Pre-purchase Information Acquisition and Credible Advertising*

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ABSTRACT

Consumers can decide whether to acquire more information about their valuations prior to purchase. In this paper we examine pricing and advertising strategies when consumers can engage in pre-purchase information acquisition. We show that consumer information acquisition can increase valuation heterogeneity and undermine a firm’s ability to extract consumer surplus. As a result, interestingly, a higher product quality can exert a non-monotonic impact on equilibrium information acquisition, hurt firm profitability, and lead to lower consumer surplus. We also demonstrate that pre-purchase information acquisition can be an endogenous mechanism to enable credible advertising in a cheap-talk setting. We show that quality claims in advertisements can be informative even when the firm can freely misrepresent its advertising message. Informative advertising can arise because a higher perceived quality can not only increase consumers’ expected value, but also induce more information acquisition and thus hurt the firm’s ability to extract consumer surplus. This novel explanation for the credibility of cheap-talk advertising is distinguished from those identified in the literature (e.g., matching between firm types and heterogeneous consumers, restrictive communication on multidimensional attributes). Moreover, we show that a higher quality can soften competition by inducing more information acquisition, thus benefiting the rival firm’s profitability.

Key words: advertising, cheap talk, information acquisition, pricing, strategic communication
1 Introduction

Consumers can invest time and efforts to gather relevant information before deciding whether to make a purchase. Pre-purchase information acquisition can help reduce valuation uncertainty and improve decisions. Many consumers visit physical stores to examine products (e.g., clothes, electronics, furniture, shoes, toys) that they order online subsequently (Kuksov and Liao 2018, Jing 2018). Such showrooming activity happened for 73% of people in a survey by Accenture (2013). Analogously, consumers may engage in webrooming and use the Internet to collect information for products and services (e.g., cruises, flights, hotels, restaurants) before visiting a physical/online store. For example, it is common for people to employ search engines (e.g., Google) to plan trips, learn about individual needs, compare alternatives, and so on. Many users start with generic queries (e.g., hotels), narrow down to branded keywords (e.g., Hilton Las Vegas), and may take multiple days or weeks to click around and then possibly proceed to the conversion stage.\(^1\) It is becoming prevalent that consumers read reviews/ratings in online platforms (e.g., tripadvisor.com) as the first step in their decision-making process. People may also expand the scope/source of information acquisition: they may consult friends or relatives with similar experiences, attend seminars or take courses before deciding whether to subscribe to health-related programs, study legal documents prior to home purchase, or pay for professional opinions on critical investment decisions.

Information acquisition is usually separated in time from actual purchase along the decision funnel. The growth in the number and variety of products/attributes lengthens and complicates consumers’ decision process. Nevertheless, the ubiquity of information-gathering devices (i.e., search engines, review/video platforms, social media, mobile networks) makes it unprecedentedly feasible to initiate information collection substantially before making a purchase. Therefore, it is common that consumers decide whether to search for product information without knowing the actual price they need to pay to secure a purchase. Posted prices may not be guaranteed, terms for price quotes or promotions can be ambiguous, and unexpected payments (e.g., taxes, handling fees, delivery charges) may be imposed. Similar tricks can be adopted by online sellers who shroud actual prices that are revealed only in the checkout page (Gabaix and Laibson 2006). More generally, prices are revised from time to time, especially in markets where dynamic/surge pricing is conventionally implemented (e.g., airlines, hotels, demand-sharing platforms). Online sellers such as Amazon can modify their prices several million times a day, or every several minutes for an average product (Mehta, Detroja and Agashe 2018). Recent years have also seen examples of brick-and-mortar stores removing shelve prices to enable the adoption of dynamic/surge pricing (Kristof 2017).

\(^1\)Rutz and Bucklin (2011) find that 88% Google searches for a major lodging chain involved generic keywords. Similar pattern can be seen for financial services: 76% on weekdays and 54% for weekends (Joo et al. 2014).
These observations raise important research questions. First, how should a firm price its offering when consumers may engage in pre-purchase information acquisition on product valuation? Relatedly, how would product quality influence equilibrium behavior (i.e., information acquisition and pricing), firm profit, and consumer surplus? As consumers need to gauge the benefit and cost of information acquisition, their ex post valuations would be endogenously determined. In addition, because information acquisition activities normally occur before the final purchase stage, prices would directly influence consumers’ purchase decisions but not their initial decisions on whether to acquire information. Nevertheless, when consumers make their information acquisition decisions, they need to take into account the price they expect to encounter (as well as their anticipated purchase decision). These considerations distinguish current research from the standard pricing problem where consumer preferences are exogenous.

Second, how should a firm advertise its product quality to directly influence the interaction between consumer information acquisition and firm pricing? In practice most advertisements focus on subjective quality perceptions rather than objective attributes (Abernethy and Butler 1992). It is common to observe commercials that are abundant with celebrity endorsements, inspiring stories, interesting concepts, and exquisite scenes, but contain no direct information on technical attributes. Many firms seek to enhance consumers’ subjective perceptions by making direct quality claims. For example, supermarkets may boast about the freshness of food supply, and hotels may exaggerate their services or security. However, these claims are typically unverifiable. This yields a striking puzzle: if consumers are credulous of advertising messages, firms would easily overstate their quality claims and thus render advertising uninformative, but if consumers simply disregard the content of advertisements, it is not clear why firms would want to spend significant resources into such advertising campaigns. Therefore, the critical question is whether and how plain claims in advertising can informatively and credibly convey a firm’s true quality to the consumers.

We tackle these issues in this research. We consider a simple model in which consumers are ex ante homogenous and uncertain about their product valuations in a monopoly market. The quality of the product is subjective in that it can exert a stochastically positive influence on consumer valuation, and that it cannot be verified or demonstrated. In particular, the quality of the product determines the probability that a consumer has high product valuation. A consumer can choose to incur a cost to acquire relevant product information and become informed of her idiosyncratic valuation, before deciding whether to make a purchase. As more consumers acquire information, their heterogeneity in valuation would increase and hence the firm’s ability to appropriate consumer surplus would be undermined. Conversely, as the price becomes overly low or sufficiently high, the consumers would have less incentive for information acquisition. We show that the interaction of pre-purchase information acquisition and firm pricing may lead to a unique mixed-strategy equilibrium.
Interestingly, when product quality is known to the consumers, it can exhibit a non-monotonic impact on the equilibrium proportion of consumers who choose to search for product information. As product quality increases, firm profitability may be hurt because a higher quality may lead more consumers to acquire information. Furthermore, consumer surplus can be negatively influenced by product quality, because greater quality may induce higher prices.

More importantly, we identify pre-purchase information acquisition as a novel mechanism that may yield credible advertising. When product quality is unverifiable and unknown to the consumers, the firm may misrepresent its quality claims. However, we show that there may exist conditions under which the firm reports its quality truthfully even though it can freely choose not to do so. This is because firm types of different qualities may have differential preferences over consumer information acquisition and perceived quality may influence the consumers’ incentive for information acquisition. In particular, information acquisition is relatively less harmful for a high-quality firm than for a low-quality firm, because the consumers are more likely to have high valuation for high-quality products. As a result, high-quality claims can be credible if more consumers are induced to search for product information: the low-quality type may not want to imitate the high type because that would result in more searching consumers. Conversely, the high type may prefer to shun away from the low type because mimicking would reduce the non-searching consumers’ expected value. In other words, pre-purchase information acquisition can endogenously discipline the firm’s quality claims, making advertising credible and hence informative.

We consider two directions to extend our basic model. We demonstrate that the main results are robust to continuous valuations: the mixed-strategy equilibrium continues to arise and product quality still exerts non-monotonic impacts on equilibrium behavior and welfare. We also investigate two alternative models on duopoly competition, depending on whether the consumers are informed of the additional firm’s price when deciding whether to search for information. We show that the basic model’s main insights are not restricted to monopoly markets. Moreover, we demonstrate that a firm may become better off as the rival firm’s quality increases, because competition can be softened as consumer information acquisition endogenously increases ex post differentiation.

It is important to distinguish pre-purchase information acquisition from two related concepts. The term search is typically used to represent the gathering of product information (e.g., quality, price) where purchase is infeasible without search (e.g., Wernerfelt 1994, Villas-Boas 2009, Guo and Meng 2015).2 The unique role of search is to ensure the purchase opportunity, whereas the information-gathering role is only secondary.3 Another related term is deliberation which has been

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2There are exceptions where purchase is possible without search (e.g., Kuksov and Villas-Boas 2010, Branco et al. 2012, Ke et al. 2016).

3Nevertheless, where no confusion arises, the terms “pre-purchase information acquisition,” “information acquisition,” and “search for product information” are used interchangeably throughout the paper.
used to capture cognitive and physical activities of acquiring valuation-related information at the
purchase occasion, e.g., information retrieval/processing, introspection, retrospection, anticipation,
and inspection (Guo and Zhang 2012, Guo 2016, Guo and Wu 2016). That is, deliberation refers
to the contemplation about product valuation in the “last-minute” of the purchase process where
all attributes are known (e.g., technical specifications, price). By contrast, when consumers engage
in pre-purchase information acquisition, they are not fully informed of the price.

There is a literature on the credibility of costless communication between agents (i.e., cheap
talk). Crawford and Sobel (1982) show that cheap talk can be partially credible if the extent of
sender bias is exogenously restricted. Subsequent studies identify various mechanisms to endogenize
the credibility of cheap-talk communication. Bagwell and Ramey (1993) demonstrate that costless
communication can serve as a matching mechanism when the interests of firm types and consumer
segments are aligned. In particular, high-quality firms may want to attract high-type consumers
who purchase fewer units at high margins, whereas low-quality firms may prefer to attract low-type
consumers who buy more at low prices. In addition, consumers differ in their tradeoff between
quality and price, and low-type consumers may not like high-quality firms that charge high prices.
Gardete (2013) considers misrepresentation of quality claims in vertically differentiated markets
and shows that partial informativeness can arise in a semi-separating equilibrium. More generally,
costless communication can be credible when there are multiple information receivers who differ in
their preferred type of information sender (Farrell and Gibbons 1989). Another mechanism that may
attain informativeness in cheap-talk persuasion is multidimensional communication. Chakraborty
and Harbaugh (2010, 2014) show that credibility can endogenously arise when the product involves
both horizontal and vertical characteristics.

The mechanism of informative advertising we identify in this paper departs from those in the
literature.4 We consider ex ante homogenous consumers who can purchase without information
acquisition. This removes the possibility of heterogeneous preferences among multiple information
receivers, and renders the issue of matching products to consumers irrelevant (Bagwell and Ramey
1993, Gardete 2013). In addition, in our setup the quality types are vertically differentiated and
a high-quality product is always “good news” for all consumers. Moreover, we consider only one-
dimensional uncertainty such that credibility does not result from restrictive communication as in
Chakraborty and Harbaugh (2010, 2014). Therefore, we contribute to the literature by identifying
pre-purchase information acquisition as a new mechanism for credible communication.

4Another role of cheap-talk advertising is to facilitate the coordination and the signaling among consumers
(Wernerfelt 1990). Recently, Kuksov et al. (2013) investigate the interaction between firm advertising and consumer
communication when consumer preferences for functional and self-expressive attributes are correlated and brand
identity is determined by the composition of users. See also Zhu and Dukes (2015) on how competition may influence
communication in media stances.
There are other means of strategic communication. A large stream of research work in Economics and Marketing investigate costly signaling devices under which observable actions are taken to credibly convey private information. For example, Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986) consider the role of advertising spending as a signal of product quality. The key difference between costly signaling and cheap-talk models hinges on whether the content/act of communication can directly influence the information sender’s payoffs.

Another type of strategic communication is truthful disclosure where misrepresentation is illegal or verification is feasible.\footnote{See also Guo (2009) on quality disclosure formats in distribution channels, and Guo and Zhao (2009) on how competition may influence quality disclosure.} Anderson and Renault (2006) examine optimal advertising content in disclosing horizontal attributes. The analysis is extended in Anderson and Renault (2013) to account for the disclosure of both vertical and horizontal attributes. Mayzlin and Shin (2011) show that a high-quality firm may choose strategically not to reveal its quality even though disclosure is costless. This is because consumer information acquisition and firm disclosure are substitutes (in revealing the firm’s quality) such that the former can cause strategic withholding if disclosure is restrictive due to limited bandwidth in advertising. However, our finding is qualitatively different: the firm credibly conveys its quality in equilibrium even though truthful communication is not presumed. This is because consumer information acquisition and cheap-talk advertising are complements in our model (in the sense that a higher advertised quality can induce more information acquisition about valuation) such that the former can endogenously generate advertising credibility.

