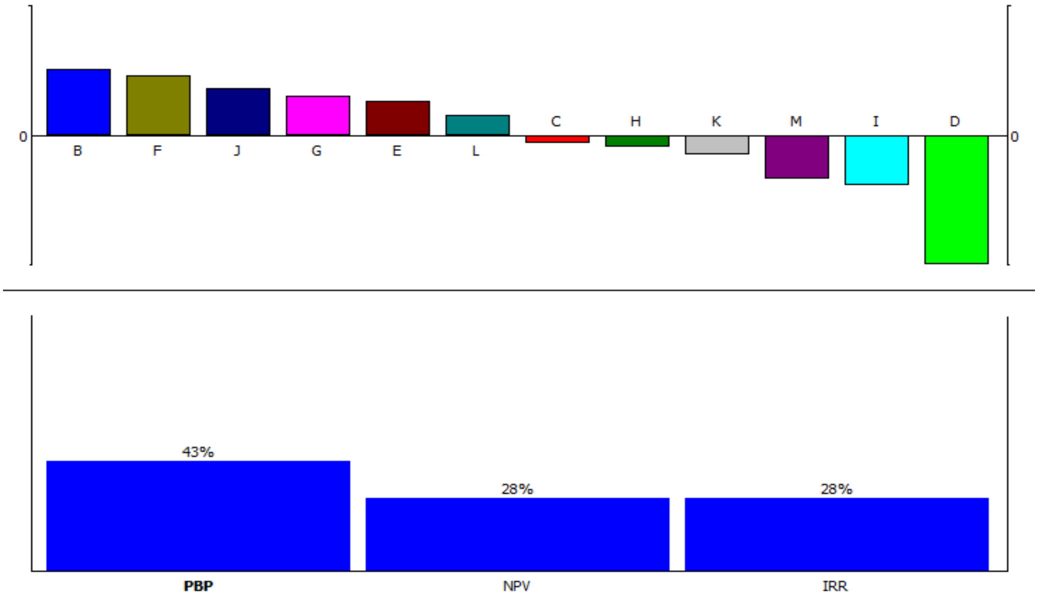
C.20. PROMETHEE final ranking of the alternatives for the Conservative DM – without DM preference functions (+10% NPV)



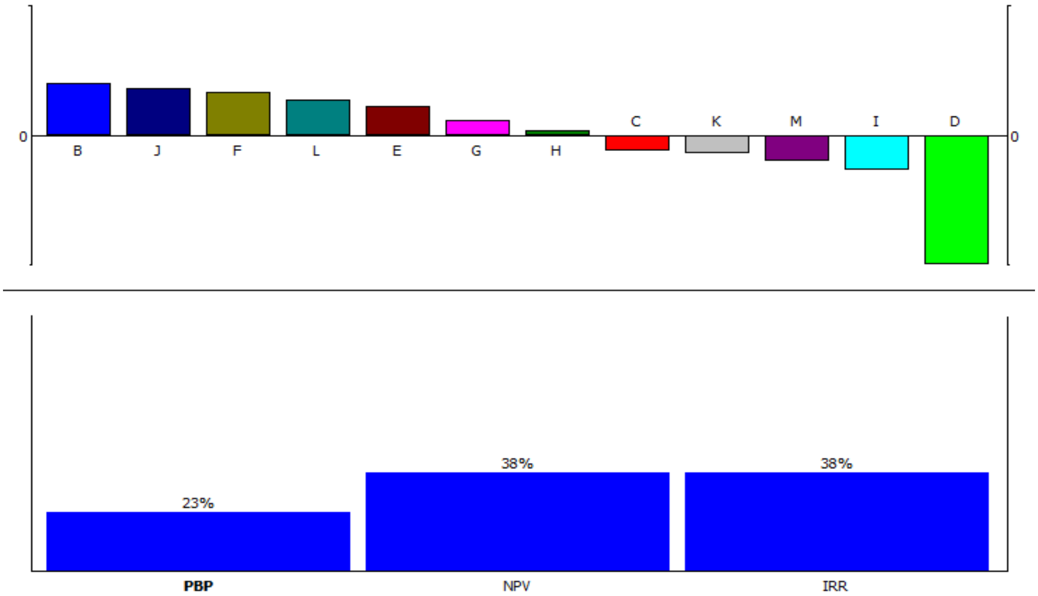
C.21. PROMETHEE final ranking of the alternatives for the Conservative DM – without DM preference functions (-10% IRR)



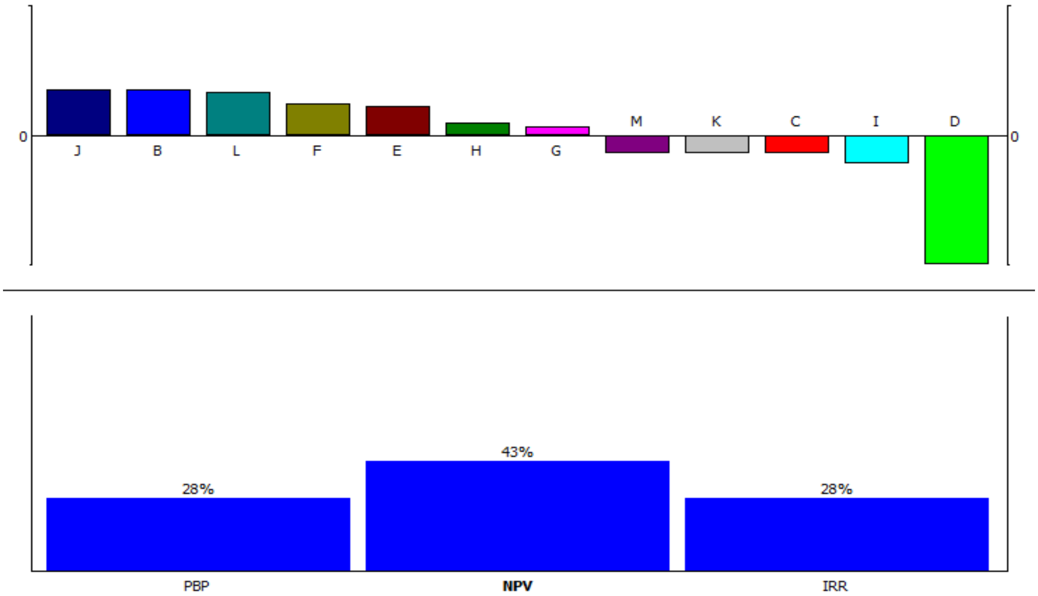
C.22. PROMETHEE final ranking of the alternatives for the Moderate DM – without DM preference functions (+10% PBP)



