The Monetary Approach to the Exchange Rate:
An empirical look at the relationship between the UK and the US

Miguel Maria Vidigal Dias, 25966
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Abstract

Using data from 1982 to 2017 for the UK and the US and using a co-integrated SVAR model, this study finds some evidence of the applicability of the Monetary Approach to the Exchange Rate in a long-run setting, although not being able to find such a strong relationship in the short-run. The model also finds some encouraging results when used in a forecasting exercise and compared to a random-walk. Furthermore, the application of restrictions for a forward-looking variant to the monetary approach is also tested, albeit the results show the non-applicability of said restrictions.

Keywords: Exchange Rates; Monetary; VAR; Co-Integration

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1 Introduction & Motivation

The exchange rate between two currencies has been at the focus of economic research for a long time. A large chunk of the analysis started after the collapse of the Bretton Woods system (as exchange rates were determined in a different fashion) in the early 1970s. Its studies are of the utmost importance, as it is one of the main indicators of the performance of an economy, and not only that but can influence said performance, and predicting its behavior and the forces behind it has always been a rather tough exercise. Although many models have been suggested and empirically tested, a clear cut consensus on what are the variables that most drive the exchange rate movement patterns and its values at a given point in time has mostly yet to have been reached.

One of the models that has been proposed to explain the variations, in this case, of the nominal exchange rate, is the one brought forth by Bilson (1978), which uses some macroeconomic fundamentals, in log terms, namely the nominal interest rates, the money supplies and the outputs for two different countries (considering flexible prices), and theorizes that the log country differences of these variables will make it able to determine the nominal exchange rate. This model is particularly interesting because, amongst other things, it can be utilized to measure the direct impact of the variables in the exchange rate, and thus may facilitate in the assessment of how a policy shift may affect the exchange rate between two economies.

So, in order to try to pick up on previous work in testing out this approach, this model will be used to test out the exchange rate between two of the most relevant currencies in the global economy, the British Pound and the US Dollar, as the exchange rate between them with this particular model has yet to have been studied in such a great detail. The amount of information available on the variables the model focuses on is also a great boost in confidence that the empirical results regarding the fit and explanatory power of the model will have additional
robustness to them as well. Overall, this study adds to the available literature due to its sample size and testing out the Monetary Approach in various ways, with a great deal of detail, which will be explain up ahead.

Considering this, the study will be focusing on the following: Firstly, what will be done is to define the methodology, both theoretical and empirical, to be able to estimate the static MAER\(^1\) model within a co-integrated VAR framework, and then to estimate not only the long but also the short-run coefficients to look for the theoretical relationships between the variables. After this, the application for a variant of the model, the Forward-Looking MAER will be tested through restrictions on the estimation of the variables. Lastly, a forecasting exercise will be carried out with the first-mentioned model, to evaluate its predicting behavior when compared to a random walk.

2 Literature Review

To start off the review on the relevant literature regarding the topic at hand, one should first distinguish between the two types of relevant literature that can be examined: the first one should be literature regarding the actual nominal exchange rate between the US Dollar and British Pound, so as to get a better sense of how well the model that is going to be applied will work; the second one is regarding the actual model itself, the monetary approach to the exchange rate determination, and how it has been applied and utilized throughout the years to explain its behavior. First focusing solely on the methodology literature review, earlier reviews of this type of model can be seen in studies such as Taylor (1995), which amongst others, identifies the model of a monetary approach with flexible prices and compares its forecasting performance with other models and against is the standard of long-run forecasting predictions

\(^{1}\)Monetary Approach to the Exchange Rate
One of the biggest inspirations regarding this work is the study by Loría et al. (2010) in which the nominal exchange rate between the Mexican peso and the US dollar is studied through a monetary approach model with the caveat that the main focus is not only establishing a long-run relationship between the exchange rate and the fundamentals (i.e. the macroeconomic variables that the model encompasses), but also in a short-run manner, to see if the model’s expected outcomes are also verified in the contemporaneously and if they adjust accordingly, and not only that but also how the exchange rate might react to shocks in fundamentals in the short to medium-run. Other applications such as this, that apply the MAER models for exchange rate determination come in recent literature such as Chin et al. (2007), that applies this approach to the Phillipines, Effiong (2014), which uses a very similar procedure to that of Loría et al. (2010) but applies it to Nigeria, Papadamou and Markopoulos (2012) which use a similar model to determine and explain the behaviors of the exchange rate between Norwegian Kroner and the US Dollar but adding in oil prices; while not all of these use the VAR methodology setting to investigate their chosen countries and respective exchange rate, they are very similar in the way that they all use an equation very closely related to the one from the MAER.

Also contributing a lot to this study is the work in MacDonald and Taylor (1993), that just like the previous one, uses the monetary approach to build upon the relationship between the macroeconomic variables and the exchange rate, through the PPP equation and the money supply to money demand equation in the economy, and then adds to it the UIP relationship, this is, the equation that states (if it holds) that the ratio between the interest rates in two different countries must be the same as the ratio between the expected spot exchange rate at a certain point in time and the spot exchange rate today. Approaches such as the previous one (and others) are compiled in more studies by McDonald, mainly from MacDonald (2007), where more elaborate...
studies of the forward-looking model for the exchange rate are explained even more in-depth, and where not only the empirical methods are studied for the creation of VAR and then using a VECM to study the co-integration relationships of the variables, but also evaluates the various empirical studies that have come up regarding this study and makes considerations regarding the forecasting ability of these types of models.

For the second part of the literature review, the focus will now be placed on studies that have researched models that try to determine the behavior of the nominal exchange rate between the UK and the US, as to be able to get a grasp not only on if the model that is being investigated will is regarded as a good fit (or which are) for the study and forecast of this specific exchange rate. To lay some initial groundwork for the model types used to determine the nominal exchange rate, this part covering mostly just the pound sterling side of things, one can take a look first at Fisher et al. (1990), that compiles several models used by various entities in the UK, such as the Bank of England, Her Majesty’s Treasury and the National Institute of Economic and Social Research, in which variations of the model of the monetary approach is used by these institutions (or at least some of its components, as is the case for the PPP equation for some of them), serving as a good starting point of validation for the type of model that will henceforth be explored.

