Co-investment, Uncertainty, and Opportunism: 

*Ex-Ante and Ex-Post Remedies*

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Abstract

Caused largely by the recent technological changes towards digitalization, infrastructure investment in network industries has become the main issue for regulatory intervention. In this paper, we study the impact of co-investment between an incumbent and an entrant on the roll-out of network infrastructures under demand uncertainty. We show that if the entrant can wait to co-invest until demand is realized, the incumbent’s investment incentives are reduced, and total coverage can be lower than in a benchmark with earlier co-investment. We consider two remedies to correct these distortions: (i) co-investment options purchased *ex ante* by the entrant from the incumbent, and (ii) risk premia paid *ex post* by the entrant. We show that co-investment options cannot fully reestablish total coverage, while premia can do so in most cases, though at the cost of less entry. Finally, we show that an appropriate combination of *ex-ante* and *ex-post* remedies can improve welfare.

Keywords: Co-investment, Uncertainty, Opportunism, Options, Risk premia.

JEL Codes: L96, L51.

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1 Introduction

The Need for New Networks and New Policy

Since the early 1990s, the European Commission has substantially reformed network industries in the European Union (EU). This policy aimed to restructure the vertical organization of these industries, open their downstream markets to competition, enhance cost efficiency and service quality, and boost investment in infrastructure (Alesina et al., 2005).

Given the peculiar market structure of network industries, for decades the regulatory approach in the EU – and similarly in other countries like Australia, New Zealand, and the US in the early nineties – was based on the analysis of the presence of significant market power and the implementation of specific obligations on incumbent firms to open their infrastructure to third parties. The main idea was to develop retail market competition to benefit final consumers with lower prices, considering that the infrastructure was, in most cases, already in place and only had to be maintained.

New technological trends in network industries have recently questioned the role of this regulatory approach. In several industries, existing infrastructures should not be simply maintained; they must be replaced by new advanced networks. In the energy sector, the International Energy Agency (2010) estimates investment needs in Europe to create new future-proof infrastructures – the so-called smart grids – to be around €480bn. For the telecommunications industry, the Boston Consulting Group (2016) estimates a cost of €660bn to deploy fiber networks in the EU27, and 25 years needed to complete the investment. However, these new infrastructures are essential to the economy as a whole, as they represent “general-purpose technologies” (Bresnahan and Trajtenberg, 1995), capable of creating feedback loops of productivity gains and growth across major economic sectors on a massive scale (Czernich et al., 2011).

The roll-out of fiber networks in the telecoms sector, providing ultra-fast access to fixed and mobile Internet, is particularly urgent. With the COVID-19 pandemic, we are facing
an unprecedented situation on a global scale. Entire countries are in lockdown, and people everywhere are asked to minimize their physical (social) interactions. Therefore, fast broadband connections have become our lifelines: people connect to school and work environments, often with whole families doing so simultaneously. However, the coronavirus outbreak has shown how weak the state of telecom infrastructures in several countries is. For example, in Europe and other countries, big digital platforms, such as Netflix and YouTube, have been asked to reduce their video quality to limit the strain on Internet service providers.

According to recent OECD data, as of June 2019, fiber-based connections made up only 27% of the total number of broadband connections in the OECD.¹ In Europe, the diffusion of ultra-fast broadband is higher than the OECD average, but still far from reaching the entire EU population. The most recent data from the European Commission (2019) shows that ultra-fast broadband is available to 60% of households. Therefore, a large part of the EU population does not have access to advanced broadband services.

To encourage the deployment of fiber across Europe, the European Union adopted a new regulatory framework in 2018, the new European Electronic Communications Code (EECC),² with measures aimed at providing market players with strong(er) investment incentives. To stimulate investment while preserving a competitive environment, the EECC offers incumbent firms the choice to share the heavy burden of investment, that is, to invite new entrants to “co-invest” in the deployment of their new high-speed broadband infrastructures, instead of being subject to access obligations.³

The opportunities for cooperative investment by operators have recently become a prominent topic of discussion. Co-investment is seen as a solution to the financial constraints faced by firms and the asymmetric risk allocation between incumbents and entrants, which slow down the roll-out of new fiber-based infrastructures.

When a firm deploys a new infrastructure in a local area, it will often face uncertainty

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about the level of demand for the services that its network will deliver. Under traditional access regulation, potential entrants can wait and ask for wholesale access when uncertainty is resolved. In contrast, if the new network is constructed under a co-investment agreement, potential co-investors take their decision under the same level of uncertainty as the incumbent. However, the EECC allows entrants to ask for co-investment later, when the network is already deployed (ex post).\textsuperscript{4} Hence, entrants can wait until enough information about demand has become available, which reduces their risk and transfers it to early investors. In this paper, we study the impact of such a wait-and-see option on investment and co-investment incentives and analyze regulatory provisions that may help avoid the distortions that ex-post co-investment generates.\textsuperscript{5} Our goal is to provide an assessment of these policy measures and understand their potential impact in spurring fiber deployment. Hence, the starting point of our analysis is not the classical social welfare perspective which determines a socially optimal level of coverage. Rather, we follow the aim of the new EU regulatory framework to expand fiber connections (from a base that is too low) and assess how new co-investment policies and alternative regulatory instruments could help reaching this target.

**Summary of main findings**

We model a geographical market, where an incumbent decides to deploy a new infrastructure in local areas with different sunk costs to be covered. An entrant can decide to co-invest with the incumbent in some or all areas, taking on half of the investment cost. Some areas (e.g., urban areas) are cheap to cover, but as the firms turn to more outlying areas, with lower population density (e.g., rural areas), the investment cost increases.

When it makes its investment decision, the incumbent faces uncertainty about the demand level in each local area. Demand becomes known after the network is deployed. Firms

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\textsuperscript{4}Article 76(a) of the EECC states that a co-investment offer should be “open at any moment during the lifetime of the network to any provider of electronic communications networks or services.”

\textsuperscript{5}Annex IV of the Directive 2018/1972 states that “the determination of the financial consideration to be provided by each co-investor needs to reflect the fact that early investors accept greater risks and engage capital sooner.”
make profits in a local area (gross of the investment cost), which depend on the level of demand and the local market structure (monopoly or duopoly). We adopt a general approach and use reduced-form profits.

As a benchmark, we first consider the situation where the entrant makes its co-investment decision before the network is rolled out (i.e., *ex ante*), and therefore faces the same demand uncertainty as the incumbent. We show that in equilibrium, the entrant offers to co-invest in the least costly areas of the market. If monopoly profits are higher than the sum of duopoly profits, there are also monopoly areas where the incumbent invests alone.

We then study the case where the entrant can wait for demand to be realized before deciding on co-investment (i.e., co-investment takes place *ex post*). We show that this creates two distortions compared to the benchmark. First, it reduces the probability of entry in co-investment areas, while increasing the probability of entry in monopoly areas (market structure distortion). Second, it reduces the incumbents investment incentives, and as a consequence, it can lead to lower coverage than in the benchmark (total coverage distortion). This happens in particular, when the level of demand uncertainty is high. We show that when the total coverage distortion is sufficiently strong, *ex-post* co-investment decreases welfare, relative to the *ex-ante* benchmark.

Two possible remedies to curb the entrant’s opportunism and correct these distortions are (i) a co-investment option purchased *ex ante* by the entrant from the incumbent, to be exercised *ex post*, and (ii) a risk premium paid *ex post* by the entrant.

We show that compared to the case of *ex-post* co-investment with no remedy, the co-investment option does not affect the entrant’s co-investment decision in the areas covered by the incumbent. Therefore, it cannot correct the market structure distortion. However, the option reduces (but does not fully eliminate) the coverage distortion by making investment more attractive for the incumbent. By contrast, a positive risk premium increases coverage while reducing the entrant’s willingness to co-invest and enter *ex post*. Only a negative premium (i.e., a subsidy paid by the incumbent to the entrant) could reduce the market
structure distortion.