2 Basic Model

2.1 Setup

We consider a market with a monopoly firm selling a single product (or service) to a unit mass of consumers. The firm’s fixed and marginal costs are, without loss of generality, normalized to zero. The product has a quality level $q \in (0, 1)$. A consumer can have either a high product valuation $v_1$, or a low valuation $v_0$, where $v_1 > v_0 > 0$. The product quality can increase the likelihood that a consumer’s valuation is high. In particular, with probability $q$ the product yields gross utility $v_1$, and with probability $1 - q$ the gross utility is $v_0$. The distribution of product valuation is identical and independent across consumers. That is, the consumers are ex ante homogenous, although their ex post valuation can be heterogeneous if some consumers choose to acquire information about their valuation. This assumption is intentionally made to rule out ex ante consumer heterogeneity as a potential driver for informative advertising. Each consumer demands at most one unit of the prod-
uct, and the utility of not buying is normalized to zero. Similar setup of product quality/valuation can be seen in the literature (e.g., Schmalensee 1982, Moorthy and Srinivasan 1995).

In the basic model we assume that the consumers know the product quality. Nevertheless, although the valuation distribution with a given quality is common knowledge, the product valuation is not a priori known. In order to learn the product valuation prior to purchase, the consumers can engage in various information acquisition activities. For example, consumers can inspect/experience the product in physical stores before deciding whether to buy it online (i.e., showroming), or browse online reviews/ratings before visiting a retail store (i.e., webrooming). Consumers can also seek advice from friends or relatives with relevant experiences, and/or consult professional opinions. A consumer can decide whether to invest in these costly information gathering activities. Only if a consumer decides to invest does she incur a cost $c > 0$ and learn her valuation for the product.\footnote{This assumption can be readily relaxed to accommodate scenarios when information acquisition does not resolve all valuation uncertainty. This can be done by re-defining $v_1$ and $v_0$ as the expected product valuations integrating over all residual uncertainty that cannot be resolved by pre-purchase information acquisition.} If a consumer decides not to search for information, she maintains her prior belief. The cost of acquiring information can represent, for instance, the disutility of spending time or cognitive/physical efforts. A fundamental feature of the model is that the consumers can still buy regardless of their information acquisition decision. Hence, the search decision affects only the consumers’ information about valuation but not their ability to buy the product.\footnote{See also Lynch and Ariely (2000) on pre-purchase information acquisition for quality-related attributes that may lead to higher valuation differentiation.}

The timing of the game is presented in Figure 1. First, the consumers independently make their decision on information search, and the firm sets the price $p$. In the second stage of the game, after observing $p$, each consumer decides whether to purchase the product, conditional on her first-stage decision and outcome of information acquisition.

Note that the consumers’ search and the firm’s pricing decisions are made simultaneously. This assumption does not mean that these two decisions are made at the same time in practice. It simply represents scenarios under which the consumers are not informed of the price the firm is setting when they decide whether to search for information, and vice versa. On the one hand, many information-acquisition activities (e.g., showroming, webrooming, keyword search) precedes the actual purchase...
occasion, whereas the actual price a consumer will encounter cannot be perfectly foreseen. Tricks for
shrouded pricing can be exercised by physical or online stores (Gabaix and Laibson 2006). Service
providers (e.g., airlines, hotels, demand-sharing systems) may adopt dynamic/surge pricing, and
online sellers such as Amazon can adjust their prices every several minutes for an individual product
(Mehta, Detroja and Agashe 2018). Retail stores can even remove shelve prices to implement
dynamic/surge pricing as in online stores (Kristof 2017). On the other hand, this timing reflects
the practice that consumer information acquisition is a latent decision. It means that consumer
information acquisition is unobservable to the firm and thus exerts no direct impact on the firm’s
price setting decision. As a result, it is as if the consumers’ information-acquisition and the firm’s
price-setting decisions are de facto made simultaneously.

It is also important to emphasize that we are not assuming that search can reveal information
only on valuation but not on price: what is needed for our results is that the consumers do not
know the actual price before deciding whether to search. Moreover, the simultaneous-move setup
allows us to rule out the informational role of pricing. This is a common assumption in the
informative advertising literature that can help isolate the informational effect of advertisements
and avoid imposing beliefs when pricing and advertising messages are inconsistent (e.g., Bagwell
and Ramey 1993, Gardete 2013). This assumption is especially useful in Section 3 when we focus
on the informativeness of advertising in influencing consumer information acquisition.

A summary of the main notations is presented in Table 1.

2.2 Optimal Consumer Decisions

We start with the consumers’ optimal purchase decision in the last stage of the game. If a consumer
has decided not to acquire product information, she maintains the prior belief and will buy the
product if and only if $E[v|q] - p \geq 0$, where $E[v|q] = qv_1 + (1-q)v_0$ captures the expected
gross utility from purchasing the product. If the consumer has acquired product information, the
purchase decision depends on the realized product valuation. In particular, the consumer will buy
the product if and only if $v_1 - p \geq 0$ for the high-valuation product, and $v_0 - p \geq 0$ for the
low-valuation product, respectively.

Consider now the consumers’ decision on information acquisition in the first stage of the game.
Let $\hat{p}$ be the consumers’ expectation on the price charged by the firm. Note that it is never optimal
for the firm to set a price below the consumers’ lowest possible valuation. That is, the anticipated
price must satisfy $\hat{p} \geq v_0$. This implies that a consumer’s anticipated surplus is always zero in case

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[8] Guo and Wu (2016) investigate the signaling role of pricing in the alternative timing in which the price is observed
before consumer information acquisition (i.e., deliberation).
the product turns out to be of low value: she will either buy the product at a price that completely extracts her surplus, or she will choose the outside option with zero utility. As a result, a consumer decides to search for product information if and only if

$$ q (v_1 - \hat{p}) - c \geq \max \{E [v|q] - \hat{p}, 0\}, $$

(1)
where the left-hand side represents the expected utility of search, and the right-hand side captures the best of the two possible scenarios that the consumer may face if she does not search. In the absence of product information, the consumer receives the maximum utility between buying the product and resorting to the outside option. In particular, if \( \hat{p} \leq E[v|q] \) the consumer believes that she will buy the product should she decide not to search, and if \( \hat{p} > E[v|q] \) she anticipates that she will not make an uninformed purchase.

We consider these two scenarios in turn. Consider first the scenario when the anticipated price is \( \hat{p} \leq E[v|q] \). A consumer would decide to acquire product information if and only if

\[
q(v_1 - \hat{p}) - c \geq E[v|q] - \hat{p}.
\]

This condition can be simplified as \( \hat{p} \geq v_0 + \frac{c}{1-q} \). Alternatively, consider the case when the consumer believes that the product is too expensive to warrant a purchase if no product information is available, i.e., \( \hat{p} \geq E[v|q] \). In this case the consumer would acquire product information if and only if

\[
q(v_1 - \hat{p}) - c \geq 0,
\]

which can be rearranged as \( \hat{p} \leq v_1 - \frac{c}{q} \).

Therefore, the consumers’ optimal information acquisition decision depends on the anticipated price. When the anticipated price is low enough, the consumers are willing to buy without searching for product information. As the anticipated price increases, the consumers prefer to acquire more information to guide their purchase decision. However, when the anticipated price is overly high, the information value of search would be suppressed and not be enough to justify the search cost. The consumers would then decide not to search and not to purchase.

**Lemma 1** The consumers decide to acquire product information only when the product quality is intermediate, i.e., \( q \in [q_0, q_1] \), where the boundary points \( q_0 \) and \( q_1 \) are given in the Appendix. When \( q < q_0 \) or \( q > q_1 \), the consumers do not acquire product information for any anticipated price \( \hat{p} \).

When the product quality is very low, acquiring information is not valuable because the product is unlikely to be of high valuation to ensure a purchase. On the other hand, when the product quality becomes sufficiently high, the consumers would act without information acquisition either. In this case the consumers do not need to be informed of the specific product valuation to justify the purchase. It is only when the product quality is intermediate that information acquisition may be preferable, depending on the anticipated price. Intuitively, knowing the product valuation can improve the consumers’ expected payoff only if the valuation uncertainty is large enough and the acquired information can guide the purchase decision.

As can be seen from the Appendix, one necessary condition for information acquisition to be desirable is that the information search cost must be small enough vis-a-vis the valuation range, i.e., \( c < (v_1 - v_0)/4 \). When this condition is not met, information search is never optimal for any quality and any price. As a result, we retain this assumption for the remainder of the paper.
2.3 Equilibrium Outcome

We now derive the equilibrium outcome when the product quality is known. Recall that the consumers decide on information acquisition before knowing the price, and similarly the firm sets the price without knowing the consumers’ information search decision. Therefore, we will analyze the parties’ best responses to each other’s behavior, and then determine the equilibrium outcome by jointly solving the parties’ optimization problems.

Consider first the case when the product quality is either very low or very high. According to Lemma 1, if \( q \in (0, q_0) \cup (q_1, 1) \), the consumers would never acquire information for any anticipated price. Given this, the firm would charge \( p = E[v|q] \) that completely extracts consumer surplus. No profitable deviation for either party can arise and a pure-strategy equilibrium exists. That is, when uncertainty about product valuation is sufficiently small, the unique equilibrium involves no information acquisition.

Consider then the intermediate quality range, \( q \in [q_0, q_1] \). In this case no pure-strategy equilibrium can exist. To see this, suppose all consumers decide to acquire information. This means that the ex post product valuation of some consumers of size \( q \) is \( v_1 \), and that of the other consumers of size \( 1 - q \) is \( v_0 \), respectively. Given this, the firm would optimally charge either \( p = v_1 \) or \( p = v_0 \). However, according to Section 2.2, at either of these anticipated prices, the consumers are better off deviating and saving the cost of information acquisition. That is, if \( \hat{p} = v_1 \) the consumers will give up search and forgo the purchase opportunity, and if \( \hat{p} = v_0 \), the consumers will buy without acquiring information. Conversely, suppose all consumers decide not to acquire information. The optimal response for the firm would be to set \( p = E[v|q] \). But at this anticipated price the consumers are better off acquiring product information to decide whether to purchase. As a result, no pure-strategy equilibrium can exist from which the parties find no profitable deviation.

It follows that, for the intermediate quality range, there exists only mixed-strategy equilibrium under which some consumers acquire information and the others do not. Denote \( \alpha \in [0, 1] \) as the proportion of consumers who decide to acquire information. This gives rise to three segments of consumers at the purchase stage, with (expected) product valuation \( v_1, v_0, \) and \( E[v|q] \), and segment size \( \alpha q \), \( \alpha(1-q) \), and \( 1-\alpha \), respectively. As a result, the optimal price would take one of these three possible values. This yields the firm’s profit function:

\[
\pi (p; \alpha) = \begin{cases} 
  v_0, & \text{if } p = v_0 \\
  (\alpha q + 1 - \alpha) E[v|q], & \text{if } p = E[v|q] \\
  \alpha q v_1, & \text{if } p = v_1.
\end{cases}
\]

(2)
The lowest price \( p = v_0 \) can be set such that all consumers are sure to purchase the product irrespective of their decision, or the outcome, of information acquisition. Alternatively, the firm can raise the price to \( p = E [v|q] \) and target those consumers who decide not to search for product information and those with high valuation. Finally, the firm can charge a high price \( p = v_1 \) and sell only to the consumers who find that the product is of high valuation after information search.

**Lemma 2** When \( q \leq \bar{q} \), the optimal price is given by:

\[
p = \begin{cases} 
E [v|q], & \text{if } \alpha \leq \alpha_0 \\
v_0, & \text{if } \alpha_0 \leq \alpha \leq \frac{v_0}{qv_1} \\
v_1, & \text{if } \alpha \geq \frac{v_0}{qv_1} 
\end{cases}
\]

when \( q \geq \bar{q} \), the optimal price is given by:

\[
p = \begin{cases} 
E [v|q], & \text{if } \alpha \leq \alpha_1 \\
v_1, & \text{if } \alpha \geq \alpha_1 
\end{cases}
\]

where \( \bar{q} \equiv \frac{2v_0}{\sqrt{v_1(5v_1-4v_0)-(v_1-2v_0)}}, \quad \alpha_0 \equiv \frac{E[v|q]-v_0}{(1-q)E[v|q]}, \quad \text{and } \alpha_1 \equiv \frac{E[v|q]}{qv_1+(1-q)E[v|q]} \).