Building upon this, Cheung et al. (2005) and Cheung et al. (2017) also tackle the determination and forecast the nominal exchange rate between the GBP and the USD, using for this purpose several models, including some that are used to get to the main model that will be explored further into this study, such as the PPP equation. They also use different models such as a sticky-price monetary model or a productivity differential model. These studies present their relevance due mostly to the study of the forecasting powers of the PPP equation for the nominal exchange rate being tested and therefore testing out its validity predicting the behavior
of this exchange rate.

Turning then to Alquist and Chinn (2008), that among others, explores the same exchange rate as this study by using a model very similar to the one that is the focus here. What this explores is the fitness of the model for the same countries like the study at hand but adds to the model some new components in order to further enhance perhaps the overall fit and forecasting powers.

To finish off this literature review, it is also important to mention that, since one of the components of this study will also to be forecasting with the final model achieved, other papers besides some of the already highlighted ones (Taylor, 1995; MacDonald and Taylor, 1993; Cheung et al., 2005) have also focused on this type of exercise and will also be taken into account, such as, for example, Reinton and Ongena (1999), that focus on forecasting with a single equation of the monetary exchange models.

3 Data & Methodology

3.1 Deriving the theoretical model – the Monetary Approach to the Exchange Rate

Following the reasoning presented first by Bilson (1978) and then utilized in such studies as Taylor (1995), MacDonald and Taylor (1993) and Loría et al. (2010), the determination of the essential equations of the model that will be used will mainly come from the PPP equation (with this equation holding being one of the main assumptions of this model). Considering values marked with * as US variables and the remaining ones as UK variables (making the home country the UK in this analysis), one can first consider the real exchange rate equation:

\[ \theta = \frac{E^*}{P} \]  

(1)
This shows that the real exchange rate is given by the price ratio of the home country’s price index and foreign country’s (\( P \) and \( P^* \)) multiplied by the nominal exchange rate (E). Now, as said before, if the PPP holds and, therefore, the LOOP\(^2\) hypothesis is to hold as well, then the real exchange rate (\( \theta \)) has to be equal to 1, and so:

\[
E = \frac{P}{P^*} \quad \text{or in log terms } e = \ln\left(\frac{p}{p^*}\right)
\]  

(2)

Equation 2, or the log PPP equation, is then one of the main building blocks of the MAER model. Turning then to the monetary side of the model, which will require representation for both countries’ money markets:

\[
\frac{M}{P} = L(i,Y) \quad \frac{M^*}{P^*} = L(i^*,Y^*)
\]  

(3)

The equations above in (3) represent, respectively, the home and foreign money market (in this case, the United Kingdom and the United States), and tell us that the amount of real money (seeing as that one has M, the money supply, divided by \( P \), the price level) in a country is a function of both the interest rate and output, which creates the money demand function (\( L \)). This can be represented in log terms as:

\[
p = m - l, \quad p^* = m^* - l^*
\]  

(4)

If a linear relationship (for log terms) is assumed, such as \( l(i,y) = \alpha + \eta y - \gamma i \) and then treating log money demand, \( l \), endogenously, equation 5 is created:

\[
l^* - l = \eta(y^* - y) + \gamma(i - i^*)
\]  

(5)

Now, if one plugs equations 3 and 4 into equation 5 (all in log terms), and then inputs equation 5 into the expression from before, the result will be the main equation of the MAER. Also, to get the same relationship as the \( m \) and \( i \) variables, the order of the output variables is

\[^2\text{LOOP is the Law of One Price, which states that (given certain assumptions about both countries) the price levels should equivalent when taken into account in the same currency, i.e. the real exchange rate is 1}\]
switched and \( \eta \) converted into \( \theta \), with \( \theta = -\eta \), and therefore \( \theta < 0 \):

\[
e = (m - m^*) + \theta(y - y^*) + \gamma(i - i^*)
\]  

The previous equation (Loría et al., 2010) represents the main relationships for the monetary approach to the exchange rate. The predicted relationships will then be, in order to keep the money market conditions in equilibrium (3), that an increase in one of the variables will cause the price level to adjust, which will then cause the exchange rate to adjust as well (equation 2). This means that when the GDP differential by increases the nominal exchange rate decreases, and increases when there is an increase in the interest rate or money supply differentials.

Regarding the econometric methodology used for the MAER, the fairly standard methods will be applied. For the recovery of the structural coefficients of the model, the representation of the structural matrix of coefficients follows a representation as the ones presented by, for example, Stock and Watson (2001), and therefore, the Structural Model, in its simplest form (and assuming just one lag), can be represented by:

\[
Ay_t = A^1y_{t-1} + B\epsilon_t
\]  

Since the goal is to establish a long-run model but also to recover the short-run effects between these variables to test whether or not the relationship in (6) can be found, one first has to estimate the Vector Error Correction Model, and then define the restrictions in the structural factorization matrices in (7), \( A \) and \( B \). This is done in such fashion due to the individual and the difference variables being not following a stationary path, meaning that the best procedure to follow will be one as proposed by Amisano and Giannini (1997), in which a VAR representation can be taken as a starting ground in order for then to specify the structural factorization. As one is dealing with non-stationary series\(^3\), what is done first is to determine the co-integration rank

\(^3\)This is proven in the Estimation section, that both the individual country variables and their differences are \( I(1) \)
(if there is one) (Johansen, 1995), to then estimate the VEC, which would make it possible to
analyze the variables in levels (if they co-integrate with each other), and lastly, estimate the
structural factorization of the model, converting it into a SVEC (a Structural VEC), where one
can specify the short-run restrictions and capture the contemporaneous effects.