Finally, we discuss combining remedies by implementing co-investment options in some areas and risk premia in others. First, we show that a negative risk premium (i.e., subsidies) allows correcting partially – but not fully – the market structure distortion in the areas with less entry than in the benchmark. Second, we show that to correct the total coverage distortion, one should implement co-investment options in areas with moderate cost to be covered and a risk premium in the most outlying and costlier areas. Using a numerical example, we show that introducing such a combination of *ex-ante* and *ex-post* remedies can increase total welfare, compared to the outcome under *ex-post* co-investment.

**Literature review**

Our paper contributes to two different strands of literature.

The first strand of literature relates to the role of uncertainty for capital investment decisions. This literature (see Dixit and Pindyck (1994)) emphasizes the interaction of (i) uncertainty over future returns to capital, (ii) irreversibility, and more importantly, for this paper, (iii) the opportunity to delay the investment. The opportunity to delay generates for the firm a (call or put) option that should be accounted for. The approach used in this literature is based on the real option approach, which is founded on Bernanke (1983)’s “bad news principle,” later developed by Abel et al. (1996). This principle states that the value of an option depends only on the size of the potential downward move (i.e., bad states of demand), but not on the size of the upward move (i.e., good states of demand). We contribute to this literature in several ways. First, all of these models consider a single firm investing in a new asset, hence a pure monopolistic setting. Instead, we consider a setting where an incumbent and an entrant – which holds the option – can reach a co-investment agreement for deploying a new infrastructure, while competing at the retail level. Second, we model the presence of a call option, but do not consider the put option, i.e., the option to disinvest, since it is not relevant in our setting: once the ultra-fast broadband network
is deployed, there is no gain in ripping it out again. The value of the option to invest later arises from avoiding the downside of low returns, i.e., from not co-investing when demand turns out to be low.

The real option approach has been followed by Hausman (1997 and 1999) to propose a redefinition of access regulation by including the *ex-post* option value of network investment as a mark-up on the per-unit access charge. Differently from these studies, we model this possibility as an *ex-ante* charge that buys the right to ask for co-investment later on, i.e., we put a price on the option that otherwise is given for free to the entrant.

The second strand of literature to which we contribute relates to co-investment and other risk-sharing agreements in network industries.\(^6\) Nitsche and Wiethaus (2011) model co-investment as a joint venture decision, where firms jointly decide on an investment in quality that maximizes their expected joint profits. In the presence of demand uncertainty, they show that co-investment can be beneficial in terms of both investment incentives and consumer welfare, compared to standard access regulation (e.g., LRIC access pricing). Different from their paper, we model co-investment as non-cooperative coverage decisions rather than as a joint quality choice. Modeling investment as a coverage decision rather than a quality choice seems more realistic in the context of fiber deployments: the quality of the technology (e.g., in terms of speed) is given, and firms decide which geographical areas to cover. We also capture the fact that under the new European regulatory framework, co-investment is a decision made by the entrant, possibly *ex post*, and we investigate remedies to the resulting distortions.

Inderst and Peitz (2012) analyze cost-sharing agreements between an incumbent firm and an entrant in the form of long-term contracts concluded before the investment is made, as opposed to contracting taking place after the network has been constructed. In their model, investment corresponds to a quality improvement, which is different from the coverage decision we consider in our paper. The authors show that long-term contracts reduce the

\(^6\)See Sand-Zantman (2017) for a survey on the economics of cooperative agreements for the deployment of network infrastructures.
duplication of investment and may lead to higher quality. However, they assume that coordination at the investment level directly implies reduced competition in the areas covered. They do not consider an *ex-post* agreement and the distortions it would cause.

Our benchmark model with *ex-ante* co-investment draws on Bourreau et al. (2018), in which the authors focus on co-investment as an alternative to standard access regulation. Different from that paper, we consider the possibility that co-investment can take place *ex post*, as the European legal framework permits. This allows potential entrants to wait until the level of demand has been revealed, cherry-picking profitable areas. We show that this may make investment unprofitable, and we then consider realistic policy measures to reestablish investment incentives.

In the literature, several formal studies have investigated the impact of access regulation on investment incentives by incumbent and entrant firms in the presence of demand uncertainty, for example, Hori and Mizuno (2006), Klumpp and Su (2010) and Inderst and Peitz (2013). In a recent companion paper, Bourreau et al. (2020), we compare various regulated access schemes under demand uncertainty. However, none of these papers considers co-investment, as we do here, since their focus is on standard access pricing regulation. Hence, while in our previous paper, we focused on how to redesign access pricing to make an incumbent invest in expanding its own network, in the present paper, we focus on the timing and extent of entrants’ co-investment decisions, a different policy issue.

The rest of the paper is organized as follows. In Section 2, we set up the model. In Section 3, we solve for the coverage equilibrium when co-investment takes place *ex-ante*, which constitutes our benchmark, and when it takes place *ex-post*. In Section 4, we consider two remedies: co-investment options and risk premia. Section 5 concludes.
2 Model Setup

We consider a country consisting of a continuum of areas $z \in \mathbb{R}_+$, with identical distribution of uncertain demand, but different sunk costs of coverage by a new network infrastructure. The cost of covering area $z$ is $c(z)$, where $c(0) = 0$, $c(.)$ is continuous and strictly increasing, and $\lim_{z \to +\infty} c(z) = +\infty$. We denote by $C(z) = \int_0^z c(x) \, dx$ the total cost of covering the areas $[0, z]$.

There is one incumbent, firm 1, and one potential entrant, firm $e$. We assume sequential investment decisions. The incumbent first decides on the areas where it will invest. Then, firm $e$ can co-invest in any area covered by the incumbent, taking on half of the investment cost. Finally, firms compete in local areas, and profits are realized. Firm $e$ can only enter through co-investment. Under co-investment, the entrant can access the infrastructure under the same conditions as the incumbent (and in particular, at the same marginal cost of access).

Firms can adjust their prices according to local competitive conditions. In each co-investment area, firms 1 and $e$ make the expected symmetric retail duopoly profits $\pi^d > 0$, with corresponding social welfare $w^d$, where welfare is defined as the sum of consumer surplus and industry profits. In the areas covered by the incumbent and where the entrant does not co-invest, the incumbent makes a higher expected profit $\pi^m \geq \pi^d$; we denote by $w^m$ the social welfare in these areas. The profit $\pi^m$ can correspond to the monopoly profit, or a profit smaller than the monopoly profit in case there is competition from an old technology. For simplicity, in the following we refer to these areas as monopoly areas, where the incumbent derives monopoly profits.

The level of demand in each local area is uncertain \textit{ex-ante} when the incumbent makes its investment decision. Information about the level of demand is revealed after the infrastructure is deployed and then observed by all market players. As a benchmark, we will first consider the case where the entrant also makes its co-investment decision under demand uncertainty, prior to the deployment of the infrastructure (\textit{ex-ante} co-investment). Then, we will allow the entrant to wait for demand to be realized (\textit{ex-post} co-investment).
The demand levels in the different areas of the country are identically distributed, but may not be independent. In any given area, we assume that the demand level, \( \delta \), is uniformly distributed over \([1 - \sigma, 1 + \sigma]\), with \( \sigma \in (0, 1) \). The expected level of demand is \( \mathbb{E}[\delta] = 1 \), and \( \sigma^2/3 \) is its variance. We interpret \( \sigma \) as the degree of demand uncertainty. The expected profit for an area of type \( \tau = d, m \) is then \( \mathbb{E}[\delta \pi^\tau] = \pi^\tau \).

Our assumptions are in line with the real characteristics of fiber deployment in many countries, especially in the EU. Indeed, in the next few years, the ultra-fast broadband market will still be characterized by a high degree of demand uncertainty, as reflected in the relatively low willingness to pay for fiber based on traditional usage cases and the uncertainty about which new services will anchor demand for ultra-fast broadband. From the supply perspective, most telecom companies are currently providing offers with a very low level of differentiation. These conditions however are more likely to change in the next few years. Indeed, demand uncertainty will probably diminish, while from the supply side, wide possibilities for product differentiation will emerge due to new fiber-based services like the Internet of Things (such as smart homes, smart factories, cloud computing, etc.).

3 Co-Investment

In this section, we solve for the equilibrium of the (co-)investment game when co-investment takes place \( \text{ex ante} \), before demand is realized, and then when it takes place \( \text{ex post} \).