This lemma characterizes the firm’s optimal pricing decision in response to the consumers’ information acquisition behavior. Figure 2 illustrates how the firm’s optimal price is influenced by the level of quality and the information search intensity in the \((q, \alpha)\) space. When the quality is sufficiently low, it is very likely that the consumers search to find a low-valuation product. In this case the firm’s main tradeoff is between charging \( p = v_0 \) and selling to all consumers, and setting \( p = E [v|q] \) and targeting consumers except those in the low-valuation segment. When the product is of sufficiently high quality, the firm compares setting \( p = v_1 \) and selling only to the consumers who find the product to be of high valuation, with setting \( p = E [v|q] \) and targeting also the consumers who decide not to search. In either case, it is optimal to charge a price equal to the expected value \( E [v|q] \) only if not many consumers choose to acquire information (i.e., \( \alpha \leq \min \{\alpha_0, \alpha_1\} \)).

The indifference curve between charging \( p = v_0 \) and \( p = E [v|q] \), and that between \( p = v_1 \) and \( p = E [v|q] \), are depicted by the segments OB and BC in Figure 2, respectively.

Next, we analyze the consumers’ optimal information acquisition decision in response to the firm’s anticipated pricing behavior. Let \( \beta_0 \in [0, 1] \) and \( \beta_1 \in [0, 1] \) be the probabilities that the firm charges \( \hat{p} = v_0 \) and \( \hat{p} = v_1 \), respectively. The consumers decide to acquire information if and only if the expected payoff of search exceeds that of no search:

\[
\beta_0 (E [v|q] - v_0) + (1 - \beta_0 - \beta_1) q (v_1 - E [v|q]) - c \geq \beta_0 (E [v|q] - v_0), \tag{3}
\]
It is evident from (3) that searching for product information can improve the expected payoff (gross of the search cost), only when the firm charges $\hat{p} = E[v|q]$ and the consumers search to find the product to be of high valuation. When the anticipated price is $\hat{p} = v_0$, information acquisition is unnecessary, because it is always optimal to purchase the product irrespective of the decision or the outcome of search (i.e., the part $\beta_0 (E[v|q] - v_0)$ appears in both sides of (3)). When the firm charges $\hat{p} = v_1$, the consumers cannot benefit from information acquisition either, because their expected surplus would always be zero. Only when the firm charges the intermediate price, can information acquisition improve the purchase decision and hence increase the expected payoff: the searching consumers purchase the product if and only if it is found to be of high valuation, thus avoiding the non-searching consumers’ unwanted incidence of buying a product of low valuation.

Let $\beta = \beta_0 + \beta_1$ be the probability of charging the extreme prices ($v_0$ and $v_1$). It follows from (3) that search is beneficial if and only if $\beta$ is not too high. This is illustrated by Figure 3, where the solid curve represents the value of $\beta$ that makes the two sides of (3) equal to each other. When the product is of overly low or overly high quality, information acquisition is dominated irrespective of firm price. In the middle range, the net benefit of information acquisition is non-monotonic, first increasing and then decreasing as the quality becomes higher. For a given $q$, the consumers decide to search if and only if the firm’s probability of charging the extreme prices is below a threshold.

It follows from (3) that the net benefit of information acquisition is negative if the firm charges $\hat{p} = E[v|q]$ with zero probability (i.e., $\beta = 1$). This means that there does not exist an equilibrium in which the firm mixes between $\hat{p} = v_0$ and $\hat{p} = v_1$ only. Moreover, there does not exist an equilibrium in which the firm charges each of the three possible prices with strictly positive probability. This is because, as can be seen from Lemma 2, there does not exist an $\alpha$ such that the firm is indifferent
between all three price levels. As a result, under the only feasible mix-strategy equilibrium, the consumers are indifferent between searching and not searching, and the firm mixes between $p = v_0$ and $p = E[v|q]$, or between $p = v_1$ and $p = E[v|q]$.

**Proposition 1** The equilibrium outcome with known product quality is given by:

(i) When $q < q_0$ or $q > q_1$, the consumers buy the product without acquiring information, and the firm sets $p^* = E[v|q]$;

(ii) When $q \in [q_0, \overline{q}]$, the consumers acquire information with probability $\alpha^*_0$, and the firm mixes between $p^* = v_0$ and $p^* = E[v|q]$ with probability $\beta^*$ and $1 - \beta^*$, respectively;

(iii) When $q \in [\overline{q}, q_1]$, the consumers acquire information with probability $\alpha^*_1$, and the firm mixes between $p^* = v_1$ and $p^* = E[v|q]$ with probability $\beta^*$ and $1 - \beta^*$, respectively, where $\beta^* \equiv 1 - \frac{c}{q(1-q)(v_1-v_0)}$.

This proposition completely characterizes the equilibrium outcome when the product quality is known to the consumers. When consumer valuation uncertainty is small enough (i.e., $q$ is sufficiently low or sufficiently high), in equilibrium the consumers acquire no information for sure and the firm can extract all consumer surplus. When the product quality is intermediate, the only equilibrium is mixed: some consumers decide to acquire information while the others do not, and the firm’s prices are such that the consumers are indifferent whether to search or not. Intuitively, when the product quality is not too high (i.e., $q \in [q_0, \overline{q}]$), the firm has no incentive to target only the high-valuation consumers, and the equilibrium prices are mixed between $v_0$ and $E[v|q]$. On the other hand, when the product quality becomes higher (i.e., $q \in [\overline{q}, q_1]$), it is not profitable to cover the low-valuation...
consumers any more, and the equilibrium prices are mixed between $v_1$ and $E[v|q]$. In either case the non-searching consumers are targeted as the marginal consumers with positive probability.

Mixed strategies in equilibrium pricing are typically interpreted as temporary promotions. Price competition with discrete demand is one standard mechanism to yield mixed pricing in equilibrium (e.g., Varian 1980, Narasimhan 1988). In contrast, here the mixed-strategy equilibrium arises for a monopoly because pre-purchase information acquisition leads to not only endogenous consumer preference but also “concurrent” price setting. Moreover, as will be demonstrated in Section 4.1, we can obtain the mixed-strategy equilibrium even for continuous valuations.

We now conduct comparative statics analyses to evaluate the impacts of product quality $q$ on the equilibrium outcome.

**Proposition 2** When $q \in [q_0, \bar{q}]$, the equilibrium information acquisition probability $\alpha_0^*$ strictly increases in $q$. When $q \in [\bar{q}, q_1]$, the equilibrium information acquisition probability $\alpha_1^*$ strictly increases in $q$ if $v_1 < 4v_0$, and first decreases and then increases in $q$ if $v_1 > 4v_0$.

This proposition presents the impact of quality on the equilibrium probability of acquiring information. Recall from (3) and Figure 3 that the net benefit of information acquisition first increases and then decreases with quality in the range $q \in [q_0, q_1]$. One might then be tempted to expect a similar impact on the equilibrium probability of information acquisition. However, Proposition 2 shows that this intuition is not confirmed. Figure 4 depicts the two different cases characterized in the proposition. In either case the consumers’ equilibrium likelihood of information acquisition always increases with quality in the range $q \in [q_0, \bar{q}]$. A similar pattern would emerge for the range $q \in [\bar{q}, q_1]$ if, as in the left panel, the valuation range is not wide (i.e., $v_1 < 4v_0$). In contrast, if the valuation range is wide enough (i.e., $v_1 > 4v_0$), as shown in the right panel, there would be a non-monotonic relationship between equilibrium search probability and quality.

This is because the consumers’ search behavior here is influenced not only by the net benefit of information—in equilibrium they are indifferent between searching and not searching—but also by the need to balance the firm’s pricing behavior. The proportion of searching consumers has to be such that the firm is indifferent mixing between the equilibrium prices. When $q \in [q_0, \bar{q}]$, the firm mixes between $p = v_0$ and $p = E[v|q]$. As can be seen from (2), the firm’s profit of charging $p = v_0$ is constant, whereas that of charging $p = E[v|q]$ is unequivocally increasing in $q$ and decreasing in $\alpha$. Therefore, as the product quality increases, more consumers would choose to search to make the firm’s profits unchanged across the equilibrium prices, i.e., $\alpha_0^*$ is unambiguously increasing in $q$.

In contrast, when $q \in [\bar{q}, q_1]$, the proportion of searching consumers needs to ensure the same profits between $p = v_1$ and $p = E[v|q]$. Note that $\pi(v_1; \alpha)$ is increasing in both $q$ and $\alpha$. As a
result, the equilibrium impact of product quality on the search probability $\alpha_1^*$ depends on whether $\pi(v_1; \alpha)$ or $\pi(E[v|q]; \alpha)$ is raised relatively more by a higher $q$. A higher quality increases both $\pi(v_1; \alpha)$ and $\pi(E[v|q]; \alpha)$ through its positive effect on the number of searching consumers who find high valuation. In addition, product quality positively affects the non-searching consumers’ expected value $E[v|q]$, thus resulting in an additional impact on $\pi(E[v|q]; \alpha)$. Therefore, when $q$ is sufficiently high or when the valuation range is not very large such that $E[v|q]$ is sufficiently close to $v_1$, an increasing $q$ can exert a larger impact on $\pi(E[v|q]; \alpha)$ than on $\pi(v_1; \alpha)$. This would then lead more consumers to search for product information. Conversely, when $q$ is not too high and the valuation range is sufficiently large, the valuation gap between the non-searching consumers and the high-valuation consumers would be large enough such that a higher $q$ increases $\pi(v_1; \alpha)$ more than $\pi(E[v|q]; \alpha)$. This would then yield the negative relationship between $\alpha_1^*$ and $q$.

The expected equilibrium price strictly increases with quality. When $q < q_0$ or $q > q_1$, the equilibrium price is equal to $E[v|q]$. When $q \in [q_0, \bar{q}]$, the expected equilibrium price is $\beta^* v_0 + (1 - \beta^*) E[v|q] = v_0 + \frac{c}{1-q}$. Recall from Section 2.2 that this is the highest price to induce (all) consumers to purchase the product without search. However, here a strictly positive proportion of consumers may not buy if they search to find a low valuation. When $q \in [\bar{q}, q_1]$, the expected equilibrium price is $\beta^* v_1 + (1 - \beta^*) E[v|q] = v_1 - \frac{c}{q}$. If the firm charged this price for sure, all consumers would search and those with a high valuation would buy. In contrast, here both the proportion of non-searching consumers and their likelihood of not buying are strictly positive. Therefore, due to the equilibrium mixed strategy, the firm charges the same expected prices but has lower sales, in comparison to the (pure-strategy) equilibrium outcome in the alternative setup in which the firm sets the price prior to consumer information retrieval/gathering (e.g., Guo and Wu 2016). In other words, pre-purchase information acquisition, compared with on-purchase deliberation, can hurt firm profitability.

Figure 4: Equilibrium Behavior of Information Acquisition

$v_1 < 4v_0$ ($v_1 = 3/5$, $v_0 = 1/5$, $c = 1/20$)  
$v_1 > 4v_0$ ($v_1 = 2$, $v_0 = 1/5$, $c = 1/10$)
Proposition 3 The equilibrium profit strictly decreases in \( q \) at \( q = q_0 \).

Interestingly, the firm’s equilibrium profit may decrease with the product quality. This is due to the discontinuous drop of the equilibrium profit at \( q = q_0 \). When \( q < q_0 \), the consumers’ incentive for information acquisition is sufficiently low such that the firm can charge \( p^* = E[v|q] \) to extract all consumer surplus (part (i) of Proposition 1). However, when \( q \geq q_0 \), information acquisition would be valuable enough and the consumers would become ex post heterogeneous in equilibrium. This would undermine the firm’s ability to extract consumer surplus: the firm’s equilibrium profit under part (ii) of Proposition 1 is reduced to \( v_0 \).\(^9\) However, consumer information acquisition per se does not necessarily yield this result. It can be shown that the equilibrium profit is strictly increasing in quality in the above-mentioned alternative setup of consumer deliberation (e.g., Guo and Wu 2016). When consumer information search follows firm pricing, the firm can always price to prevent search and thus extract all consumer surplus (if that is desirable). In comparison, here information acquisition would take place inevitably under the mixed-strategy equilibrium, because it cannot be directly influenced by firm pricing. Therefore, it is pre-purchase information acquisition that leads to the harmful effect of a higher quality.

Proposition 4 The equilibrium consumer surplus is zero when \( q < q_0 \) or when \( q > \overline{q} \). When \( q \in [q_0, \overline{q}] \), the equilibrium consumer surplus strictly increases in \( q \) if the valuation range \( v_1 - v_0 \) is sufficiently large, and first increases and then decreases in \( q \) if \( v_1 - v_0 \) is sufficiently small.