The co-integrated VAR equation can be represented following Johansen (1995), one can
define a VEC representation as (considering a VAR of order $p$):

$$\Delta y_t = \Pi y_{t-1} + \sum_{j=1}^{p-1} [\Gamma_j \Delta y_{t-j} + \epsilon_t]$$

(8)

3.2 Deriving the theoretical model – the FMAER

Building upon the already defined equation of the static flexible-price monetary approach to
the exchange rate, and following the works of MacDonald and Taylor (1993), in order to test
two variants of the model, this work will also try to apply the various restrictions in order to get
a grasp on if the FMAER (Forward-Looking Monetary Approach to the Exchange Rate) model
can be applied for the Dollar/Pound Exchange Rate. Starting off by defining the Uncovered
Interest Rate Parity, to add to the model, in log terms:

$$i - i^* = E(e_{t+1}|I_t) - e_t \iff i - i^* = E(\Delta e_{t+1}|I_t)$$

(9)

Considering the equation above (9) - where $E(.)|I_t$ denotes the conditional expected
value given a set of information $I_t$ - and inserting it into the final equation of the static model
(6) will then yield the basis for the new equation that represents the FMAER, effectively sub-
stituting the interest rate differentials by the expectation of the evolution of the exchange rate.

Using the aforementioned equation (9), one can get to a new exchange rate determination
equation:

$$e_t = (1 + \gamma)^{-1} x_t + \gamma(1 + \gamma)^{-1} E(e_{t+1}|I_t), \text{ with } x_t = (m - m^*) + \theta(y - y^*)$$

(10)
Which, solving forward and imposing the transversality condition, \( \lim_{j \to \infty} e_t = [\gamma(1 + \gamma)^{-1}]^j E(e_{t+j}|I_t) \) yields:

\[
e_t = (1 + \gamma)^{-1} \sum_{j=0}^{\infty} [\gamma(1 + \gamma)^{-1}]^j E(x_{t+j}|I_t)
\] (11)

This last equation will then be the basic equation of the FMAER, which involves solving the expected variables forward. This will then have an implication, as pointed out by the analysis applied by Campbell and Shiller (1987), and that is that the exchange rate should have a co-integration relationship with \( x_t \) (the forcing variables). To check for this, one turns to the following equation:

\[
e_t - x_t = \sum_{j=0}^{\infty} [\gamma(1 + \gamma)^{-1}]^j E(\Delta x_{t+j}|I_t)
\] (12)

As MacDonald and Taylor (1993) point out, the right-hand side should be \( I(0) \), if the variables it contains (in \( x_t \)), are \( I(1) \) (which will be checked), and also if the forecasting errors are stationary, which they also point out should be the case. This implies that \( e_t \), being an \( I(1) \) series, should be co-integrated with the variables in \( x_t \) and therefore, such an equation should be verified:

\[
L_t = e_t - (m_t - m_t^*) - \theta(y_t - y_t^*) \sim I(0)
\] (13)

And thus, one of the steps in testing out the validity of this variant of the model, as specified by the literature, is to check for a co-integration equation between the above variables. This will also not be inconsistent with a co-integration equation present in the static version of the model, the MAER, meaning that the results achieved for the previous specifications are not rendered invalid.

The next step then will be, following again Campbell and Shiller (1987)’s method, will be to test for a more specific restriction. Starting by testing the stationarity of both \( L_t \) and \( x_t \), if this is guaranteed, then there is a Wold representation (Hannan, 2009) through which a VAR can be constructed such that by defining \( s_t \), with lag length \( p \), it is represented as:
\[ s_t = F_{s_{t-1}} + z_t \text{ with } s_t = \left[ \Delta x_t, ..., \Delta x_{t-p+1}, L_t, ..., L_{t-p+1} \right] \] (14)

\[ L_t = g'_t s_t, \Delta x_t = h'_t s_t \] (15)

Setting up the equation to forecast \( s_t \), and defining \( H_t \) as an information set that contains the contemporaneous and lags of the variables defined in \( s_t \) one then has that:

\[ E(s_{t+k}|H_t) = F^k s_t \] (16)

If then one applies both sides of equation 12 to \( H_t \), while also using the law of iterated expectations\(^4\) and furthermore adds (14)-(16), the expression that needs to hold as a restriction for the VAR model is reached (Campbell and Shiller, 1987; MacDonald and Taylor, 1993):

\[ g'_s t = \sum_{j=0}^{\infty} \left( \frac{\gamma}{1 + \gamma} \right)^j h'_F s_t \rightleftharpoons g'_s t = h'_F F(I - \psi F)^{-1} s_t, \psi = \frac{\gamma}{1 + \gamma} \] (17)

\[ g'_s - h'_F F(I - \psi F)^{-1} = 0 \rightleftharpoons g'_s (I - \psi F) - h'_F F = 0 \] (18)

Through simplification, one can define a “theoretical spread variable” (MacDonald and Taylor, 1993) using the previous restriction in 18 and thus, the hypothesis test then becomes equivalent to testing if the initially created spread variable \( L_t \) will behave in such a fashion that the FMAER model can be properly defined.

\[ L^*_t = h'_F F(I - \psi F)^{-1} s_t \] (19)

\[ H_0 : L_t = L^*_t \] (20)

As for the actual tests, the parameters that need to be included, this is, the elasticities of both the output and the interest rate variables have to be firstly estimated to test for the restriction above; the first one can be directly be estimated from the co-integration equation that will be tested out with the Johannsen method for \( L_t \), (MacDonald and Taylor, 1993); as for the second one, several elasticities will be used to test out the restrictions (MacDonald and Taylor, 1993; 4Meaning that \( E(E(.|I_t)|H_t) = E(.|H_t) \), if \( H_t \subseteq I_t \)
Bilson, 1978), including the one coming from the first variant of the model, the MAER.