3.1 Benchmark: \textit{ex-ante} co-investment

As a benchmark, we first assume that the incumbent and the entrant both make their investment decisions \( \text{ex ante} \) under demand uncertainty. We solve the game backwards, starting from the entrant’s decision.\(^7\)

\(^7\)The analysis in the benchmark is similar to Bourreau, Cambini and Hoernig (2018), though slightly more general since we do not make the assumption that firms (co-)invest in intervals of areas.
**Entrant’s decision.** If the entrant co-invests in a given area $z$ covered by the incumbent, it makes the expected profit $\pi_e(z) = \pi^d - c(z)/2$. The entrant decides to co-invest in the area if and only if $\pi_e(z) \geq 0$, that is, if $z \leq \bar{z}^c \equiv c^{-1}(2\pi^d)$.

**Incumbent’s decision.** The incumbent decides whether to invest in a given area, taking into account whether it will later be matched by the entrant’s co-investment.

In any area $z \leq \bar{z}^c$, the entrant will co-invest, and hence, the incumbent will face competition. The incumbent’s expected profit in such an area is $\pi_1(z) = \pi^d - c(z)/2$, which is positive for all $z \leq \bar{z}^c$. Thus, the incumbent will invest in all the areas $z \leq \bar{z}^c$.

In areas $z > \bar{z}^c$, the entrant will not co-invest. The incumbent will enjoy a monopoly position and obtain the expected profit $\pi_1(z) = \pi^m - c(z)$. This is positive if and only if $z \leq \bar{z}^m \equiv c^{-1}(\pi^m)$. There is a set of monopoly areas in equilibrium if $\bar{z}^m > \bar{z}^c$, which happens if $\pi^m > 2\pi^d$ (i.e., when duopoly profits are low because services are close substitutes).

Summing up:

**Proposition 1** *Under ex-ante co-investment, the incumbent and the entrant cover the areas* $[0, \bar{z}^c]$. *Furthermore,*

1. If $\pi^m \leq 2\pi^d$, there are no monopoly areas.

2. If $\pi^m > 2\pi^d$, the incumbent also covers the monopoly areas $(\bar{z}^c, \bar{z}^m]$.

In this benchmark with *ex-ante* co-investment, total coverage in equilibrium is given by $\max\{\bar{z}^c, \bar{z}^m\}$. The market structure is characterized by a duopoly in the co-investment areas $[0, \bar{z}^c]$ and a monopoly in the monopoly areas $(\bar{z}^c, \bar{z}^m]$ (when they exist). When service are more differentiated, as may happen in a few years, no monopolistic areas will be present. These are more likely to emerge in the short term while fiber-based offers remain more homogeneous. In this latter case, there is competition in the areas that are relatively cheap to cover, and only one provider (or none) in the more costly areas. In the next subsection, we study how the equilibrium is affected when co-investment takes place *ex post.*
3.2 Ex-post co-investment

We now consider the case where the entrant can wait and ask for co-investment in any given area ex post, after demand is realized.

**Entrant’s decision.** In a given area $z$ covered by the incumbent, firm $e$ observes the realized demand $\delta$, and then asks for co-investment if and only if $\delta \pi^d \geq c(z)/2$, that is, if demand is high enough, $\delta \geq c(z)/(2\pi^d)$.

For the sake of the analysis, we introduce some notations. We define $\delta(z) \equiv \min \{1 + \sigma, \max \{1 - \sigma, c(z)/(2\pi^d)\}\}$ as the minimum level of demand for entry to occur.\(^8\) We also define $\bar{z}$ as the solution of $c(z)/(2\pi^d) = 1 - \sigma$ and $\tilde{z}$ as the solution of $c(z)/(2\pi^d) = 1 + \sigma$. Since $c(\cdot)$ is strictly increasing, $\delta(\cdot)$ is strictly increasing for $z \in [\bar{z}, \tilde{z}]$, with $\delta(\bar{z}) = 1 - \sigma$ and $\delta(\tilde{z}) = 1 + \sigma$. We have $0 \leq \bar{z} \leq \bar{z}^e \leq \tilde{z}$, and we have $\bar{z} < \bar{z}^m$ if and only if $\pi^m > 2(1 + \sigma)\pi^d$.

The (ex-ante) probability that the entrant enters via co-investment in an area $z$ covered by the incumbent is then given by

$$p^e(z) = \int_{\delta(z)}^{1+\sigma} \frac{d\delta}{2\sigma} = \frac{1 + \sigma - \delta(z)}{2\sigma}, \quad (1)$$

with $p^e(z) = 1$ for all $z \leq \bar{z}$ and $p^e(z) = 0$ for all $z \geq \tilde{z}$. From (1), entry via co-investment is less likely in more costly areas (with a larger $z$). Higher uncertainty makes entry less likely in cheaper areas ($\delta(z) < 1$) but more likely in costlier areas ($\delta(z) > 1$), because in the latter it raises the upside value.

For further reference, let $\delta^e(z) \equiv (1 + \sigma + \delta(z))/2$ and $\delta^n(z) \equiv (1 - \sigma + \delta(z))/2$ represent the expected levels of demand conditional on entry and no entry, respectively. Note that we have $p^e(z)\delta^e(z) + (1 - p^e(z))\delta^n(z) = \mathbb{E}[\delta] = 1$. The entrant’s expected profit can then be written as $\hat{\pi}_e(z) = p^e(z)(\delta^e(z)\pi^d - c(z)/2)$.

We can now compare the probability of entry with ex-post co-investment with the prob-

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\(^8\)The minimum level of demand $\delta$ is bounded from below and from above because of our assumptions on the support of the distribution.
ability of entry in the benchmark.

**Lemma 1** Compared to the benchmark with ex-ante co-investment, if the entrant can wait for demand to be realized, it is:

1. as likely to co-invest in the least costly areas \( z \leq \bar{z} \);
2. less likely to co-invest in areas with intermediate costs \( z \in (\bar{z}, \bar{z}^c) \);
3. more likely to co-invest in the more costly areas \( z \in (\bar{z}^c, \pi] \).

**Proof.** In the benchmark, the entrant co-invests (with probability one) in the areas \( z \leq \bar{z}^c \), and does not co-invest (hence, co-invests with probability zero) in the areas \( z > \bar{z}^c \). When it can wait for demand to be realized, the entrant co-invests with probability one in the areas \( z \leq \bar{z} \), as in the benchmark. If \( z \in (\bar{z}, \bar{z}^c) \), the entrant co-invests with probability \( p^e(z) < 1 \), that is, with a lower probability than in the benchmark. Finally, if \( z \geq \bar{z}^c \), the entrant co-invests with probability \( p^e(z) > 0 \), that is, with a higher probability than in the benchmark.

This result shows that the possibility of *ex-post* co-investment introduces a market structure distortion: compared to the *ex-ante* benchmark, there is less entry in low-cost areas, and more entry in high-cost areas.

**Incumbent’s decision.** We now solve for the incumbent’s coverage decision. In a given area \( z \), the entrant will co-invest if \( \delta \geq \delta(z) \). Thus, the incumbent’s expected profit in the area is given by

\[
\hat{\pi}_1(z) = \int_{1-\sigma}^{\delta(z)} (\delta \pi^m - c(z)) \frac{d\delta}{2\sigma} + \int_{\delta(z)}^{1+\sigma} (\delta \pi^d - c(z)/2) \frac{d\delta}{2\sigma}.
\]

In the low states of demand \( (\delta < \delta(z)) \), the entrant does not co-invest and the incumbent enjoys a monopoly position, whereas in the high states of demand \( (\delta \geq \delta(z)) \), the entrant co-invests and the firms compete in the retail market.
The incumbent’s expected profit can be rewritten as

\[
\hat{\pi}_1(z) = \pi^m - c(z) - p^e(z) \left[ \delta^e(z) \left( \pi^m - \pi^d \right) - \frac{c(z)}{2} \right].
\]  

(2)

If \( z \leq \bar{z} \), we have \( \hat{\delta}(z) = 1 - \sigma \) and hence, \( \hat{\pi}_1(z) = \pi^d - c(z)/2 \geq 0 \), where the inequality comes from the fact that \( z \leq \bar{z} \leq \bar{z}^c \). If \( z \geq \bar{z} \), we have \( \hat{\delta}(z) = 1 + \sigma \) and \( \hat{\pi}_1(z) = \pi^m - c(z) \), which is positive if and only if \( z \leq \bar{z}^m \).