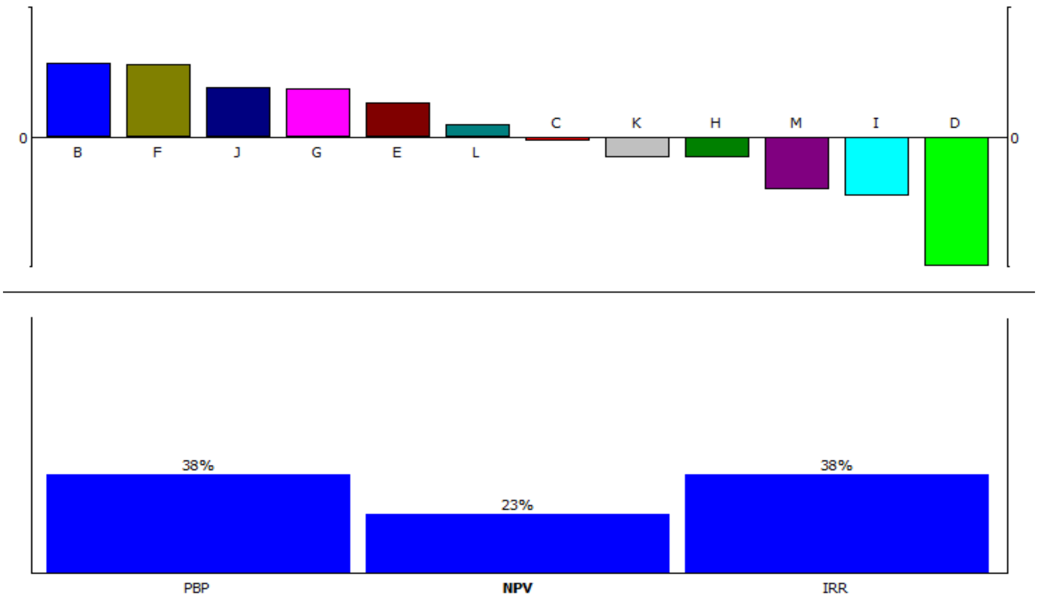
C.23. PROMETHEE final ranking of the alternatives for the Moderate DM – without DM preference functions (-10% PBP)



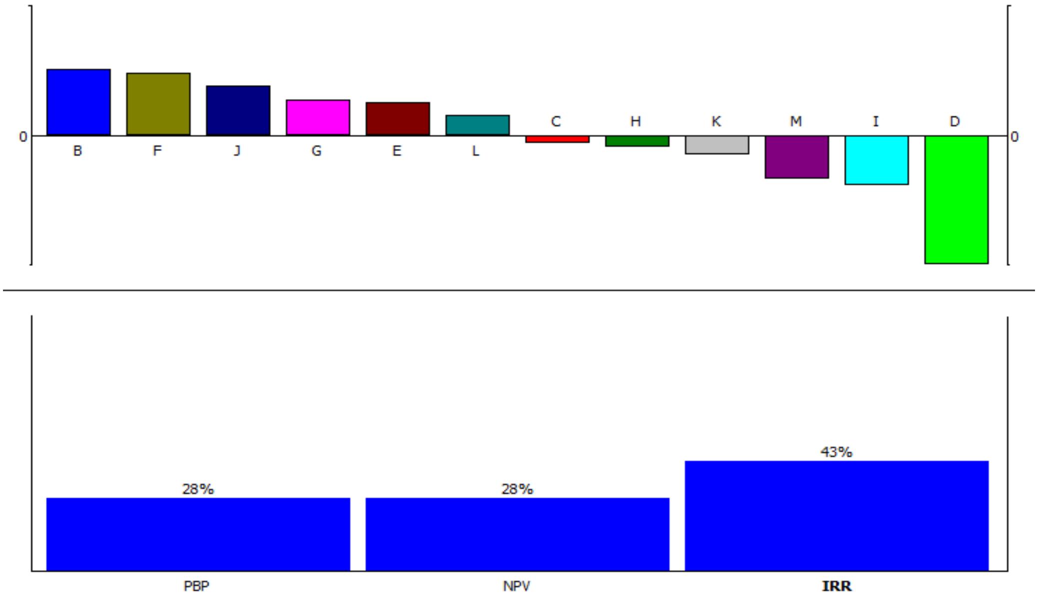
C.24. PROMETHEE final ranking of the alternatives for the Moderate DM – without DM preference functions (+10% NPV)



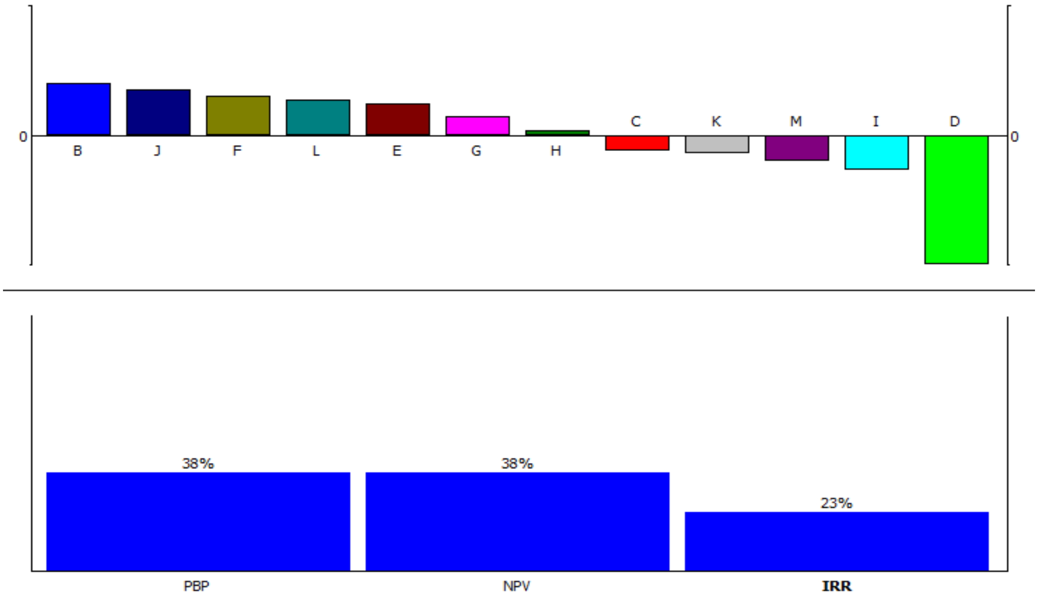
C.25. PROMETHEE final ranking of the alternatives for the Moderate DM – without DM preference functions (-10% NPV)