### 3.3 Data

Regarding the data set that will be utilized, it is concentrated in four main variables that will be then transformed into log form and the difference of the log variables for the two countries: the average quarterly nominal exchange rate between the two currencies ($/£) retrieved from the FRED database; the money supply will be represented by the quarterly M2 (Cheung et al., 2017) (retrieved from FRED for the US and the Bank of England for the UK⁵); the quarterly Constant Price GDP (Effiong, 2014) - with the GDP variables (current prices) coming from, again, the FRED database, and the GDP Deflator (base year 2012), from OECD; and finally the interest rate represented by the quarterly average interest rate for 3-Month Treasury Bonds (Cheung et al., 2017) for both the UK and the US (US-Yahoo Finance; UK-Bank of England). This all amounts to a sample size of 139 observations, from 1982Q4 to 2017Q2.⁶ To then get the data ready for the estimation and restriction testing exercises, the retrieved data is seasonally adjusted as to not have seasonality interfere with the coefficients obtained.

### 4 Estimation & Results

#### 4.1 Estimation of the MAER model

The first step in the estimation process of this work (as the process will involve estimating a long-run relationship between the variables utilizing a co-integration technique) is to check for the adequacy of the data in terms of stationarity because, in order to estimate a VAR model in error correction form, one has to first guarantee that the variables do not follow a stationary

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⁵M2 for the UK is given by the Retail component of M4, as defined by the Bank of England.

⁶This sample size is due to the data availability. More on this in the limitations sub-section of the paper.
path. For this purpose, tests are run for both the individual and difference variables (Table 1).

Table 1: Unit root testing for individual variables and country differences

<table>
<thead>
<tr>
<th></th>
<th>ADF</th>
<th>PP</th>
<th>DF-GLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-1.090</td>
<td>-1.298</td>
<td>-0.588</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>-6.688</td>
<td>-11.61</td>
<td>-6.464</td>
</tr>
<tr>
<td>$y^*$</td>
<td>-1.828</td>
<td>-1.967</td>
<td>-0.864</td>
</tr>
<tr>
<td>$\Delta y^*$</td>
<td>-5.356</td>
<td>-8.086</td>
<td>-5.385</td>
</tr>
<tr>
<td>$i$</td>
<td>-1.860</td>
<td>-1.404</td>
<td>-1.637</td>
</tr>
<tr>
<td>$\Delta i$</td>
<td>-6.612</td>
<td>-6.541</td>
<td>-5.979</td>
</tr>
<tr>
<td>$i^*$</td>
<td>-1.339</td>
<td>-1.454</td>
<td>-0.922</td>
</tr>
<tr>
<td>$\Delta i^*$</td>
<td>-6.071</td>
<td>-10.929</td>
<td>-6.025</td>
</tr>
<tr>
<td>$m$</td>
<td>-3.020</td>
<td>-2.781</td>
<td>0.122</td>
</tr>
<tr>
<td>$\Delta m$</td>
<td>-9.882</td>
<td>-9.909</td>
<td>-8.995</td>
</tr>
<tr>
<td>$m^*$</td>
<td>-0.9280</td>
<td>-1.320</td>
<td>-1.258</td>
</tr>
<tr>
<td>$\Delta m^*$</td>
<td>-7.903</td>
<td>-13.52</td>
<td>-7.547</td>
</tr>
<tr>
<td>$e$</td>
<td>-3.002</td>
<td>-2.306</td>
<td>-3.047</td>
</tr>
<tr>
<td>$\Delta e$</td>
<td>-8.307</td>
<td>-7.896</td>
<td>-7.193</td>
</tr>
<tr>
<td>$(y - y^*)$</td>
<td>-3.349</td>
<td>-3.349</td>
<td>-2.750</td>
</tr>
<tr>
<td>$\Delta(y - y^*)$</td>
<td>-7.903</td>
<td>-13.52</td>
<td>-7.547</td>
</tr>
<tr>
<td>$(i - i^*)$</td>
<td>-2.539</td>
<td>-1.891</td>
<td>-2.374</td>
</tr>
<tr>
<td>$\Delta(i - i^*)$</td>
<td>-5.499</td>
<td>-5.480</td>
<td>-11.39</td>
</tr>
<tr>
<td>$(m - m^*)$</td>
<td>-0.750</td>
<td>-0.392</td>
<td>0.176</td>
</tr>
<tr>
<td>$\Delta(m - m^*)$</td>
<td>-9.657</td>
<td>-9.688</td>
<td>-6.721</td>
</tr>
</tbody>
</table>

Note: Every test is estimated with a constant and a trend except for the tests regarding $i - i^*$, $i^*$, $i$, and their respective first differences (estimated with just a constant). Values in bold indicate a rejection of the null hypothesis of having a unit root at a 5% level.

ADF: Augmented Dickey-Fuller; PP: Phillips Perron; DF-GLS: Dickey-Fuller GLS

As it can be seen from the table above (focusing especially on the country differences and exchange rate variables), the tests indicate that the series present unit roots in levels but do not in first difference, indicating that they are $I(1)$. Knowing this, one can now turn to the actual estimation of the VAR model in its error correction form, which will be done in three different stages: the first one is to determine not (MacDonald and Taylor, 1993) only the lag
length through the appropriate tests - Schwarz, Akaike, and Hannah-Quinn Information criteria (Loría et al., 2010). The lag length chosen was 1 lag for the VEC model, and results for these statistics can be found in the Appendix (Appendix 1).

Regarding the co-integration exercise, the fairly common in literature tests are applied, such as the TRACE and the Maximum Eigenvalue statistics (testing co-integration vectors with no trend and no intercept). The results are presented in Table 2, which points to the presence of a single co-integration equation (Plotted in Appendix 8). An additional step, in order to determine the appropriate type of co-integration function, is to look at the Schwarz and Akaike Information criteria, which will indicate the most valid way to estimate the equation. (values can be found in the annexes in Appendix 2).

Table 2: Co-integration rank testing: TRACE test and Max Eigenvalue statistics

<table>
<thead>
<tr>
<th></th>
<th>TRACE 5% Critical Value</th>
<th>Max Eigenvalue 5% Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = 0$</td>
<td>54.864*</td>
<td>40.175</td>
</tr>
<tr>
<td>$r = 1$</td>
<td>12.887</td>
<td>24.276</td>
</tr>
<tr>
<td>$r = 2$</td>
<td>3.315</td>
<td>12.321</td>
</tr>
<tr>
<td>$r = 3$</td>
<td>0.492</td>
<td>4.130</td>
</tr>
</tbody>
</table>

Note: * indicates a rejection of the null hypothesis of $r$ being equal to that value (5% level).