For \( z \in (\bar{z}, \bar{z}) \), the last term in (2) represents the externality from firm \( e \)'s possible entry on firm \( 1 \)'s profit. Entry occurs with probability \( p^e(z) \). If there is entry, the externality on firm \( 1 \)'s profit is equal to the lost profits in the high-demand states \( \delta^e(z)(\pi^m - \pi^d) \) less the cost savings from co-investment \( c(z)/2 \). Thus, while entry has a negative effect on the incumbent’s profit in low-cost areas, it may have a positive effect in high-cost areas due to large cost savings. This implies that in these areas, firm \( 1 \)'s profit may not always be decreasing with \( z \).

The overall equilibrium arises from considering the monopolistic profit \( \pi^m \) that the incumbent obtains in absence of entry and the negative externality that entry generates on the incumbent. The following proposition characterizes the equilibrium.

**Proposition 2** The incumbent’s profit-maximizing coverage with ex-post co-investment is given by the following conditions:

1. If \( \pi^m < \left( 2 + \frac{\sigma}{2 - \sigma} \right) \pi^d \), the incumbent invests in the areas \([0, \bar{z}^o]\), with \( \bar{z}^o \in (\bar{z}, \bar{z}^c) \).
2. If \( \left( 2 + \frac{\sigma}{2 - \sigma} \right) \pi^d \leq \pi^m < 2 (1 + \sigma) \pi^d \), the incumbent invests in the areas \([0, \bar{z}^o]\), with \( \bar{z}^o \in [\bar{z}^c, \bar{z}^m) \).
3. If \( \pi^m \geq 2 (1 + \sigma) \pi^d \), the incumbent invests in the areas \([0, \bar{z}^o]\), with \( \bar{z}^o = \bar{z}^m \).

The entrant’s probability of entry via co-investment in an area \( z \) covered by the incumbent is given by (1).
Proof. See the Appendix.

The incumbent’s investment decision depends on the degree of competition. When fiber-based services offered by the firms are more differentiated, the equilibrium reported in condition 1 of Proposition 2 is more likely to emerge, while when fiber-based services are less differentiated, the other conditions are more likely to hold.

We can now compare the equilibrium outcome described in Proposition 2 to the equilibrium in the benchmark given in Proposition 1.

Proposition 3 When the entrant can wait for demand to be realized, the incumbent covers fewer areas than in the ex-ante benchmark if $\pi^m < 2 (1 + \sigma) \pi^d$. The entrant co-invests less often in the areas with intermediate costs and more often in the more costly areas.

Proof. Immediate.

When it co-invests ex post, the entrant can observe the realized demand in each local area and cherry-pick which areas to enter. This creates two distortions compared to the ex-ante benchmark. First, there is a market structure distortion. As shown in Lemma 1, allowing for ex-post co-investment decreases the probability of entry in benchmark co-investment areas and increases the probability of entry in benchmark monopoly areas. Second, there is a total coverage distortion. Since there is more entry in monopoly areas, the incumbent’s investment incentives are reduced, and total coverage is lower than in the benchmark unless the degree of demand uncertainty is sufficiently low.

The incumbent fares worse compared to the benchmark, as co-investment is less likely to occur in the bad states of demand and there is more entry in the good states. Conversely, the entrant obtains a higher profit if it can wait for demand to realize.

In the benchmark with ex-ante co-investment, the incumbent makes (at least weakly) more profit than the entrant; this is because the two firms make the same profits in the co-investment areas and the incumbent can earn additional profits in the monopoly areas.
Can *ex-post* co-investment hurt the incumbent and benefit the entrant to such an extent that the entrant obtains a higher expected profit than the incumbent? With an example, we show that this can be the case. Consider the realistic short-term case that fiber-based services are relatively homogeneous, i.e. \( \pi^m > 2\pi^d \). As a numerical example, assume that \( \pi^m = 2.5, \pi^d = 1, c(z) = z \) and \( \sigma \in [0, 1] \). The incumbent’s and the entrant’s total expected profits are given by

\[
\hat{\Pi}_i = \int_0^{z^o} \hat{\pi}_i(z)dz,
\]

for \( i = 1, e \), where \( z^o \) is the equilibrium coverage defined in Proposition 2. With our specific assumptions for \( \pi^m, \pi^d \) and \( c(z) \), we find that \( \hat{\Pi}_1(\sigma) > \hat{\Pi}_e(\sigma) \) for \( \sigma \in (0, 0.46) \) and \( \hat{\Pi}_1(\sigma) < \hat{\Pi}_e(\sigma) \) for \( \sigma > 0.46 \). When the degree of demand uncertainty \( \sigma \) increases, the magnitude of the market structure and total coverage distortions becomes larger; the incumbent is thus hurt more by *ex-post* co-investment, while the entrant benefits more. If the degree of uncertainty is sufficiently high, the entrant obtains a higher total expected profit than the incumbent by waiting.

In our framework, we assumed that firm 1 is the investor and that firm \( e \) can only enter via co-investment. Now, consider the case where who is the first investor and who is the co-investor is determined endogenously through a timing game. If the degree of uncertainty is low, each firm would prefer to be the first investor as it makes higher profits than the co-investor (\( \hat{\Pi}_1(\sigma) > \hat{\Pi}_e(\sigma) \)), and we would obtain a preemption game. By contrast, if the degree of uncertainty is high, each firm would prefer to let the other firm invest first and co-invest later (as \( \hat{\Pi}_1(\sigma) < \hat{\Pi}_e(\sigma) \)). Firms play a waiting game instead, delaying any investment. This delay is caused exclusively by allowing later co-investment.

The impact of *ex-post* co-investment on welfare, aggregated over all areas, compared to the benchmark, is *a priori* ambiguous. On the one hand, welfare is reduced because there is less entry in co-investment areas and possibly lower total coverage. On the other hand, there is more entry in monopoly areas (when they exist), which improves welfare. However, we can show the following result when the investment cost is linear.
Proposition 4 Assume that $c(z) = kz$. If the total coverage distortion is sufficiently strong, i.e., $z^o$ sufficiently low, ex-post co-investment decreases welfare compared to the ex-ante benchmark. Otherwise, it increases welfare.

Proof. See the Appendix.

Proposition 4 shows that allowing for ex-post co-investment may be welfare-increasing, when the distortion that it entails, in terms of total coverage, is sufficiently small. However, when ex-post co-investment leads to strong market structure and total coverage distortions, the possibility for entrants to wait before co-investing harms welfare.

With a specific example, we show that ex-post co-investment harms welfare in particular if there is sufficiently strong demand uncertainty. We adopt the demand specification from Singh and Vives (1984), with local quantity competition. The inverse demand for firm $i = 1, e$ is given by $p_i = \alpha - q_i - \gamma q_j$, where $\alpha > 0$ and $\gamma \in [0, 1]$. The marginal cost is set to 0 for both firms. We calibrate the values of $\alpha$ and $\gamma$ such that $\pi^m = 2.5$ and $\pi^d = 1$ as in the previous example, i.e. we assume that the degree of service differentiation is low; we obtain $\alpha = 3.16$ and $\gamma = 0.72$. Total expected welfare is given by

$$W = \int_0^{z^o} \left[ w^m - c(z) - p^e(z)\delta^e(z)(w^m - w^d) \right] dz,$$

with $w^d = 5.54$ and $w^m = 3.75$. In this numerical example, total welfare is higher with ex-post co-investment for $\sigma \in (0, 0.26)$ and lower for $\sigma \in (0.26, 1)$.

In the rest of this paper, we focus on situations where ex-post co-investment harms welfare, and study possible ex-ante or ex-post remedies to reduce or eliminate the adverse effects of waiting.