The consumers’ equilibrium surplus is positive only if the product quality is in the range \([q_0, \overline{q}]\). In this case the equilibrium is in mixed-strategy such that the consumers have the same expected surplus no matter they decide to search or not. It then follows from the right-hand side of (3) that the consumers’ equilibrium surplus is equal to their expected valuation, \( E[v|q] \), minus the expected equilibrium price, \( \beta^* v_0 + (1 - \beta^*) E[v|q] = v_0 + \frac{c}{1-q} \). A higher quality increases the expected valuation as well as the expected equilibrium price. As shown in the left panel of Figure 5, the impact on the expected valuation is relatively stronger when the valuation range is sufficiently large. Nevertheless, when the valuation range is not very large, a higher quality can increase the

\(^9\)We would still get the result in Proposition 3 even when \( v_0 < 0 \): part (ii) of Proposition 1 would be empty and the equilibrium profit at \( q = q_0 \) would be characterized by part (iii). To see this, note that the firm would then charge the same price (under the mixed-strategy equilibrium) as that for \( q < q_0 \) (i.e., \( p^* = E[v|q] \)) but less consumers would buy because of information acquisition. Note also that, when \( v_0 < 0 \) (or marginal cost is higher than \( v_0 \)), if endogenous consumer information acquisition were not allowed, the firm prefers to provide information to the consumers and thus price to the high-valuation consumers rather than the uninformed average consumers. Therefore, the underlying mechanism here differs from that in the literature (e.g., Xie and Shugan 2001, Shulman, Coughlan and Savaskan 2009) whereby a firm may prefer to sell to uninformed over informed consumers when the possible valuations are not too low relative to the firm’s marginal cost (which is normally set to zero).
expected equilibrium price more than the expected valuation. As a result, as demonstrated in the right panel, the consumers can be hurt as the quality increases.

3 Informative Advertising

3.1 Setup

We now investigate how the firm can credibly advertise its quality. To this end, we assume that the firm has private information about its own quality \( q \in Q \equiv \{q_L, q_H\} \), where \( 0 \leq q_L < q_H \leq 1 \). We will call the high- and the low-quality firm as the type-\( H \) and type-\( L \) firm, respectively. The prior probability that the firm is type \( H \) is \( \lambda \in (0, 1) \), which is common knowledge. The firm can communicate its quality to the consumers by engaging in informative advertising. In particular, the firm can send an advertising message \( m \) to the consumers. The content of the message is cheap talk in the sense that it can be freely set irrespective of the firm’s quality type, and that neither the cost of advertising nor the cost of the advertising content vary across firm types. This stands in contrast to alternative communication mechanisms such as costly signaling or truthful disclosure.

The timing of the game is depicted in Figure 6. At the beginning of the game the firm is endowed by nature with a product of quality \( q \), which is unknown to the consumers. After observing its product quality, the firm determines the message to send to the consumers about its quality. The rest of the game is the same as that we consider in Section 2 with known quality. Let the consumers’ belief, prior to the search decision (at stage 2), that the firm is of type \( H \) be \( b_1(m) \equiv \widehat{Pr}(H|m) \).
At the time of purchase, those consumers who decide not to search are still uncertain about their product valuations. Define $b_2(m, p) \equiv \hat{P}_r(H|m, p)$ as the non-searching consumers’ belief, prior to the purchase decision (at stage 3), that the firm is of type $H$.

The solution concept we adopt is the Perfect Bayesian Equilibrium (PBE). This requires that in equilibrium the consumers update their beliefs about firm type according to the Bayes’ rule whenever applicable. That is, the consumers’ belief updating is consistent with the firm’s equilibrium strategies. The consumers can make inference about the firm’s optimal behavior, and understand that the firm may have an incentive for misrepresentation in the cheap-talk advertising.

We focus on separating equilibrium in which the firm’s advertising message is informative such that it can credibly convey the firm’s type to the consumers. That is, the firm’s equilibrium advertising message satisfies $m^*(q) = q$, for $q \in Q$. Moreover, under the PBE we must have $b_1^*(q_H) = 1$ and $b_1^*(q_L) = 0$, i.e., in equilibrium the consumers have correct beliefs about firm type.

We assume that the (non-searching) consumers’ belief is influenced by advertising only, i.e., $b_2(m, p) = b_1(m)$, $\forall p$. This is a restriction on the off-equilibrium belief. Note that PBE requires that $b_1^*(m^*) = b_2^*(m^*, p^*)$, where $m^*$ and $p^*$ are the equilibrium advertising message and price, respectively. That is, in equilibrium the (non-searching) consumers cannot receive an informative advertising message that is later contradicted by the message conveyed by the price. More importantly, this restriction permits us to concentrate on the informational role of advertising. It is a common assumption in games of asymmetric information with potentially multiple signals (e.g., Hart and Tirole 1990, Caminal and Vives 1996). We intentionally make this assumption, because it allows us to show that cheap-talk advertising can be informative even when pricing is not.

---

**Figure 6: Timing of the Game with Costless Quality Advertising**

10 As in models of cheap talk, there always exists a “babbling” equilibrium in which the advertising message is uninformative and the consumers maintain their prior belief about firm type.

11 The advertising message can be informative even if its content is factually irrelevant or even wrong. In addition, it does not matter what specific language is used for the advertising message. What is critical for the advertising to be informative is that different messages are sent across quality types, and that the mapping from quality type to advertising message can be understood by the consumers. For example, the firm can send the message “the sky is green” when its quality is high, and the message “the sea is yellow” when its quality is low. Nevertheless, to comply with tradition and to ease exposition, the set of quality types, $Q$, is used for the set of informative messages.

12 In the absence of advertising, price alone cannot credibly signal the firm’s type in the current setup because no single-crossing condition is imposed.
3.2 Equilibrium Outcome

Under the equilibrium of informative advertising, the type of the firm is credibly communicated to the consumers. As a result, along the equilibrium path, the subgame starting from stage 2 is equivalent to the game of complete information we investigate in Section 2. This means that in equilibrium the consumers’ decisions are the same as those characterized under known quality, because their beliefs do not vary with the price. Let $\tilde{q}(m)$ be the consumers’ believed quality after receiving the advertising message $m$. In the informative equilibrium we would have $\tilde{q}^*(m) = m$, and the consumers’ equilibrium probability of search would be given by $\alpha^*(\tilde{q})$ as in Proposition 1.

Given that the consumers believe in the firm’s advertising message, the firm’s expected profit function (at stage 1) when its true quality is $q$ and the consumers’ believed quality is $\tilde{q}$ is:

$$
\pi (p; q, \tilde{q}) = \begin{cases} 
  v_0, & \text{if } p = v_0 \\
  [\alpha^*(\tilde{q})q + 1 - \alpha^*(\tilde{q})] E[v|\tilde{q}], & \text{if } p = E[v|\tilde{q}] \\
  \alpha^*(\tilde{q})qv_1, & \text{if } p = v_1.
\end{cases}
$$

The key to sustain the informative advertising equilibrium is to ensure that the firm is indeed willing to report its type truthfully, given that its claimed type is to be believed by the consumers. Truth telling (i.e., $m^*(q) = q$) is desirable for the firm if and only if the following incentive compatibility conditions are satisfied:

\begin{align*}
\text{IC}_1 & : \max_p \pi (p; q_L, \tilde{q}_L) \geq \max_p \pi (p; q_L, \tilde{q}_H), \\
\text{IC}_2 & : \max_p \pi (p; q_H, \tilde{q}_H) \geq \max_p \pi (p; q_H, \tilde{q}_L).
\end{align*}

The left-hand sides in the incentive compatibility conditions represent the firm’s maximized profits when its advertising message truthfully communicates its quality to the consumers. The right-hand sides capture the firm’s best possible profits when its true type is $q_L$ but it deviates to claim to be $q_H$, or when its true type is $q_H$ but it deviates to claim to be $q_L$, respectively.

**Proposition 5** Informative advertising cannot credibly arise in equilibrium unless the quality levels are $q_L \in (0, \bar{q})$ and $q_H \in (\bar{q}, q_1)$.

This proposition characterizes the parameter regions of the qualities under which the informative advertising equilibrium cannot be sustained. In particular, it shows that the incentive compatibility conditions cannot be satisfied unless the quality of type $L$ is in the interval $(0, \bar{q})$ and that of type
Figure 7: Parameter Regions of Misrepresentation of Advertising Message

$H$ is $q_H \in (\overline{q}, q_1)$. This is represented by the shaded area in Figure 7. The other areas in the figure capture the parameter regions under which at least one of the incentive compatibility conditions is violated, where the capital letters $L$ and $H$ denote the firm type(s) that would benefit from misrepresenting its quality type in each region.

Consider first the regions where the high type’s quality is sufficiently low or sufficiently high (i.e., $q_H < q_0$ or $q_H > q_1$) such that no consumer would acquire information if they were informed of the high-type’s quality. The high type would be able to completely extract the consumers’ surplus by charging $E[v|q_H]$, if it could successfully separate from the low type. However, the low-quality firm would have an incentive to claim to be the high type. This imitation can result in a higher profit for the low-type firm through increasing the consumers’ willingness to pay from $E[v|q_L]$ to $E[v|q_H]$, and/or through reducing the proportion of searching consumers from $\alpha^*(q_L)$ to zero.

In the regions where $q_H \in [q_0, \overline{q}]$, the type-$H$ firm would induce a proportion $\alpha_0^*(q_H)$ of consumers to search for product information if it truthfully reveals its type to the consumers. In this case the high-quality firm is indifferent between selling to all consumers and pricing at the non-searching consumers’ expected value. A smaller number of searching consumers would increase the expected revenue of targeting the non-searching consumers, but does not change the profit of serving all consumers. As a result, the high type would prefer to understate its quality, even though that would lower the consumers’ expected value. This is because, if the consumers believe that the firm is of the low type, they would reduce their equilibrium probability of search to $\alpha_0^*(q_L)$ (when $q_L \in [q_0, \overline{q}]$) or even zero (when $q_L < q_0$). Conversely, the low-type firm does not want to overstate its quality because that would increase the number of searching consumers.

When the qualities of both types are in the interval $[\overline{q}, q_1]$, under complete information the
consumers’ probability of information acquisition would be such that the firm is indifferent between selling only to the high-valuation consumers and targeting also the non-searching consumers. A larger number of searching consumers would increase the expected revenue serving only the high-valuation consumers, but would decrease the profit of pricing to cover the non-searching consumers. Therefore, the low-type firm can increase its profit by misleading the consumers to believe that it is the high type. This is because the low-type firm can sell to more high-valuation consumers if imitating the high type increases the number of searching consumers, or serve more non-searching consumers at a higher price if otherwise. Similarly, the high-type firm may want to mimic the low-type firm’s advertising message if that can lead more consumers to acquire information.

Next, we investigate the remaining two parameter regions under which informative advertising may emerge in equilibrium. We will identify conditions under which neither firm type has an incentive to deviate from the informative equilibrium.

**Proposition 6** In the region where \( q_L < q_0 \) and \( q_H \in (q, q_1) \), informative advertising can credibly arise in equilibrium, if and only if \( \frac{v_1 - v_0}{v_0} \) is low (or \( \frac{v_1 - v_0}{v_0} \) is high and \( q_L \) is high) and \( q_H \) is high.

This proposition shows that there exist conditions to sustain the informative advertising equilibrium. In this case the consumers do not acquire information if they know the product is the low-quality type, whereas a proportion \( \alpha^*_1(q_H) \) of consumers would search if they believe that the firm is type \( H \). As a result, by claiming to be the high type, the low-type firm could increase its profit margin from \( E[v|q_L] \) to \( E[v|q_H] \), whereas its expected demand would drop from one to \( \alpha^*_1(q_H)q_L + 1 - \alpha^*_1(q_H) \). The improvement in profit margin, \( \frac{E[v|q_H]}{E[v|q_L]} = \frac{1+q_H(v_1-v_0)/v_0}{1+q_L(v_1-v_0)/v_0} \), is increasing in the ratio of valuations, \( \frac{v_1-v_0}{v_0} \). Conversely, the loss in expected demand, \( (1-q_L)\alpha^*_1(q_H) = \frac{(1-q_L)(1+q_H(v_1-v_0)/v_0)}{1+q_H(2-q_H)(v_1-v_0)/v_0} \), is decreasing in \( \frac{v_1-v_0}{v_0} \). Therefore, when \( \frac{v_1-v_0}{v_0} \) is low, imitating the high-quality type is not profitable because the gain in profit margin cannot compensate the loss in demand. When \( \frac{v_1-v_0}{v_0} \) becomes high, the low-type firm may want to mimic the high type unless its quality is not too low. This is because the expected payoff under truth telling would increase faster than the best deviating profit as \( q_L \) becomes higher.