After the estimation of the model itself, some tests have to be run in order to check for its validity. For this purpose, a battery of tests is run to check for autocorrelation and heteroskedasticity present in the residuals, with tests such as the Portmanteau Test and LM tests being run for Autocorrelation purposes (both tests measuring up to 6 lags), the White Test being run for Heteroskedasticity. The model passes all of these tests (Appendices 3, 4, 5 and 6). Furthermore, the stability of the model is also observed, through the roots of the characteristic polynomial, which are all under 1 (in absolute value), and therefore the model is stable.

The estimation of the VAR then leaves us with one co-integration equation, which to verify the results of the MAER proposed by Bilson (1978), has to have a positive coefficient for
the elasticities of both the money supply and the interest rate differences terms, and a negative one for the output term: 
\[ e = \beta_1 (m - m^*) + \beta_2 (i - i^*) + \beta_3 (y - y^*) \]
where \( \beta_1 > 0, \beta_2 > 0, \beta_3 < 0. \) The results of the co-integration equation obtained for the estimated Vector Error Correction model are presented below.

Table 3: Co-integration vector (standard errors in brackets)

\[
\begin{align*}
\epsilon_{t-1} &= 0.012(i - i^*)_{t-1} + 0.523(m - m^*)_{t-1} - 0.688(y - y^*)_{t-1} \\
&= (0.026) (0.093) (0.095)
\end{align*}
\]

This estimation (in 3) then confirms the results of the monetary model equation for the long-run relationship in model, although not entirely: as the standard error for the interest rate differential variable suggests, there is not enough evidence to reject the null hypothesis that this variable is not relevant in this equation. In Figure 1 one can see the fit of the co-integration equation.

The next step, after having correctly estimated the model, is to estimate the Structural part, taking into account the following equation for the estimation of the matrix of contemporaneous effects: 
\[ A \epsilon_t = B u_t \]
one can apply some constraints to the matrices, as to recover only the theorized equation (6) from the MAER (Loría et al., 2010), in order to capture the contemporaneous effects of the macroeconomic fundamentals in the nominal exchange rate. This will make the model over-identified, and an LR test to check for the applicability of those restrictions is applied (since the matrix \( A \) has more restrictions than those required), and the model passes it (Appendix 7)

The matrices estimated are the ones presented below:
\[
\begin{bmatrix}
1 & a_{12} & a_{13} & a_{14} \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\epsilon_{et} \\
\epsilon_{(i-i^*)t} \\
\epsilon_{(m-m^*)t} \\
\epsilon_{(y-y^*)t}
\end{bmatrix}
= 
\begin{bmatrix}
b_{11} & 0 & 0 & 0 \\
0 & b_{22} & 0 & 0 \\
0 & 0 & b_{33} & 0 \\
0 & 0 & 0 & b_{44}
\end{bmatrix}
\begin{bmatrix}
\epsilon_{et} \\
\epsilon_{(i-i^*)t} \\
\epsilon_{(m-m^*)t} \\
\epsilon_{(y-y^*)t}
\end{bmatrix}
\]

Where the coefficients need to verify the following conditions in order to follow the
findings of the MAER model (Chin et al., 2007; Loría et al., 2010): $a_{12}, a_{14} > 0$ and $a_{13} < 0$.

The equation below represents the coefficients found (for the now estimated Structural VEC model):

$$
\epsilon_{e_t} = -0.002\epsilon_{(i-i^*)_t} - 0.132\epsilon_{(y-y^*)_t} + 1.234\epsilon_{(m-m^*)_t}
$$

As it can be seen in the equation above, the model estimated does not confirm the relationships predicted by the flexible-price monetary model of the nominal exchange in the immediate short-run (that is, the contemporaneous effects) for all of the variables; this is because the coefficient for the interest rate difference presents a negative sign when it should present a positive one.

Although the effect from the interest rate difference between the two countries is not seen in the immediate responses of the nominal exchange rate between the two countries, in the short to medium-run, the Impulse Response Function for this relationship shows a positive trend after the first two values and starting in the 18th value, it actually becomes positive, which shows signs of achieving the theorized sign towards the medium to long-run. Regarding the other two variables, the Impulse Responses present the same signs as those theorized, although as one can see through the graphs, only the Money Supply variable presents some statistical significance in this short-run testing. (Figure 1).

Figure 1: Impulse Responses (from the Structural Effects) for $e$ (95% Confidence Bands)
4.2 Restriction application for the Forward-Looking Model

The next step in the estimation part of this exercise will then be to check if the data behaves in such a way that the restrictions for the estimation of a Forward-Looking model (MacDonald and Taylor, 1993) can be applied.

First, to check if there is any applicability of the model, one must check if there is a co-integration relationship between the nominal exchange rate, money supply, and output variables, as to guarantee that the variable composed by these three, $L_t$ is $I(0)$ (as discussed in the previous section). The same method of testing that was used in the co-integration stage for the regular MAER model will be applied and the results can be seen in Table 5; these confirm that a co-integration relationship between these variables does exist, and one can move on from this to the testing of the more narrowly defined forward-looking restriction, which will be applied to the coefficients of the VAR estimated for $L_t$ and $\Delta x_t$ (with 2 lags, as maximized by the common tests of AIC, BIC, and SIC, showed in Appendix 9) (MacDonald and Taylor, 1993; Campbell and Shiller, 1987).

Table 5: Co-integration rank testing (for $L_t$): TRACE test and Max Eigenvalue statistics

<table>
<thead>
<tr>
<th>r</th>
<th>TRACE</th>
<th>5% Critical Value</th>
<th>Max Eigenvalue</th>
<th>5% Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>58.425*</td>
<td>35.193</td>
<td>39.958*</td>
<td>22.300</td>
</tr>
<tr>
<td>1</td>
<td>18.468</td>
<td>20.262</td>
<td>12.459</td>
<td>15.892</td>
</tr>
</tbody>
</table>

Note: * indicates a rejection of the null hypothesis of $r$ being equal to that value (5% level).