4 Remedies

In this section, we study potential remedies which could be applied either ex ante or ex post to fix the market structure and total coverage distortions arising with ex-post co-investment,
and therefore, improve welfare. First, we analyze an *ex-ante* remedy, whereby the potential entrant would be required to purchase *ex ante* a co-investment option to be able to co-invest *ex post*. Second, we consider a risk premium, an *ex-post* remedy that has been envisaged by regulatory authorities.

### 4.1 Co-investment options

We first consider the case where the incumbent and the entrant can enter *ex ante* into an option contract for co-investment, which can be enforced *ex post*. More specifically, if the entrant wants to co-invest *ex post* rather than *ex ante* in a given area, it must make a payment to the incumbent *ex ante*, conditional on the incumbent actually investing in the area. Thus, instead of co-investing *ex ante*, the entrant buys an option, which it can later use (by co-investing via the payment of \( c(z)/2 \)) or not (by staying out).

#### Market structure distortion

As we have seen, with *ex-post* co-investment the entrant is less likely to enter co-investment areas and more likely to enter monopoly areas, compared to the benchmark. Whereas the latter distortion is actually welfare-enhancing, the former harms welfare. Therefore, it would be welfare improving to increase entry in co-investment areas. However, a co-investment option, once bought, has no effect on later entry, because it does not change the *ex-post* co-investment incentives of the entrant. To correct this inefficiency, we would need to subsidize the entrant; we will discuss this possibility in the subsection on the risk premium below.

#### Total coverage distortion

We now study to which extent co-investment options can correct the total coverage distortion. First, we assume low monopoly profits, i.e., \( \pi^m < 2\pi^d \). In this case, in the benchmark, there is co-investment in the areas \( z \leq z^c \), but there are no monopoly areas. If the entrant can wait for demand to realize, a total coverage distortion arises, as the incumbent covers only
the areas \([0, \bar{z}^0]\), with \(\bar{z}^0 < \bar{z}^c\). We look at whether, in this case, selling a co-investment option can restore total coverage up to the benchmark level \(\bar{z}^c\).

Consider the areas where investment does not occur with \textit{ex-post} co-investment, whereas it does in the \textit{ex-ante} benchmark, i.e., areas \(z \in (\bar{z}^0, \bar{z}^c)\), where \(\hat{\pi}_1(z) < 0\) but \(\pi_1(z) \geq 0\).

The expected value for the entrant of a co-investment option in such an area, conditional on the incumbent covering the area, is equal to the difference in expected profits for the entrant between the two cases with \textit{ex-post} and \textit{ex-ante} co-investment:

\[
V(z) = \hat{\pi}_e(z) - \pi_e(z) = p^e(z) \left[ \delta^e(z)\pi^d - \frac{c(z)}{2} \right] - \left[ \pi^d - \frac{c(z)}{2} \right],
\]

which simplifies to \(V(z) = (1 - p^e(z))^2 \sigma \pi^d \geq 0\).

By exercising the co-investment option, the entrant gains by not making losses in the low-demand states. The option value \(V(z)\) is the maximum amount that the entrant is willing to pay to wait instead of co-investing \textit{ex ante}.

The \textit{ex-ante} benchmark coverage can be reestablished if the incumbent’s losses when the option is exercised and co-investment occurs \textit{ex post}, can be recovered via the maximum price of the option, i.e., if \(V(z) \geq -\hat{\pi}_1(z)\), or

\[
\left( \pi^d - \frac{c(z)}{2} \right) + \left( \pi^m - 2\pi^d \right) (1 - p^e(z)) \delta^a(z) \geq 0.
\]

The first term is positive for \(z \leq \bar{z}^c\) and equal to zero at \(z = \bar{z}^c\), whereas the second term is strictly negative, as \(\pi^m < 2\pi^d\) and \(p^e(z) < 1\) for \(z \in [\bar{z}^0, \bar{z}^c]\). Therefore, this condition does not hold for \(z\) close to \(\bar{z}^c\). So, it is not possible to design a co-investment option that fully restores the incumbent’s infrastructure coverage in this case.

Now, consider the other case, where \(\pi^m > 2\pi^d\). In the benchmark, there are monopoly areas \(z \in [\bar{z}^c, \bar{z}^m]\), where the incumbent invests but the entrant does not co-invest. With \textit{ex-post} co-investment, the total coverage distortion arises if \(\pi^m < 2(1 + \sigma)\pi^d\), in which case the incumbent covers only the areas \([0, \bar{z}^o]\), with \(\bar{z}^o < \bar{z}^m\).
The value of an option for the entrant in an area \( z \in (\bar{z}_o, \bar{z}_m) \) corresponds to the gains from co-investing when demand turns out to be high, while \textit{ex ante} it would not co-invest:

\[
V(z) = \hat{\pi}_e(z) = p^e(z) \left[ \delta^e(z) \pi^d - \frac{c(z)}{2} \right],
\]

which simplifies to \( V(z) = (p^e(z))^2 \sigma \pi^d \geq 0 \).

The incumbent’s \textit{ex-ante} coverage incentives can be reestablished by selling a co-investment option if \( V(z) \geq -\hat{\pi}_1(z) \), that is, if

\[
(\pi^m - c(z)) - (\pi^m - 2\pi^d) p^e(z) \delta^e(z) \geq 0.
\]

The first term is always positive and equal to zero at \( z = \bar{z}_m \), whereas the second term is strictly negative for \( 2\pi^d < \pi^m < 2(1 + \sigma) \pi^d \) (the latter implying that \( p^e(z) > 0 \)). Thus, as \( z \) approaches \( \bar{z}_m \), this condition is not satisfied. Therefore, the co-investment option cannot restore the incumbent’s investment incentives in the most outlying areas.

The following proposition summarizes our findings:

**Proposition 5** Selling a co-investment option:

- \textit{does not change the entrant’s ex-post co-investment choices in areas covered by the incumbent;}

- \textit{increases coverage in ex-ante co-investment areas and ex-ante monopoly areas, but cannot fully reestablish benchmark coverage.}

Because the option is bought \textit{ex-ante}, it does not affect the entrant’s co-investment incentives \textit{ex-post}, and therefore it cannot fix the market structure distortion. Furthermore, the entrant’s \textit{ex-post} opportunism dissipates industry profits and makes it impossible to design options that fully solve the total coverage distortion.
4.2 Risk premia

In regulatory discussions, it has been proposed that late co-investors should be charged a “risk premium” on top of the co-investment fee paid by early co-investors, in order not to reward opportunistic behavior.\(^9\) In this subsection, we study this remedy, and determine which lump-sum risk premium, defined \textit{ex ante}, should be applied \textit{ex post} to the late co-investor in order to achieve the benchmark outcome.

We assume that co-investment takes place \textit{ex post}. In a given area \(z\) covered by the incumbent, after demand has been observed, the entrant decides whether to co-invest.\(^10\) If it does, and only then, it has to pay to the incumbent, apart from half of the investment cost, an additional fee, \(R(z)\). Here, we include the possibility that the entrant receives a subsidy (i.e., a negative fee) from the incumbent.\(^11\) A risk premium \(R(z) > 0\) aims to encourage the incumbent to invest in the area, while a subsidy \(R(z) < 0\) aims to make the entrant co-invest. The payment is set by the regulator and announced before investment and co-investment decisions are taken. We assume that it depends on the area, but cannot be made conditional on the level of demand.

Investment and co-investment decisions

We study how the premium affects the entrant’s \textit{ex-post} co-investment decision, and then, in turn, the incumbent’s investment decision.

\textbf{Entrant’s decision.} In an area \(z\) covered by the incumbent, with realized demand \(\delta\), the entrant enters via co-investment if and only if \(\delta \pi^d \geq R(z) + c(z)/2\), where \(R(z)\) represents the \textit{ex-post} premium for the area, that is, if

\[
\delta \geq \delta_R(z) \equiv \min\{1 + \sigma, \max\{1 - \sigma, (R(z) + c(z)/2) / \pi^d\}\},
\]

\(^9\) A risk premium for late co-investment has already been implemented in France.

\(^{10}\) We maintain the assumption that the entrant can decide whether to co-invest in each single area.