The high-quality firm would earn an expected payoff of \( \alpha^*_1(q_H)q_Hv_1 \) if it truthfully communicates its type to the consumers. This payoff is strictly increasing in its product quality \( q_H \). In comparison, the high-type firm’s expected profit would be \( E[v|q_L] \) if it chose to misreport its quality type. As a result, truth telling is compatible with the high-type firm’s incentive if \( q_H \) is high.

The shaded area in Figure 8 represents the parameter values (in the region \( q_L < q_0 \) and \( q_H \in (q, q_1) \)) that sustain the incentive compatibility conditions for both firm types. For the set of parameter values where the informative advertising equilibrium cannot be supported, the capital
Figure 8: Incentive Compatibility Conditions for Informative Advertising ($q_L < q_0$)

letter $L$ or $H$ captures the firm type that has the incentive for quality misrepresentation. The left panel demonstrates the scenario when $\frac{v_1 - v_0}{v_0}$ is low such that type $L$ does not want to deviate from truthful communication. As the right panel shows, if $\frac{v_1 - v_0}{v_0}$ is high, the low-type firm may want to lie about its quality unless its quality is not too low. In either case the high-type firm’s quality needs to be high enough to ensure its incentive compatibility for truthful advertising.

**Proposition 7** In the region where $q_L \in (q_0, \overline{q})$ and $q_H \in (\overline{q}, q_1)$, informative advertising can credibly arise in equilibrium, if and only if $q_L$ is low and $\frac{v_1 - v_0}{v_0}$ is high.

This proposition confirms that informative advertising can also be credible in the alternative region in which both quality types in equilibrium induce some consumers to acquire information. In this case, if the low-quality firm truthfully communicates its quality, it would be indifferent between setting $p = v_0$ and $p = E[v|q_L]$, and its expected profit would be equal to $v_0$. If it deviated to overstate its quality, it would be able to earn an expected payoff of $[\alpha_1^*(q_H)q_H + 1 - \alpha_1^*(q_H)] E[v|q_H]$, which is increasing in $q_L$. As a result, it is necessary that $q_L$ is not too high for the low-quality firm to truthfully advertise its quality.

The high type’s equilibrium expected profit is $\alpha_1^*(q_H)q_H v_1 = [\alpha_1^*(q_H)q_H + 1 - \alpha_1^*(q_H)] E[v|q_H]$. Imitating the low type would be desirable if that leads more consumers to search (i.e., $\alpha_0^*(q_L) > \alpha_1^*(q_H)$). If instead fewer consumers would acquire information, the high-quality firm could consider mimicking the low type while charging $p = E[v|q_L]$ to target the non-searching consumers. This would increase the high type’s expected demand from $\alpha_1^*(q_H)q_H + 1 - \alpha_1^*(q_H)$ to $\alpha_0^*(q_L)q_H + 1 - \alpha_0^*(q_L)$, but its profit margin would drop from $E[v|q_H]$ to $E[v|q_L]$. It can be shown that the gain in demand
is decreasing, and that of the loss in margin is increasing, in $\frac{v_1-v_0}{v_0}$. As a result, the high-quality firm would be better off advertising truthfully if $\frac{v_1-v_0}{v_0}$ is high.

Figure 9 illustrates the set of parameter values (in the region $q_L \in (q_0, \bar{q})$ and $q_H \in (\bar{q}, q_1)$) under which the incentive compatibility conditions can be satisfied. It shows that the low type is better off sending a truthful advertising message as long as its quality is lower than the upper bound of the shaded region, and the high-quality firm prefers truthful advertising as long as its quality is above the lower bound of the shaded region.

**Proposition 8** Under the informative advertising equilibrium, the consumers’ ex ante surplus may decrease in the probability $\lambda$ that the firm is the $H$ type.

The equilibrium ex ante consumer payoff is the weighted average of the complete-information surplus across the two quality types. Recall from Proposition 4 that the equilibrium consumer surplus is strictly positive only when $q \in (q_0, \bar{q})$. This means that the consumers may enjoy a higher equilibrium surplus when the product quality is reduced. As a result, the consumers may become worse off as the probability of facing the high-quality firm increases. This surprising result would arise when the low type’s quality is $q_L \in (q_0, \bar{q})$ and that of the high type is $q_H \in (\bar{q}, q_1)$.

### 4 Extensions

We extend our basic model along two directions: continuous value distribution and competition. We will show that the main results are robust to these extensions.
4.1 Continuous Valuation

Consider the following alternative setup. The product consists of two attributes that are valued as \( u_1(q) \) and \( u_2(q) \), respectively, where \( u_1(q) > u_2(q) \) are both increasing in the quality \( q \in (0, 1) \). The overall product valuation is \( v = \theta u_1(q) + (1 - \theta) u_2(q) \), where \( \theta \) is the relative weight between the attributes. The weight \( \theta \) is uniformly distributed between zero and one. This means that product valuation is uniformly distributed between \( u_2(q) \) and \( u_1(q) \). For simplicity, we will focus on a specific setting: \( u_1(q) = q \) and \( u_2(q) = q^2 \). This captures the same features as those of the two-point setup in the basic model: a higher quality stochastically increases product valuation, whereas the valuation uncertainty (i.e., \( q - q^2 \)) is enlarged as the quality \( q \) becomes intermediate, but converges to zero as \( q \) approaches zero or one. As in Section 2, the consumers have identical and independent distribution for \( \theta \) (or \( v \)) prior to search, but can learn about the weight/valuation by incurring a cost \( c > 0 \). The other assumptions are also retained.

Let us start with characterizing the firm’s optimal behavior. Conditional on the price \( p \) and the proportion \( \alpha \in [0, 1] \) of searching consumers, the profit function becomes

\[
\pi(p; \alpha) = \begin{cases} 
\alpha \frac{q - p}{q(1 - q)} + 1 - \alpha \, p, & \text{if } p \leq E[v|q] \\
\alpha \frac{q - p}{q(1 - q)} p, & \text{if } p > E[v|q],
\end{cases}
\]

where \( E[v|q] \equiv q(1 + q)/2 \) is the expected valuation for the non-searching consumers. It follows that the optimal price is given by

\[
p(\alpha) = \max \left\{ \min \left\{ \frac{q - (1 - \alpha) q^2}{2\alpha}, q(1 + q)/2 \right\}, q^2 \right\}.
\]

Consider then the consumers’ optimal response to the anticipated price \( \hat{p} \). Upon searching to learn about \( v \), a consumer would buy the product if and only if \( v \geq \hat{p} \). The expected payoff of search is then

\[
\int_0^{\hat{p}} \frac{q - \hat{p}}{q(1 - q)} \, dv = \frac{(q - \hat{p})^2}{2q(1 - q)} - c.
\]

This is a better choice than no search if and only if

\[
\frac{(q - \hat{p})^2}{2q(1 - q)} - c \geq \max \left\{ E[v|q] - \hat{p}, 0 \right\}.
\]

Note that the optimal price is never higher than \( E[v|q] \). The above condition for search can then be simplified as \( \hat{p} \geq q^2 + \sqrt{2q(1 - q)c} \).

Consider the interaction between the firm and the consumers. If all consumers search, the firm would optimally charge \( p(1) = \max \{ q/2, q^2 \} \). In anticipation of this, the consumers would choose to search if and only if the cost \( c \) is not higher than the gross benefit of search:

\[
GBS_1(q) = \begin{cases} 
\frac{q(1 - 2q)^2}{8(1 - q)}, & \text{if } q \in (0, 1/2] \\
0, & \text{if } q \in [1/2, 1).
\end{cases}
\]
Conversely, if no consumer acquires information, the optimal price would be \( p(0) = q(1 + q)/2 \). The condition for preferring search would then be that \( c \) is less than or equal to \( GBS_0(q) = q(1 - q)/8 \). It is evident that both \( GBS_1(q) \) and \( GBS_0(q) \) are single peaked (i.e., first increasing and then decreasing), and that \( GBS_1(q) < GBS_0(q) \), for \( q \in (0, 1) \). This implies that, if \( c \) is not too high, there are two solutions \( q_0 \) and \( q_1 \) for \( GBS_0(q) = c \), and \( q_2 \) and \( q_3 \) for \( GBS_1(q) = c \), where \( q_0 < q_2 < q_3 < 1/2 < q_1 \). This is illustrated in Figure 10.

We are ready to derive the equilibrium outcome. If \( q \in (0, q_0) \cup (q_1, 1) \), we have \( GBS_1(q) < c \) and \( GBS_0(q) < c \) and thus the unique equilibrium involves zero consumer search (\( \alpha^* = 0 \)). However, if \( q \in (q_0, q_2) \cup (q_3, q_1) \), no pure-strategy equilibrium can exist because \( GBS_1(q) < c \) but \( GBS_0(q) > c \). Under the mixed-strategy equilibrium, the price is such that the consumers are indifferent between searching and not searching (i.e., \( p^* = q^2 + \sqrt{2q(1 - q)c} \)), and the proportion of searching consumers is such that the firm optimally charges the equilibrium price (i.e., \( p(\alpha^*) = p^* \)), which implies that \( \alpha^* = \frac{q - q^2}{q^2 + 2\sqrt{2q(1 - q)c}} \). Finally, if \( q \in (q_2, q_3) \), we have \( GBS_1(q) > c \) and \( GBS_0(q) > c \), resulting in the unique equilibrium under which all consumers search (\( \alpha^* = 1 \)). These results are summarized in the following proposition.

**Proposition 9** The equilibrium outcome with known product quality under continuous valuation is given by:

(i) If \( q \in (0, q_0) \cup (q_1, 1) \), the consumers buy the product without acquiring information (\( \alpha^* = 0 \)), and the firm sets \( p^* = E[v|q] = q(1 + q)/2 \);

(ii) If \( q \in (q_0, q_2) \cup (q_3, q_1) \), the consumers acquire information with probability \( \alpha^* = \frac{q - q^2}{q^2 + 2\sqrt{2q(1 - q)c}} \), and the firm sets \( p^* = q^2 + \sqrt{2q(1 - q)c} \);

(iii) If \( q \in (q_2, q_3) \), all consumers acquire information (\( \alpha^* = 1 \)), and the firm sets \( p^* = q/2 \).
This proposition demonstrates that the equilibrium outcome is qualitatively similar to that under discrete valuation (Proposition 1). There are two minor differences, both of which are driven by the continuity of current profit function. The first is that, under the mixed-strategy equilibrium, the consumers continue to randomize between searching and not searching, but the firm charges a specific price with probability one. Second, there may exist a unique equilibrium where all consumers search (for intermediate quality range) here but not in the discrete-valuation setup.

The continuous-valuation setup yields some interesting results on equilibrium welfare as those under discrete valuation. In particular, the firm’s equilibrium profit strictly decreases at $q = q_0$ (as in Proposition 3). Again this is because of the discontinuous change in the consumers’ equilibrium search behavior, and more importantly, due to the firm’s inability to directly influence consumer information acquisition. When $q < q_0$, no consumer searches in equilibrium and the firm charges $p^* = E[v|q]$ to extract all consumer surplus. In contrast, when $q$ is above but sufficiently close to $q_0$, the price that ensures indifference between searching and not searching (i.e., $p^* = q^2 + \sqrt{2q(1-q)c}$) is slightly below $E[v|q]$. But to induce the firm to set this price, the proportion of searching consumers has to be sufficiently above zero: otherwise the optimal price would be at the boundary solution $p = E[v|q]$. This would then lead to the discontinuous drop in the firm’s equilibrium profit.

In addition, similar to that in Proposition 4, the consumers’ equilibrium surplus is non-monotonic in the quality $q$. This can be readily seen from the observations that $E(CS)$ is continuous for all $q \in (0,1)$ (because of the continuity of $p^*$), $E(CS) = 0$ for $q < q_0$ or $q > q_1$, and $E(CS) > 0$ for $q \in (q_0, q_1)$ (because then $E[v|q] > p^* = \max\{q^2 + \sqrt{2q(1-q)c}, q_2\}$). We can also show that $E(CS)$ first increases and then decreases in $q$ within the range $(q_0, q_1)$.