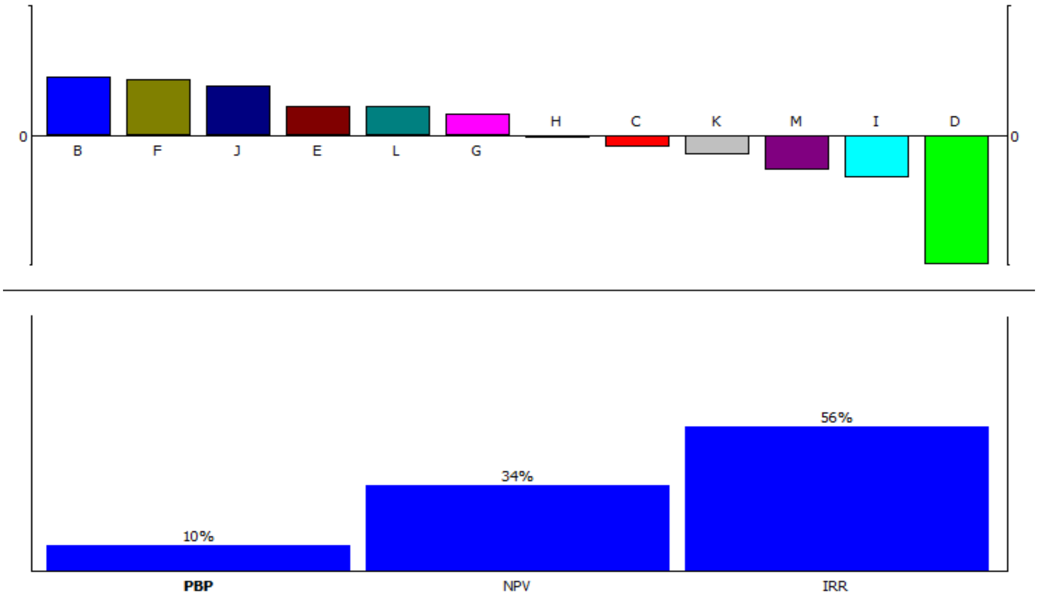
C.26. PROMETHEE final ranking of the alternatives for the Moderate DM – without DM preference functions (+10% IRR)



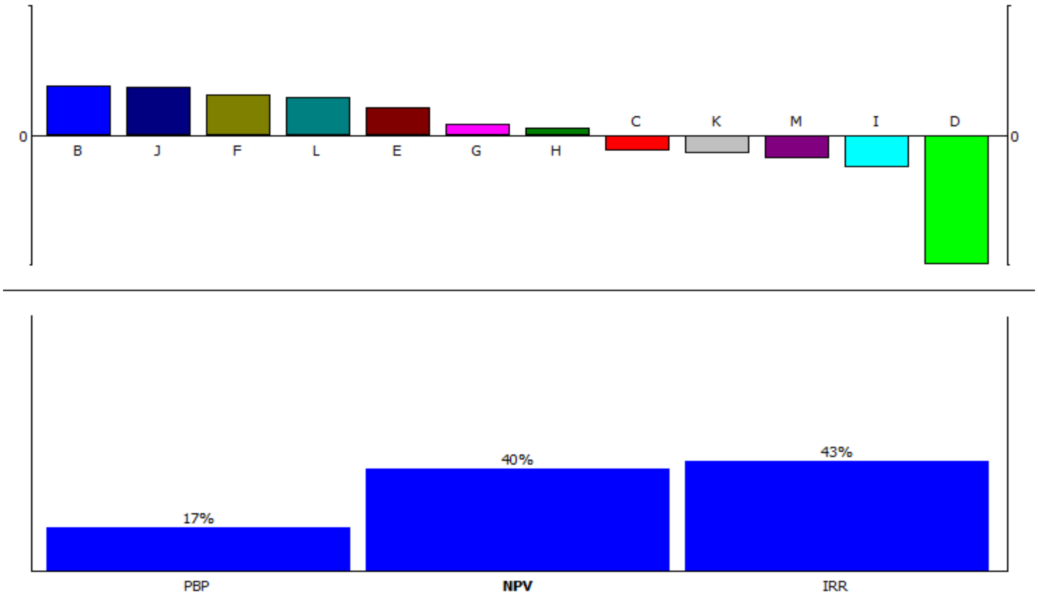
C.27. PROMETHEE final ranking of the alternatives for the Moderate DM – without DM preference functions (-10% IRR)