\(^{11}\) Of course, the government could also subsidize the entrant’s co-investment, but this is not our focus here.
where $\delta_R(z)$ is bounded from below by $1 - \sigma$ and from above by $1 + \sigma$. For $z \in [\hat{z}, \bar{z}]$, we have $\hat{\delta}_R(z) > \hat{\delta}(z)$ if $R(z) > 0$, and $\hat{\delta}_R(z) < \hat{\delta}(z)$ if $R(z) < 0$.

The *ex-ante* probability that the entrant enters area $z$ is then

$$p^e_R(z) = \int_{\hat{\delta}_R(z)}^{1+\sigma} \frac{d\delta}{2\sigma} = \frac{1 + \sigma - \hat{\delta}_R(z)}{2\sigma}.$$ 

Compared to the benchmark, the premium reduces the probability of entry in area $z$ if it is positive, whereas it increases the probability of entry if it is negative, i.e., if it is a subsidy. This is because, in contrast to the option price that is paid *ex-ante*, the premium is paid *ex-post*, and therefore affects the co-investment decision of the entrant.

We define $\delta^e_R(z) \equiv (1 + \sigma + \hat{\delta}_R(z))/2$ as the expected level of demand conditional on entry. Since the threshold level of demand that allows entry, $\hat{\delta}_R(z)$, is increasing in $R(z)$, $\delta^e_R$ is also increasing in $R(z)$: as the premium reduces the probability of entry, entry takes place in higher states of demand on average.

**Incumbent’s decision.** Since the entrant co-invests in area $z$ if $\delta \geq \hat{\delta}_R(z)$, the incumbent’s expected profit is

$$\hat{\pi}_1(z, R(z)) = \int_{1-\sigma}^{\hat{\delta}_R(z)} (\delta \pi^m - c(z)) \frac{d\delta}{2\sigma} + \int_{\hat{\delta}_R(z)}^{1+\sigma} (\delta \pi^d - c(z)/2) \frac{d\delta}{2\sigma} + p^e_R(z)R(z)$$

$$= \pi^m - c(z) - p^e_R(z) \left[ \delta^e_R(z) \left( \pi^m - \pi^d \right) - \frac{c(z)}{2} - R(z) \right],$$

with $\hat{\pi}_1(z, 0) = \hat{\pi}_1(z)$. The premium $R(z) > 0$ has a direct positive effect on the incumbent’s expected profits, as a direct source of profit conditional on entry. However, the premium also has indirect negative effects, which work through the probability of entry $p^e_R(z)$, which is lower with a higher premium, and the expected level of demand conditional on entry, $\delta^e_R(z)$, which increases with the premium. Because of these contradictory direct and indirect effects, the impact of a higher premium on the incumbent’s profit is *a priori* ambiguous.

The following technical lemma states the effect of the premium on the incumbent’s ex-
Lemma 2: The premium $R(z)$ has the following effects on the incumbent’s expected profits:

1. If $\pi^m \geq 2\pi^d$, profits $\hat{\pi}_1(z, R)$ are strictly increasing in $R$, with maximum value $\pi^m - c(z)$ when $p_R'(z) = 0$.

2. If $\frac{2-4\sigma}{1-\sigma} \pi^d \leq \pi^m < 2\pi^d$, profits $\hat{\pi}_1(z, R)$ are maximized at $\hat{R}(z) = \frac{1}{3\pi^d - \pi^m} - c(z) / 2$, with maximum value

   $$\hat{\pi}_1(z, \hat{R}(z)) = \pi^m - c(z) + \frac{(1 + \sigma)^2 (2\pi^d - \pi^m)^2}{4\sigma (3\pi^d - \pi^m)}.$$  

3. If $\pi^d < \pi^m < \frac{2-4\sigma}{1-\sigma} \pi^d$, profits $\hat{\pi}_1(z, R)$ are maximized when $p_R'(z) = 1$, with maximum value $(2 - \sigma) \pi^d - c(z)$.

Proof. See Appendix.

Lemma 2 shows that the incumbent’s expected profits are not monotonically increasing in the premium. Therefore, setting a higher premium will not always improve firm 1’s investment incentives.

We can now discuss the impact of the premium on market structure and total coverage.

Market structure distortion

Compared to the ex-ante benchmark, ex-post co-investment distorts the market structure in two ways: (i) there is more entry in monopoly areas when they exist, which happens if $\pi^m > 2\pi^d$, and (ii) there is less entry in co-investment areas.

For given coverage levels, the first distortion is actually welfare-enhancing and there is no need to correct this via a premium.

By contrast, the second distortion, which occurs because the entrant co-invests ex-post only when demand is high ($\delta \geq \hat{\delta}(z)$), harms welfare. If we wish to correct this distortion and
replicate the *ex-ante* outcome of certain co-investment, the entrant must be paid a subsidy equal to its losses at the lowest demand state, that is, we should set:

\[ R(z) = (1 - \sigma)\pi^d - c(z)/2 \leq 0. \] (5)

Since this subsidy is financed by the incumbent, it cannot exceed the latter’s expected profits if investment is to occur, i.e., we must have \( \pi^d - c(z)/2 + R(z) \geq 0 \) or \((2 - \sigma)\pi^d - c(z) \geq 0\).

This condition holds at \( z = \bar{z} \), since \( c(\bar{z}) = 2(1 - \sigma)\pi^d \), but not at \( z = \bar{z}^c \), where \( c(\bar{z}^c) = 2\pi^d \).

Thus, for areas close to \( \bar{z}^c \) the subsidy necessary to make the entrant co-invest *ex-post* with probability 1 prevents the incumbent from investing in the first place. This means that we cannot totally correct the market structure distortion with a subsidy paid by the incumbent to the entrant.

**Total coverage distortion**

A distortion in total coverage arises when \( \pi^m < 2(1 + \sigma)\pi^d \), in which case total coverage is lower with *ex-post* co-investment than in the benchmark. To determine whether a premium can correct this distortion, we use Lemma 2 and check whether a premium can restore investment incentives while maintaining entry.

The following proposition summarizes our findings:

**Proposition 6** *Area-specific co-investment risk premia paid by the entrant to the incumbent:*

- reduce the probability of entry in the areas covered by the incumbent;
- reestablish coverage in some but not all *ex-ante* co-investment areas if \( \pi^m < 2\pi^d \);
- reestablish coverage in (almost) all *ex-ante* monopoly areas if \( \pi^m \geq 2\pi^d \).

**Proof.** See the Appendix. ■

One notable result is that while payments from the entrant to the incumbent are needed when fiber-based services are sufficiently homogeneous (i.e., \( \pi^m \geq 2\pi^d \)), as is more likely in
the short term, a subsidy to the entrant may be needed to raise coverage when services are very differentiated (i.e., $\pi^m < 2\pi^d$), as may happen in the near future with the development of new Internet of Things services. This is due to a coordination failure under *ex-post* co-investment: Duopoly profits in this case are more than enough to cover investment costs *ex ante*, but the incumbent knows that the entrant will not always enter.

### 4.3 Combining *ex-ante* and *ex-post* remedies

As we have seen, introducing co-investment options (an *ex-ante* remedy) or a premium (an *ex-post* remedy) can partially, but not fully, restore coverage in the *ex-ante* benchmark. In this sub-section, we discuss the possibility of combining both remedies, that is, to implement co-investment options for some areas and a premium for others.

Remember that the market outcome under *ex-post* co-investment is inefficient compared to the benchmark due to two distortions: a market structure distortion and a total coverage distortion.

The market structure distortion means that, compared to the benchmark, there is more entry in monopoly areas $z \geq \bar{z}_c$ and less entry in co-investment areas $z < \bar{z}_c$. We want to correct the latter distortion but not the former, since it is welfare-enhancing. As we have seen, the only way to increase entry is to use subsidies (a negative premium).

By contrast, there are two ways to correct the total coverage distortion, using either options or premia. Options allow increasing investment incentives without affecting entry, whereas premia do so by decreasing entry.

Let us be more specific by focusing on the case where there are *ex-ante* monopoly areas (i.e., $\pi^m > 2\pi^d$) and a total coverage distortion (i.e., $\pi^m < 2(1+\sigma)\pi^d$). Then, total coverage under *ex-post* co-investment, $\bar{z}^0$, lies between $\bar{z}_c$ and $\bar{z}_m$, which corresponds to Case 2 in Proposition 2. Let us also assume that $c(z) = z$.