Moreover, the firm may credibly advertise the quality $q \in Q \equiv \{q_L, q_H\}$ to the consumers. For example, consider the case when $c$ is neither too low nor too high (i.e., $\frac{5\sqrt{3}-11}{16} < c < 1/32$) such that the range $(q_2, q_3)$ is null. It can be shown that credible advertising cannot be sustained in equilibrium unless $q_L < q_0$ and $q_H \in (q_0, q_1)$. When the quality levels fall into these respective ranges, the low-quality type may not want to imitate the high type because that would lead to a higher proportion of searching consumers. Conversely, it may not be desirable for the high type to mimic the low type’s quality claim because that would decrease the (non-searching) consumers’ expected valuation. Therefore, credible advertising can arise as in the discrete-valuation setup.

### 4.2 Competition

We extend the basic model to consider a duopoly market with two competing firms, $i \in \{a, b\}$. The consumers’ valuation for the product of firm $a$ is $x$, and that for firm $b$ remains $v \in \{v_1, v_0\}$, where $v_1 > v_0 > x > 0$. Both firms’ costs are normalized to zero. Firm $a$’s price ($p_a$) is set before that of
firm $b$ ($p_b$). Firm $b$ is akin to the monopoly firm in Section 2. We can interpret firm $b$ as an entrant to a market where the incumbent firm $a$ has a known valuation and serves as the price leader. The other assumptions are similar to those in the basic model. We will investigate two alternative cases, depending on whether the consumers are informed of $p_a$ at the time of deciding whether to search.

### 4.2.1 Informed $p_a$

The timing of the game is presented in Figure 11. In comparison to the game in Figure 1, we add a first stage where firm $a$ sets its price. Firm $a$ can be interpreted as the price leader whose price is not updated as frequently as that of the rival firm. As a result, the consumers are informed of $p_a$ (but not $p_b$) before deciding whether to incur the cost $c$ to learn about $v$.

Given $p_a$, let the consumers’ surplus of buying from firm $a$ be $y = x - p_a$. Let us also define $\tilde{v}_1 = v_1 - y$ and $\tilde{v}_0 = v_0 - y$ as the (possible) relative valuations of firm $b$. Consider the sub-game starting from the second stage. The equilibrium outcome will be the same as that in the monopoly setup by replacing $v_1$ and $v_0$ with $\tilde{v}_1$ and $\tilde{v}_0$, respectively. This would be the case for any first-stage positive price that would be set by firm $a$. Therefore, all the insights we obtain there continue to hold for firm $b$ in this competitive setting.

What about firm $a$? Its expected payoff can be derived in light of the (sub-game) equilibrium outcome characterized in Proposition 1. When $q < q_0$ or $q > q_1$, no consumer will search and firm $b$ will set $p_b = E[\tilde{v}|q] = q \tilde{v}_1 + (1-q)\tilde{v}_0$. The anticipated profit for firm $a$ would be zero (even if it charges a price equal to zero). When $q \in [q_0, q]$, the consumers will search with probability $\alpha^*_0$, and firm $b$ will mix between $\tilde{v}_0$ and $E[\tilde{v}|q]$ (with probability $\beta^*$ and $1-\beta^*$, respectively). Firm $a$ would then have an expected payoff $\pi_a = \alpha^*_0(1-q)(1-\beta^*)(x-y) = \frac{c}{q(1-q)\tilde{v}_1 + (1-q)\tilde{v}_0}(x-y)$. Furthermore, when $q \in [q, q_1]$, the consumers’ search probability will be $\alpha^*_1$, and firm $b$ will randomize between $\tilde{v}_1$ and $E[\tilde{v}|q]$ (with probability $\beta^*$ and $1-\beta^*$, respectively). Firm $a$’s expected payoff would become $\pi_a = [\alpha^*_1(1-q) + (1-\alpha^*_1)\beta^*](x-y) = \frac{2q(1-q)\tilde{v}_1 + (1-q)(1-2q)\tilde{v}_0-c}{q(2-q)\tilde{v}_1 + (1-q)\tilde{v}_0}(x-y)$.

**Proposition 10** The equilibrium profit for firm $a$ can strictly increase in $q$.

This result can be readily seen by comparing firm $a$’s expected payoff across the second-stage
equilibrium regimes. It is straightforward that firm \(a\) anticipates to improve its profit from zero to positive, as \(q\) increases from the range \((0, q_0)\) to \([q_0, \bar{q}]\). In addition, the expected payoff of firm \(a\) would increase discontinuously at \(q = \bar{q}\). This is because, although the search probability is continuous at \(q = \bar{q}\) (Figure 4), firm \(b\) would raise its prices discontinuously such that firm \(a\) would see a jump in its expected demand across these two mixed-strategy regimes. Therefore, by the envelope theorem, firm \(a\) would have a higher first-stage equilibrium profit as \(q\) increases across the boundaries of these three second-stage equilibrium regimes.

The positive impact of \(q\) on firm \(a\)’s equilibrium payoff is counter-intuitive, because it suggests that a firm can benefit from the improvement in the rival firm’s quality. The underlying mechanism for this interesting result is that pre-purchase information acquisition can lead to endogenous consumer preference and hence endogenous differentiation between the competing firms. As \(q\) increases, the consumers may have a higher incentive for information acquisition. This can soften competition by enhancing ex post differentiation (recall that the consumers are ex ante homogenous). Therefore, a higher \(q\) can have a positive influence on the equilibrium payoff of firm \(a\).

### 4.2.2 Uninformed \(p_a\)

The timing of the alternative case is presented in Figure 12. Firm \(a\) continues to be the price leader and firm \(b\) the follower. However, the consumers are uninformed of both \(p_a\) and \(p_b\) before deciding whether to search. Therefore, from an informational perspective (as indicated by the dashed box), the consumers act as if their search decisions are made simultaneously with each of the firms’ pricing decisions, and vice versa. In addition, the consumers are aware that the firms’ prices are made sequentially. For simplicity, we assume that \(x\) is not too low relative to \(v_0\).

We first show that not all consumers search in equilibrium. Suppose otherwise that \(\alpha = 1\). Given the consumers’ surplus \(y\) of buying from firm \(a\), firm \(b\) will optimally set \(p_b = \tilde{v}_1\) or \(p_b = \tilde{v}_0\), resulting in the consumers’ expected payoff \(y - c\) or \(E[v|q] - \tilde{v}_0 - c = q(v_1 - v_0) + y - c\), respectively. However, if the consumers deviate and choose not to search, they would save the cost \(c\) while their gross expected payoff would remain unchanged (i.e., \(y\) if the optimal price for firm \(b\) is \(p_b = \tilde{v}_1\), or
between searching and not searching. Moreover, the proportion \( \alpha \) offers \( y \) if \( p_b = \tilde{v}_0 \). This proves that there is no equilibrium where all consumers search.

We then identify conditions under which the equilibrium involves zero consumer search. Given \( \alpha = 0 \) and \( y = x - p_a \), firm \( b \) will set \( p_b = E[\tilde{v}|q] \), yielding an expected consumer payoff \( y \). At these prices, if the consumers deviate to acquire information about \( v \), their expected payoff would become \( q(v_1 - p_b) + (1 - q)(x - p_a) - c = q(1 - q)(v_1 - v_0) + y - c \). As a result, not searching is indeed desirable and would be sustained in equilibrium, if and only if \( q(1 - q)(v_1 - v_0) < c \). This is the same condition as that in Proposition 1 (i): \( q < q_0 \) or \( q > q_1 \). Similarly, no pure-strategy equilibrium can exist if this condition is not satisfied.

Next, we derive the mixed-strategy equilibrium for \( q \in [q_0, q_1] \). For any \( y \), the optimal price for firm \( b \) can only be \( \tilde{v}_1, \tilde{v}_0 \), or \( E[\tilde{v}|q] \). Similar to the basic model, for the consumers to be indifferent between searching and not searching, the probability that firm \( b \) charges \( E[\tilde{v}|q] \) must be equal to \( 1 - \beta = \frac{c}{q(1 - q)(v_1 - v_0)} \in (0, 1) \). This is again because the consumers’ gross expected payoff of searching is the same as that of not searching if \( p_b = \tilde{v}_1 \) or \( p_b = \tilde{v}_0 \). Therefore, in equilibrium firm \( a \) cannot charge a positive price with probability one. If otherwise, firm \( b \) must mix between \( E[\tilde{v}|q] \) and \( \tilde{v}_1 \) or between \( E[\tilde{v}|q] \) and \( \tilde{v}_0 \). But then firm \( a \) can improve its profit by cutting its price slightly such that firm \( b \)’s indifference between its prices would be broken: firm \( b \) would strictly prefer \( \tilde{v}_1 \) over \( E[\tilde{v}|q] \), or \( E[\tilde{v}|q] \) over \( \tilde{v}_0 \), respectively.

For a given \( \alpha \), firm \( a \) may consider two possible values for its offer \( y \leq x \). One possibility is to offer a relatively small \( y \) just enough to make firm \( b \) (slightly) prefer \( p_b = E[\tilde{v}|q] \) over \( p_b = \tilde{v}_0 \). This leads to \( y_l(\alpha) = \max \left\{ \frac{v_0 - (\alpha q + 1 - \alpha) E[\tilde{v}|q]}{\alpha (1 - q)}, 0 \right\} \). Alternatively, firm \( a \) can raise \( y \) just enough to induce firm \( b \) to choose \( p_b = \tilde{v}_1 \) over \( p_b = E[\tilde{v}|q] \). This implies another offer firm \( a \) may consider: \( y_h(\alpha) = \max \left\{ \frac{(\alpha q + 1 - \alpha) E[\tilde{v}|q] - \alpha v_0}{1 - \alpha}, 0 \right\} \). Under the mixed-strategy equilibrium, firm \( a \) must randomize between \( y_l \) (with probability \( 1 - \beta = \frac{c}{q(1 - q)(v_1 - v_0)} \)) and \( y_h \) (with probability \( \beta \)), inducing firm \( b \) to charge \( E[\tilde{v}|q] \) and \( \tilde{v}_1 \) with the corresponding probabilities such that the consumers are indifferent between searching and not searching. Moreover, the proportion \( \alpha \) of searching consumers must guarantee that firm \( a \) is indifferent between offering \( y_l \) and \( y_h \):

\[
\alpha(1 - q)[x - y_l(\alpha)] = (1 - \alpha q)[x - y_h(\alpha)].
\tag{10}
\]

**Proposition 11** The equilibrium outcome with uninformed \( p_a \) is given by:

(i) If \( q \in (0, q_0) \cup (q_1, 1) \), the consumers do not acquire information (\( \alpha^* = 0 \)), firm \( a \) sets \( p_a^* = 0 \), and firm \( b \) sets \( p_b^* = E[\tilde{v}|q] - x \);

(ii) If \( q \in (q_0, q_1) \), the consumers acquire information with probability \( \alpha^* \) that solves (10), firm \( a \) offers \( y_l(\alpha^*) \) and \( y_h(\alpha^*) \) with probability \( 1 - \beta^* = \frac{c}{q(1 - q)(v_1 - v_0)} \) and \( \beta^* \), and firm \( b \) follows by charging
\[ p_b^* = E[\tilde{v}|q] \text{ and } p_b^* = \tilde{v}_1 \text{ correspondingly.} \]

This proposition confirms that the equilibrium outcome for intermediate \( q \) continues to be in mixed strategies. Similar to the other settings we consider in the paper, some consumers choose to search while the others do not. Nevertheless, not all firms explicitly randomize their prices. In contrast to the setup with informed \( p_a \), it is now firm \( a \) that engages in mixed pricing. Here firm \( b \) follows a pure-strategy equilibrium behavior, responding optimally to the (realized) price of firm \( a \). However, because firm \( a \)'s equilibrium prices are mixed, from the consumers’ perspective it is as if firm \( b \) is randomizing between two different prices as well. This difference in the nature of the equilibrium outcome is driven by the effective timing of the games: the consumers are moving simultaneously with firm \( b \) (with informed \( p_a \)) or with the duopoly (with uninformed \( p_a \)). Despite this, the equilibrium welfare implication for firm \( a \) is similar to that in the other competitive setting: due to softened competition, firm \( a \) can benefit from an increase in the quality of the rival firm.

5 Conclusion

Consumers with imperfectly-known product valuation can bear the uncertainty and make an uninformed purchase, or they can seek to acquire more relevant information before making the buying decision. Pre-purchase information acquisition activities are normally costly and can lead to endogenous heterogeneity in ex post valuations. Prices can exert direct influence on purchase decisions but not on decisions on pre-purchase information search. Nevertheless, firms can directly influence consumer search through investing in advertising campaigns to communicate their quality. However, many quality claims in advertising are unverifiable. So it is unclear how a firm should price and advertise its product when consumers can engage in pre-purchase information acquisition.