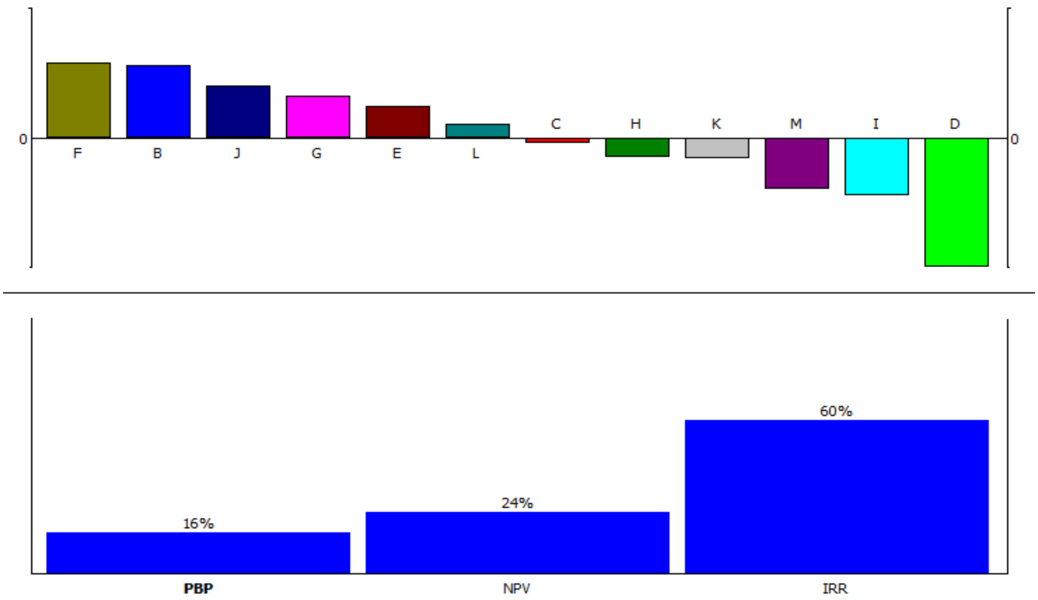
C.28. PROMETHEE final ranking of the alternatives for the Aggressive DM – without DM preference functions (-10% PBP)



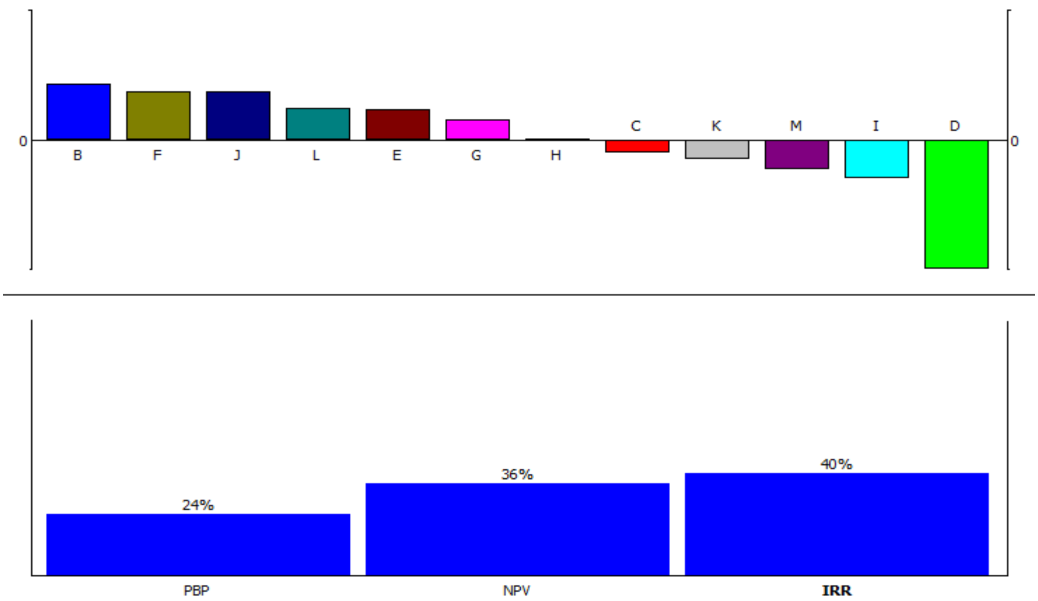
C.29. PROMETHEE final ranking of the alternatives for the Aggressive DM – without DM preference functions (+10% NPV)



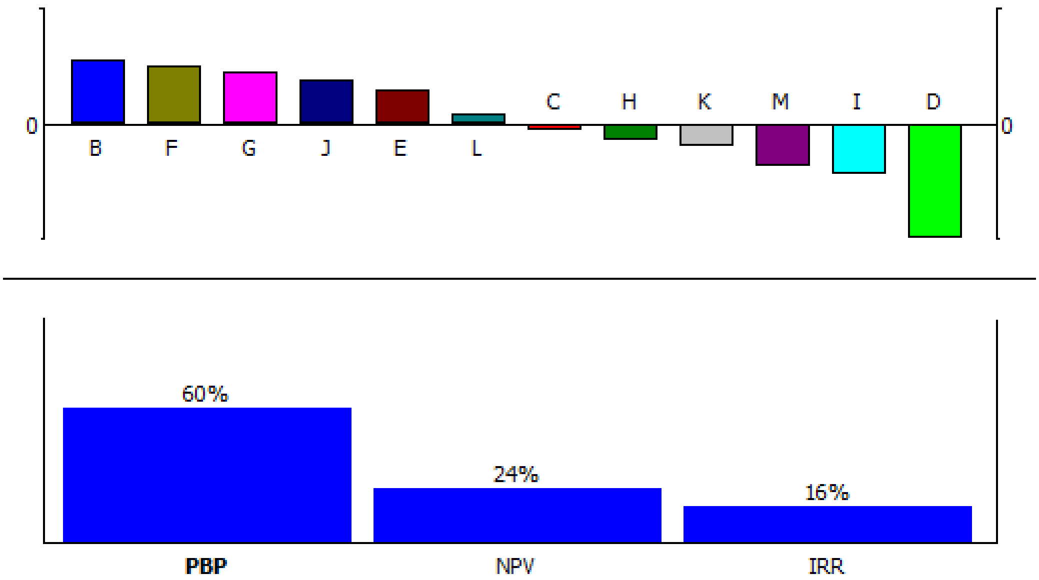
C.30. PROMETHEE final ranking of the alternatives for the Aggressive DM – without DM preference functions (+10% IRR)