In order to test for this restriction, as pointed out in the Methodology section, an estimation for the interest rate differential coefficient (for the already defined monetary model equation) needs to be utilized. As MacDonald and Taylor (1993) point out, one can use the coefficient estimated through the previous co-integration exercise, but to offer some robustness to the test, other values will be used as well. The following are the LR statistics (equivalent to the
Wald test statistics utilized in MacDonald and Taylor (1993), which under the null hypothesis have a $\chi^2$ distribution with 4 degrees of freedom\(^7\) to test for the restriction imposed on the VAR model, which will, in turn, indicate, similarly, if the spread variable found in the data follows a similar path to the theoretically defined spread variable, i.e. $H_0 : L_t = L_t^*$:

\[
\gamma = 0.01261 \implies \text{LR Test Statistic: } 1124.2754 \sim \chi^2(4)
\]

\[
\gamma = 0.025 \implies \text{LR Test Statistic: } 1039.0908 \sim \chi^2(4)
\]

\[
\gamma = 0.05 \implies \text{LR Test Statistic: } 859.0018 \sim \chi^2(4)
\]

This indicates a clear rejection of the null hypothesis\(^8\) that was previously defined in the Methodology section, in line with the results from MacDonald and Taylor (1993), which means that the forward-looking approach restrictions fail and this variation of the model cannot be totally applied to the data as is. Moreover, if one takes a look at the behavior of both the theoretical and the actual spread variables ($L_t$ and $L_t^*$), it is easily observable that both variables differ from each other quite immensely, which points to the rejection of the null hypothesis being not only due to issues such as data imperfections but due to economically important behaviors amongst the variables themselves Campbell and Shiller (1987). The following graph plots the relationship between the two spread variables for $\gamma = 0.01261$ (Figure 2):

With this information, there is no sign of applicability of the full set of restrictions of the FMAER, (which would then be applied to a VAR with $L_t$ and $\Delta x_t$ in its final form), and therefore, following the reasoning from MacDonald and Taylor (1993), this particular set of data only seems fit to coherently estimate the static monetary approach to the exchange rate model (the MAER) and not the forward-looking one, as the necessary restrictions could not have been applied further.

---

\(^7\)Four restrictions are ultimately imposed on the VAR, coming from the equation in (18)

\(^8\)Measured against a 5% critical value of 9.49
4.3 Forecast Evaluation with the MAER

Following the likes of MacDonald and Taylor (1993); Reinton and Ongena (1999) and others, one can now evaluate the capabilities of the estimated VEC model (the MAER Long-Run relationship estimation) by testing out its forecast capacity and comparing it to predictions by a simple random walk model. The forecasts will be done considering the period of 2012Q3 to 2017Q2, with a total of 20 data points to run the out-of-sample forecasts on.

The forecast results will be evaluated given four different measures (contrary to, for example, MacDonald and Taylor (1993) which utilizes only RMSE to evaluate the quality of forecasts) of error minimization: RMSE (Root Mean Squared Error), MAE (Mean Absolute Error), MAPE (Mean Absolute Percentage Error) and Theil’s Inequality Coefficient. This is done to evaluate in a thorough manner the quality of the forecast predictions, and therefore, the model’s overall capability to explain changes in the nominal exchange rate - and consequently, how good the fit of the model is, given the data. Moreover, this will be done taking into account
several horizons: 1 quarter ahead, 2 quarters ahead, 4 quarters ahead (a year) and 8 quarters ahead (2 years); this will then give us a more comprehensive look at how the model performs when forecasting starting in the short-run and going all the way into more of a long-run setting. The results from the forecast estimation are presented in Table 6.

<table>
<thead>
<tr>
<th></th>
<th>1Q</th>
<th>2Q</th>
<th>4Q</th>
<th>8Q</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Random Walk</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.0324</td>
<td><strong>0.0570</strong></td>
<td><strong>0.0936</strong></td>
<td>0.1368</td>
</tr>
<tr>
<td>MAE</td>
<td>0.0245</td>
<td>0.0449</td>
<td><strong>0.0787</strong></td>
<td><strong>0.1053</strong></td>
</tr>
<tr>
<td>MAPE</td>
<td>6.9716</td>
<td>13.3339</td>
<td><strong>25.0200</strong></td>
<td>37.6751</td>
</tr>
<tr>
<td>Theil IC</td>
<td>0.0387</td>
<td><strong>0.0676</strong></td>
<td><strong>0.1096</strong></td>
<td>0.1613</td>
</tr>
<tr>
<td><strong>Vector Error-Correction</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td><strong>0.0315</strong></td>
<td>0.0585</td>
<td>0.1012</td>
<td><strong>0.1346</strong></td>
</tr>
<tr>
<td>MAE</td>
<td><strong>0.0235</strong></td>
<td><strong>0.0444</strong></td>
<td>0.0841</td>
<td>0.1059</td>
</tr>
<tr>
<td>MAPE</td>
<td><strong>6.6841</strong></td>
<td><strong>13.2497</strong></td>
<td>26.8208</td>
<td><strong>37.6388</strong></td>
</tr>
<tr>
<td>Theil IC</td>
<td><strong>0.0376</strong></td>
<td>0.0693</td>
<td>0.1178</td>
<td><strong>0.1573</strong></td>
</tr>
</tbody>
</table>

Note: Values in bold indicate a smaller statistic when compared to the corresponding one for the other model.