The first research issue we investigate in this paper is about the impact of pre-purchase information acquisition on firm pricing, and about how product quality may influence equilibrium information acquisition, firm profit, and consumer surplus. We consider a monopoly market in which ex ante homogenous consumers have uncertain product valuations and product quality determines the likelihood that the product is of high valuation. In general pre-purchase information acquisition can increase valuation heterogeneity, and may undermine the firm’s surplus extraction. On the other hand, the consumers’ incentive for information acquisition is non-monotonic in the anticipated price. As a result of the interaction of pre-purchase information acquisition and the firm’s pricing effort to extract consumer surplus, a unique mixed-strategy equilibrium may arise. This yields surprising results on the comparative statics analysis of product quality. Interestingly, the impact of quality on the equilibrium number of searching consumers can be non-monotonic.
Another surprising result is that an increasing quality may not necessarily be beneficial for the firm’s equilibrium profit. This is because greater quality may also imply higher valuation uncertainty, which can induce more information acquisition and reduce the firm’s ability to appropriate consumer surplus. Moreover, consumer surplus can be unfavorably affected by product quality. This unexpected finding obtains because higher quality increases equilibrium prices.

The second contribution of this paper is to present a novel explanation for the puzzling phenomenon that unverifiable quality claims in advertising can be informative. We demonstrate that the credibility of quality claims can arise endogenously when consumers’ valuations are influenced by pre-purchase information acquisition. This is because, relatively speaking, a high-quality firm may suffer less from consumer information search than a low-quality firm. In other words, inducing a higher perceived quality from consumers can be a “two-edged sword” for the firm, increasing consumers’ expected value while suppressing the firm’s ability to extract consumer surplus. It is this tradeoff that makes the firm report its quality truthfully.

It is important to emphasize that, under this mechanism of endogenous information acquisition, consumers can always buy even without search. Thus our explanation for informative advertising is fundamentally different from the traditional one in the literature that is based on the matching of firm and consumer types (e.g., Bagwell and Ramey 1993, Gardete 2013). Moreover, informative advertising in our paper results in the complete-information equilibrium. This outcome is socially more efficient than the partially informative equilibrium that may arise from restrictive communication under multidimensional cheap talk (e.g., Chakraborty and Harbaugh 2010, 2014).

The current research can be extended in several directions. Even though we expect the insights in the current paper will continue to hold, new findings may emerge. One simplifying assumption we make is that the firm can offer a take-it-or-leave-it posted price. It may be interesting to extend the analysis to other pricing mechanisms such as negotiation (e.g., automobile, real estate) and auction (e.g., media, procurement). The second direction for future study is to consider other types of strategic communication. For instance, when it is not legally feasible to misrepresent quality, firms can decide strategically whether to disclose their quality truthfully or to keep silent about it. It may be interesting to investigate whether pre-purchase information acquisition can result in strategic withholding of quality information. The third extension direction is to investigate the dynamics of consumer-firm interaction. We have taken in this paper a static approach as the first step to study the interaction between consumer information acquisition and firm pricing. It would be interesting, while non-trivial, to explore other issues that may arise in a dynamic setting (e.g., revolution of the mixed-strategy equilibrium, firm incentive for price commitment). Another fruitful direction for extension is to consider other competitive settings. We have considered in Section 4.2 two setups of competition where valuation uncertainty and information search involve only one firm. There
would be many new issues to consider if consumers are uncertain about both firms’ valuations. For example, how would consumers determine the sequence of information acquisition (e.g., starting with the high-quality or the low-quality firm)? How could firms use comparative advertising to influence consumer information acquisition? We hope that tractable models can be developed in future research to address these challenging issues.
Appendix

Proof of Lemma 1: Information acquisition takes place if and only if \( \tilde{p} \in [v_0 + \frac{c}{1-q}, v_1 - \frac{c}{q}] \). The interval is non-empty if and only if \( v_1 - \frac{c}{q} \geq v_0 + \frac{c}{1-q} \). This is equivalent to \( q \in [q_0, q_1] \), where \( q_0 = \frac{1}{2} \left( 1 - \sqrt{1 - \frac{4c}{v_0-v_1}} \right) \) and \( q_1 = \frac{1}{2} \left( 1 + \sqrt{1 - \frac{4c}{v_0-v_1}} \right) \). Note also that the necessary condition for \( q_0 < q_1 \) is \( c < (v_1 - v_0) / 4 \).

Proof of Lemma 2: Comparing the firm’s profits at \( p = v_0 \) and \( p = E[v|q] \) results in

\[
\pi(p = v_0; \alpha) \leq \pi(p = E[v|q]; \alpha) \\
\Leftrightarrow v_0 \leq (\alpha q + 1 - \alpha) E[v|q] \\
\Leftrightarrow \alpha \leq \alpha_0 \equiv \frac{E[v|q]-v_0}{(1-q)E[v|q]} = \frac{q v_0}{q (1-q) v_1 + (1-q)^2 v_0}.
\]

Next, comparing the firm’s profits at \( p = v_1 \) and \( p = E[v|q] \) leads to

\[
\pi(p = v_1; \alpha) \geq \pi(p = E[v|q]; \alpha) \\
\Leftrightarrow \alpha q v_1 \geq (\alpha q + 1 - \alpha) E[v|q] \\
\Leftrightarrow \alpha \geq \alpha_1 \equiv \frac{E[v|q]}{q v_1 + (1-q) v_0} = \frac{q v_1}{q (2-q) v_1 + (1-q)^2 v_0}.
\]

Similarly, the comparison of the firm’s profits at \( p = v_1 \) and \( p = v_0 \) yields

\[
\pi(p = v_1; \alpha) \geq \pi(p = v_0; \alpha) \\
\Leftrightarrow \alpha q v_1 \geq v_0 \\
\Leftrightarrow \alpha \geq \frac{v_0}{q v_1}.
\]

It can be verified that \( \alpha_0 \leq \alpha_1 \leq \frac{v_0}{q v_1} \) when \( q \leq \bar{q} \), and that \( \frac{v_0}{q v_1} \leq \alpha_1 \leq \alpha_0 \) when \( q \geq \bar{q} \), where \( \bar{q} = \frac{2 v_0}{\sqrt{v_1 (5 v_1 - 4 v_0) - (v_1 - 2 v_0)}} \). Therefore, when \( q \leq \bar{q} \), the optimal price is given by \( p = E[v|q] \) if \( \alpha \leq \alpha_0 \), \( p = v_0 \) if \( \alpha_0 \leq \alpha \leq \frac{v_0}{q v_1} \), and \( p = v_1 \) if \( \alpha \geq \frac{v_0}{q v_1} \). When \( q \geq \bar{q} \), the optimal price is \( p = E[v|q] \) if \( \alpha \leq \alpha_1 \) and \( p = v_1 \) if \( \alpha \geq \alpha_1 \).

Note that we have \( \bar{q} \in [q_0, q_1] \) if and only if the information search cost is low enough: \( c < v_0 \left[ \sqrt{v_1 (5 v_1 - 4 v_0) (v_1^2 - v_1 v_0 + v_0^2) + v_1 (v_1^2 - 5 v_1 v_0 + 3 v_0^2)} \right]^{2 (v_1 - v_0) (v_1 + v_0)^2} \). We assume that this condition holds throughout the paper, which will considerably simplify exposition.

Proof of Proposition 1: When \( q < q_0 \) or \( q > q_1 \), the proof follows directly from the discussion in
the text. When \( q \in [q_0, q_1] \), the only equilibrium is in mixed strategy under which the consumers are indifferent between searching and not searching, and the firm mixes between \( p = v_0 \) and \( p = E[v|q] \), or between \( p = v_1 \) and \( p = E[v|q] \). Moreover, the consumers’ probability of search is such that the firm is indifferent between the equilibrium prices, and the firm’s probabilities of charging the equilibrium prices are such that the consumers are indifferent between searching and not searching.

In particular, when \( q \in [q_0, q] \), it follows from Lemma 2 that the firm is indifferent between charging \( p = v_0 \) and \( p = E[v|q] \) if and only if \( \alpha = \alpha_0 \). In addition, if \( \alpha = \alpha_0 \), charging \( p = v_1 \) is dominated by \( p = v_0 \) or \( p = E[v|q] \). Furthermore, if the consumers’ probability of search is such that the firm’s profits are the same at \( p = v_1 \) and at \( p = E[v|q] \) (i.e., \( \alpha = \alpha_1 \)), the firm is better off deviating to charge \( p = v_0 \). This means that the only equilibrium is when the firm mixes between \( p = v_0 \) and \( p = E[v|q] \) (i.e., \( \beta_1 = 0 \)). It then follows from (3) that the consumers are indifferent between searching and not searching if and only if

\[
\beta_0 = \beta^* \equiv 1 - \frac{c}{q(1-q)(v_1-v_0)}.
\]

Similarly, when \( q \in [q, q_1] \), it can be readily proved that the only equilibrium is when the firm mixes between \( p = v_1 \) and \( p = E[v|q] \) (i.e., \( \beta_0^* = 0 \)), and that the indifference condition for the firm is \( \alpha = \alpha_1 \). The indifference condition for the consumers, \( \beta_1^* = \beta^* \), follows from (3).

**Proof of Proposition 2:** When \( q \in [q_0, q] \), the consumers’ equilibrium information acquisition probability is \( \alpha_0^* = \frac{E[v|q]-v_0}{(1-q)E[v|q]} = \frac{q_0 q}{(1-q)q + (1-q)^2 v_0} \). It follows that \( \frac{d\alpha_0^*}{dq} = \frac{(v_1-v_0)[v_0 + q^2 (v_1-v_0)]}{(1-q)^2 [v_0 + q (v_1-v_0)]} > 0 \).

When \( q \in [q, q_1] \), the equilibrium information acquisition probability is \( \alpha_1^* = \frac{E[v|q]}{q v_1 + (1-q)v_0} = \frac{q_0 q}{q2 - q + (1-q)^2 v_0} \). It follows that \( \frac{d\alpha_1^*}{dq} = \frac{(v_1-v_0)(q^2 v_1 - (1-q)^2 v_0)}{v_0 (v_0 - q^2 (v_1-v_0))} > 0 \). It can be verified that \( q^2 v_1 - (1-q)^2 v_0 \) is strictly increasing in \( q \), negative for \( q < q' \equiv \sqrt{v_0 \sqrt{v_1 + v_0}} \), and positive for \( q > q' \). In addition, we have \( q < q_1 \), which implies that \( \frac{d\alpha_1^*}{dq} > 0 \) when \( q \) is sufficiently close to \( q_1 \). Moreover, it can be shown that \( q^2 v_1 - (1-q)^2 v_0 \) evaluated at \( q = q \) is positive if \( v_1 < 4v_0 \) and negative if \( v_1 > 4v_0 \). This means that \( \frac{d\alpha_1^*}{dq} |_{q=q} > 0 \) if \( v_1 < 4v_0 \), and \( \frac{d\alpha_1^*}{dq} |_{q=q} < 0 \) if \( v_1 > 4v_0 \).

**Proof of Proposition 4:** It is straightforward that the equilibrium consumer surplus is zero except in the range \( q \in [q_0, q] \). In this range the equilibrium consumer surplus is given by:

\[
E(CS) = \beta^* \{ E[v|q] - v_0 \} = q(v_1 - v_0) - \frac{c}{1-q}.
\]

It follows that \( \frac{dE(CS)}{dq} = v_1 - v_0 - \frac{c}{(1-q)^2} \). It can be verified that this derivative, when evaluated at \( q = q_0 \), is positive. It is easy to see that the second-order derivative of \( E(CS) \) with respect to \( q \), \( \frac{d^2E(CS)}{dq^2} \), is negative. It can also be verified that \( \frac{dE(CS)}{dq} |_{q=q} = v_1 - v_0 - \frac{c}{(1-q)^2} \) is positive if \( v_1 - v_0 \) is sufficiently large, and negative if \( v_1 - v_0 \) is sufficiently small.
Proof of Proposition 5: We will consider each of the parameter regions where at least one of the firm types has an incentive to misrepresent its true type. Consider first the regions where $q_H < q_0$ or $q_H > q_1$. Under the informative advertising equilibrium, for the high-quality product, no consumer would acquire information and its expected profit would be $E[v|q_H]$. The low-type firm’s expected profit would be $E[v|q_L]$, $v_0$, or $[\alpha^*_1(q_L)q_L + 1 - \alpha^*_1(q_L)]E[v|q_L]$, when $q_L < q_0$ or $q_L > q_1$, respectively. Under the informative advertising equilibrium, no consumer would acquire information for the low type and its expected profit is $E[v|q_L]$. In contrast, for the high type, the consumers’ equilibrium probability of search would be $\alpha^*_0(q_H)$ and the firm’s equilibrium profit would be $[\alpha^*_0(q_H)q_H + 1 - \alpha^*_0(q_H)]E[v|q_H] = v_0$. However, the high type can profitably deviate by claiming to be the low type and earning a higher profit $E[v|q_H]$. It is straightforward that the high type does not have an incentive to imitate the low type.