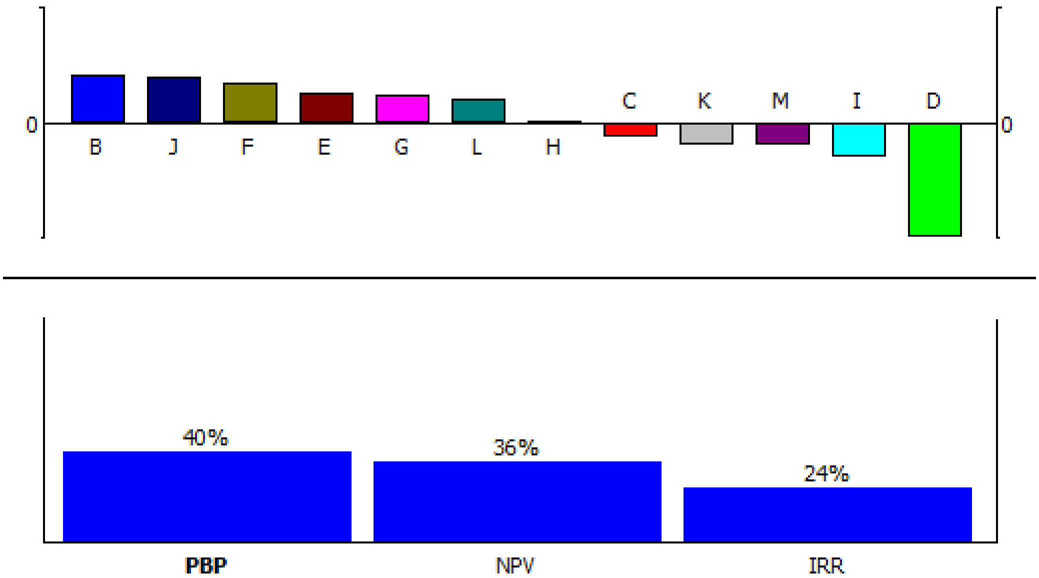
C.31. PROMETHEE final ranking of the alternatives for the Aggressive DM – without DM preference functions (-10% IRR)



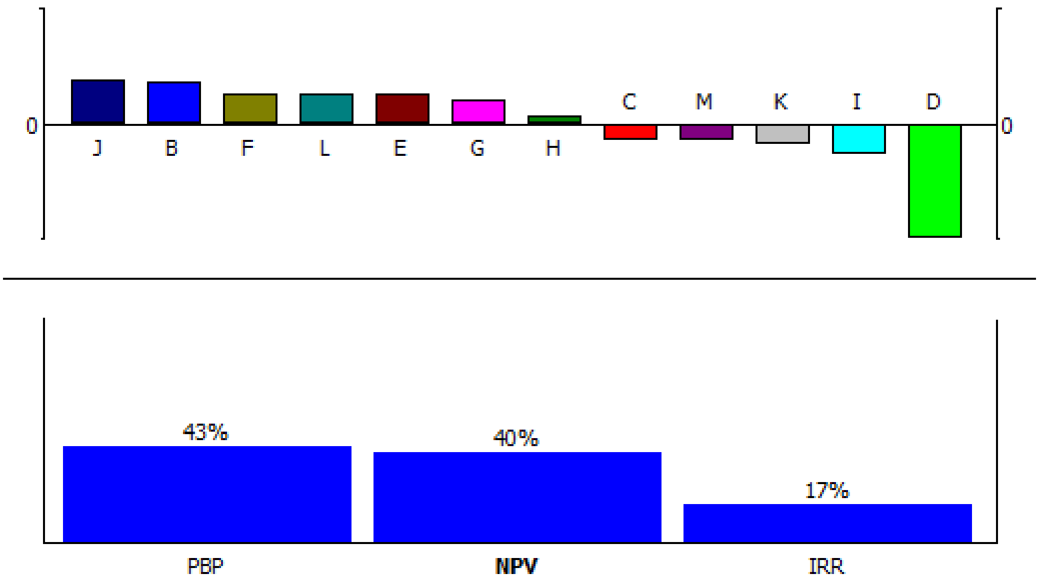
C.32. PROMETHEE final ranking of the alternatives for the Conservative DM – with DM preference functions (+10% PBP)



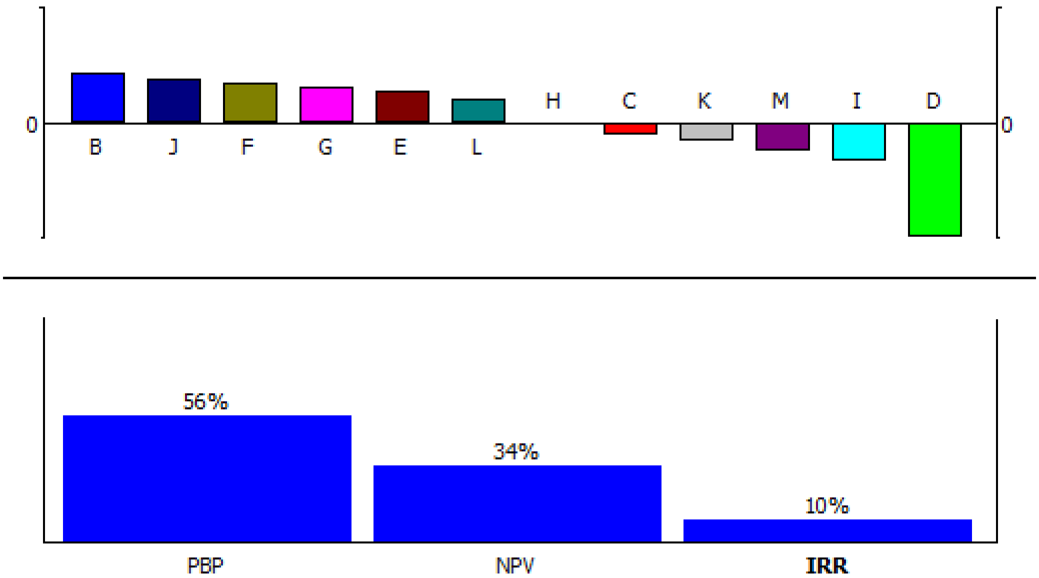
C.33. PROMETHEE final ranking of the alternatives for the Conservative DM – with DM preference functions (-10% PBP)