First, consider the market structure distortion in the areas $[\hat{z}, \bar{z}_c]$. Let $\hat{z} = c^{-1}((2 - \sigma)\pi^d) = (2 - \sigma)\pi^d$. The incumbent makes a positive profit with the premium given by (5)
if and only if $z \leq \hat{z}$. Therefore, in the areas $z \in [\hat{z}, \check{z}]$, it is possible to replicate the \textit{ex-ante} benchmark with a probability 1 of entry, without undermining investment, by setting the subsidy given by (5). By contrast, in the areas $z \in [\check{z}, \check{z}^c]$, entry can only be partially replicated by setting the maximum subsidy such that the incumbent has an incentive to cover the area, i.e., $\hat{\pi}_1(z, R(z)) = 0$.

Let us now consider the total coverage distortion. With \textit{ex-post} co-investment, the incumbent covers up to the area $\bar{z}_o \in [\check{z}^c, \bar{z}_m)$. Since options do not distort entry, whereas the premium does, the idea is to use options to restore coverage for the largest set of areas as possible. The incumbent’s profit in a given area $z$, including the price of the option, i.e., $\hat{\pi}_1(z) + V(z)$, which is given by (3), is decreasing in $z$, positive at $\bar{z}_o$ and negative at $\bar{z}_m$. Thus, there is exists a unique $\hat{z} \in (\bar{z}_o, \bar{z}_m)$ such that $\hat{\pi}_1(z) + V(z) \geq 0$ if and only if $z \leq \hat{z}$. Therefore, in the areas $[\bar{z}_o, \hat{z}]$, we can restore coverage using options, without affecting entry.

In the areas $[\hat{z}, \bar{z}_m]$, we can use a premium to restore coverage up the \textit{ex-ante} marginal area, $\bar{z}_m$, without cutting off entry, as shown by our analysis in Sub-Section 4.3.

Thus, by combining \textit{ex-ante} and \textit{ex-post} remedies in different areas, total coverage is fully restored. The market structure in co-investment areas is partially, but not fully, replicated. There is still more entry in monopoly areas than in the benchmark.

As an illustration, we use the same numerical example as in Section 3, with $\pi^m = 2.5$, $\pi^d = 1$, $c(z) = z$, and we also set $\sigma = 0.4$. Hence, we consider the current conditions in the fiber-based market where services are not very differentiated. Figure 1 below shows the probability of entry in the benchmark (in black), with \textit{ex-post} co-investment (in red), and when the optimal combination of \textit{ex-ante} and \textit{ex-post} remedies is implemented (in blue). In the areas $z \in [\hat{z}, \bar{z}_c]$, we introduce a subsidy (i.e., a negative premium) to correct the market structure distortion and increase the probability of entry. In the areas $z \in [\bar{z}_o, \hat{z}]$, we implement options to restore coverage without affecting entry, and in the areas $z \in [\hat{z}, \bar{z}_m]$ a premium, which ensures investment, but with a lower probability of entry.

To compare the three scenarios in terms of total welfare, we use the same Singh and
Figure 1: Probability of entry under three scenarios.

Vives (1984) demand specification as in Section 3, and the same numerical example where the local welfare in duopoly and monopoly areas are $w^d = 5.54$ and $w^m = 3.75$, respectively. We find that total welfare over all areas is $W^b = 9.83$ in the \textit{ex-ante} benchmark, $W^{\text{ex-post}} = 9.39$ with \textit{ex-post} co-investment, and $W^{\text{remedies}} = 10.06$ with the combination of remedies. Therefore, introducing a combination of \textit{ex-ante} and \textit{ex-post} remedies increases total welfare, compared to the outcome under \textit{ex-post} co-investment. It even leads to higher welfare than in the benchmark, because the possibility of \textit{ex-post} co-investment stimulates entry in monopoly areas.

5 Conclusions

In this paper, we have studied the impact of co-investment on the roll-out of network infrastructures under demand uncertainty. In line with the new EU regulatory framework, we assume that the policy maker’s main goal is to expand fiber connections. We therefore
assess how new regulatory tools for co-investment could favor fiber-based network expansion. Compared to a benchmark where the incumbent and the entrant both take their (co-)investment decisions under uncertainty, we have shown that if the entrant can wait to co-invest until after the level of demand has become known, it can opportunistically cherry-pick which areas to enter. Two distortions then arise. First, there is less entry in *ex-ante* co-investment areas and more entry in *ex-ante* monopoly areas (market structure distortion). Second, the incumbent’s investment incentives are reduced, and total coverage tends to be lower than in the benchmark (total coverage distortion). If these distortions are sufficiently strong, total welfare is lower with *ex-post* co-investment.

We considered two possible remedies to correct these distortions and improve welfare when co-investment takes place *ex post*: (i) a co-investment option purchased *ex ante* by the entrant from the incumbent, and (ii) a risk premium paid *ex post* by the entrant. We showed that the co-investment option does not affect entry, and that it cannot fully reestablish total coverage. By contrast, a negative premium (i.e., a subsidy from the incumbent to the entrant) can partially correct the market structure distortion, and a positive premium can also reestablish total coverage in most cases, but at the cost of less entry. Finally, we discussed the possibility to combine the use of options and risk premia in different areas, and showed through a numerical example that it can improve welfare compared to both the *ex-ante* and *ex-post* co-investment outcomes.

From a policy perspective, our findings show that options can be complementary to a risk premium to cope with the opportunism induced by allowing late co-investment. Our results suggest that they would constitute a relevant remedy in areas with intermediate costs where the risk of opportunistic entry may discourage investment.

In our analysis, we considered only one potential entrant. It would be interesting to extend our framework to multiple entrants and study the mechanism through which co-investment options are sold in this case, as well as their potential foreclosure effects among co-investors and access seekers. One could also consider a market for the trade of co-investment
options. We leave these topics to future research.

References


Appendix

Proof of Proposition 2.

First, note that the profit function of firm 1 is continuous in \( z \geq 0 \). For \( z \in [0, \bar{z}] \), \( \hat{\pi}_1(z) = \pi^d - c(z)/2 \) is strictly decreasing in \( z \), with \( \hat{\pi}_1(\bar{z}) = \sigma \pi^d > 0 \). Thus, the incumbent will at least cover all the areas \([0, \bar{z}]\). For \( z \geq \bar{z} \), \( \hat{\pi}_1(z) = \pi^m - c(z) \) is strictly decreasing from \( \hat{\pi}_1(\bar{z}) = \pi^m - 2(1 + \sigma)\pi^d \), and the incumbent covers the areas \([\bar{z}, \bar{z}_m] \) if \( \bar{z}_m \geq \bar{z} \Leftrightarrow \hat{\pi}_1(\bar{z}) \geq 0 \). Finally, for \( z \in (\bar{z}, \bar{z}) \) the derivative of profits is

\[
\frac{d\hat{\pi}_1(z)}{dz} = \left[ (1 - 3\sigma) \pi^d + \delta(z) (\pi^m - 3\pi^d) \right] \frac{\delta'(z)}{2\sigma},
\]

where \( \delta'(z) > 0 \). Therefore, the derivative of \( \hat{\pi}_1(z) \) has the sign of the term in brackets.

For \( \pi^m \leq 3\pi^d \), the term in brackets is weakly decreasing in \( z \in [\bar{z}, \bar{z}] \), which implies that \( \hat{\pi}_1(z) \) is either first increasing then decreasing, or always increasing or decreasing. By contrast, for \( \pi^m > 3\pi^d \), this term is increasing in \( z \), which means that \( \hat{\pi}_1(z) \) is either always increasing or decreasing, or first decreasing and then increasing, in which case it has a local minimum at \( \hat{z} \) with \( \delta(\hat{z}) = \frac{3\pi^d - 1}{\pi^m/\pi^d - 3} \). The latter case occurs if and only if \( \bar{z} \leq \hat{z} \leq \bar{z} \), or equivalently \( \frac{2 + 6\sigma}{1 + \sigma} \leq \pi^m/\pi^d \leq \frac{2}{1 - \sigma} \), which implies that \( \hat{\pi}_1(\hat{z}) \geq 0 \) and therefore also that \( \hat{\pi}_1(z) > 0 \) for all \( z \in [\bar{z}, \bar{z}] \).