Consider then the region $q_0 < q_L < q_H < q$. Under the informative advertising equilibrium, no consumer would acquire information for the low type and its expected profit is $E[v|q_L]$. In contrast, for the high type, the consumers’ equilibrium probability of search would be $\alpha^*_0(q)$ and the firm’s equilibrium profit would be $[\alpha^*_0(q)q + 1 - \alpha^*_0(q)]E[v|q] = v_0$, for type $q \in Q$. However, the high type can earn a profit $[\alpha^*_0(q_L)q_H + 1 - \alpha^*_0(q_L)]E[v|q_L]$, where the two terms on the right-hand side capture the low type’s profit if it deviates to charge $E[v|q_H]$ or $v_1$, respectively. This follows from $v_0 = [\alpha^*_0(q_H)q_H + 1 - \alpha^*_0(q_H)]E[v|q_H]$ and $v_0 > \alpha^*_0(q_H)q_Hv_1$ for $q_0 < q_H < q$.

In each of the regions $q_L < q_0 < q_H < q$ and $q_0 < q_L < q_H < q$, type $L$ does not have incentive to mis-claim its type. It suffices to show that $v_0 > \max\{[\alpha^*_0(q_H)q_L + 1 - \alpha^*_0(q_H)]E[v|q_H], \alpha^*_0(q_H)q_Lv_1\}$, where the two terms on the right-hand side capture the low type’s profit if it deviates to charge $E[v|q_H]$ or $v_1$, respectively. This follows from $v_0 = [\alpha^*_0(q_H)q_H + 1 - \alpha^*_0(q_H)]E[v|q_H]$ and $v_0 > \alpha^*_0(q_H)q_Hv_1$ for $q_0 < q_H < q$.

Finally, consider the region $q < q_L < q_H < q_1$. Under the informative advertising equilibrium, the consumers’ probability of search would be $\alpha^*_1(q)$ and the firm’s expected profit would be $[\alpha^*_1(q)q + 1 - \alpha^*_1(q)]E[v|q] = \alpha^*_1(q)qv_1$, for $q \in Q$. If the low type claims to be the high type, its expected profit would become $\max\{[\alpha^*_1(q_L)q_H + 1 - \alpha^*_1(q_H)]E[v|q_H], \alpha^*_1(q_H)q_Lv_1\}$. When $\alpha^*_1(q_H) > \alpha^*_1(q_L)$, the deviation would lead to more consumer search and hence higher expected profit since $\alpha^*_1(q_H)q_Lv_1 > \alpha^*_1(q_L)q_Hv_1$. When $\alpha^*_1(q_H) < \alpha^*_1(q_L)$, the expected profit under deviation would still be higher since $[\alpha^*_1(q_H)q_L + 1 - \alpha^*_1(q_H)]E[v|q_H] > [\alpha^*_1(q_L)q_L + 1 - \alpha^*_1(q_L)]E[v|q_L]$.

Consider then the high-type’s incentive of deviation in this region. Its expected profit under deviation would become $\max\{[\alpha^*_1(q_L)q_H + 1 - \alpha^*_1(q_L)]E[v|q_L], \alpha^*_1(q_L)q_Hv_1\}$. When $\alpha^*_1(q_L) > \alpha^*_1(q_H)$, the deviation would result in more consumer search and hence higher expected profit since $\alpha^*_1(q_H)q_Lv_1 > \alpha^*_1(q_H)q_Hv_1$. Alternatively, when $\alpha^*_1(q_L) < \alpha^*_1(q_H)$, the expected profit under deviation would be lower. Note that the difference of type-$H$ firm’s profits between truth telling and
deviation is \([\alpha_1^*(q_H)q_H + 1 - \alpha_1^*(q_H)] E[v|q_H] - [\alpha_1^*(q_L)q_H + 1 - \alpha_1^*(q_L)] E[v|q_L]\). It can be verified that this is positive.

**Proof of Proposition 6:** Under the informative advertising equilibrium, for the low-quality product, no consumer would acquire information and its expected profit would be \(E[v|q_L]\). For the high-quality product, the consumers’ equilibrium probability of search would be \(\alpha_1^*(q_H)\) and the firm’s equilibrium profit would be \([\alpha_1^*(q_H)q_H + 1 - \alpha_1^*(q_H)] E[v|q_H] = \alpha_1^*(q_H)q_H v_1\).

Consider first the low-type firm’s incentive of deviation. If it deviates to claim to be the high type, its expected profit would be \([\alpha_1^*(q_H)q_L + 1 - \alpha_1^*(q_H)] E[v|q_H]\) if it sets \(p = E[v|q_H]\), and \(\alpha_1^*(q_H)q_L v_1\) if \(p = v_1\) is charged. It can be readily verified that \([\alpha_1^*(q_H)q_L + 1 - \alpha_1^*(q_H)] E[v|q_H] > \alpha_1^*(q_H)q_L v_1\) if and only if \(\frac{q_H v_1 + (1 - q_L)E[v|q_H]}{q_H v_1 + (1 - q_L)E[v|q_H]} < 1\), which indeed holds because \(v_1 > E[v|q_H]\) and \(q_H > q_L\). So the low-type firm’s best deviating strategy is to charge \(p = E[v|q_H]\).

Therefore, the difference of type-\(L\) firm’s profits between truth telling and best deviation is \(E[v|q_L] - [\alpha_1^*(q_H)q_L + 1 - \alpha_1^*(q_H)] E[v|q_H]\). This can be simplified as \(\frac{A-Bq_L}{q_H(2-q_H)\Delta + v_0}\), where \(\Delta = v_1 - v_0\), \(A = -(1 - q_H)q_H^2\Delta^2 + q_H v_0 \Delta + v_0^2\), and \(B = -2q_H(1 - q_H)\Delta^2 - (1 - 2q_H)v_0 \Delta + v_0^2\). It can be readily shown that \(A > B\), \(A > 0\) if and only if \(\Delta_{v_0} < \frac{1 + \sqrt{5 - 4q_H}}{2q_H(1 - q_H)}\), \(B > 0\) if and only if \(\Delta_{v_0} < \frac{1 + \sqrt{5 - 4q_H}}{2q_H(1 - q_H)}\). It follows that \(A - Bq_L > 0\) when \(A > 0\), or when \(A < 0\) and \(q_L > A/B\). As a result, the incentive compatibility condition for the low type not to deviate is:

\[
\frac{\Delta_{v_0}}{v_0} < \frac{1 + \sqrt{5 - 4q_H}}{2q_H(1 - q_H)} \land \left(\frac{\Delta_{v_0}}{v_0} > \frac{1 + \sqrt{5 - 4q_H}}{2q_H(1 - q_H)} \land q_L > \frac{-(1 - q_H)q_H^2\Delta^2 + q_H v_0 \Delta + v_0^2}{-2q_H(1 - q_H)\Delta^2 - (1 - 2q_H)v_0 \Delta + v_0^2}\right).
\]

Consider then the high-type firm’s incentive of deviation. If it deviates to claim to be the low type, its expected profit would be \(E[v|q_L]\), which is constant in \(q_H\). Note that type-\(H\) firm’s profit under truth telling, \(\alpha_1^*(q_H)q_H v_1\), is strictly increasing in \(q_H\). Therefore, the incentive compatibility condition for the high type not to deviate is:

\[
q_H > \frac{2q_L \Delta^2 + v_0 \Delta - v_0^2 + \sqrt{v_1[(5\Delta + v_0)v_0^2 + 4q_L(q_L \Delta + 2v_0)\Delta^2]} - 2\Delta(1 + q_L)\Delta + 2v_0}{2\Delta(1 + q_L)\Delta + 2v_0}.
\]

**Proof of Proposition 7:** Under the informative advertising equilibrium, for the low-quality product, the consumers’ probability of information acquisition would be \(\alpha_0^*(q_L)\) and the firm’s expected profit would be \([\alpha_0^*(q_L)q_L + 1 - \alpha_0^*(q_L)] E[v|q_L] = v_0\). For the high-quality product, the consumers’ equilibrium probability of search would be \(\alpha_1^*(q_H)\) and the firm’s equilibrium profit
would be \([\alpha_1^*(q_H)q_H + 1 - \alpha_1^*(q_H)]\) \(E[v|q_H] = \alpha_1^*(q_H)q_Hv_1\).

Similar to the region where \(q_L < q_0\) and \(q_H \in (\overline{q}, q_1)\), the low-type firm’s best deviating strategy is to charge \(p = E[v|q_H]\). As a result, the difference of type-\(L\) firm’s profits between truth telling and best deviation is \(v_0 - [\alpha_1^*(q_H)q_L + 1 - \alpha_1^*(q_H)]\) \(E[v|q_H]\). So the incentive compatibility condition for the low type not to deviate is:

\[
q_L < 1 - \frac{E[v|q_H] - v_0}{\alpha_1^*(q_H)E[v|q_H]} = \frac{q_H(v_1 - v_0)[v_0 - q_H(1 - q_H)(v_1 - v_0)] + v_0^2}{q_Hv_1 + (1 - q_H)v_0}.
\]

Consider then the high-type firm’s incentive of deviation. If imitating the low type leads to a larger number of searching consumers (i.e., \(\alpha_0^*(q_L) > \alpha_1^*(q_H)\)), the high-type firm can profitably deviate from truthful advertising by claiming to the be low type and charging \(p = v_1\). If the deviation results in a smaller number of searching consumers, the best deviating strategy is to charge \(p = E[v|q_L]\). As a result, the high-quality firm would not deviate if \([\alpha_1^*(q_H)q_H + 1 - \alpha_1^*(q_H)]\) \(E[v|q_H] > [\alpha_0^*(q_L)q_H + 1 - \alpha_0^*(q_L)]\) \(E[v|q_L]\). This can be re-written as:

\[
\frac{v_1 - v_0}{v_0} > \frac{q_H - 2(1 - q_L)q_H^2 + \sqrt{q_H^2 + (1 - 2q_L)q_H}[4(1 - q_L)q_H^3 - 8(1 - q_L)q_H^2 + (5 - 6q_L)q_H + q_L^2]}{2q_H^2(1 - 2q_H)q_H - q_H(3 - 2q_H)q_H + q_H}.
\]

**Proof of Proposition 11:** Given the discussion in the text, it suffices to show that there is an \(\alpha \in (0, 1)\) that solves (10). Note first that \(y_f(\alpha)\) is weakly increasing in \(\alpha\), equal to zero for \(\alpha < E[v|q] - v_0\), and converging to max \(\{\frac{v_0 - qE[v|q]}{1 - q}\}\) as \(\alpha \to 1\). In addition, \(y_h(\alpha)\) is weakly decreasing in \(\alpha\), equal to zero for \(\alpha > \frac{E[v|q]}{qv_1 + (1 - q)E[v|q]} \in (0, 1)\), and converging to \(E[v|q]\) as \(\alpha \to 0\). Therefore, there must be a unique \(\hat{\alpha} \in (0, 1)\) such that \(y_f(\alpha) < y_h(\alpha)\) for \(\alpha < \hat{\alpha}\) and \(y_f(\alpha) \geq y_h(\alpha)\) for \(\alpha \geq \hat{\alpha}\). Moreover, let \(\hat{\alpha}\) be the unique solution to \(y_f(\alpha) = x\). Since \(x\) is sufficiently close to \(v_0\), \(y_f(\alpha) \leq \max \{\frac{v_0 - qE[v|q]}{1 - q}, 0\} < v_0\), and \(y_h(\alpha)\) converges to \(E[v|q] > x\) as \(\alpha \to 0\), we must have \(\hat{\alpha} \in (0, \hat{\alpha})\).

Consider the two sides of (10). At \(\alpha = \hat{\alpha}\), we have \(y_f(\alpha) = y_h(\alpha)\) such that the left-hand side is smaller than the right-hand side. However, at \(\alpha = \hat{\alpha}\), the left-hand side is strictly positive but the right-hand side is equal to zero. As a result, there must be an \(\alpha \in (\hat{\alpha}, \hat{\alpha})\) that solves (10), because of the continuity of both sides in \(\alpha\).
References