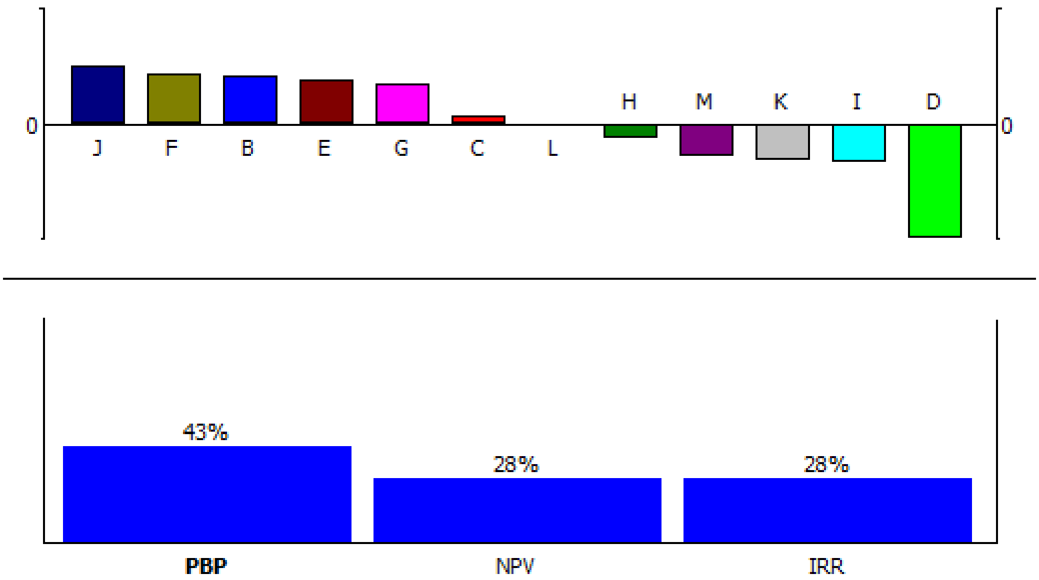
C.34. PROMETHEE final ranking of the alternatives for the Conservative DM – with DM preference functions (+10% NPV)



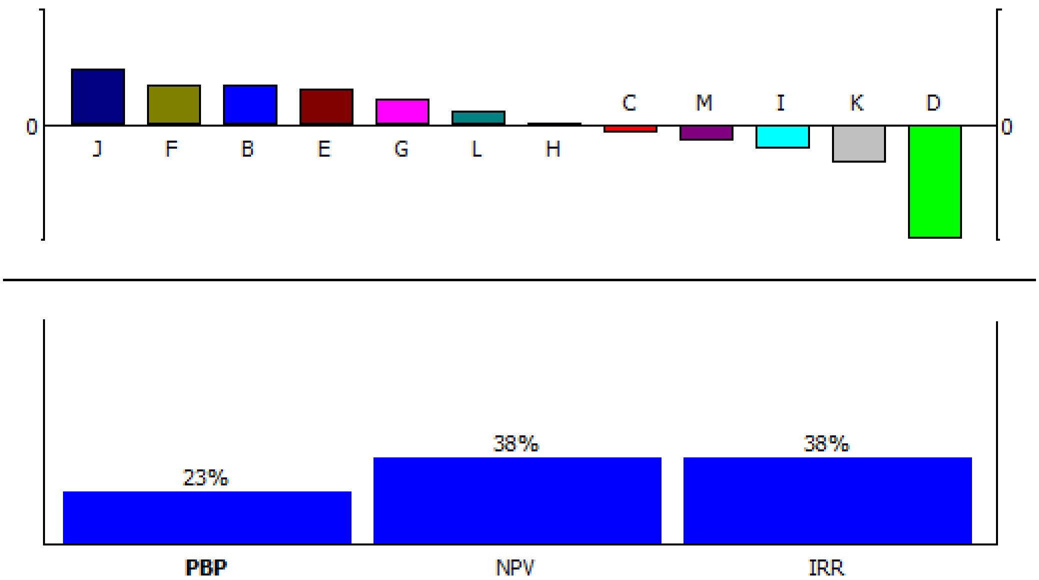
C.35. PROMETHEE final ranking of the alternatives for the Conservative DM – with DM preference functions (-10% IRR)



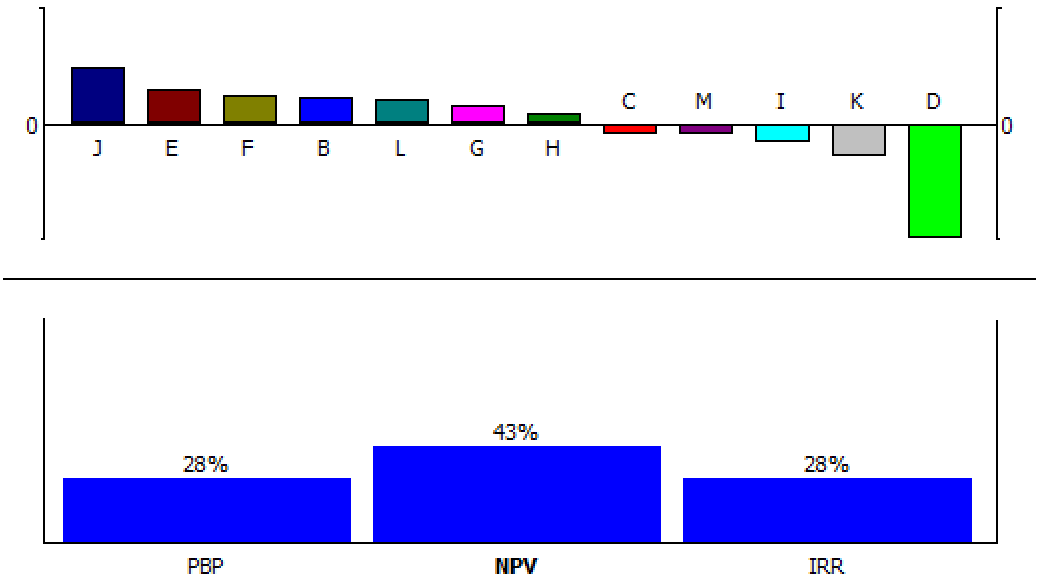
C.36. PROMETHEE final ranking of the alternatives for the Moderate DM – with DM preference functions (+10% PBP)