Taking into account the table above, one can consider the following as the main takeaways regarding the forecast carried out by the model: Regarding the four measurements presented, when considering the forecast one quarter ahead, the model beats out the random-walk in all of them, showing an aptitude for short-run prediction. When considering more medium-run scenarios, the forecasts are not as positive as the ones before; considering the two quarters ahead forecast, the results are split, since the monetary approach model presents a lower MAE and MAPE statistics, but larger RMSE and Theil IC, while in the four quarters ahead prediction (a full year) the model is beaten in all statistics presented. As for the two-years ahead forecast, the results are much more positive, seeing as that the model is only slightly beaten out by the simple random-walk when considering the MAE statistic.
5 Conclusion and Discussion

5.1 Conclusion

This study allowed not only one to evaluate the fit of the monetary approach to the exchange rate model to the USD/GBP relationship, with a rather comprehensive quarterly data set (1982Q4-2017Q2), both in short and long-run horizons, but it also made possible to test for the applicability of the forward-looking variant of this monetary approach. It also provided a co-integrated final long-run model, utilized to test the forecast prediction power of this approach when compared to a random walk.

Regarding the first aforementioned point, the normal variant of the monetary approach (Bilson, 1978), there is some evidence that supports this variant of the model in a co-integration fashion, therefore establishing a long-run relationship between the nominal exchange rate and the other macroeconomic variables (Chin et al., 2007; Effiong, 2014; Loría et al., 2010), as the coefficients estimated presented the theorized relationships, although here, in regards to the interest rate variable, there is a caveat that there is not enough evidence to reject the null hypothesis that it has no effect in the long-run co-integration vector estimated; as for the short-run effects in the static model, it did not perform as well as the one by Loría et al. (2010), as it was not able to find the exact contemporaneous relationship as it did for the co-integration exercise. The only variable that did not behave as predicted was the interest rate one, but by analyzing the Impulse Responses, the variable did show a positive trend, and it eventually became positive as predicted, but only towards the medium to long-run. It is worth mentioning that by looking at the same graphs (1), both the interest and output variables did not show a very significant coefficient.

As for the Forward-Looking variant, although a co-integration relationship between the variables that formed the spread variable $L_t$ was found (the four main variables excluding $i_t$), as
this was not enough to estimate the model correctly, following MacDonald and Taylor (1993), after imposing restrictions on the VAR formed by the aforementioned variable and $x_t^9$, these were rejected, using several values for the elasticity of the log difference of interest rates ($\gamma$), and consequently, one concludes that the forward-looking variant of the monetary approach can not be utilized with this particular set of data, as the actual spread variable behaves in a completely different manner than the theoretical one, $L_t^*$ (Graph 2).

Regarding the forecasts, the model displayed some adequacy regarding short-run and more long-run estimation (1 quarter ahead and 8 quarters ahead) as it minimizes almost all of the statistics for both of these horizons; but when it comes to the medium-run, then as the results are not as positive (half of the statistics are minimized for the model in the 2 quarter ahead and none in the 4 quarter ahead), then the conclusion is that although the model did not completely beat out the random-walk, it did show some promise regarding its forecast capabilities, especially in the short-run. In contrast to MacDonald and Taylor (1993), in regards to the RMSE, this study was then not able to verify the same adequacy in the forecasts as they do.

5.2 Limitations and Next Steps

Although this paper was able to deliver some rather robust results regarding the usefulness and fit of the static flexible-price monetary model of exchange rate determination, albeit not very positive, and its forward-looking variant (in this case, the non-applicability of all of its restrictions), there are still some limitations regarding the techniques and data used for the study. The first one that can be identified is the data that was utilized and how that may influence the results of the test since, although the one used was the one that was found to be the most fitting, different representations of the interest rate variables, the output variables or the money supply variables may have yielded some different results.

---

9The composition of these variables is presented in the Methodology section of the paper
Also needing to be taken into account are the assumptions that have to be made in order to consider this model viable, and these are, mainly, that both the PPP and the Monetary Market Conditions (this last one, in both countries) are in equilibrium and therefore hold; such is the case for the UIP as well, as when testing the applicability of the FMAER, it is also integrated into the model; if not, then the results may be invalid, as these are integral parts of the theoretical model construction.

Lastly, it should also be considered that, due to constrictions regarding the time periods for the data available, the initial time period theorized to be studied - the post-Bretton Woods era - was not available, which would have brought an even greater robustness to this exercise.

As for next steps, reworks of the model should perhaps include new data to represent the variables, as for example a different measure for the interest rate variables (with perhaps a different maturity) or another measure for the output variable, which may be substituted for industrial production (as is the case in some of the literature) which was not done here, as the GDP Data was found to be a more fitting variable. Moving further, it would also be useful to compare the model’s forecast results to the ones achieved by other models for exchange rate determination, such as the Portfolio-Balance Approach, to get a better sense as to if these models would be better substitutes to explain the behavior of this exchange rate. Lastly, regarding forecasts, other ways of measuring its prediction capability can also be applied, as exemplified by Moosa and Burns (2013), which considers the direction of change as criteria for the capacity of the forecasting procedure.
6 References


7 Appendix

Appendix 1: Lag order determination statistics for the VEC Model (Static MAER)

<table>
<thead>
<tr>
<th>Lags</th>
<th>AIC</th>
<th>SIC</th>
<th>HQC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-4.304</td>
<td>-4.216</td>
<td>-4.268</td>
</tr>
<tr>
<td>1</td>
<td>-14.878</td>
<td>-14.439*</td>
<td>-14.699*</td>
</tr>
<tr>
<td>3</td>
<td>-14.901</td>
<td>-13.760</td>
<td>-14.437</td>
</tr>
</tbody>
</table>

Note: * indicates the minimum value for the criterion, and therefore, the lag chosen by that specific criterion.