Thus, for all values of \( \pi^m \) there will be a unique \( \bar{z}^o > \bar{z} \) such that \( \hat{\pi}_1(\bar{z}^o) = 0 \), and we have \( \hat{\pi}_1(\bar{z}^o) > 0 \) for \( z < \bar{z}^o \) and \( \hat{\pi}_1(\bar{z}^o) < 0 \) for \( z > \bar{z}^o \). The incumbent covers the areas \([0, \bar{z}^o] \), where \( \bar{z}^o \) is defined by

\[
c(\bar{z}^o) = \frac{2\pi^d}{3\pi^d - \pi^m} \left( (1 - 3\sigma) \pi^d + \sqrt{(1 - \sigma)^2 (2\pi^d - \pi^m)^2 + 4\sigma^2 \pi^d (2 (1 + \sigma) \pi^d - \pi^m)} \right).
\]

For \( \pi^m < \frac{4 - \sigma}{2\pi - \sigma} \pi^d \), we have \( \hat{\pi}_1(\bar{z}^c) = \frac{1}{4} (2 - \sigma) \pi^m - \frac{1}{4} (4 - \sigma) \pi^d < 0 \), thus \( \bar{z}^o < \bar{z}^c \). For \( \frac{4 - \sigma}{2\pi - \sigma} \pi^d < \pi^m < 2(1 + \sigma) \pi^d \), we have \( \hat{\pi}_1(\bar{z}) < \hat{\pi}_1(\bar{z}^m) < 0 < \hat{\pi}_1(\bar{z}^c) \), with \( \bar{z}^c < \bar{z}^o < \bar{z}^m < \bar{z} \), while for \( \pi^m \geq 2(1 + \sigma) \pi^d \), we have \( \hat{\pi}_1(\bar{z}^m) = 0 \), with \( \bar{z}^o = \bar{z}^m \geq \bar{z} \). ■
Proof of Proposition 4.

First, note that if \( \bar{z}^o < \bar{z}^c \), which corresponds to Case 1 in Proposition 2, aggregate welfare is always lower under \textit{ex-post} co-investment than in the benchmark. Indeed, \textit{ex-post} co-investment has no positive effect on welfare through entry in monopoly areas, only negative effects due to less entry in co-investment areas and lower total coverage.

Second, consider Case 3 in Proposition 2, where \( \bar{z}^o = \bar{z}^m \). The difference in aggregate welfare between \textit{ex-post} and \textit{ex-ante} co-investment is given by

\[
\hat{W} - W = \int_0^{\bar{z}^c} \left[ \hat{w}(z) - w^d \right] dz + \int_{\bar{z}^c}^{\bar{z}^m} \left[ \hat{w}(z) - w^m \right] dz,
\]

where \( \hat{w}(z) = w^m - p^e(z)\delta^e(z)(w^m - w^d) \) and \( \hat{w}(z) \in (w^m, w^d) \). The sign of \( \hat{W} - W \) is a priori ambiguous. However, when \( c(z) = kz \), this expression simplifies to

\[
\hat{W} - W = \frac{2}{3k} (w^d - w^m) \pi^d \sigma^2 > 0.
\]

Starting from \( \bar{z}^o = \bar{z}^c \), where \( \hat{W} < W \), and increasing \( \bar{z}^o \), leads to a higher welfare, as total coverage increases and there is (more) entry in monopoly areas. When \( \bar{z}^o = \bar{z}^m \), we have \( \hat{W} > W \) as shown above. Therefore, by continuity, if \( \bar{z}^o \) is low enough, we have \( \hat{W} < W \); otherwise, we have \( \hat{W} > W \). ■

Proof of Lemma 2.

We have

\[
\frac{\partial \hat{\pi}_1}{\partial R} = \frac{(1 + \sigma)\pi^d + (\pi^m - 3\pi^d) \delta_R(z)}{2\sigma\pi^d},
\]
and $\partial^2 \hat{\pi}_1 / \partial R^2 \leq 0$ (i.e., profits are concave in $R$) if and only if $\pi^m \leq 3\pi^d$. If $\pi^m > 2\pi^d$, profits are increasing in $R$ since 

$$\frac{\partial \hat{\pi}_1}{\partial R} \geq \frac{(1 + \sigma) - \delta_R(z)}{2\sigma} \geq 0,$$

as $\delta_R(z) \leq 1 + \sigma$. Firm 1’s expected profit is thus maximized when $\delta_R(z) = 1 + \sigma$, i.e., $p^e_R(z) = 0$, where $\hat{\pi}_1 = \pi^m - c(z)$.

For $\pi^m < 2\pi^d$, the candidate for an interior maximum is $\hat{R} = \frac{(\sigma + 1)(\pi^d)^2}{3\sigma^2 - \pi^m} - c(z)/2$, with expected profits

$$\hat{\pi}_1(z, \hat{R}) = \pi^m - c(z) + \frac{(\sigma + 1)^2 (2\pi^d - \pi^m)^2}{4\sigma (3\pi^d - \pi^m)}.$$

This candidate is the maximizer if $1 - \sigma \leq \left(\frac{\hat{R} + c(z)}{2}\right) / \pi^d \leq 1 + \sigma$, or $\frac{2-4\sigma}{1-\sigma} \pi^d \leq \pi^m \leq 2\pi^d$.

Finally, for $\pi^m < \frac{2-4\sigma}{1-\sigma} \pi^d$, firm 1’s profits are maximized if $(R + c(z)/2) / \pi^d = 1 - \sigma$, i.e., when $p^e_R(z) = 1$, with expected profits

$$\pi^m - c(z) - 1 \left[1 \left(\pi^m - \pi^d\right) - (1 - \sigma) \pi^d\right] = (2 - \sigma) \pi^d - c(z).$$

Proof of Proposition 6.

First, if $\pi^d < \pi^m < \frac{2-4\sigma}{1-\sigma} \pi^d$, the incumbent’s investment incentives are restored by subsidizing the entrant (i.e., with $R < 0$). The idea is that with sufficient product differentiation, total profits are increased by entry so that the incumbent can give up some profit for a subsidy and still be better off with co-investment due to the sharing of investment costs. This allows to raise coverage above $\bar{z}^o$, up to the area $\bar{z}^P$ given by $c(\bar{z}^P) = (2 - \sigma) \pi^d$. Still, since $\bar{z}^P < \bar{z}^c$, the subsidy can only partially fix the total coverage distortion.

Second, if $\frac{2-4\sigma}{1-\sigma} \pi^d < \pi^m < 2\pi^d$, total coverage can be raised up to the area $\bar{z}^P$ where...
\( \tilde{\pi}_1(\hat{z}^P, \hat{R}(\hat{z}^P)) = 0 \), that is,
\[
c(\hat{z}^P) = \pi^m + \frac{(1 + \sigma)^2 (2\pi^d - \pi^m)^2}{4\sigma (3\pi^d - \pi^m)}.
\]

For \( \pi^m < 2\left(2\frac{\sqrt{3} - 2\sigma + 2\sigma^2}{(1 - \sigma)^2}\right)\pi^d \), this involves a subsidy, while for larger values of \( \pi^m \) the entrant pays a premium to the incumbent. Still, since \( c(\hat{z}^P) < 2\pi^d = c(\hat{z}^c) \), the premium cannot reestablish coverage of all \textit{ex-ante} co-investment areas.

Third, and finally, if \( \pi^m \geq 2\pi^d \) there is no premium that allows to reestablish coverage at \( z = \hat{z}^m \) while maintaining entry. However, in all the other \textit{ex-ante} monopoly areas \( z \in (\hat{z}^0, \hat{z}^m) \) not covered under \textit{ex-post} investment, there is a premium \( R(z) \) that is high enough such that the incumbent’s expected profits become positive, since the monopoly profit can be approached arbitrarily closely if the premium is high enough, while not completely cutting off entry. Thus, coverage of all these areas can be guaranteed, though at the cost of a lower probability of entry. ■