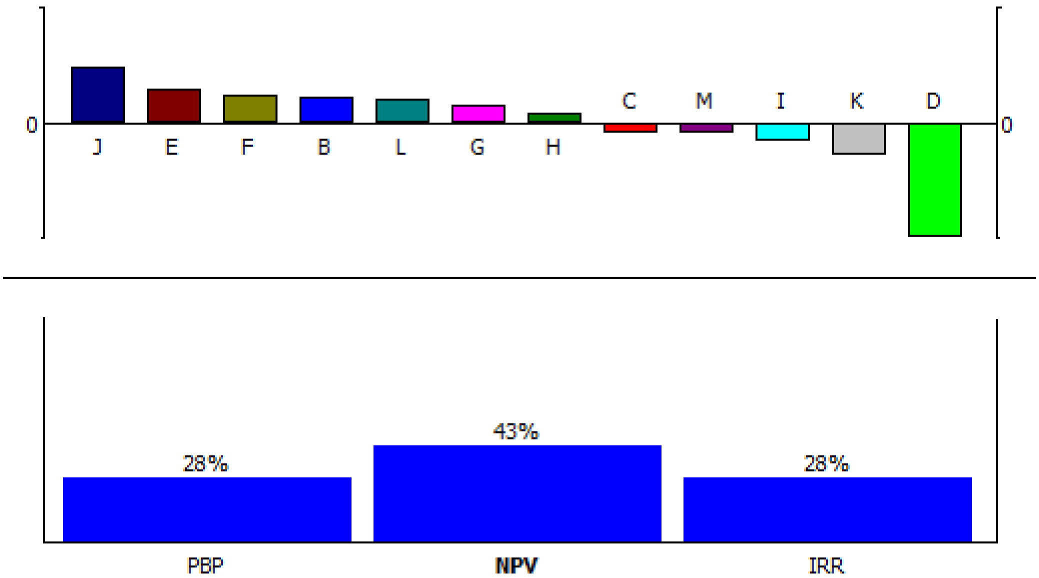
C.37. PROMETHEE final ranking of the alternatives for the Moderate DM – with DM preference functions (-10% PBP)



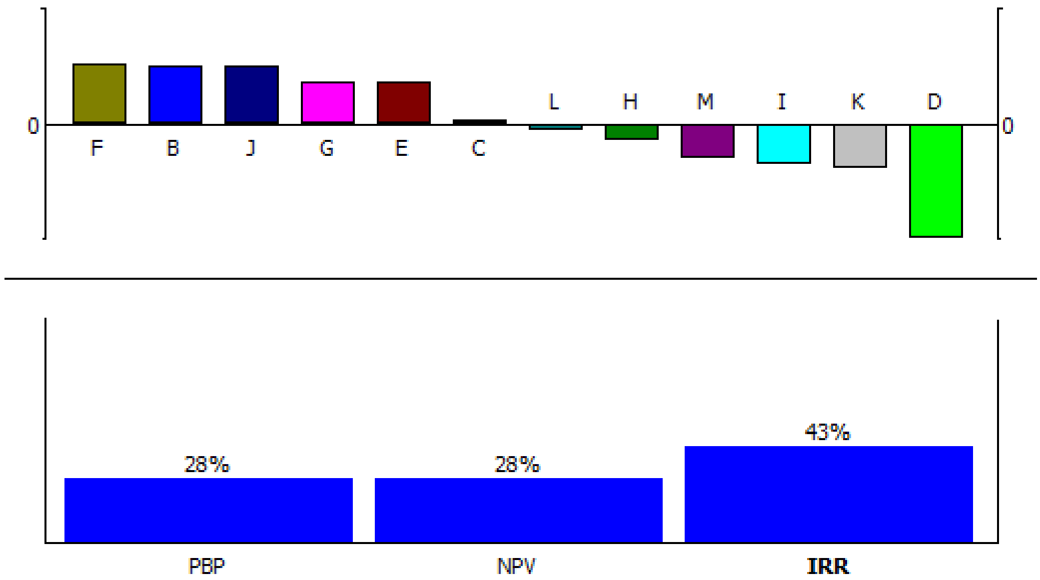
C.38. PROMETHEE final ranking of the alternatives for the Moderate DM – with DM preference functions (+10% NPV)



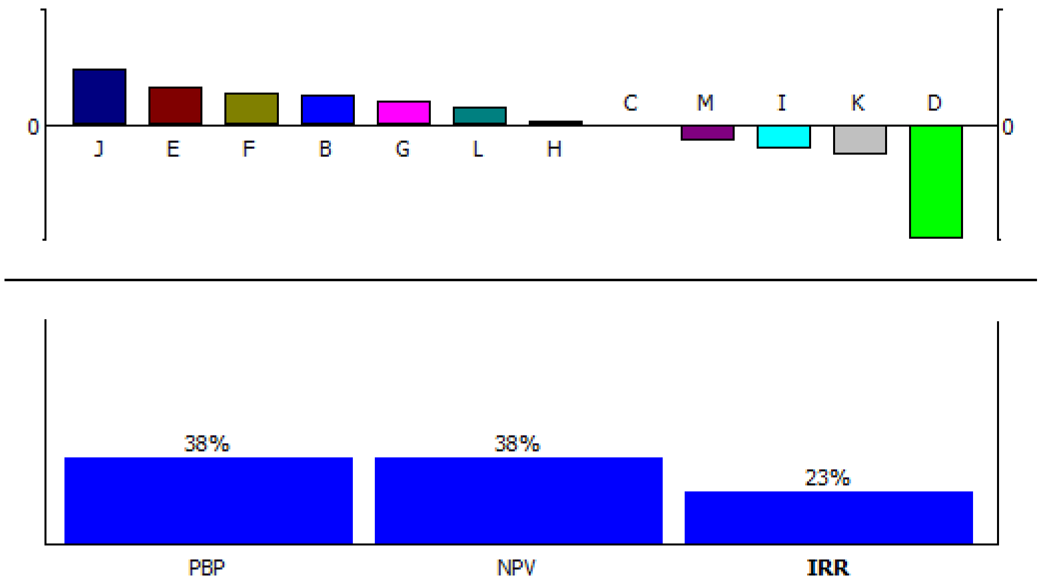
C.39. PROMETHEE final ranking of the alternatives for the Moderate DM – with DM preference functions (-10% NPV)