Appendix 2: Co-Integration Equation Type Evaluation - Akaike and Schwarz Information Criteria

<table>
<thead>
<tr>
<th></th>
<th>No Intercept</th>
<th>Intercept</th>
<th>Intercept</th>
<th>Intercept</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Trend</td>
<td>No Trend</td>
<td>No Linear Trend</td>
<td>Linear Trend</td>
<td>Quadratic Trend</td>
</tr>
</tbody>
</table>

Note: * represents the minimized value for regarding each criterion.
Appendix 3: Portmanteau Autocorrelation Test - No Residual Autocorrelation up to lag $h$

<table>
<thead>
<tr>
<th>Lags</th>
<th>Q-Stat</th>
<th>Prob</th>
<th>Adj. Q-Stat</th>
<th>Prob</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>22.849</td>
<td>0.741</td>
<td>23.173</td>
<td>0.724</td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>37.391</td>
<td>0.749</td>
<td>38.041</td>
<td>0.724</td>
<td>44</td>
</tr>
<tr>
<td>4</td>
<td>55.165</td>
<td>0.653</td>
<td>56.350</td>
<td>0.610</td>
<td>60</td>
</tr>
<tr>
<td>5</td>
<td>61.497</td>
<td>0.886</td>
<td>62.921</td>
<td>0.859</td>
<td>76</td>
</tr>
<tr>
<td>6</td>
<td>73.441</td>
<td>0.923</td>
<td>75.412</td>
<td>0.895</td>
<td>92</td>
</tr>
</tbody>
</table>

Note: All values presented do not reject the null hypothesis of no residual autocorrelation up to lag $h$ (at a 5% Critical Value)

Appendix 4: LM Autocorrelation test at lag $h$

<table>
<thead>
<tr>
<th>Lags</th>
<th>LRE Stat</th>
<th>Prob</th>
<th>df</th>
<th>Rao F-stat</th>
<th>df</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23.087</td>
<td>0.111</td>
<td>16</td>
<td>1.460</td>
<td>(16, 382.5)</td>
<td>0.111</td>
</tr>
<tr>
<td>2</td>
<td>23.056</td>
<td>0.112</td>
<td>16</td>
<td>1.458</td>
<td>(16, 382.5)</td>
<td>0.112</td>
</tr>
<tr>
<td>3</td>
<td>15.225</td>
<td>0.508</td>
<td>16</td>
<td>0.953</td>
<td>(16, 382.5)</td>
<td>0.508</td>
</tr>
<tr>
<td>4</td>
<td>18.629</td>
<td>0.288</td>
<td>16</td>
<td>1.171</td>
<td>(16, 382.5)</td>
<td>0.289</td>
</tr>
<tr>
<td>5</td>
<td>6.800</td>
<td>0.977</td>
<td>16</td>
<td>0.421</td>
<td>(16, 382.5)</td>
<td>0.977</td>
</tr>
<tr>
<td>6</td>
<td>12.966</td>
<td>0.675</td>
<td>16</td>
<td>0.809</td>
<td>(16, 382.5)</td>
<td>0.675</td>
</tr>
</tbody>
</table>

Note: All values presented do not reject the null hypothesis of no residual autocorrelation at lag $h$ (at a 5% Critical Value)

Appendix 5: LM Autocorrelation test up to lag $h$

<table>
<thead>
<tr>
<th>Lags</th>
<th>LRE Stat</th>
<th>df</th>
<th>Prob</th>
<th>Rao F-stat</th>
<th>df</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23.087</td>
<td>16</td>
<td>0.111</td>
<td>1.460</td>
<td>(16, 382.5)</td>
<td>0.112</td>
</tr>
<tr>
<td>2</td>
<td>40.038</td>
<td>32</td>
<td>0.156</td>
<td>1.265</td>
<td>(32, 447.8)</td>
<td>0.156</td>
</tr>
<tr>
<td>3</td>
<td>55.990</td>
<td>48</td>
<td>0.200</td>
<td>1.178</td>
<td>(48, 452.7)</td>
<td>0.201</td>
</tr>
<tr>
<td>4</td>
<td>81.041</td>
<td>64</td>
<td>0.074</td>
<td>1.291</td>
<td>(64, 444.7)</td>
<td>0.075</td>
</tr>
<tr>
<td>5</td>
<td>91.401</td>
<td>80</td>
<td>0.180</td>
<td>1.157</td>
<td>(80, 432.4)</td>
<td>0.185</td>
</tr>
<tr>
<td>6</td>
<td>98.159</td>
<td>96</td>
<td>0.420</td>
<td>1.023</td>
<td>(96, 418.5)</td>
<td>0.429</td>
</tr>
</tbody>
</table>

Note: All values presented do not reject the null hypothesis of no residual autocorrelation up to lag $h$ (at a 5% Critical Value)

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Appendix 6: Heteroskedasticity Tests

<table>
<thead>
<tr>
<th></th>
<th>Chi-sq</th>
<th>df</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>White Test</td>
<td>111.098</td>
<td>100</td>
<td>0.211</td>
</tr>
<tr>
<td>No Cross-Terms</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>White Test</td>
<td>232.902</td>
<td>200</td>
<td>0.055</td>
</tr>
<tr>
<td>Cross-Terms</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Both values are rejected at a 5% Critical Value.

Appendix 7: Over-identification test for the SVEC Model - LR Test

<table>
<thead>
<tr>
<th></th>
<th>Chi-sq</th>
<th>df</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR Test</td>
<td>1.1985</td>
<td>3</td>
<td>0.7534</td>
</tr>
</tbody>
</table>

Note: This tests the rejection of the null hypothesis of the applicability of the over-identification scheme, which here is not rejected (at a 5% critical value).

Appendix 8: Co-Integration Vector - MAER Model
Appendix 9: Lag order determination statistics for the VAR Model (containing $L_t$ and $\Delta x_t$)

<table>
<thead>
<tr>
<th>Lags</th>
<th>AIC</th>
<th>SIC</th>
<th>HQC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-5.479</td>
<td>-5.434</td>
<td>-5.461</td>
</tr>
<tr>
<td>1</td>
<td>-9.198</td>
<td>-9.066</td>
<td>-9.144</td>
</tr>
<tr>
<td>2</td>
<td>-9.301*</td>
<td>-9.081*</td>
<td>-9.212*</td>
</tr>
<tr>
<td>3</td>
<td>-9.275</td>
<td>-8.967</td>
<td>-9.150</td>
</tr>
<tr>
<td>4</td>
<td>-9.275</td>
<td>-8.878</td>
<td>-9.114</td>
</tr>
</tbody>
</table>

Note: * indicates the minimum value for the criterion, and therefore, the lag chosen by that specific criterion