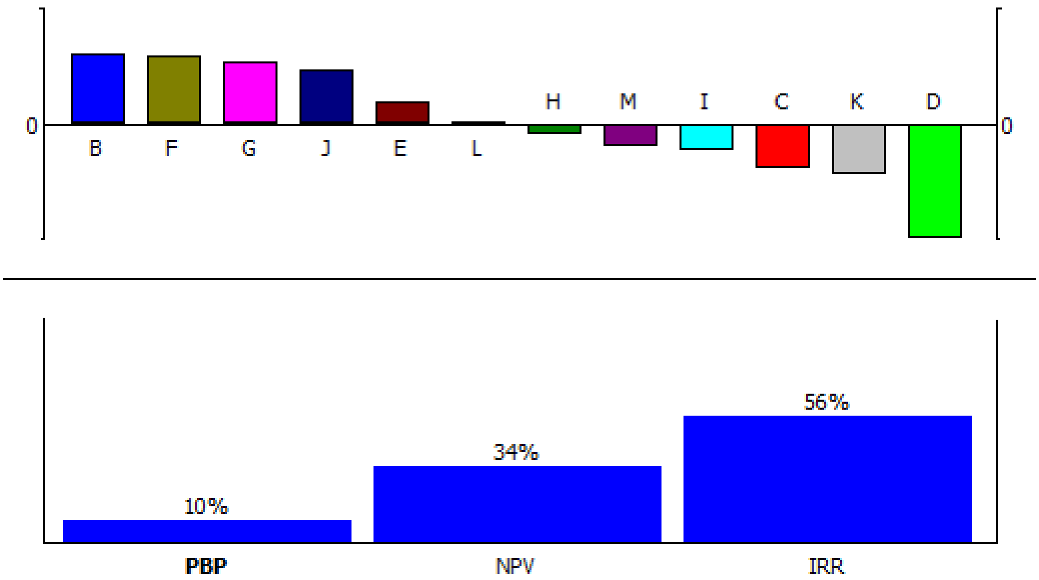
C.40. PROMETHEE final ranking of the alternatives for the Moderate DM – with DM preference functions (+10% IRR)



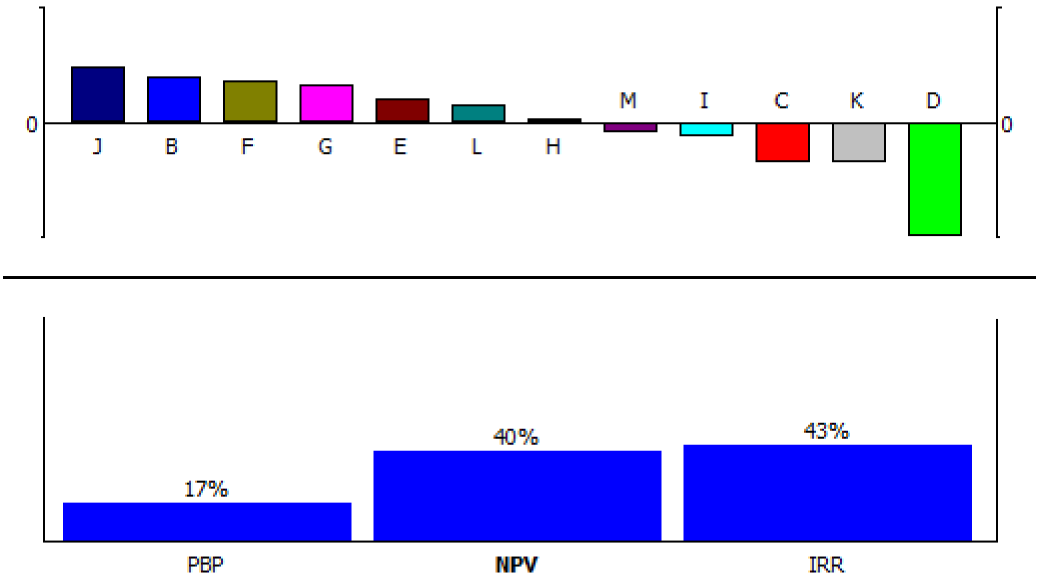
C.41. PROMETHEE final ranking of the alternatives for the Moderate DM – with DM preference functions (-10% IRR)



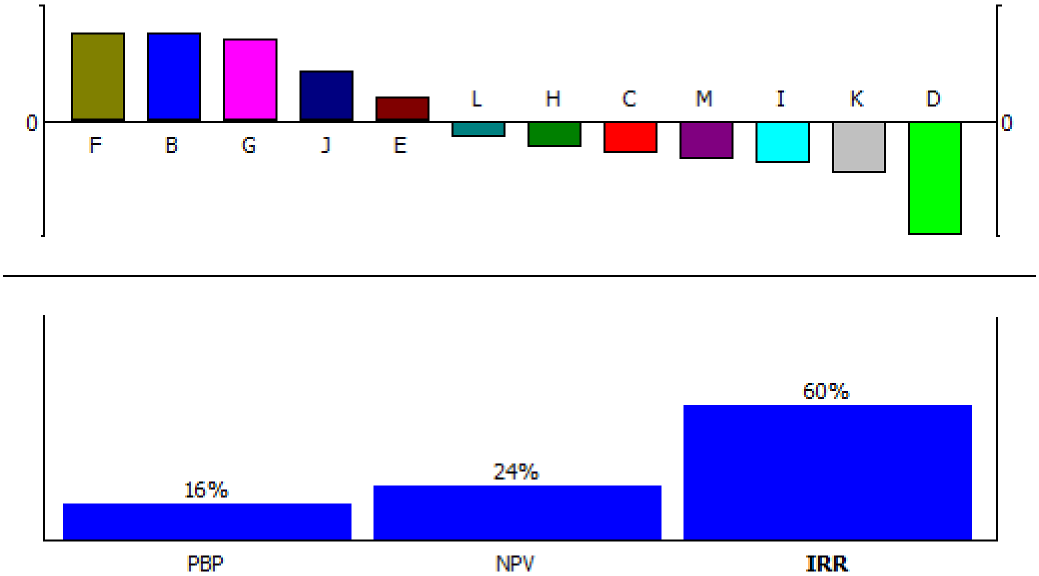
C.42. PROMETHEE final ranking of the alternatives for the Aggressive DM – with DM preference functions (-10% PBP)



C.43. PROMETHEE final ranking of the alternatives for the Aggressive DM – with DM preference functions (+10% NPV)



C.44. PROMETHEE final ranking of the alternatives for the Aggressive DM – with DM preference functions (+10% IRR)



C.45. PROMETHEE final ranking of the alternatives for the Aggressive DM – with DM preference functions (-10% IRR)